Avalanche Statistics

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RD51 Collaboration Meeting, 14 October 2008

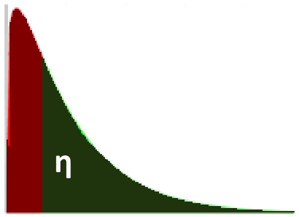
- The random nature of the electron multiplication process leads to fluctuations in the avalanche size
  → probability distribution $P(n, x)$ that an avalanche contains n electrons after a distance x from its origin.
- Together with the fluctuations in the ionization process, avalanche fluctuations set a fundamental limit to detector resolution

Motivation

- Exact shape of the avalanche size distribution $P(n, x)$ becomes important for small numbers of primary electrons.
- Detection efficiency

$$\eta = \sum_{n=n_T}^{\infty} P(n,x)$$

is affected by $P(n, x)$



Outline

- Review of avalanche evolution models and the resulting distributions
- Results from single electron avalanche simulations in Garfield using the recently implemented microscopic tracking features

Assumptions

- homogeneous field $\boldsymbol{E} = (E, 0, 0)$
- avalanche initated by a single electron
- space charge and photon feedback negligible

Assumption

- ionization probability *a* (per unit path length) is the same for all avalanche electrons
- *a* = *α* (Townsend coefficient)
- In other words: the ionization mean free path has a mean λ = 1/α and is exponentially distributed

$$\rho(l) = \alpha e^{-\alpha l}$$

Mean avalanche size

$$G = \bar{n}(x) = e^{\alpha x}$$

Distribution

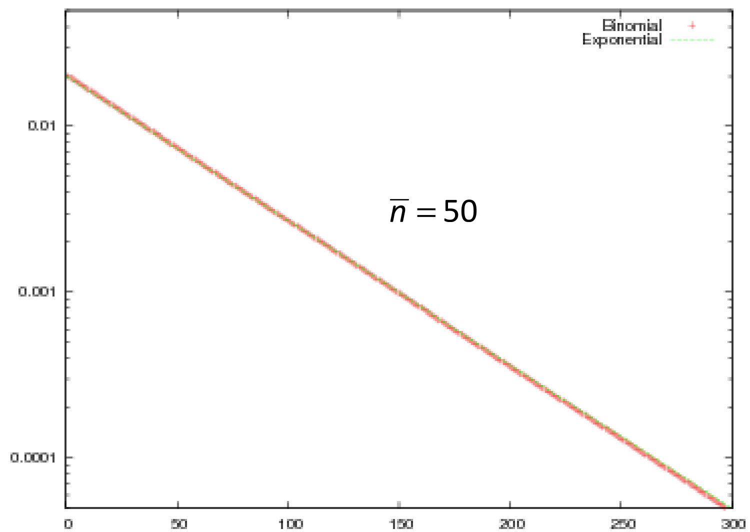
- The avalanche size follows a **binomial** distribution

$$P(n,x) = \frac{1}{\bar{n}(x)}\left(1 - \frac{1}{\bar{n}(x)}\right)^{n-1}$$

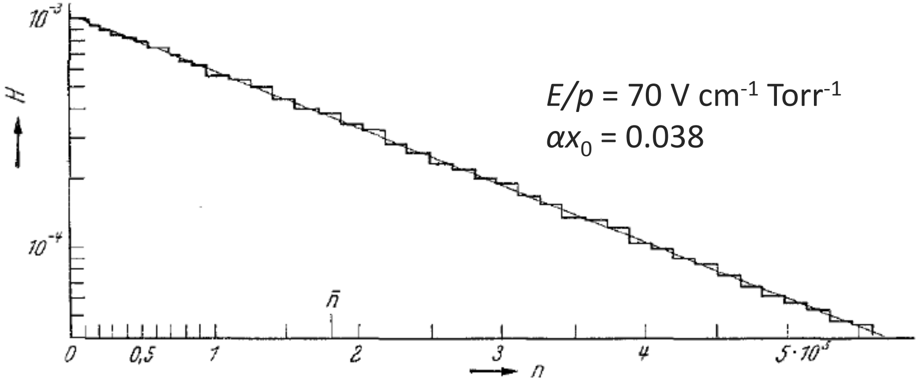
- For large avalanche sizes, $P(n,x)$ can be well approximated by an **exponential**

$$P(n) = \frac{1}{\bar{n}}\exp(-n/\bar{n})$$

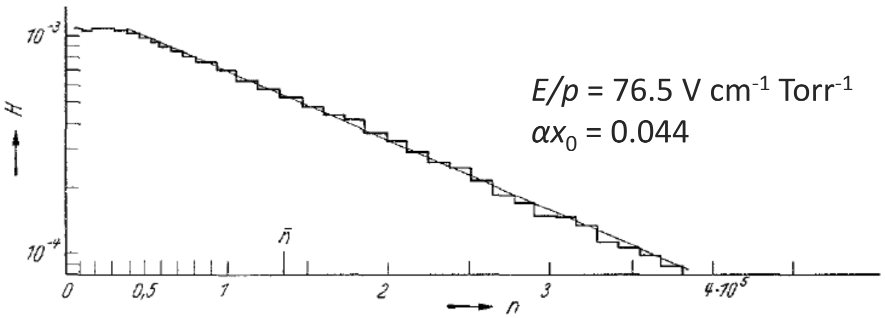
- Efficiency $\eta = e^{-n/\bar{n}}$



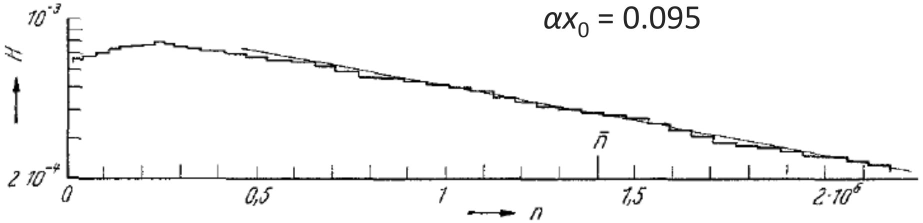
- measurements in methylal by H. Schlumbohm → significant deviations from the exponential at large reduced fields
- „rounding-off“ characterized by parameter αx_0 ($x_0 = U_i/E$)



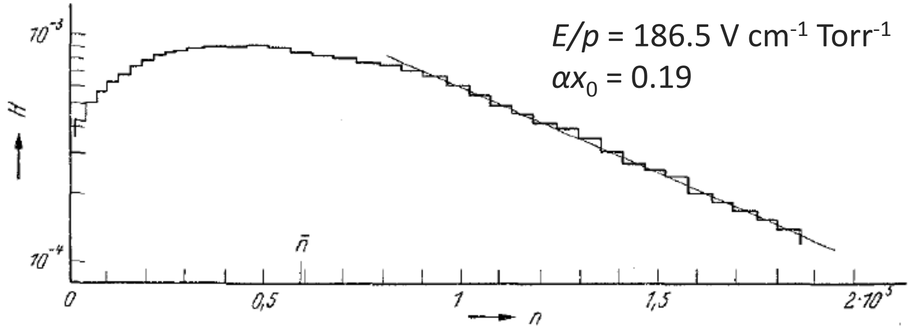
E/p = 70 V cm^{-1} Torr^{-1}
αx_0 = 0.038



