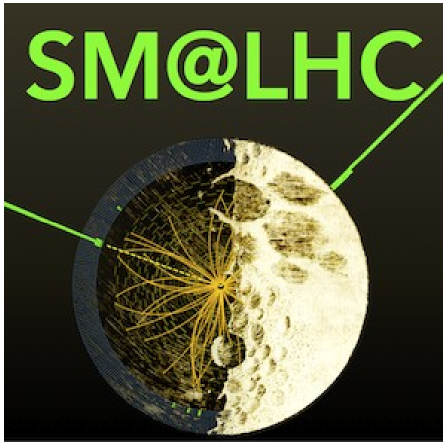
Multiparton Interactions - Theory.

Jonathan Gaunt, DESY



SM@LHC 2015, Florence, Italy, 23rd April 2015

HELMHOLTZ
ASSOCIATION

DESY

Outline

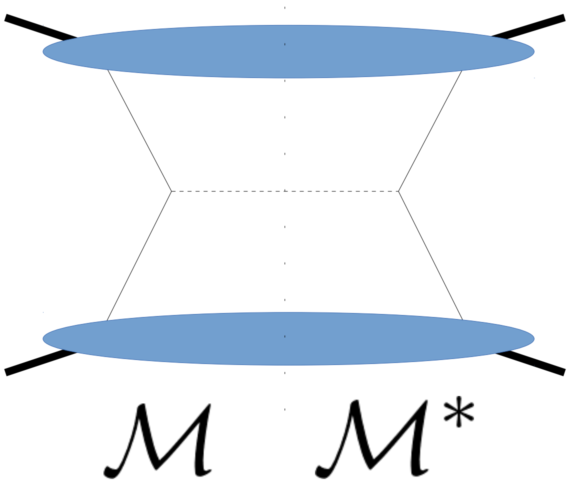
I will briefly review the theory description of multiple interactions (MPI) and double parton scattering (DPS), plus some recent developments.

- Why do we often ignore DPS/MPI? When should we take it into account?
- Theoretical expression for the DPS cross section in terms of two-parton distributions (2pGPDs). Approximations made leading to Pythia/Herwig models of MPI, and DPS 'pocket formula'.
- Effects recently studied by theory community in context of DPS, that are outside scope of DPS pocket formula:
  - Parton pair generation via perturbative splitting. Will discuss graphs in which parton pairs from one or both protons are perturbatively generated.
  - Interference and correlation effects in spin, colour, flavour.
- Cancellation of Glauber modes in DPS.

Why/when do we ignore MPI?

Protons contain large numbers of QCD partons → in each LHC pp collision, it is likely that there will be several parton-parton interactions (MPI).

Consider production of some particle A (A = Z, W, H, new physics, etc.). Typically we do not concern ourselves with MPI when calculating cross sections for this process:



Total cross section:

$$\sigma = \hat{\sigma}_{ij\to A}(\hat{s} = x_A x_B s) \otimes f_i(x_A) \otimes f_j(x_B)$$

Parton distribution functions (PDFs)

Differential transverse momentum:

$$W^{\mu\nu} \propto C_f^{\mu\nu}\left(\hat{k}_A, \hat{k}_B\right) \int d^2\mathbf{b}_T e^{i\mathbf{p}_T\cdot\mathbf{b}_T} \tilde{f}\left(x_A, \mathbf{b}_T; \zeta_A\right) \tilde{f}\left(x_B, \mathbf{b}_T; \zeta_B\right)$$

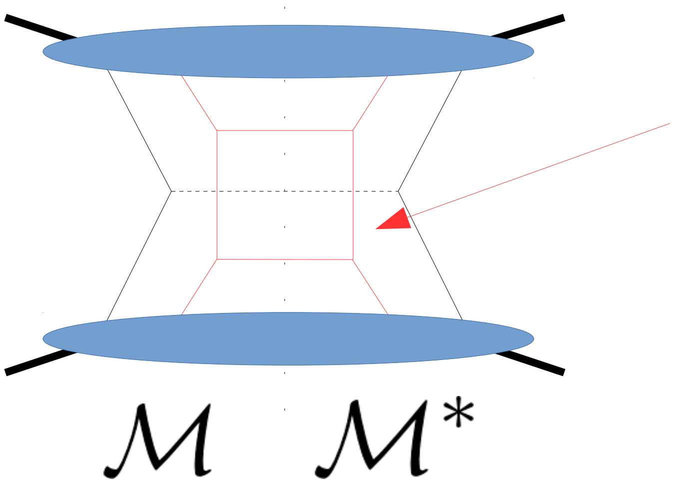
Transverse momentum dependent PDFs (TMDs)

The PDFs and TMDs are single parton distributions

Why/when do we ignore MPI?

Protons contain large numbers of QCD partons → in each LHC pp collision, it is likely that there will be several parton-parton interactions (MPI).

Consider production of some particle A (A = Z, W, H, new physics, etc.). Typically we do not concern ourselves with MPI when calculating cross sections for this process:



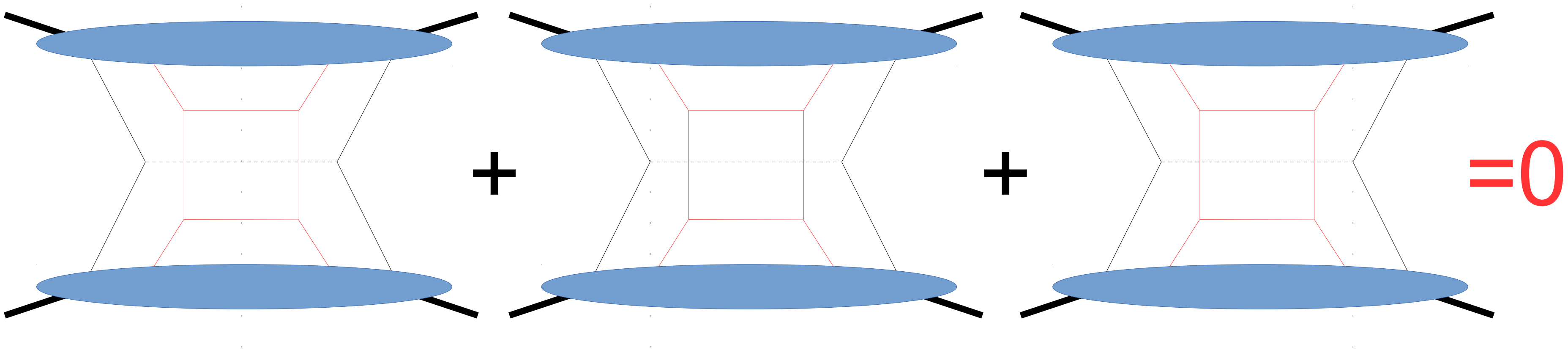
Q. Why do we not also need to calculate this process with an additional scattering (and indeed processes with arbitrary extra scatterings) to obtain the V production cross section?

Why/when do we ignore MPI?

A. Unitarity!

When we say cross section for production of A, what we really mean is inclusive cross section: pp → A + X.

X can be anything, we sum over all possibilities for X.



