LHC ski 2016

A first discussion of 13 TeV results
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Ultra-high multiplicity Electro-Weak production at FCC

Perturbation theory meltdown and Beyond

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- The model: the Weak sector of the SM: SU(2) + Higgs
- Investigate scattering processes at ~100 TeV CoM energies E
- Concentrate on n~100s of Higgses and W,Z's produced in the final state.
  n times lambda >> 1 or n times alpha_weak >>1.
- Two distinct classes of high-E processes with such final states are of interest:
- Non-perturbative (B+L)-violating processes (sphalerons and instantons => tunneling)
- Ordinary perturbative high-n processes (expansion around standard perturbative vacuum)
- Sphaleron mass is a new scale in the SM at ~10 TeV so that at > 30 TeV a possibility of new non-perturbative dynamics in the SM
- Perturbative high-E behaviour presents an easier problem to tackle
- Our trusted weakly coupled perturbation theory breaks down: Amplitudes~n!

FIRST: Perturbative large-n amplitudes

- 1*->n on mass threshold at tree level: Recursion relations & classical solutions general technique -Brown 1992
- Results in factorial growth of amplitudes in:
- (a) unbroken phi^4 theory
- (b) scalar theory with the VEV
- (c) Gauge-Higgs theory (spontaneously broken gauge theory)
- Perturbative growth generalises to more realistic 2 -> n

Tree-level amplitudes in phi^4 on mass threshold

Brown 9209203

The generating function of tree amplitudes on multiparticle thresholds is a classical solution. It solves an ordinary differential equation with no source term,

$$d_t^2\phi + M^2\phi + \lambda\phi^3 \;=\; 0\,.$$

The solution contains only positive frequency harmonics, i.e. the Taylor expansion in $z(t)$,

$$\phi_{\rm cl}(t) \;=\; z(t) \;+\; \sum_{n=2}^{\infty} d_n\, z(t)^n\,, \qquad\qquad z \;:=\; z_0\, e^{iMt}$$

Coefficients d_n determine the actual amplitudes by differentiation w.r.t. z,

$$\mathcal{A}_{1\to n} \;=\; \left(\frac{\partial}{\partial z}\right)^n \phi_{\rm cl}\Big|_{z=0} \;=\; n!\, d_n \qquad \textbf{Factorial growth!!}$$

$$\phi_{\rm cl}(t) \;=\; \frac{z(t)}{1-\frac{\lambda}{8M^2}\, z(t)^2} \qquad\qquad \mathcal{A}_{1\to n} \;=\; n!\,\left(\frac{\lambda}{8M^2}\right)^{\frac{n-1}{2}}$$

Example 2: apply to phi^4 with SSB (Higgs-like)

Brown 9209203

$$\mathcal{L}(h) \;=\; \frac{1}{2}\,(\partial h)^2 - \frac{\lambda}{4}\,\left(h^2 - v^2\right)^2 \;,$$

The classical equation for the spatially uniform field $h(t)$,

$$d_t^2 h \;=\; -\lambda\, h^3 + \lambda v^2\, h \,,$$

again has a closed-form solution with correct initial conditions $h_{\rm cl} = v + z + \ldots$

$$h_{\rm cl}(t) \;=\; v\, \frac{1 + \frac{z(t)}{2v}}{1 - \frac{z(t)}{2v}} \,, \quad \text{where} \quad z(t) \;=\; z_0\, e^{iM_h t} \;=\; z_0\, e^{i\sqrt{2\lambda}\, v\, t}$$

$$h_{\rm cl}(t) \;=\; 2v \sum_{n=0}^{\infty} \left(\frac{z(t)}{2v}\right)^n d_n \;=\; v \,+\, 2v \sum_{n=1}^{\infty} \left(\frac{z(t)}{2v}\right)^n \;,$$

i.e. with $d_0 = 1/2$ and all $d_{n\geq 1} = 1$.

$$\mathcal{A}_{1\to n} \;=\; \left(\frac{\partial}{\partial z}\right)^n h_{\rm cl}\Big|_{z=0} \;=\; n!\,(2v)^{1-n} \qquad \textbf{Factorial growth!!}$$

Gauge-Higgs theory: Tree-level threshold amplitudes

VVK 1404.4876

These equations are solved by iterations (numerically) with Mathematica. The double Taylor expansion of the generating functions takes the form:

$$
\begin{aligned}
h_{\mathrm{cl}}(z, w^a) &= 2v \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} d(n, 2k) \left(\frac{z}{2v}\right)^n \left(\frac{w^a w^a}{(2v)^2}\right)^k , \\
A^a_{L\,\mathrm{cl}}(z, w^a) &= w^a \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} a(n, 2k) \left(\frac{z}{2v}\right)^n \left(\frac{w^a w^a}{(2v)^2}\right)^k ,
\end{aligned}
$$

where $d(n, 2k)$ and $a(n, 2k)$ are determined from the iterative solution of EOM.

By repeatedly differentiating these with respect to z and w^a for the Higgs to n Higgses and m longitudinal Z bosons threshold amplitude we get,

$$
\mathcal{A}(h \to n \times h + m \times Z_L) = (2v)^{1-n-m}\, n!\, m!\, d(n, m) ,
$$

and for the longitudinal Z decaying into n Higgses and $m+1$ vector bosons,

$$
\mathcal{A}(Z_L \to n \times h + (m+1) \times Z_L) = \frac{1}{(2v)^{n+m}}\, n!\, (m+1)!\, a(n, m) .
$$

Factorial growth reemains (in n and in m) !

(More) physical 2 -> n processes

- Our discussion so far involved an initial state of a single highly virtual boson $1^* \to n$.

- In reality should look at physical scattering processes which are $2 \to n$ with two on-shell initial particles.

- In the pure ϕ^4 scalar field theory, both in the unbroken and in the broken phase, it is actually known that tree-level amplitudes on the multiparticle threshold for $2 \to n$ processes are **exactly vanishing**.

- But the pure scalar ϕ^4 theory (with a single self-coupling constant) is known to be a special case; this vanishing was expected to hold in the SM only for special fine-tuned values of the couplings (Vector and Higgs masses). *Voloshin; Smith; Argyres, Kleiss, Papadopoulous 1992-94*

- The process $VV \to nH$ in the Gauge-Higgs theory retains the $n!$ growth.
Jaeckel and VVK 1411.5633.

IIb. Off the multi-particle threshold

- Tree level recursion relations & classical equations
- Non-relativistic kinematics in the multi-particle final state
- Integration over the n-particle phase space
- The holy grail

- VVK 1411.2925 following the approach of
  Libanov, Rubakov, Son & Troitsky 9407381 and Son 9505338 in unbroken phi^4

Off the mass-threshold in phi^4 (Higgs-like)

$$-\left(\partial^\mu \partial_\mu + M_h^2\right)\varphi \;=\; 3\lambda v\,\varphi^2 \;+\; \lambda\,\varphi^3$$

This classical equation for $\varphi(x) = h(x) - v$ determines directly the structure of the recursion relation for tree-level scattering amplitudes:

$$(P_{\rm in}^2 - M_h^2)\,\mathcal{A}_n(p_1\ldots p_n) \;=\; 3\lambda v \sum_{n_1,n_2}^{n} \delta^n_{n_1+n_2} \sum_{\mathcal{P}} \mathcal{A}_{n_1}(p_1^{(1)},\ldots,p_{n_1}^{(1)})\,\mathcal{A}_{n_2}(p_1^{(2)}\ldots p_{n_2}^{(2)})$$

$$+\lambda \sum_{n_1,n_2,n_3}^{n} \delta^n_{n_1+n_2+n_3} \sum_{\mathcal{P}} \mathcal{A}_{n_1}(p_1^{(1)}\ldots p_{n_1}^{(1)})\,\mathcal{A}_{n_2}(p_1^{(2)}\ldots p_{n_2}^{(2)})\,\mathcal{A}_{n_3}(p_1^{(3)}\ldots p_{n_2}^{(3)})$$

Away from the multi-particle threshold, the external particles 3-momenta $\vec{p}_i$ are non-vanishing. In the non-relativistic limit, the leading momentum-dependent contribution to the amplitudes is proportional to $E_n^{\rm kin}$ (Galilean Symmetry),

$$\mathcal{A}_n(p_1\ldots p_n) \;=\; \mathcal{A}_n + \mathcal{M}_n\,E_n^{\rm kin} \;:=\; \mathcal{A}_n + \mathcal{M}_n\,n\,\varepsilon\,,$$

$$\varepsilon = \frac{1}{n\,M_h}\,E_n^{\rm kin} \;=\; \frac{1}{n}\,\frac{1}{2M_h^2}\,\sum_{i=1}^{n}\vec{p}_i^{\,2}\,.$$

In the non-relativistic limit we have $\varepsilon \ll 1$.

