

QCD resummation: recent progresses and current challenges

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Fixed order QCD and resummation

- High energy strong interaction can be very well described by perturbative QCD (PT) through a power series in the (small) coupling constant
- Each extra power of the coupling corresponds to an extra real emission and relative virtual corrections
- Each real emission has a different scale for its strong coupling which is of the order of its transverse momentum
- In a PT expansion the coupling is evaluated at some common scale of the order of the hard scale(s) of the process - this ensures the coupling to be small (high transverse momentum)
- The PT expansion can be taken and the hard event is usually very well described by the first few terms in the power series

Fixed order QCD and resummation

- When the transverse momentum of the QCD radiation is constrained to be small, the strong coupling becomes large and an arbitrary amount of QCD emissions become equally important - Need for a all-orders description of the reaction
- The large coupling manifests itself in terms of large **single** logarithms in the perturbative series

$$\alpha_s(k_t^2) \sim \frac{\alpha_s(Q^2)}{1 - \alpha_s(Q^2)\beta_0 \ln \frac{Q^2}{k_t^2}}$$

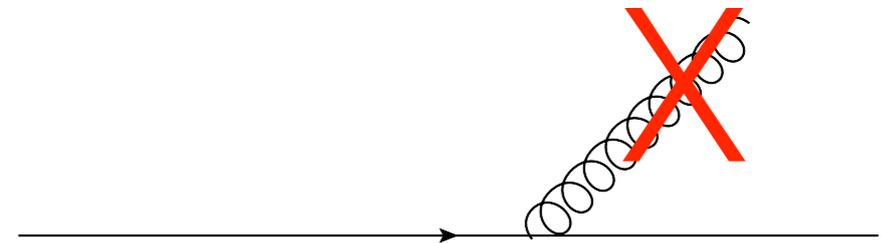
- One way to account for the large coupling effects is to sum up all these logarithms to all orders in PT series

Fixed order QCD and resummation

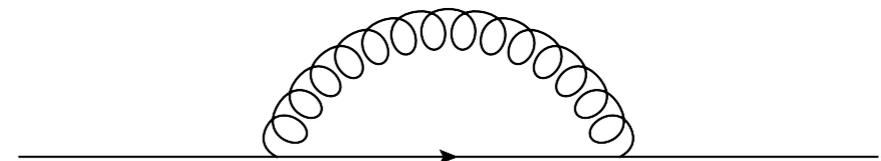
- Constraining the real radiation's kinematics leads to additional (e.g. **double**) logarithms
 - real emission forced to be soft and/or collinear to the emitter
 - virtual corrections are unaffected

e.g. soft-collinear case

$$\frac{dk_t}{k_t} d\eta \alpha_s(k_t^2)$$



$$-\frac{dk_t}{k_t} d\eta \alpha_s(k_t^2)$$



$$P(k_t < v) \sim 1 - \frac{\#\alpha_s C_F}{2\pi} \ln^2 v + \dots$$

Fixed order QCD and resummation

- In the perturbative regime these logarithms can become as large as (breakdown of the PT below this limit)

$$L \sim \frac{1}{\alpha_s}$$

- This makes “higher order” corrections as large as leading order ones, i.e. $(\alpha_s L)^n L \sim \alpha_s L^2$
- The PT series breaks down and the probability of the reaction diverges logarithmically in the large L limit instead of being suppressed
- The **resummation** of the large logarithms to all perturbative orders restores the correct physical (Sudakov) suppression and rescues the predictive power of perturbation theory

Logarithmic accuracy

- **Double** logarithms due to constraining the radiation kinematics commonly happen to exponentiate exactly (formalised with the concept of rIRC safety)
 - non-exponentiating observables are avoided because of issues with the simulation in event generators, e.g. JADE algorithm
- For such observables we can define a new perturbative order by expressing the cross section as an exponential function

$$\Sigma(v) = \int_0^v \frac{1}{\sigma_{\text{Born}}} \frac{d\sigma}{dv'} dv' \sim e^{\overset{\text{LL}}{\alpha_s^n L^{n+1}} + \overset{\text{NLL}}{\alpha_s^n L^n} + \overset{\text{NNLL}}{\alpha_s^n L^{n-1}} + \dots}$$

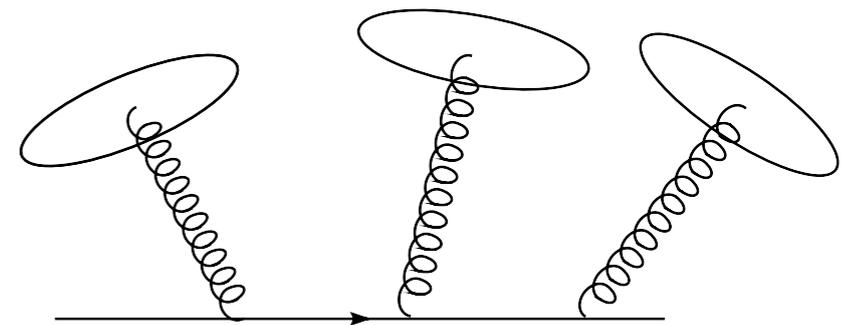
- In the region where $L \sim 1/\alpha_s(Q)$, LL are enhanced w.r.t. the Born, NLL are as large as the Born cross section itself, NNLL count as NLO corrections, and so on

The zero-jet rate

- In productions of EW/H bosons, veto on any accompanying jet activity is often required to suppress the background from decay of (heavy) coloured objects
 - At NLL, strong angular ordering ensures exponentiation in momentum space

$$\Sigma(p_{t,\text{veto}}) \sim \sigma_0 \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{i=n} \int [dk_i] |M(k_i)|^2 (\Theta(p_{t,\text{veto}} - k_{ti}) - 1)$$

$$= \sigma_0 \exp \left[- \int [dk_i] |M(k_i)|^2 \Theta(k_{ti} - p_{t,\text{veto}}) \right]$$

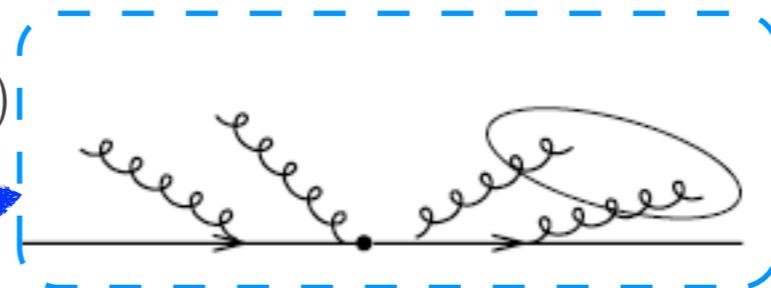


- Details of the jet (e.g. jet radius dependence) enter at NNLL

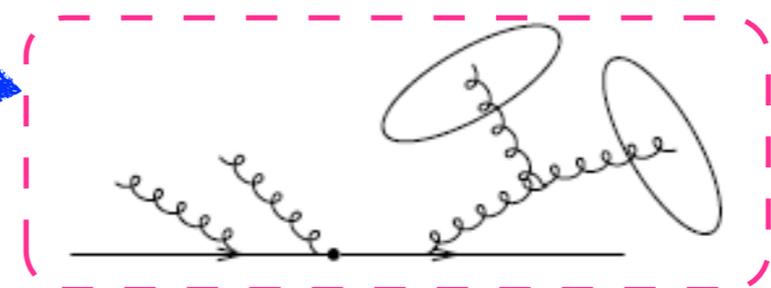
$$\Sigma(p_{t,\text{veto}}) = \mathcal{L}(p_{t,\text{veto}}) |\mathcal{M}_B|^2 \times e^{-R(p_{t,\text{veto}})} \mathcal{F}(p_{t,\text{veto}})$$

$$\mathcal{F}(p_{t,\text{veto}}) = 1 + \mathcal{O}(\text{NNLL})$$

Banfi, Monni, Salam, Zanderighi (2012)



$$\sim \alpha_s^n \ln^{n-1} \frac{M}{p_{t,\text{veto}}} R^2$$



$$\sim \alpha_s^n \ln^{n-1} \frac{M}{p_{t,\text{veto}}} \ln R$$

The zero-jet rate

- Same all-order structure for any colour-singlet production
 - NNLL predictions exist for several processes (matching with NNLO necessary)

H, Z: Banfi, Monni, Salam, Zanderighi (2012) WW: Jaiswal, Okui (2014)

H: Becher, Neubert, Rothen (2013)

Z, WW, WWW: Becher, Frederix, Neubert, Rothen (2014)

