CoLorFul NNLO

Completely Local Subtractions for Fully Differential Predictions at NNLO

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in collaboration with

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HOCTools NNLO workshop, Athens

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NEW SZÉCHENYI PLAN

LHCphenOnet

Outline

- The problem and our goals
- Our method: recipe for a general subtraction scheme at any order in perturbation theory
- Main difficulty: integrating the counter terms
- Light in the tunnel: cancellation of poles
- Some new predictions
- Conclusions

Problem

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$$\equiv \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{VV}}_{m} J_{m}$$

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  - in σ^{RV} kinematical singularities as one parton becomes unresolved yielding ϵ-poles at $O(\epsilon^{-2}, \epsilon^{-1})$ after integration over phase space + explicit ϵ-poles at $O(\epsilon^{-2}, \epsilon^{-1})$
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personal opinion: general solution is not yet available

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- ✓ option to constrain subtraction near singular
  regions (important check)

Recipe

Structure

of subtractions is governed by the jet functions

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$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

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RR,A_2 regularizes doubly-unresolved limits

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042
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RR,A_1 regularizes singly-unresolved limits

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RR,A_{12} removes overlapping subtractions

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Use known ingredients

- Universal IR structure of QCD (squared) matrix elements
  - ϵ-poles of one-loop amplitudes:

$$|\mathcal{M}_m^{(1)}(\{p\})\rangle = \boldsymbol{I}_0^{(1)}(\{p\},\epsilon)|\mathcal{M}_m^{(0)}(\{p\})\rangle + \mathrm{O}(\epsilon^0)$$

$$\boldsymbol{I}_0^{(1)}(\{p\},\epsilon) = \frac{\alpha_\mathrm{s}}{2\pi}\sum_i\left[\frac{1}{\epsilon}\gamma_i - \frac{1}{\epsilon^2}\sum_{k\neq i}\boldsymbol{T}_i\cdot\boldsymbol{T}_k\left(-\frac{4\pi\mu^2}{s_{ik}}\right)^\epsilon\right]$$

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  - ϵ-poles of two-loop amplitudes:

$$|\mathcal{M}_m^{(2)}(\{p\},\epsilon)\rangle = \boldsymbol{I}_0^{(1)}(\{p\})|\mathcal{M}_m^{(1)}(\{p\})\rangle + \boldsymbol{I}_0^{(2)}(\{p\})|\mathcal{M}_m^{(0)}(\{p\})\rangle + \mathrm{O}(\epsilon^0)$$

$$\begin{aligned}
\boldsymbol{I}_{\mathrm{R.S.}}^{(2)}(\epsilon,\mu^2;\{p\}) &= -\frac{1}{2}\boldsymbol{I}^{(1)}(\epsilon,\mu^2;\{p\})\left(\boldsymbol{I}^{(1)}(\epsilon,\mu^2;\{p\}) + 4\pi\beta_0\frac{1}{\epsilon}\right) \\
&+ \frac{e^{+\epsilon\psi(1)}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(2\pi\beta_0\frac{1}{\epsilon}+K\right)\boldsymbol{I}^{(1)}(2\epsilon,\mu^2;\{p\}) \qquad (19) \\
&+ \boldsymbol{H}_{\mathrm{R.S.}}^{(2)}(\epsilon,\mu^2;\{p\}) \;,
\end{aligned}$$

S. Catani 1998

G. Sterman, M.E.Tejeda-Yeomans 2003,S. Moch, M. Mitov 2007

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  - soft and collinear factorization of QCD matrix elements

tree-level 3-parton splitting, double soft current:

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- Extension over whole phase space using momentum mappings (not unique):

$$\{p\}_{n+s} \to \{\tilde{p}\}_n$$

Momentum mappings

$$\{p\}_{n+s} \longrightarrow \{\tilde{p}\}_n$$

- implement exact momentum conservation
- recoil distributed democratically

  ⇒ can be generalized to any number s of unresolved partons

- different mappings for collinear and soft limits
  - collinear limit $p_i || p_r$: $\{p\}_{n+1} \xrightarrow{\mathsf{C}_{ir}} \{\tilde{p}\}_n^{(ir)}$
  - soft limit $p_s \to 0$: $\{p\}_{n+1} \xrightarrow{\mathsf{S}_{s}} \{\tilde{p}\}_n^{(s)}$

