STATUS OF $|V_{cb}|$ AND $|V_{ub}|$

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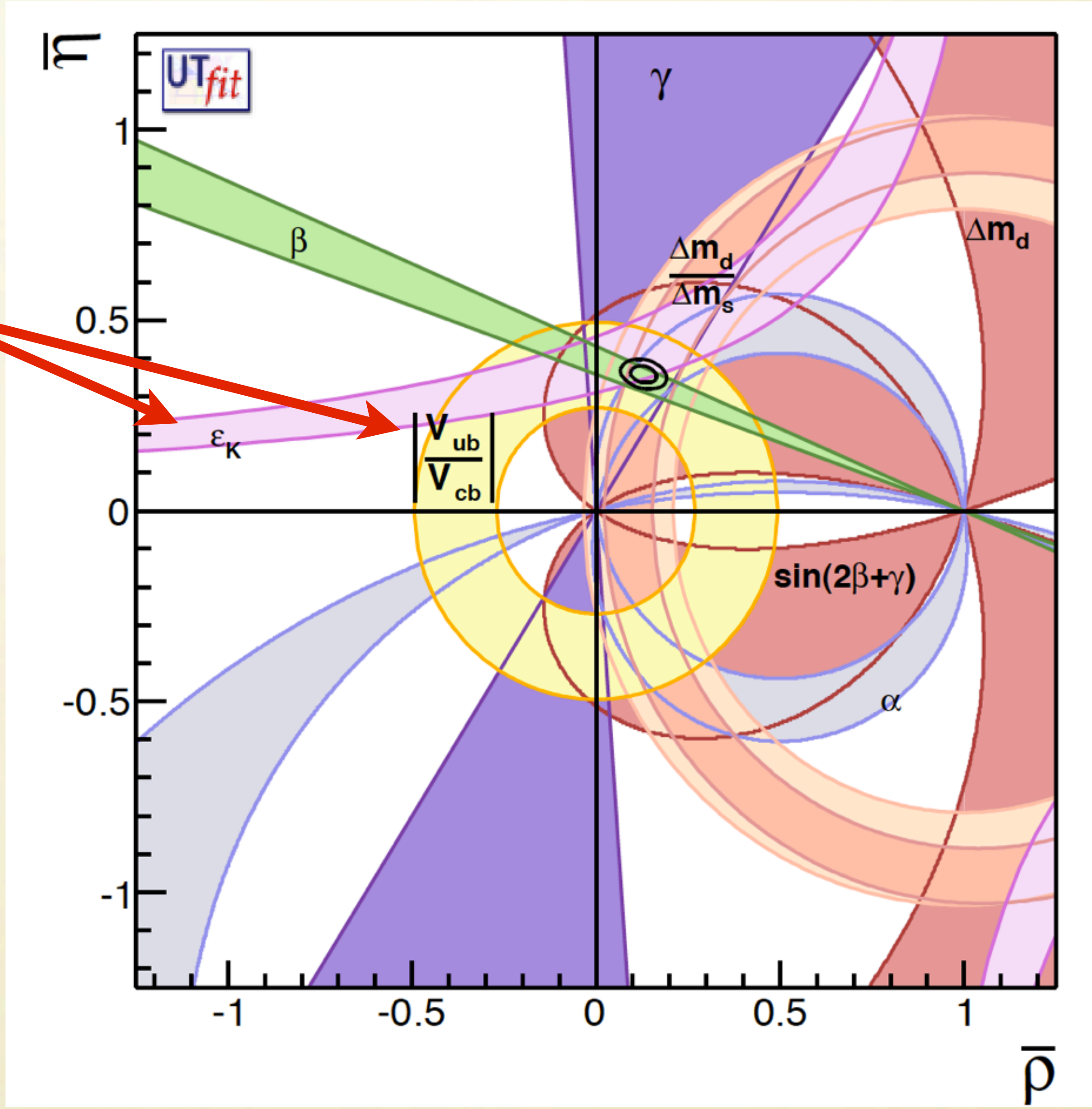
IMPORTANCE OF $|V_{cb}|$

V_{cb} and V_{ub} play important role in the determination of UT

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2 \left[1 + O(\lambda^2)\right]$$

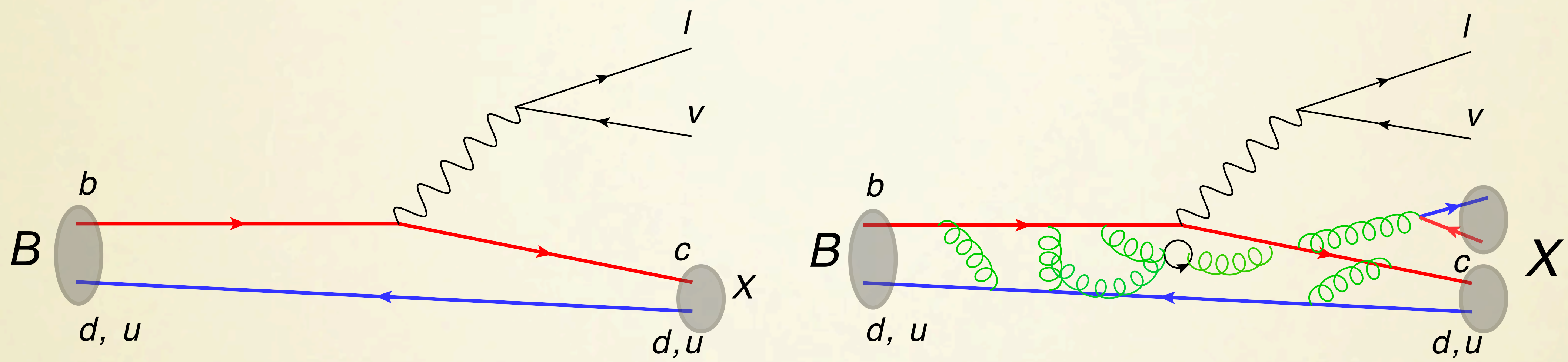
V_{cb} already dominant error in $B_s \to \mu^+\mu^-$, $K \to \pi\nu\nu$, ε_K



Since several years there is a tension between the exclusive and inclusive determinations of $|V_{ub}|$ and $|V_{cb}|$

INCLUSIVE $|V_{cb}|$

Inclusive decays: basics



- ***Simple idea:*** inclusive decays do not depend on final state, long distance dynamics of the B meson factorizes. An OPE allows to express it in terms of B meson matrix elements of local operators

- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: ***double series in α_s, Λ/m_b***

- Lowest order: decay of a free *b*, linear Λ/m_b absent. Depends on $m_{b,c}$, 2 parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

$$\mu_\pi^2(\mu) = \frac{1}{2M_B}\left\langle B\middle|\bar{b}\left(i\vec{D}\right)^2 b\middle|B\right\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2M_B}\left\langle B\middle|\bar{b}\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b\middle|B\right\rangle_\mu$$

OBSERVABLES IN THE OPE

$$
\begin{aligned}
M =& M_0\Big[1 + c_1(r)\frac{\alpha_s}{\pi} + c_2(r)\frac{\alpha_s^2}{\pi^2} \\
&- \frac{\mu_\pi^2}{2m_b^2}\left(1 + c_\pi^{(1)}(r)\frac{\alpha_s}{\pi}\right) \\
&+ \frac{\mu_G^2}{m_b^2}\left(c_G^{(0)}(r) + c_G^{(1)}(r)\frac{\alpha_s}{\pi}\right) \\
&+ c_D(r)\frac{\rho_D^3}{m_b^3} + c_{LS}(r)\frac{\rho_{LS}^3}{m_b^3} \\
&+ O\left(\alpha_s^3, \alpha_s^2\frac{\Lambda^2}{m_b^2}, \alpha_s\frac{\Lambda^3}{m_b^3}, \frac{\Lambda^4}{m_b^4}\right)\Big]
\end{aligned}
$$

NEW

$$
r = \frac{m_c^2}{m_b^2}
$$

OPE valid for inclusive enough measurements, away from perturbative singularities ➠ semileptonic width, moments

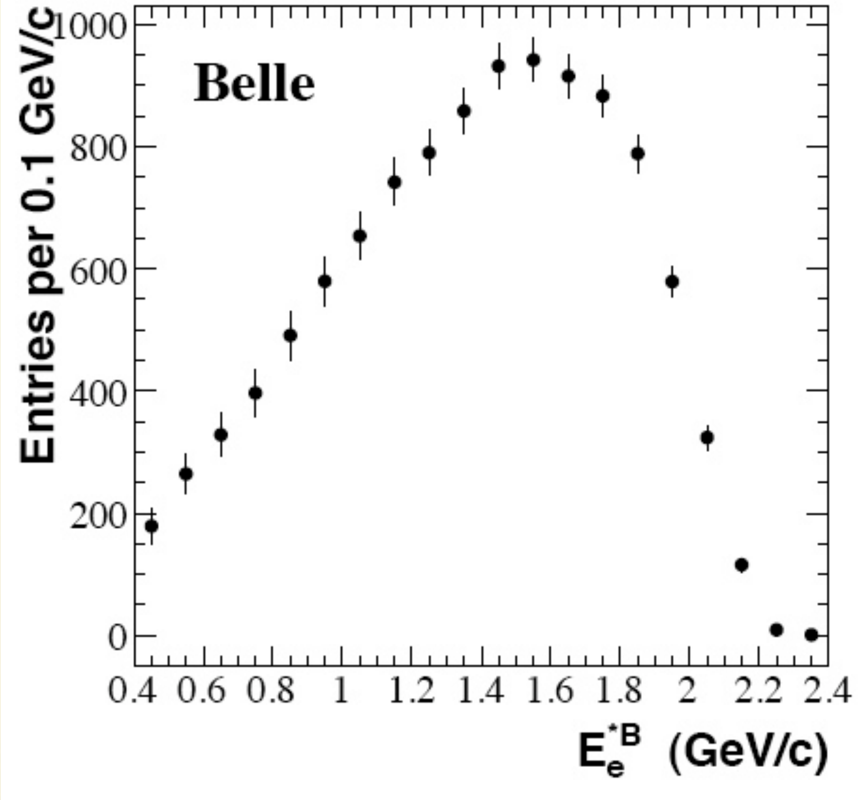
The fit presented here includes 6 non-pert parameters

$m_{b,c},\quad \mu^2_{\pi,G},\quad \rho^3_{D,LS}$

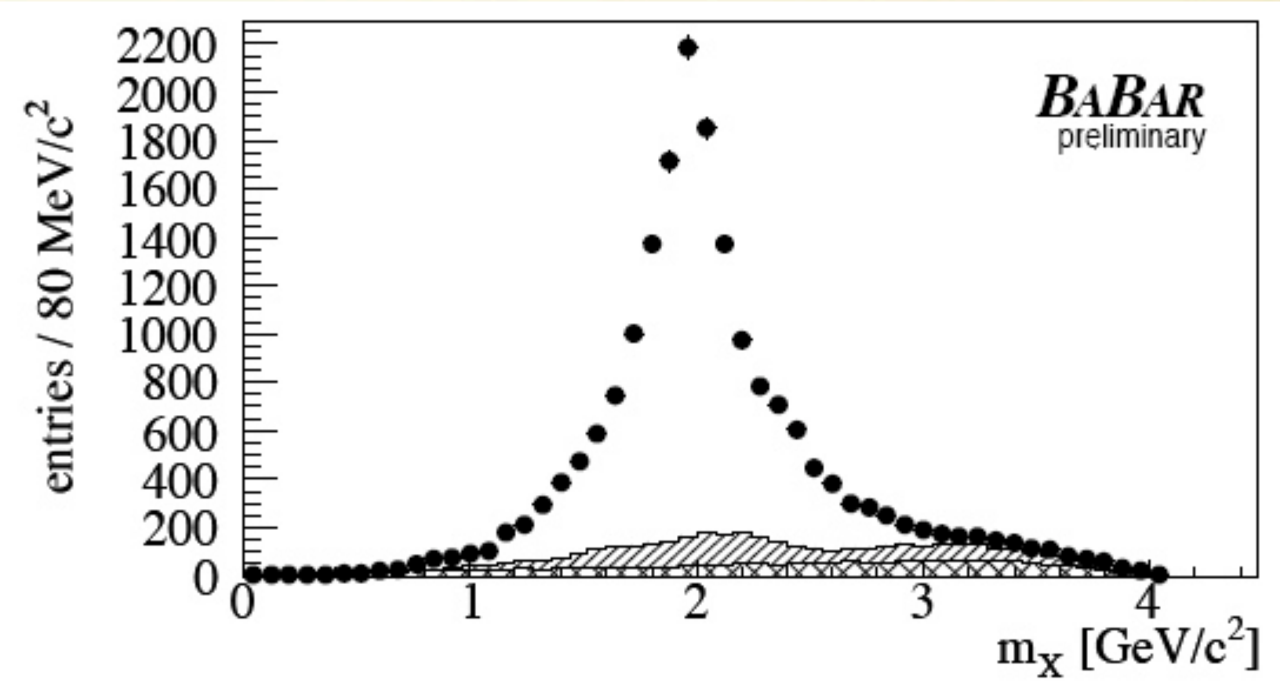
and all known corrections up to $O(\Lambda^3/m_b^3)$

Extraction of the OPE parameters

E_l spectrum



m_x spectrum



Global ***shape*** *parameters (first moments of the distributions) tell us about B structure, m_b and m_c, total* ***rate*** *about $|V_{cb}|$*

OPE parameters describe universal properties of the B meson and of the quarks → useful in many applications (rare decays, V_{ub},...)

Let's focus on:

1. Status of higher order corrections
2. Estimate of residual theoretical errors
3. Additional constraints in the fits

HIGHER ORDER EFFECTS

- Reliability of the method depends on our ability to control higher order effect and quark-hadron duality violations.
- **Purely perturbative corrections** complete at NNLO, small residual error Melnikov, Biswas, Czarnecki, Pak, PG
- **Higher power corrections** $O(1/m_Q^{4,5})$ known

  Mannel,Turczyk,Uraltsev 2010
- **Mixed corrections** perturbative corrections to power suppressed coefficients completed at $O(\alpha_s/m_b^2)$

  Becher, Boos, Lunghi, Alberti, Ewerth, Nandi, PG

Higher power corrections

Mannel,Turczyk,Uraltsev **1009.4622**

Proliferation of non-pert parameters and powers of $1/m_c$ starting $1/m^5$. At $1/m_b{}^4$

$$2M_B m_1 = \langle ((\vec{p})^2)^2 \rangle$$
$$2M_B m_2 = g^2 \langle \vec{E}^2 \rangle$$
$$2M_B m_3 = g^2 \langle \vec{B}^2 \rangle$$
$$2M_B m_4 = g \langle \vec{p} \cdot \mathrm{rot}\, \vec{B} \rangle$$