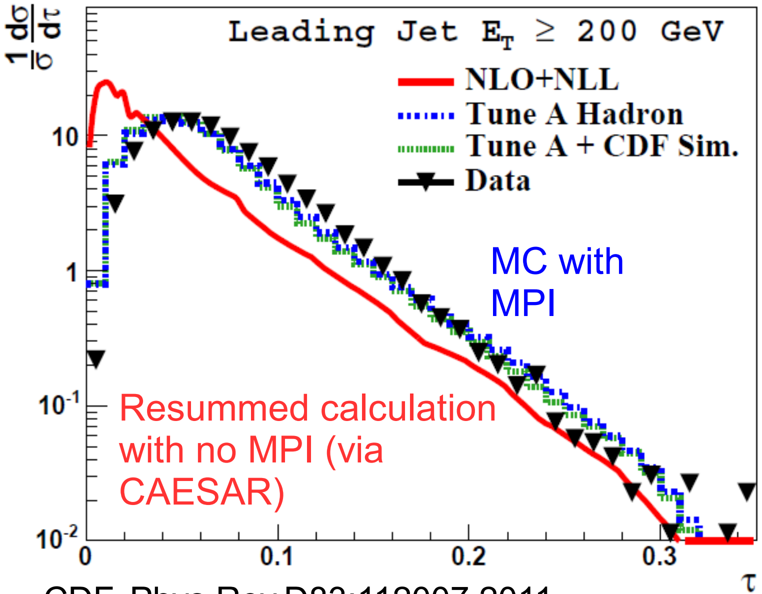
MPI

Absorptive processes

Bodwin, Phys. Rev. 31 (1985) 2616.
Collins Soper Sterman Nucl. Phys. B261 (1985) 104, Nucl. Phys. B308 (1988) 833.

MPI sensitive observables

Beware: if you are not sufficiently inclusive on X → become sensitive to additional scatters.



CDF, Phys.Rev.D83:112007,2011

Good example of such an observable is transverse thrust:

$$T_\perp \equiv \max_{\vec{n}_T} \frac{\sum_{i=1}^{n} |\vec{q_\perp}_{,i} \cdot \vec{n}_T|}{\sum_{i=1}^{n} |\vec{q_\perp}_{,i}|} \qquad \tau \equiv 1 - T_\perp$$

Additional uncorrelated scatters make event more spherical and raise τ – observable sensitive to MPI.

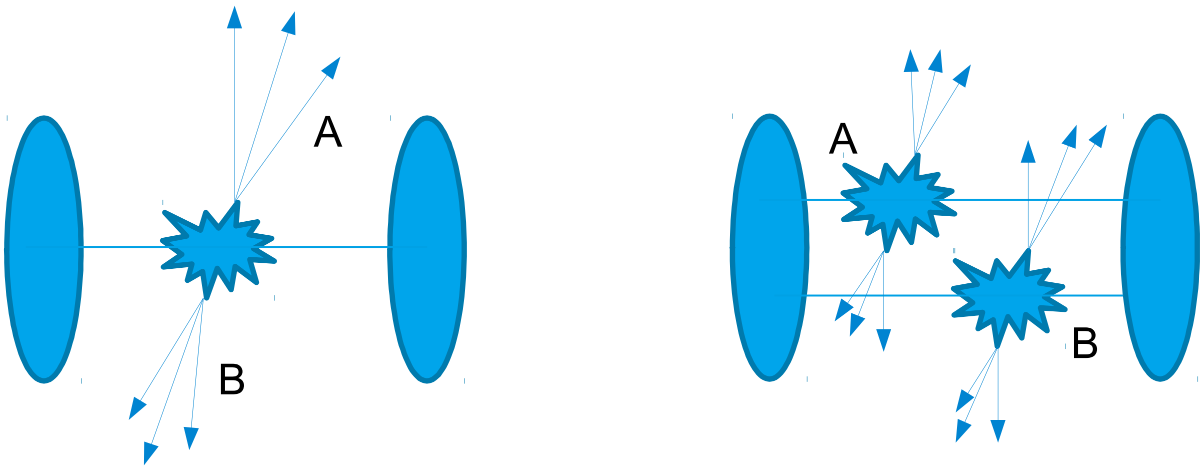
Other observables sensitive to MPI – beam thrust, hadronic transverse energy.

Formally these observables do not obey standard factorisation due to Glauber exchange between spectators – close connection between MPI and Glauber exchanges. JG, JHEP 2014:110, 2014

Double Parton Scattering

Another observable that is sensitive to whether additional scatters occurred (in this case 1 additional scatter) – production of two sets of hard objects A and B, with associated scales Q_A and Q_B, $p + p \rightarrow A + B + X$

The two sets can either be produced in a single scattering (SPS) or in a double parton scattering (DPS) process:



In terms of the total cross section, the DPS mechanism is power suppressed with respect to SPS:

$$\sigma_{DPS}/\sigma_{SPS} \sim \Lambda^2/Q^2$$

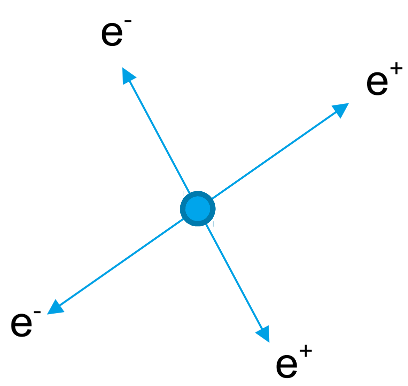
However: could still be important if SPS process is suppressed by small/multiple coupling constants (e.g. same sign WW, new physics signals).

Double Parton Scattering

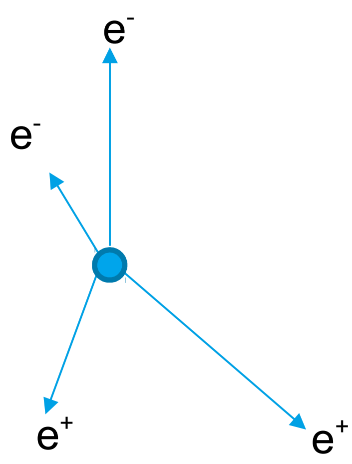
DPS populates the final state phase space in a different way from SPS. In particular, it tends to populate the region of small $\mathbf{q}_A$, $\mathbf{q}_B$ – competitive with SPS in this region.

e.g. $pp \rightarrow \underbrace{e^+ e^-}_{A} \underbrace{e^+ e^-}_{B}$

DPS:

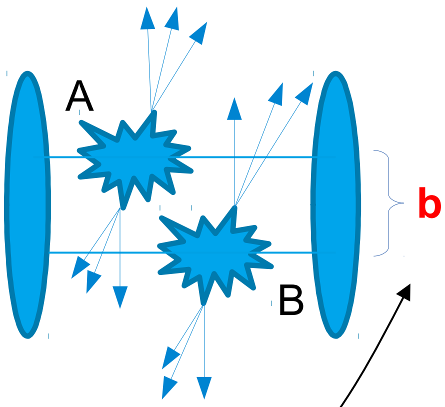


SPS:



In the region with $\mathbf{q}_A$, $\mathbf{q}_B$ small, DPS and SPS are comparable for any process!
This is exploited by experiments to measure DPS.