Off the mass threshold

$$\varepsilon = \frac{1}{n\,M_h}\,E_n^{\rm kin} \;=\; \frac{1}{n}\,\frac{1}{2M_h^2}\,\sum_{i=1}^{n}\vec{p}_i^{\,2}$$

$$\mathcal{A}_n(p_1\ldots p_n) \;=\; n!\,(2v)^{1-n}\,\left(1-\frac{7}{6}\,n\,\varepsilon \;-\; \frac{1}{6}\,\frac{n}{n-1}\,\varepsilon+\mathcal{O}(\varepsilon^2)\right).$$

An important observation is that by exponentiating the order-$n\varepsilon$ contribution, one obtains the expression for the amplitude which solves the original recursion relation to all orders in $(n\varepsilon)^m$ in the large-n non-relativistic limit,

$$\mathcal{A}_n(p_1\ldots p_n) \;=\; n!\,(2v)^{1-n}\,\exp\left[-\frac{7}{6}\,n\,\varepsilon\right]\,,\quad n\to\infty\,,\quad \varepsilon\to 0\,,\quad n\varepsilon = \text{fixed}\,.$$

Simple corrections of order ε, with coefficients that are not-enhanced by n are expected, but the expression is correct to all orders $n\varepsilon$ in the double scaling large-n limit. The exponential factor can be absorbed into the z variable so that

$$\varphi(z) = \sum_{n=1}^{\infty} d_n\,\left(z\,e^{-\frac{7}{6}\,\varepsilon}\right)^n\,,$$

remains a solution to the classical equation and the original recursion relations.

Can now integrate over the phase-space

- VVK 1411.2925

IIc. The Holy Grail

In the non-rel. limit for perturbative Higgs bosons only production we obtained:

$$\boxed{\sigma_n \propto \exp\left[n\left(\log\frac{\lambda n}{4} - 1\right) + \frac{3n}{2}\left(\log\frac{\varepsilon}{3\pi} + 1\right) - \frac{25}{12}\, n\,\varepsilon\right]}$$

More generally, in the large-n limit with $\lambda n =$ fixed and $\varepsilon =$ fixed, one expects

$$\boxed{\sigma_n \propto \exp\left[\frac{1}{\lambda}\, F_{\mathrm{h.g.}}(\lambda n, \varepsilon)\right]}$$

[e.g. Libanov, Rubakov, Troitsky review 1997]

where the *holy grail* function $F_{\mathrm{h.g.}}$ is of the form,

$$\frac{1}{\lambda}\, F_{\mathrm{h.g.}}(\lambda n, \varepsilon) = \frac{\lambda n}{\lambda}\left(f_0(\lambda n) + f(\varepsilon)\right)$$

In our higgs model, i.e. the scalar theory with SSB,

$$\boxed{\begin{array}{rclr} f_0(\lambda n) & = & \log\dfrac{\lambda n}{4} - 1 & \text{at tree level} \\ f(\varepsilon) & \to & \dfrac{3}{2}\left(\log\dfrac{\varepsilon}{3\pi} + 1\right) - \dfrac{25}{12}\,\varepsilon & \text{for } \varepsilon \ll 1 \end{array}}$$

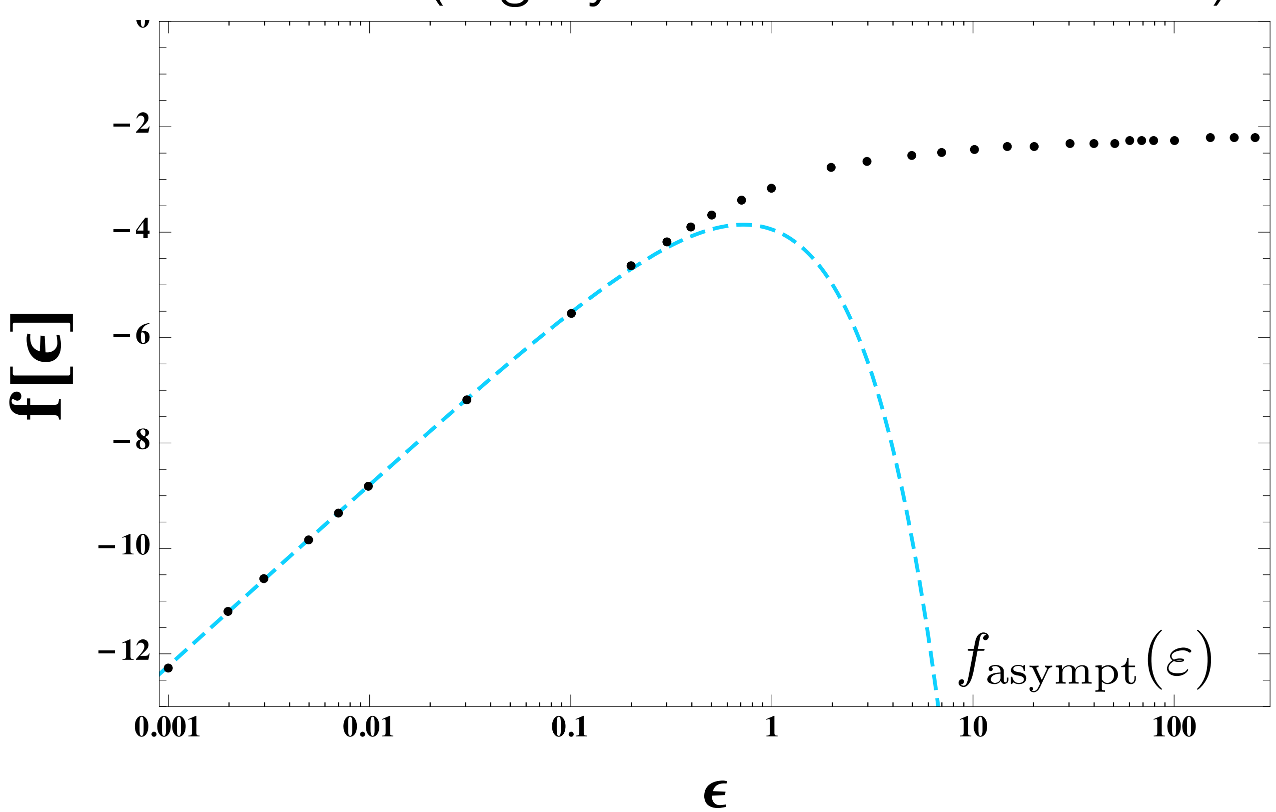
- VVK 1504.05023

Large-n limit with λn = fixed and ε = fixed,

$$\boxed{\frac{1}{n}\log\sigma_n \;=\; \frac{1}{\lambda n}F_{\rm h.g.}(\lambda n,\varepsilon) \;=\; f_0(\lambda n)+f(\varepsilon)}$$

