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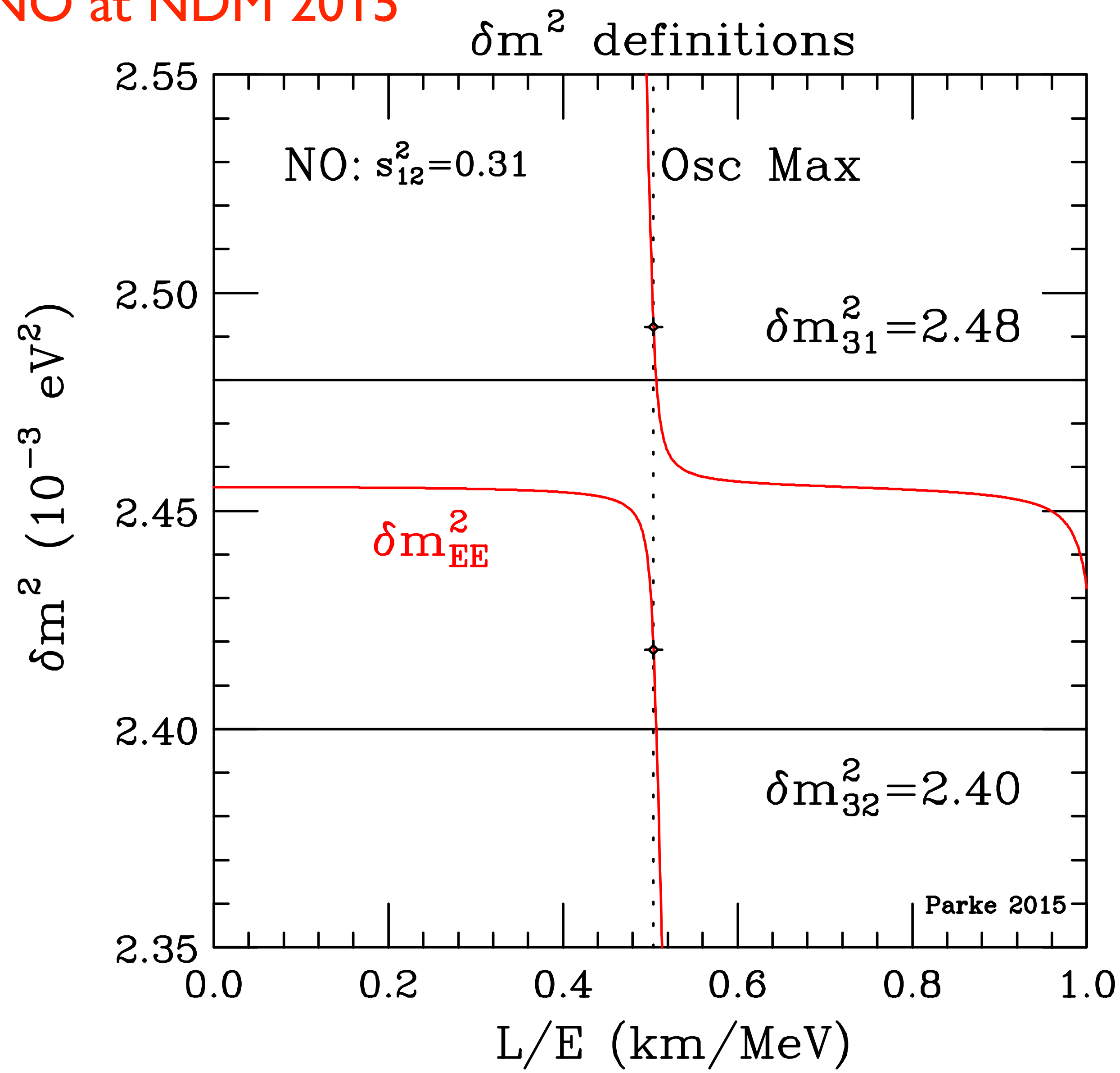