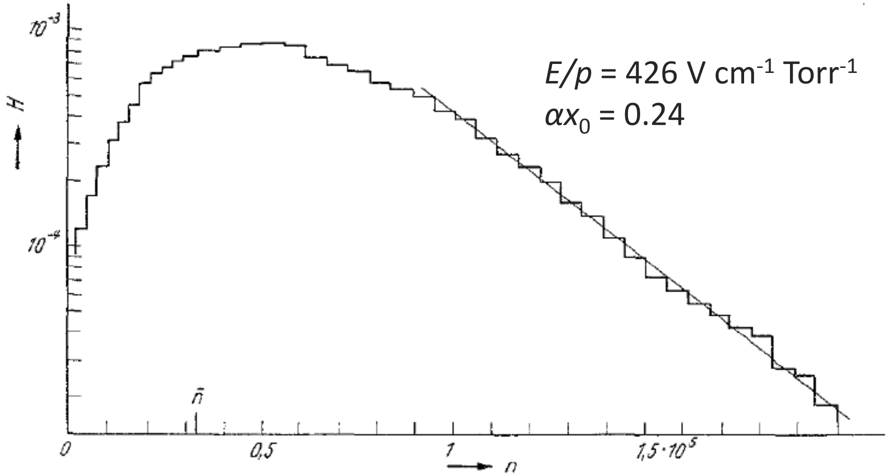
E/p = 76.5 V cm^{-1} Torr^{-1}
αx_0 = 0.044



E/p = 105 V cm^{-1} Torr^{-1}
αx_0 = 0.095



E/p = 186.5 V cm^{-1} Torr^{-1}
αx_0 = 0.19

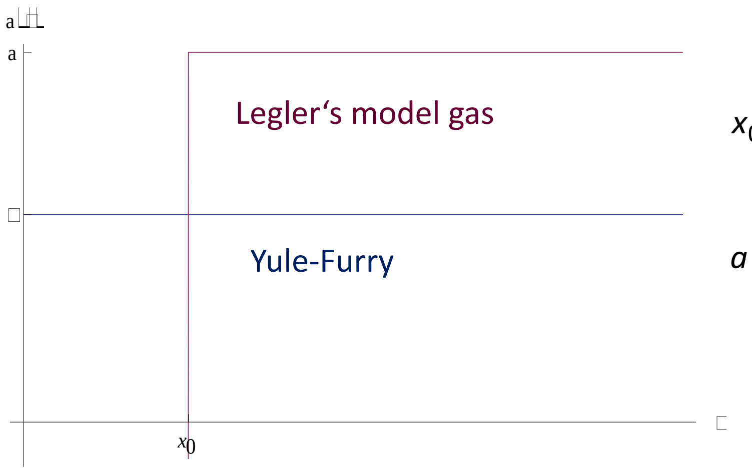


E/p = 426 V cm^{-1} Torr^{-1}
αx_0 = 0.24

H. Schlumbohm, *Zur Statistik der Elektronenlawinen im ebenen Feld*, Z. Physik **151**, 563 (1958)

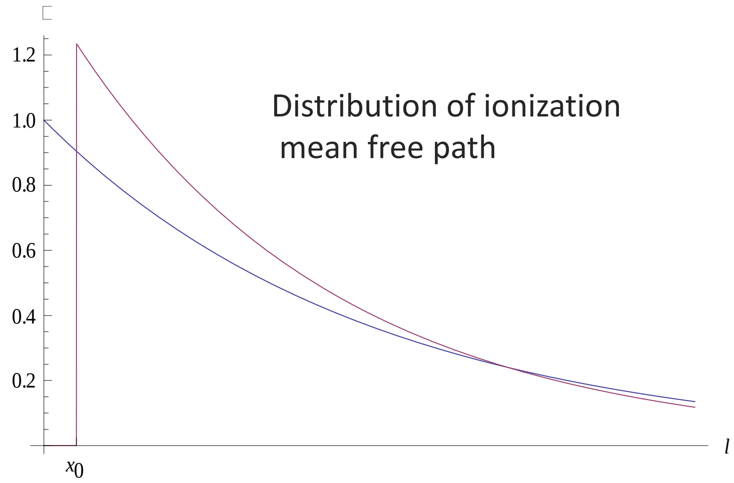
Legler's approach

Electrons are created with energies below the ionization energy eU_i and lose most of their kinetic energy after an ionizing collision
→ electron has to gain energy from the field before being able to ionize
→ *a* depends on the distance ξ since the last ionizing collision



$$x_0 \approx \frac{U_i}{E}$$

$$a = \frac{\alpha}{2e^{-\alpha x_0} - 1}$$



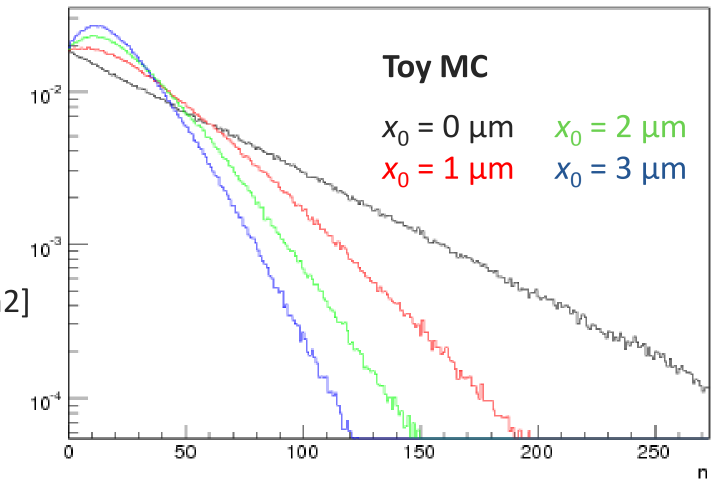
W. Legler, *Die Statistik der Elektronenlawinen in elektronegativen Gasen, bei hohen Feldstärken und bei großer Gasverstärkung*, Z. Naturforschg. **16a**, 253-261 (1961)

Mean avalanche size

$$\bar{n}(x) \propto e^{\alpha x}$$

Distribution

The **shape** of the distribution is characterized by the **parameter αx_0** □ [0, ln2]
αx_0 □ 1 → Yule-Furry
With increasing αx_0 the distribution becomes more „rounded",
maximum approaches mean



$$P(n,x)=\int \rho(l)dl \sum_{n'=1}^{n-1} P(n-n',x-l)P(n',x-l)$$

→ moments of the distribution can be calculated (as shown by Alkhazov)
→ allows (very) approximative reconstruction of the distribution (convergence problem)

