EFT of Gravity and Dark Energy

Alessandro Codello

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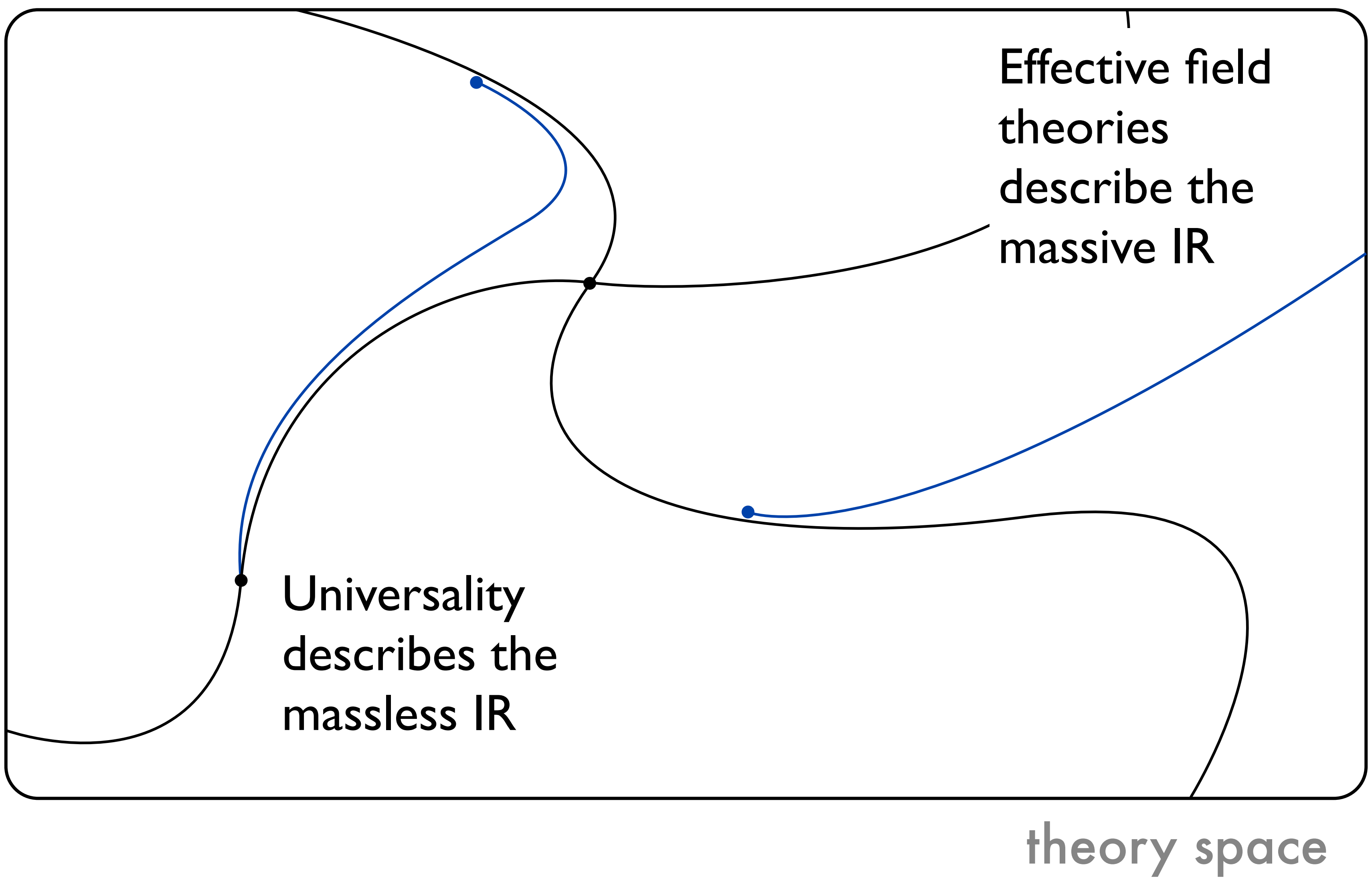
in collaboration with:
R K Jain
J Joergensen, F Sannino, O Svendsen
R Percacci, A Tonero, L Rachwal

Outline of the talk

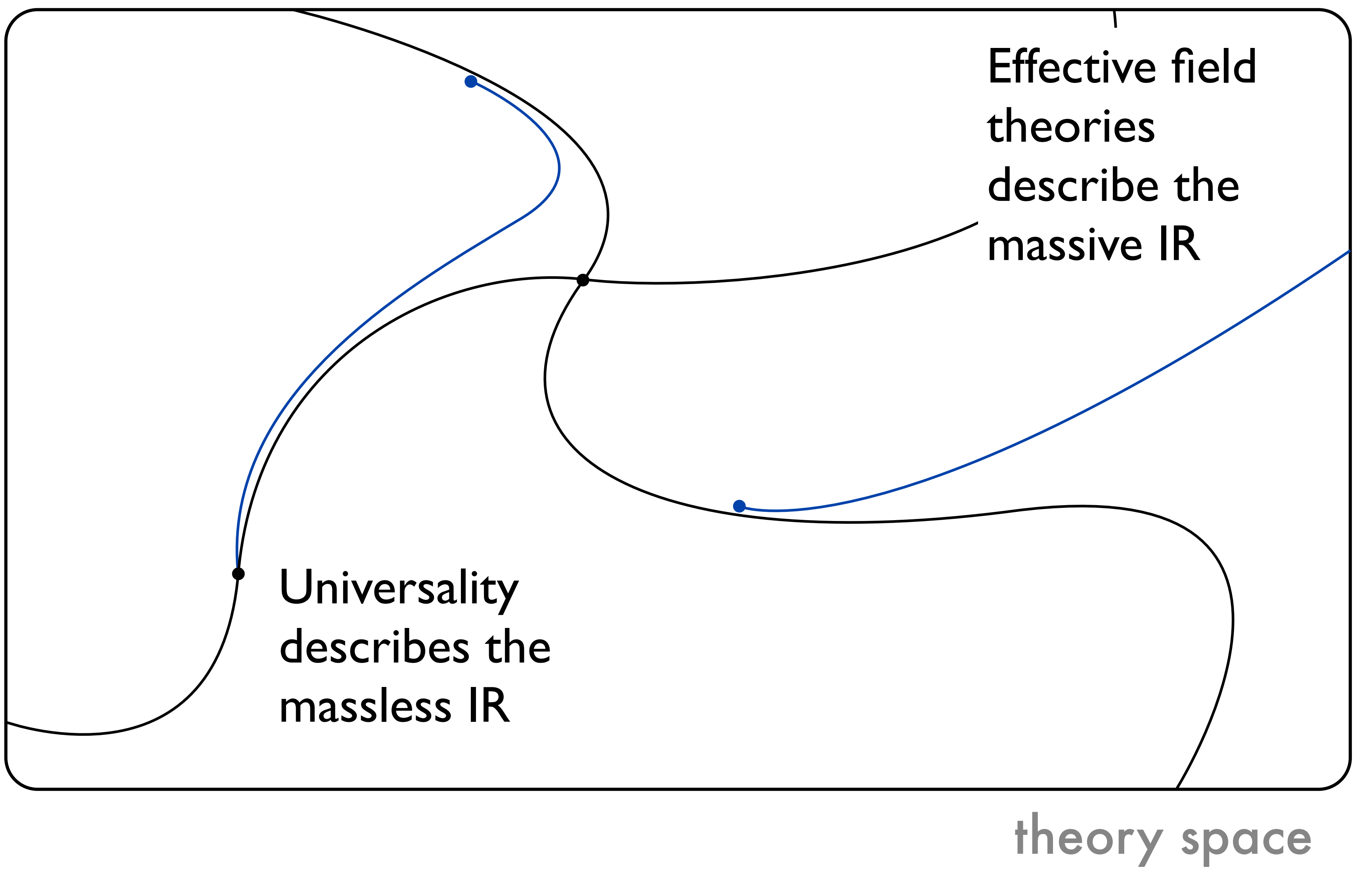
- Effectivity vs Universality
- Covariant EFT of gravity
- Phenomenological parameters
- Adding matter
- LO quantum corrections
- Effective Friedmann equations
- Dark energy
- Marginally deformed Starobinsky

Effectivity vs Universality



Two main reasons why mathematical modeling of nature actually works

Effectivity vs Universality



Massive IR lies in the broken phase (G to G/H)

Characteristic large scale M at which G is broken

EFT of Gravity

- The theory of small fluctuations of the metric

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + \sqrt{16\pi G} h_{\mu\nu} = g_{\mu\nu} + \frac{1}{M} h_{\mu\nu}$$

- Planck's scale is the characteristic scale of gravity

$$M \equiv \frac{1}{\sqrt{16\pi G}} = \frac{M_{Planck}}{\sqrt{16\pi}}$$

$$M_{Planck} = \frac{1}{\sqrt{G}} = 1.2 \times 10^{19}\,\mathrm{GeV}$$

- Classical theory (CT) is successful over many orders of magnitude

EFT of Gravity

$$S_{eff}[g] = M^2 \left[I_1[g] + \frac{1}{M^2} I_2[g] + \frac{1}{M^4} I_3[g] + ... \right]$$

$$I_1[g] = \int d^4x \sqrt{g} \left[M^2 c_0 - c_1 R \right]$$

$$I_2[g] = \int d^4x \sqrt{g} \left[c_{2,1} R^2 + c_{2,2} \mathrm{Ric}^2 + c_{2,3} \mathrm{Riem}^2 \right]$$

$$I_3[g] = \int d^4x \sqrt{g} \left[c_{3,1} R \Box R + c_{3,2} R_{\mu\nu} \Box R^{\mu\nu} + c_{3,3} R^3 + ... \right]$$

Derivative expansion of the bare (UV) action

Covariant EFT of Gravity

$$e^{-\Gamma[g]} = \int_{1PI} \mathcal{D}h_{\mu\nu}\, e^{-S_{eff}[g+\frac{1}{M}h]}$$

$$= \int_{1PI} \mathcal{D}h_{\mu\nu}\, e^{-M^2\left\{I_1[g+\frac{1}{M}h]+\frac{1}{M^2}I_2[g+\frac{1}{M}h]+\ldots\right\}}$$

EFT: saddle point expansion in $\frac{1}{M^2}$

Covariant EFT of Gravity

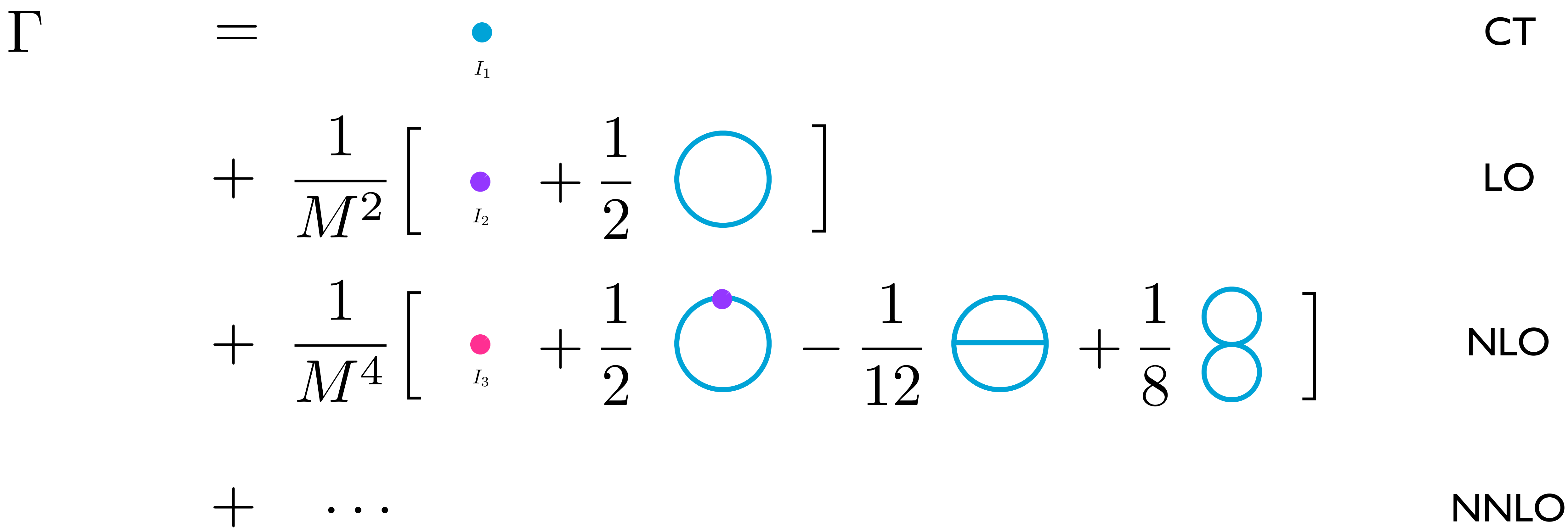
$$\Gamma[g] \;=\; I_1[g]$$ CT

$$+\frac{1}{M^2}\left\{I_2[g]+\frac{1}{2}\mathrm{Tr}\log I_1^{(2)}[g]\right\}$$ LO

$$+\frac{1}{M^4}\left\{I_3[g]+\frac{1}{2}\mathrm{Tr}\left[\left(I_1^{(2)}[g]\right)^{-1}I_2^{(2)}[g]\right]+\text{2-loops with }I_1[g]\right\}$$ NLO

