

# Higgs to $Z + \gamma$ at NLO (with full top-mass dependence)

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Jan. 18th, 2015

# Next-to-Leading Order QCD and Light-Fermion Corrections to the Decay Width $H \rightarrow Z\gamma \rightarrow \gamma l^+ l^-$

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**ABSTRACT:** We present the analytic calculation of the following two sets of contributions at the NLO to the decay width of a Higgs boson into a photon and a leptonic pair: the QCD and the light-fermion corrections. The result is expressed in terms of Generalized Harmonic Polylogarithms of maximum weight 4. ....

**KEYWORDS:** Higgs decay, Feynman diagrams, Multi-loop calculations.

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\*On leave of absence from Dipartimento di Fisica, Università di Roma "La Sapienza".

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Numerical evaluation: (Spira, Djouadi, and Zerwas, 1992).

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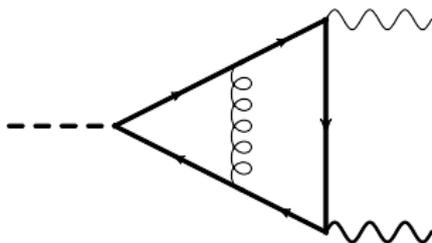
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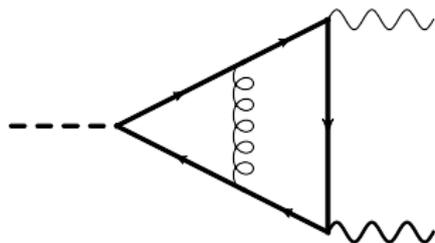
Numerical evaluation: (Spira, Djouadi, and Zerwas, 1992).

When calculating the cross section we consider  $Z \rightarrow l\bar{l}$   
so this makes it the narrow width approximation of  $H \rightarrow \gamma l\bar{l}$

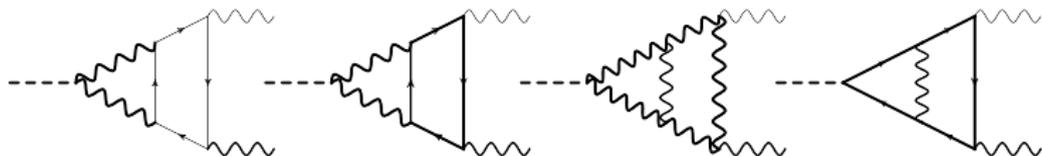
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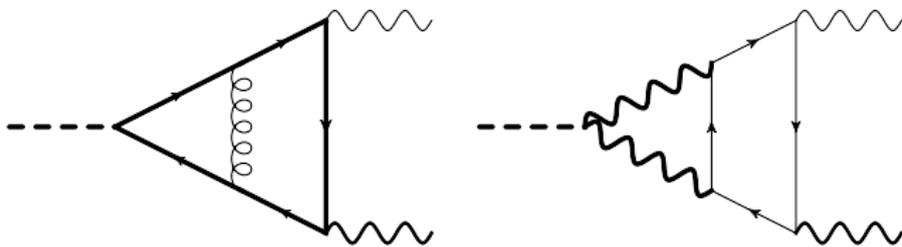


And the 'weak' contributions are given by



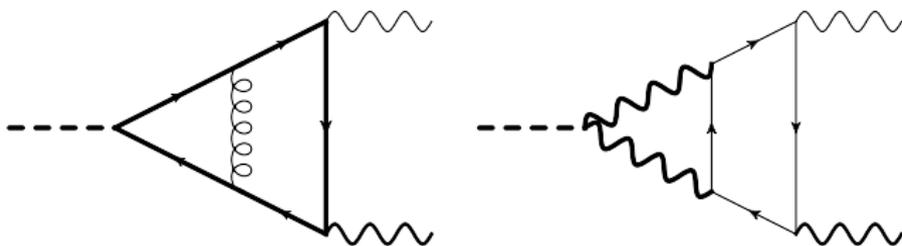
(We disregard  $Hq\bar{q}$ -couplings, internal  $H$ s, etc.)

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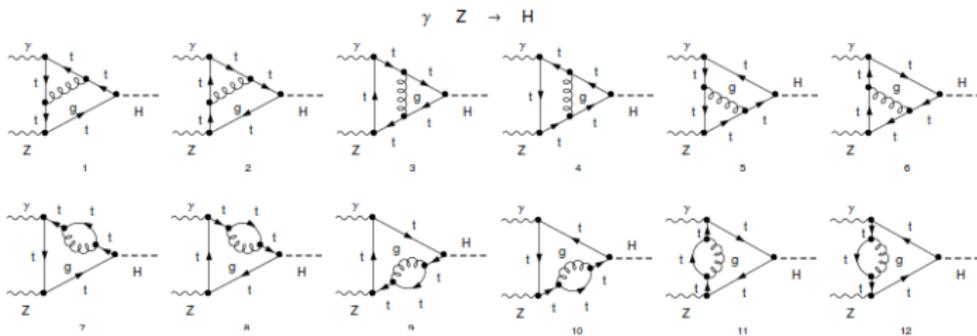


Why the massless fermions?