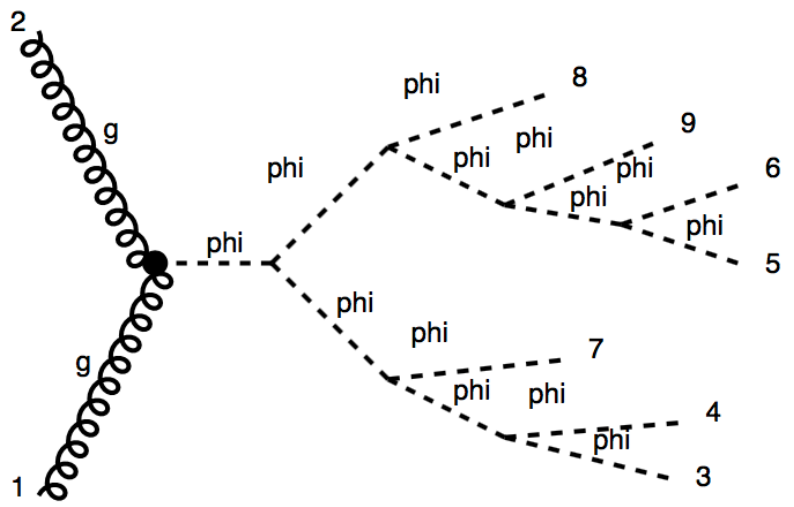
$$\begin{array}{rcll} f_0(\lambda n) & = & \log\frac{\lambda n}{4}-1 & \text{at tree level} \\ f_{\rm asympt}(\varepsilon) & = & \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right)-\frac{25}{12}\,\varepsilon & \text{for } \varepsilon \ll 1 \end{array}$$

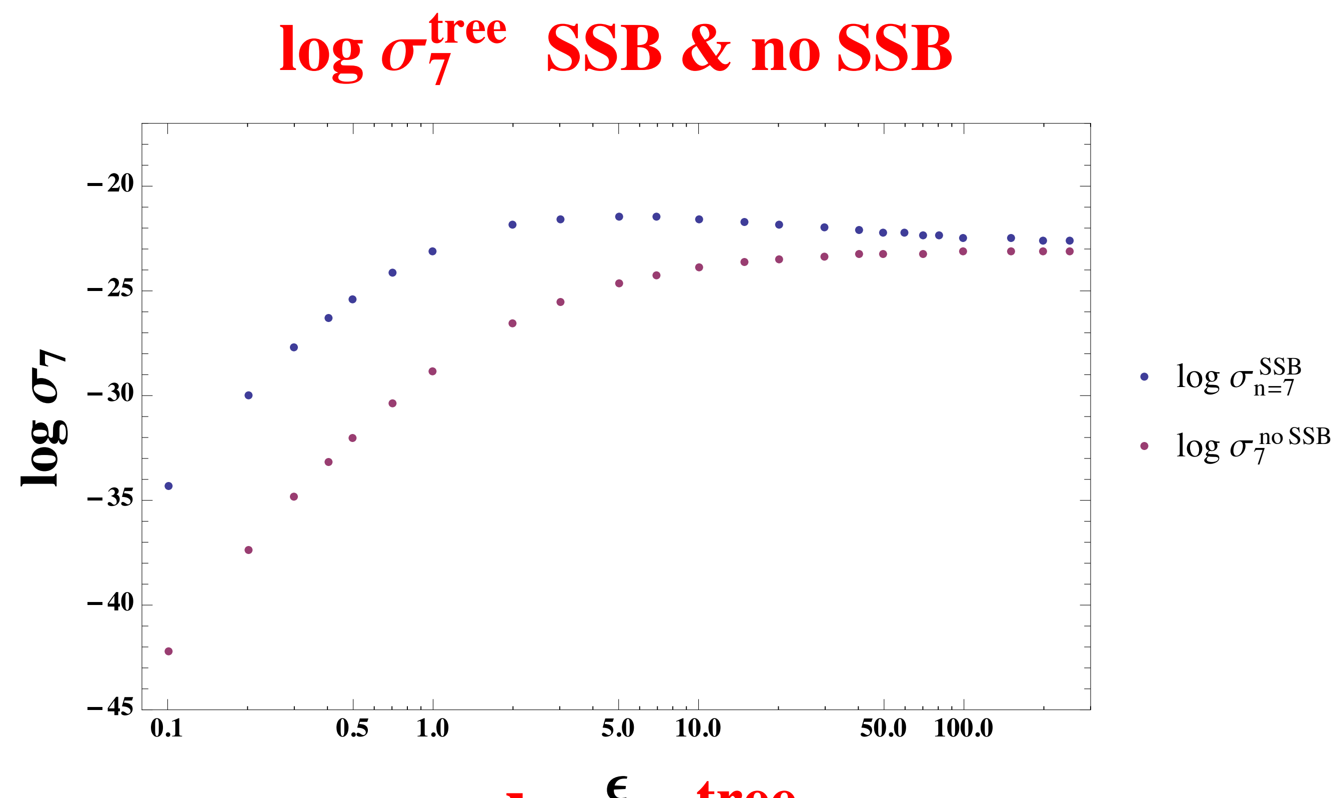
f(eps) asymptotes to a const at large eps
(highly relativistic final state)



computed w MadGraph 2-> 7 for any epsilon and scaled to large n

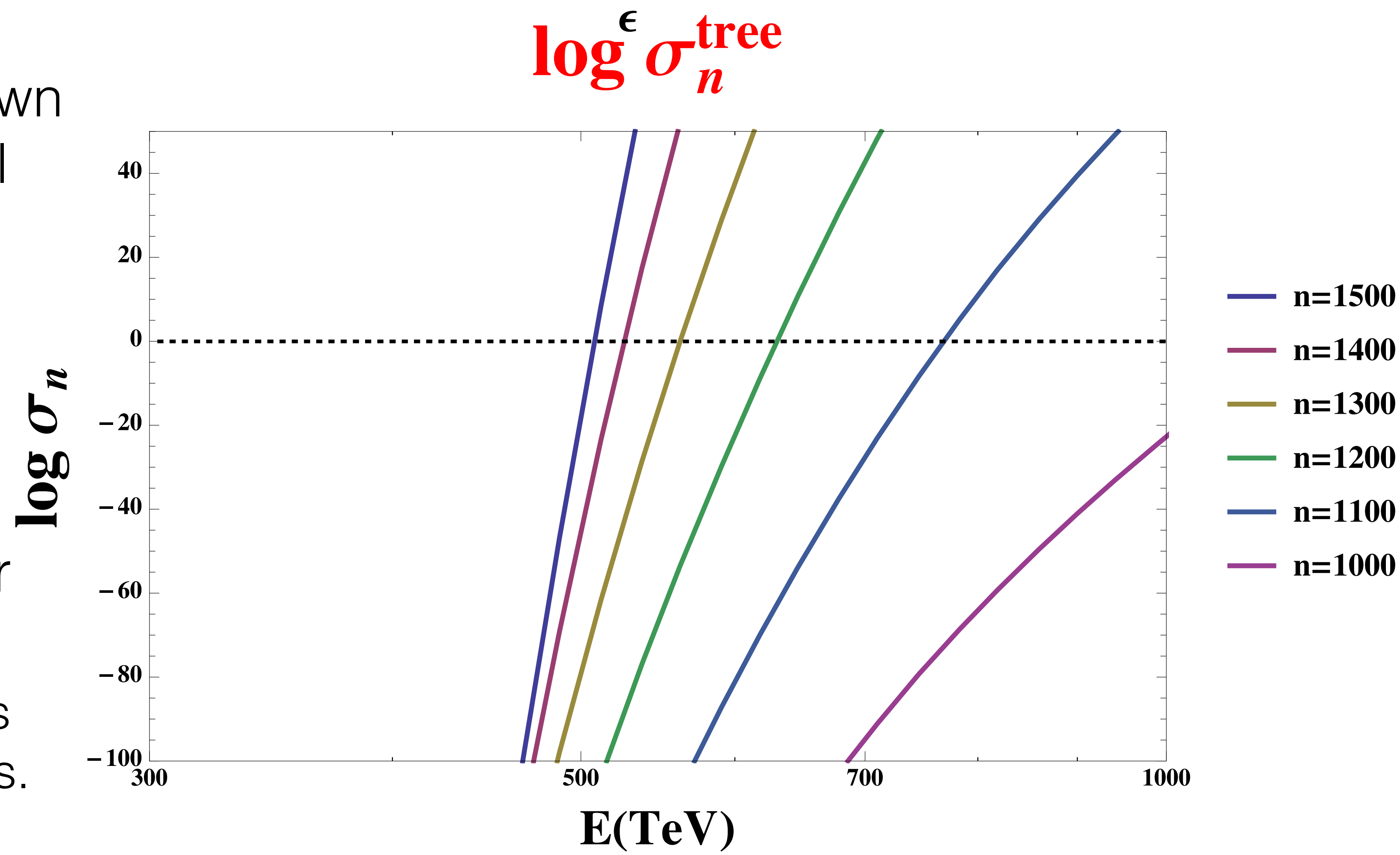
1. Compute cross-sections with MadGraph 2 -> 5,6,7 at all energies (i.e. arbitrary epsilon)





2. Scale to large n using the known n-dependence in the holy grail

- VVK 1504.05023



+ C.Degrande, VVK, O.Mattelaer in preparation:
including triangles, boxes, … hexagons in the gluon fusion multi-higgs processes.