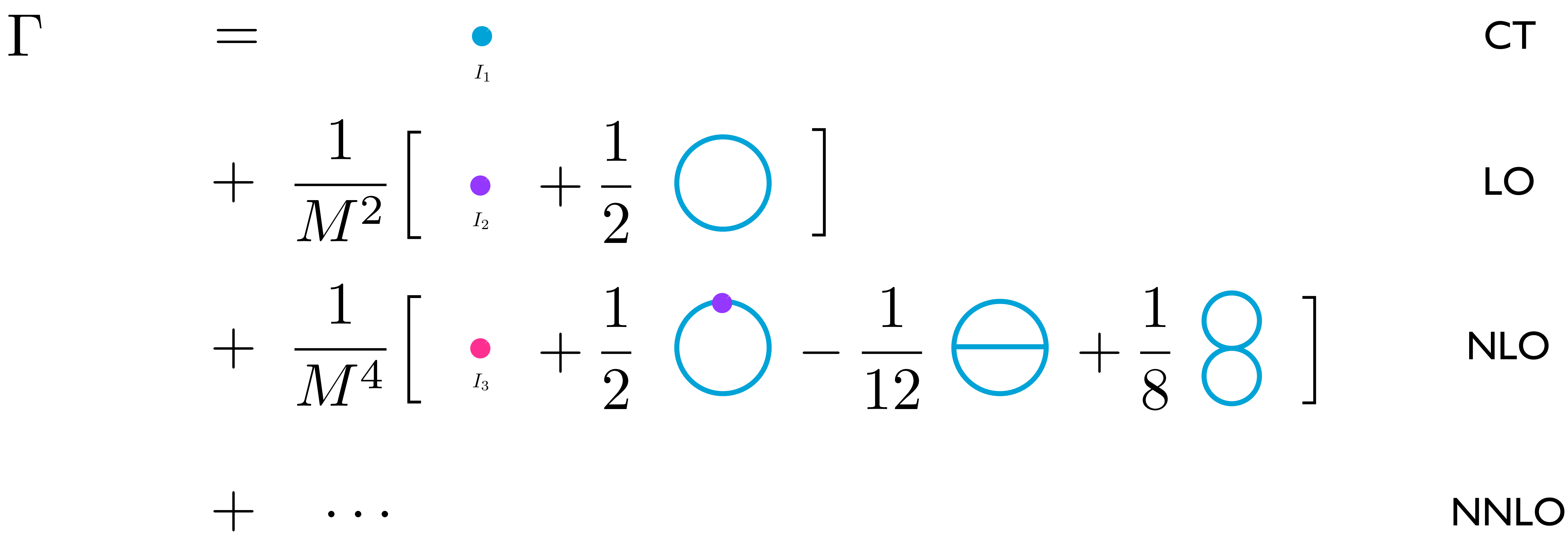
$$+\ldots$$ NNLO

Covariant EFT of Gravity



Covariant EFT of Gravity

The EFT recipe in three lines



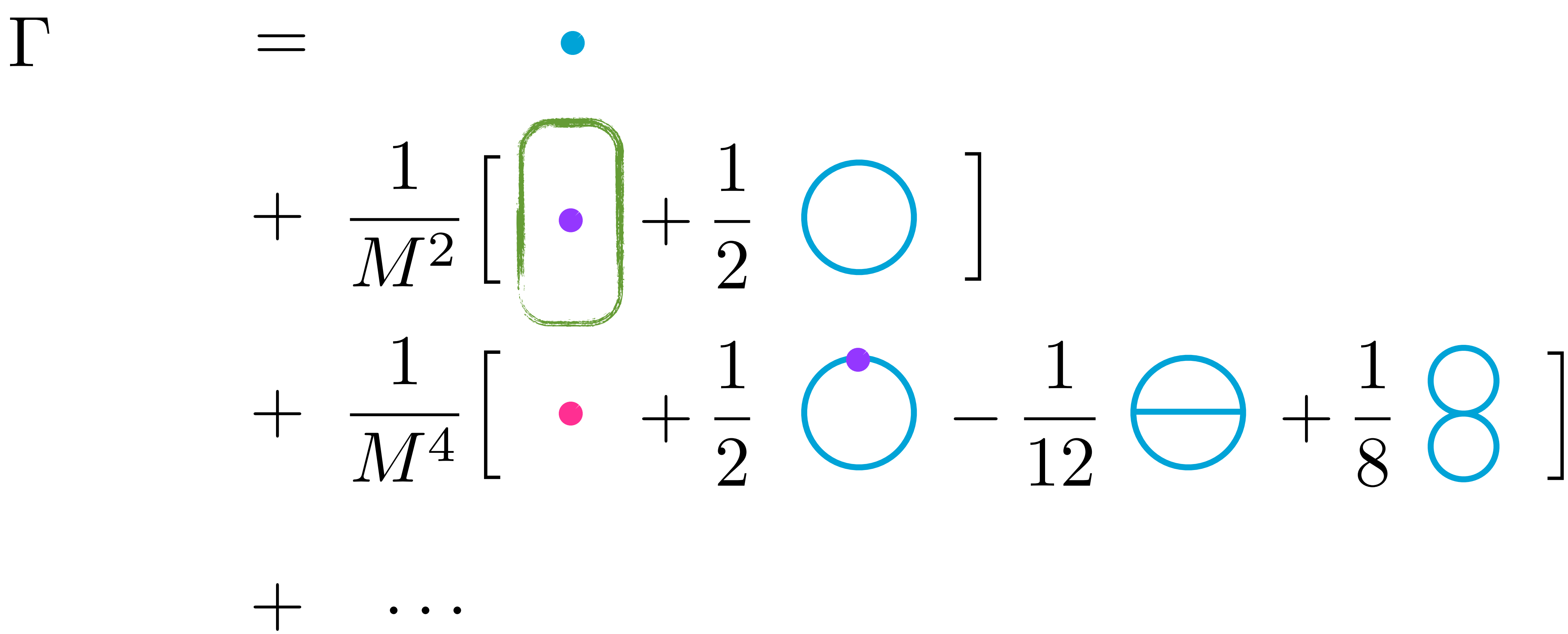
1) the general lagrangian of order E^2 is to be used both at tree level and in loop diagrams

2) the general lagrangian of order $E^{n\geq 4}$ is to be used at tree level and as an insertion in loop diagrams

3) the renormalization program is carried out order by order

Covariant EFT of Gravity

What do we already know?



CT

LO

NLO

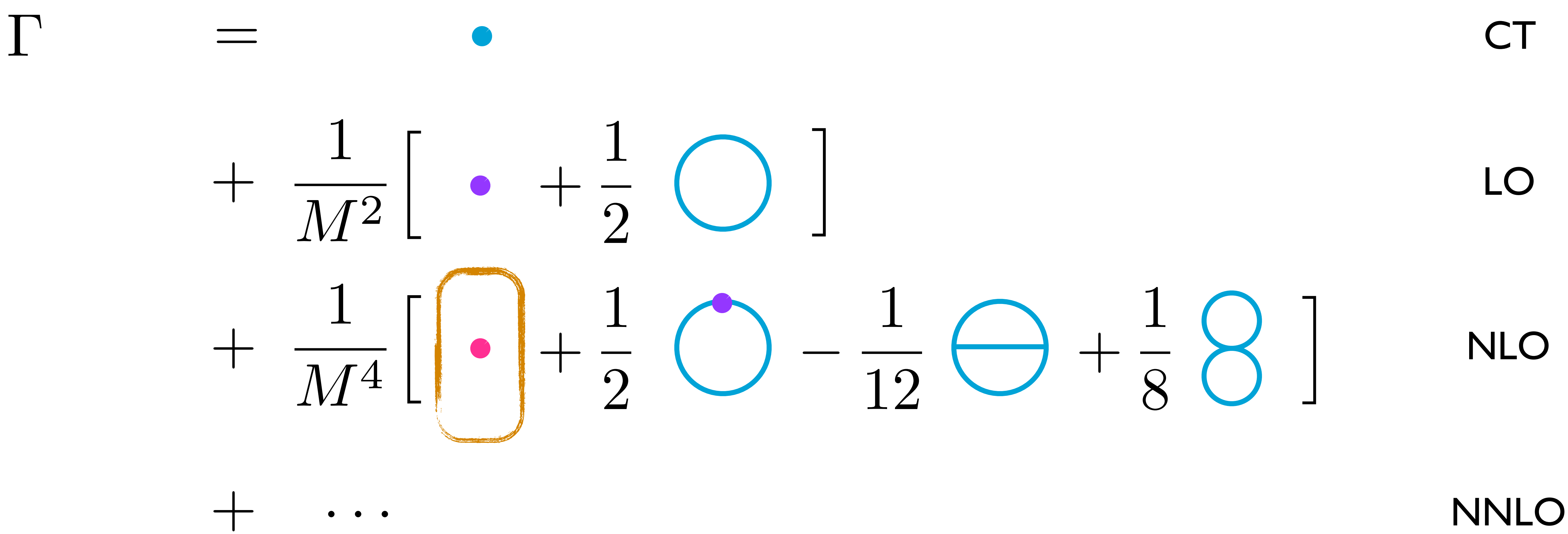
NNLO

UV divergencies and renormalization

G. ’t Hooft and M. J. G. Veltman, Annales Poincare Phys. Theor. A 20 (1974) 69
G. W. Gibbons, S. W. Hawking and M. J. Perry, Nucl. Phys. B 138 (1978) 141
S. M. Christensen and M. J. Duff, Nucl. Phys. B 170 (1980)

Covariant EFT of Gravity

What do we already know?