↓ $\bar{n} >> 1$

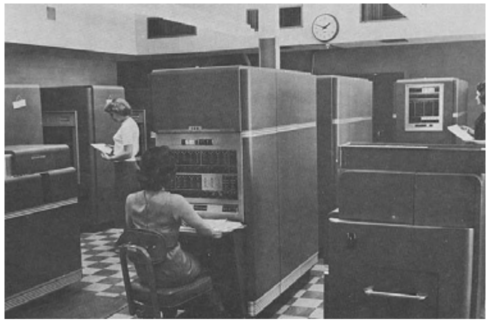
$$P(n,x)=\frac{1}{\bar{n}(x)}\varphi(\nu), \quad \nu=\frac{n}{\bar{n}}$$

G. D. Alkhazov, *Statistics of Electron Avalanches and Ultimate Resolution of Proportional Counters,* Nucl. Instr. Meth. **89**, 155-165 (1970)

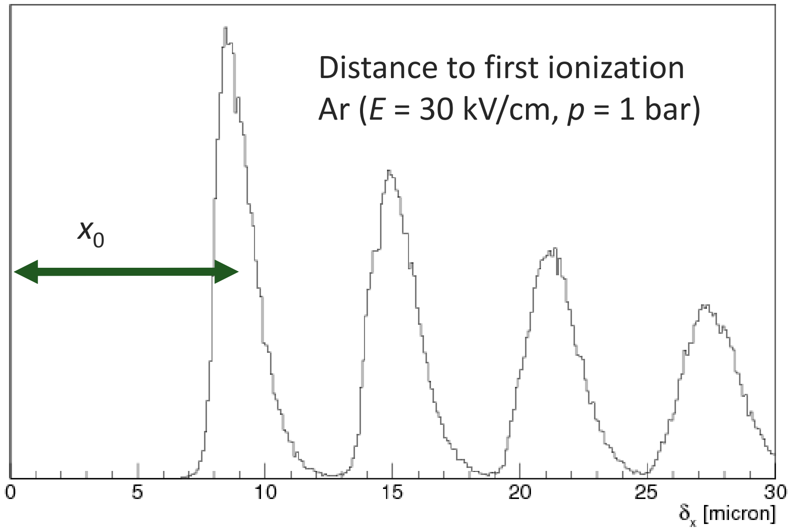
↓

$$\varphi(\nu)=\frac{1}{\alpha\nu}\int_{\nu}^{\infty} d\nu \rho\left(\frac{1}{\alpha}\ln\frac{\nu'}{\nu}\right)\int_{0}^{\nu'} d\nu' \varphi(\nu'-\nu'')\varphi(\nu'')$$

→ no closed-form solution
numerical solution difficult



„Die Rechnungen wurden mit dem Magnettrommelrechner IBM 650 (...) durchgeführt."



- „bumps“ seem to indicate avalanche evolution in steps
- an electron is stopped after a typical distance $x_0 \propto 1/E$ of the order of several µm
- with probability p it ionizes, with probability $(1 - p)$ it loses its energy in a different way after each step

$$p = e^{\alpha x_0} - 1$$

Mean avalanche size after *k* steps

$$\bar{n}_\mathrm{k} = (1+p)^k$$

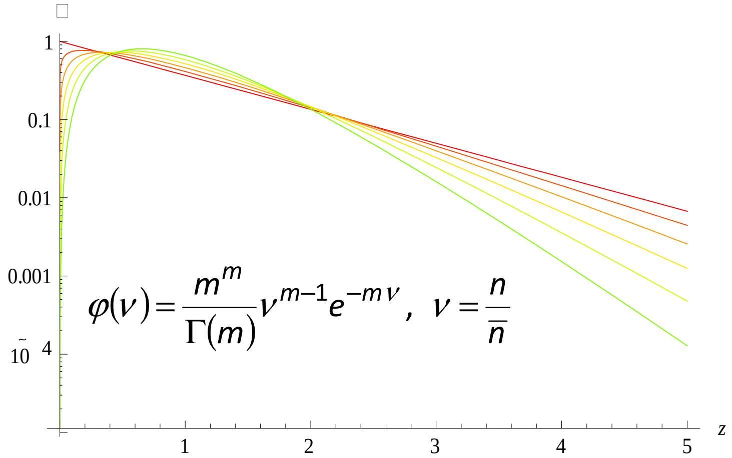
Distribution

$$P_{k+1}(n) = (1-p)P_k(n) + p\sum_{n'=0}^{n-1} P_k(n-n')P_k(n')$$

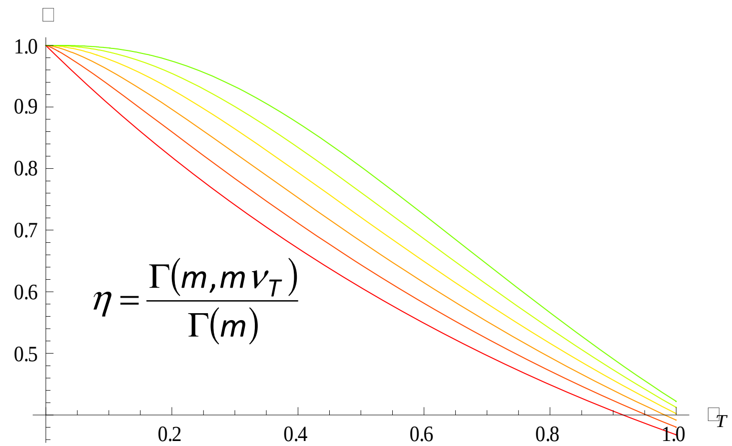
moments can be calculated, but no solution in closed form

p = 1 → delta distribution
p small → exponential

Pólya distribution



Efficiency



Good agreement with experimental avalanche spectra

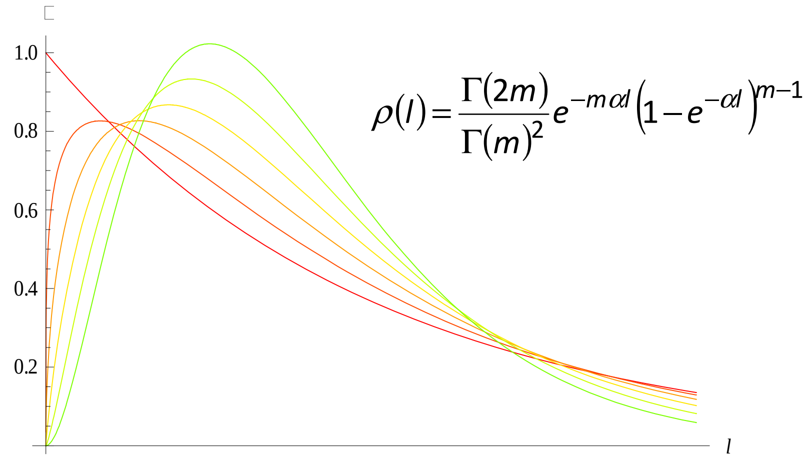
Problem: no (convincing) physical interpretation of the parameter *m*