Loop corrections to tree-level amplitudes@threshold

The 1-loop corrected threshold amplitude for the pure n Higgs production:

$$\boxed{\phi^4 \text{ with SSB}: \quad \mathcal{A}_{1\to n}^{\text{tree+1loop}} = n!\,(2v)^{1-n}\left(1 + n(n-1)\frac{\sqrt{3}\lambda}{8\pi}\right)}$$

There are strong indications, based on the analysis of leading singularities of the multi-loop expansion around singular generating functions in scalar field theory, that the 1-loop correction exponentiates,

Libanov, Rubakov, Son, Troitsky 1994

$$\boxed{\mathcal{A}_{1\to n} = \mathcal{A}_{1\to n}^{\text{tree}} \times \exp\left[B\,\lambda n^2 + \mathcal{O}(\lambda n)\right]}$$

in the limit $\lambda \to 0$, $n \to \infty$ with λn^2 fixed. Here B is determined from the 1-loop calculation (as above) – *Smith; Voloshin 1992*):

$$\boxed{\phi^4 \text{ with SSB}: \quad B = +\frac{\sqrt{3}}{8\pi}\,,}$$

$$\phi^4 \text{ w. no SSB}: \quad B = -\frac{1}{64\pi^2}\left(\log(7+4\sqrt{3}) - i\pi\right)\,,$$

In the Higgs model, 1st equation leads to the exponential enhancement of the tree-level threshold amplitude at least in the leading order in $n^2\lambda$.

RETURN to the beginning of the section:

In the non-rel. limit for perturbative Higgs bosons only production we obtained:

$$\boxed{\sigma_n \propto \exp\left[n\left(\log\frac{\lambda n}{4}-1\right)+\frac{3n}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right)-\frac{25}{12}\,n\,\varepsilon\right]+2\lambda n^2 B}$$

More generally, in the large-n limit with $\lambda n =$ fixed and $\varepsilon =$ fixed, one expects

$$\boxed{\sigma_n \propto \exp\left[\frac{1}{\lambda}\,F_{\mathrm{h.g.}}(\lambda n,\varepsilon)\right]}$$

where the *holy grail* function $F_{\mathrm{h.g.}}$ is of the form,

$$\frac{1}{\lambda}\,F_{\mathrm{h.g.}}(\lambda n,\varepsilon) = \frac{\lambda n}{\lambda}\,(f_0(\lambda n)+f(\varepsilon))$$

In our higgs model, i.e. the scalar theory with SSB,

$$\begin{aligned} f_0(\lambda n) &= \log\frac{\lambda n}{4}-1+\lambda n\,\frac{\sqrt{3}}{4\pi} \\ f(\varepsilon) &\to \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right)-\frac{25}{12}\,\varepsilon \qquad \text{for } \varepsilon \ll 1 \end{aligned}$$

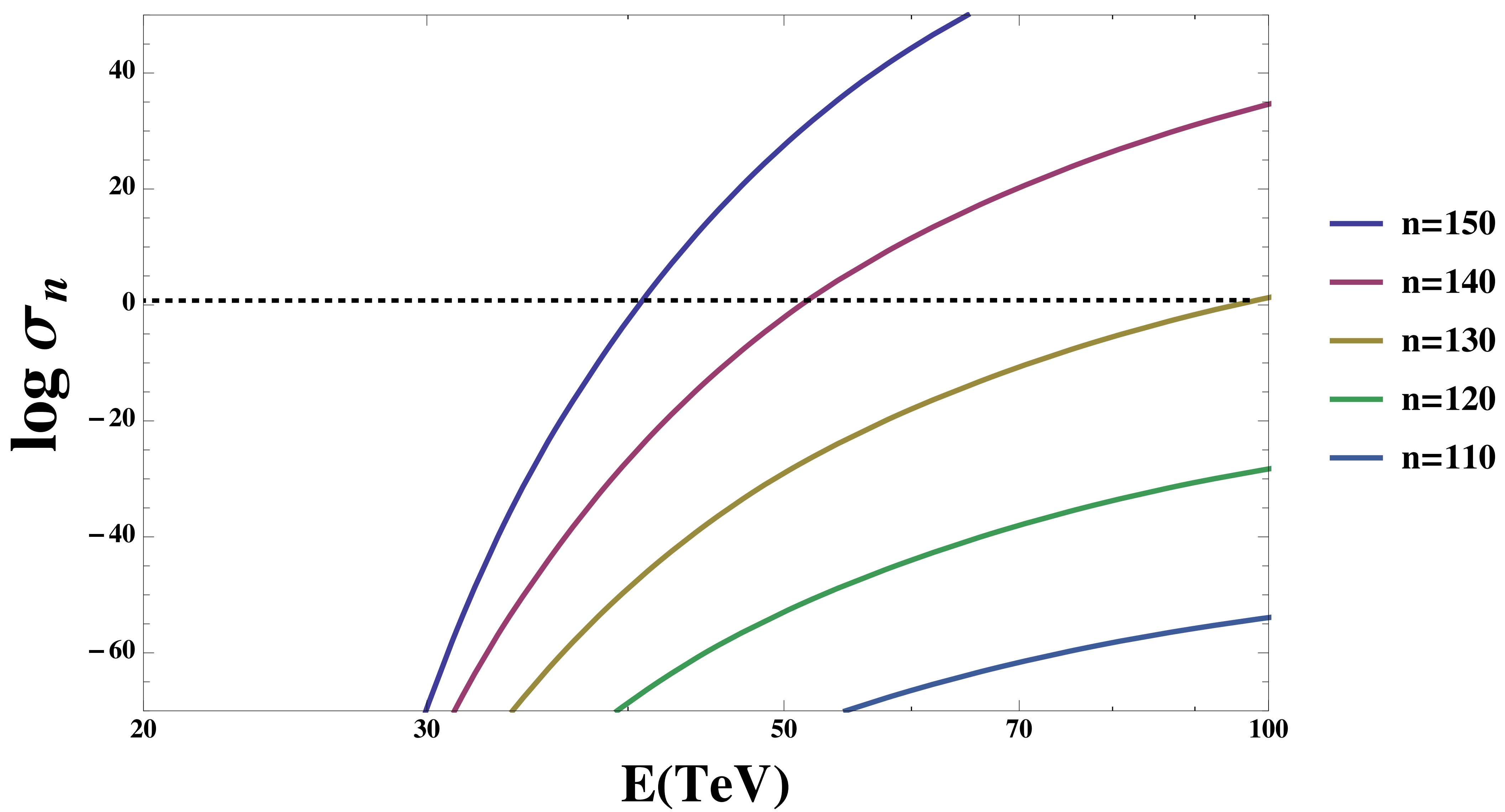
a significant enhancement
though higher orders unknown!

