Testing the homogeneity and isotropy of the Universe with recent observations

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In **collaboration** with
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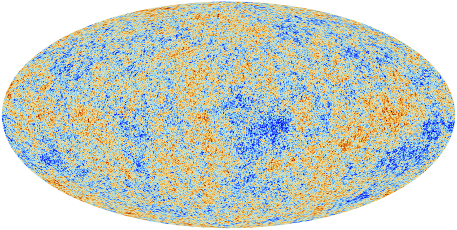
Motivation



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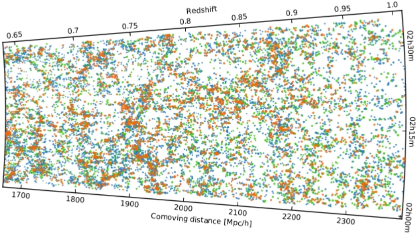


credit: VIPERS

The CMB and large scale structure show that the Universe is nearly homogeneous and isotropic on large scales.

Motivation

The metric which describes a perfectly homogeneous and isotropic Universe is the **Friedmann-Lemaitre-Robertson-Walker metric**

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + d\Omega^2\right)$$

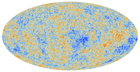
Inserting the FLRW metric into the Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{3T_{\mu\nu}}$$

one obtains the Friedmann equations:

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} - \frac{\Lambda}{3} &= \frac{8\pi G}{3}\rho \\ 3\frac{\ddot{a}}{a} + 4\pi G\rho - \Lambda &= 0 \end{aligned}$$

Motivation



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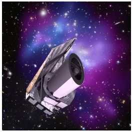
evolve in time → average over inhomogeneities



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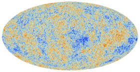
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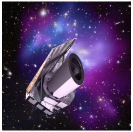
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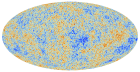

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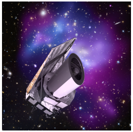

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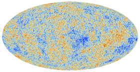

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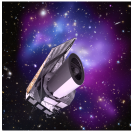

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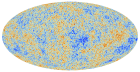

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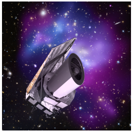

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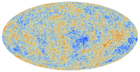
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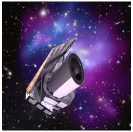
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- Interesting! It could explain the present apparent acceleration of the Universe without finely tuned Λ, extra scalar fields or modifications of gravity
- Difficult: defining univocally an averaging procedure, finding a metric for the coarse-grained spacetime
- Up to now it seems that the effect cannot account for observed Λ, but very interesting anyway for precision cosmology

This talk: testing the FLRW metric

Consistency test for FLRW

of *C. Clarkson, B. Bassett and T. Hui-Ching Lu (2008)*

Comoving distance in a general FLRW model with curvature:

$$D(z) = \frac{c}{H_0\sqrt{-\Omega_K}} \sin\left(\sqrt{-\Omega_K}\int_0^z dz' \frac{H_0}{H(z')}\right)$$

$\Omega_K =$ curvature parameter today.

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Ω_K = curvature parameter today.
Invert the previous equation $\Rightarrow$ expression for Ω_K independent of the specific cosmology:

$$\Omega_K(z) = \frac{[H(z)D_{,z}(z)]^2 - 1}{[H_0 D(z)]^2} = \begin{cases} const & \text{for FLRW} \\ \Omega_K(z) & \text{otherwise} \end{cases}$$

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By measuring independently $H(z)$, $D(z)$ and H_0 one can test whether Ω_K deviates from a constant.

Data sets

19 $H(z)$ data from passively evolving galaxies, *Moresco et al. (2012)*

- select most massive red elliptical galaxies, with no signature of star formation $\rightarrow t_{gal} \sim t_{Universe}$
- compute their ages $t_{gal} \Rightarrow dt_{gal}/dz$
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- compute SNIa distance modulus

$$\mu(z) = m(z) - M = 5\log_{10}(d_L(z)) + 25$$

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H_0 from HST and Wide Field Camera 3, *Riess et al. (2011)*:

$$H_0 = 73.8 \pm 2.4 \ \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

Computing $\Omega_K(z)$

- Collect measured H_0, and $d_L(z_i)$, $H(z_j)$ for different sets of $\{z_i\}$, $\{z_j\}$

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- discrete: $\begin{cases} \text{simple binning} \\ \text{binning + principal component decomposition} \end{cases}$