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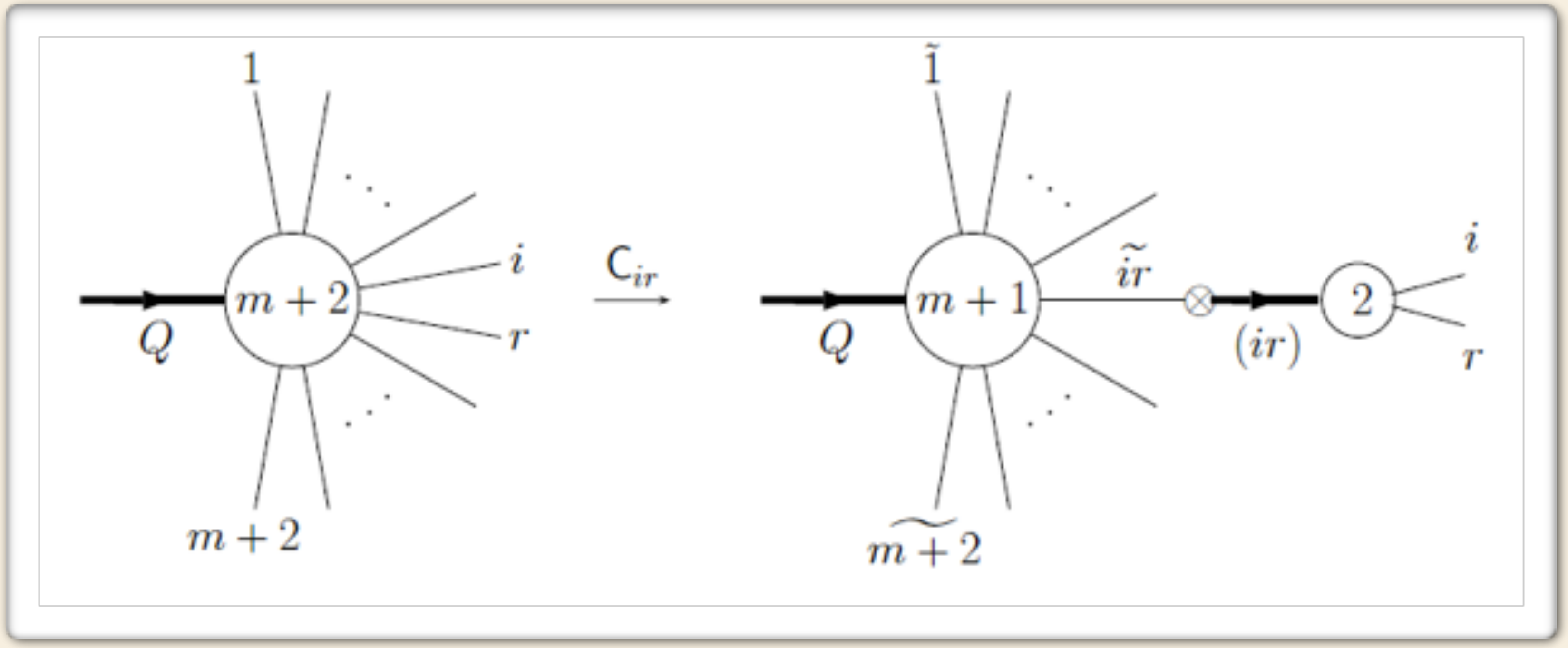

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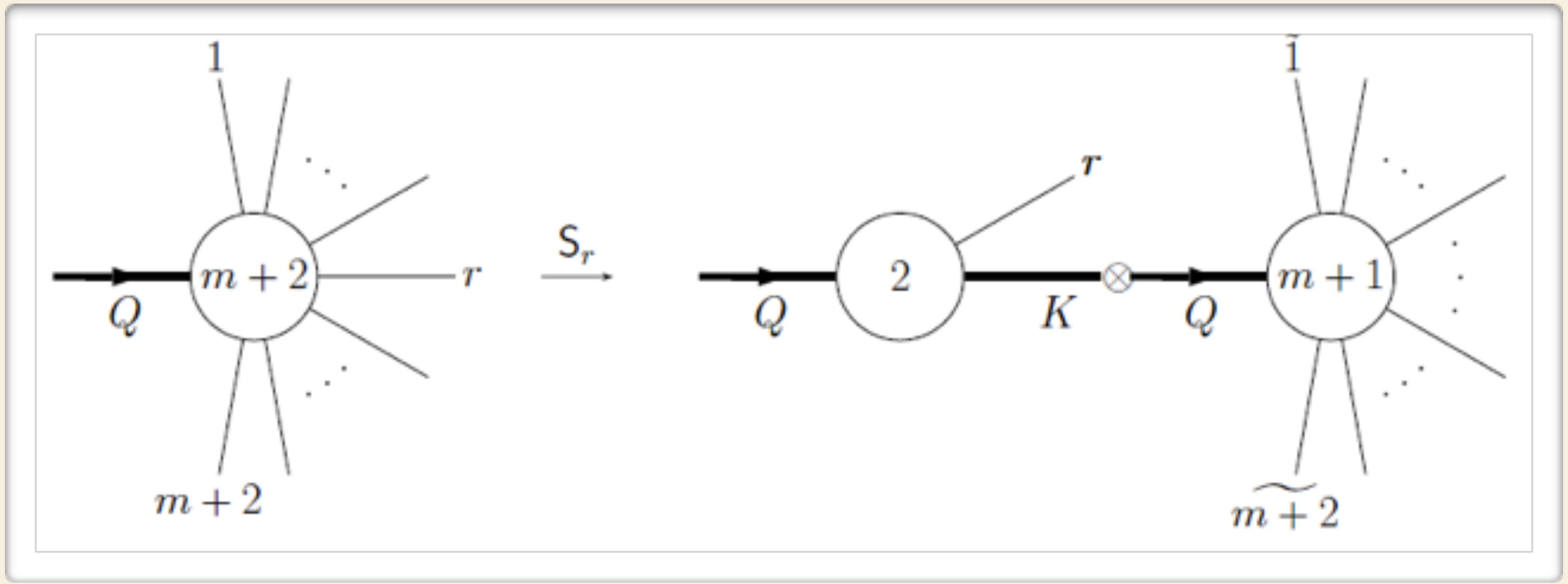