1. Compute cross-sections with MadGraph 2 -> 5,6,7 at all energies (i.e. arbitrary epsilon)

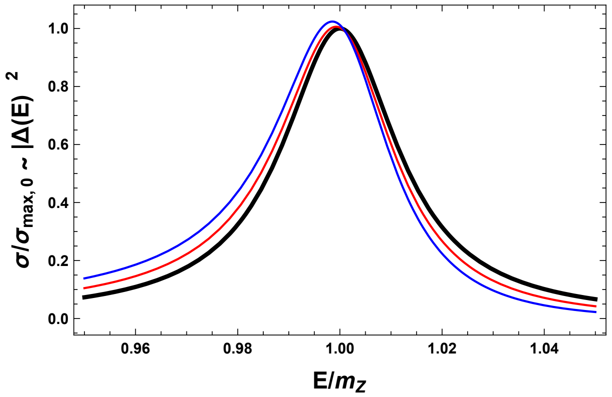
2. Scale to large n using the known n-dependence in the holy grail including the leading-loop factor to the exponent $+\lambda n \frac{\sqrt{3}}{4\pi}$

- VVK 1504.05023

$\log \sigma_n^{\text{loop}}$



- The perturbative cross section grows with energy, ultimately violating:
  - naive perturbative unitarity: $$\sum_{n,\text{inelastic}} \int d\Phi_{n,m} |\mathcal{A}_{n,m}|^2 \leq 8\pi (l_{\max}+1)^2 \overset{?}{\sim} 8\pi$$
  - the observed form of the Z-peak via the Kallen-Lehmann spectral representation:



$$\Delta(p) = \frac{Z}{p^2 - m^2} + \sum_{n\geq 2} \int_{(nm)^2}^{\infty} ds \, \frac{\int d\Phi_n |\mathcal{A}(1\to n)|^2(s)}{p^2 - s}$$

$$\int_{(nm)^2}^{\infty} ds \, \frac{\int d\Phi_n |\mathcal{A}(1\to n)|^2(s)}{p^2 - s} = \frac{1}{m^2}\mathcal{C}_1 + \frac{p^2}{m^4}\mathcal{C}_2 + \ldots \quad , \quad |\mathcal{C}_1| < 1 \quad , \quad |\mathcal{C}_2| < 1$$

  - the cosmic ray limit: upper bound comes from assuming that the effective cross section for inelastic scattering of cosmic rays is of the size of the universe. In this case high energy cosmic rays will be severely attenuated in conflict w observation.

- pert. SM cross sections exceed these bounds at energies:

$$\begin{aligned}
E &\lesssim 810\ \text{TeV} && \text{naive unitarity limit} \\
E &\lesssim 830\ \text{TeV} && \text{cosmic limit} \\
E &\lesssim 300\ \text{TeV} && \text{asymptotic series truncation heuristic} \\
E &\lesssim 100\ \text{TeV} && \text{adding } \simeq \varepsilon^2 \text{ term in } f_{\text{our pert.}}(\varepsilon) \sim (1/n)\log\sigma_n \\
E &\lesssim 35\ \text{TeV} && \text{include} \qquad \text{loop factor} \simeq \lambda n \frac{\sqrt{3}}{4\pi} \text{ in } f_0(\lambda n)
\end{aligned}$$

- Jaeckel, VVK 1411.5633

Conclusions for the perturbative part

- At (not too high) high energies perturbative Standard Model exhibits a formal breakdown. Perturbative unitarity is broken. OPTIONS:

- At high energies (multiplicities) the Standard Model is fundamentally non-perturbative (?)

- The theory classicalizes: the ultra-high multiplicity processes will completely dominate everything else. At high energies above some E_critical start producing more and more of soft quanta. No longer able to probe shorter and shorter length scales with higher and higher energies (?)

- New physics beyond the Standard Model has to set in before the cross-sections become large (?)

- New theoretical approaches & computational techniques have to be developed to determine the relevant energy scale - almost as exciting as probing this at the FCC -

NEXT consider intrinsically Non-perturbative high-E large-n processes

Baryon + Lepton number violation [much older story]

- Electroweak vacuum has a nontrivial structure (!) [SU(2)-sector]

- The saddle-point at the top of the barrier is the *sphaleron.* New EW scale ~ 10 TeV

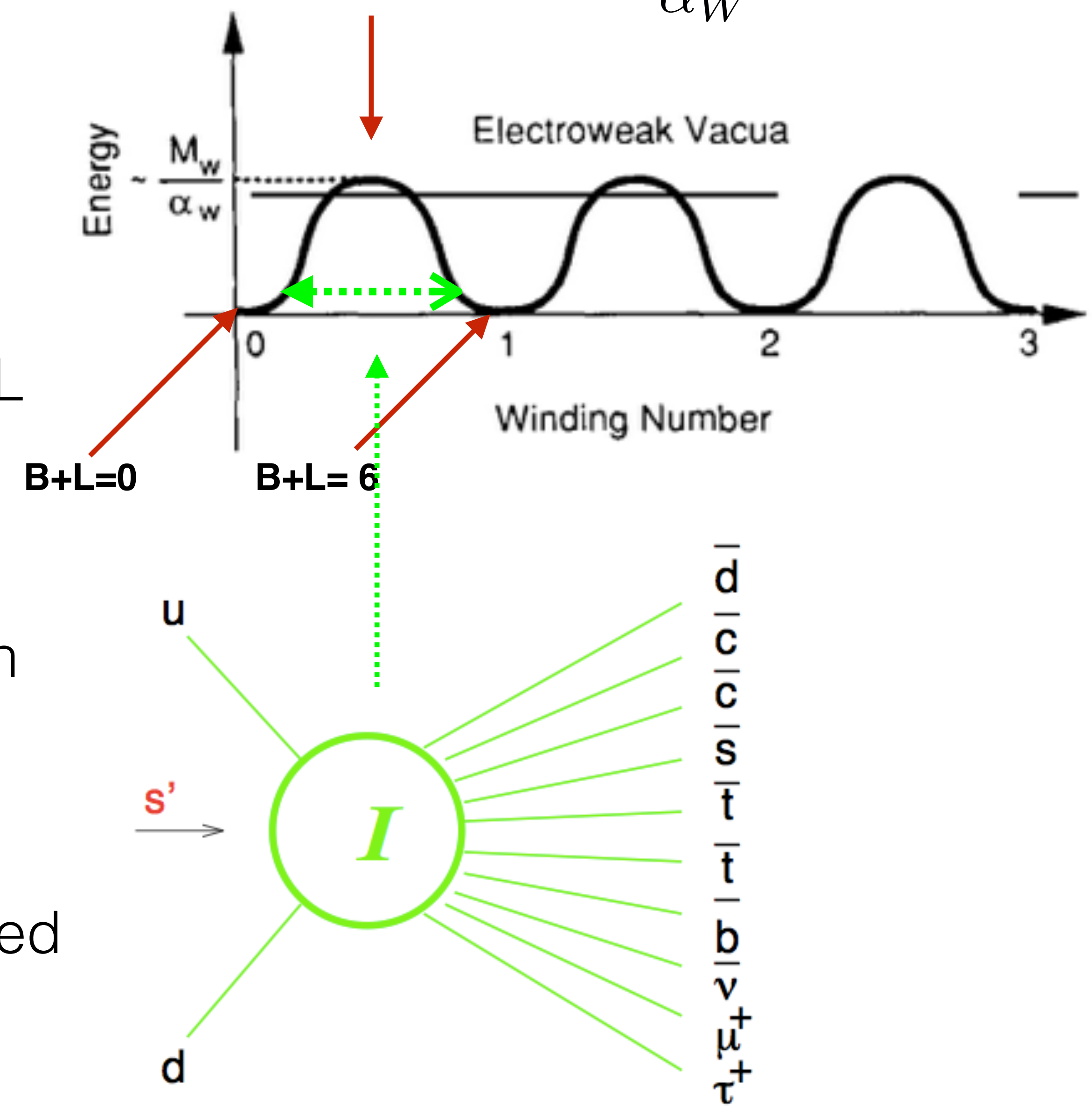
- Transitions between the vacua change B+L (result of the ABJ anomaly):
  Delta (B+L)= 3 x (1+1) ; Delta (B-L)=0

- *Instantons* are tunnelling solutions between the vacua. They mediate B+L violation

- 3 x (1 lepton + 3 quarks) = 12 fermions
  12 left-handed fermion doublets are involved

- There are EW processes which are not described by perturbation theory!

$$E_{\rm sph} \; = c_{\rm sph} \, \frac{m_W}{\alpha_W} \approx 10\,{\rm TeV}$$



$$\boxed{q + q \to 7\bar{q} + 3\bar{l} + n_W W + n_Z Z + n_h H}$$

Instanton approach

- All instanton contributions come with an exponential suppression due to the instanton action:

$$\mathcal{A}^{\rm inst} \propto e^{-S^{\rm inst}} = e^{-2\pi/\alpha_w - \pi^2\rho^2 v^2}, \quad \sigma^{\rm inst} \propto e^{-4\pi/\alpha_w} \simeq 5 \times 10^{-162}$$

- This is precisely the expected semiclassical price to pay for a quantum mechanical tunnelling process.