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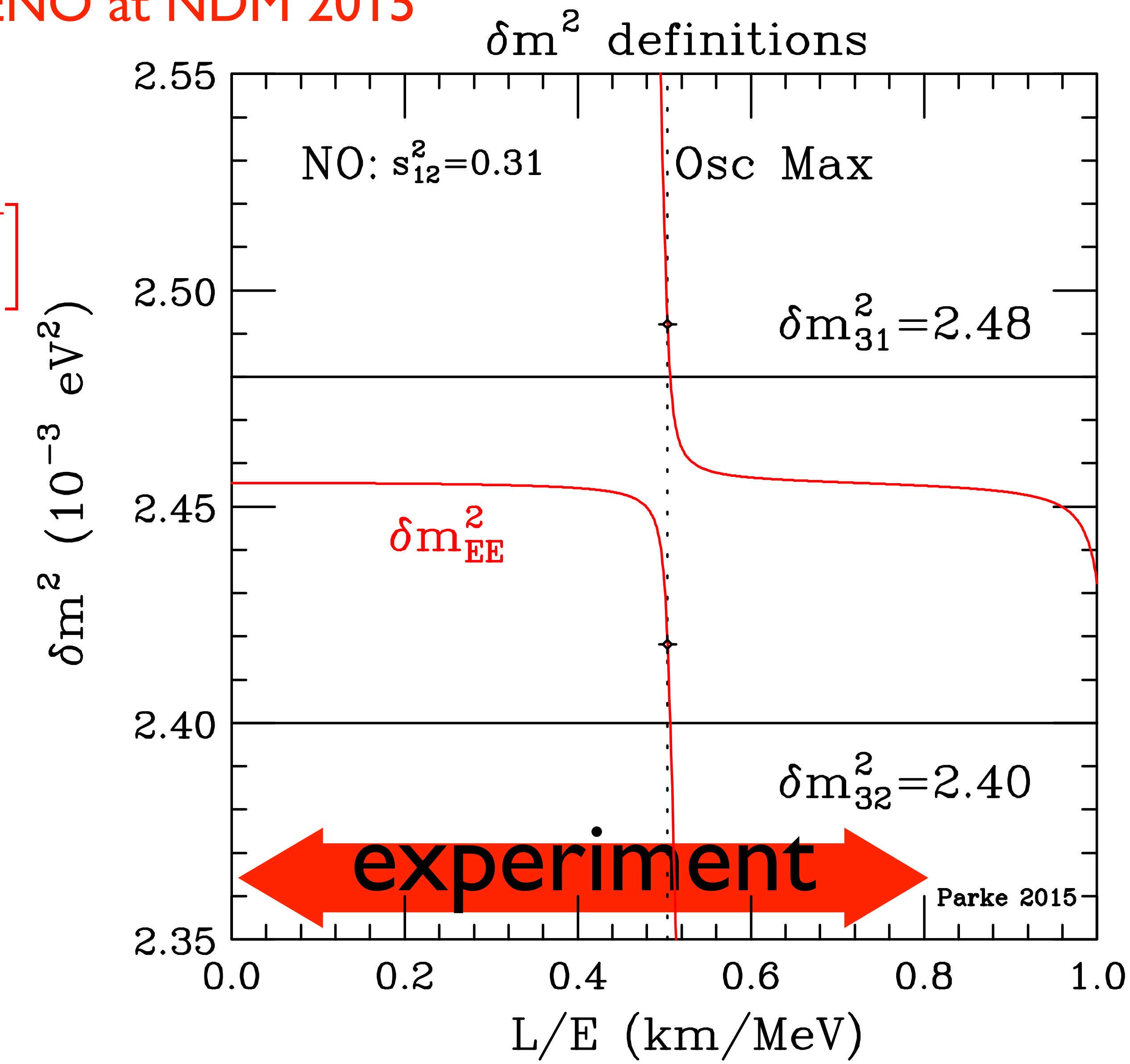