Total Cross Section for DPS



Assuming only the factorisation of the hard processes A and B, the total DPS cross section may be written as:

Symmetry factor — Two-parton generalised PDF (2pGPD)

$$\sigma_D^{(A,B)} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_h^{ik}(x_1, x_2, \mathbf{b}; Q_A, Q_B) \Gamma_h^{jl}(x_1', x_2', \mathbf{b}; Q_A, Q_B) \times \hat{\sigma}_{ij}^{A}(x_1, x_1') \hat{\sigma}_{kl}^{B}(x_2, x_2') \mathrm{d}x_1 \mathrm{d}x_1' \mathrm{d}x_2 \mathrm{d}x_2' \mathrm{d}^2\mathbf{b}$$

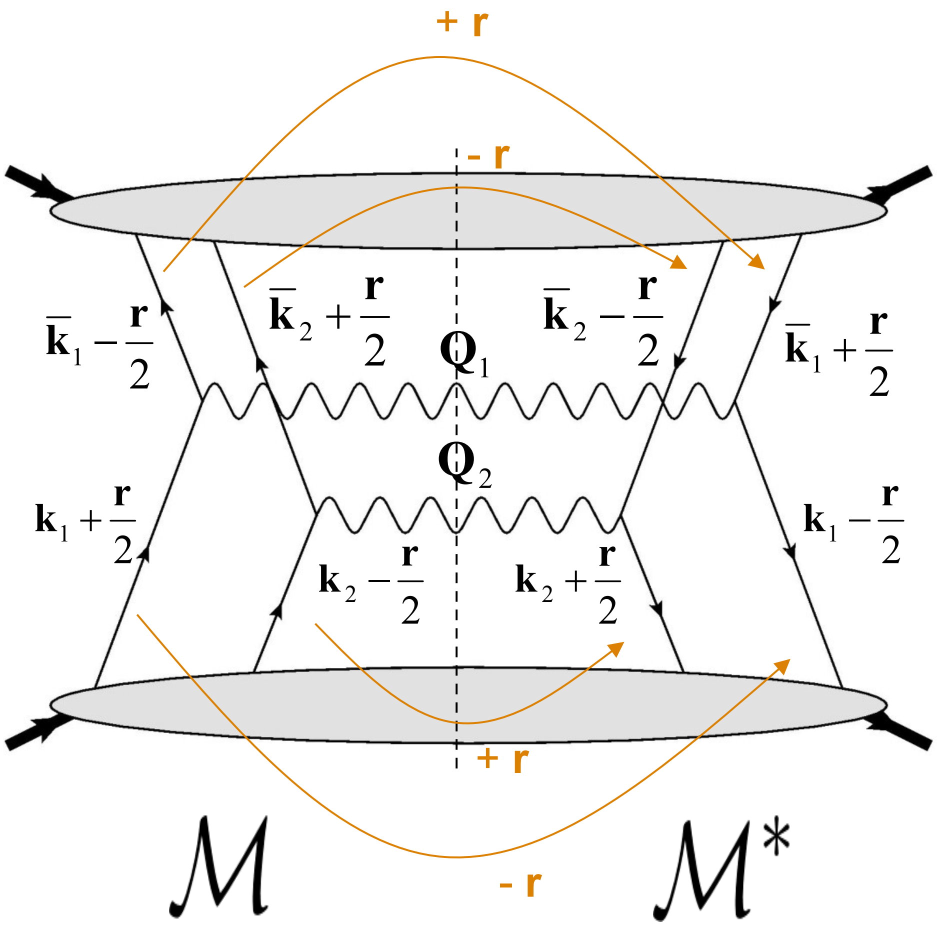
Parton level cross sections

Paver, Treleani, Nuovo Cim. A70 (1982) 215.
Mekhfi, Phys. Rev. D32 (1985) 2371.
Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))

b = separation in transverse space between the two partons

In this formula the two 2pGPDs are integrated over a common **b** – cannot express DPS cross section in terms of parton distributions independently integrated over their impact parameter arguments, as in single scattering case.

DPS – transverse momentum picture



Key point: transverse momentum of partons does not have to be equal in amplitude and conjugate!

Most general transverse momentum configuration of partons entering hard scatters

r = momentum imbalance of a parton line between amplitude and conjugate

$$\sigma = \int \frac{d^2\mathbf{r}}{(2\pi)^2} D_h^{p_1 p_2}(x_1, x_2, \mathbf{r}) D_h^{p_3 p_4}(x_1, x_2, -\mathbf{r})$$

Fourier transform of **b**-space 2pGPD wrt **b**

Simplifying assumptions for DPS cross section

If one ignores correlations between partons in the proton:

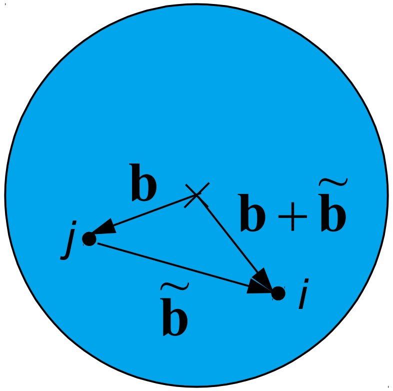
$$D_p^{ij}(x_1,x_2;\mathbf{b}) = \int d^2\tilde{\mathbf{b}}\, D_p^i(x_1;\tilde{\mathbf{b}}+\mathbf{b})\, D_p^j(x_2;\tilde{\mathbf{b}})$$

Impact parameter dependent PDFs

or equivalently

$$D_p^{ij}(x_1,x_2;\mathbf{\Delta}) \approx D_p^i(x_1;\mathbf{\Delta})\, D_p^j(x_2;-\mathbf{\Delta})$$

'GPD'



Common 'lore': approximately valid at low *x*, due to the large population of partons at such x values.

Further approximation that is often made: $D_p^i(x_1;\tilde{\mathbf{b}}) = D_p^i(x_1)\, F(\tilde{\mathbf{b}})$

$$\Rightarrow D_p^{ij}(x_1,x_2;\mathbf{b}) = D_p^i(x_1) D_p^j(x_1) \int d^2\tilde{\mathbf{b}}\, F(\tilde{\mathbf{b}}+\mathbf{b})\, F(\tilde{\mathbf{b}})$$

Several MCs (PYTHIA, HERWIG) use these approximations to model MPI

$$\Rightarrow \sigma_D^{(A,B)} = \frac{\sigma_S^{(A)}\sigma_S^{(B)}}{\sigma_{eff}}$$

Some refinements – e.g. *x* dependent proton size: Corke, Sjöstrand, JHEP 05 (2011) 009)

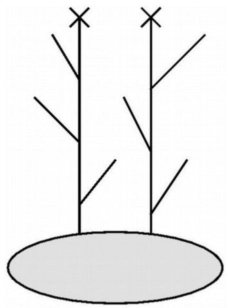
Almost all phenomenological estimates of DPS use this equation

Parton splitting effects

Two possibilities for how a parton pair in the proton could have arisen:

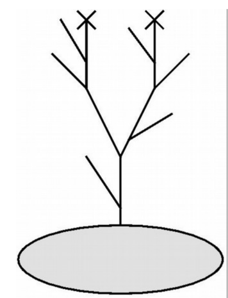
1) Pair generated already at the perturbative level:

.