- ▶ It is (presumably) bigger than the other weak contributions due to the number of fermionic flavours.
- ▶ It contains no elliptic functions (so the answer is expressible in terms of Goncharov polylogs).

# The strong contribution:

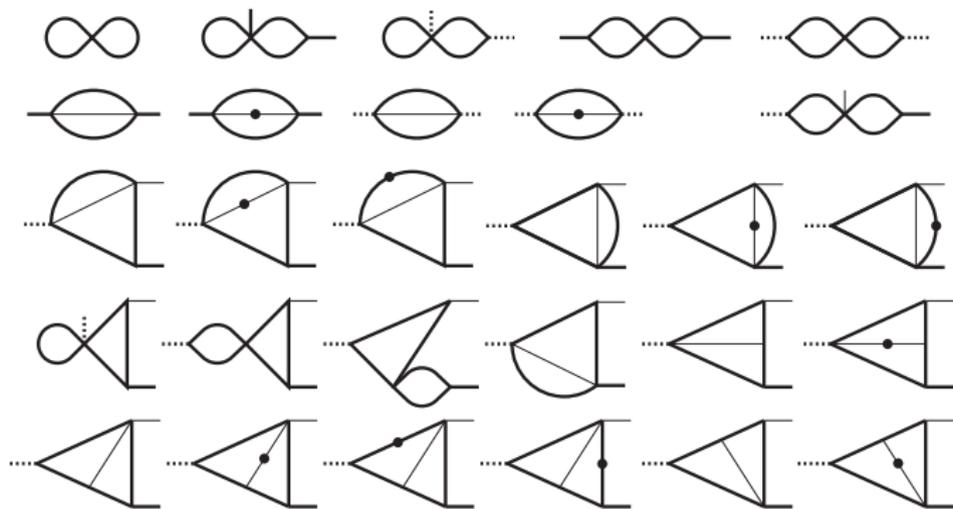
12 Feynman diagrams contribute:



Feynman diagrams are not the bottleneck

Use IBP identities (FIRE5 ArXiv:1408.2372) to reduce all Feynman integrals to master integrals.

There are 28 masters in total:



Find analytical expressions for the masters using differential eqs.

“The Henn Method:” (arXiv:1304.1806)

Find a basis such that

$$dg(\epsilon, \bar{x}) = \epsilon dA(\bar{x})g(\epsilon, \bar{x})$$

$g$  is a vector of 28 masters,  $A$  is a matrix of logs.

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The solution at each order is given by Goncharov polylogs

$$G(a_1, a_2, \dots, a_n; x) = \int_0^x \frac{dt}{t - a_1} G(a_2, \dots, a_n; t)$$

(If  $A$  is expressible without roots).

We have the kinematic variables  $m_H$ ,  $m_Z$ ,  $m_t$   
but use  $m_t^2$ ,  $x$ ,  $y$ , such that

$$m_Z^2 = -m_t^2 \frac{(1-x)^2}{x} \qquad m_H^2 = -m_t^2 \frac{(1-y)^2}{y}$$

This removes all roots in the problem.

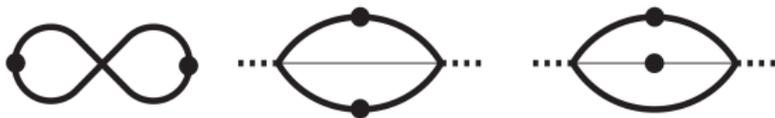
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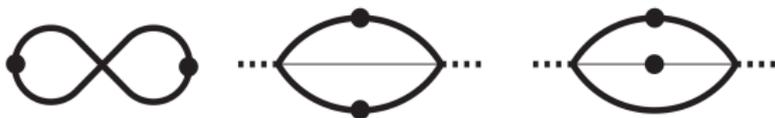
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A good boundary point is  $m_Z = m_H = 0$   
corresponding to  $x = y = 1$ .

This can be found using 'expansion by subgraphs'.



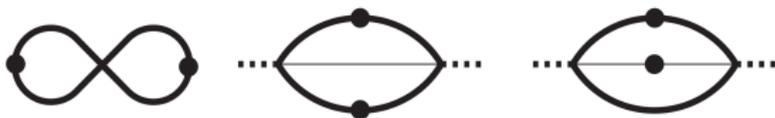
$$f \propto (m_t^2)^{2\epsilon} \left\{ I_{220}, -m_H^2 I_{221}, -\sqrt{m_H^4 - 4m_H^2 m_t^2} (I_{221} + 2I_{212}) \right\}$$



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$$df(\epsilon, y) = \epsilon dA(y) f(\epsilon, y)$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2\log(y-1) - \log(y) & -2\log(y) \\ \log(y) & 3\log(y) & -2\log(y-1) + 4\log(y) - 6\log(y+1) \end{bmatrix}$$