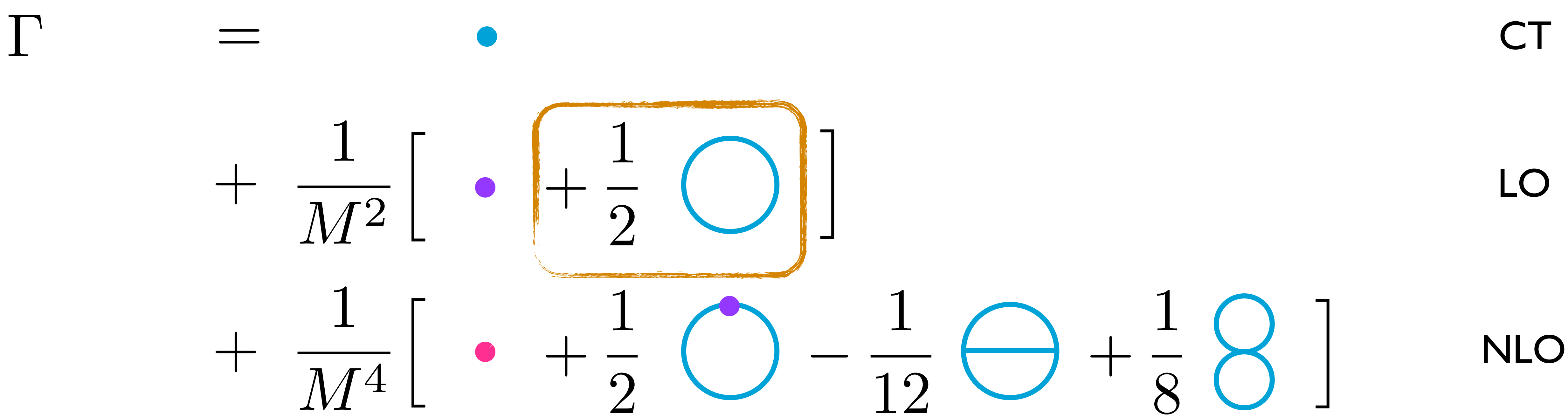


Two loops UV divergencies

M.H. Goroff and A. Sagnotti, Nucl.Phys.B266, 709 (1986)
A. E. M. van de Ven, Nucl. Phys. B378, 309 (1992)

Covariant EFT of Gravity

What do we already know?



$+ \cdots$ NNLO

Finite LO terms

Leading logs

J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)

A. C., J. Joergensen, F. Sannino and O. Svendsen, JHEP 1502 (2015) 050

Conformal anomaly

S. Deser, M. J. Duff and C. J. Isham, Nucl. Phys. B 111, 45 (1976)

R.J. Riegert, Phys. Lett. B 134 (1984) 56

Four graviton vertex in Minkowski space

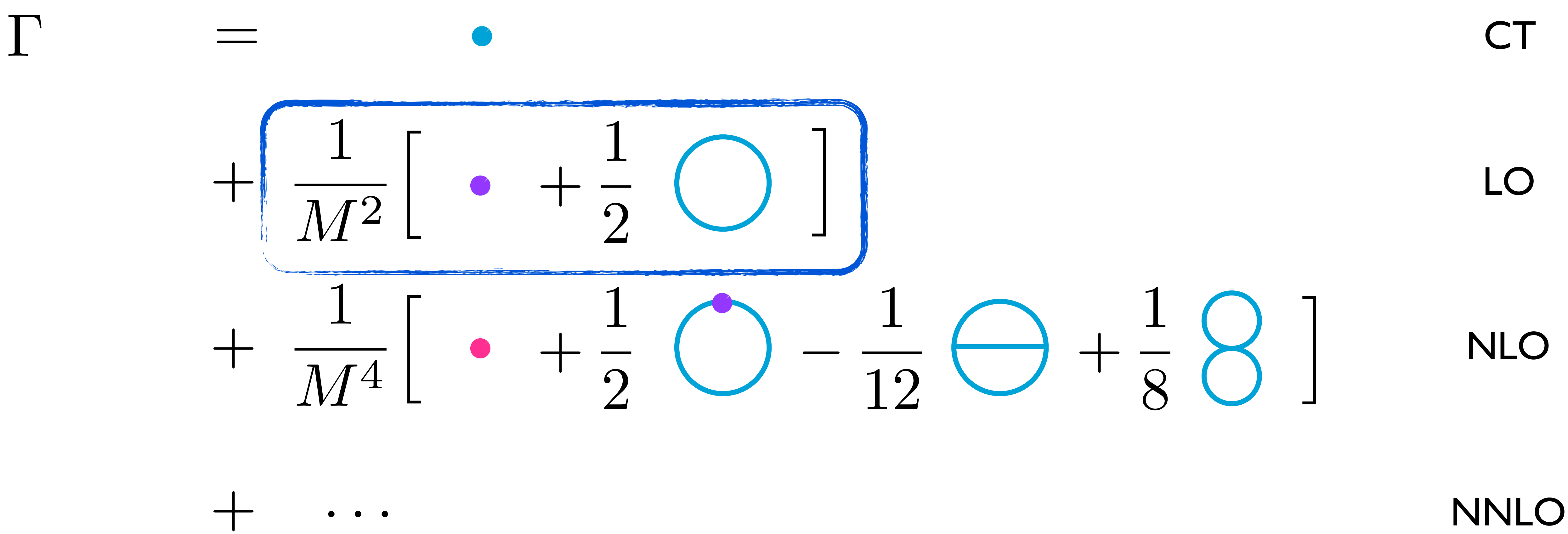
D. C. Dunbar and P. S. Norridge, Nucl. Phys. B 433, 181 (1995)

Curvature square terms

A. C. and R. K. Jain, in preparation

Covariant EFT of Gravity

LOQG: the only QG we will ever observe!



Even if we have a fundamental theory its is generally difficult to compute phenomenological parameters...

Phenomenological parameters

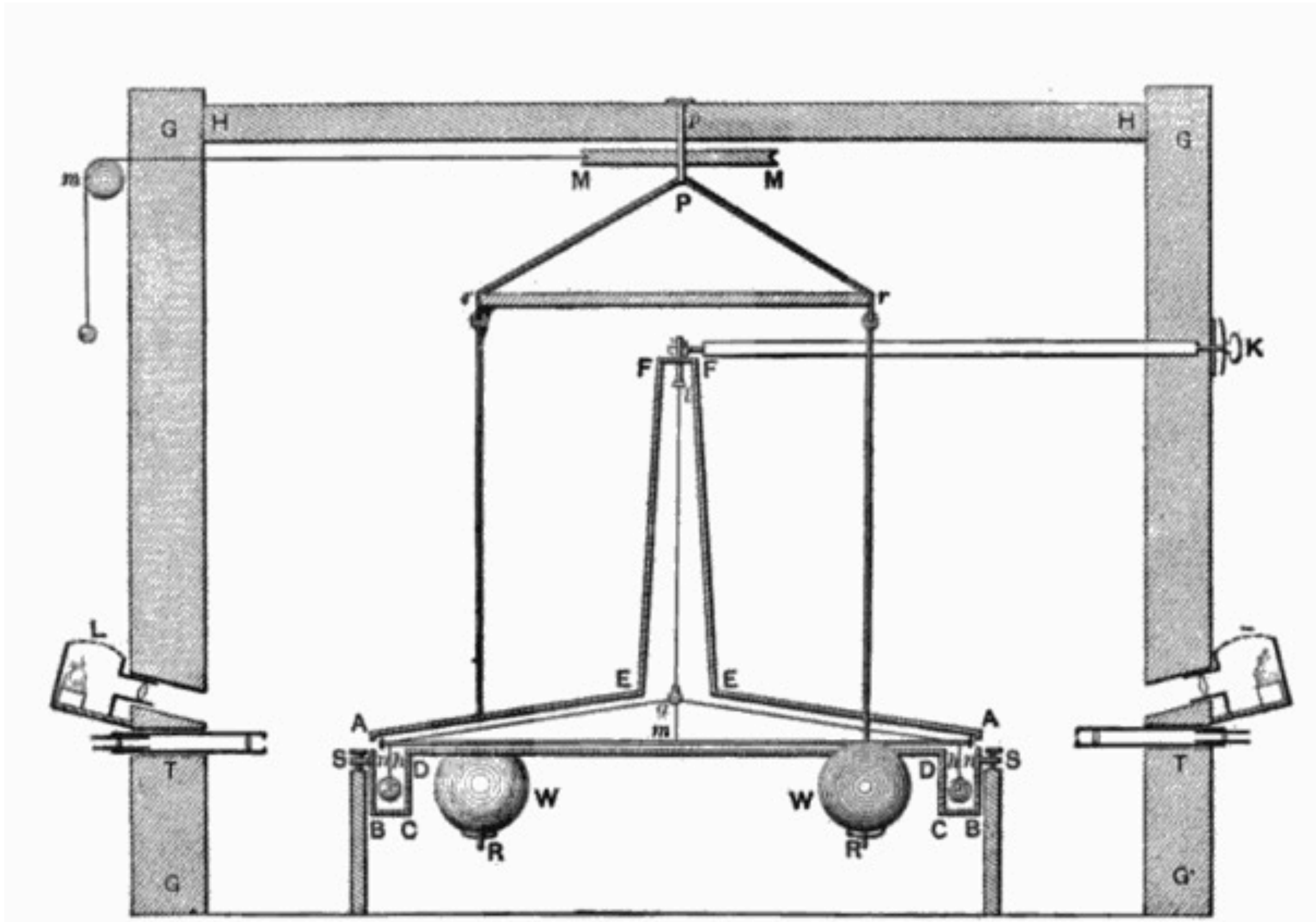
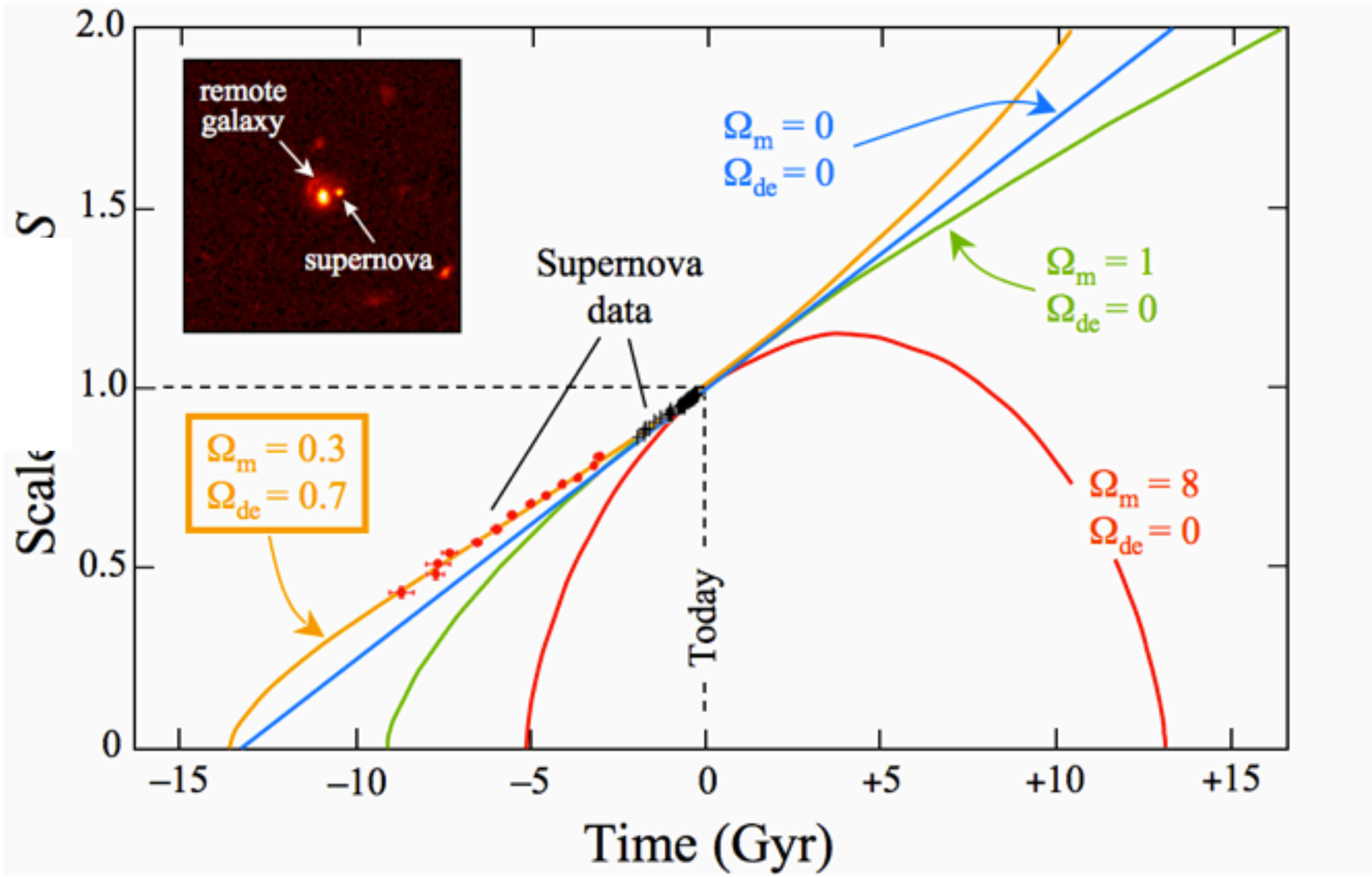


Fig. 1

$$G = 6.67428 \times 10^{-11} \mathrm{m}^3 \mathrm{kg}^{-1} \mathrm{s}^{-2}$$

Cavendish 1797 (1% off best value!)