Momentum mappings

define subtractions

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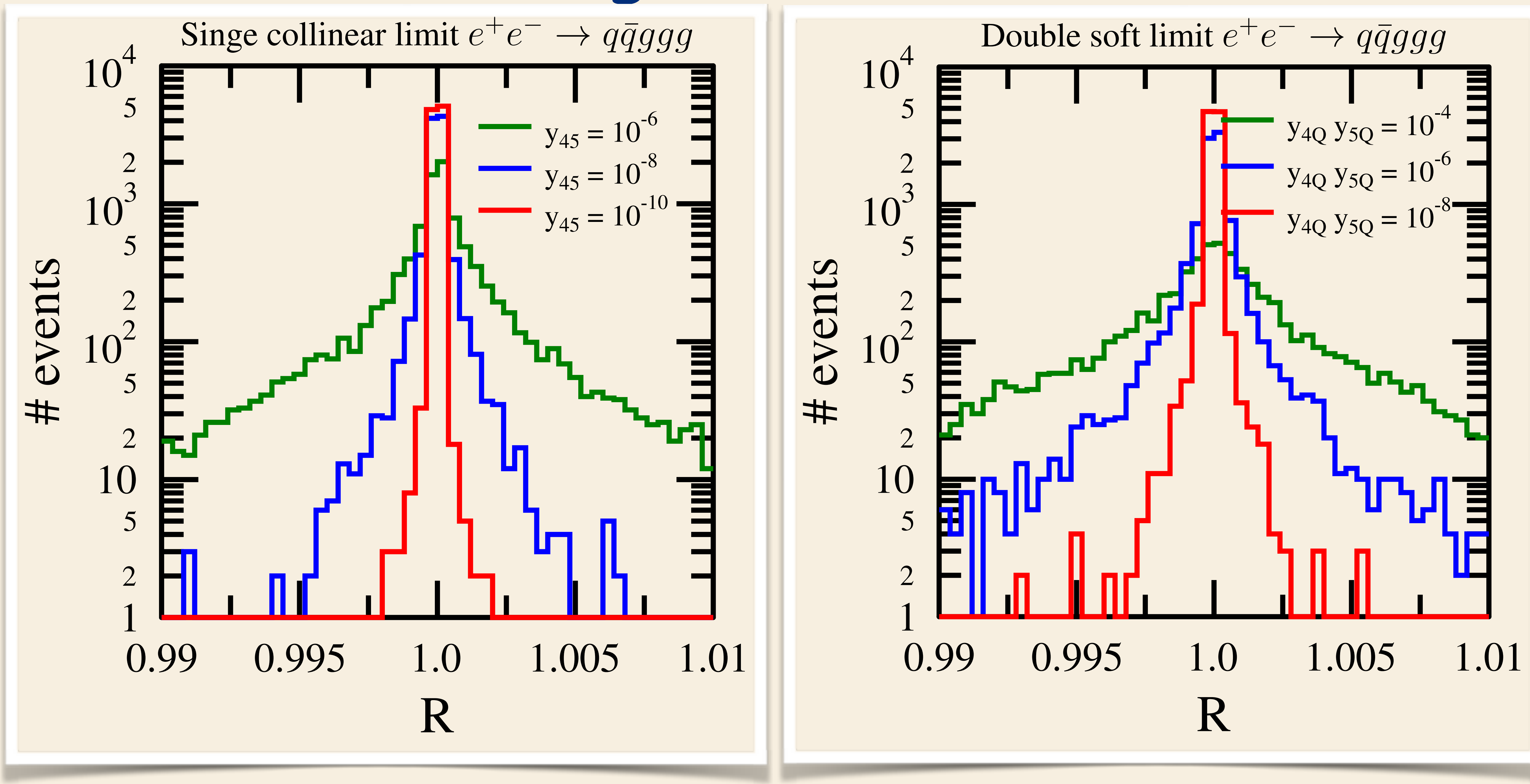
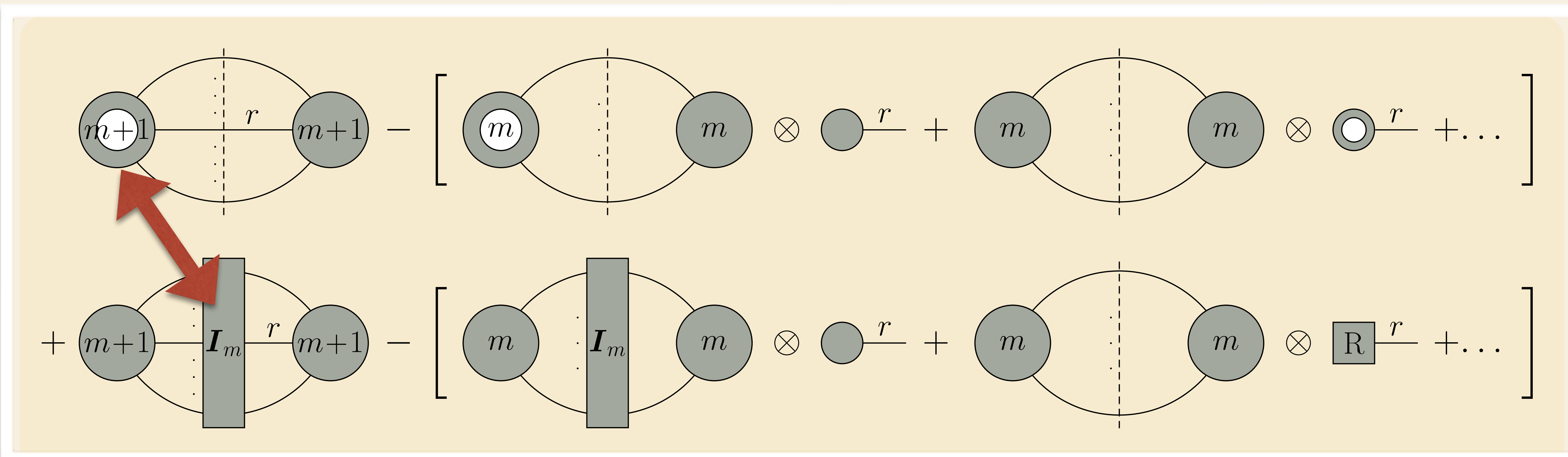
Kinematic singularities cancel in RR