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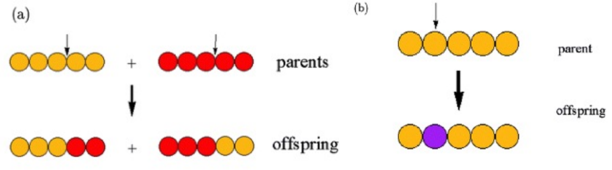
- We tried **4 different techniques**: 2 discrete and 2 continuous ones
- discrete: { simple binning / binning + principal component decomposition
- continuous: { genetic algorithms / Padé approximation

Best model-independent reconstruction: Genetic Algorithms

Based on principles of evolution through natural selection:



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Probability of reproductive success $\propto$ fitness of individual

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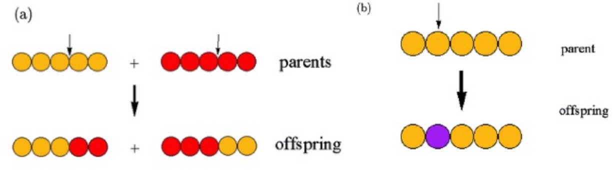

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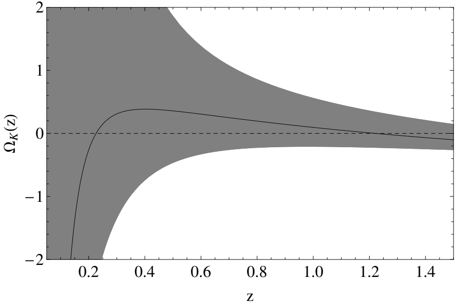
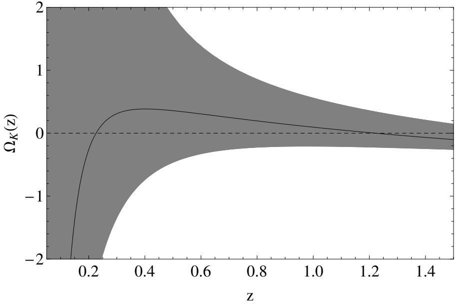
In our case:

- fitness of individual $\rightarrow$ χ^2 function
- initial population $\rightarrow$ set of functions, the grammar, and set of operators
- in each generation, crossover and mutation are applied
- process repeated several thousand times until e.g. max number of generations or desired convergence reached.



Error regions made with the path integral formalism of *Nesseris & Garcia-Bellido (2012)*

Genetic Algorithms



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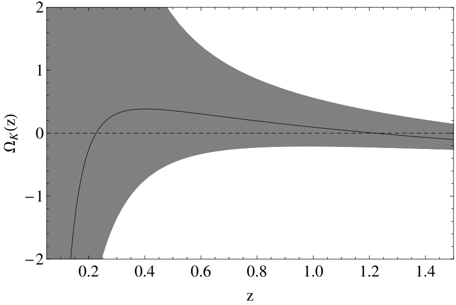

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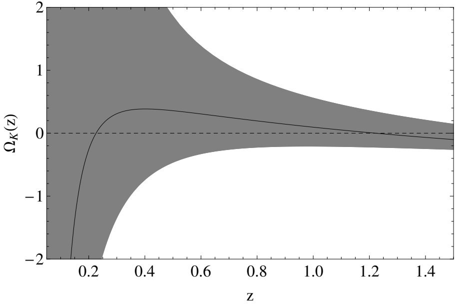

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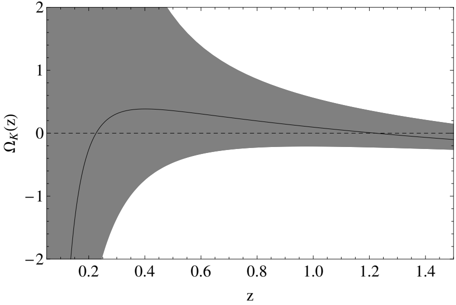

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- reconstruction consistent with $\Omega_K = 0$

Error regions computation in GA

Analytical method devised in *Nesseris & Garcia-Bellido (2012) based on* **path integral formalism**.