- For the B+L violating process

$$q + q \to 7\bar{q} + 3\bar{l} + n_W W + n_Z Z + n_h H$$

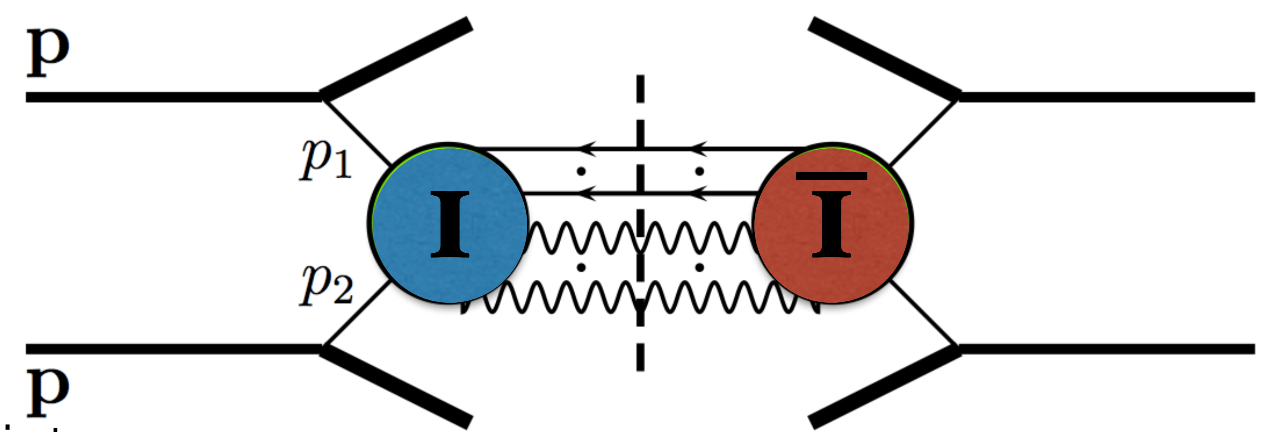
- at leading order, the instanton acts as a point-like vertex with a large number n of external legs => n! factors in the amplitude.

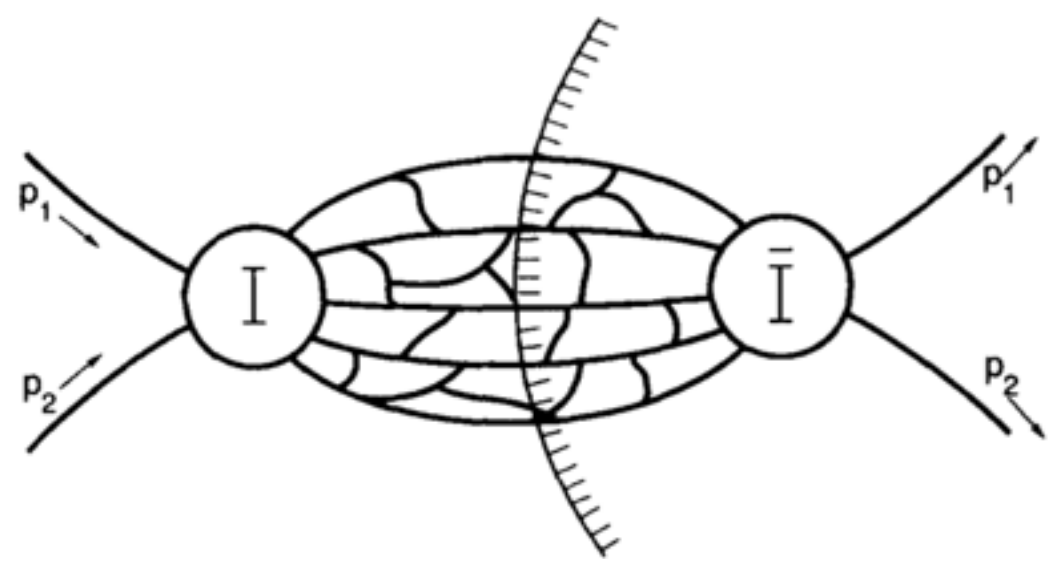
- As the number of W's, Z's and H's produced in the final state at sphaleron-like energies is allowed to be large, ~ 1/alpha, **the instanton crossection receives exponential enhancement with energy**

Ringwald 1990 => McLerran, Vainshtein, Voloshin 1990 => ….

Instanton-Antiinstanton valley VVK & Ringwald 1991

- Crossection is obtained by |squaring| the instanton amplitude.
- Final states have been instrumental in combatting the exp. suppression.
- Now also the interactions between the final states (and the improvement on the point-like I-vertex) are taken into account.
- Use the Optical Theorem to compute *Im* part of the FES amplitude in around the **Instanton-Antiinstanton configuration**.
- Higher and higher energies correspond to shorter and shorter I-Ibar separations R. At R=0 they annihilate to perturbative vacuum.
- The suppression of the crossection is gradually reduced with energy.





Instanton-Antiinstanton optimistic estimate

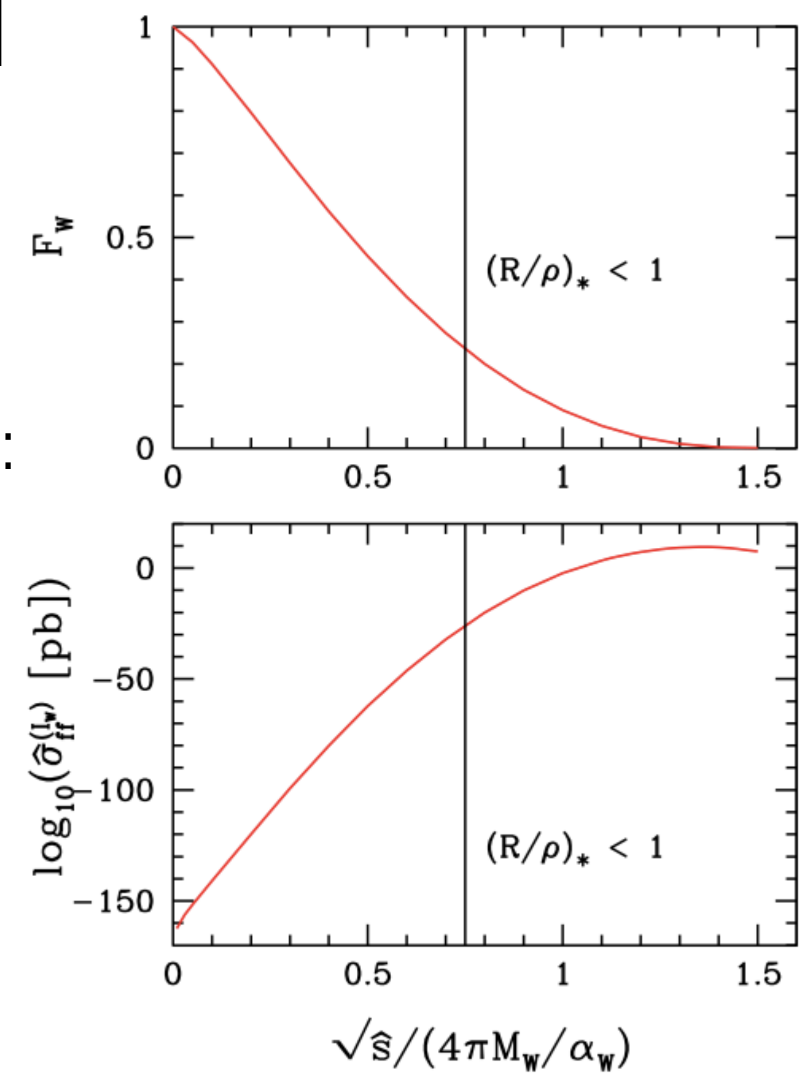
VVK & Ringwald 1991

$$\hat{\sigma}_{qq}^{\text{inst}} \approx \frac{1}{m_W^2}\left(\frac{2\pi}{\alpha_W}\right)^{7/2} \times \exp\left[-\frac{4\pi}{\alpha_W} F_{\text{hg}}\left(\frac{\sqrt{\hat{s}}}{4\pi m_W/\alpha_W}\right)\right]$$