R = subtraction/RR

Cancellation of singularities in RV

Poles cancel vertically pairwise



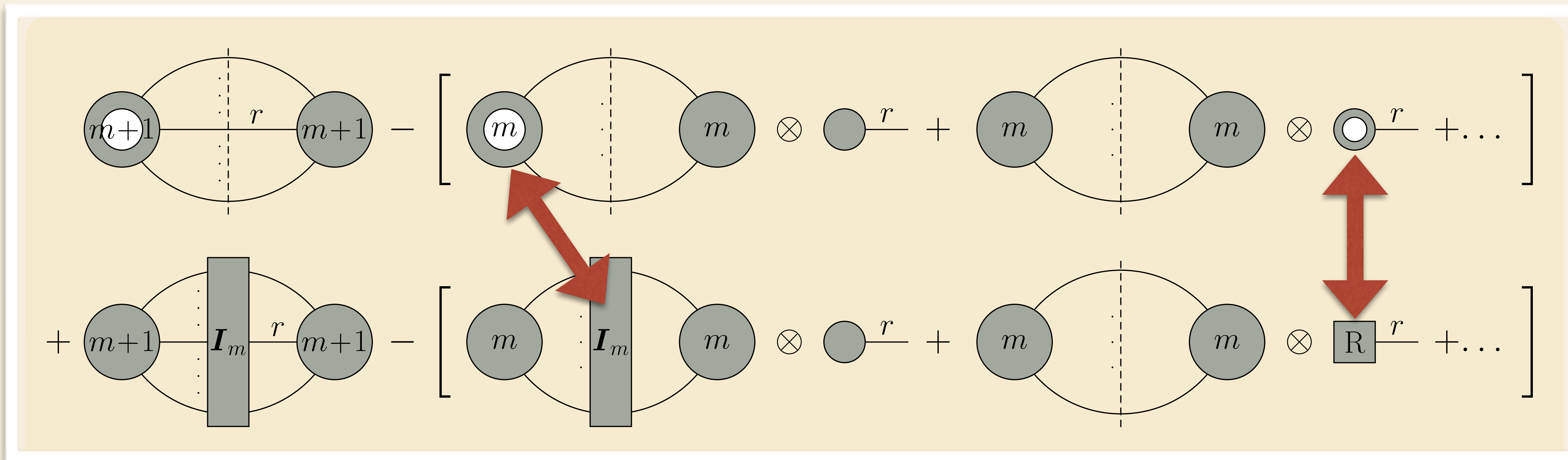
$$\sigma_{m+2}^{\mathrm{NNLO}} = \int_{m+2} \left\{ \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} J_m - \left(\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} J_{m+1} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} J_m \right) \right\}$$

$$\sigma_{m+1}^{\mathrm{NNLO}} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma_{m+1}^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right) J_{m+1} - \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] J_m \right\}$$

$$\sigma_{m}^{\mathrm{NNLO}} = \int_{m} \left\{ \mathrm{d}\sigma_{m}^{\mathrm{VV}} + \int_2 \left(\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} \right) + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