Byrne‘s approach:

$$\alpha(n,x)=\alpha\left(1+\frac{m-1}{n(x)}\right) \longrightarrow \text{space-charge effect}$$

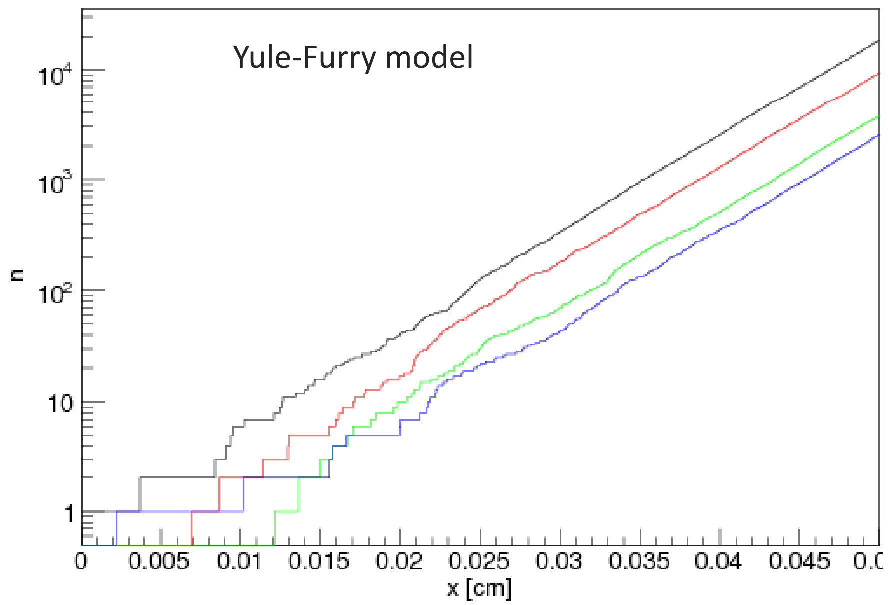
J. Byrne, *Statistics of Electron Avalanches in the Proportional Counter,* Nucl. Instr. Meth. **74**, 291-296 (1969)

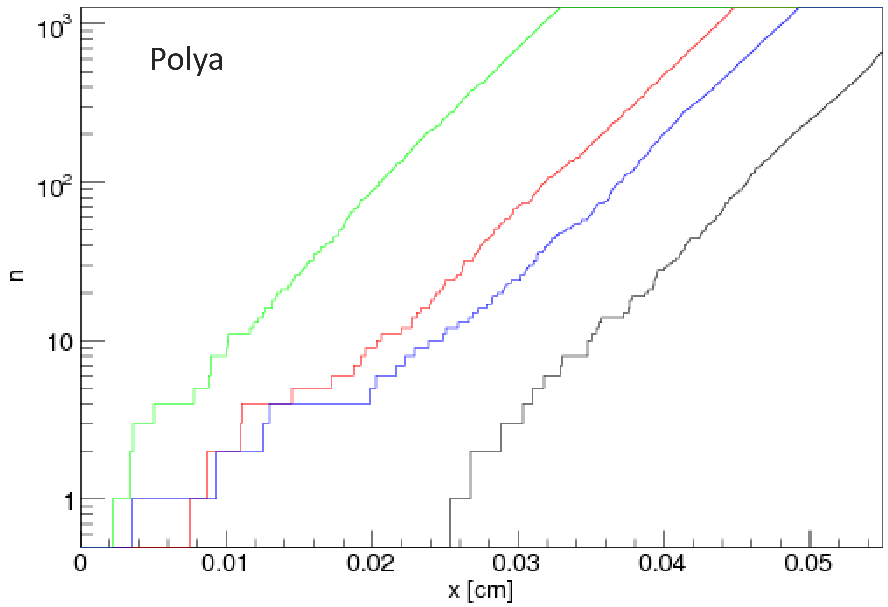
Distribution of ionization mean free path



- The avalanche size statistics is determined by fluctuations in the early stages.

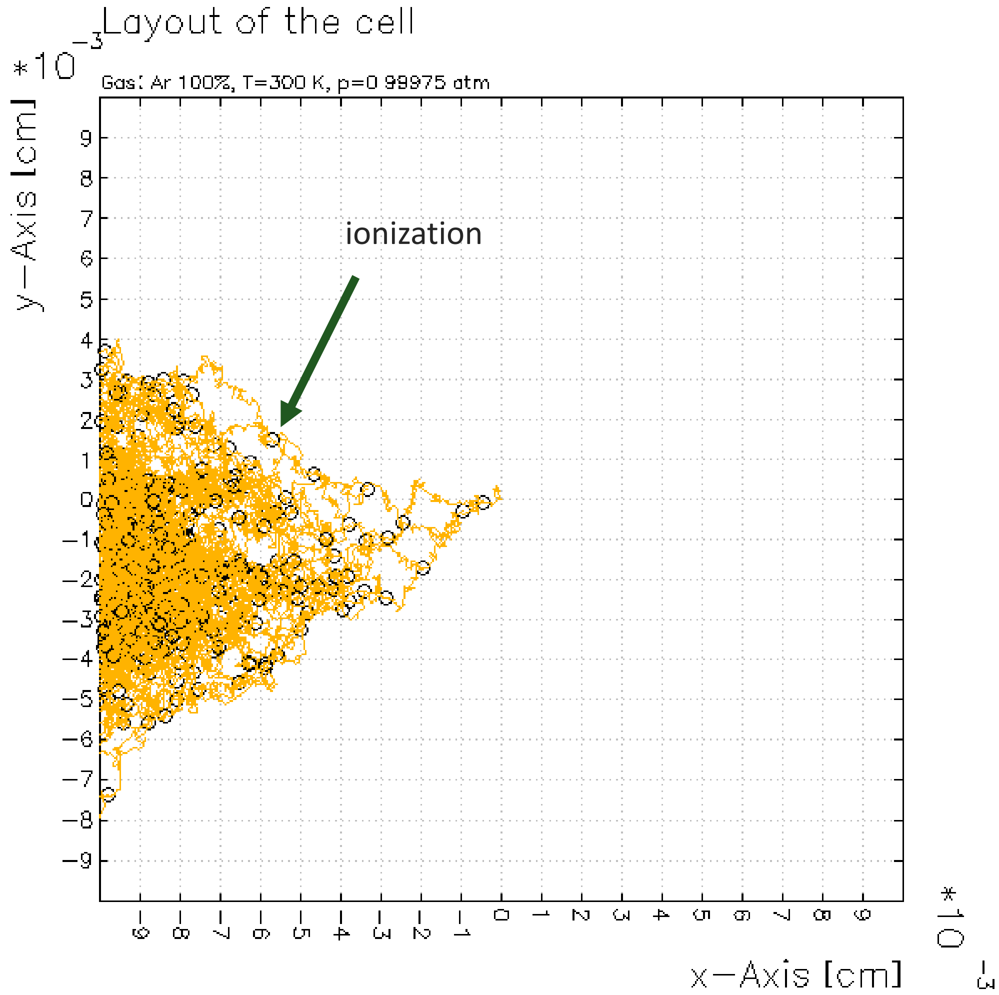
- After the avalanche size has become sufficiently large, a stationary electron energy distribution should be attained. Hence, for n □ $10^2 - 10^3$ the avalanche is expected to grow exponentially.