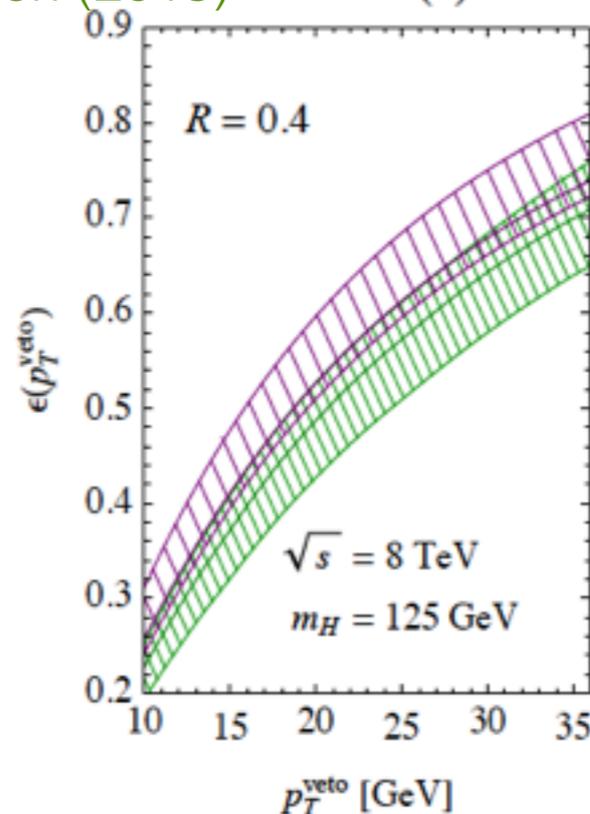
H: Stewart, Tackmann, Walsh, Zuberi (2013) (NNLL+NLO automated for colour-singlet production in MC@NLO)

WH: Boughezal, Focke, Li, Liu (2014)

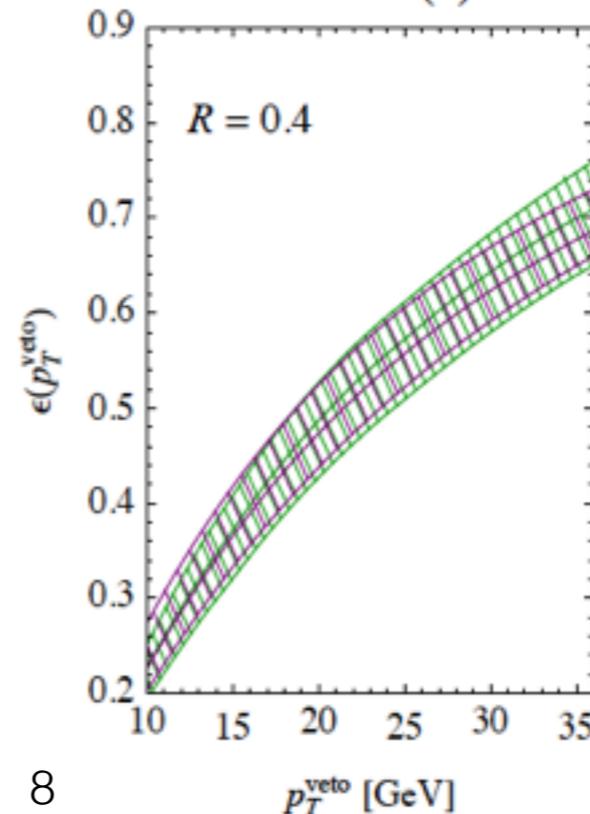
WZ, ZZ: Wang, Li, Liu (2015)

- Few corrections beyond NNLL also known, e.g. Higgs production

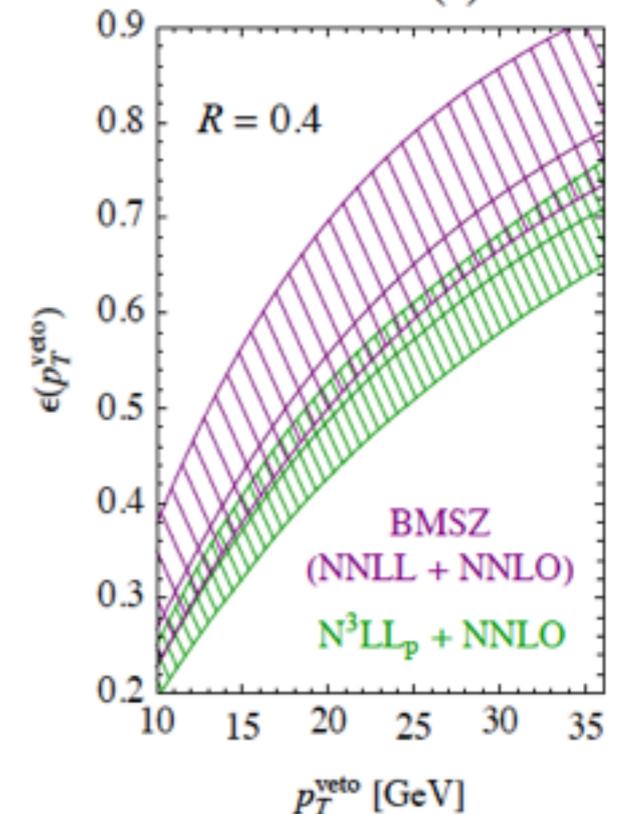
Becher, Neubert, Rothen (2013) Scheme (a)



Scheme (b)



Scheme (c)



Going to higher jet multiplicities

- How far can one go beyond this simple configuration ?
- Moving to higher tagged-jet multiplicities (e.g. notably the one-jet rate) requires to attack a number of problems
 - Higher number of hard (Born) legs, i.e. more emitters: QCD coherence allows cross-talk between different hard legs
 - Possible issues with factorisation of the measurement function: complicated observable's definition
 - Pattern of radiation inside (and outside) the tagged jet(s): non-global and clustering logarithms
 - Multi-scale problems: more sources of large logarithms should be treated simultaneously