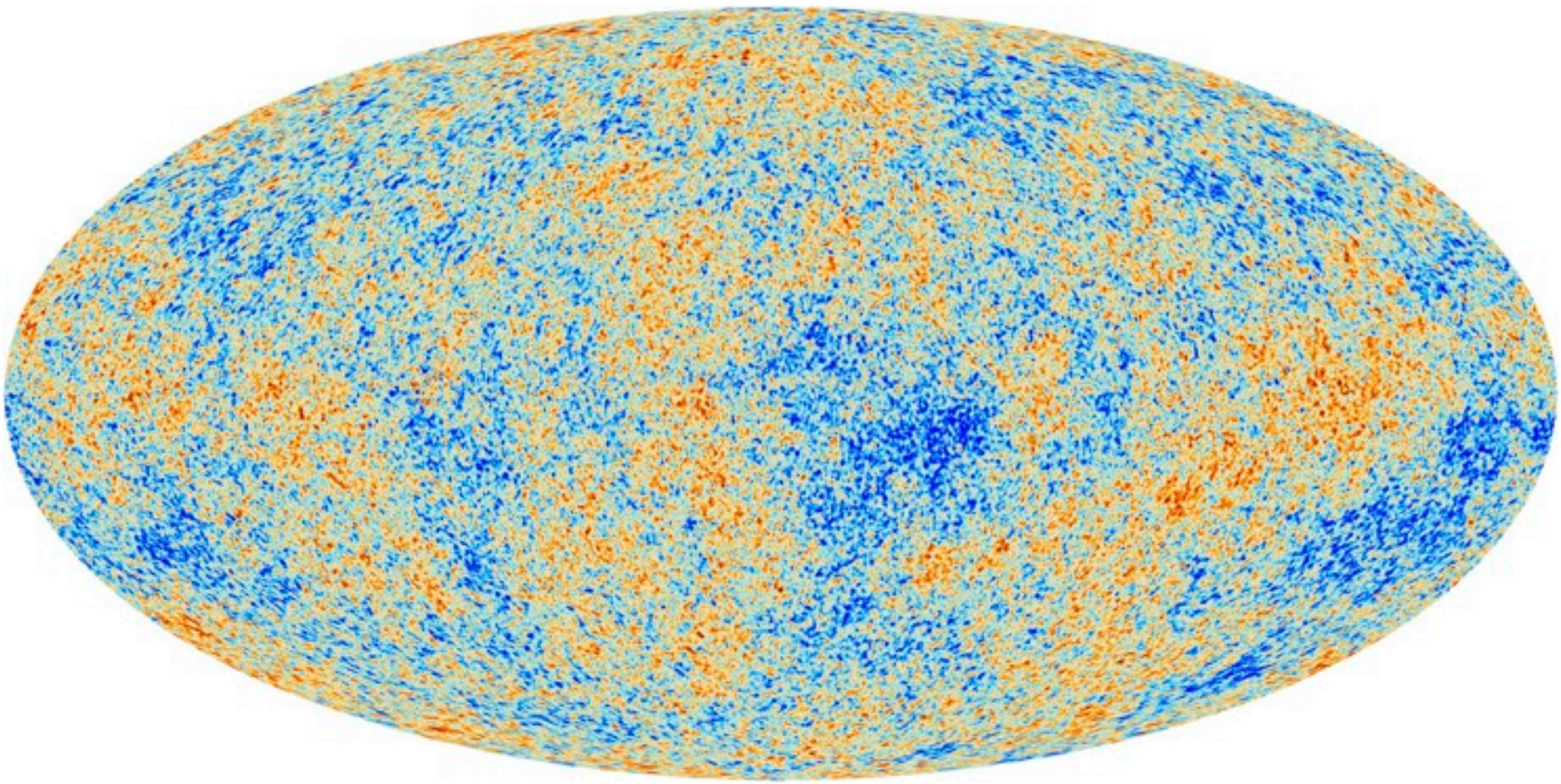
Phenomenological parameters



$$\Lambda = 10^{-47}\,\mathrm{GeV}^4$$

Supernova Cosmology Project

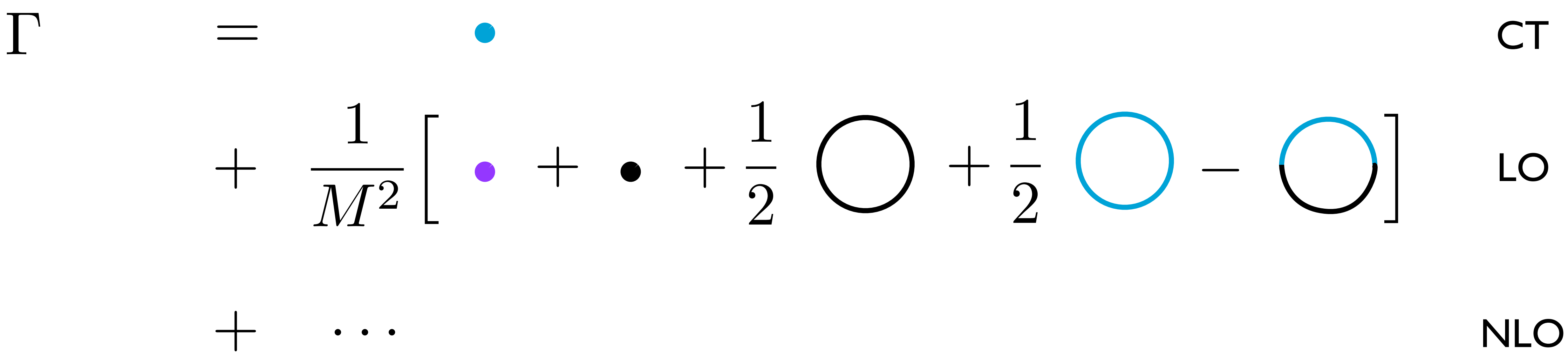
Phenomenological parameters



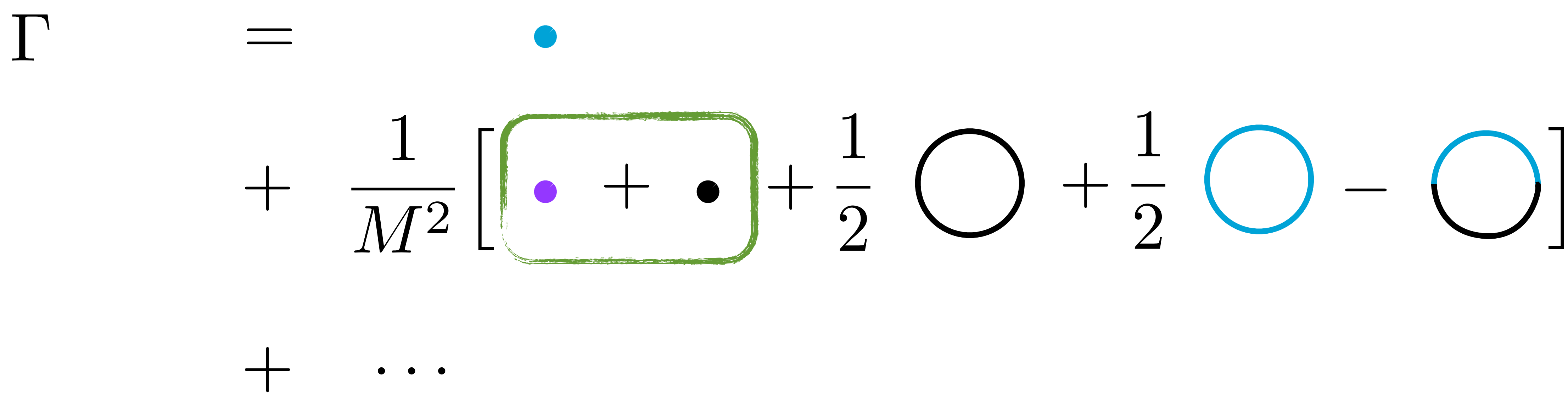
$\xi \equiv c_{R^2}$ $\quad\quad$ $\xi(k_*) \sim 10^9$ $\quad\quad$ $k_* \equiv 0.05\,\mathrm{Mpc}^{-1} \sim 10^{-40}\mathrm{GeV}$

Planck mission

Adding matter



Adding matter



CT

LO

NLO

UV divergencies and renormalization with matter

Scalars

A. O. Barvinsky, A. Y. .Kamenshchik and I. P. Karmazin, Phys. Rev. D 48 (1993) 3677

...

Gauge

S. P. Robinson and F. Wilczek, Phys. Rev. Lett. 96, 231601 (2006)

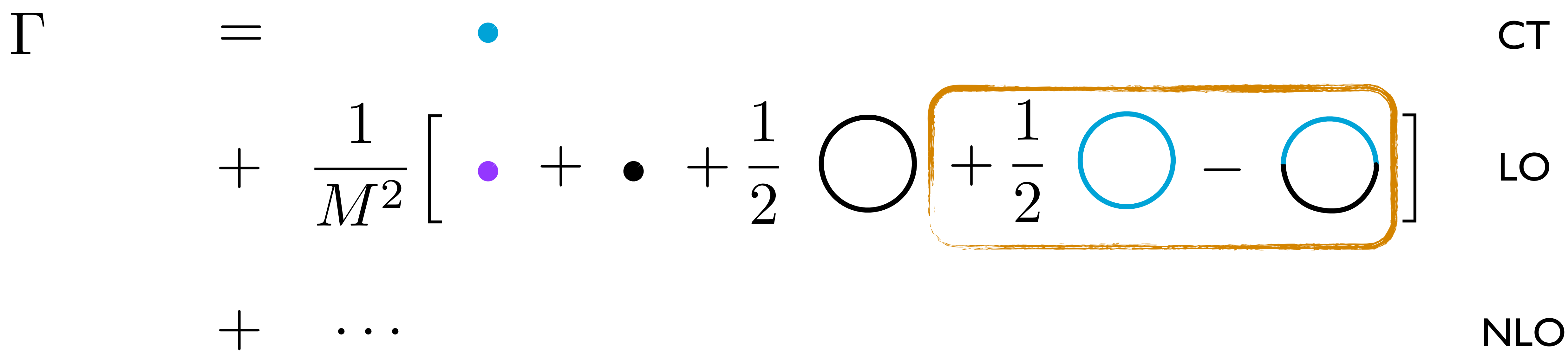
...

Yukawa

A. Rodigast and T. Schuster, Phys. Rev. Lett. 104, 081301 (2010)

...