$$\simeq (5.28 \times 10^{15}\ \text{fb}) \times \exp\left[-\frac{4\pi}{\alpha_W} F_{\text{hg}}\left(\frac{\sqrt{\hat{s}}}{4\pi m_W/\alpha_W}\right)\right]$$

Ringwald 2002

The holy grail function F



First few terms in the energy-expansion of the holy grail:

$$F_W(\epsilon) = 1 - \frac{3^{4/3}}{2}\,\epsilon^{4/3} + \frac{3}{2}\,\epsilon^2 + \mathcal{O}(\epsilon^{8/3}) + \ldots$$

$$\epsilon = \sqrt{\hat{s}}/(4\pi m_W/\alpha_W) \simeq \sqrt{\hat{s}}/(30\ \text{TeV})$$

Mattis, Phys. Rept.1992

is a comprehensive review of the 90's literature on the holy grail

Instanton-Antiinstanton optimistic estimate

VVK & Ringwald 1991

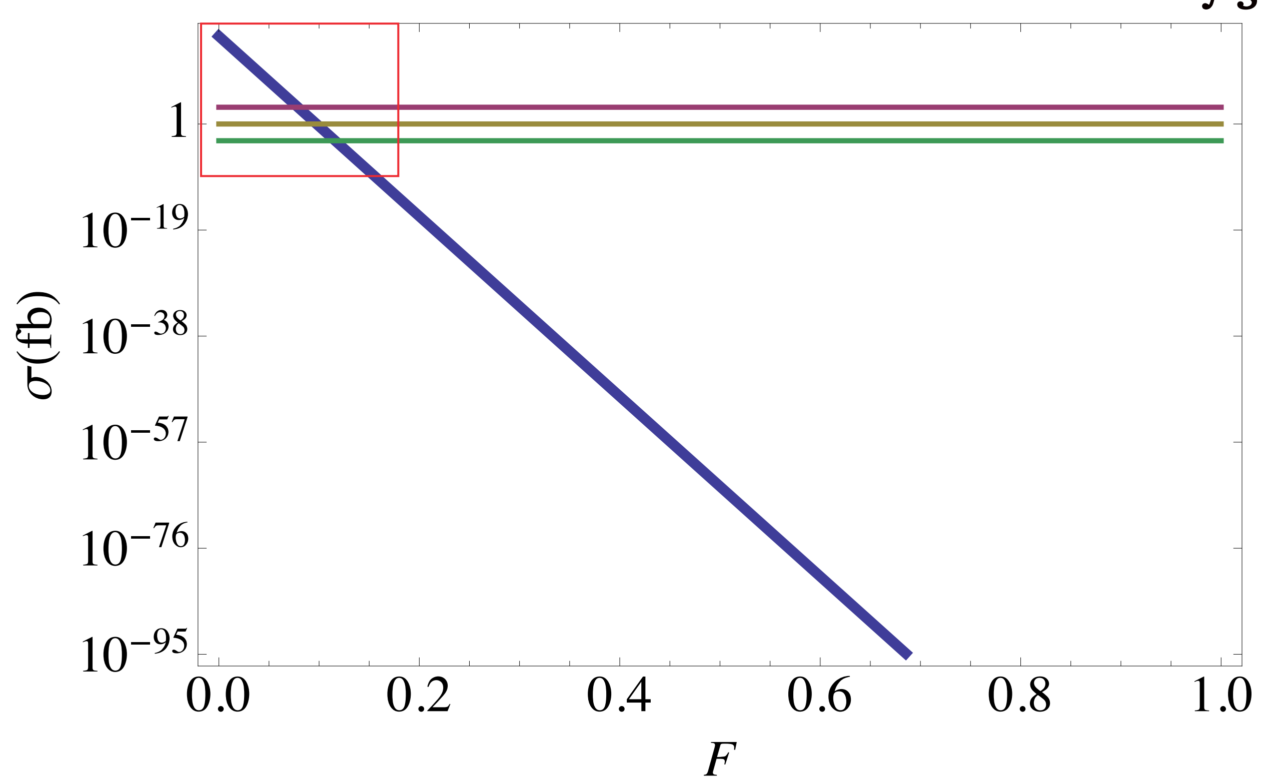
$$\hat{\sigma}_{qq}^{\text{inst}} \approx \frac{1}{m_W^2}\left(\frac{2\pi}{\alpha_W}\right)^{7/2} \times \exp\left[-\frac{4\pi}{\alpha_W} F_{\text{hg}}\left(\frac{\sqrt{\hat{s}}}{4\pi m_W/\alpha_W}\right)\right]$$

$$\simeq (5.28 \times 10^{15}\ \text{fb}) \times \exp\left[-\frac{4\pi}{\alpha_W} F_{\text{hg}}\left(\frac{\sqrt{\hat{s}}}{4\pi m_W/\alpha_W}\right)\right]$$