Coloured final states

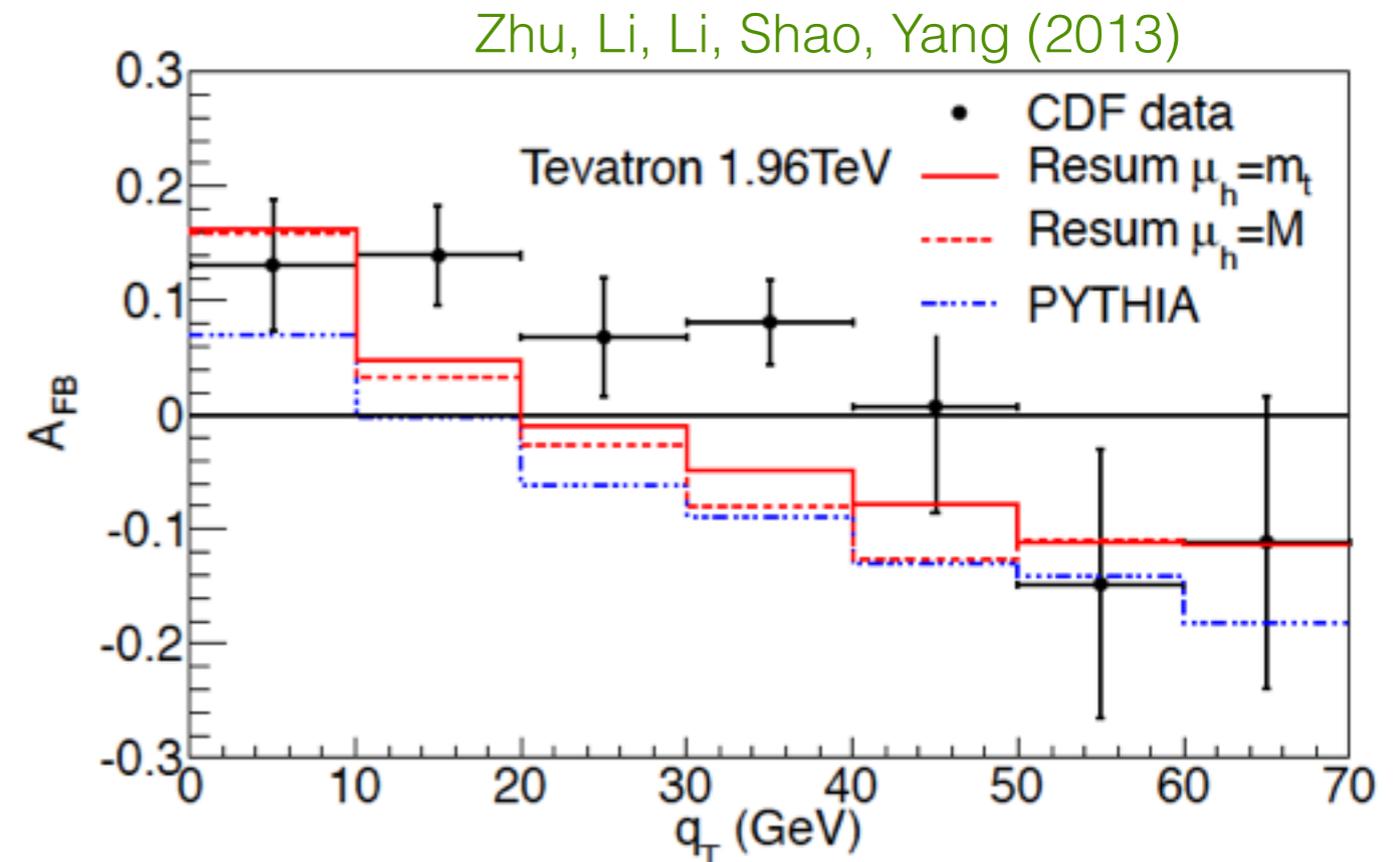
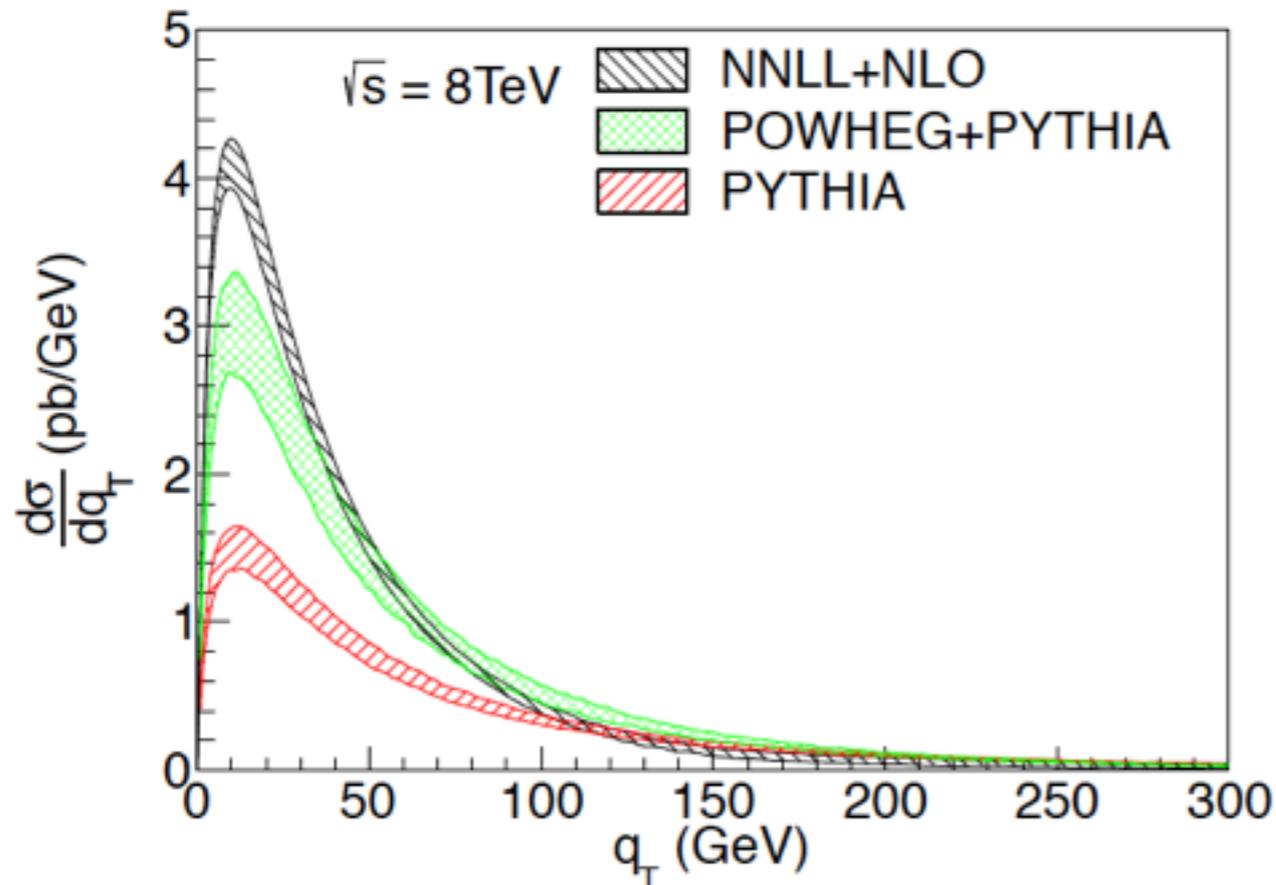
- The presence of strongly interacting partons in both initial and final states leads to some complications
 - Final (Initial) - Final state interference effects due to wide-angle soft radiation give rise to NLL contributions
 - Botts, Sterman (1989)
 - Sterman et al. (1996 - 2000)
 - Banfi, Marchesini, et al. (2000 - 2002)
 - Bonciani, Catani, Mangano, Nason (2003)
 - Same radiation also responsible for azimuthal correlation of final state, i.e. non-abelian exponentiation does not apply because of the azimuth integral
- Recently extended to NNLL for top-pair transverse momentum
 - Zhu, Li, Li, Shao, Yang (2012 - 2013)
 - Catani, Grazzini, Torre (2014)

$$\frac{d\sigma^{(\text{sing})}(P_1, P_2; \mathbf{q}_T, M, y, \Omega)}{d^2\mathbf{q}_T dM^2 dy d\Omega} = \frac{M^2}{2P_1 \cdot P_2} \sum_{c=q, \bar{q}, g} \left[d\sigma_{c\bar{c}}^{(0)} \right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} S_c(M, b)$$

$$\times \sum_{a_1, a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} [(\mathbf{H} \Delta) C_1 C_2]_{c\bar{c}; a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

Coloured final states

- Impact substantial in the low transverse momentum region



Matching with NNLO necessary at large q_T
(large K factor)

- Wide-angle interference effects at NLL can be automated for all rIRC safe observables

Banfi, Salam, Zanderighi 2004

- Is it possible to parametrise this structure at NNLL using the known two-loop singularities (single poles) of multiparton amplitudes ?

Catani (1998)

Dixon, Magnea, Sterman (2008)

Gardi, Magnea (2009)

Becher, Neubert (2009)

Ferrogia, Neubert, Pecjak, Yang (2009)

Factorisation

- Sudakov resummation is based on factorisation properties of soft and collinear singularities w.r.t. the hard matrix element

$$|\mathcal{M}(\{\tilde{p}\}, k_1, \dots, k_n)|^2 \simeq |M_{\text{Born}}(\{\tilde{p}\})|^2 |M(k_1, \dots, k_n)|^2 + \dots$$

- To gain a full factorisation of the cross section, one needs to factorise the measurement function for the observable - often this can be done in some conjugate space, e.g. thrust

$$1 - T \simeq \sum_{i=1}^n \frac{k_{ti}}{Q} e^{-\eta_i} \quad \rightarrow \quad \Theta(1 - T < \tau) = \int \frac{d\nu}{2\pi i \nu} e^{\nu\tau} \prod_{i=1}^n e^{-\nu \frac{k_{ti}}{Q} e^{-\eta_i}}$$

- If the observable is factorisable (a factorisation theorem exists), resummation is achieved via evolution equations for each of the kinematical subprocesses
- Factorisation requirement limits the range of observables which can be resummed. **Is it actually necessary ?**

Factorisation

- Factorisation of the measurement function is an **unnecessary** requirement (although it can often make one's life easier)
 - Several observables cannot be factorised in a trivial way as a product of contributions due to individual emissions (e.g. thrust major, two-jet rate in e^+e^- with a k_t algorithm,...)
 - The observables for which a factorisation theorem exists is a small subset of the resumable ones
- All one needs is a specific property of the observable known as recursive InfraRed and Collinear (rIRC) safety (see backup)

Banfi, Salam, Zanderighi (2001-2004)

 - This property together with QCD coherence set the ground for an approach that does not rely on any factorisation of the measurement function
 - Easily automated: CAESAR @ NLL, ARES @ NNLL

Banfi, Salam, Zanderighi (2001-2004)
Banfi, McAslan, Monni, Zanderighi (2014)

Automated NNLL resummation

- General method for [global, rIRC*](#) observables

Banfi, McAslan, Monni, Zanderighi (2014)

- currently worked out for two hard Born legs (i.e. DY-like, e+e-) - can deal with observables with different logarithmic structures

correction type	$p_{t,\text{veto}}$	$1 - T$	B_T	B_W	C	ρ_H	T_M	O	$y_3^{\text{Dur.}}$	$y_3^{\text{Cam.}}$
\mathcal{F}_{NLL}	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$\delta\mathcal{F}_{\text{rap}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x
$\delta\mathcal{F}_{\text{wa}}$	x	x	x	x	✓	x	x	x	✓	✓
$\delta\mathcal{F}_{\text{hc}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x
$\delta\mathcal{F}_{\text{rec}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x
$\delta\mathcal{F}_{\text{clust}}$	✓	x	x	x	x	x	x	x	✓	✓
$\delta\mathcal{F}_{\text{correl}}$	✓	x	✓	✓	x	x	✓	✓	✓	✓

- No factorisation required: jet observables are treated in the same way as other event shapes (e.g. two-jet rates)

Banfi, McAslan, Monni, Zanderighi (to appear)

- e+e- encode main conceptual issues: extension to more hard legs requires some technical work

*Observables with cancellations away from the Sudakov region (e.g. qT) require some gimmicks (in progress)

non-global logarithms

- Non global logarithms are due to wide-angle gluon radiation that propagates near the edge of the limited rapidity region where the measurement is performed (e.g. a jet)

Dasgupta, Salam (2001)