2) Pair generated by a 1→2 perturbative splitting:

.

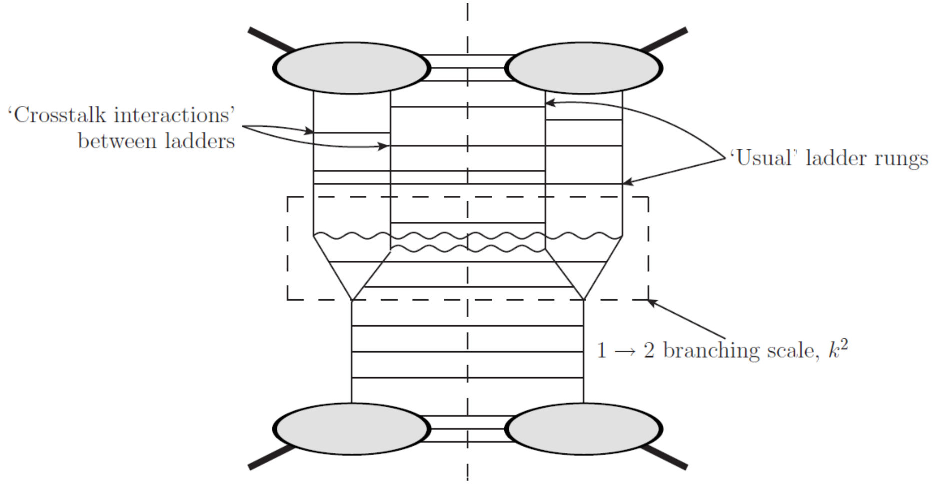


These two processes correspond to very different distributions in impact parameter space:

$$\Gamma(x_1, x_2, b) \simeq D(x_1)D(x_2)G(b) \qquad \Gamma(x_1, x_2, b) \propto \alpha_s \frac{D(x_1+x_2)}{x_1+x_2} P\left(\frac{x_1}{x_1+x_2}\right)\frac{1}{b^2}$$

Parton splitting and radiation can occur at all scales – in general these effects will break x_1-x_2-**b** factorisation in the 2pGPD.

Perturbative splitting in one proton – 2v1 graphs



'2v1' Graphs in which a perturbative splitting occurs in only one proton have been extensively studied – established that such graphs can contribute to DPS cross section, and LL evolution effects worked out.

BDFS, Eur.Phys.J. C72 (2012) 1963
Ryskin, Snigirev, Phys.Rev.D83:114047,2011
JG, JHEP 1301 (2013) 042

- Geometrical '$1/\sigma_{eff}$' prefactor for these graphs is twice as big as 2v2 graphs with no 1→2 splitting.
- Numerical studies imply 2v1 cross section is sizeable ($\sigma_{2v1}/\sigma_{2v1}$ ~0.3-1.5 depending on scale and x values), but gives differential cross sections very similar to 2v2. JG, Maciula, Szczurek Phys. Rev. D 90 (2014) 054017, BDFS, Eur.Phys.J. C74 (2014) 2926
- This mechanism has been investigated in the context of Pythia 8 using a reweighting procedure – good fit to hard + soft MPI observables, although no conclusive discrimination between 2v1 model and default model yet. Blok, Gunnellini, arXiv:1503.08246

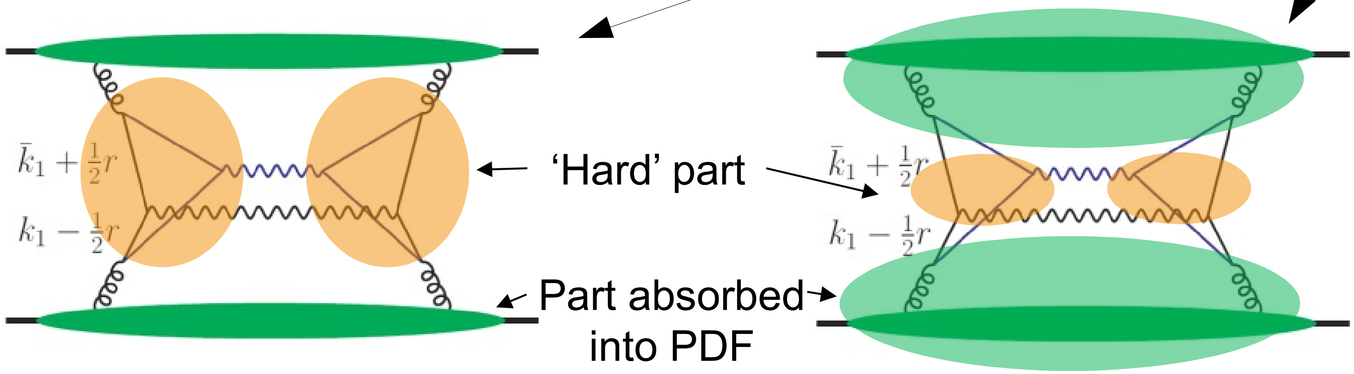
‘1v1’ or ‘Double Perturbative Splitting’ Diagrams

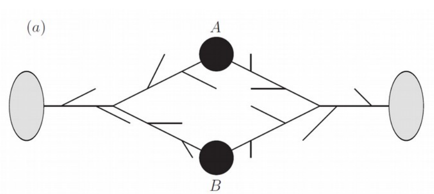
What about '1v1' graphs in which we have a perturbative splitting in both protons?

Trying to calculate this graph in a naive way using the DPS framework yields problematic quadratic divergences!

$$\int \frac{d^2b}{b^4} = ?$$

This is related to the fact that we can regard 1v1 graph as SPS loop correction or DPS





There is no natural power suppressed ($\propto \frac{\Lambda^2}{Q^4}\left[\alpha_s \log\left(\frac{Q^2}{\Lambda^2}\right)\right]^n$) part of the 1v1 graph that we can separate off as DPS ➔ regard all of these graphs as SPS?

JG and Stirling, JHEP 1106 048 (2011) & arXiv:1202.3056
Manohar, Waalewijn Phys.Lett. 713 (2012) 196–201.
BDFS, Eur.Phys.J. C72 (2012) 1963

Total Cross Section for DPS

Advantage: we avoid double counting between DPS and SPS!

Potentially concerning implication:

> The cross section can no longer be written as parton level cross sections convolved with overall 2pGPD factors for each hadron.