- `Microscopic_Avalanche` procedure in Garfield available since May 2008 performs tracking of all electrons in the avalanche at molecular level (Monte Carlo simulation derived from Magboltz).

- Information obtained from the simulation
  - total numbers of electrons and ions in the avalanche
  - coordinates of ionization events
  - electron energy distribution
  - interaction rates



Goal

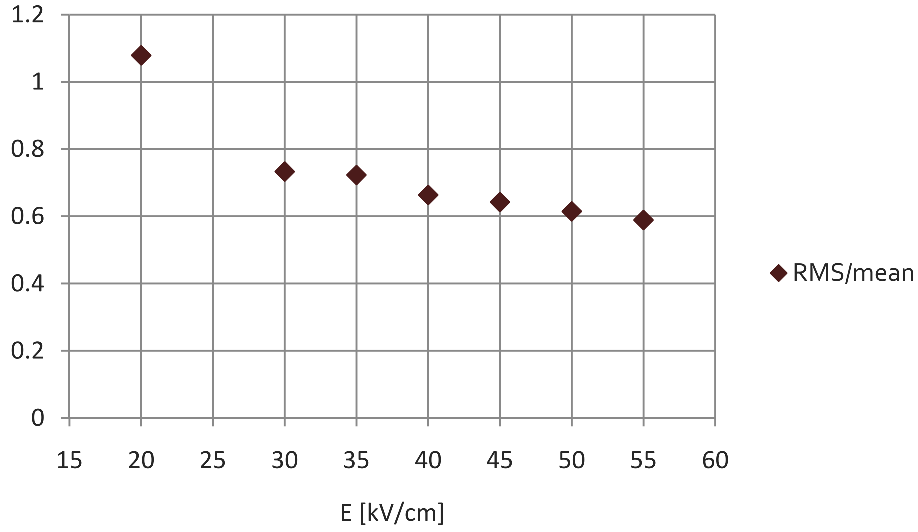
- Investigate impact of
  - electric field
  - pressure
  - gas mixture

  on the single electron avalanche spectrum

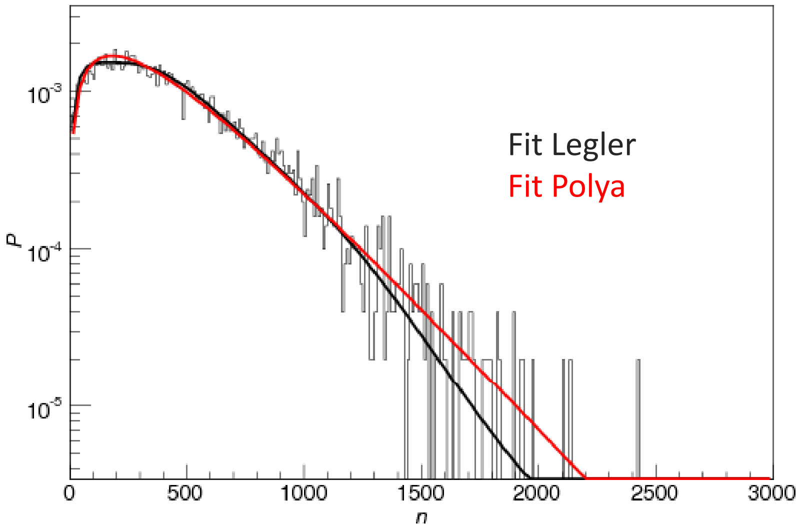
- parallel-plate geometry
- electron starts with kinetic energy ε = 1 eV

What is the effect of the electric field on the avalanche spectrum?

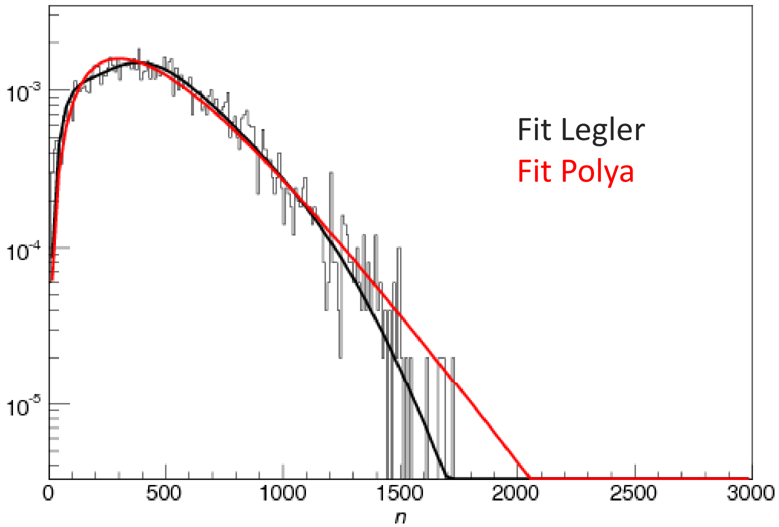
gap d adjusted such that $\langle n \rangle \approx 500$