Cancellation of singularities in RV

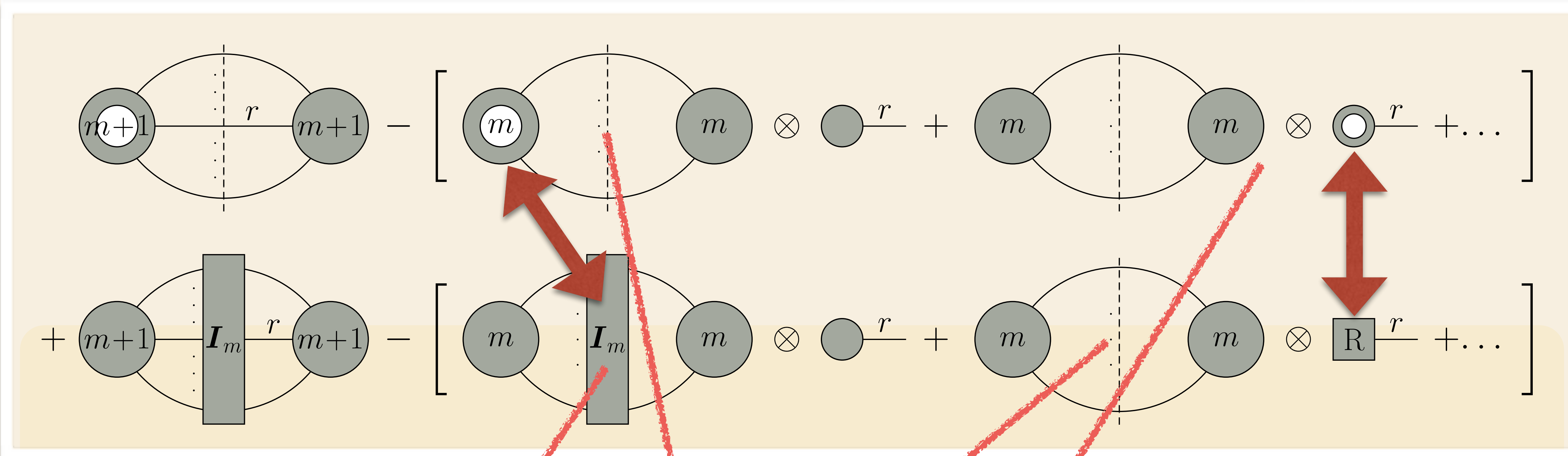
Poles cancel vertically pairwise



$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ d\sigma^{\rm RR}_{m+2} J_{m+2} - d\sigma^{\rm RR,A_2}_{m+2} J_m - \left(d\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - d\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left(d\sigma^{\rm RV}_{m+1} + \int_1 d\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[d\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 d\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ d\sigma^{\rm VV}_{m} + \int_2 \left(d\sigma^{\rm RR,A_2}_{m+2} - d\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[d\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 d\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$



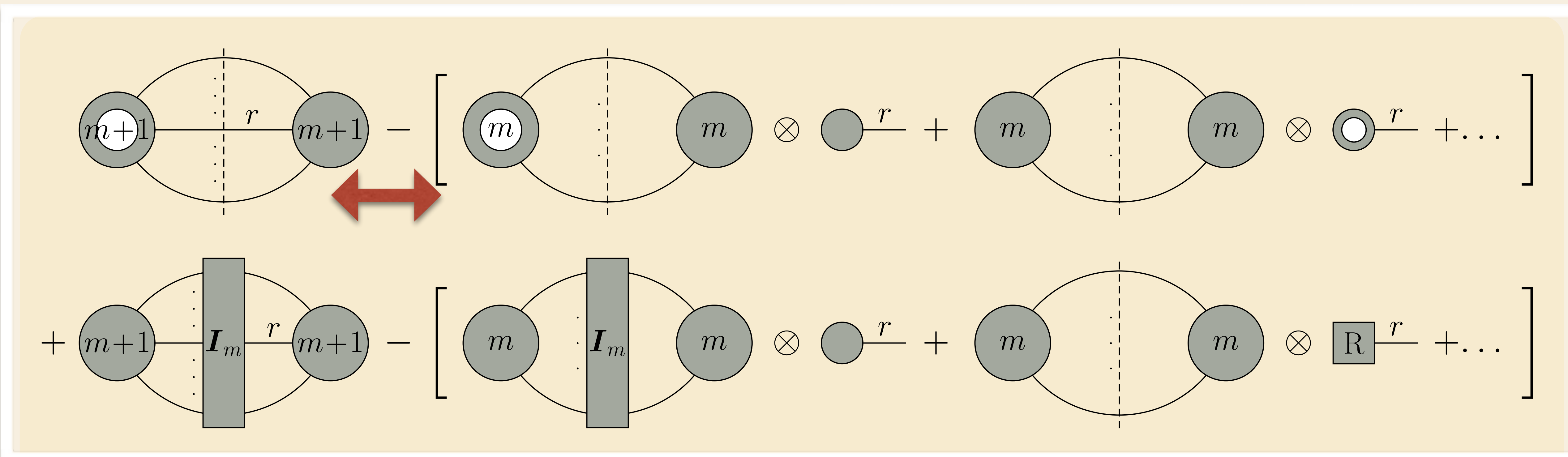

```
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~ Cir ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
e+ e- -> b  b~ b  b~
Checking pole cancellation in point 1
iterm:    1 , g (3) -> b (3) || b~(4)
UBorn: e+ e- -> g  b  b~
                \->  b  b~


Cancellation for Cir00I + Cir01:
             Cir00I                     Cir01                     norm. sum
O(e^-2) :    18.826825462152872        -18.826825462153515       -3.4155581733924357E-014
O(e^-1) :    63.517133810744149        -63.517133810746685       -3.9936272537449989E-014

Cancellation for CirR00 + Cir10:
             CirR00                     Cir10                     norm. sum
O(e^-2) :   -1.1074603213031107         1.1074603213031107       -0.0000000000000000
O(e^-1) :    39.321998994866775        -39.321998994866760        3.6139705707884122E-016
```