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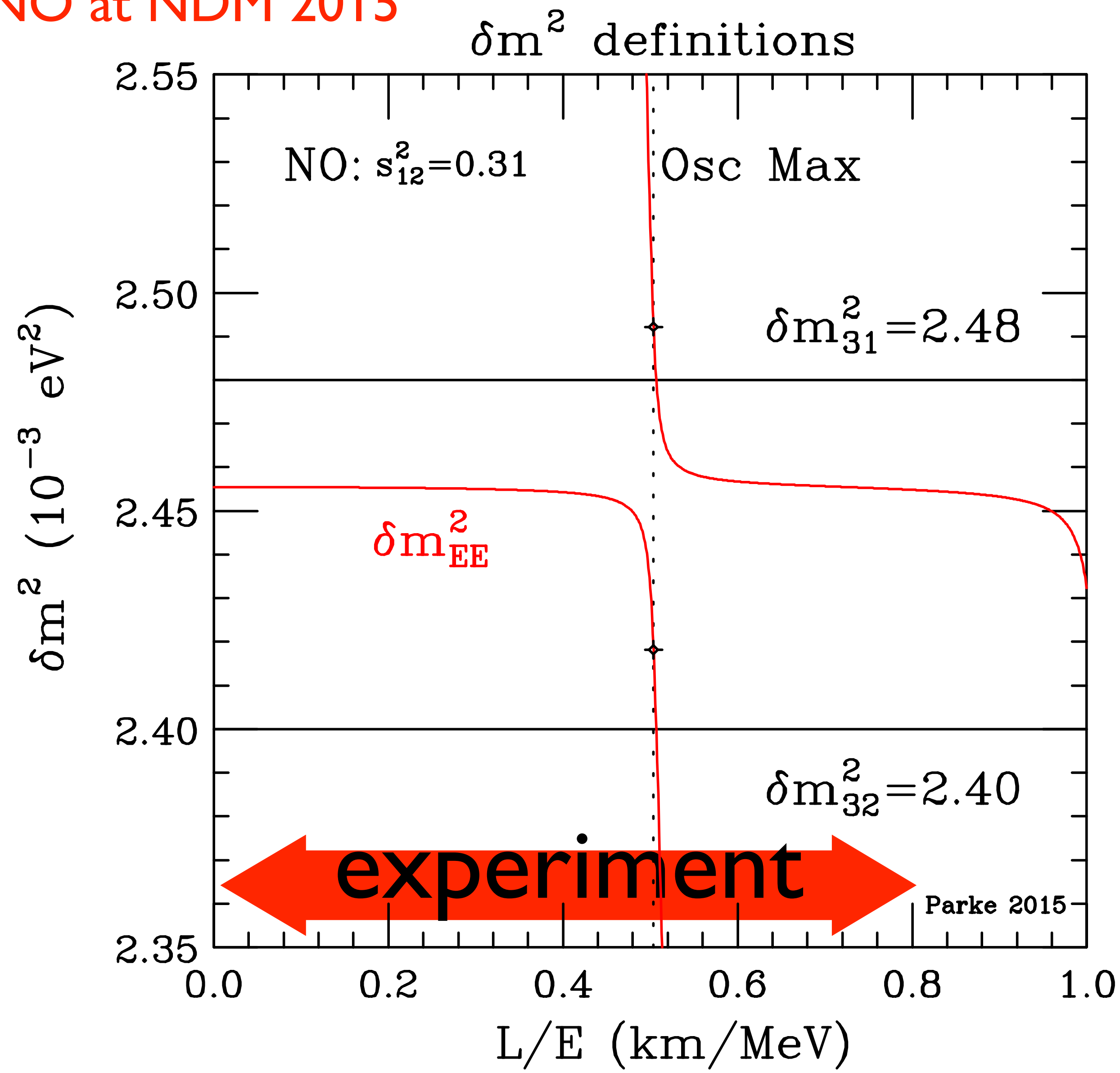

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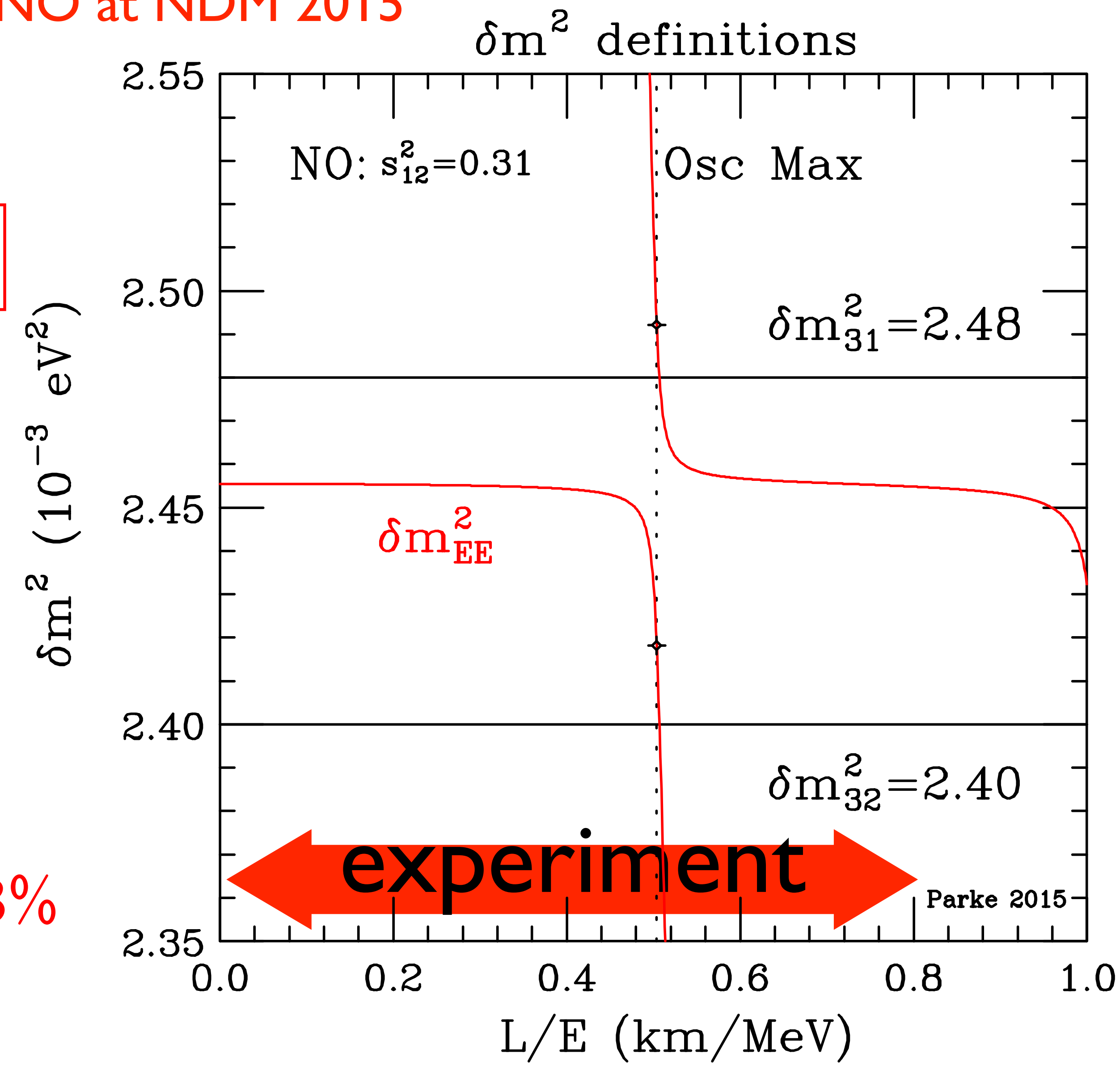