Original expression written down on slide 4

2v2 1v2 2v1

$$\sigma^{DPS} \propto \cancel{\int d^2\mathbf{b}\, \Gamma_a(\mathbf{b})\Gamma_b(\mathbf{b})} \rightarrow \int d^2\mathbf{b}\, \Gamma_{a,NP}(\mathbf{b})\Gamma_{b,NP}(\mathbf{b}) + D_{a,P}\Gamma_{b,NP}(\mathbf{b}=\mathbf{0}) + \Gamma_{a,NP}(\mathbf{b}=\mathbf{0})D_{b,P}$$

BDFS, Eur.Phys.J. C72 (2012) 1963
Manohar and Waalewijn (Phys.Lett. B713 (2012) 196–201

$$(A+B)^2 \neq A^2 + AB + BA$$

There can be no concept of the 2pGPD for an individual hadron, with an associated operator definition and evolution equation. Appropriate hadronic operators in DPS would have to involve both hadrons at once!

BDFS, Eur.Phys.J. C74 (2014) 2926
Manohar and Waalewijn (Phys.Lett. B713 (2012) 196–201

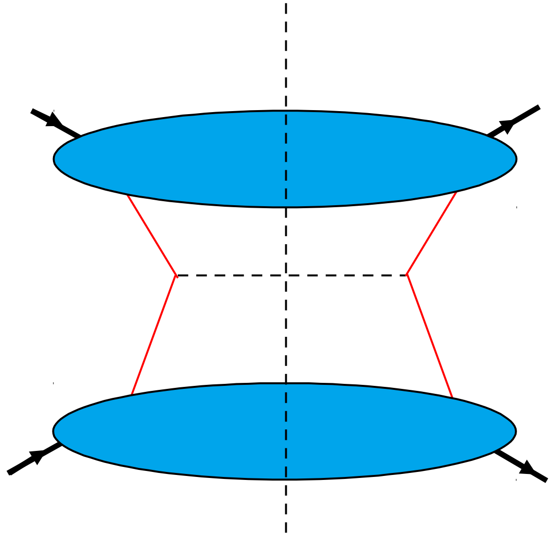
Interference contributions to proton-proton DPS

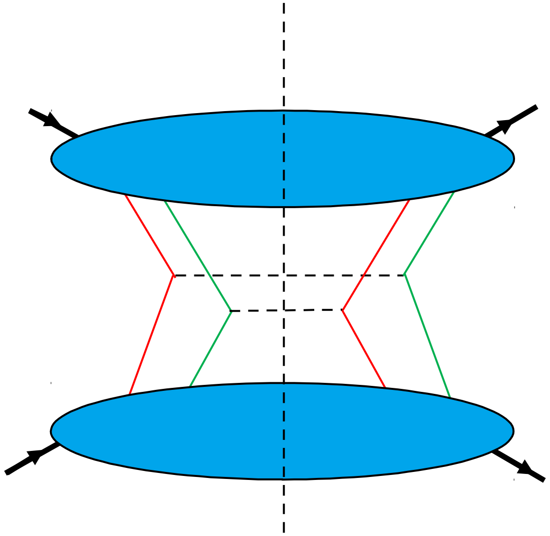
Mekhfi, Phys. Rev. D32 (1985) 2380
Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))
Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009

SPS: One parton per proton 'leaves', interacts and 'returns'.

To reform proton, parton must return with same quantum numbers.

No interference contributions to SPS cross section.





Here we have two partons per proton interacting.

Interference contributions to total cross section in which quantum numbers are swapped between parton legs. Complementary swap is required in other proton.

Can get interference contributions in colour, spin, flavour, and quark number.

Correlated parton contributions to DPS

There are also contributions to the unpolarised p-p DPS cross section associated with correlations between partons:

e.g. $$\Delta q_1 \Delta q_2 = \underbrace{q_1{\uparrow}\, q_2{\uparrow} + q_1{\downarrow}\, q_2{\downarrow}}_{\text{Same spin}} - \underbrace{q_1{\uparrow}\, q_2{\downarrow} - q_1{\downarrow}\, q_2{\uparrow}}_{\text{Opposing spin}}$$

For all of these distributions, limits on their size have been derived for the LO distributions.

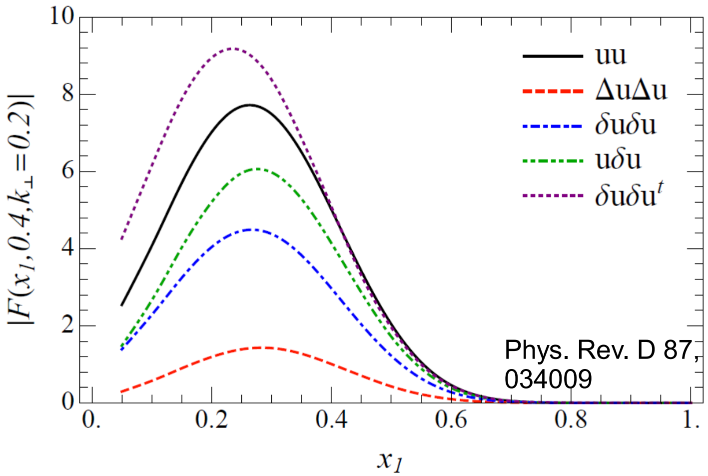
Diehl, Kasemets JHEP 1305 (2013) 150
Kasemets, Mulders Phys.Rev. D91 (2015) 014015

one example for spin case: $F_{qq} - F_{\Delta q \Delta q} \geq 2|F_{\delta q \delta q}|$

← Transverse spin correlation

Based on the probability interpretation of certain combinations of LO 2pGPDs

Spin correlations and DPS



Model calculations with 3-quark wavefunctions suggest a large degree of spin correlation for large x.

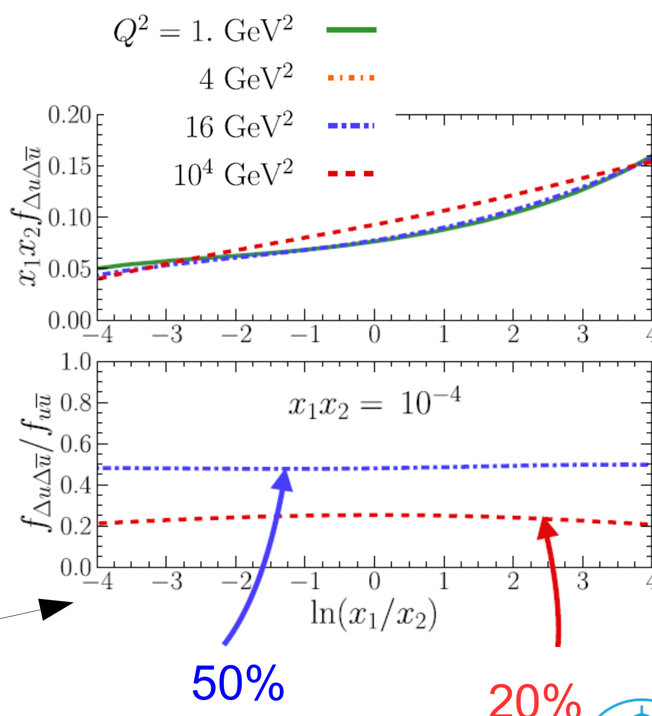
Manohar, Waalewijn, Chang, Phys. Rev. D 87, 034009 (2013)
Rinaldi, Scopetta, Traini, Vento, JHEP 1412 (2014) 028

What about the more relevant small x region?