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$
$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$
$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$
$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$
$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

can be estimated by **Lowest Lying State Saturation** approx by truncating

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

In LLSA ***good convergence*** of the HQE. First fit with $1/m^{4,5}$:

$$\frac{\delta V_{cb}}{V_{cb}} \simeq -0.35\%$$

Turczyk,PG preliminary

Heinonen,Mannel 1407.4384 have more systematic approach

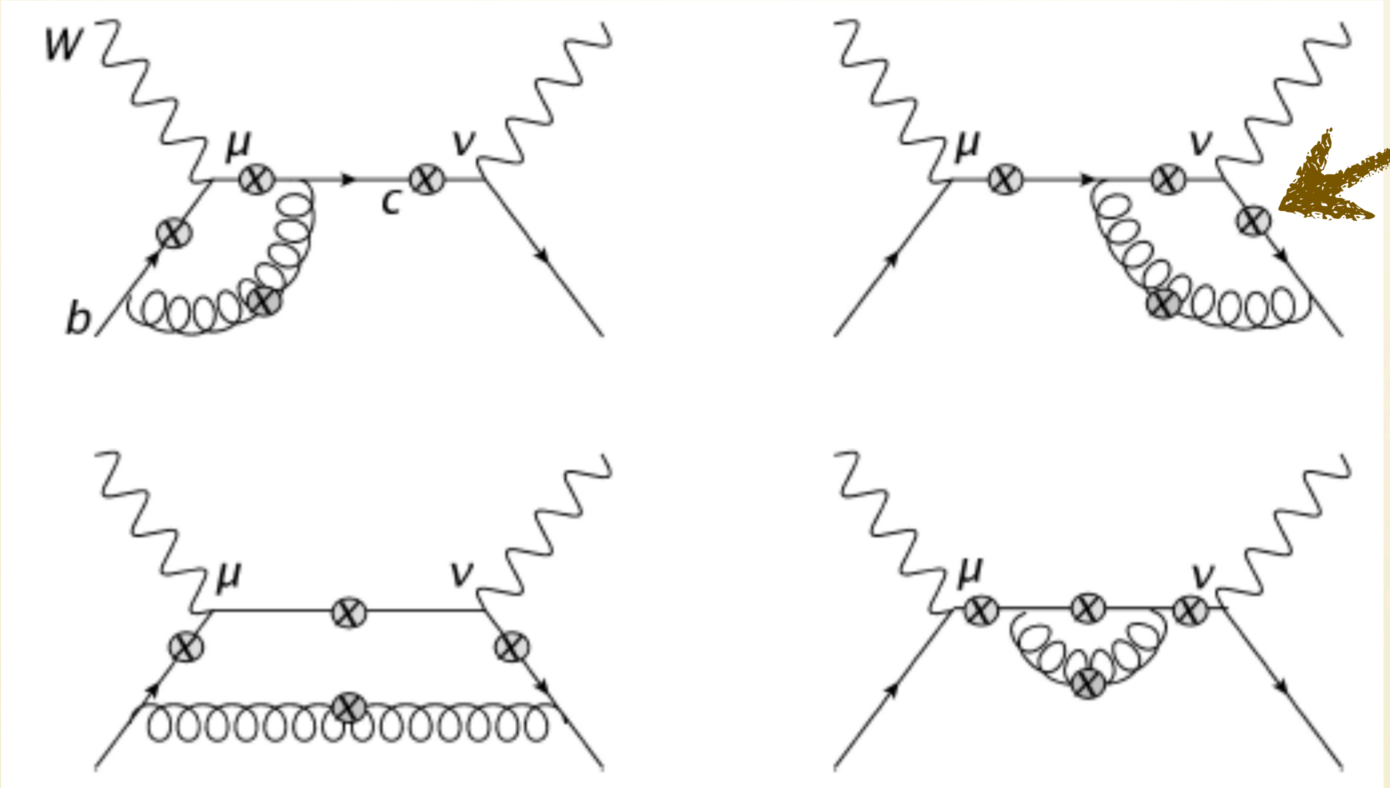
LLSA might set the scale of effect, not yet clear *how much it depends on assumptions on expectation values.* Large corrections to LLSA have been found.

Mannel, Uraltsev, PG, 2012

Allowing 80% gaussian deviations from LLSA seem to leave V_{cb} unaffected.

Matching at $O(\alpha_s)$

Boos,Becher,Lunghi 2007
Ewerth,Nandi, PG 2009
Alberti,Ewerth,Nandi,PG 2012
Alberti,Nandi,PG 2013



possible gluon insertions

QCD

HQET

$$\frac{2i}{\pi}\int d^4x\, e^{-iq\cdot x}\, T[J_L^{\dagger\mu}(x) J_L^{\nu}(0)] = \sum_i c^{(i)\mu\nu}_{\{\alpha\}}(v,q)\, O_i^{\{\alpha\}}(0)$$

Taylor expansion around on-shell b quark matched onto HQET local operators. Analytic formulae. RPI relations reproduced. Unlike μ_π, μ_G gets renormalized, therefore Wilson coefficients scale-dependent.

Numerical results

In on-shell scheme (m_b=*4.6*GeV, m_c=*1.15*GeV) without cuts

$$\Gamma_{B\to X_c\ell\nu} = \Gamma_0\left[\left(1-1.78\,\frac{\alpha_s}{\pi}\right)\left(1-\frac{\mu_\pi^2}{2m_b^2}\right)-\left(1.94+2.42\,\frac{\alpha_s}{\pi}\right)\frac{\mu_G^2(m_b)}{m_b^2}\right]$$

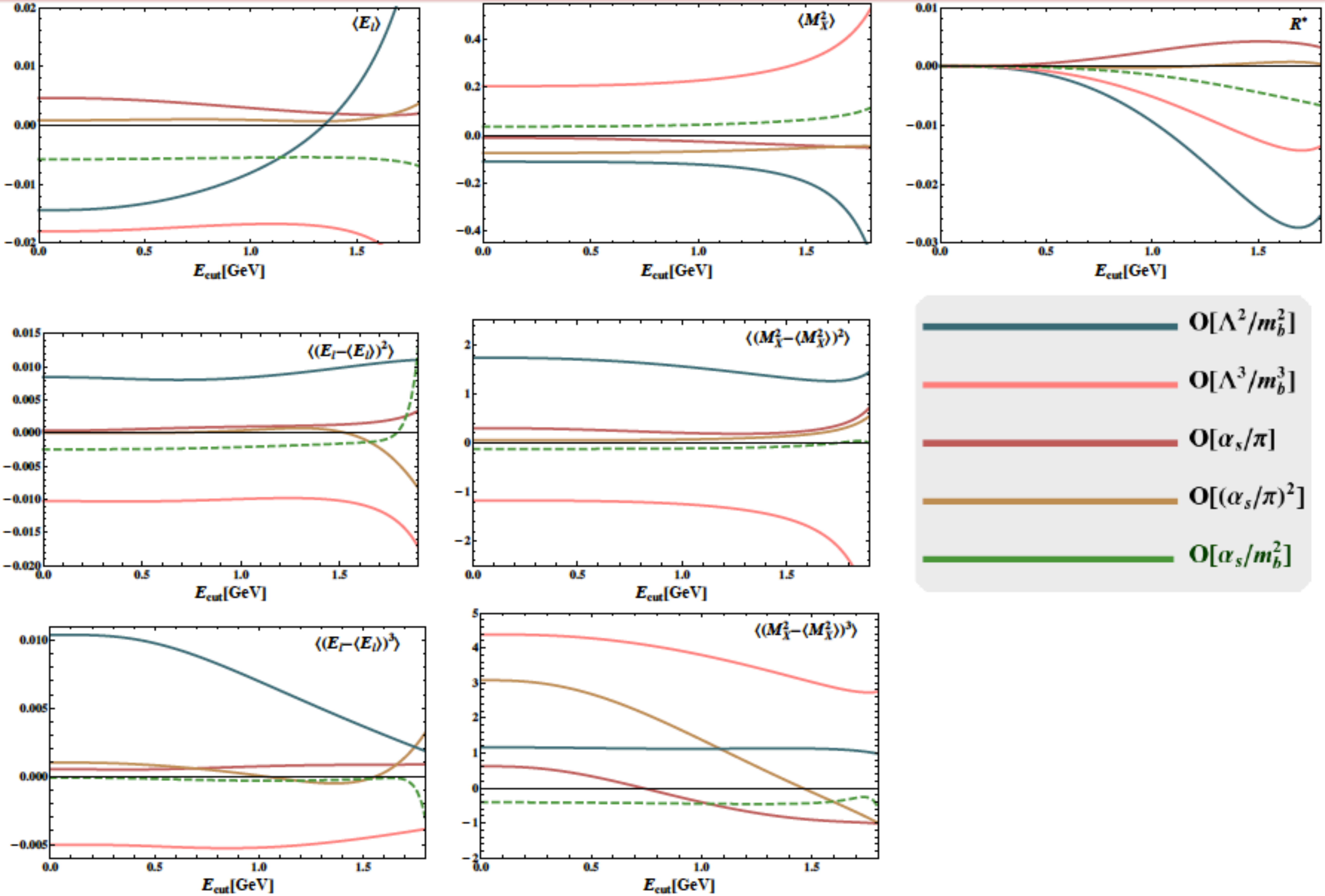
$$\langle E_\ell\rangle = 1.41\text{GeV}\left[\left(1-0.02\,\frac{\alpha_s}{\pi}\right)\left(1+\frac{\mu_\pi^2}{2m_b^2}\right)-\left(1.19+4.20\,\frac{\alpha_s}{\pi}\right)\frac{\mu_G^2(m_b)}{m_b^2}\right]$$

$$\ell_2 = 0.183\,\text{GeV}^2\left[1-0.16\,\frac{\alpha_s}{\pi}+\left(4.98-0.37\,\frac{\alpha_s}{\pi}\right)\frac{\mu_\pi^2}{m_b^2}-\left(2.89+8.44\,\frac{\alpha_s}{\pi}\right)\frac{\mu_G^2(m_b)}{m_b^2}\right]$$

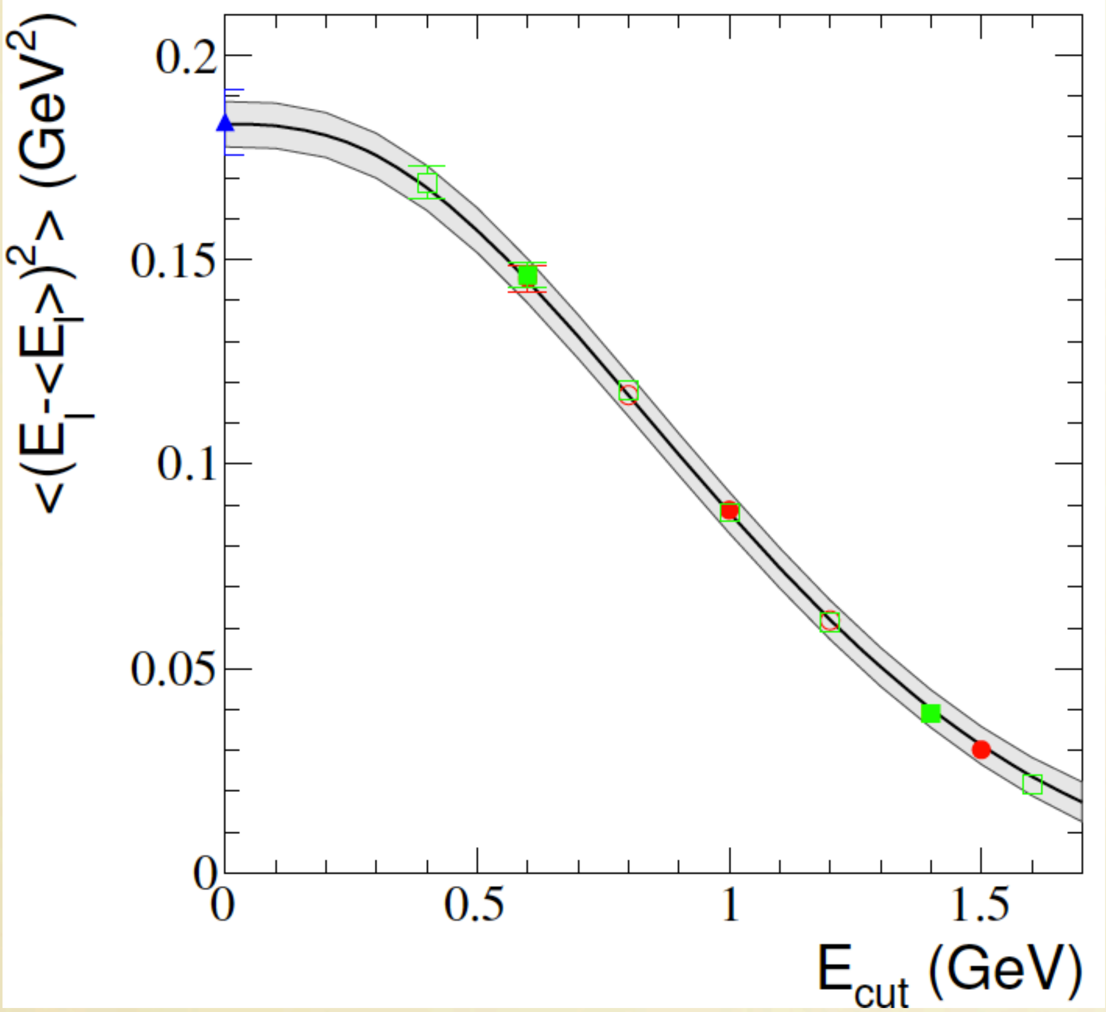
Similar results in the kinetic scheme. NLO corrections generally *O(15-20%)* of tree level coefficients, **shifts in some cases larger than experimental error**. Impact on V_{cb} requires new fit of semileptonic moments.

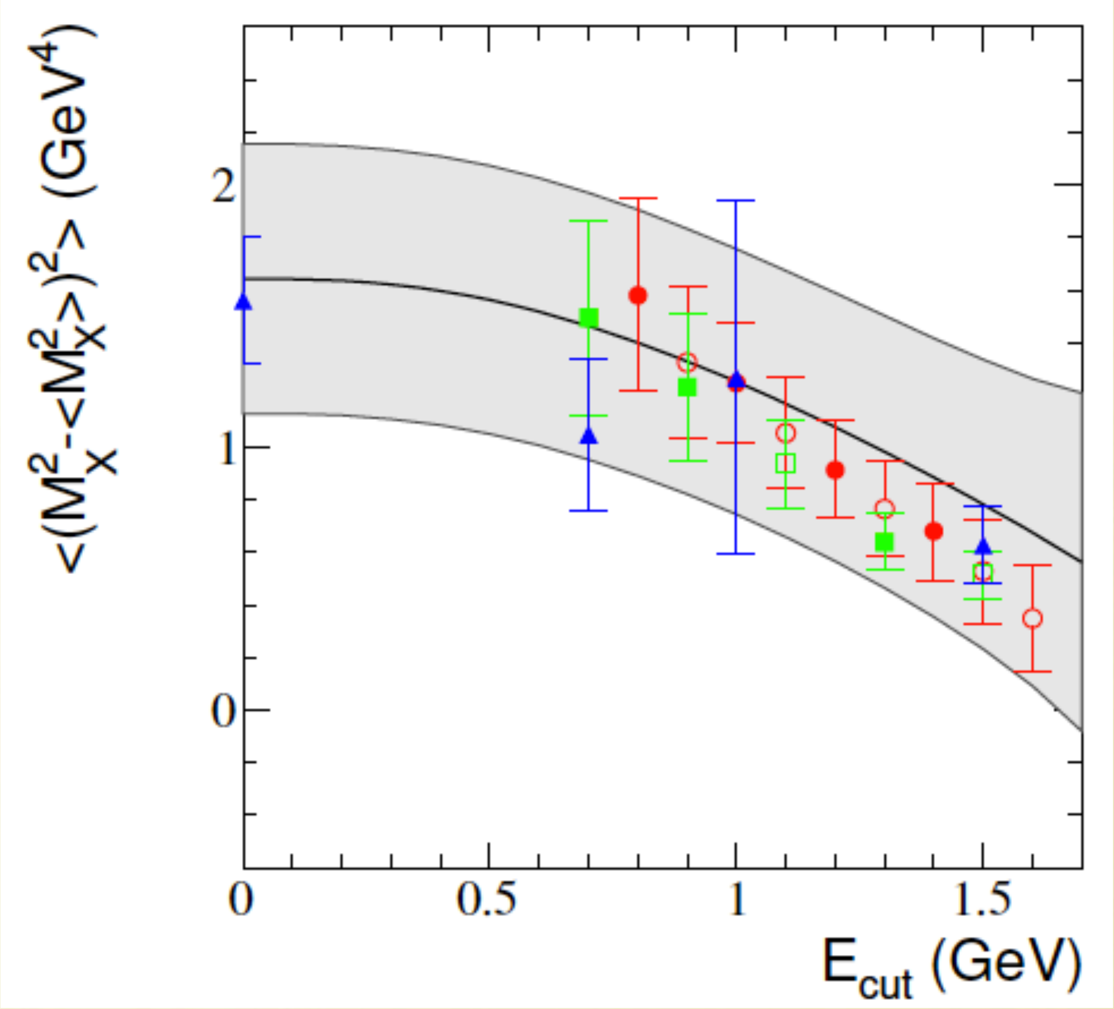
Mannel, Pivovarov, Rosenthal (1405.5072) have computed the μ_G correction to the width in the limit m_c=0 and find compatible result.