- Requires treatment of correlated splittings to all orders

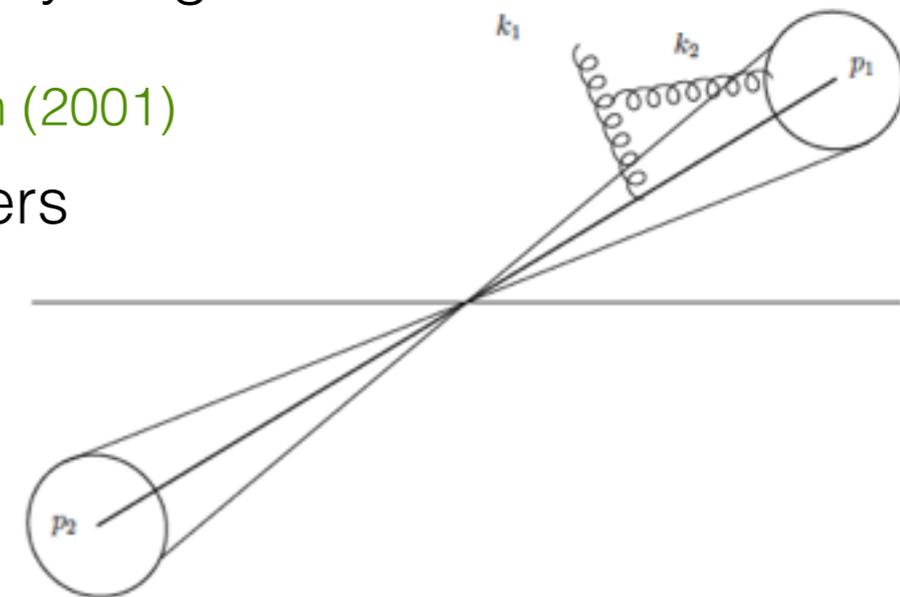
- Up to recently performed in the planar limit @ NLL

$$E\partial_E G_{ab}(E) = \int \frac{d^2\Omega_k}{4\pi} \bar{\alpha}_s w_{ab}(k) [u(k) G_{ak}(E) \cdot G_{kb}(E) - G_{ab}(E)] \quad \bar{\alpha}_s := \alpha_s N_c / \pi$$

Dasgupta, Salam (2001); Banfi, Marchesini, Smye (2002)

- Extension to the finite N_c case obtained through analogy between BMS and BK equations

Weigert (2003)



BK equation

BMS equation

B-JIMWLK equation

$$E\partial_E G_{pq}(E) = \int \frac{d\Omega_k}{4\pi} \frac{\alpha_s}{\pi} w_{pq}(k) \left\langle u(k) \left(\hat{G}_{pk} \hat{G}_{kq} N_c - \frac{G_{pq}}{N_c} \right) - 2C_f \hat{G}_{pq} \right\rangle (E)$$

$N_c \rightarrow \infty$

- Numerical solution challenging, but possible... Hatta, Ueda (2014)

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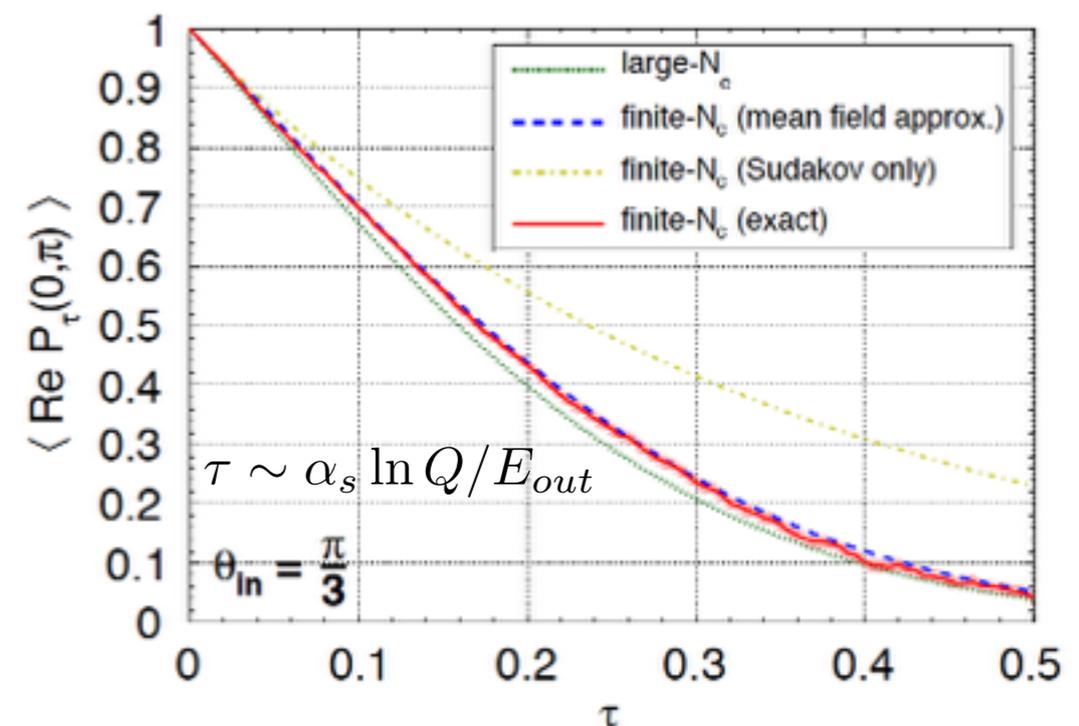
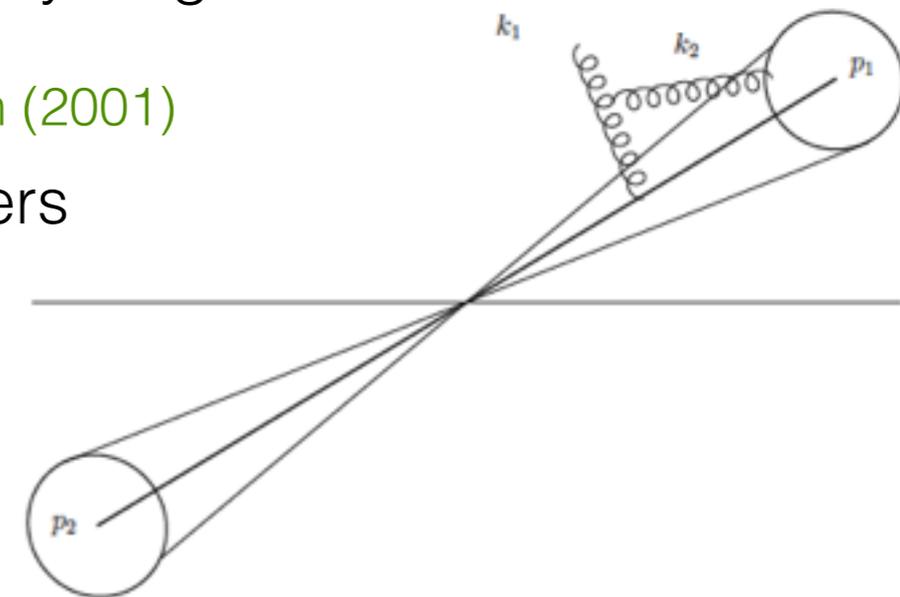
- Extension to the finite N_c case obtained through analogy between BMS and BK equations

- ... and corrections found to be small

e.g. measure the energy outside of a jet with fixed angular opening in $e+e-$

Hatta, Ueda (2014)

- Can we go beyond NLL ?



non-global logarithms

- Recently the problem has been reformulated in terms of a *soft colour-density matrix*, which satisfies a linear RGE equation

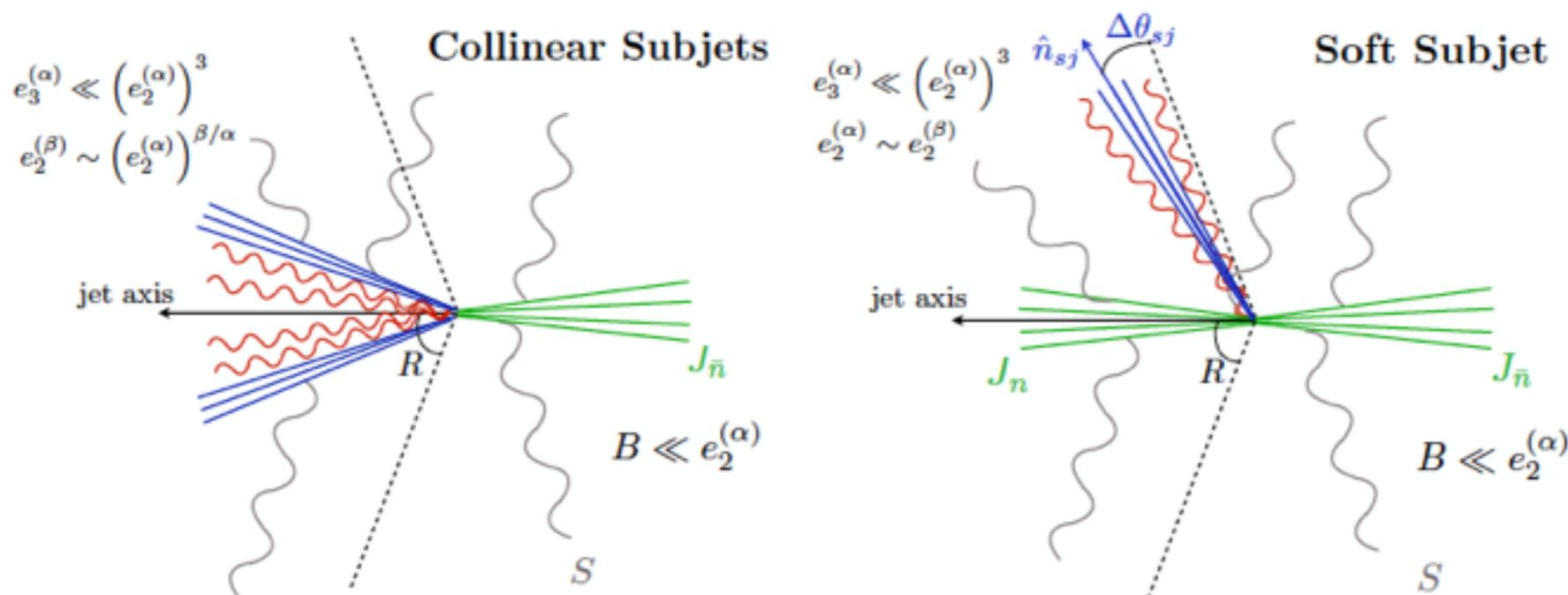
Caron-Huot (2015)