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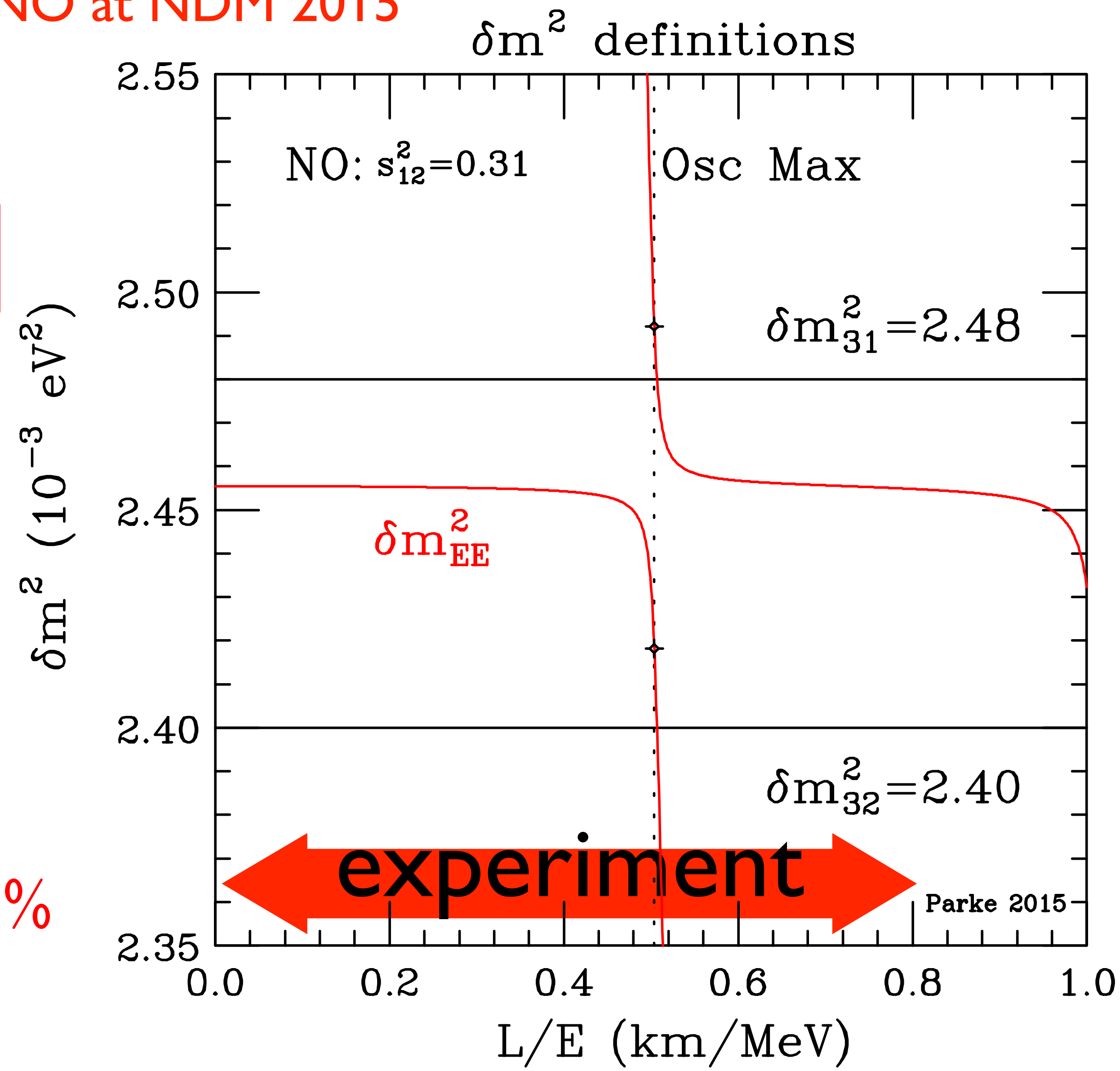