New Contributions $\mathcal{O}(\alpha_s/m_b^2)$: R



Theoretical errors

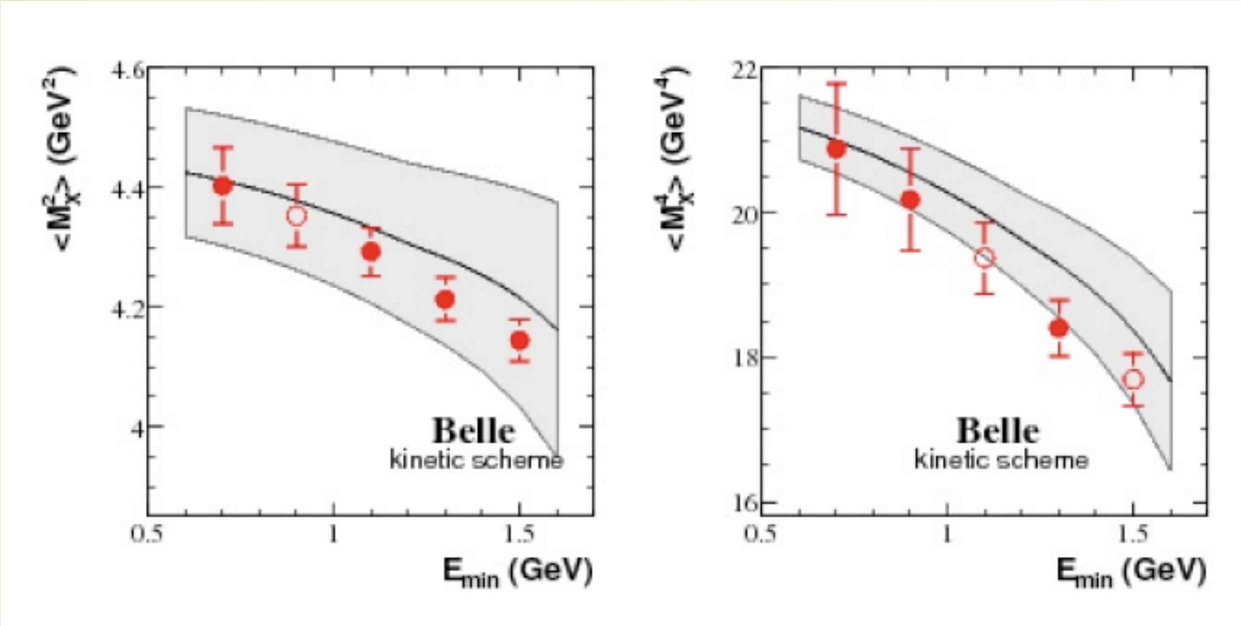


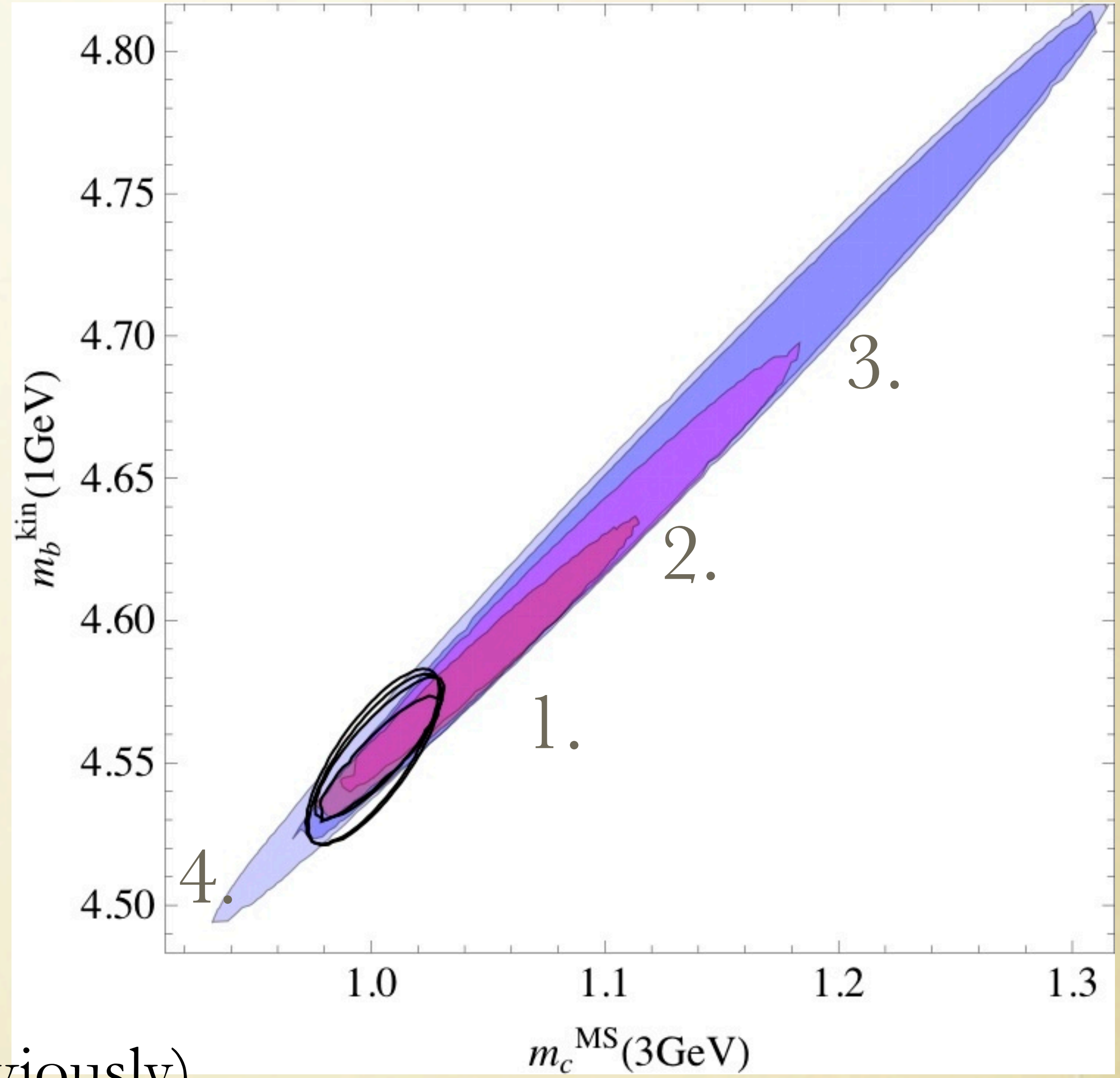


Theoretical errors are generally the **dominant** ones in the fits.
We estimate them in a **conservative** way by mimicking higher orders varying the parameters by fixed amounts.

Duality violation, expected here to be suppressed, would manifest as inconsistency in the fit.

Theoretical correlations





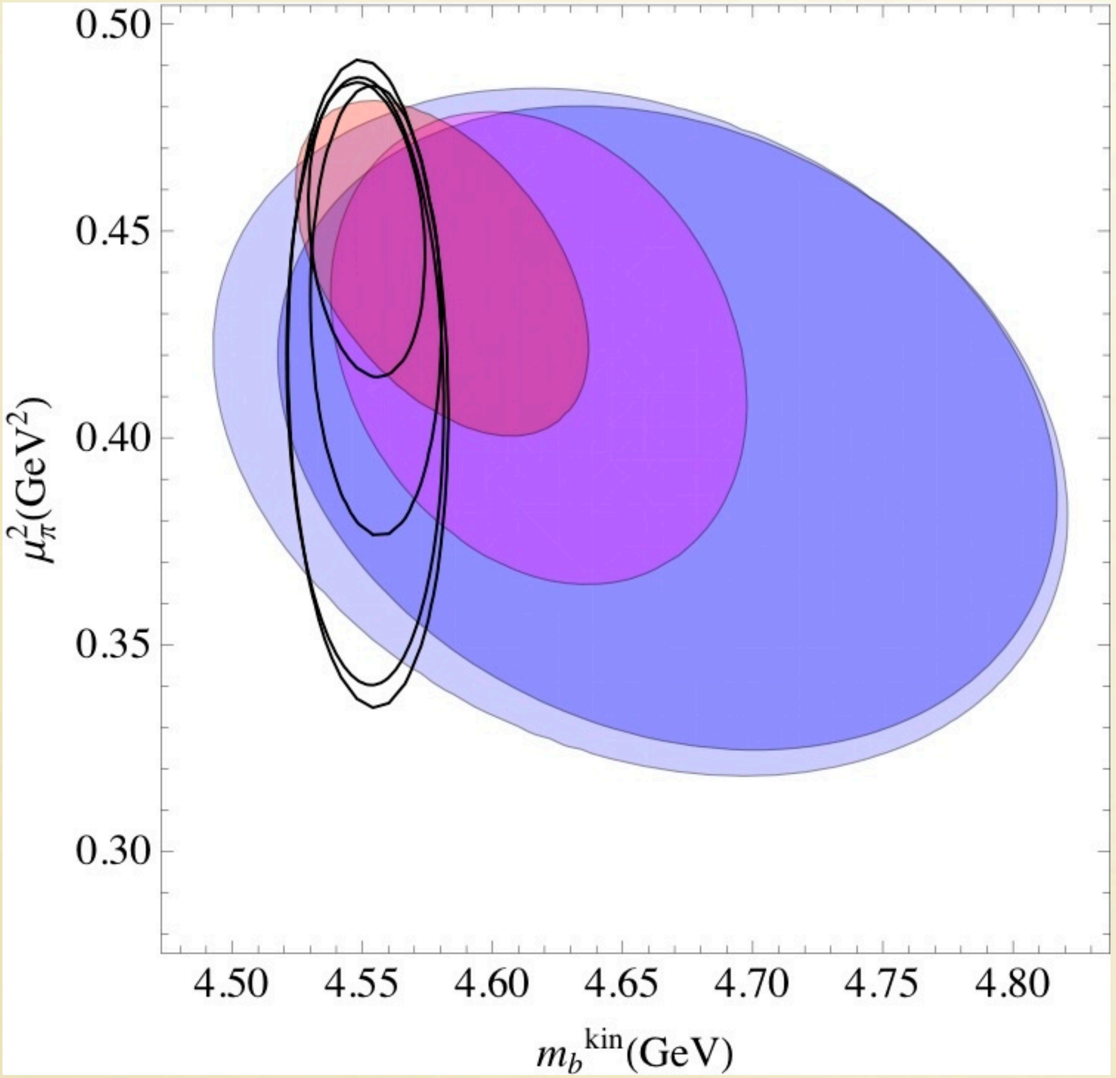
Correlations between theory errors of moments with different cuts difficult to estimate

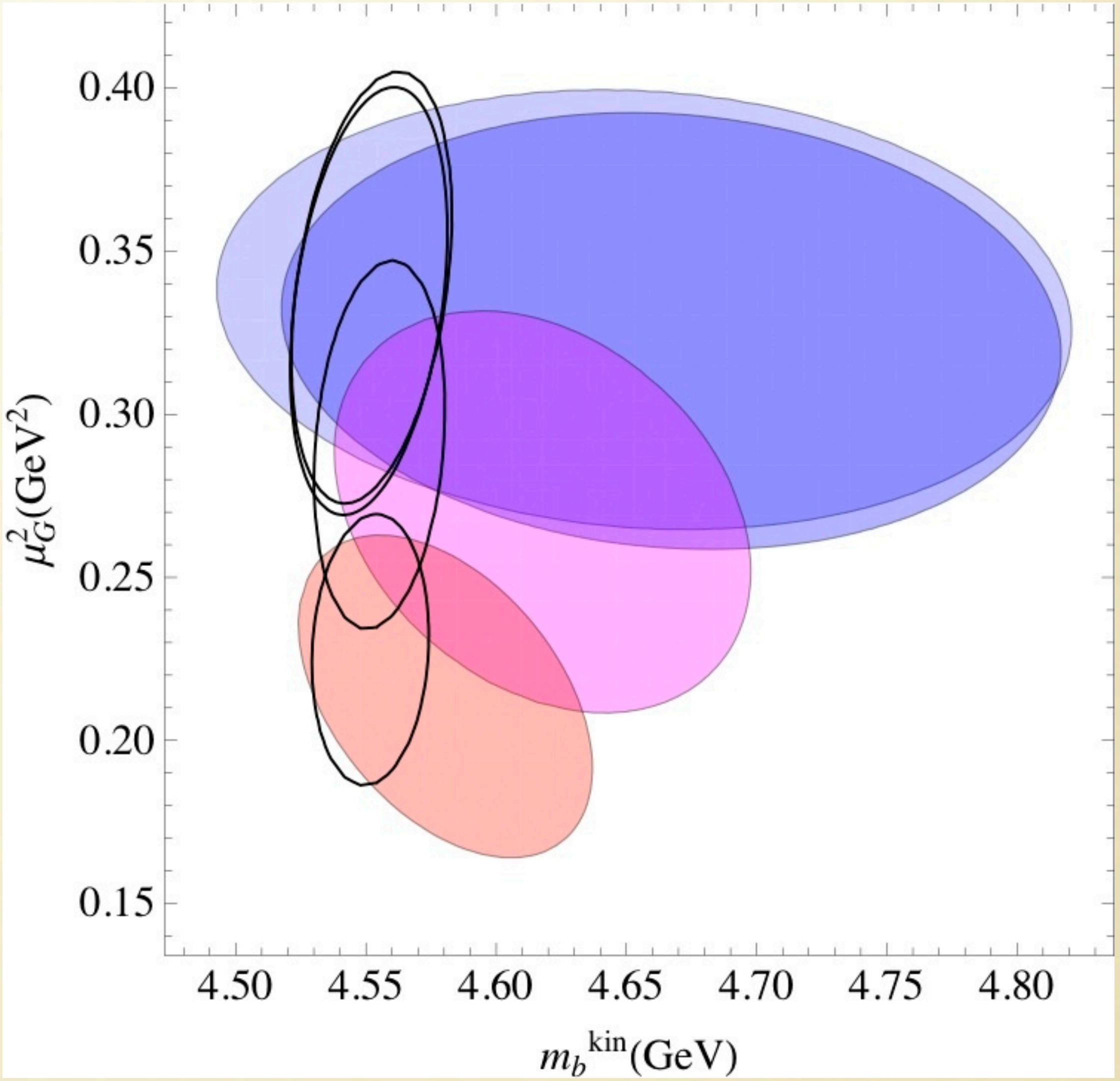
1. 100% correlations (unrealistic but used previously)
2. corr. computed from low-order expressions
3. constant factor $0<\xi<1$ for 100MeV step
4. same as 3. but larger for larger cuts