Adding matter



Finite LO terms with matter

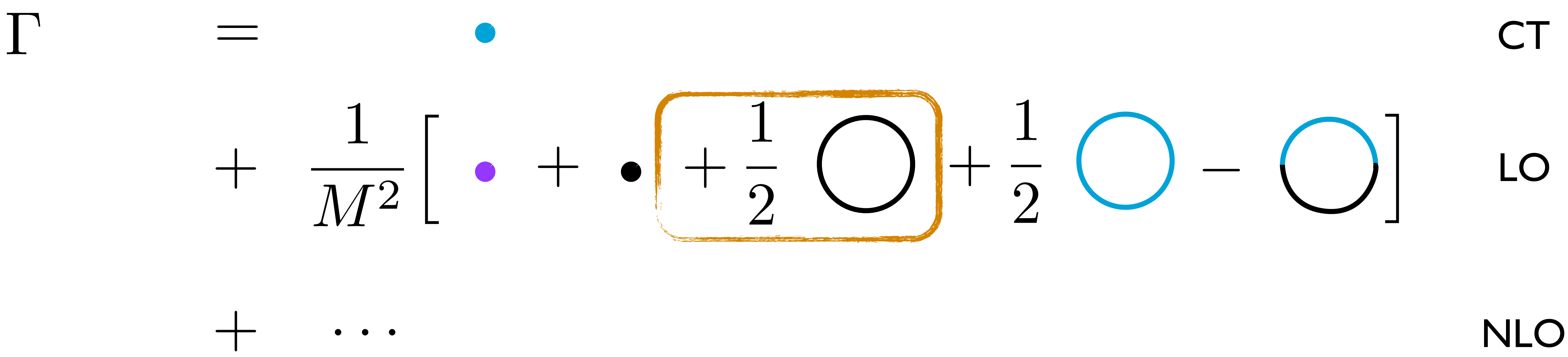
Flat space corrections to Newton's potential

J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)
N.E.J. Bjerrum-Bohr, J.F. Donoghue and B.R. Holstein (2003b) Phys. Rev. D 67
I.B. Khriplovich, G.G. Kirilin (2004) J. Exp. Theor. Phys. 98, 1063-1072

Covariant leading logs

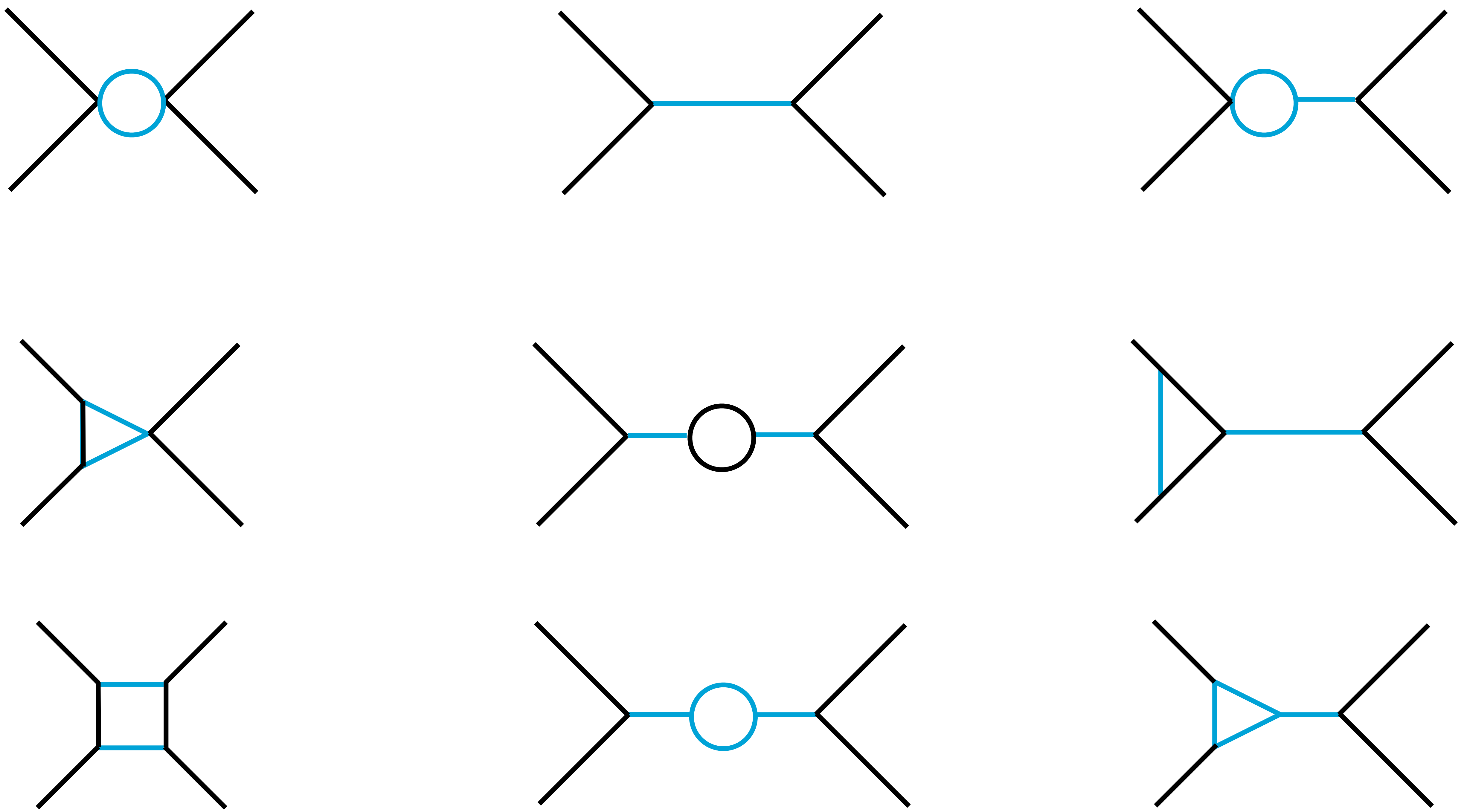
A C, R Percacci, L Rachwal and A Tonero in preparation

Adding matter

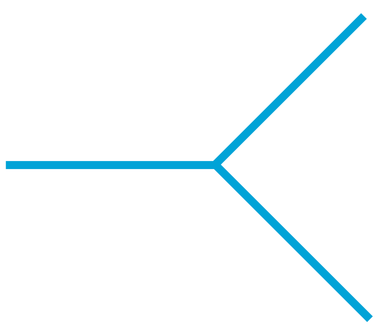


Matter induced effective action

Corrections to Newton's interaction



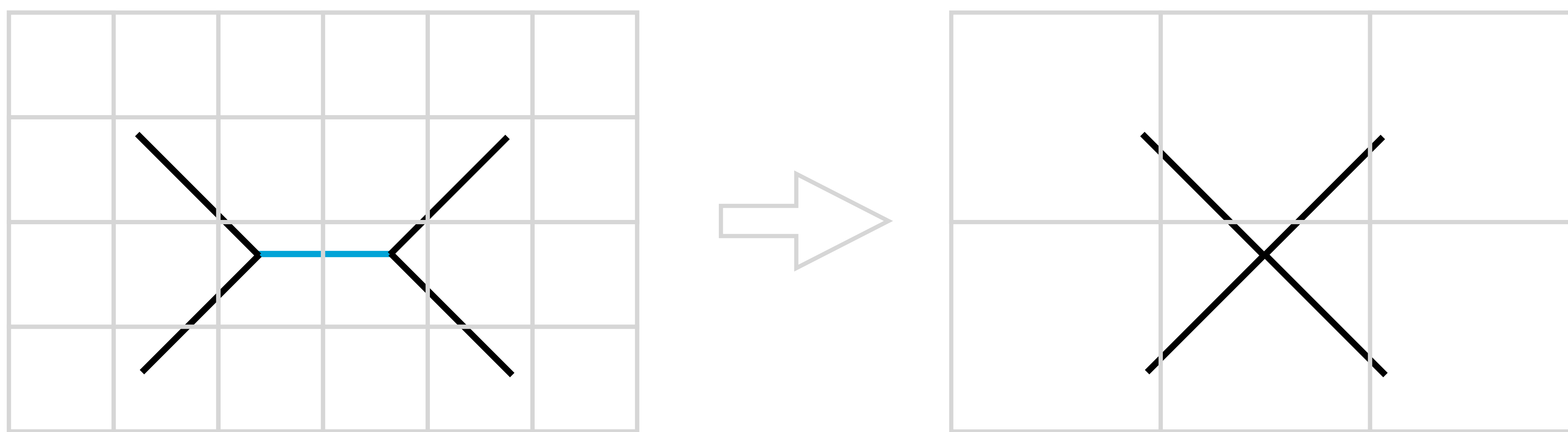
Corrections to Newton's interaction



$$
\begin{aligned}
= \; & \frac{i\kappa}{2}\Bigg(P_{\alpha\beta,\gamma\delta}\Big[k^\mu k^\nu + (k-q)^\mu (k-q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu}q^2\Big] \\
& +2q_\lambda q_\sigma\Big[I^{\lambda\sigma,}{}_{\alpha\beta}I^{\mu\nu,}{}_{\gamma\delta} + I^{\lambda\sigma,}{}_{\gamma\delta}I^{\mu\nu,}{}_{\alpha\beta} - I^{\lambda\mu,}{}_{\alpha\beta}I^{\sigma\nu,}{}_{\gamma\delta} - I^{\sigma\nu,}{}_{\alpha\beta}I^{\lambda\mu,}{}_{\gamma\delta}\Big] \\
& +\Big[q_\lambda q^\mu\Big(\eta_{\alpha\beta}I^{\lambda\nu,}{}_{\gamma\delta} + \eta_{\gamma\delta}I^{\lambda\nu,}{}_{\alpha\beta}\Big) + q_\lambda q^\nu\Big(\eta_{\alpha\beta}I^{\lambda\mu,}{}_{\gamma\delta} + \eta_{\gamma\delta}I^{\lambda\mu,}{}_{\alpha\beta}\Big) \\
& -q^2\Big(\eta_{\alpha\beta}I^{\mu\nu,}{}_{\gamma\delta} + \eta_{\gamma\delta}I^{\mu\nu,}{}_{\alpha\beta}\Big) - \eta^{\mu\nu}q^\lambda q^\sigma\left(\eta_{\alpha\beta}I_{\gamma\delta,\lambda\sigma} + \eta_{\gamma\delta}I_{\alpha\beta,\lambda\sigma}\right)\Big] \\
& +\Big[2q^\lambda\Big(I^{\sigma\nu,}{}_{\alpha\beta}I_{\gamma\delta,\lambda\sigma}(k-q)^\mu + I^{\sigma\mu,}{}_{\alpha\beta}I_{\gamma\delta,\lambda\sigma}(k-q)^\nu \\
& -I^{\sigma\nu,}{}_{\gamma\delta}I_{\alpha\beta,\lambda\sigma}k^\mu - I^{\sigma\mu,}{}_{\gamma\delta}I_{\alpha\beta,\lambda\sigma}k^\nu\Big) \\
& +q^2\Big(I^{\sigma\mu,}{}_{\alpha\beta}I_{\gamma\delta,\sigma}{}^{\nu} + I_{\alpha\beta,\sigma}{}^{\nu}I^{\sigma\mu,}{}_{\alpha\delta}\Big) + \eta^{\mu\nu}q^\lambda q_\sigma\Big(I_{\alpha\beta,\lambda\rho}I^{\rho\sigma,}{}_{\gamma\delta} + I_{\gamma\delta,\lambda\rho}I^{\rho\sigma,}{}_{\alpha\beta}\Big)\Big] \\
& +\Big\{\left(k^2 + (k-q)^2\right)\Big(I^{\sigma\mu,}{}_{\alpha\beta}I_{\gamma\delta,\sigma}{}^{\nu} + I^{\sigma\nu,}{}_{\alpha\beta}I_{\gamma\delta,\sigma}{}^{\mu} - \frac{1}{2}\eta^{\mu\nu}P_{\alpha\beta,\gamma\delta}\Big) \\
& -\Big(k^2\eta_{\gamma\delta}I^{\mu\nu,}{}_{\alpha\beta} + (k-q)^2\eta_{\alpha\beta}I^{\mu\nu,}{}_{\gamma\delta}\Big)\Big\}\Bigg)
\end{aligned}
$$

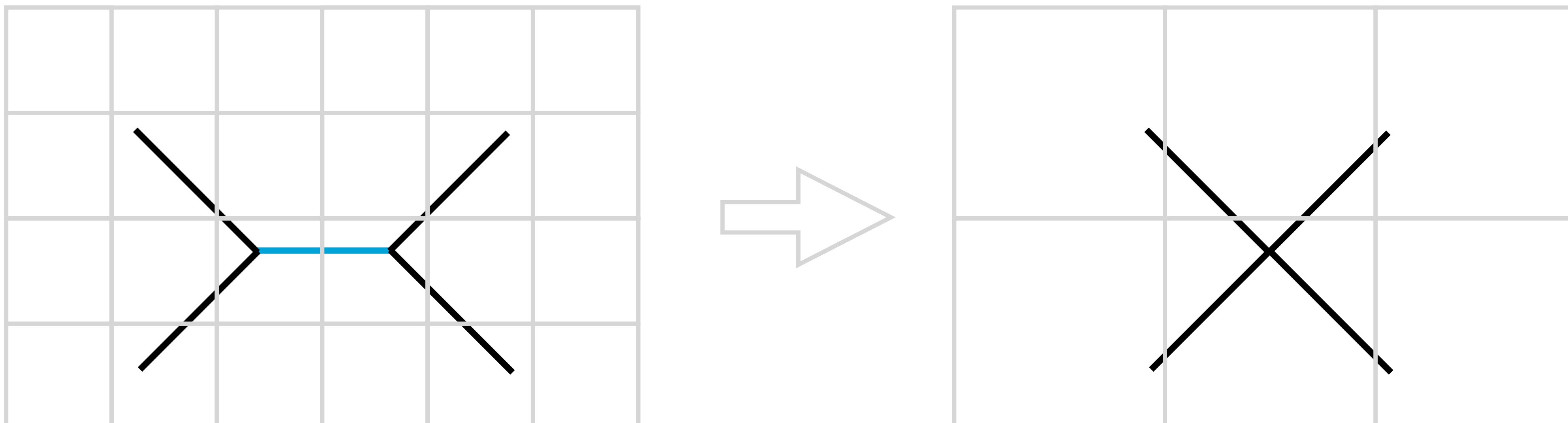
the truth behind Feynman diagrams...