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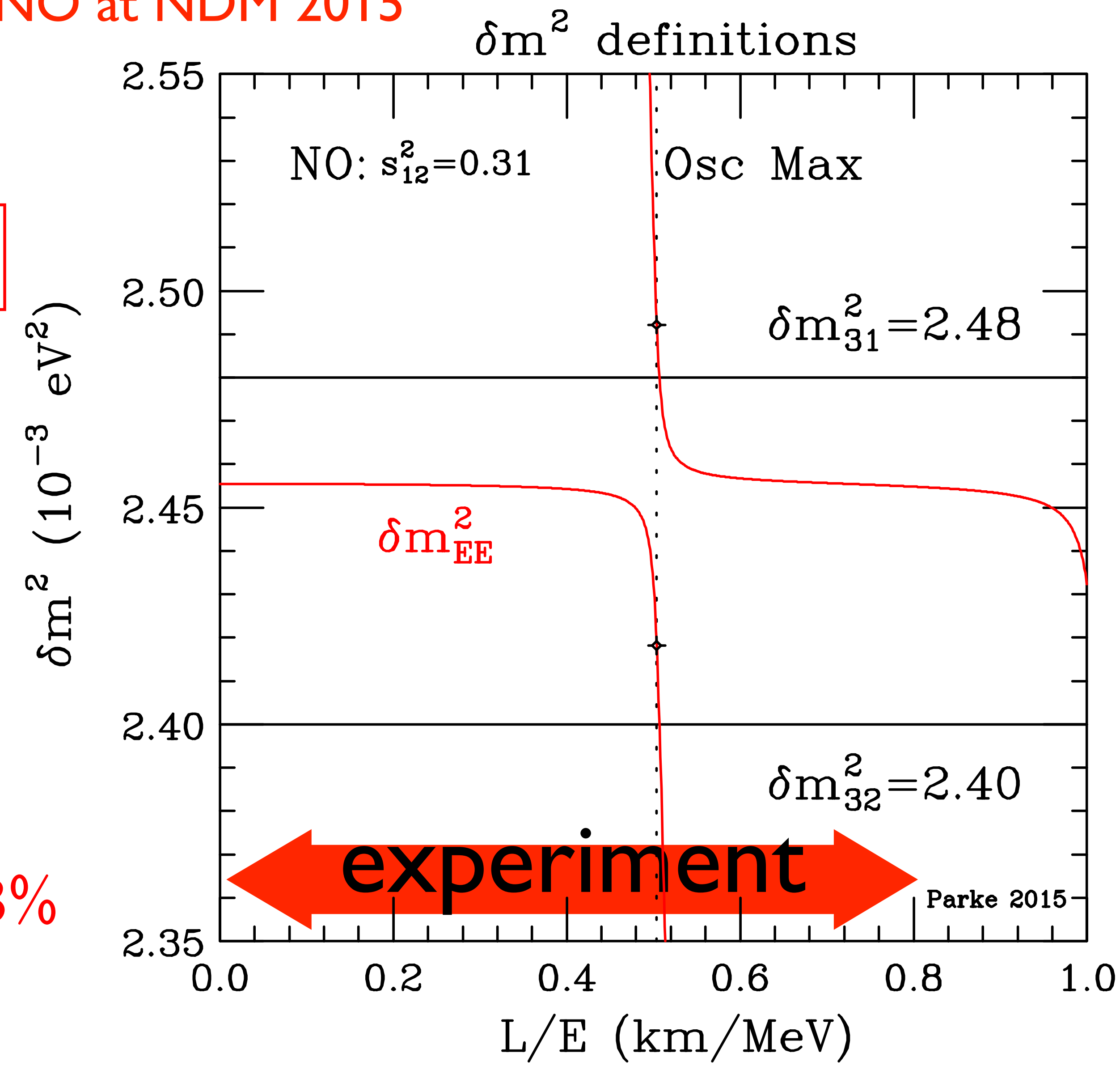