always assume different central moments uncorrelated

Schwanda, PG 2013

Theoretical correlations





Schwanda, PG 2013

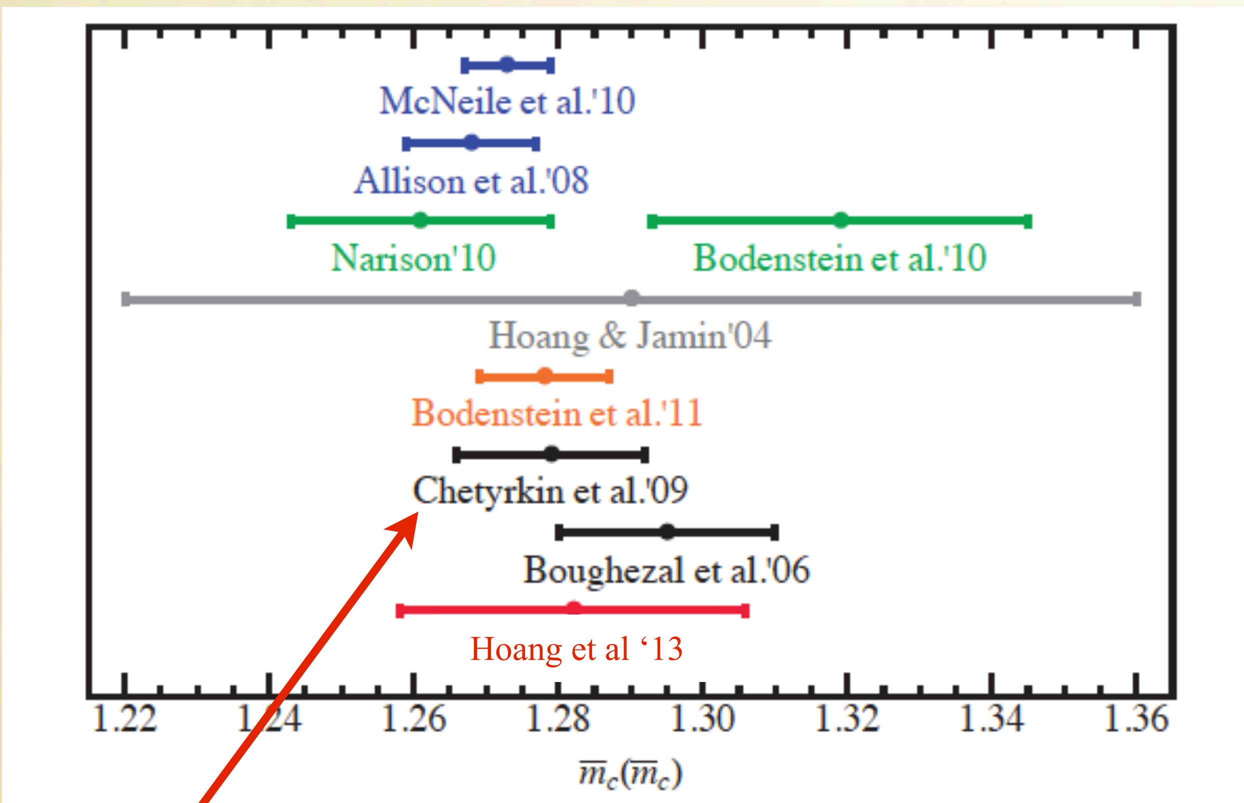
New semileptonic fit

Alberti, Healey, Nandi, PG, 1411.6560

- **updates** the fit in Schwanda, PG, 1307.4551
- kinetic scheme calculation based on 1107.3100; hep-ph/0401063
- NNLO partonic: it includes all $O(\alpha_s^2)$ corrections Czarnecki, Pak, Melnikov, Biswas, PG
- includes new $O(\alpha_s/m_b^2)$ complete corrections, not the $O(1/m_Q^{4,5})$
- reassessment of theoretical errors, realistic correlations
- **external constraints**: precise heavy quark mass determinations, mild constraints on μ^2_G from hyperfine splitting and ρ^3_{LS} from sum rules

Previous global fits: Buchmuller, Flaecher hep-ph/0507253, Bauer et al, hep-ph/0408002 (1S scheme)

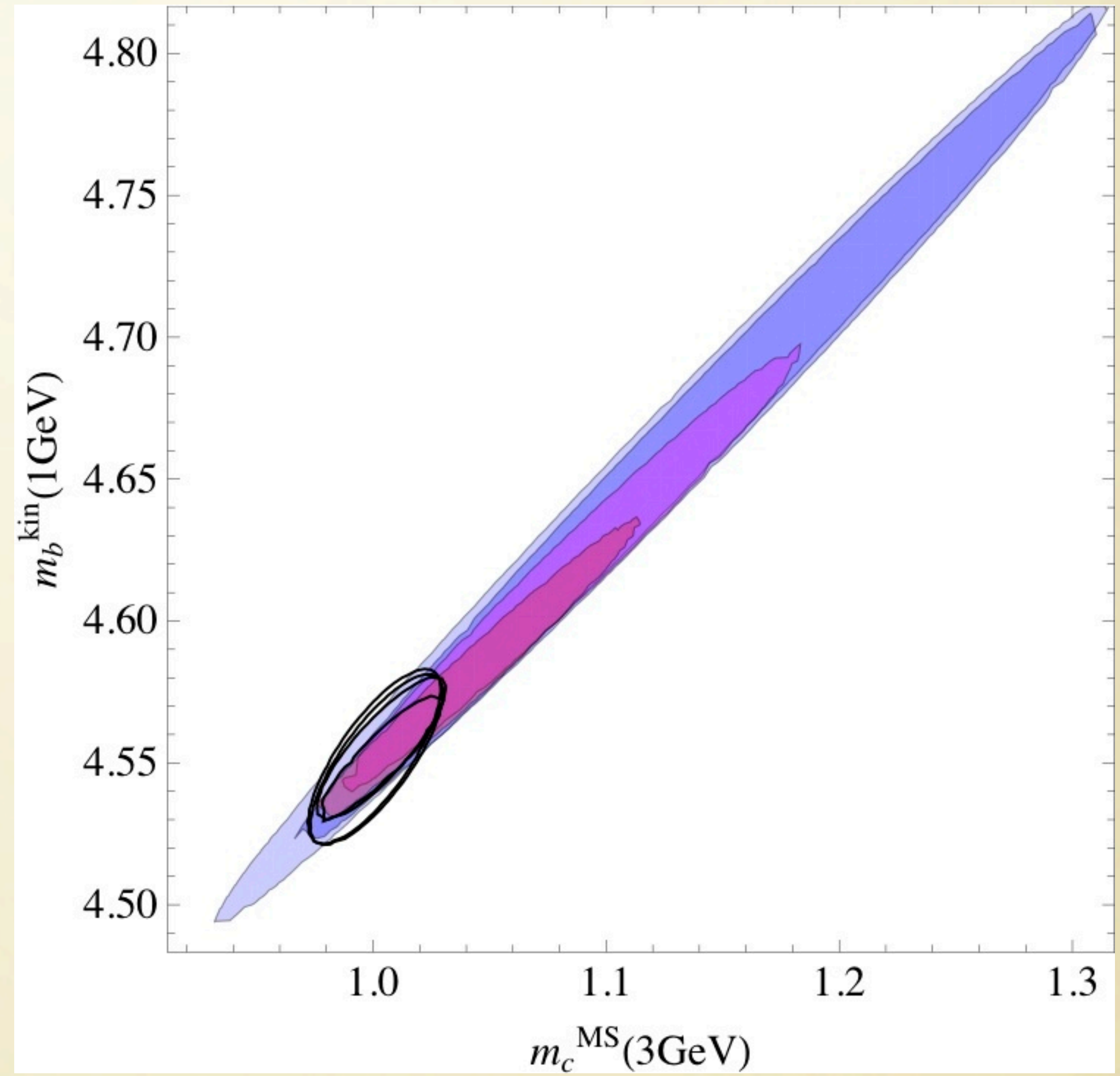
CHARM MASS DETERMINATIONS



our default choice

sum rules studies of $\sigma(e^+e^- \to \text{hadrons})$

almost all at NNNLO



Remarkable improvement in recent years.

m_c can be used as precise input to fix m_b instead of radiative moments

FIT RESULTS

NEW 1411.6560

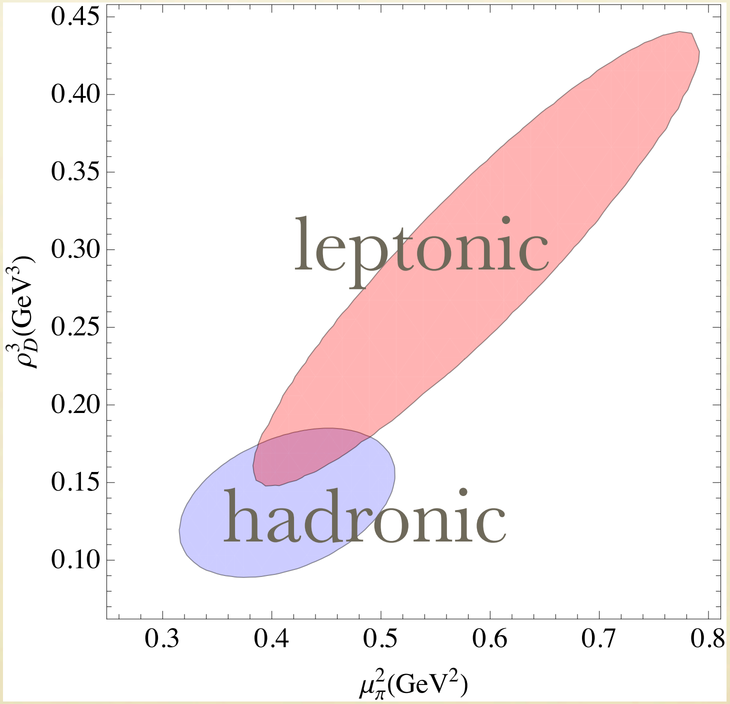
m_b^{kin}	$\overline{m}_c(3\,\mathrm{GeV})$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.553	0.987	0.465	0.170	0.332	-0.150	10.65	42.21
0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78

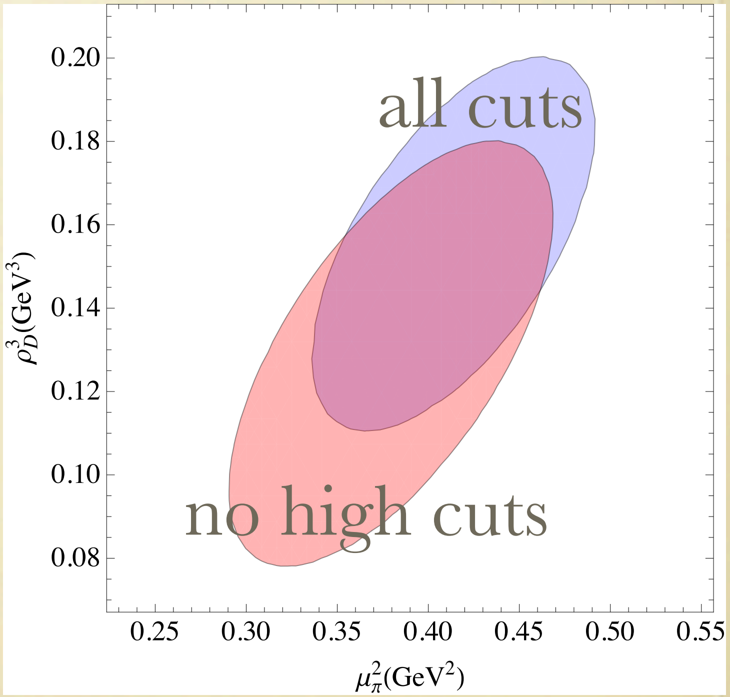
Schwanda PG 2013

m_b^{kin}	$m_c^{(3\mathrm{GeV})}$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}(\%)$	$10^3\lvert V_{cb}\rvert$
4.541	0.987	0.414	0.154	0.340	-0.147	10.65	42.42
0.023	0.013	0.078	0.045	0.066	0.098	0.16	0.86

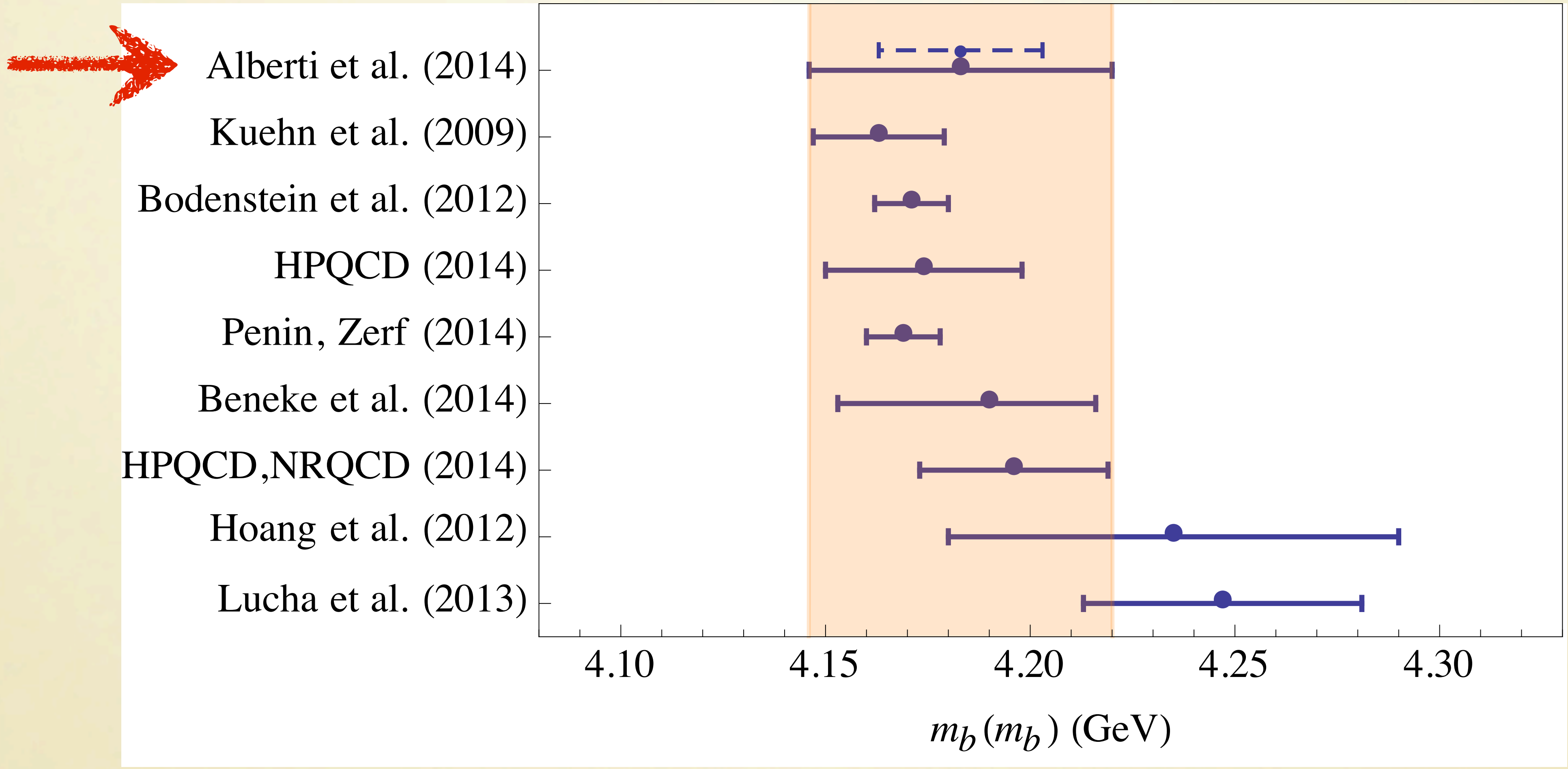
Without mass constraints $m_b^{kin}(1\mathrm{GeV}) - 0.85\,\overline{m}_c(3\mathrm{GeV}) = 3.714 \pm 0.018\,\mathrm{GeV}$

- results depend little on assumption for correlations and choice of inputs, 2% determination of V_{cb}
- 20-30% determination of the OPE parameters