Common 'lore' – spin correlations present at low scale quickly washed out by separate evolution of two partons.

Assumption has been tested by Diehl, Kasemets, Keane in JHEP 1405 (2014) 118

Put in maximum spin correlations at starting scale of 1 GeV and evolve using two chain evolution to larger scales



Indeed spin correlations die out, but not so quickly for certain distributions – e.g. $\Delta u \Delta \bar{u}/(u\bar{u})$

DESY

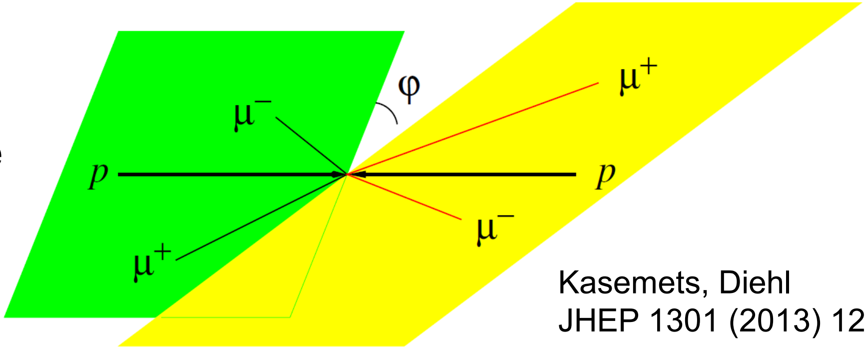
Spin correlations and their effect on DPS processes

Spin correlations can can change both normalisation and shapes of differential DPS cross sections.

In Double Drell Yan producing lepton pairs:

Longitudinal spin correlations change overall rate of process and distribution in lepton rapidities

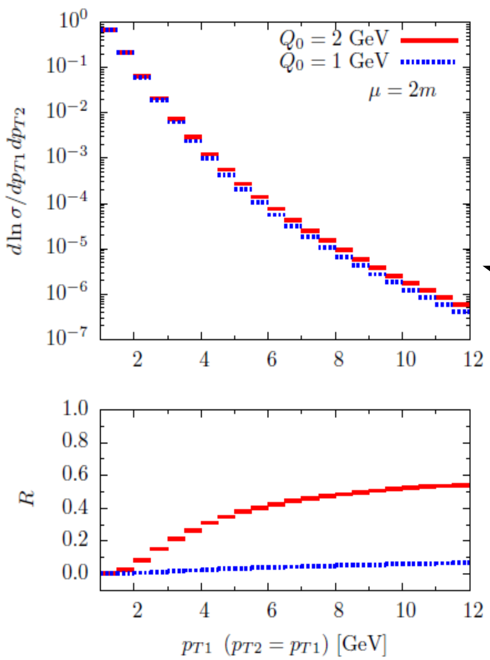
Transverse spin correlations cause azimuthal correlations between lepton planes



Kasemets, Diehl
JHEP 1301 (2013) 121

NOTE: it is often assumed that DPS produces two sets of final state particles that are completely uncorrelated in the transverse plane.

In double open charm production:



Spin correlations affect shape of double differential distribution in charm quark p_Ts

(plot produced including two-chain evolution)

Echevarria, Kasemets, Mulders, Pisano, JHEP 1504 (2015) 034

Sudakov Suppression of Colour Interference Distributions

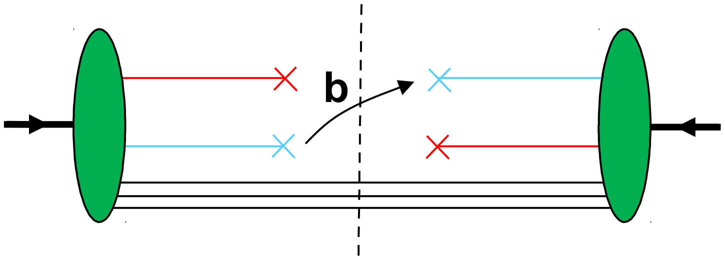
For the 2pGPD with finite **b**, every colour interference distribution is Sudakov suppressed:

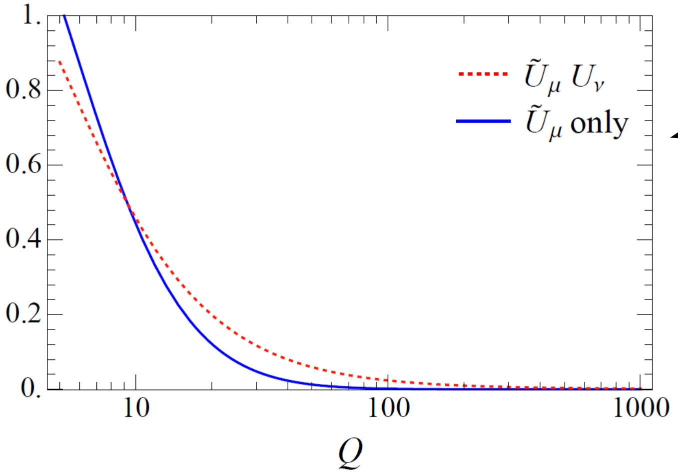
$$\sim \exp\left(\frac{\alpha_s}{2\pi}\underbrace{\left(C_R^I - C_V^I\right)}_{<0}\ln^2\left(\mathbf{b}^2 Q^2\right)\right)$$

Mekhfi and Artru, Phys.Rev. D37 (1988) 2618–2622
Diehl, Ostermeier and Schäfer (JHEP 1203 (2012) 089)
Manohar and Waalewijn, Phys.Rev. D85 (2012) 114009

Physical explanation: Movement of colour by large transverse distance **b** in hadron between amplitude and conjugate.

Manohar and Waalewijn, Phys.Rev. D85 (2012) 114009

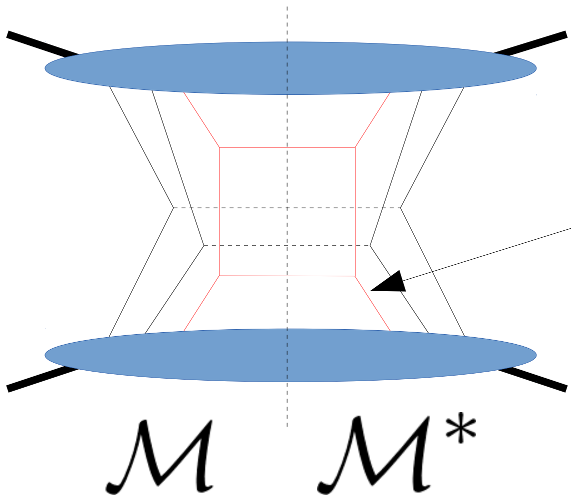




Suppression is strong for large scales, but is only ~½ for Q = 10 GeV.

Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009

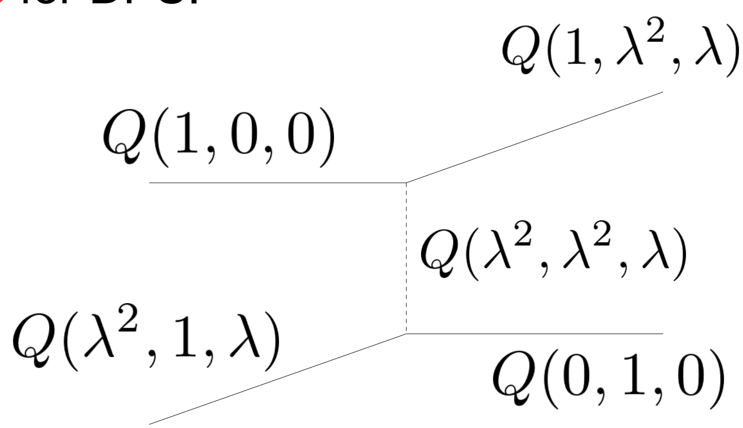
Glaubers in DPS



Earlier we implicitly assumed that the unitarity cancellation of additional soft spectator-spectator interactions in X goes through for DPS, just as it does for SPS.

As mentioned before, this is related to the issue of the cancellation of Glauber gluon exchanges for DPS.

Glauber gluons are soft gluons that have much larger transverse components than lightcone components – naturally mediate soft MPI (+potentially other effects):



$$(\lambda \ll 1)$$

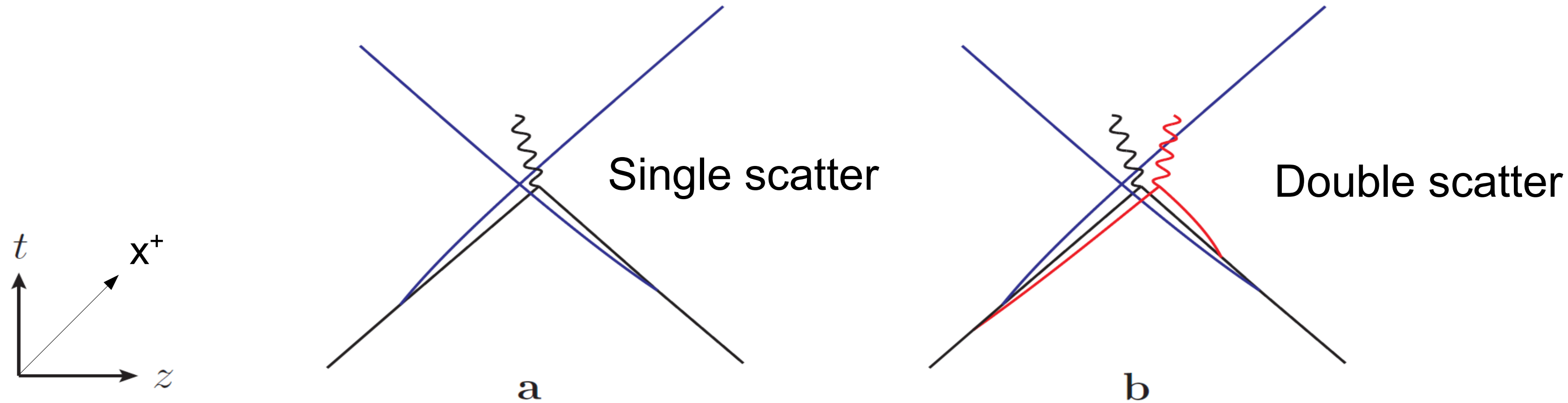
Do Glauber modes cancel for DPS?

Glauber in DPS – all-order analysis

All-order analysis for double Drell-Yan can be done using the same method as used by CSS for single Drell-Yan (light-cone perturbation theory)

Collins Soper Sterman Nucl. Phys. B308 (1988) 833.

We find Glaubers also cancel for double scattering. Very rough explanation – in terms of the lightcone coordinates, two scatters in DPS take place at the same point → from the point of view of larger scale Glauber modes, DPS looks quite similar to SPS.



Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))

Diehl, Gaunt, Ostermeier, Plößl, Schäfer, to appear

Summary

- Various observables can be defined at the LHC that are sensitive to soft or hard MPI. Process with one extra interaction, DPS, is interesting as a signal, and as a background to rare processes.
- 2pGPDs to DPS pocket formula, or MC MPI models → take additional scatters to be essentially uncorrelated.
- Parton splitting effects in DPS: 'Single splitting' contribution extensively studied, of comparable size to nonsplitting contribution. 'Double splitting' contribution has overlap with SPS – treat as pure SPS?
- There are interference and correlated parton contributions to DPS in colour, flavour and spin space.
  - Spin effects not necessarily negligible, and can change both normalisation and shapes of differential DPS cross sections.
  - Colour interference contributions to DPS are Sudakov suppressed.
- Glauber cancellation in DPS.

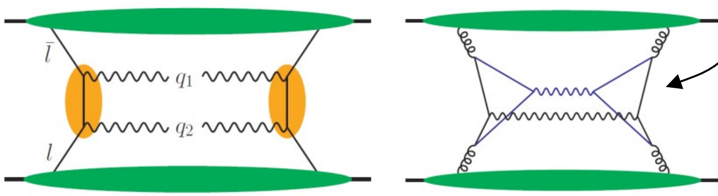
Backup Slides

Power Counting

Some relevant diagrams:

Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))

$s\sigma$ | $\left.\frac{s\,d\sigma}{d^2\mathbf{q}_1 d^2\mathbf{q}_2}\right|_{|\mathbf{q}_1|\sim|\mathbf{q}_2|\sim\Lambda}$

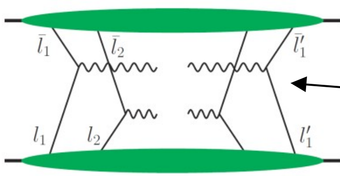


1v1 Diagram

1 | $\frac{1}{\Lambda^2 Q^2}$

Everything except SPS power suppressed

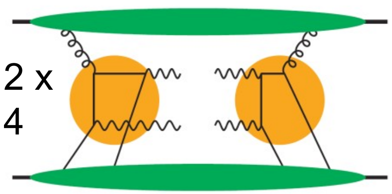
'DPS' and 'SPS' both leading power!

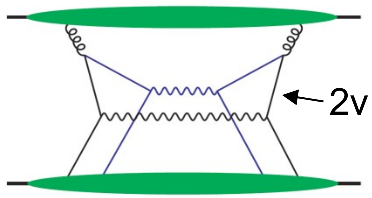


2v2 Diagram

$\frac{\Lambda^2}{Q^2}$ | $\frac{1}{\Lambda^2 Q^2}$

Twist 2 x Twist 4

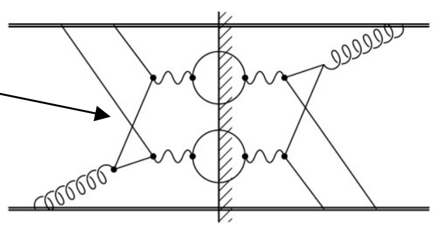




2v1 Diagram

$\frac{\Lambda^2}{Q^2}$ | $\frac{1}{Q^4}$

(Twist 3)^2 (in total cross section)



Many twist 3 distributions suppressed due to helicity nonconservation in associated diagrams