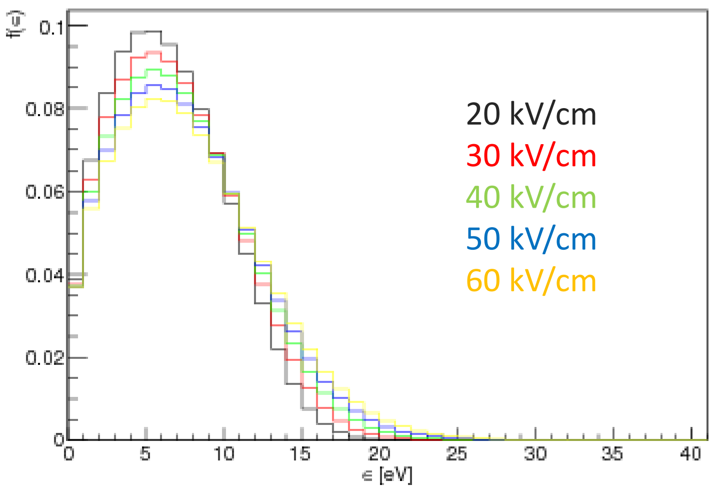
E = 30 kV/cm, p = 1 bar

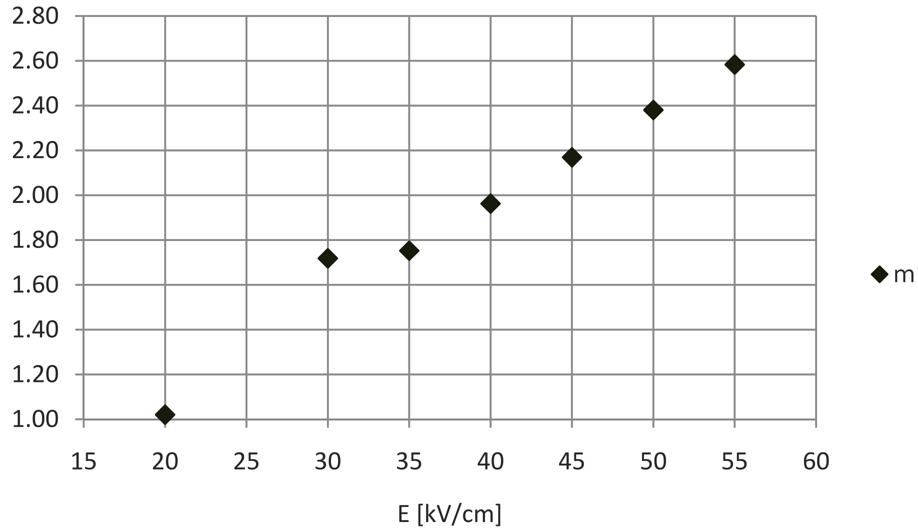


E = 55 kV/cm, p = 1 bar

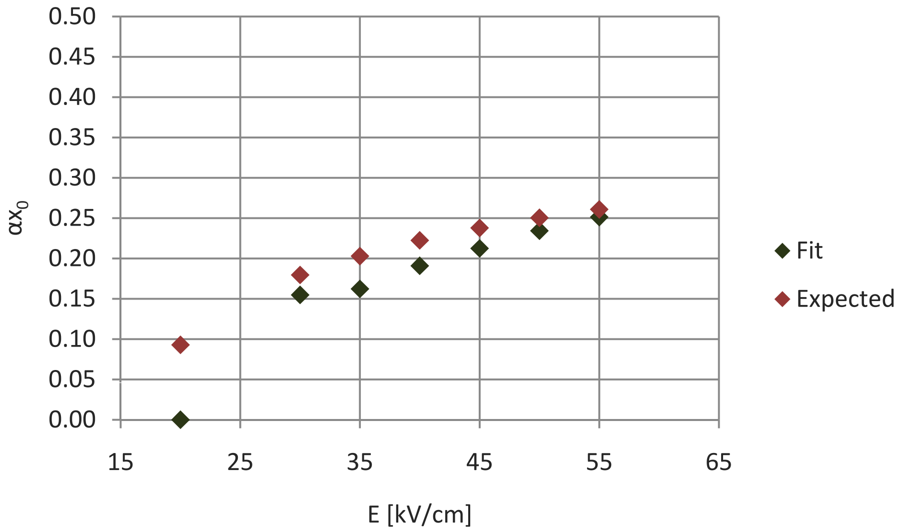


energy distribution





with increasing field, the energy distribution is shifted towards higher energies where ionization is dominant



introduce **attachment coefficient η** (analogously to α)

Mean avalanche size

$$\bar{n}(x) = e^{(\alpha-\eta)x}$$

→ effective Townsend coefficient α - η

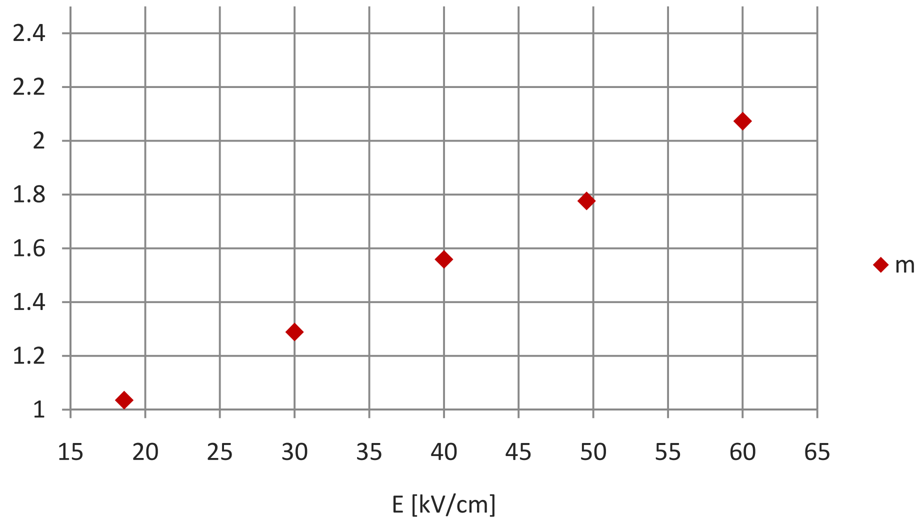
Distribution for constant α and η

$$P(n,x) = \begin{cases} \dfrac{\eta}{\alpha}\dfrac{\bar{n}-1}{\bar{n}-\eta/\alpha} & n=0 \\ \bar{n}\left(\dfrac{1-\eta/\alpha}{\bar{n}-\eta/\alpha}\right)^2\left(\dfrac{\bar{n}-1}{\bar{n}-\eta/\alpha}\right)^{n-1} & n>0 \end{cases}$$

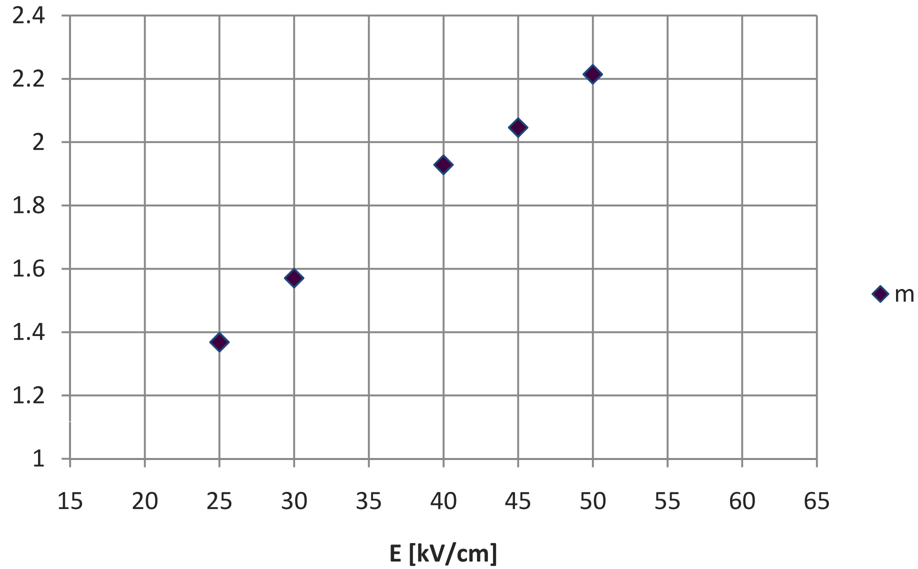
→ distribution remains essentially exponential