To calculate the 1σ error δf_i around the best-fit $f_{bf}(x)$ at a point x_i:

$$CI(x_i, \delta f_i) \equiv \int_{f_{bf}(x_i)-\delta f_i}^{f_{bf}(x_i)+\delta f_i} df_i \frac{1}{(2\pi)^{1/2}\sigma_i} \exp\left(-\frac{1}{2}\left(\frac{y_i - f_i}{\sigma_i}\right)^2\right) = \mathrm{erf}\left(1/\sqrt{2}\right)$$

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We want a smooth function, so we assume a shape for df:

$$df(x) = a + bx + cx^2$$

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and minimise simultaneously

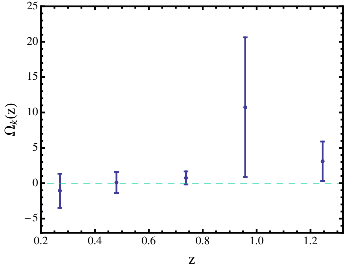
$$\chi^2_{CI}(\delta f_i) = \sum_{i=1}^{N}\left(CI(x_i, \delta f_i) - \mathrm{erf}\left(1/\sqrt{2}\right)\right)^2$$

and

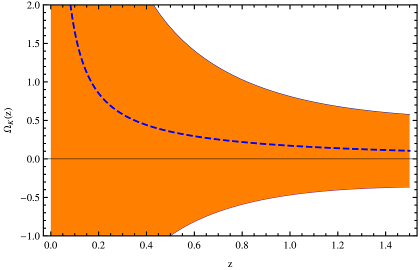
$$\chi^2(f_{bf} + df_i) \equiv \sum_{i=1}^{N}\left(\frac{y_i - (f_{bf} + \delta f_i)}{\sigma_i}\right)^2 .$$

Other model-independent methods

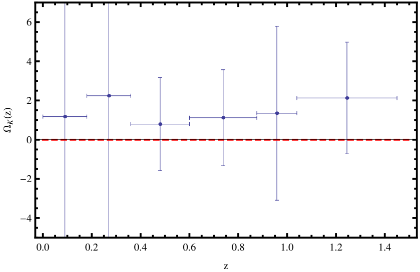
Simple binning



Padé approximation



Principal component analysis



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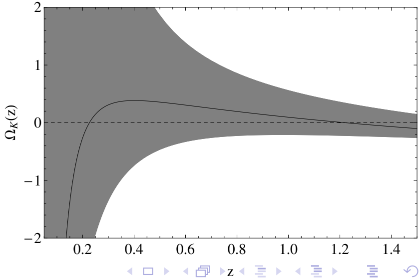

Forecasts for future experiments

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Medium-size mission of the ESA Cosmic Vision programme, launch planned for 2020. The spectroscopic survey will measure ~ 50 million galaxies with slitless spectroscopy, over $15,000$ square deg.

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Observed power spectrum:

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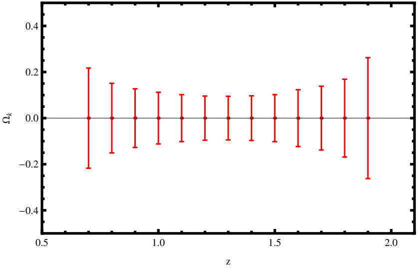
- $0.65 < z < 2.05$, $\Delta z = 0.1$
- scale-independent bias approximation, OK for large scales, as in *Orsi et al. (2011)*
- number densities as in *EM, L. Guzzo et al (2012)*, from a sophisticated simulation
- WMAP-7 ΛCDM fiducial model

Forecasts for future experiments

Euclid spectroscopy alone



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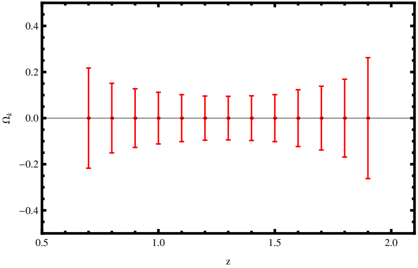

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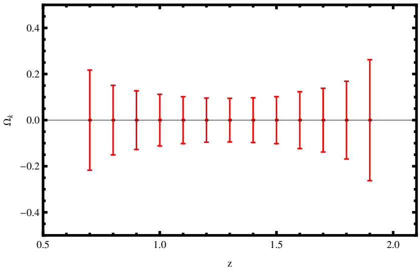

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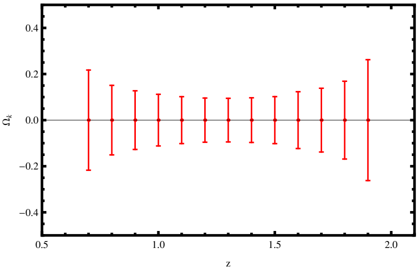

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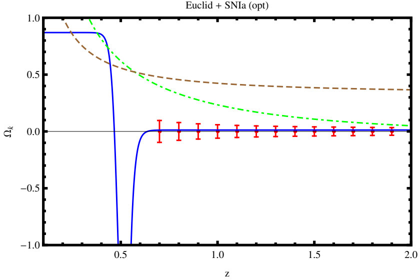
Euclid spectroscopy & 1000 SNIa



LTB, Tardis model, timescape.