Cancellation of singularities in RV

Kinematic singularities cancel horizontally



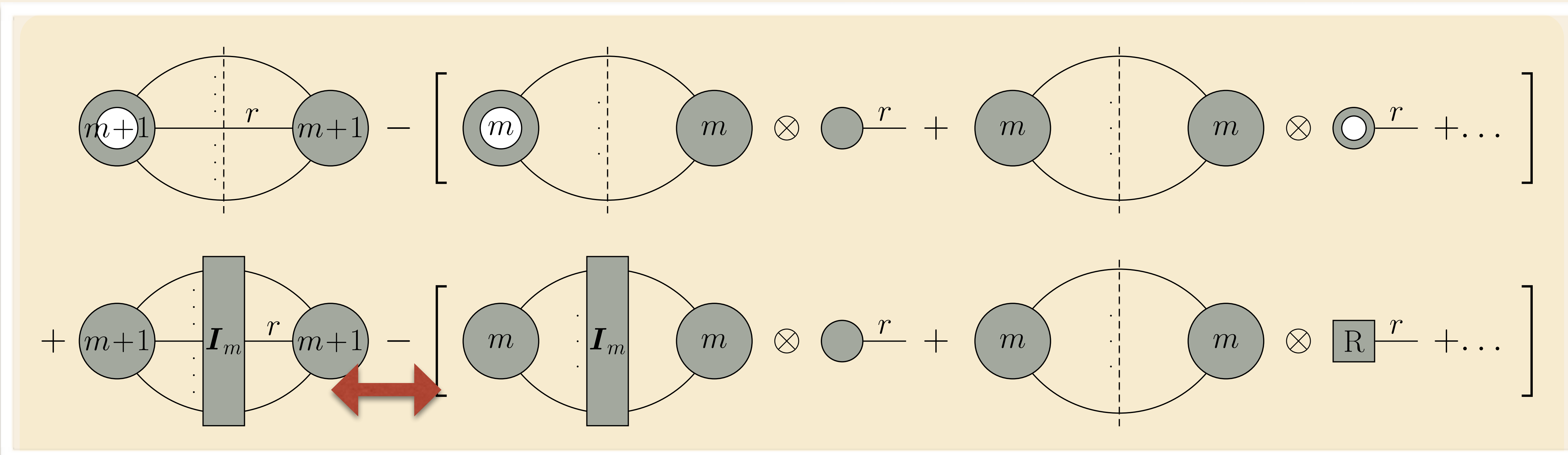
$$\sigma_{m+2}^{\mathrm{NNLO}} = \int_{m+2} \left\{ \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} J_m - \left(\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} J_{m+1} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} J_m \right) \right\}$$

$$\sigma_{m+1}^{\mathrm{NNLO}} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma_{m+1}^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right) J_{m+1} - \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] J_m \right\}$$

$$\sigma_{m}^{\mathrm{NNLO}} = \int_{m} \left\{ \mathrm{d}\sigma_{m}^{\mathrm{VV}} + \int_2 \left(\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} \right) + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