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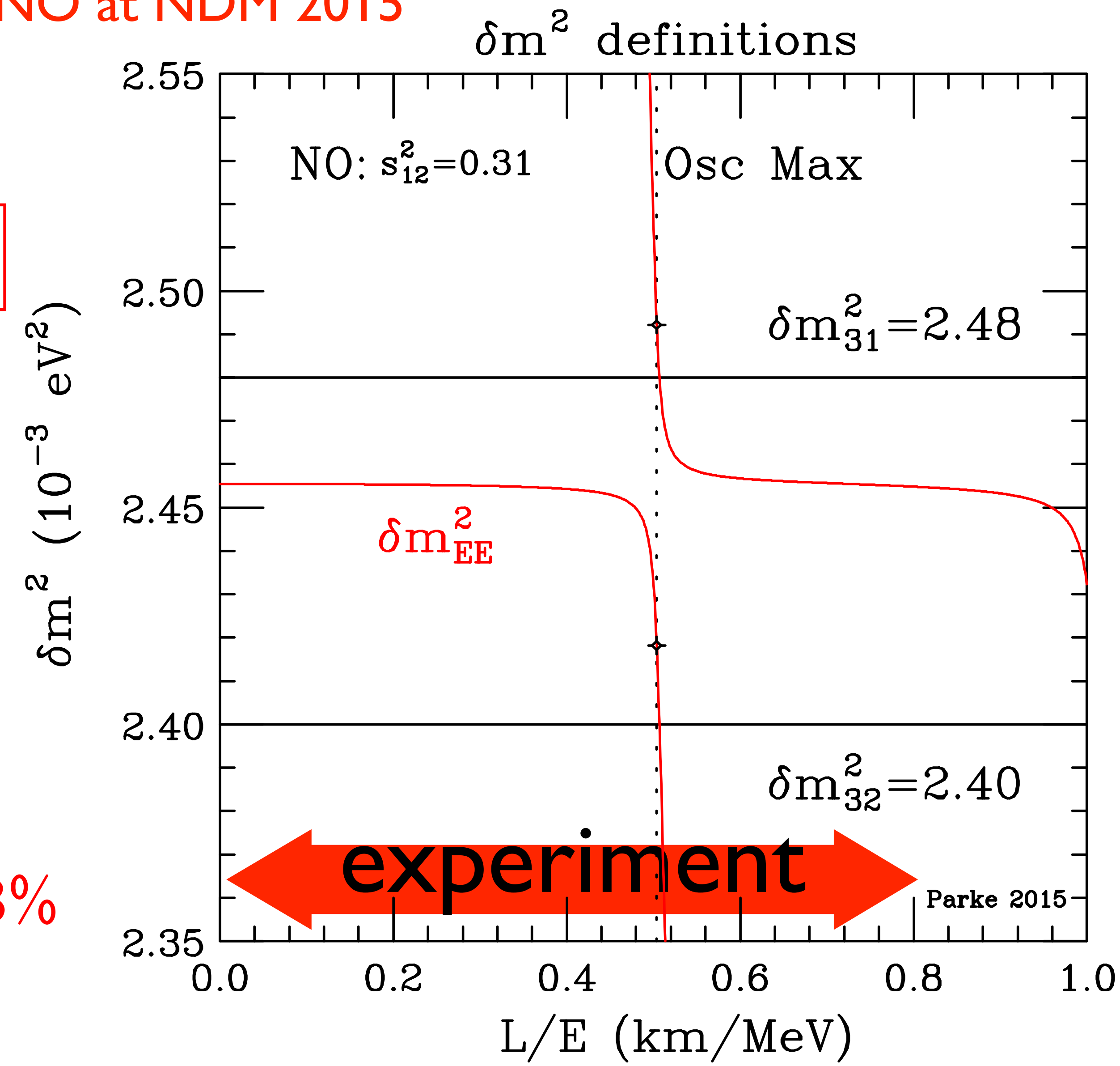

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H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, Phys. Rev. D **72**, 013009 (2005), hep-ph/0503283