$\frac{\Lambda^2}{Q^2}$ | $\frac{1}{\Lambda^2 Q^2}$

Manohar, Waalewijn Phys.Lett. 713 (2012) 196–201
Qiu, Sterman, Nucl.Phys. B353 (1991) 105-136

Differential Cross Section for DPS for q_T << Q

To calculate differential DPS cross sections for small $\mathbf{q}_A$, $\mathbf{q}_B$ where DPS is comparable with SPS, would actually require a different formula containing 'two parton transverse momentum dependent PDFs' or 2pGTMDs: Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))

2pGTMD

$$\frac{d\sigma_D^{(A,B)}}{d^2\mathbf{q}_1 d^2\mathbf{q}_2} = \frac{m}{2}\sum_{i,j,k,l}\int \Gamma_h^{ik}(x_1,x_2,\mathbf{k}_1,\mathbf{k}_2,\mathbf{b})\Gamma_h^{jl}(x_1',x_2',\bar{\mathbf{k}}_1,\bar{\mathbf{k}}_2,\mathbf{b})$$
$$\times\hat{\sigma}_{ij}^{A}(x_1,x_1')\hat{\sigma}_{kl}^{B}(x_2,x_2')\mathrm{d}x_1\,\mathrm{d}x_1'\mathrm{d}x_2\,\mathrm{d}x_2'\mathrm{d}^2\mathbf{b}$$
$$\times\prod_{i=1,2}\int d^2\mathbf{k}_i d^2\bar{\mathbf{k}}_i\delta(\mathbf{k}_i+\bar{\mathbf{k}}_i-\mathbf{q}_i)$$

(Neglecting a possible soft factor + dependence of the 2pGTMDs on rapidity regulator)

Differential cross section can also be expressed in terms of **r** space 2pGTMDs – as in total cross section, one makes the replacement:

$$\int \Gamma_h^{ik}(\mathbf{b})\Gamma_h^{jl}(\mathbf{b})d^2\mathbf{b}\to\int \Gamma_h^{ik}(\mathbf{r})\Gamma_h^{jl}(-\mathbf{r})\frac{d^2\mathbf{r}}{(2\pi)^2}$$

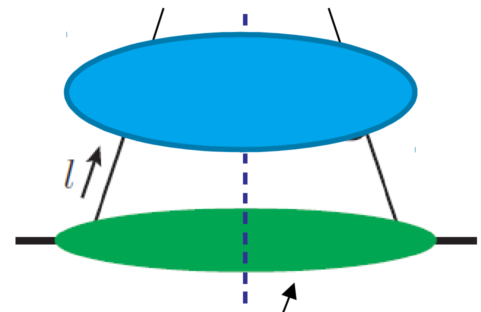
Relation between 2pGPDs and 2pGTMDs for qT >> Λ

SPS:

If |**q**| >> Λ (but still << Q), then TMD can be written in terms of collinear PDFs and a perturbative factor.

Collins, Soper, Sterman , Nucl.Phys. B250 (1985) 199
Collins, pQCD book, Ch. 13

$$F_h(x,\mathbf{k}) = T(x,\mathbf{k}) \otimes D_h(x,\mu^2=\mathbf{k}^2)$$



Collinear (single) PDF

Indeed, at double leading logarithmic order, we obtain the DDT formula for the differential SPS cross section for |**q**| >> Λ :

$$\frac{d\sigma}{dq^2\, dq_\perp^2} = \frac{d\sigma_{\text{tot}}}{dq^2} \times \frac{\partial}{\partial q_\perp^2}\left\{ D_a^q\left(x_1, q_\perp^2\right) D_b^{\bar{q}}\left(x_2, q_\perp^2\right) S_q^2\left(q^2, q_\perp^2\right) \right\}.$$

Sudakov factor

We expect there to be a similar relation between 2pGPDs and 2pGTMDs. At the double leading log level, it has been shown that the Sudakov factor for DPS is the product of Sudakov factors for SPS:

$$\pi^2 \frac{d\sigma^{(4\to4)}}{d^2\delta_{13}\, d^2\delta_{24}} = \frac{d\sigma_{\text{part}}}{d\hat{t}_1\, d\hat{t}_2} \cdot \frac{\partial}{\partial\delta_{13}^2} \frac{\partial}{\partial\delta_{24}^2} \Big\{ {}_{[2]}D_a^{1,2}(x_1,x_2;\delta_{13}^2,\delta_{24}^2) \times {}_{[2]}D_b^{3,4}(x_3,x_4;\delta_{13}^2,\delta_{24}^2) \times S_1\left(Q^2,\delta_{13}^2\right) S_3\left(Q^2,\delta_{13}^2\right) \times S_2\left(Q^2,\delta_{24}^2\right) S_4\left(Q^2,\delta_{24}^2\right) \Big\}.$$

➔ for |**q**| >> Λ there is a portion of the DPS differential σ that resembles the total σ

Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))
BDFS, Eur.Phys.J. C72 (2012) 1963

Operator definition of 2pGPDs + Mellin Moments

$$D^{q_1 q_2}_{p(0)}(\xi_1,\xi_2) = \langle P|\, 2p^+ \int \sum_{i=1}^{2} \frac{dz_i^-}{2\pi} e^{i\xi_i z_i^- p^+} dy^- d^2\boldsymbol{y}$$
$$\times \bar{\psi}_{q_2,a(0)}(y-\tfrac{1}{2}z_2)\mathcal{G}_{ab}(y-\tfrac{1}{2}z_2, y+\tfrac{1}{2}z_2)\tfrac{1}{2}\gamma^+\psi_{q_2,b(0)}(y+\tfrac{1}{2}z_2)$$
$$\times \bar{\psi}_{q_1,c(0)}(-\tfrac{1}{2}z_1)\mathcal{G}_{cd}(-\tfrac{1}{2}z_1, \tfrac{1}{2}z_2)\tfrac{1}{2}\gamma^+\psi_{q_1,d(0)}(\tfrac{1}{2}z_1)\,|P\rangle_c\big|_{z_i^+=y^+=0,\boldsymbol{z}_i=0}$$

$$M_{q,q}(\boldsymbol{y}^2) = \int_0^1 dx_1 \int_0^1 dx_2 \Big[{}^1F_{q,q}(x_1,x_2,\boldsymbol{y}) - {}^1F_{\bar{q},q}(x_1,x_2,\boldsymbol{y}) - {}^1F_{q,\bar{q}}(x_1,x_2,\boldsymbol{y}) + {}^1F_{\bar{q},\bar{q}}(x_1,x_2,\boldsymbol{y})\Big]$$
$$= \frac{2}{p^+}\int dy^- \langle p|\,\mathcal{O}_q(0,0)\,\mathcal{O}_q(y,0)|p\rangle\,. \quad (37)$$

$$M_{q,q}(\boldsymbol{y}^2) = \int d(py)\ \langle \mathcal{O}\mathcal{O}\rangle(py,y^2)\Big|_{y^2=-\boldsymbol{y}^2}.$$

$$(py)^2/(-y^2) = (\vec{p}\,\vec{y})^2/\vec{y}^{\,2} \ \le\ \vec{p}^{\,2}\,,$$