W. Legler, *Die Statistik der Elektronenlawinen in elektronegativen Gasen, bei hohen Feldstärken und bei großer Gasverstärkung*, Z. Naturforschg. **16a**, 253-261 (1961)

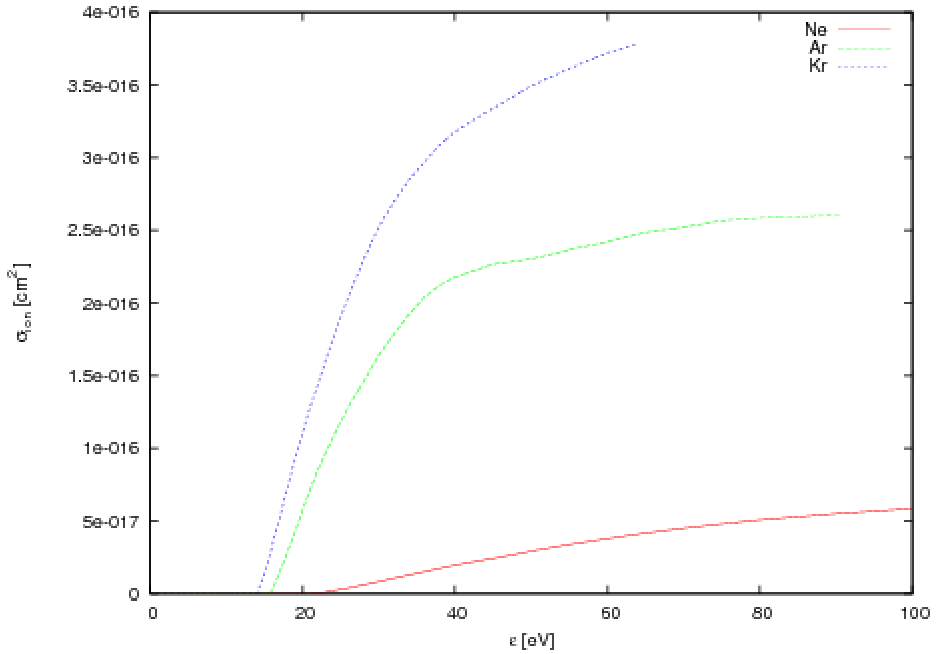
Ar (80%) + CO_2 (20%)



Ar (95%) + iC_4H_{10} (5%)

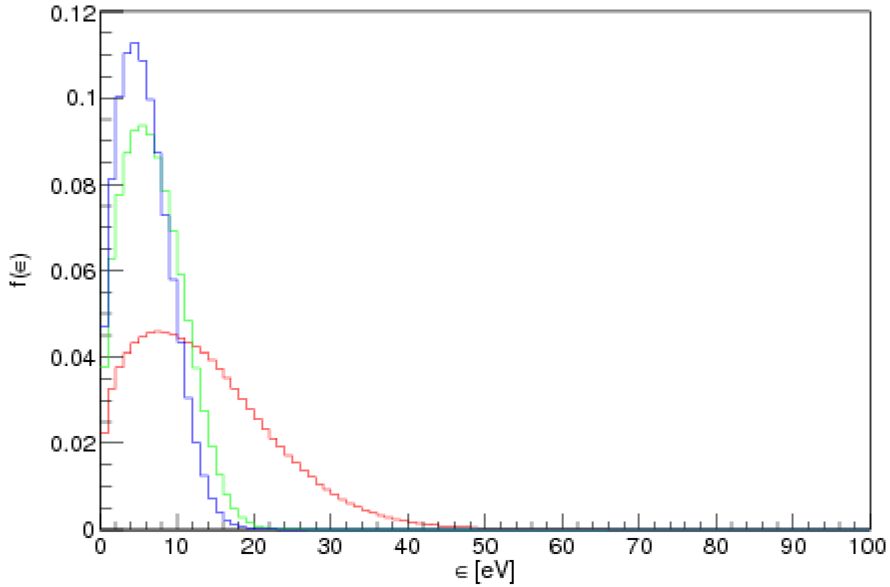


ionization cross-section (Magboltz)



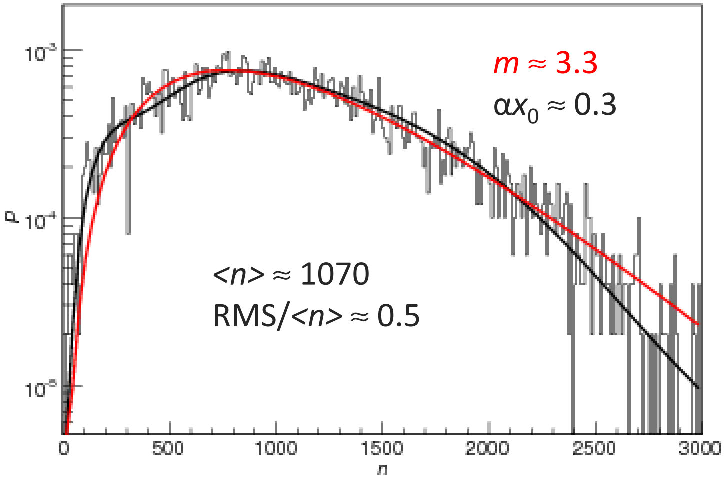
	Ionization energy [eV]
Ne	21.56
Ar	15.70
Kr	13.996

energy distribution (E = 30 kV/cm, p = 1 bar)



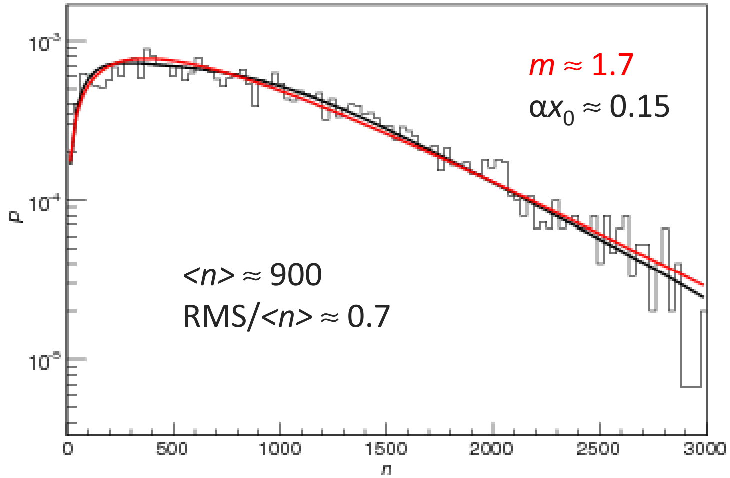
Which shape of σ(ε) yields „better" avalanche statistics?