Results: bottom mass

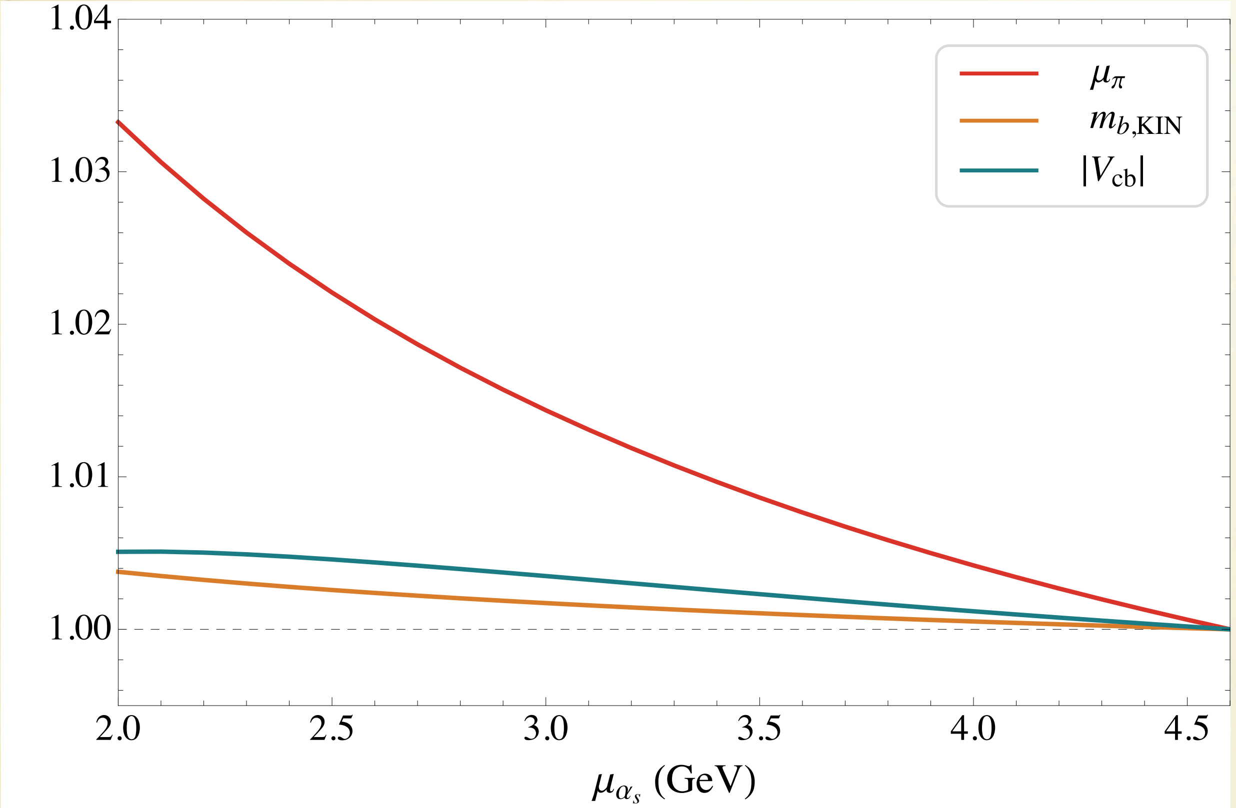


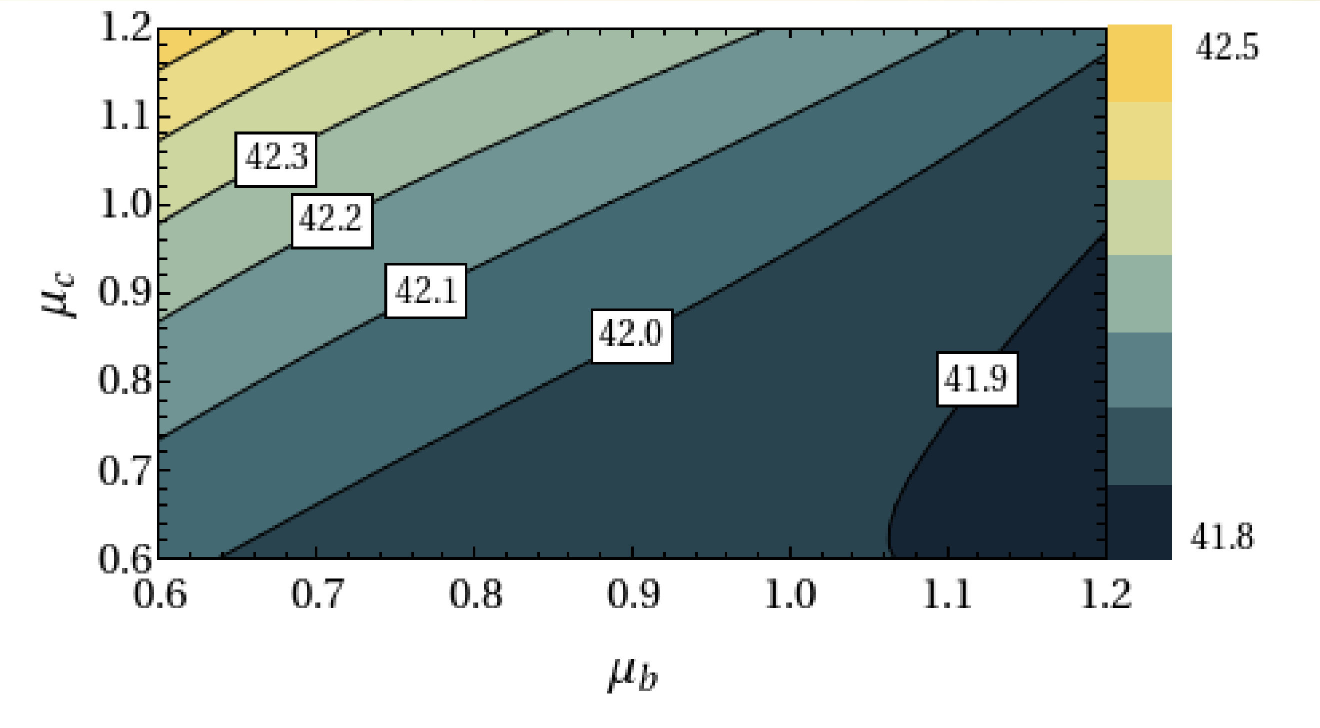
The fits give $\boldsymbol{m_b^{kin}}$**(1GeV)=4.553(20)GeV**, independent of th corr.

scheme translation error $m_b^{kin}(1\text{GeV})=m_b(m_b)+0.37(3)\text{GeV}$

$\boldsymbol{m_b(m_b)}$**=4.183(37)GeV**

FURTHER CHECKS





Dependence on strong coupling scale

Dependence on kinetic cutoffs on bottom and charm masses

Exclusive decay $B \to D^* \ell \nu$

<u>At zero recoil</u>, where rate vanishes, the ff is

$$\mathcal{F}(1) = \eta_A \left[1 + O\left(\frac{1}{m_c^2}\right) + \ldots\right]$$

Recent progress in measurement of slopes and shape parameters, exp error only ~1.3%

The ff F(1) cannot be experimentally determined. Lattice QCD is the best hope to compute it. <u>Only one</u> unquenched Lattice calculation:

F(1) =0.906(13) ➠ $\boxed{|V_{cb}|=39.04(49)_{exp}(53)_{lat}(19)_{QED}\ 10^{-3}}$

Bailey et al 1403.0635 (FNAL/MILC)

1.9% error (adding in quadrature)

~2.9σ or ~8% from inclusive determination

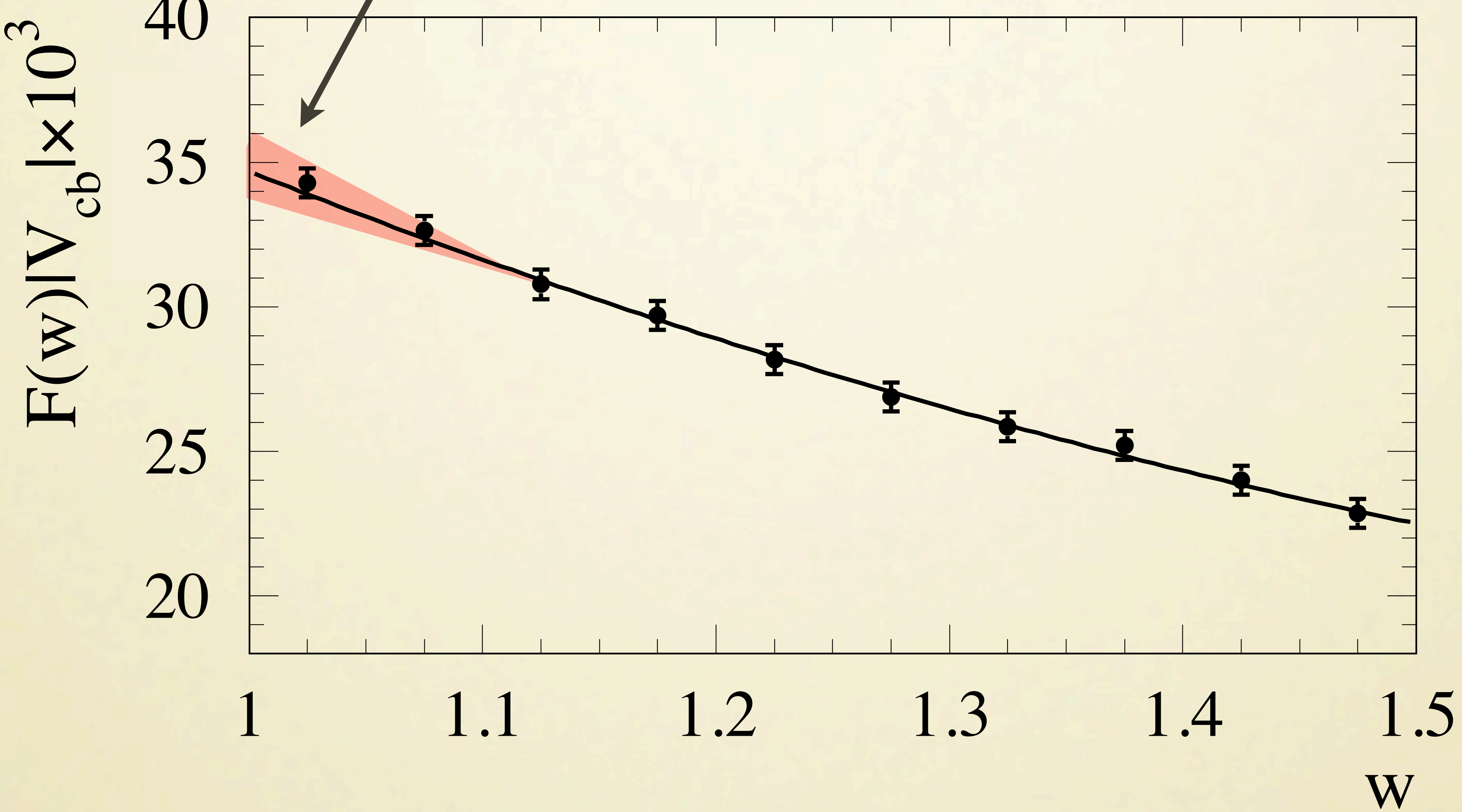
B→Dlv has larger errors: new $|V_{cb}|=38.5(2.0)\times10^{-3}$

<u>at non-zero recoil!</u> Qiu et al, 1312.0155

Comments on V_{cb}

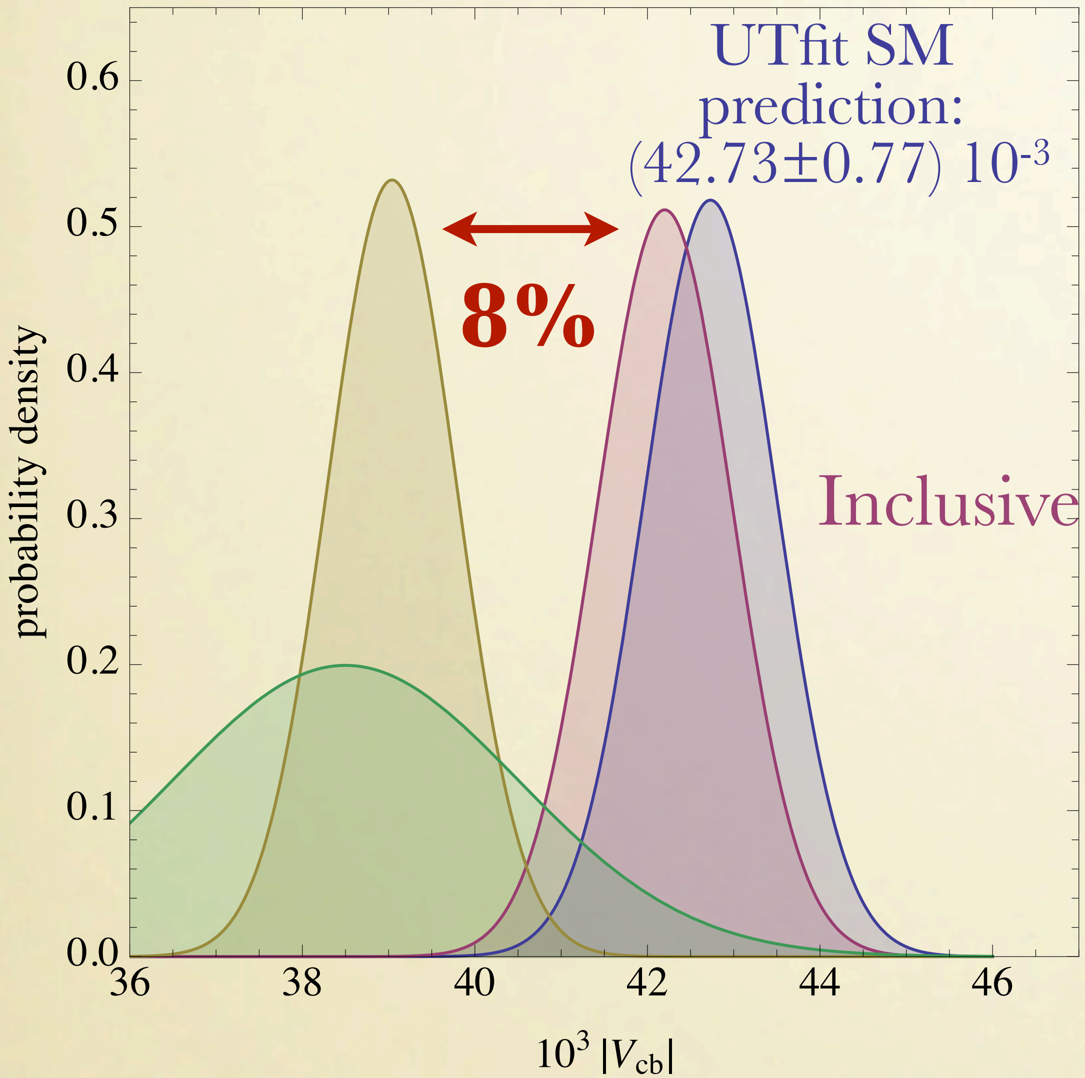
- **Heavy quark sum rules** (with BPS arguments) favor smaller F(1)=0.86(2) leading to agreement with inclusive. Difficult to improve, how good is BPS limit?
- **Extrapolations to zero recoil** by exp. coll. use Caprini et al parameterization, based on NLO HQET, and do not include a 2% uncertainty. Only 2 parameters, fits well exp data but rigid in low recoil region. Lattice simulations at non zero recoil under way.
- Matching at $1/m_Q^3$ for **lattice discretization** effects under study by FNAL/MILC. Other collaborations working on $B \to D^*$ ff.
- **Indirect $|V_{cb}|$ determinations** assuming SM+unitarity CKM: UTFit 42.05(65) 10^{-3} CKMFitter $41.4^{+2.4}_{-1.4}$ 10^{-3}

Extrapolation to zero recoil,
possible parameterization effect (qualitative picture)