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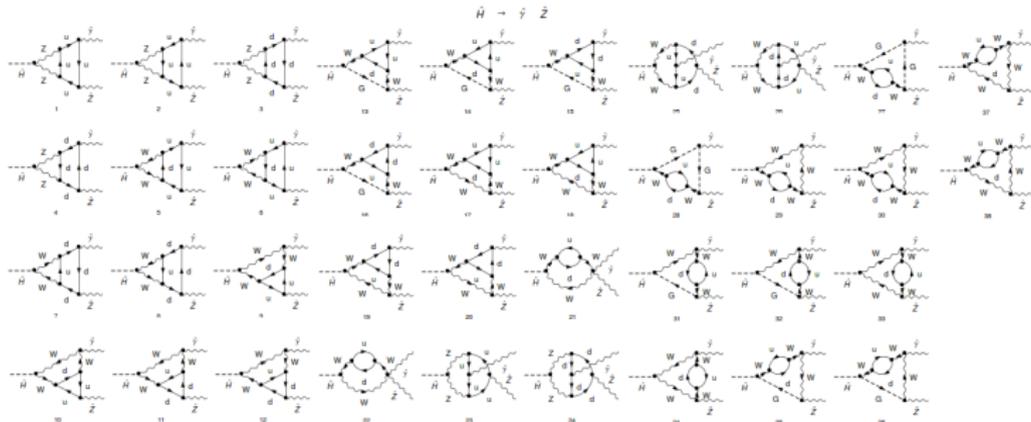
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$$\begin{aligned} f_3 = & 0 + G(0, y)\epsilon + \left( 6G(0, -1, y) + 2G(0, 1, y) - 6G(0, y)G(-1, y) - 2G(0, y)G(1, y) \right. \\ & + 2G(0, y)^2 - \frac{1}{6}\pi^2 \left. \right) \epsilon^2 + \left( \pi^2 G(-1, y) + \frac{1}{3}\pi^2 G(1, y) - 36G(-1, 0, -1, y) - 12G(-1, 0, 1, y) \right. \\ & - 36G(0, -1, -1, y) - 12G(0, -1, 1, y) + 24G(0, 0, -1, y) + 8G(0, 0, 1, y) - 12G(0, 1, -1, y) \\ & - 4G(0, 1, 1, y) - 12G(1, 0, -1, y) - 4G(1, 0, 1, y) - 12G(0, y)^2 G(-1, y) - 4G(0, y)^2 G(1, y) \\ & + 36G(0, y)G(-1, -1, y) + 12G(0, y)G(-1, 1, y) + 12G(0, y)G(1, -1, y) + 4G(0, y)G(1, 1, y) \\ & \left. + \frac{5}{3}G(0, y)^3 - \frac{2}{3}\pi^2 G(0, y) - 11\zeta_3 \right) \epsilon^3 + \mathcal{O}(\epsilon^4) \end{aligned}$$

The massless fermion contribution:

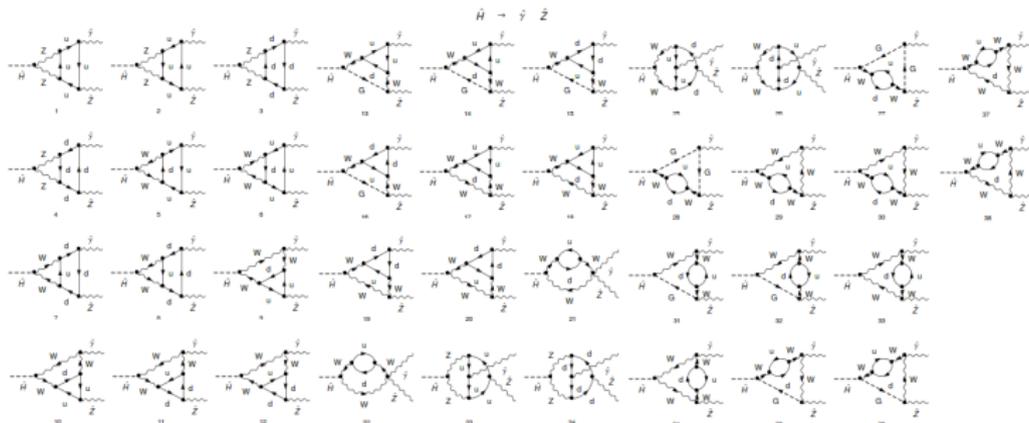
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35 master integrals in two families (planar and non-planar).  
 (Internal W and internal Z can be treated alike)

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But the Ward identity holds

$$A = \varepsilon^\mu(p_\gamma) M_\mu \Rightarrow p_\gamma^\mu M_\mu = 0$$

and the poles match those of the one-loop  
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We need to compare the result with the with the numerical one,  
and to express it in terms of log and  $\text{Li}_n$  (and  $\text{Li}_{22}$ ).

Thank you for listening ...

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