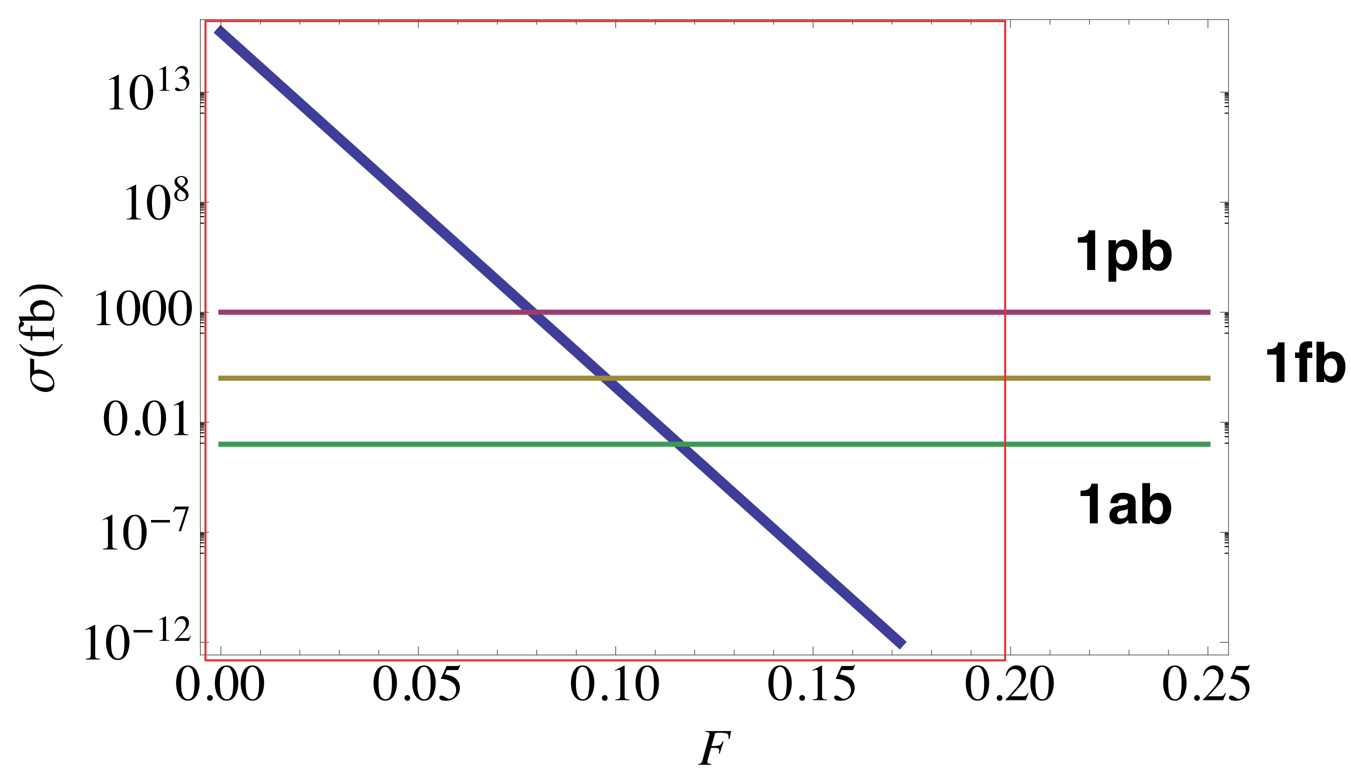
F =1 at E=0

The holy grail function F

0<F<1 at large E



The holy grail function F



The holy grail function F

Pessimistic view:

The sphaleron is a semiclassical configuration with

$$\boxed{\text{Size}_{\text{sph}} \sim m_W^{-1}\,, \quad E_{\text{sph}} = \text{few} \times m_W/\alpha_W \simeq 10\,\text{TeV}.}$$

It is 'made out' of $\sim 1/\alpha_W$ particles (i.e. it decays into $\sim 1/\alpha_W$ W's, Z's, H's).

$$\boxed{2_{\text{initial hard partons}} \to \text{Sphaleron} \to (\sim 1/\alpha_W)_{\text{soft final quanta}}}$$

The sphaleron production out of 2 hard partons is unlikely.

Assumptions:
(1) the intermediate state had to be the sphaleron;
(2) the initial state was a 2-particle state;
(3) that one cannot create $(\sim 1/\alpha_W)_{\text{soft final quanta}}$ from $2_{\text{initial hard partons}}$.

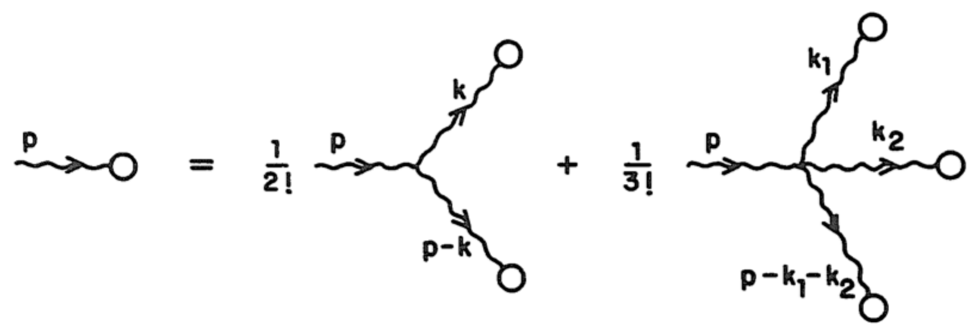
But this reasoning does not hold in classicalizing theories: (Gia's talk)

At high energies above some E_critical start producing more and more of soft quanta. No longer able to probe shorter and shorter length scales with higher and higher energies.

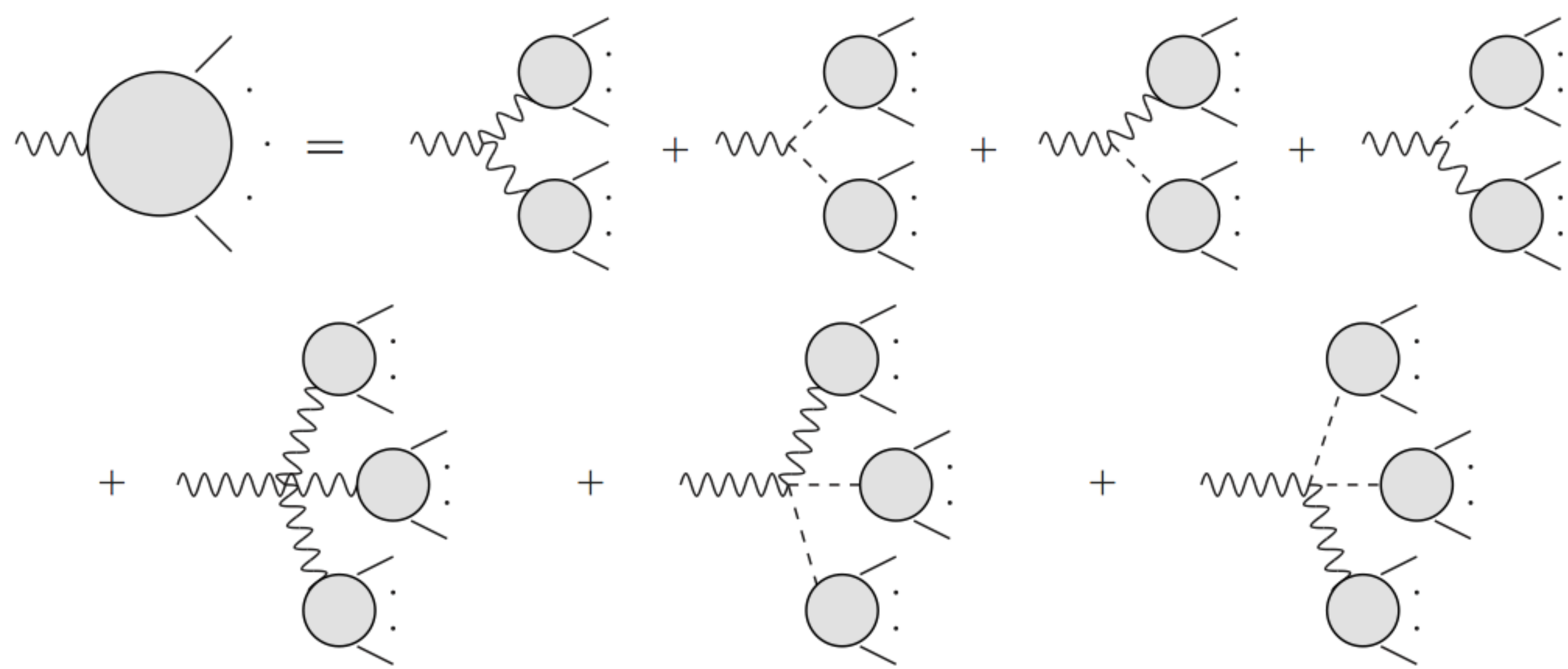
Optimistic view:

1. Use instanton, not the sphaleron as the guide. Initial hard quanta probe short distances, but are not prevented from probing larger scales as well, by emitting soft quanta. Instanton is a classical solution, thus:

Classical equation:



Berends-Giele type recursion relations for the instanton current:



- Electroweak sector of the SM is always seen as perturbative. If these instanton processes can be detected —> a truly remarkable breakthrough in realising & understanding non-perturbative EW dynamics

- **Numbers of W's, Z's and H's** produced in the final state at 30-100 TeV energies is allowed to be large, ~ 1/alpha => a technical consequence of this fact is that the instanton crossection receives an **exponential enhancement with energy**

- The B+L processes are accompanied by ~50 EW vector & H bosons; charged Lepton number can also be measured —> unique experimental signature of the final state

- The rate of the B+L processes is still not known theoretically. There are optimistic phenomenological models with ~pb or ~fb crossections, and there are pessimistic models with unobservable rates even at infinite energy.

- Very hard theoretical problem, new computational methods are needed.

- Since the final state is essentially backgroundless, the obesrvability of the rate can be always settled experimentally.

- B+L processes provide **physics opportunities** which are **completely unique to the very high energy pp machine** (100 TeV FCC pp).