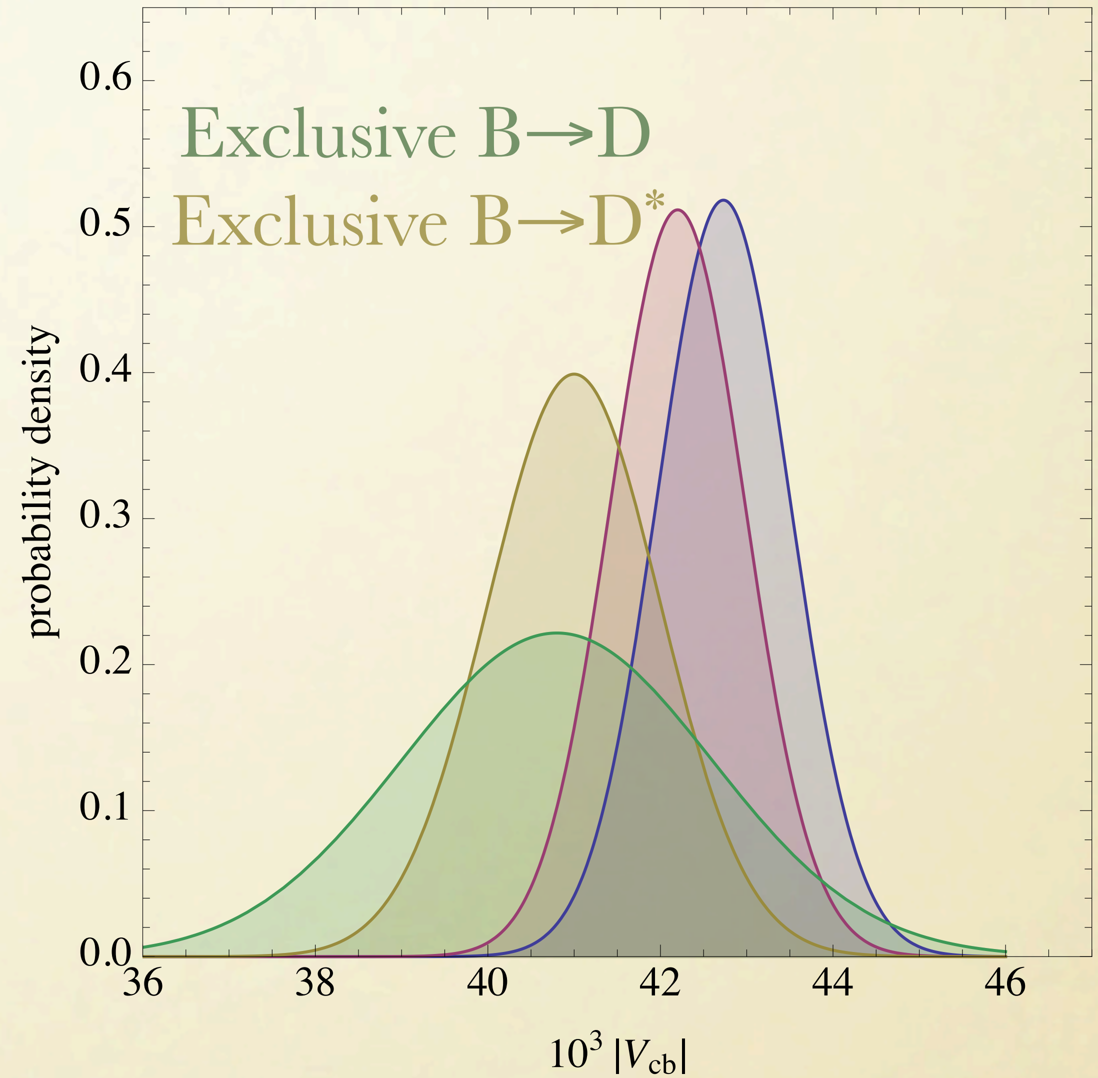


Babar form factor shape from 0705.4008

V_{cb} VISUAL SUMMARY



Latest lattice results for exclusives (FNAL/MILC)



HQSR,HQE for exclusives Mannel, Uraltsev, PG

New physics?

The difference with FNAL/MILC is **quite large**: 3σ or about 8%.
The perturbative corrections to inclusive V_{cb} total 5%, the power corrections about 4%.

Right Handed currents **disfavored** since

$$|V_{cb}|_{incl} \simeq |V_{cb}|\left(1+\frac{1}{2}|\delta|^2\right)$$

$$|V_{cb}|_{B\to D^*} \simeq |V_{cb}|\left(1-\delta\right)$$

$$|V_{cb}|_{B\to D} \simeq |V_{cb}|\left(1+\delta\right)$$

Chen,Nam,Crivellin,Buras,Gemmler,Isidori,Pokorski...

$$\delta = \epsilon_R \frac{\tilde{V}_{cb}}{V_{cb}} \approx 0.08$$

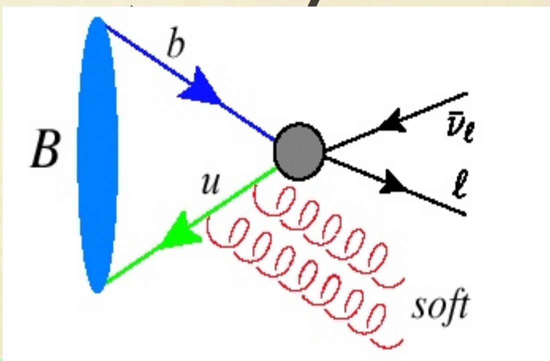
Most general SU(2) invariant dim 6 NP (without RH neutrino) can explain results, but it is incompatible with $Z\to b\bar{b}$ data

Crivellin, Pokorski 1407.1320
see also Mannel, Turczyk et al

The total $B \to X_u \ell \nu$ width

$$
\begin{aligned}
\Gamma[\bar{B} \to X_u e \bar{\nu}] \;=\; & \frac{G_F^2 m_b^5}{192\pi^3} |V_{ub}|^2 \left[1 + \frac{\alpha_s}{\pi} p_u^{(1)}(\mu) + \frac{\alpha_s^2}{\pi^2} p_u^{(2)}(r,\mu) - \frac{\mu_\pi^2}{2m_b^2} - \frac{3\mu_G^2}{2m_b^2} \right. \\
& \left. + \left(\frac{77}{6} + 8 \ln \frac{\mu_{\rm WA}^2}{m_b^2} \right) \frac{\rho_D^3}{m_b^3} + \frac{3\rho_{LS}^3}{2m_b^3} + \frac{32\pi^2}{m_b^3} B_{\rm WA}(\mu_{\rm WA}) \right] \\
& + O\left(\alpha_s \frac{\mu_{\pi,G}^2}{m_b^2}\right) + O\left(\frac{1}{m_b^4}\right)
\end{aligned}
$$

Using the results of the fit, V_{ub} could be extracted if we had the total width...



Weak Annihilation, severely constrained from D decays,
see Kamenik, PG, arXiv:1004.0114

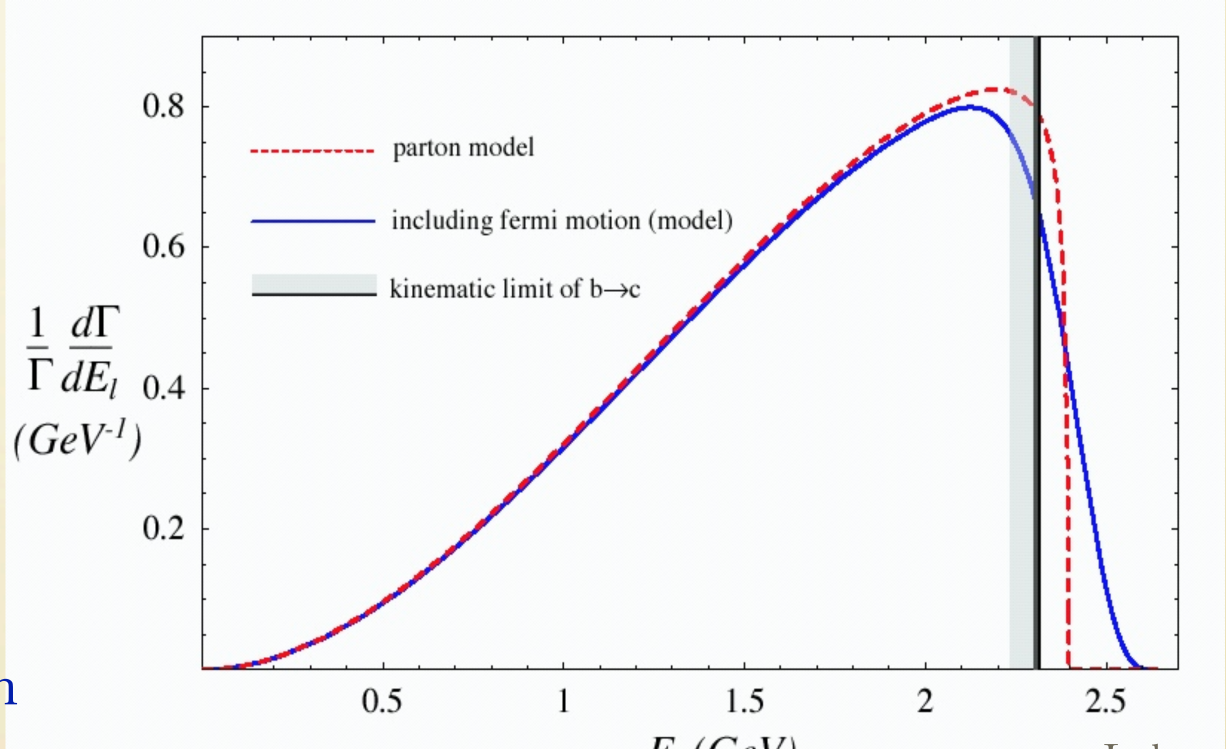
The problems with cuts

Experiments often use kinematic cuts to avoid the ~100x larger $b \to c l \nu$ background:

$m_X < M_D$ $\quad E_l > (M_B^2 - M_D^2)/2M_B$ $\quad q^2 > (M_B - M_D)^2$ ...

The cuts destroy convergence of the OPE that works so well in $b \to c$. OPE expected to work only away from pert singularities

Rate becomes sensitive to *local* b-quark wave function properties like Fermi motion. Dominant non-pert contributions can be resummed into a **SHAPE FUNCTION** $f(k+)$. Equivalently the SF is seen to emerge from soft gluon resummation



Luke

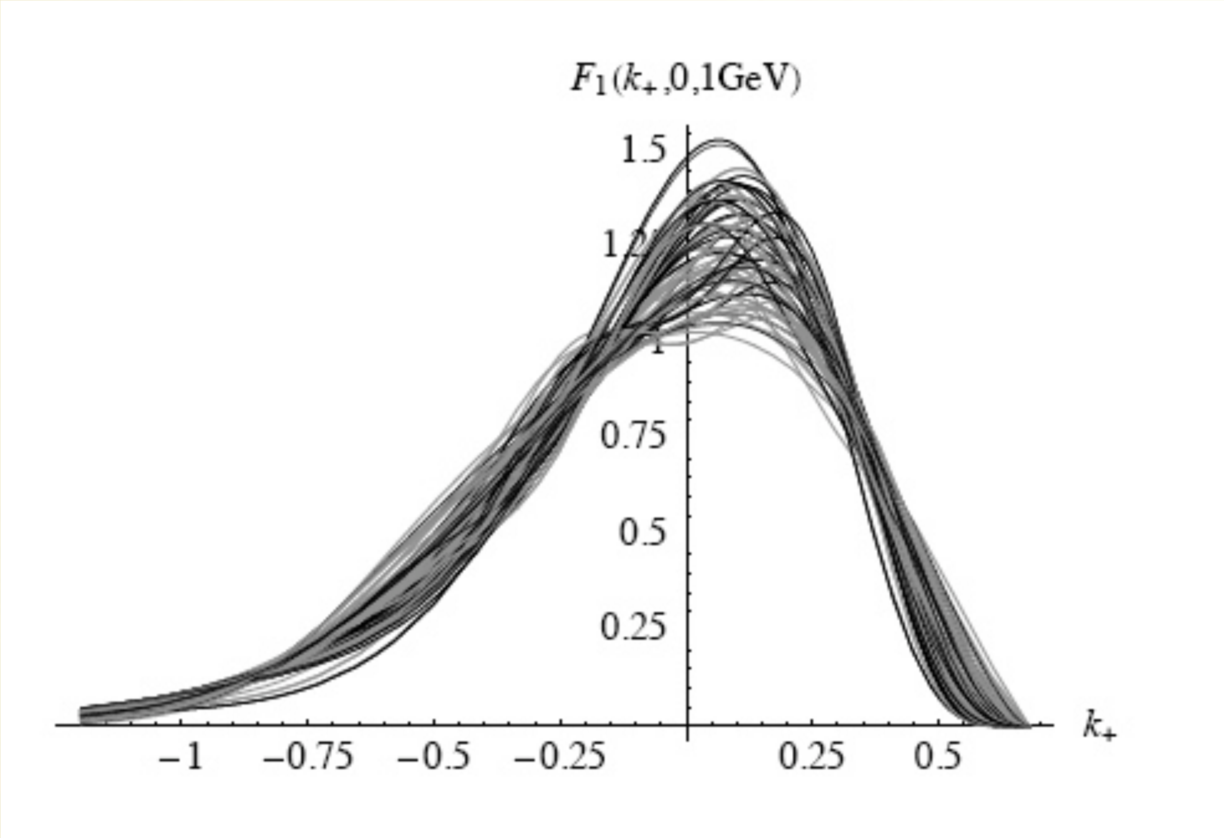
How to access the SF?

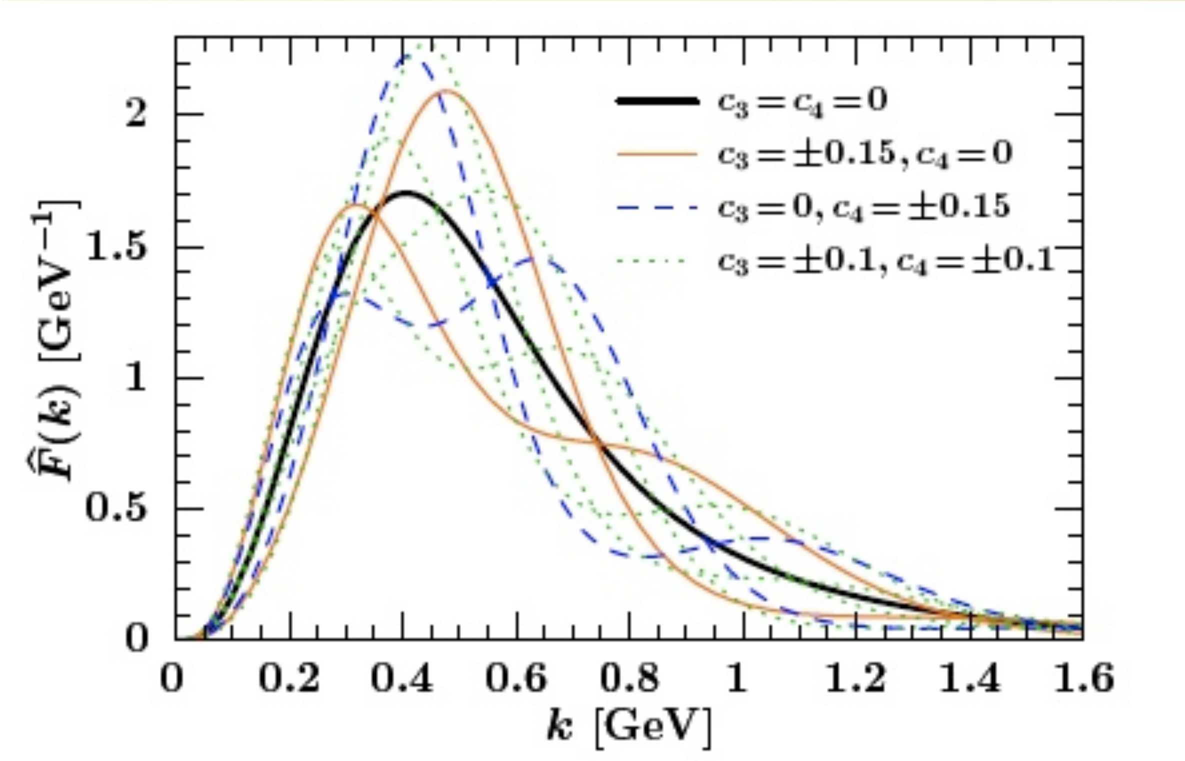
$$\frac{d^3\Gamma}{dp_+ dp_- dE_\ell} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} \int dk C(E_\ell, p_+, p_-, k) F(k) + O\left(\frac{\Lambda}{m_b}\right)$$

Subleading SFs

Prediction *based* on resummed pQCD DGE, ADFR	OPE constraints + parameterization without/with resummation GGOU, BLNP
Fit radiative data (and b→ulν) SIMBA	