Corrections to Newton's interaction



$$V = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{c^2 r} + b\frac{G\hbar}{c^3 r^2} + \cdots\right]$$

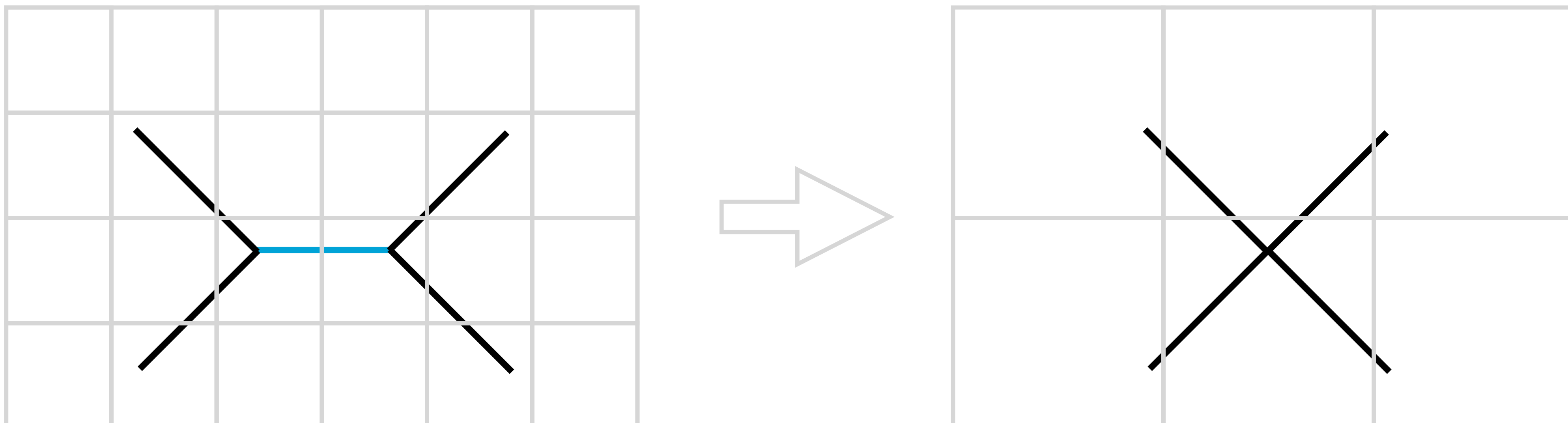
Corrections to Newton's interaction



$$V = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{c^2 r} + b\frac{G\hbar}{c^3 r^2} + \cdots\right]$$

$$[G] = \frac{\mathrm{m}^3}{\mathrm{Kg\ s}^2} \qquad [\hbar] = \frac{\mathrm{m}^2\ \mathrm{Kg}}{\mathrm{s}} \qquad [c] = \frac{\mathrm{m}}{\mathrm{s}}$$

Corrections to Newton's interaction

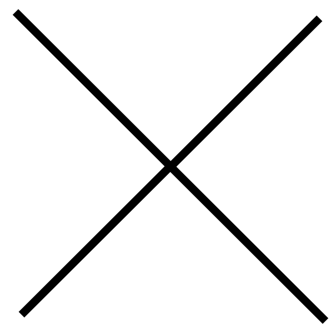


$$V = -\frac{GMm}{r}\left[1 + 3\frac{G(M+m)}{c^2 r} + \frac{41}{10\pi}\frac{G\hbar}{c^3 r^2} + \cdots\right]$$

Leading quantum corrections to Newton's potential

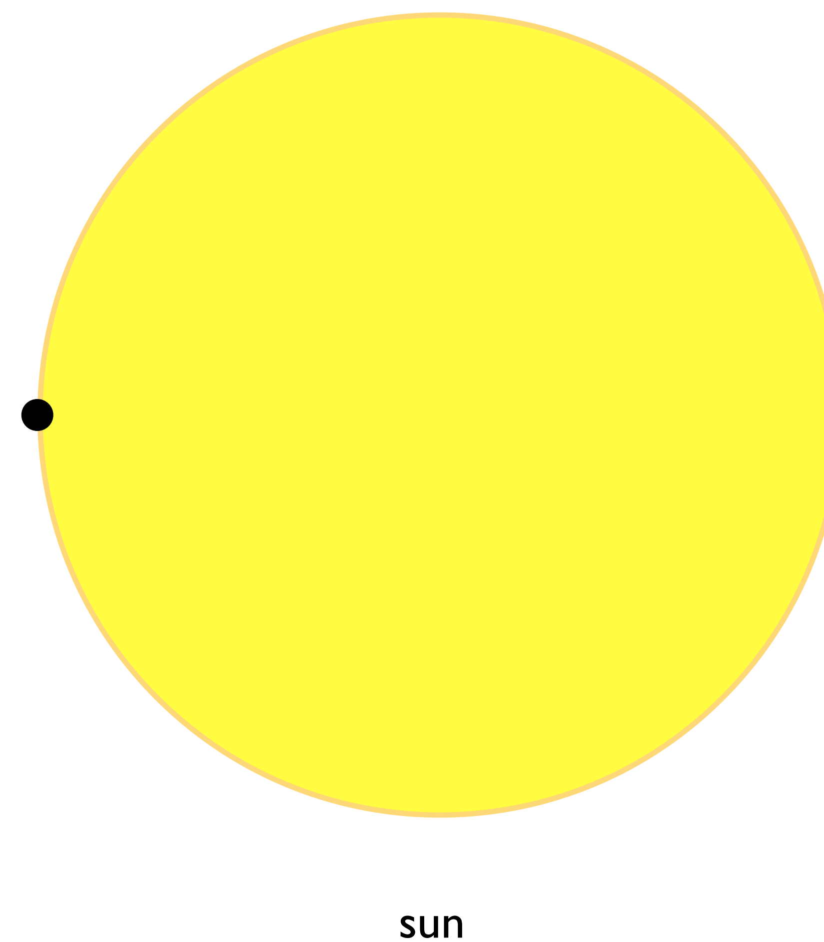
J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)

Corrections to Newton's interaction



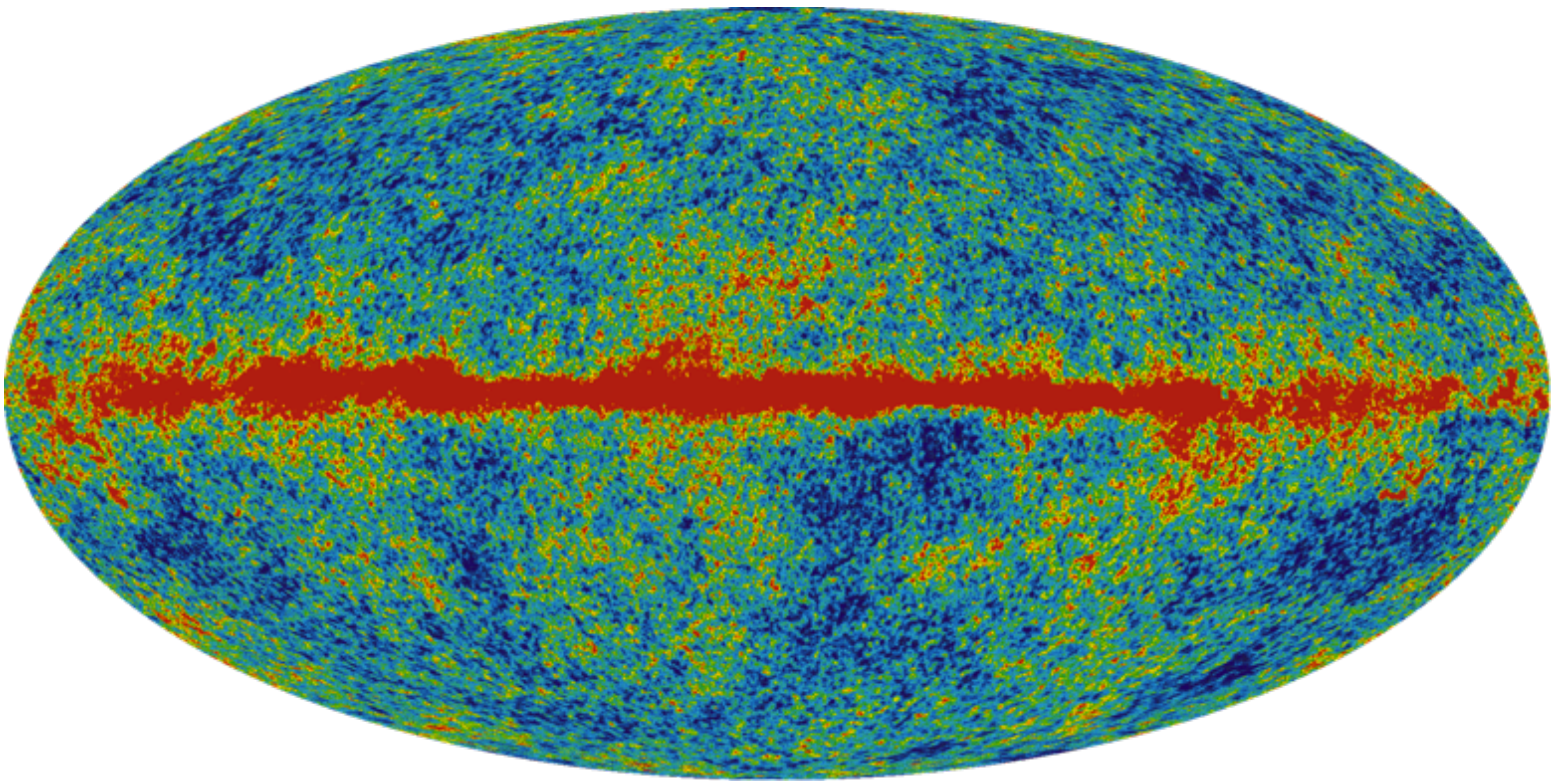
$$\frac{GM_\odot}{c^2 r_\odot} \sim 10^{-6}$$

$$\frac{G\hbar}{c^3 r_\odot^2} \sim 10^{-88}$$



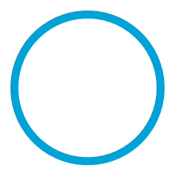
Leading quantum corrections to Newton's law are incredibly small!

Can we ever observe quantum gravity effects?