$$\sigma[U] \equiv \sum_n \int d\Pi_n [A_n^{a_1 \dots a_n}(\{p_i\})]^* U^{a_1 b_1}(\theta_1) \dots U^{a_n b_n}(\theta_n) [A_n^{b_1 \dots b_n}(\{p_i\})] u(\{p_i\}) \quad \sigma[U] = \mathcal{P} \exp \left[- \int_0^\mu \frac{d\lambda}{\lambda} K(\lambda, \alpha_s(\lambda)) \right] \sigma^{\text{ren}}[U; \mu, \alpha_s(\mu), \epsilon]$$

- The anomalous dimension K at each order is defined through non-linear equations in terms of colour generators - solvable recursively by MC techniques
- The **one-loop** case reproduces the NLL results with full colour dependence - the BMS equation is recovered in the large N_c limit
- Explicit equation for the **two-loop** K (necessary for NNLL) is given in the gluon-only case (quarks can be included)
- Potentially possible to sum non-global logarithms to NNLL !

non-global logarithms

- First understanding of non-global logarithms in a SCET framework Larkoski, Moult, Neill (2015)
 - Main idea: utilise a triple differential cross section for three different energy correlation functions to parametrise a wide-angle soft subjet



- a factorisation theorem derived for *additive* out-of-jet observable B

$$e_2^{(\beta)} = \frac{1}{E_J^2} \sum_{i < j \in J} E_i E_j \left(\frac{2p_i \cdot p_j}{E_i E_j} \right)^{\beta/2},$$

$$e_3^{(\beta)} = \frac{1}{E_J^3} \sum_{i < j < k \in J} E_i E_j E_k \left(\frac{2p_i \cdot p_j}{E_i E_j} \frac{2p_i \cdot p_k}{E_i E_k} \frac{2p_j \cdot p_k}{E_j E_k} \right)^{\beta/2}$$

$$\frac{d\sigma(B; R)}{de_2^{(\alpha)} de_2^{(\beta)} de_3^{(\alpha)}} = H(Q^2) H_{n\bar{n}}^{sj} \left(e_2^{(\alpha)}, e_2^{(\beta)} \right) J_n \left(e_3^{(\alpha)} \right) \otimes J_{\bar{n}}(B)$$

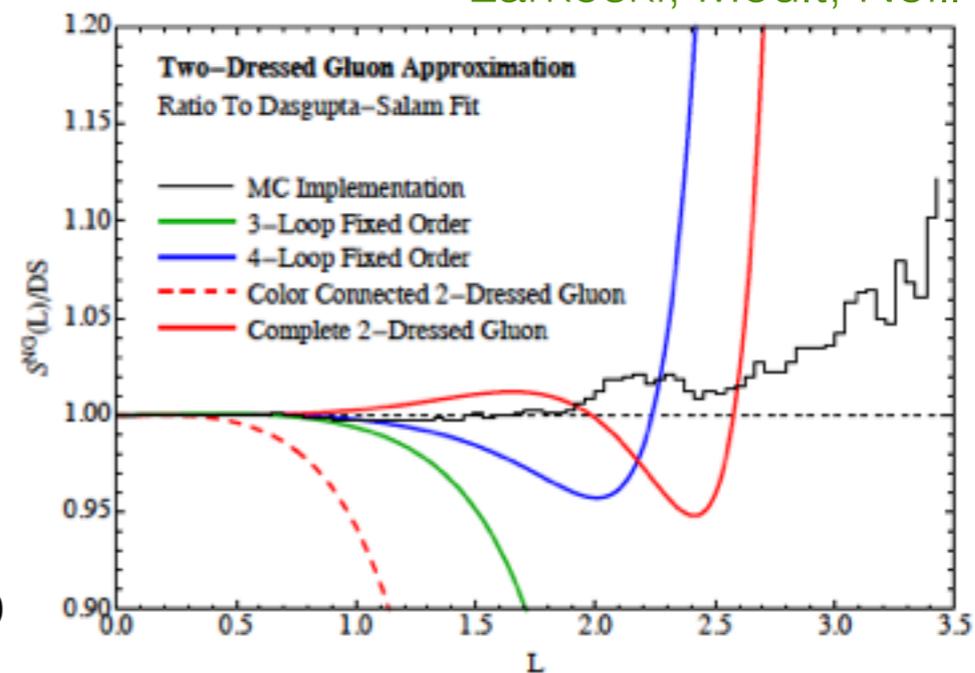
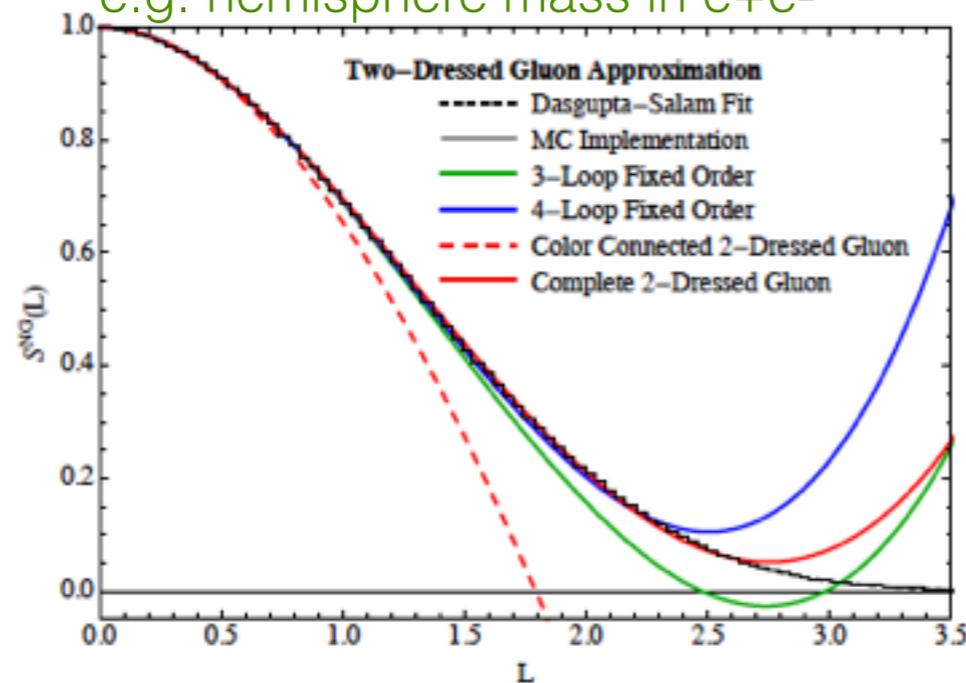
$$\otimes S_{n\bar{n}n_{sj}} \left(e_3^{(\alpha)}; B; R \right) \otimes J_{n_{sj}} \left(e_3^{(\alpha)} \right) \otimes S_{n_{sj}\bar{n}_{sj}} \left(e_3^{(\alpha)}; R \right)$$

non-global logarithms

- The single-soft-subjet case gives the all-order non-global logarithms generating from a single wide-angle soft gluon inclusive in its branchings (i.e. single-dressed-gluon approximation)
 - soft-emissions in the subjet are resolved down to the IR cutoff set by $e_3^{(\beta)}$
 - it describes NLL non-global logarithms at $\mathcal{O}(\alpha_s^2)$, and partly at higher-orders (only single-dressed-gluon contributions)
- Additional soft subjects can be included by recursive matchings, and compute non-global logarithms to higher orders in the coupling (formal logarithmic accuracy cannot be established)

e.g. hemisphere mass in $e+e-$

Larkoski, Moult, Neill (2015)