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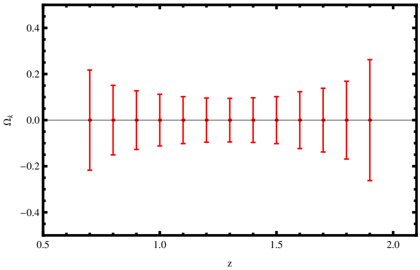

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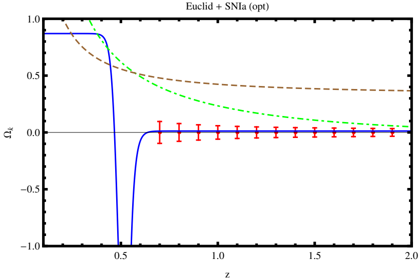

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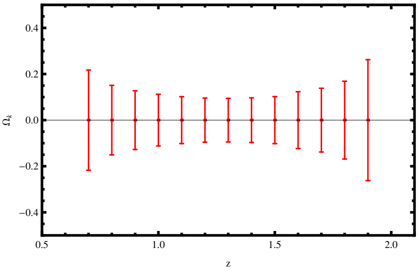
- errors improve by a factor of ~ 2

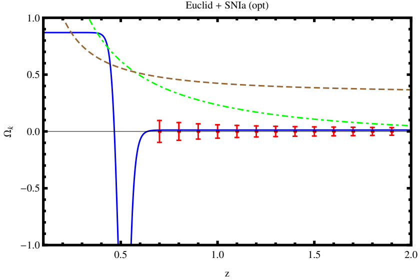
Forecasts for future experiments

Euclid spectroscopy alone



- errors are smallest when $1.1 < z < 1.5$
- at lower and higher z constraints become worse by up to a factor of 2.5

Euclid spectroscopy & 1000 SNIa



LTB, Tardis model, timescape.

- errors improve by a factor of ~ 2
- best constrained area: $z > 1.5$

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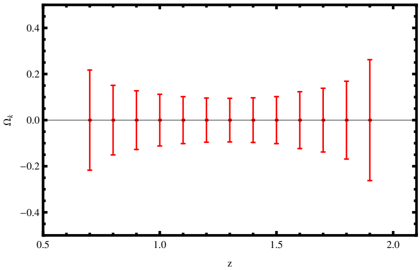

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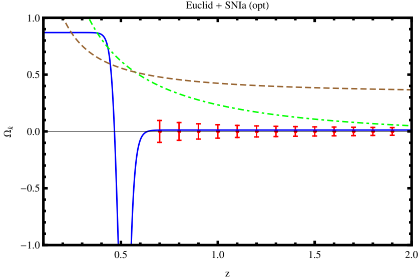

LTB, Tardis model, timescape.

- errors improve by a factor of ~ 2
- best constrained area: $z > 1.5$
- LTB cannot be distinguished, Tardis model and timescape may (but need to include error on $H(z)$, SNIa systematics etc...)

Why is Ω_K badly constrained at small z?

Model-independent proof

Expand $\Omega_K(z)$ for small z:

$$\tilde{\Omega}_K = \Omega_k + \frac{1 - \frac{H_{1,0}^2}{H_0^2}}{z^2} + \frac{\frac{2H_0 H'(0) - H_{1,0} H_1'(0)}{H_0^2} - \frac{H_1'(0)}{H_{1,0}}}{z} + \ldots$$

$' = d/dz$
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$\Rightarrow$ **divergence as z^2 unless $H_{1,0} = H_0$**

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- Padé and GA give $\sigma \sim 0.5$ for $z \leq 1$
- Future data from Euclid improve considerably the errors: Euclid only by a factor $\sim$ 10, Euclid + SNIa factor up to 40