Functional forms



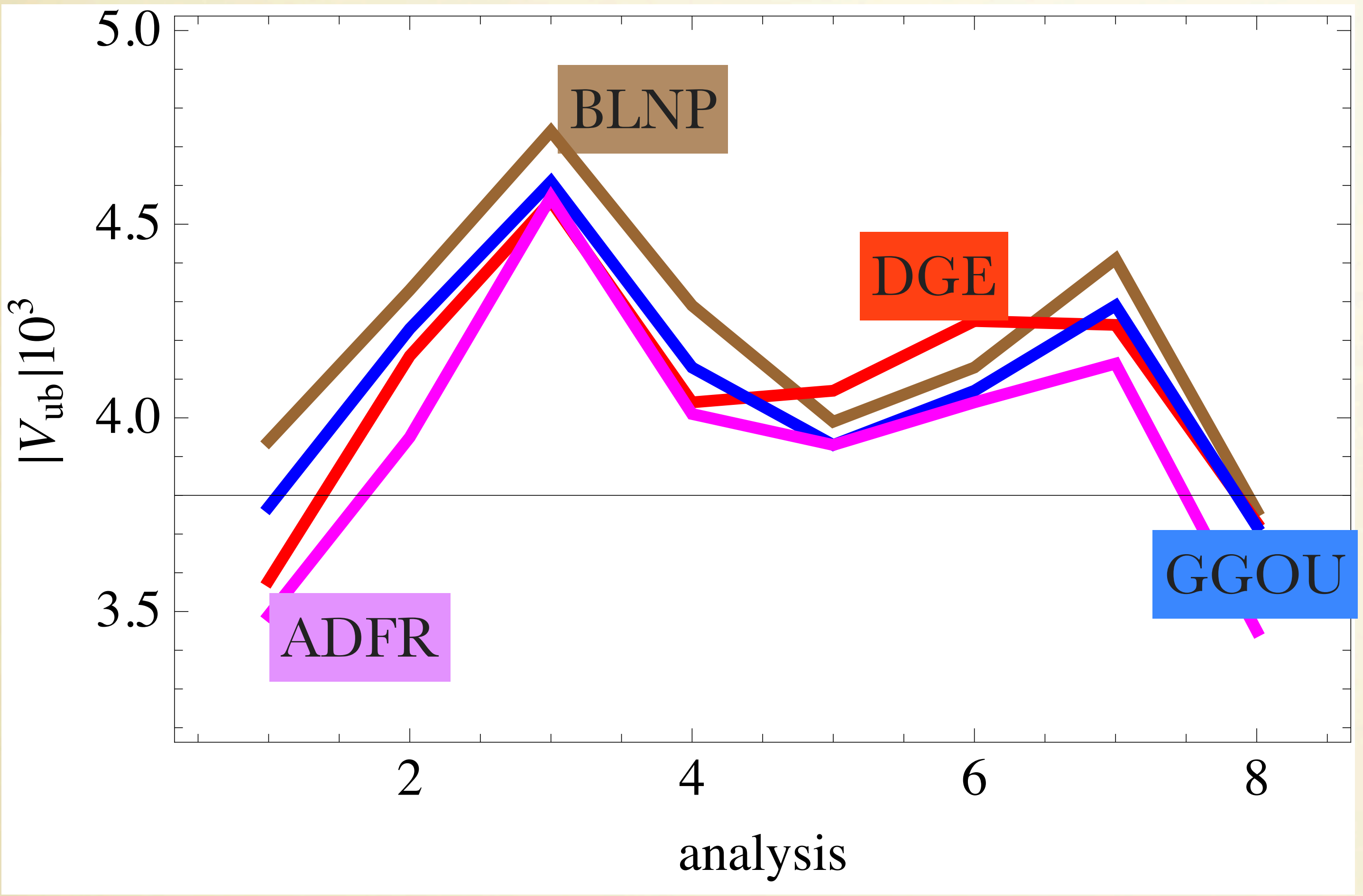


About 100 forms considered in GGOU, large variety, double max discarded. Small uncertainty (1-2%) on V_{ub}

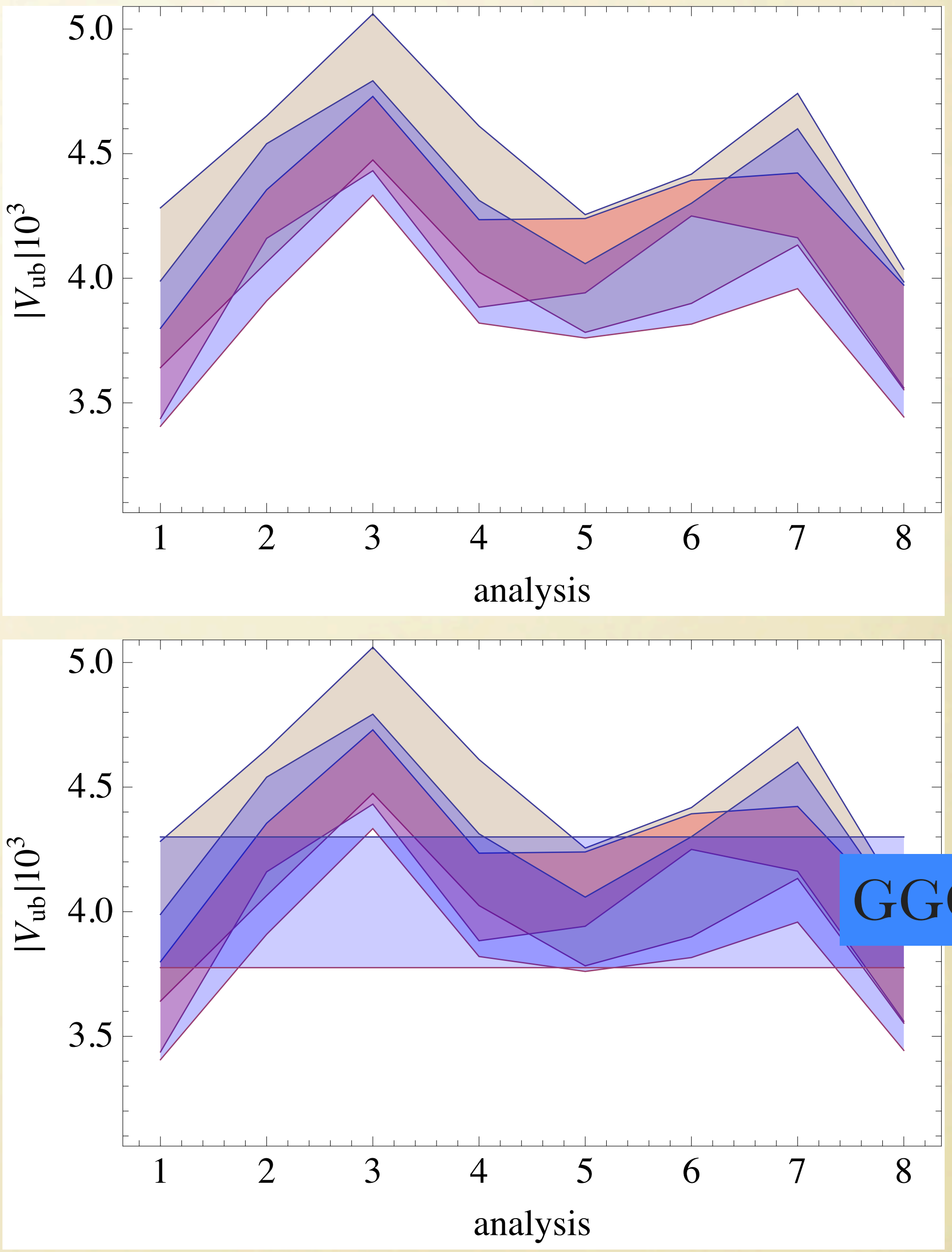
A more systematic method by Ligeti et al. arXiv:0807.1926 Plot shows 9 SFs that satisfy all the first three moments

A global comparison

0907.5386, Phys Rept



only theory errors
(without common parametric)



- ✶ common inputs (except ADFR)
- ✶ Overall good agreement **SPREAD WITHIN THEORY ERRORS**
- ✶ NNLO BLNP still missing: will push it up a bit
- ✶ Systematic offset of central values: normalization? to be investigated

V_{ub} IN THE GGOU APPROACH

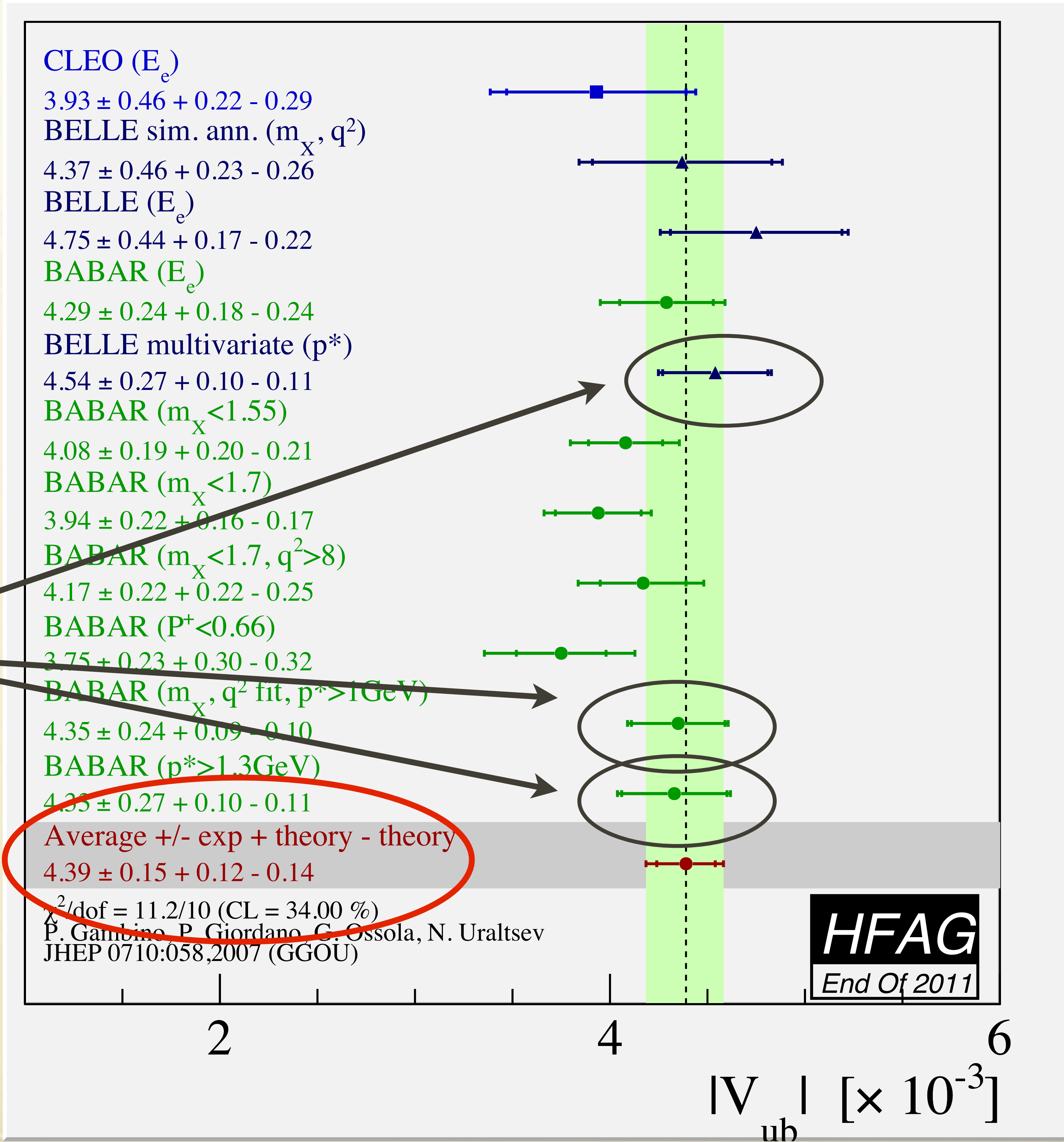
PG,Giordano,Ossola,Uraltsev

Good consistency & small th error.

5% total error

strong dependence on m_b

Recent experimental results are theoretically cleanest (2%) but based on background modelling. Signal simulation also relies on theoretical models



$|V_{ub}|$ DETERMINATIONS

Inclusive: 4-5% total error

HFAG 2012	Average $|V_{ub}|$x10^3
DGE	$4.45(15)_{ex}{}^{+15}{}_{-16}$
BLNP	$4.40(15)_{ex}{}^{+19}{}_{-21}$
GGOU	$4.39(15)_{ex}{}^{+12}{}_{-14}$

2.7-3σ from B→πlν (MILC-FNAL)
2σ from B→πlν (LCSR, Siegen)
2.5-3σ from UTFit 2014

Exclusive: 10-15% total error

$$|V_{ub}| = (3.25 \pm 0.31) \times 10^{-3}$$

Fermilab/MILC

$$|V_{ub}| = \left(3.50^{+0.38}_{-0.33}\Big|_{th.} \pm 0.11\Big|_{exp.}\right) \times 10^{-3}$$

LCSR, Khodjamirian et al, see also Bharucha

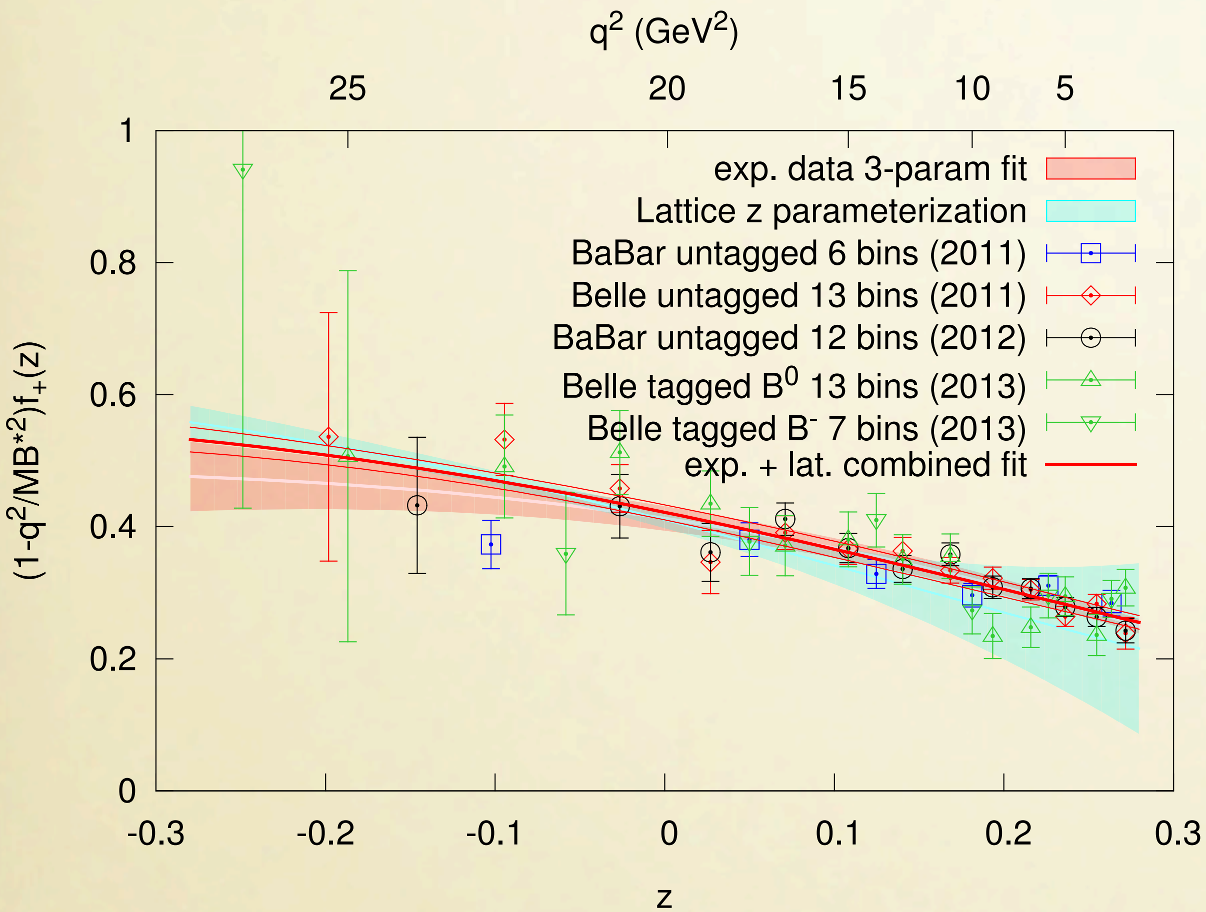
NB B→πlν data poorly consistent!