Cancellation of singularities in RV

Kinematic singularities cancel horizontally

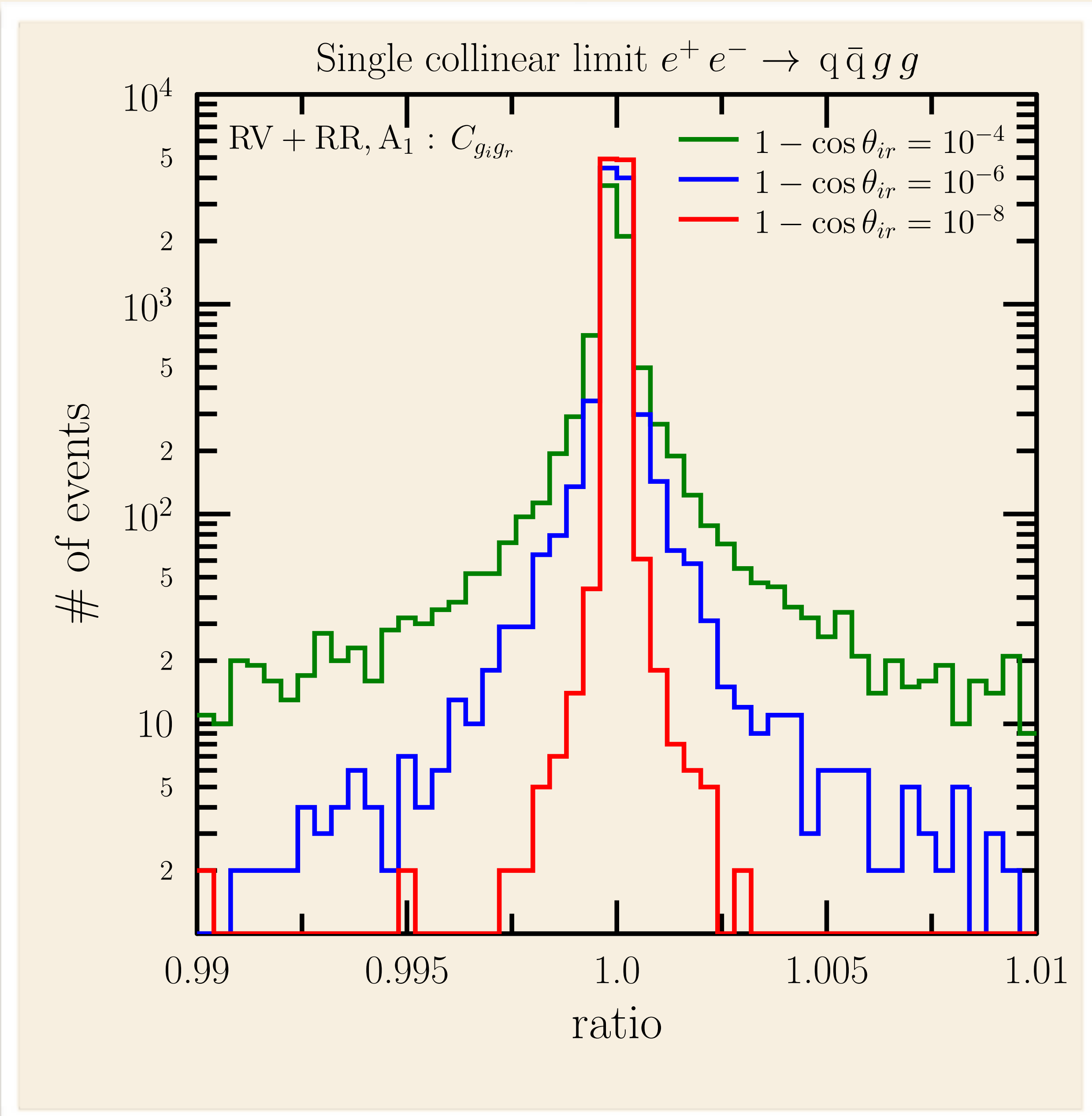


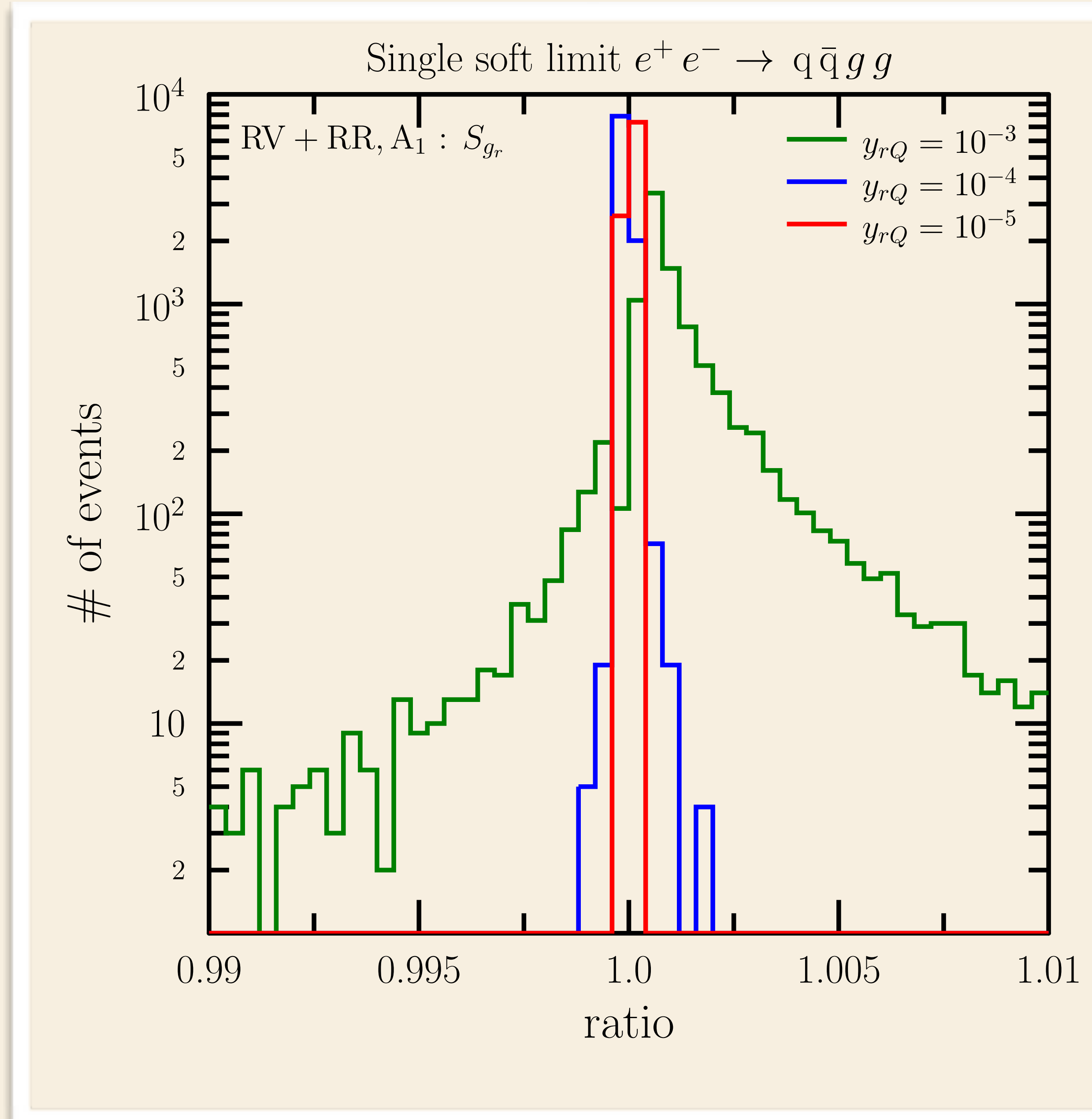
$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ d\sigma^{\rm RR}_{m+2} J_{m+2} - d\sigma^{\rm RR,A_2}_{m+2} J_m - \left(d\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - d\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left(d\sigma^{\rm RV}_{m+1} + \int_1 d\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[d\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 d\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ d\sigma^{\rm VV}_{m} + \int_2 \left(d\sigma^{\rm RR,A_2}_{m+2} - d\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[d\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 d\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

Kinematic singularities cancel in RV





R = subtraction/(RV+RR,A_1)

Regularized RR and RV contributions

can now be computed by numerical Monte Carlo integrations

(implementation for general m in progress)

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043

G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042

Z. Nagy, G. Somogyi, ZT hep-ph/0702273

Difficulty

Integrated approximate xsections