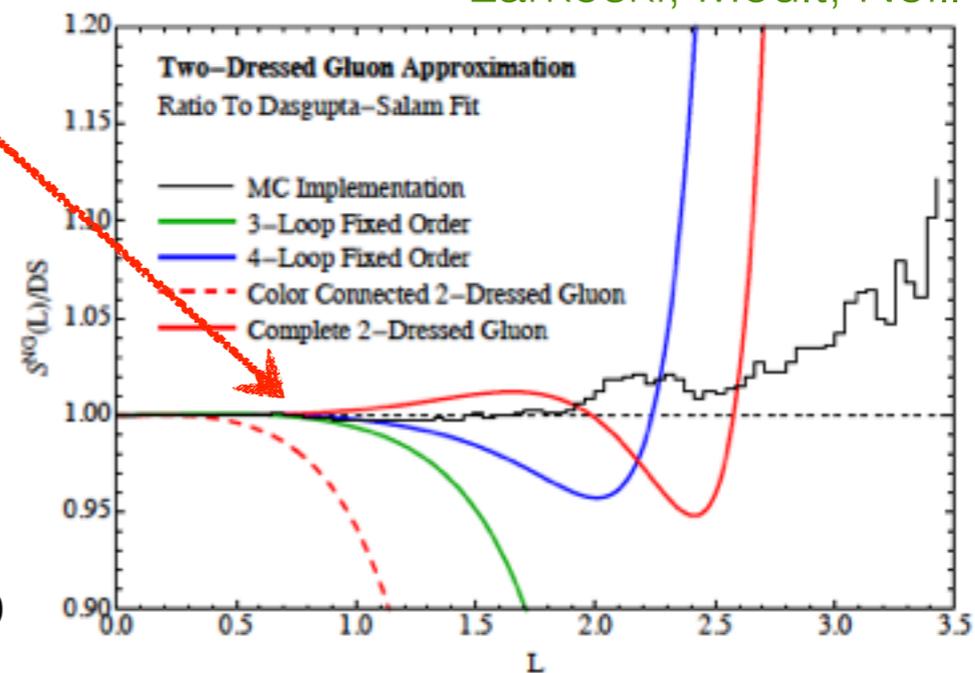
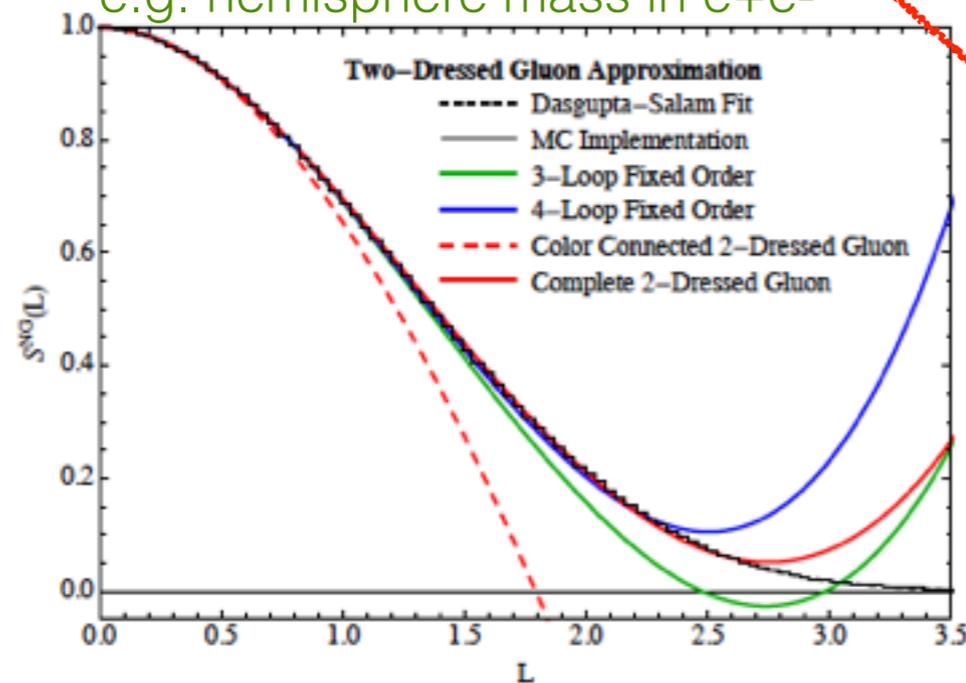


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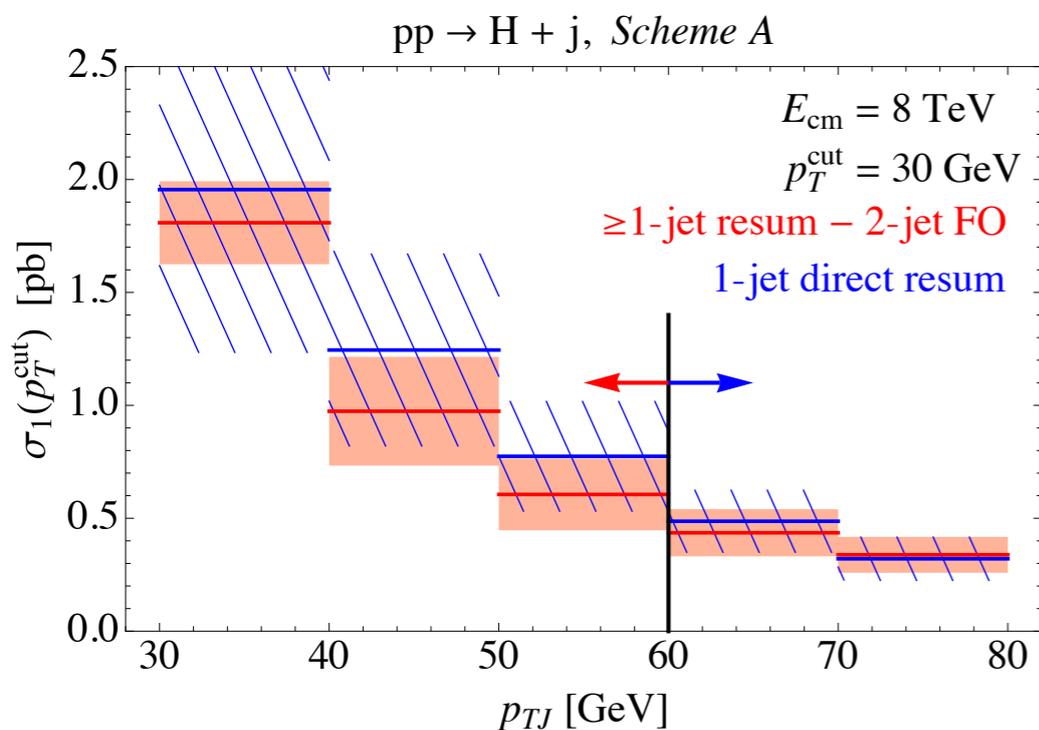
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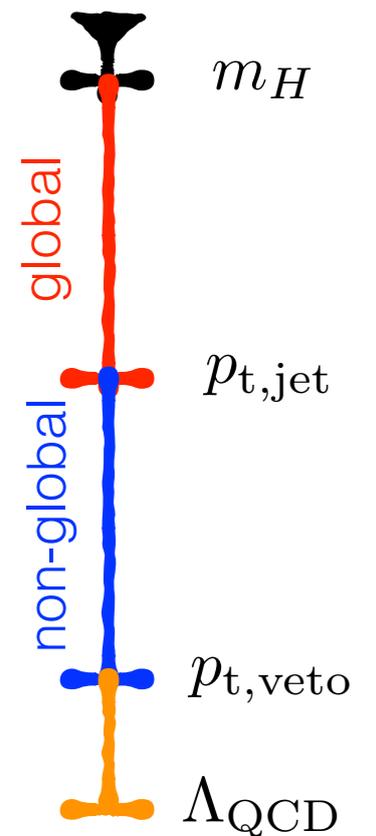


Multiple-scale problems

- Additional scales in the problem can increase the number of logarithmic sources, e.g. H+1 jet rate
- Two different (possibly large) scale gaps: each region is governed by a different resummation
 - e.g. jet at the hard scale ($p_{t,\text{jet}} \sim m_H$)



Control on terms
 $\alpha_s^{2n} L^{2n} + \alpha_s^{2n} L^{2n-1}$
 in the expansion
 Liu, Petriello (2012)
 Boughezal, Liu, Petriello, Tackmann (2013)



- Not yet clear how to describe them simultaneously in the regime

$$m_H \gg p_{t,\text{jet}} \gg p_{t,\text{veto}}$$

Inside jets

- Hadronic decays of boosted particles clustered into “fat” jets
- Jet substructure as a tool to distinguish signal from background (tagging), and/or clean jets from UE and pileup (grooming)
- Large number of grooming/tagging techniques: qualitative similarities observed with MC studies. First quantitative (analytic) understanding for the (groomed) jet mass [Dasgupta, Fregoso, Marzani, Salam \(2013\)](#)
- The plain case @ NLL:

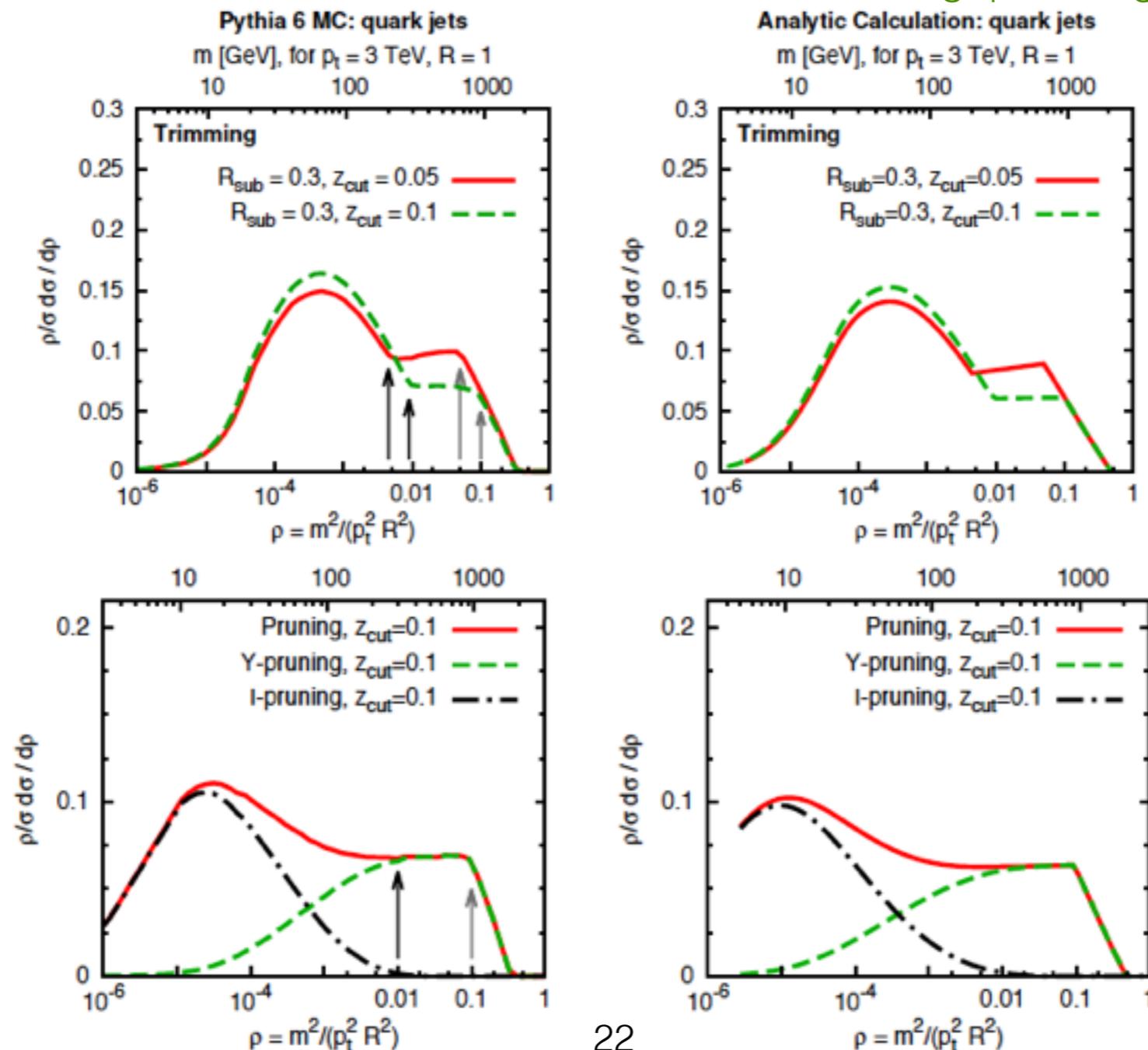
$$\Sigma(\rho) = e^{-D(\rho)} \cdot \frac{e^{-\gamma_E D'(\rho)}}{\Gamma(1 + D'(\rho))} \cdot \mathcal{N}(\rho) \quad \rho \equiv \frac{m^2}{p_t^2 R^2}$$

- Radiation pattern (logarithmic structure) affected by grooming techniques

Inside jets

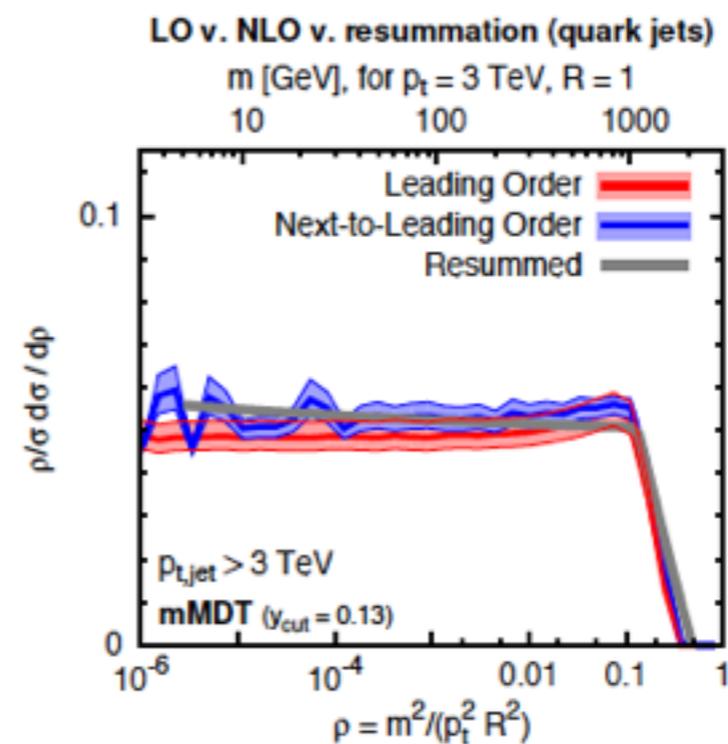
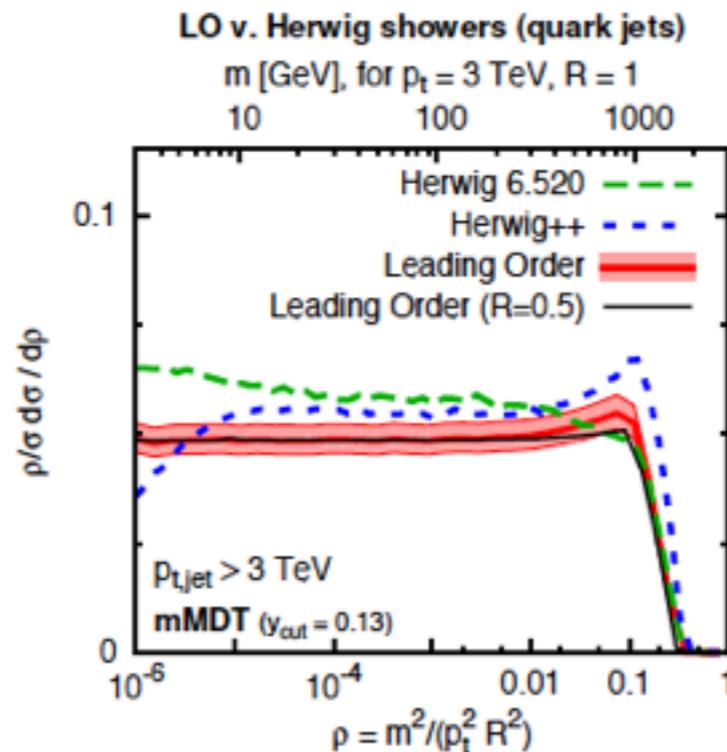
- Analytic resummation explains features of grooming techniques (e.g. trimming, pruning, mass-drop tagger)

Dasgupta, Fregoso, Marzani, Salam (2013)



Inside jets

- And helps devise new tools with a better logarithmic structure
 - e.g. modified mass drop tagger (mMDT): just collinear (single) logarithms and no non-global Dasgupta, Fregoso, Marzani, Salam (2013)



- Generalized with the Soft-drop: recursive de-clustering of a jet unless*

$$\frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\text{cut}} \left(\frac{\Delta R_{12}}{R_0} \right)^\beta \quad \text{Larkoski, Marzani, Soyez, Thaler (2014)}$$

*jet's energy loss after grooming not IRC safe for $\beta = 0$. However, LL expression finite

“Sudakov safety” Larkoski, Thaler (2013)

Larkoski, Marzani, Thaler (2015)

Small-R jets

Dasgupta, Dreyer, Salam, Soyez (2014)

- Small-R jets arise in several substructure methods (i.e. building small jets out of “fat” ones), and heavy-ion collisions
 - For small R, collinear splittings are resolved. As a result, logarithms of the form $\alpha_s^n \ln^n R^2$ appear in the perturbative expansion (e.g. 0-jet XS)
 - For such “micro-jets”, leading logarithms can be resummed through evolution equations for the micro-jet’s “fragmentation function” $f(t, z)$ in the angular variable

$$t(R, p_t) = \int_{R^2}^1 \frac{d\theta^2}{\theta^2} \frac{\alpha_s(p_t \theta)}{2\pi}$$

- Physical observables are obtained by convoluting the proper fragmentation function with the differential distributions

Small-R jets

Dasgupta, Dreyer, Salam, Soyez (2014)