Look for physical situations where LO corrections are enhanced

Curvature expansion

• $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^{d/2}} \int d^dx \sqrt{g}\, \mathrm{tr}\, \mathcal{R}\, \gamma_i\!\left(\frac{-\Box}{m^2}\right) \mathcal{R} + \ldots$

The finite physical part of the effective action is covariantly encoded in the structure functions which can be computed using the non-local heat kernel expansion

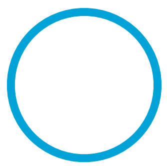
$$\gamma_i\left(\frac{X}{m^2}\right) \equiv \lim_{\Lambda_{UV}\to\infty} \int_{1/\Lambda_{UV}^2}^{\infty} \frac{ds}{s}\, s^{-d/2+2} \left[f_i(sX) - f_i(0)\right] e^{-sm^2}$$

Non-local heat kernel structure functions

Non-local heat kernel

A. O. Barvinsky and G. A. Vilkovisky, Nucl. Phys. B 282 (1987) 163
I. G. Avramidi, Lect. Notes Phys. M 64 (2000) 1
A. Codello and O. Zanusso, J. Math. Phys. 54 (2013) 013513

Curvature expansion

- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,\mathrm{tr}\Big[\mathbf{1}R_{\mu\nu}\gamma_{Ric}\Big(\frac{-\Box}{m^2}\Big)R^{\mu\nu} + \frac{\mathbf{1}}{120}R\gamma_R\Big(\frac{-\Box}{m^2}\Big)R$
  $-\frac{1}{6}R\gamma_{RU}\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{2}\mathbf{U}\gamma_U\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{12}\mathbf{\Omega}_{\mu\nu}\gamma_\Omega\Big(\frac{-\Box}{m^2}\Big)\mathbf{\Omega}^{\mu\nu}\Big]$

Explicit form for the structure functions

$$\gamma_{Ric}(u) = \frac{1}{40} + \frac{1}{12u} - \frac{1}{2}\int_0^1 d\xi\,\Big[\frac{1}{u} + \xi(1-\xi)\Big]^2 \log\big[1 + u\,\xi(1-\xi)\big]$$

$$\gamma_R(u) = -\frac{23}{960} - \frac{1}{96u} + \frac{1}{32}\int_0^1 d\xi\,\Big\{\frac{2}{u^2} + \frac{4}{u}\big[1+\xi(1-\xi)\big] - 1 + 2\xi(2-\xi)(1-\xi^2)\Big\}\log\big[1 + u\,\xi(1-\xi)\big]$$

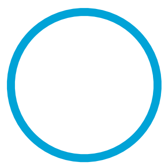
$$\gamma_{RU}(u) = \frac{1}{12} - \frac{1}{2}\int_0^1 d\xi\,\Big[\frac{1}{u} - \frac{1}{2} + \xi(1-\xi)\Big]\log\big[1 + u\,\xi(1-\xi)\big]$$

$$\gamma_U(u) = -\frac{1}{2}\int_0^1 d\xi\,\log\big[1 + u\,\xi(1-\xi)\big]$$

$$\gamma_\Omega(u) = \frac{1}{12} - \frac{1}{2}\int_0^1 d\xi\,\Big[\frac{1}{u} + \xi(1-\xi)\Big]\log\big[1 + u\,\xi(1-\xi)\big]$$

$$u \equiv \frac{-\Box}{m^2}$$

Curvature expansion

- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,\mathrm{tr}\Big[\mathbf{1}R_{\mu\nu}\gamma_{Ric}\Big(\frac{-\Box}{m^2}\Big)R^{\mu\nu} + \frac{\mathbf{1}}{120}R\gamma_R\Big(\frac{-\Box}{m^2}\Big)R - \frac{1}{6}R\gamma_{RU}\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{2}\mathbf{U}\gamma_U\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{12}\mathbf{\Omega}_{\mu\nu}\gamma_\Omega\Big(\frac{-\Box}{m^2}\Big)\mathbf{\Omega}^{\mu\nu}\Big]$

Large energy expansion $u \gg 1$

$$\gamma_{Ric}(u) = -\frac{u}{840} + \frac{u^2}{15120} - \frac{u^3}{166320} + O(u^4)$$

$$\gamma_R(u) = -\frac{u}{336} + \frac{11u^2}{30240} - \frac{19u^3}{332640} + O(u^4)$$

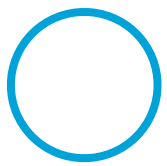
$$\gamma_{RU}(u) = \frac{u}{30} - \frac{u^2}{280} + \frac{u^3}{1890} + O(u^4)$$

$$\gamma_U(u) = -\frac{u}{12} + \frac{u^2}{120} - \frac{u^3}{840} + O(u^4)$$

$$\gamma_\Omega(u) = -\frac{u}{120} + \frac{u^2}{1680} - \frac{u^3}{15120} + O(u^4)$$

$$u \equiv \frac{-\Box}{m^2}$$

Curvature expansion

- $+\frac{1}{2}$  $= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,\mathrm{tr}\Big[\mathbf{1}R_{\mu\nu}\gamma_{Ric}\Big(\frac{-\Box}{m^2}\Big)R^{\mu\nu} + \frac{\mathbf{1}}{120}R\gamma_R\Big(\frac{-\Box}{m^2}\Big)R$
$-\frac{1}{6}R\gamma_{RU}\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{2}\mathbf{U}\gamma_U\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{12}\mathbf{\Omega}_{\mu\nu}\gamma_\Omega\Big(\frac{-\Box}{m^2}\Big)\mathbf{\Omega}^{\mu\nu}\Big]$

Low energy expansion $u \ll 1$

$$\gamma_{Ric}(u) = \frac{23}{450} - \frac{1}{60}\log u + \frac{5}{18u} - \frac{\log u}{6u} + \frac{1}{4u^2} - \frac{\log u}{2u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_R(u) = \frac{1}{1800} - \frac{1}{120}\log u - \frac{2}{9u} + \frac{\log u}{12u} + \frac{1}{8u^2} + \frac{\log u}{4u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_{RU}(u) = -\frac{5}{18} + \frac{1}{6}\log u + \frac{1}{u} - \frac{1}{2u^2} - \frac{\log u}{u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_U(u) = 1 - \frac{1}{2}\log u - \frac{1}{u} - \frac{\log u}{u} - \frac{1}{2u^2} + \frac{\log u}{u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_\Omega(u) = \frac{2}{9} - \frac{1}{12}\log u + \frac{1}{2u} - \frac{\log u}{2u} - \frac{3}{4u^2} - \frac{\log u}{2u^2} + O\left(\frac{1}{u^3}\right)$$

$$u \equiv \frac{-\Box}{m^2}$$

$C_{\alpha\beta\gamma\delta} = 0$

Cosmological effective action

$$\Gamma[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\,(R - 2\Lambda) + \xi \int d^4x \sqrt{-g}\, R^2 + \int d^4x \sqrt{-g}\, R F(\Box) R$$

$C_{\alpha\beta\gamma\delta} = 0$

Cosmological effective action

$$\Gamma[g] = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R-2\Lambda) + \xi\int d^4x\sqrt{-g}\,R^2 + \int d^4x\sqrt{-g}\,RF(\Box)R$$

$$F(\Box) = \alpha\log\frac{-\Box}{m^2}$$

$$+\,\beta\frac{m^2}{-\Box}$$

$\alpha,\ \beta,\ \gamma,\ \delta$

are calculable constants depending on effective gravitons and matter content

$$+\,\gamma\frac{m^2}{-\Box}\log\frac{-\Box}{m^2}$$

$$+\,\delta\frac{m^4}{(-\Box)^2}$$

$$+\,...$$

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Leading logs
J. F. Donoghue and B. K. El-Menoufi, Phys. Rev. D 89, 104062 (2014)

$$+\,\beta\frac{m^2}{-\Box}$$

Non-local cosmology
S. Deser and R. P. Woodard, Phys. Rev. Lett. 99, 111301 (2007)

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$$+\,\gamma\frac{m^2}{-\Box}\log\frac{-\Box}{m^2}$$

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Non-local gravity and dark energy
M. Maggiore and M. Mancarella, Phys. Rev. D 90, 023005 (2014).

$$+\,\ldots$$

Effective non-local cosmology
A. C. and K. J. Jain in preparation

Effective Friedmann equations

$$G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

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$$F(\Box) = \beta \frac{m^2}{-\Box} \quad \text{and} \quad ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2$$

$$H^2 - 16\pi G \beta m^2 \left(\frac{1}{6}\dot{U}^2 - 2H\dot{U} - 2H^2 U \right) = \frac{8\pi G}{3}\rho$$

$$\ddot{U} + 3H\dot{U} = 6\left(2H^2 + \dot{H}\right)$$

Effective Friedmann equations

$$G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$F(\Box) = \beta \dfrac{m^2}{-\Box}$ and $ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2$

$$H^2 - 16\pi G \beta m^2 \left(\frac{1}{6} \dot{U}^2 - 2H\dot{U} - 2H^2 U \right) = \frac{8\pi G}{3} \rho$$

$$\ddot{U} + 3H\dot{U} = 6\left(2H^2 + \dot{H}\right)$$

$a(t) = (t/t_0)^{2/3}$ or $a(t) = a_0 e^{\sqrt{\frac{\Lambda}{3}} t}$

$$H^2 + \frac{128\pi G}{9} \beta m^2 \Lambda \left(1 + \sqrt{3\Lambda}\,(t - t_0) - e^{-2\sqrt{3\Lambda}\,(t - t_0)}\right) = \frac{\Lambda}{3}$$

$$H^2 + \frac{128\pi G}{27} \frac{\beta m^2}{t^2} \left(1 - \frac{t_0^2}{t^2} - 4 \log \frac{t_0}{t}\right) = \frac{8\pi G}{3} \rho_m(t_0) \left(\frac{t_0}{t}\right)^2$$

Dark Energy

$\log \frac{-\Box}{m^2}$

$$\rho^{\alpha,m}_{DE}(t) = -\frac{32\alpha}{t^4}\left[\log mt + \log\left(\frac{t}{t_0}-1\right) + \frac{2}{3}\left(\frac{t}{t_0}-1\right)\right]$$

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_{DE})$$

$\frac{m^2}{-\Box}$

$$\rho^{\beta,\Lambda}_{DE}(t) = -\frac{16}{3}\beta m^2\Lambda\left(1 + \sqrt{3\Lambda}\,(t-t_0) - e^{-2\sqrt{3\Lambda}\,(t-t_0)}\right)$$