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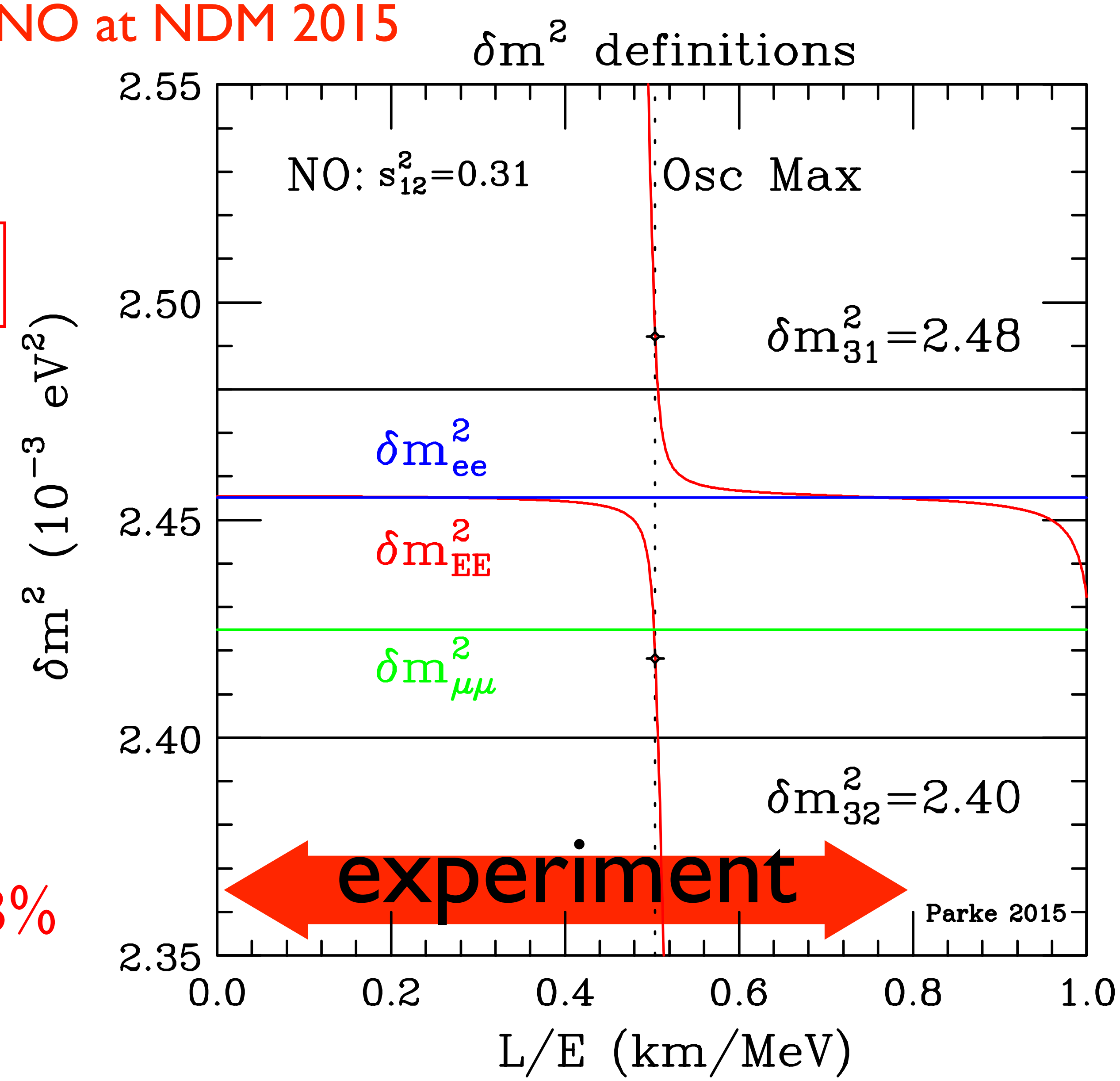

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$\sim s^2_{12}c^2_{12}\Delta^2_{21}$