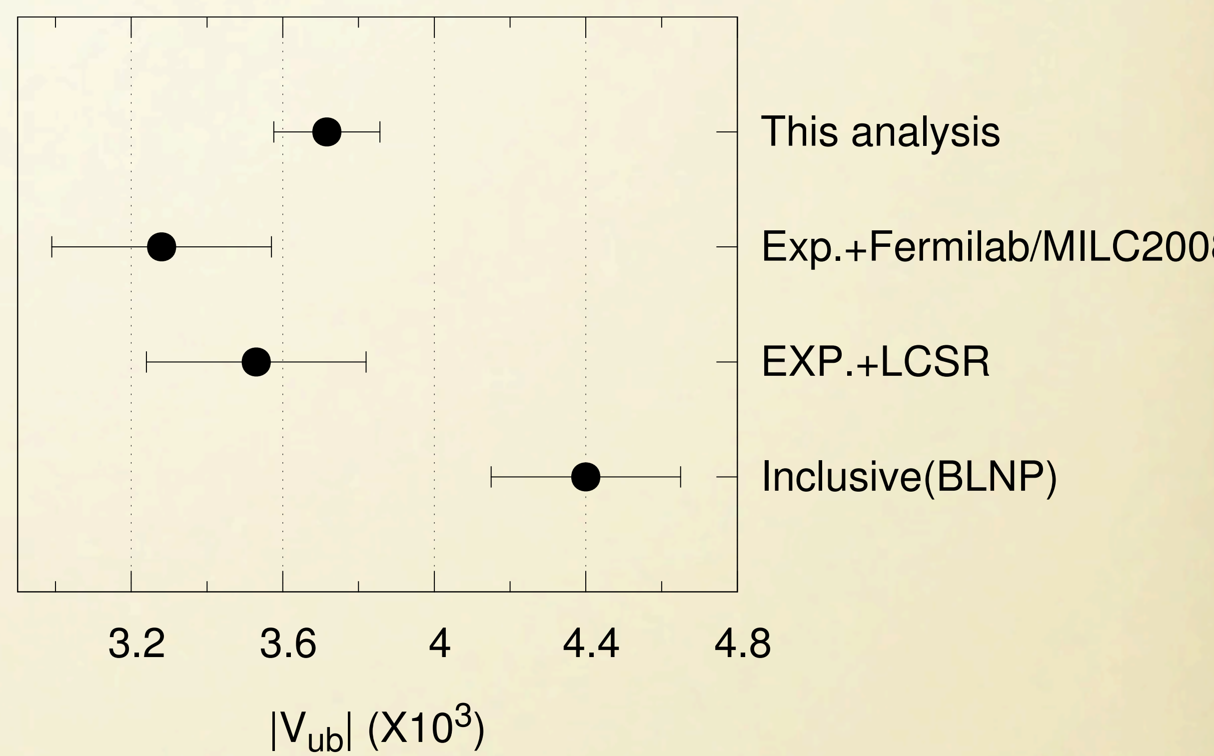
UT fit (without direct V_{ub}):
V_{ub}=3.62(12) 10^{-3}

The discrepancy here is around 25% !!

NEW FNAL/MILC RESULTS

1411.6038





$$|V_{ub}| = (3.72 \pm 0.14) \times 10^{-3}$$

Only 4% error! combined exp+lat fit has p-value=0.02, large shift wrt previous FNAL, 2.4σ from inclusive

Summary

- Improvements of OPE approach to semileptonic decays continue. All effects $O(\alpha_s \Lambda^2/m_b^2)$ implemented. **No sign of inconsistency in this approach so far**, **competitive m_b determination**. Calculation of $O(\alpha_s \Lambda^3/m_b^3)$ effects ongoing, work on higher power corrections.

- Exclusive/incl. tension in V_{cb} remains **large and mysterious** (3σ, 8%). It cannot be explained by right-handed current and in general by SU(2)-invariant new physics.

- Exclusive/incl tension in V_{ub} slightly receding because of new FNAL/MILC result. New physics explanations less constrained than for V_{cb}

- Belle-II will improve precision and allow for checks of consistency of various methods. Dedicated workshop at MITP on April 20-24.

BACK-UP SLIDES

(SEMI)LEPTONIC DECAYS TO τ

- $f_B \cdot V_{ub}$ can also be extracted in the SM from $B \to \tau\nu$, a rare decay mode measured at the B factories, which presently tends to prefer a high V_{ub}
- In the case of tau leptons charged scalars (eg from an extended Higgs sector) can contribute at tree-level. These decays are therefore sensitive probes of this New Physics.
- Recently BaBar measured $\mathcal{R}$ finding 2-3σ excess over the SM in both D and D*.
  Hard to find a NP model that can explain this result

$$\mathcal{R}\left(D^{(*)}\right) = \frac{\mathcal{B}(\overline{B} \to D^{(*)}\tau^-\overline{\nu}_\tau)}{\mathcal{B}(\overline{B} \to D^{(*)}l^-\overline{\nu}_l)}$$

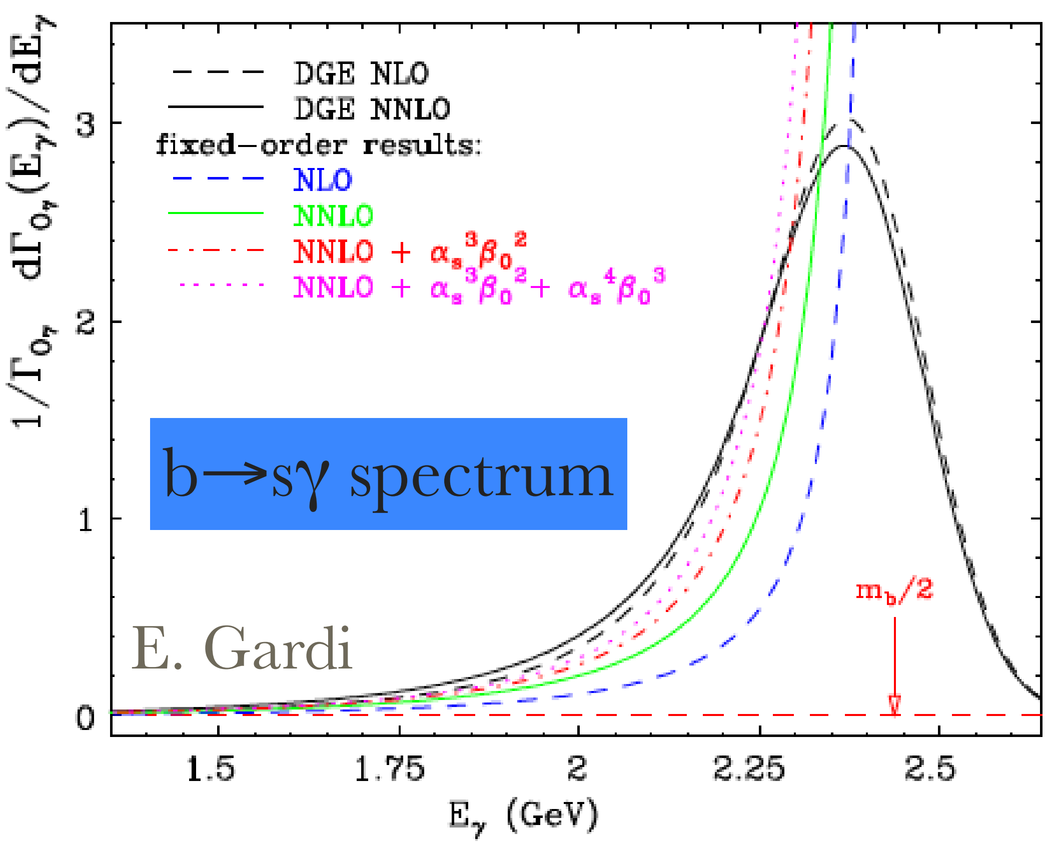
SF from perturbation theory

Resummed perturbation theory is qualitatively different: Support properties; stability! *(E. Gardi)*

b *quark* SF emerges from resummed pQCD but needs an IR prescription and power corrections for b →B

Dressed Gluon Exponentiation (DGE) by Gardi et al employs renormalon resummation to define Fermi motion. Power corrections can be partly accomodated.

Aglietti et al (ADFR) use Analytic Coupling in the IR, a model



The SF in the OPE

Local OPE has also threshold singularities and SF can be equivalently introduced resumming dominant singularities *Bigi et al, Neubert*

Fermi motion can be parameterized within the OPE like PDFs in DIS. At leading order in m_b only a single universal function of one parameter enters (SF).

Unlike resummed pQCD, ***the OPE does not predict the SF****, only its first few moments. One then* ***needs an ansatz for its functional form****.*

$$\int dk_+ \, k_+^n \, F_i(k_+, q^2) = \text{local OPE prediction} \Leftarrow \text{moments fits}$$

Two very different implementations:
PG, Giordano, Ossola, Uraltsev (GGOU)
Bosch, Lampe, Neubert, Paz (BLNP)

Several new subleading SFs appear at $O(\Lambda/m_b)$

$O(\alpha_s/m_b^2)$ EFFECTS

Boos,Becher,Lunghi 2007
Ewerth,Nandi, PG 2009
Alberti,Ewerth,Nandi,PG 2012
Alberti,Nandi,PG 2013

Hadronic tensor $$W^{\alpha\beta} = \frac{(2\pi)^3}{2m_B} \sum_{X_c} \delta^4(p_b - q - p_X) \langle \bar{B} | J_L^{\dagger\alpha} | X_c \rangle \langle X_c | J_L^\beta | \bar{B} \rangle$$

$$m_b W^{\alpha\beta} = -W_1 g^{\alpha\beta} + W_2 v^\alpha v^\beta + i W_3 \epsilon^{\alpha\beta\rho\sigma} v_\rho \hat{q}_\sigma + W_4 \hat{q}^\alpha \hat{q}^\beta + W_5 (v^\alpha \hat{q}^\beta + v^\beta \hat{q}^\beta)$$

$$W_i = W_i^{(0)} + \frac{\mu_\pi^2}{2m_b^2} W_i^{(\pi,0)} + \frac{\mu_G^2}{2m_b^2} W_i^{(G,0)} + \frac{C_F \alpha_s}{\pi} \left[W_i^{(1)} + \frac{\mu_\pi^2}{2m_b^2} W_i^{(\pi,1)} + \frac{\mu_G^2}{2m_b^2} W_i^{(G,1)} \right]$$

$W_i^{(\pi,n)}$ can be computed using **reparameterization invariance** which relates different orders in the HQET: e.g. for $i=3$ at all orders

$$W_3^{(\pi,n)} = \frac{5}{3} \hat{q}_0 \frac{dW_3^{(n)}}{d\hat{q}_0} - \frac{\hat{q}^2 - \hat{q}_0^2}{3} \frac{d^2 W_3^{(n)}}{d\hat{q}_0^2}$$ Manohar 2010

Proliferation of power divergences, up to $1/u^3$,
and complex kinematics (q^2, q_0, m_c, m_b) $W_i^{(G,1)}$ requires proper matching.

Perturbative effects

- $O(\alpha_s)$ implemented by all groups De Fazio,Neubert
- Running coupling $O(\alpha_s^2\beta_0)$ (PG,Gardi,Ridolfi) in GGOU, DGE lead to -5% & +2%, resp. in $|V_{ub}|$
- Complete $O(\alpha_s^2)$ in the SF region Asatrian,Greub,Pecjak-Bonciani,Ferroglia-Beneke,Huber, Li - G. Bell 2008
- In BLNP leads to up 8% increase in V_{ub} related to resummation, not yet included by HFAG. It is an **artefact** of this approach.

- $P_+ < 0.66$ GeV:

	$\Gamma_u^{(0)}$	μ_h	μ_i
NLO	60.37	$^{+3.52}_{-3.37}$	$^{+3.81}_{-6.67}$
NNLO	52.92	$^{+1.46}_{-1.72}$	$^{+0.09}_{-2.79}$

Greub,Neubert,Pecjak arXiv:0909.1609

- $P_+ < 0.66$ GeV:

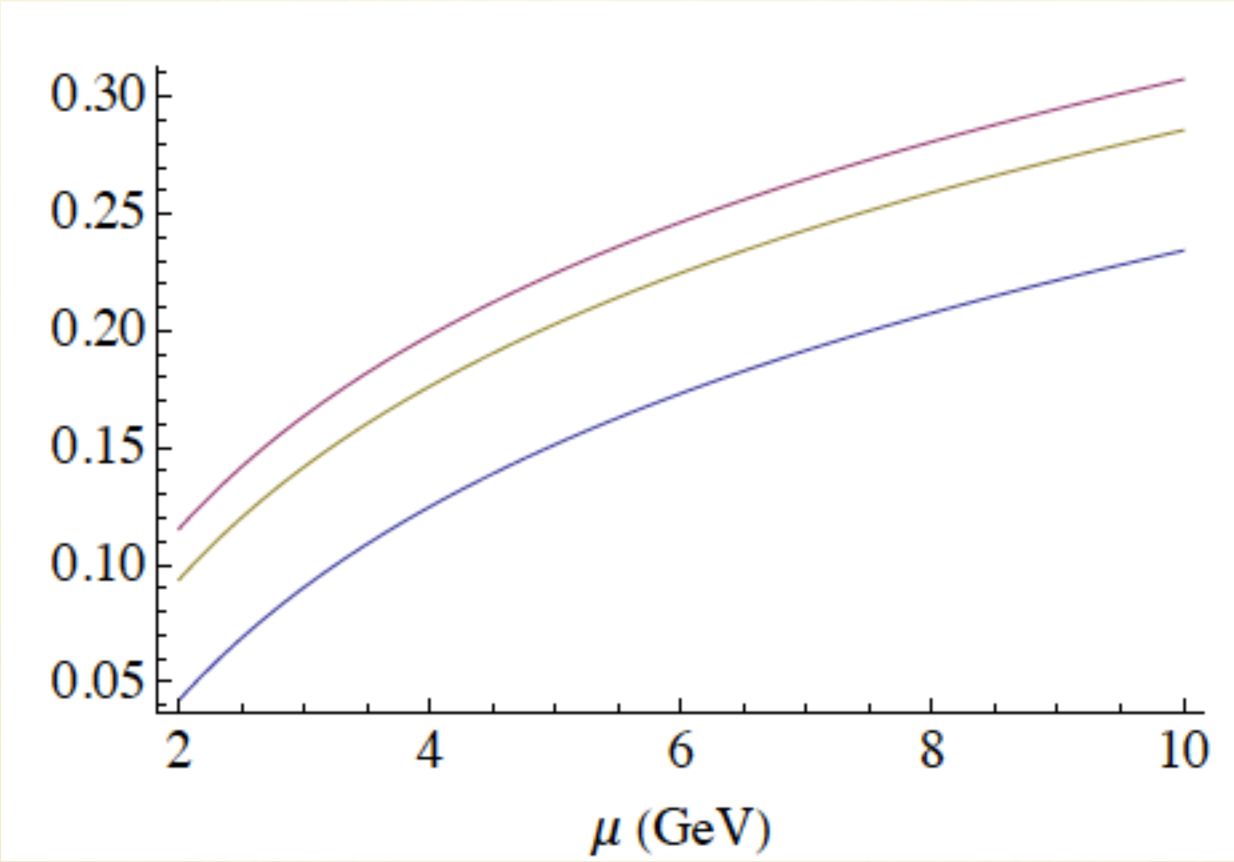
Fixed-Order	$\Gamma_u^{(0)}$	μ
NLO	49.11	$^{+5.43}_{-9.41}$
NNLO	49.53	$^{+0.13}_{-4.01}$

NEW: full phase space $O(\alpha_s^2)$ calculation

Brucherseifer,Caola,Melnikov, arXiv:1302.0444

Confirms non-BLM/BLM approx 20% over relevant phase space

μ_G^2-SCALE DEPENDENCE



Relative NLO correction to the coefficients of μ_G in the width (blue), first (red) and second central (yellow) leptonic moments as a function of the renormalization scale. Smaller corrections for smaller scale.