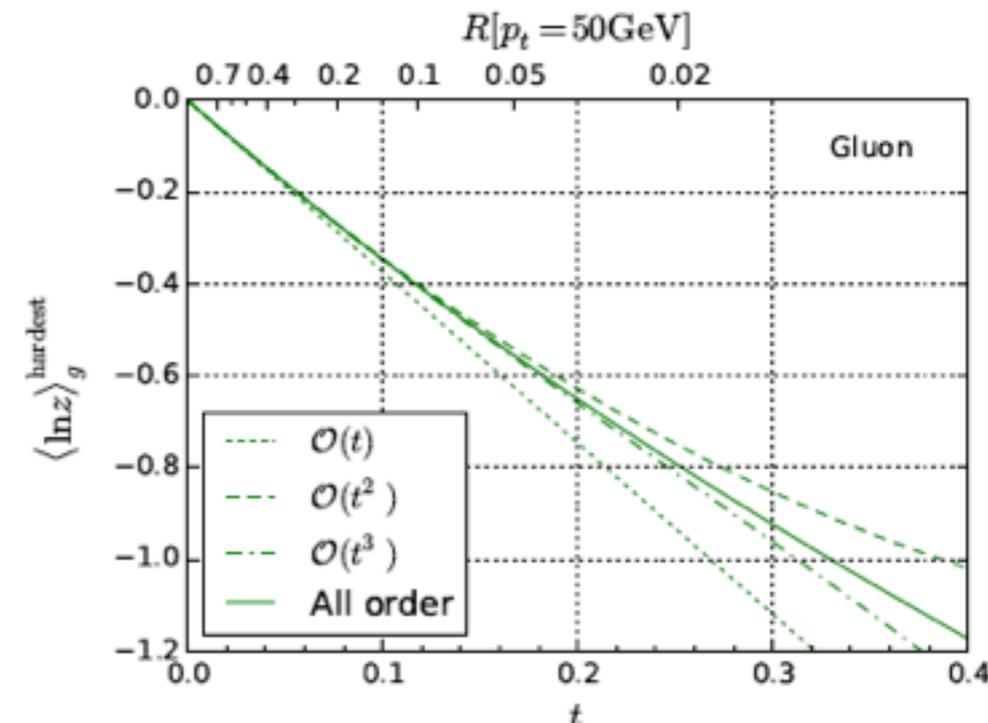
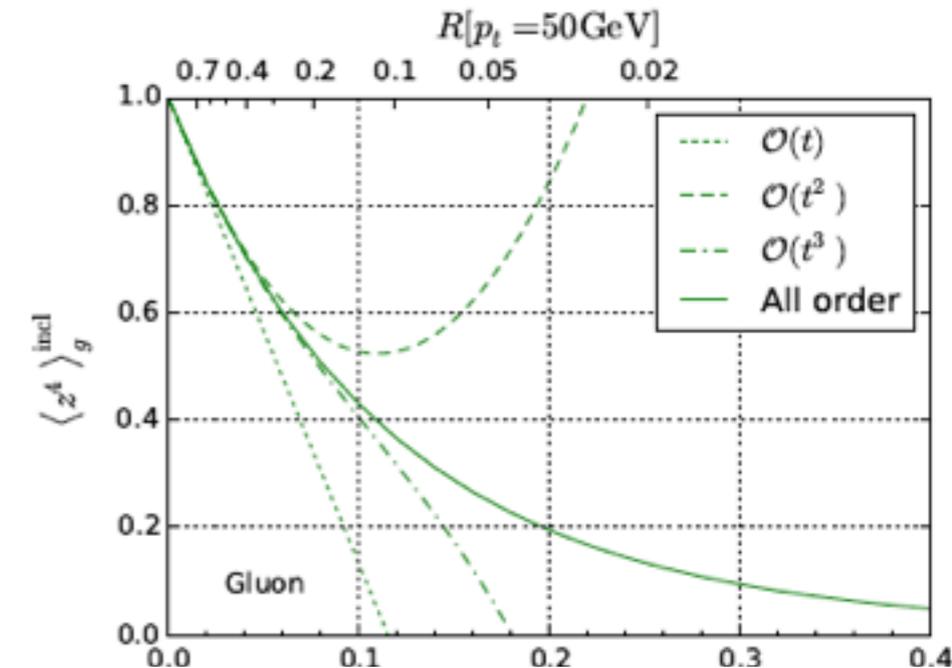
- Inclusive-jet spectrum is reduced by 30-50% for gluon jets

$$\frac{d\sigma_{\text{jet}}}{dp_t} \simeq \frac{d\sigma_i}{dp_t} \int_0^1 dz z^{n-1} f_{\text{jet}/i}^{\text{incl}}(z, t)$$

- moderate effect on 0-jet cross section

$$\mathcal{U} \equiv \frac{P(\text{no microjet veto})}{P(\text{no primary-parton veto})}$$

$$= \exp \left[-2\bar{\alpha}_s(p_t) \ln \frac{Q}{p_t} \int_0^1 dz f^{\text{hardest}}(z, t(R, p_t)) \ln z \right]$$



Other recent developments

- Progresses in threshold resummation:
 - Higgs production at N3LL [Ahrens et al.](#); [Bonvini et al.](#); [Catani et al.](#)
 - Single top production at NNLL [Yang et al.](#)
 - Stop-pair, squark and gluino production at NNLL [Broggio et al.](#); [Beenakker et al.](#)
 - Boosted top at NNLL (also mass logarithms) [Ferroglia et al.](#)
- electron-positron event shapes at NN(N)LL:
 - NNLL resummation has significant impact on coupling determinations (current NN(N)LL+NNLO fits lead to low values in tension with WA)
 - large experimental anti-correlation between alphas and NP corrections (need for observables with different - lower - sensitivity to NP effects to mitigate fit degeneracy)
 - Recently resummation extended to several event-shapes (global fit at NNLL+NNLO possible) [Banfi et al.](#); [Becher et al.](#); [Hoang et al.](#)

Thank you for your attention

Requirements on the observable

- Parametrisation for single emission and collinear splitting

$$V(\{\tilde{p}\}, \kappa_i(\zeta_i)) = \zeta_i; \quad \kappa_i(\zeta) \rightarrow \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu), \quad \mu^2 = (\kappa_{ia} + \kappa_{ib})^2 / \kappa_{ti}^2$$

- The standard requirement of **IRC safety** implies that

$$\begin{aligned} & \lim_{\zeta_{m+1} \rightarrow 0} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \dots, \kappa_m(\bar{v}\zeta_m), \kappa_{m+1}(\bar{v}\zeta_{m+1})) \\ & \qquad \qquad \qquad = V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \dots, \kappa_m(\bar{v}\zeta_m)) \\ & \lim_{\mu \rightarrow 0} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \dots, \{\kappa_{ia}, \kappa_{ib}\}(\bar{v}\zeta_i, \mu), \dots, \kappa_m(\bar{v}\zeta_m)) \\ & \qquad \qquad \qquad = V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \dots, \kappa_i(\bar{v}\zeta_i), \dots, \kappa_m(\bar{v}\zeta_m)) \end{aligned}$$

- We limit ourselves to **continuously** global observables*, i.e. the transverse momentum dependence is the same everywhere (it ensures the absence of non-global logarithms)

*Not a real limitation, although currently NNLL structure of non-global logarithms unknown

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- Impose the following conditions, known as recursive IRC (**rIRC**) safety [Banfi, Salam, Zanderighi]

$$\lim_{\bar{v} \rightarrow 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \dots, \kappa_m(\bar{v}\zeta_m)) \quad (1)$$

- The above limit must be **well defined and non-zero** (except possibly in a phase space region of zero measure)
- Condition (1) simply requires the observable to scale in the same fashion for multiple emissions as for a single emission (**IRC divergences have an exponential form**)
- It is enough to ensure the **exponentiation of double logarithms** to all orders

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$$= \lim_{\bar{v} \rightarrow 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \dots, \kappa_m(\bar{v}\zeta_m)) \quad (2.a)$$

$$\lim_{\mu \rightarrow 0} \lim_{\bar{v} \rightarrow 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \dots, \{\kappa_{ia}, \kappa_{ib}\}(\bar{v}\zeta_i, \mu), \dots, \kappa_m(\bar{v}\zeta_m))$$

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- Impose the following conditions, known as recursive IRC (**rIRC**) safety [Banfi, Salam, Zanderighi]
- Conditions (2.a) and (2.b), in addition to plain IRC safety, require that for sufficiently small \bar{v} there exists some ϵ that can be chosen *independently* of \bar{v} such that we can neglect any emissions at scales $\sim \epsilon\bar{v}$
- It allows one to establish a logarithmic hierarchy for (many) real emissions

$$\begin{aligned} & \lim_{\mu \rightarrow 0} \lim_{\bar{v} \rightarrow 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \dots, \{\kappa_{ia}, \kappa_{ib}\}(\bar{v}\zeta_i, \mu), \dots, \kappa_m(\bar{v}\zeta_m)) \\ &= \lim_{\bar{v} \rightarrow 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \dots, \kappa_i(\bar{v}\zeta_i), \dots, \kappa_m(\bar{v}\zeta_m)) \quad (2.b) \end{aligned}$$