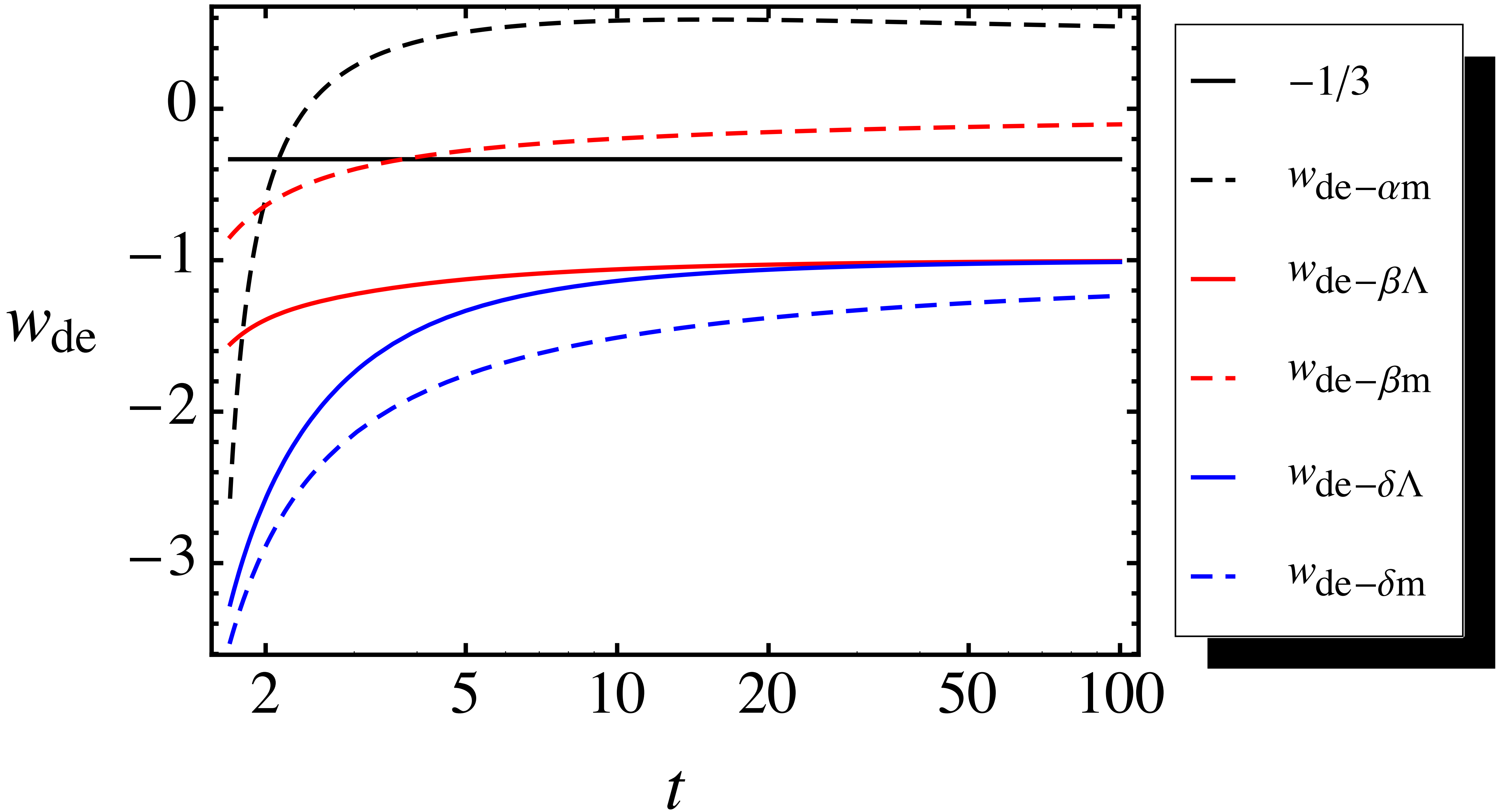
$$\rho^{\beta,m}_{DE}(t) = -\frac{16}{9}\beta m^2\frac{1}{t^2}\left(1 - \frac{t_0^2}{t^2} - 4\log\frac{t_0}{t}\right)$$

$\frac{m^4}{\Box^2}$

$$\rho^{\delta,\Lambda}_{DE}(t) = \frac{16}{9}\delta m^4\Big\{4 - 3\sqrt{3\Lambda}\,(t-t_0) + 3\Lambda(t-t_0)^2 - \left[8 - \sqrt{3\Lambda}\,(t-t_0)\right]e^{-\sqrt{3\Lambda}\,(t-t_0)} + \left[4 + 2\sqrt{3\Lambda}\,(t-t_0)\right]e^{-2\sqrt{3\Lambda}\,(t-t_0)}\Big\}$$

$$\rho^{\delta,m}_{DE}(t) = 12\delta m^4\left\{-\frac{119}{243} - \frac{26}{81}\log\frac{t_0}{t} + \frac{t_0}{t}\left(\frac{130}{243} - \frac{8}{81}\log\frac{t_0}{t}\right) - \frac{1}{27}\left(\frac{t_0}{t}\right)^2 + \frac{2}{243}\left(\frac{t_0}{t}\right)^3 - \frac{4}{243}\left(\frac{t_0}{t}\right)^4\right\}$$

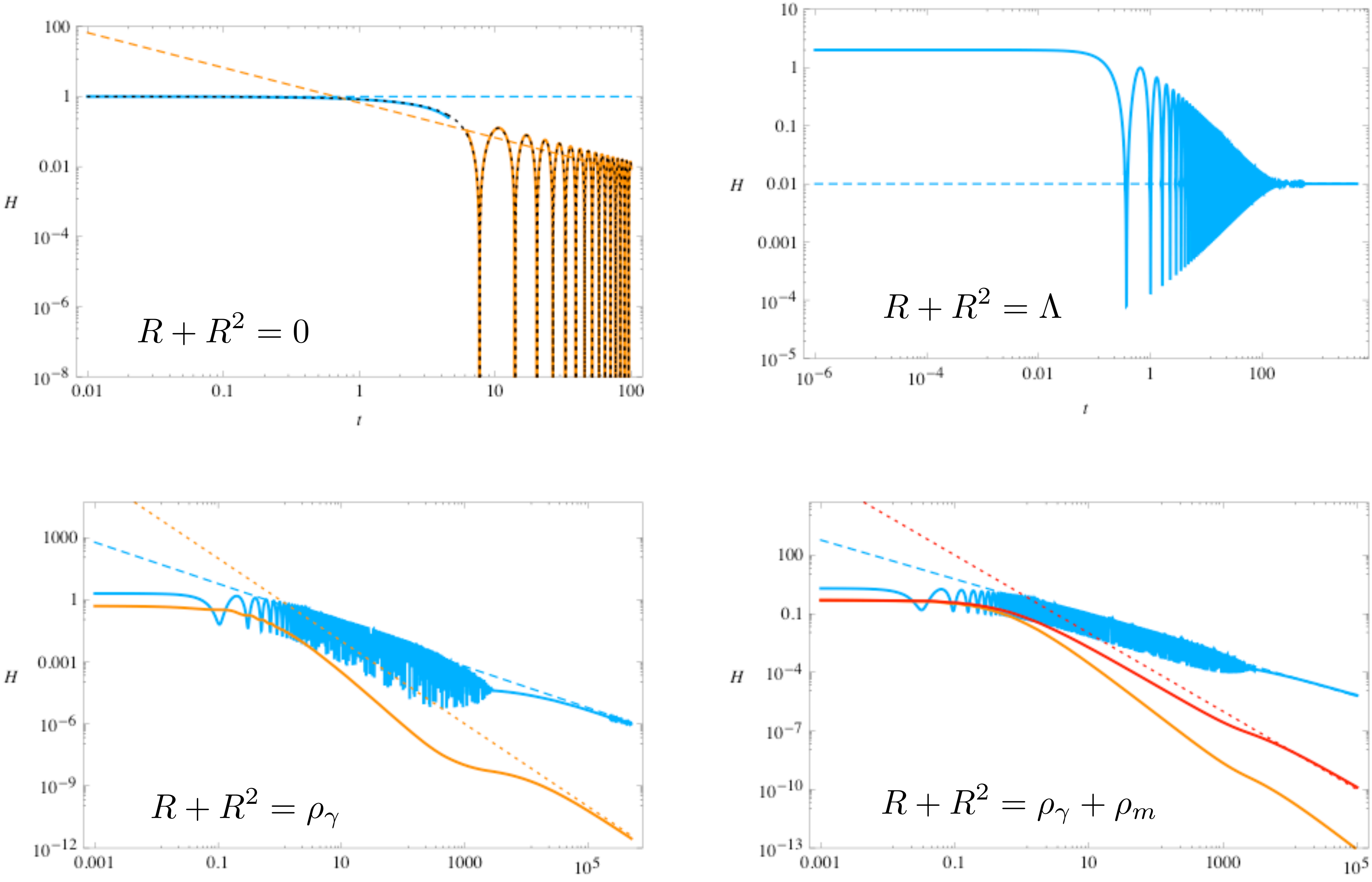
Dark Energy



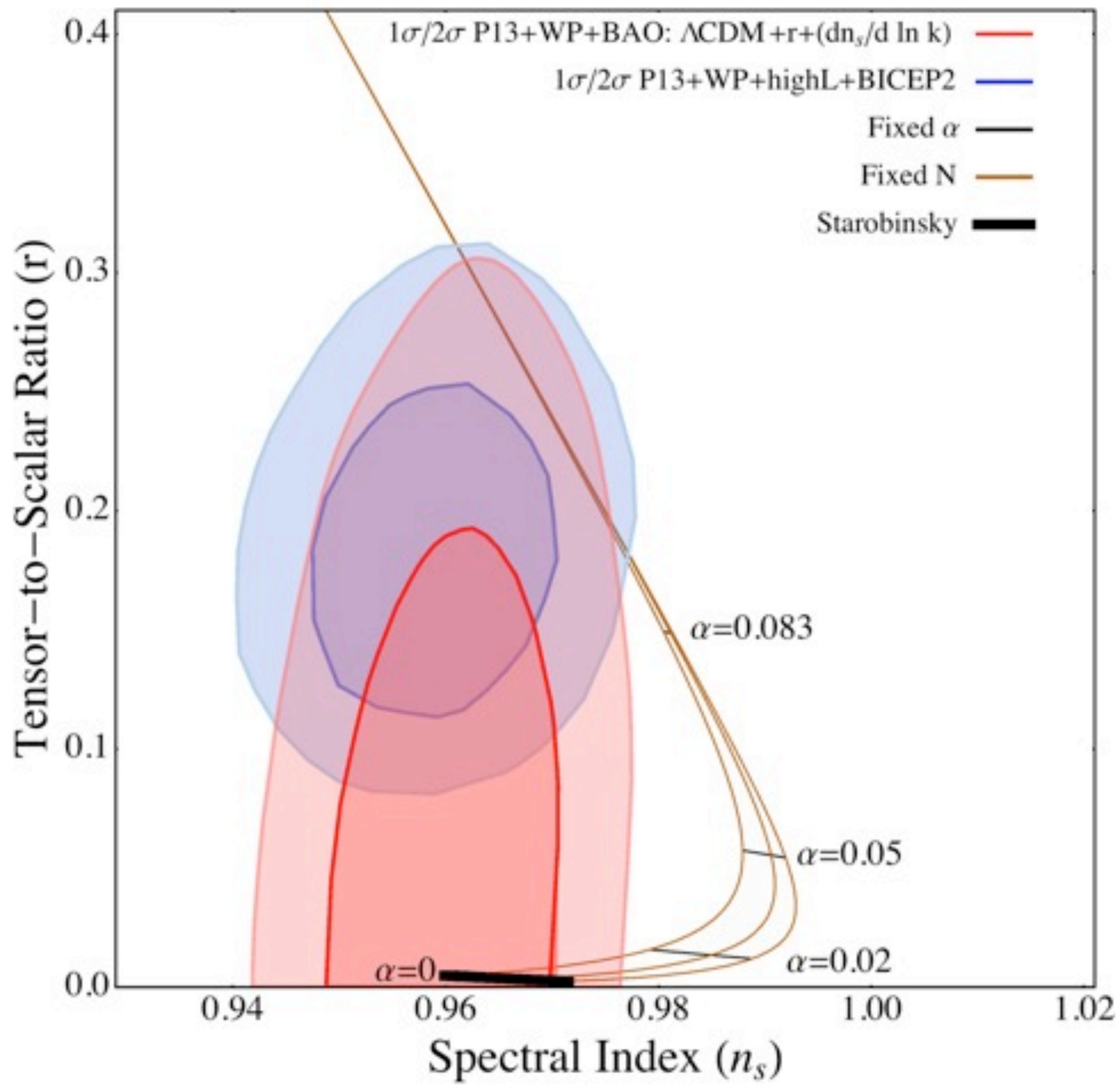
$$\dot{\rho}_{DE} + 3H(1 + w_{DE})\rho_{DE} = 0$$

$$w_{DE} = -1 - \frac{1}{3H}\left(\frac{\dot{\rho}_{DE}}{\rho_{DE}}\right)$$

Effective Friedmann equations



Marginally deformed Starobinsky



$$F(R) = \log \frac{R}{m^2}$$

Leading quantum corrections to tensor-to-scalar ratio

A. C, J. Joergensen, F. Sannino and O. Svendsen, JHEP 1502, 050 (2015)

Conclusions and Outlook

- Compute all LO terms
- Renormalization of NLO
- Conformal anomaly contribution
- Apply to cosmology
- Apply to stars/black holes
- Add the SM and constrain BSM
- Connection with high energy quantum gravity
- Falsify!

Thank you