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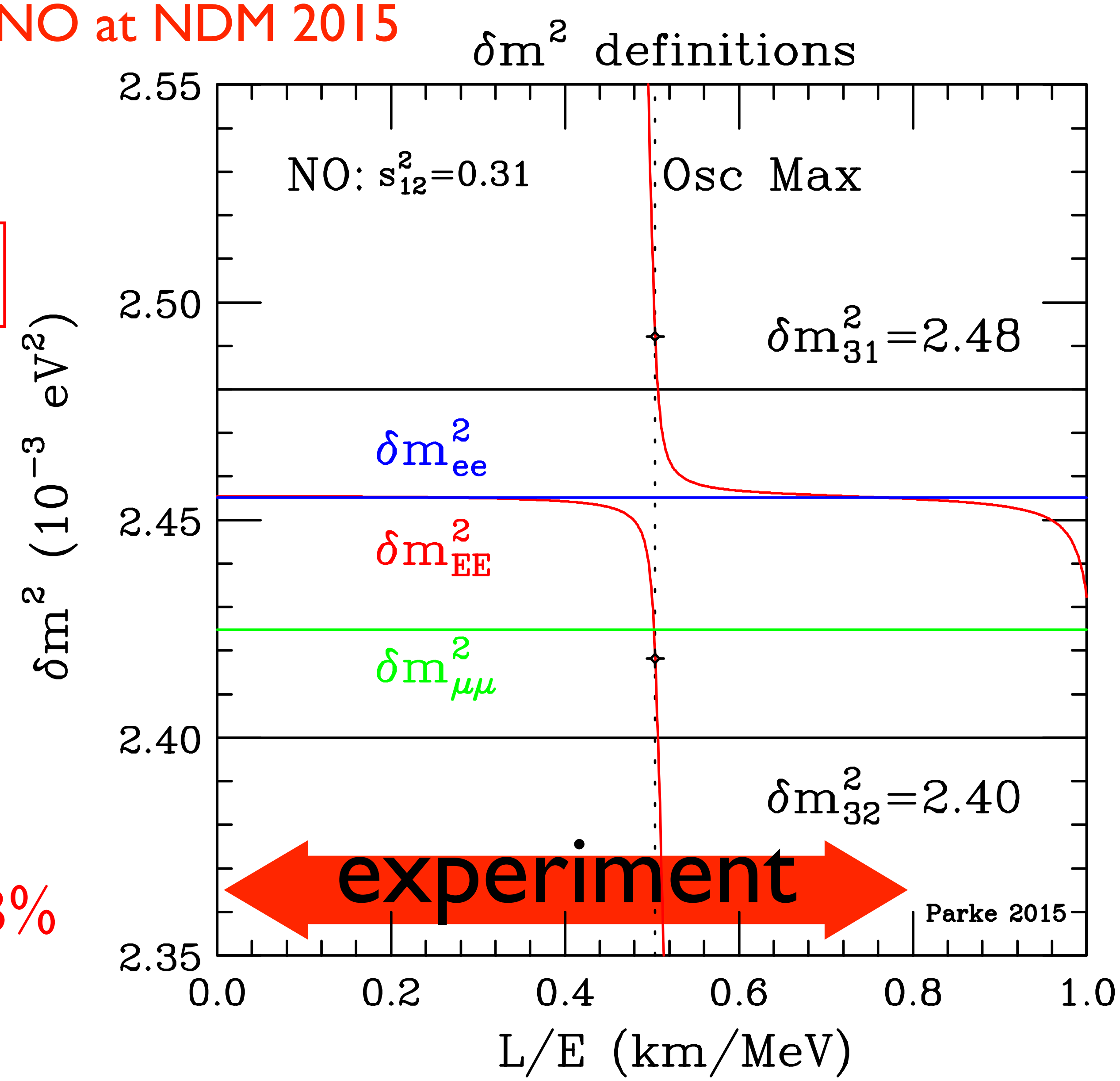

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backup:

Note, the Δ_{21} terms vanish !

$$
\begin{aligned}
c_{12}^2 \sin^2 \Delta_{31} + s_{12}^2 \sin^2 \Delta_{32} &= \sin^2 \Delta_{ee} + 0 + s_{12}^2 c_{12}^2 \Delta_{21}^2 \cos(2\Delta_{ee}) \\
&\quad -\frac{1}{6} \cos 2\theta_{12} \sin^2 2\theta_{12} \; \Delta_{21}^3 \; \sin(2\Delta_{ee}) + \mathcal{O}(\Delta_{21}^4) \\
&= 1 + \mathcal{O}(10^{-3}) \pm \mathcal{O}(10^{-5}) \quad \text{at OM}
\end{aligned}
$$

Mass Ordering effects !

where

$$\delta m_{ee}^2 \equiv c_{12}^2 \delta m_{31}^2 + s_{12}^2 \delta m_{32}^2$$

$$\Delta_{ee} = c_{12}^2 \Delta_{31} + s_{12}^2 \Delta_{32}$$

The exact expression is

$$c_{12}^2 \sin^2 \Delta_{31} + s_{12}^2 \sin^2 \Delta_{32} = \frac{1}{2}\{1 - \sqrt{(1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21})} \cos(2|\Delta_{ee}| \pm \phi)\}$$

where $\phi = \arctan(\cos 2\theta_{12} \tan \Delta_{21}) - \Delta_{21} \cos 2\theta_{12}$ (which only depends on Δ_{21}.)
The $\pm$ is the NO/IO.