Ne

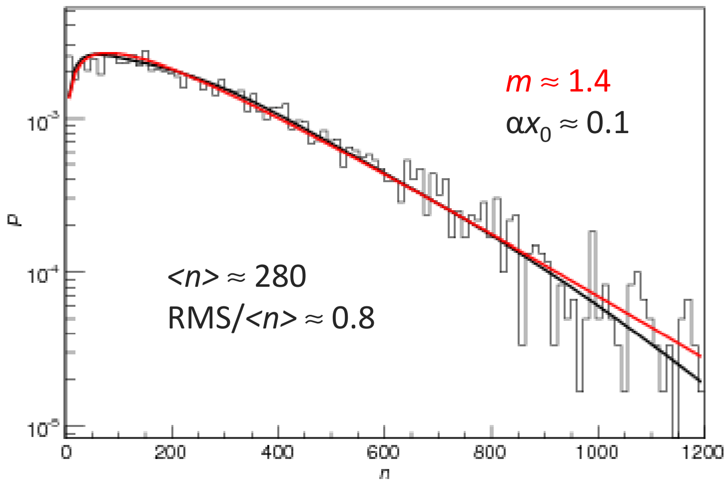


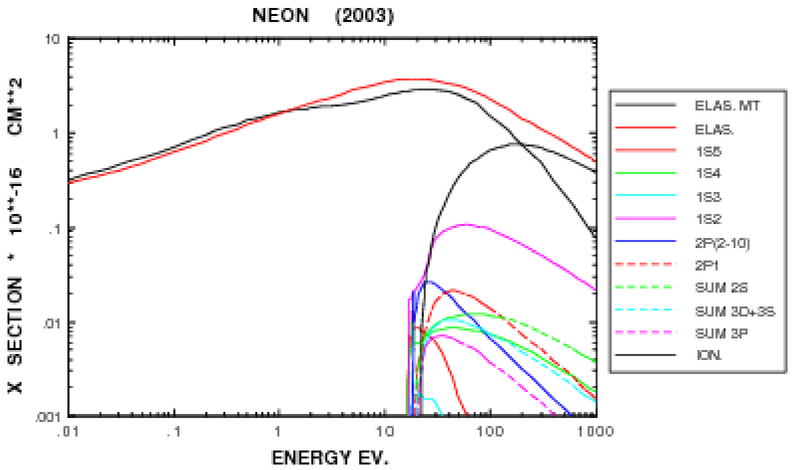
Parameters: E = 30 kV/cm, p = 1 bar, d = 0.02 cm

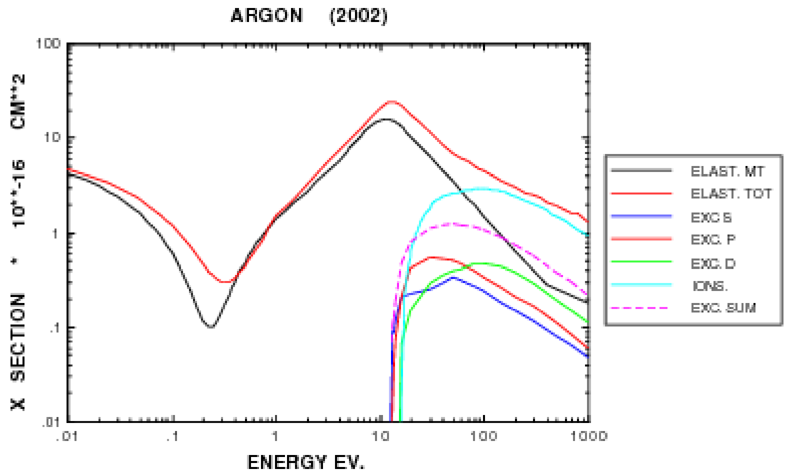
Ar

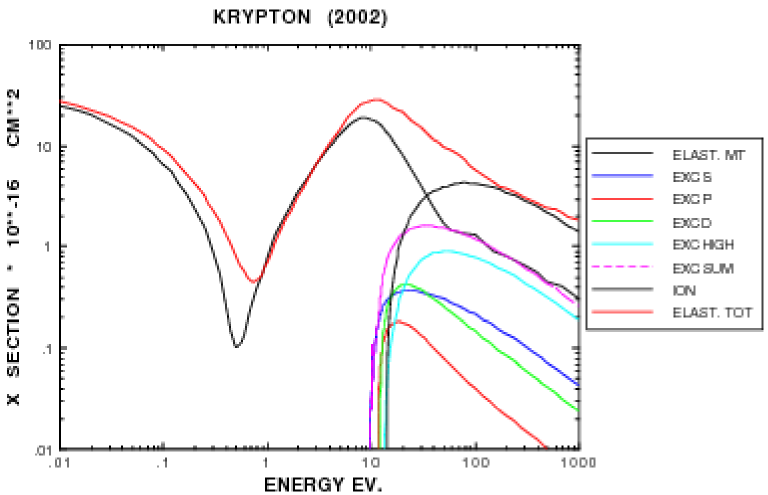


Kr




NEON (2003)
X SECTION * 10**-16 CM**2
ENERGY EV.
ELAS. MT
ELAS.
1S5
1S4
1S3
1S2
2P(2-10)
2P1
SUM 2S
SUM 3D+3S
SUM 3P
ION.


ARGON (2002)
X SECTION * 10**-16 CM**2
ENERGY EV.
ELAST. MT
ELAST. TOT
EXC S
EXC. P
EXC. D
IONS.
EXC. SUM


KRYPTON (2002)
X SECTION * 10**-16 CM**2
ENERGY EV.
ELAST. MT
EXC S
EXC P
EXC D
EXC HIGH
EXC SUM
ION
ELAST. TOT

Conclusions

- „Simple“ models (e. g. Legler‘s model gas) can provide qualitative insight into the mechanisms of avalanche evolution but are of limited use for the quantitative prediction of avalanche spectra (no analytic solution available or lack of physical interpretation).

- For realistic models, the energy dependence of the ionization/excitation cross-sections and the electron energy distribution have to be taken into account → Monte Carlo simulation is a better aproach.

- Avalanche spectra can be simulated in Garfield based on molecular cross-sections. Preliminary results confirm expected tendencies (e.g. better efficiency at higher fields).