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \Big\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \Big({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \Big) \Big\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \Big\{ \Big({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big) J_{m+1} - \Big[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \Big(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big)^{\rm A_1} \Big] J_m \Big\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \Big\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \Big({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \Big) + \int_1 \Big[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \Big(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big)^{\rm A_1} \Big] \Big\} J_m$$

After integrating over unresolved momenta & summing over unresolved colors and flavors, the subtraction terms can be written as products of insertion operators (in color space) and lower point cross sections:

$$\int_p {\rm d}\sigma^{\rm RR,A_p} = \boldsymbol{I}^{(0)}_p(\{p\}_n;\epsilon) \otimes {\rm d}\sigma^{\rm B}_n$$

Integrated approximate xsections

$$\int_p \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_p} = \int_p \left[\mathrm{d}\phi_{m+2}(\{p\}) \sum_R \mathcal{X}_R(\{p\})\right]$$

$$= \int_p \left[\mathrm{d}\phi_n(\{\tilde{p}\}^{(R)})[\mathrm{d}p_p^{(R)}] \sum_R \left(8\pi\alpha_\mathrm{s}\mu^{2\epsilon}\right)^p \mathit{Sing}_R(p_p^{(R)}) \otimes |\mathcal{M}_n^{(0)}(\{\tilde{p}\}_n^{(R)})|^2\right]$$

$$= \left(8\pi\alpha_\mathrm{s}\mu^{2\epsilon}\right)^p \sum_R \left[\int_p [\mathrm{d}p_p^{(R)}]\mathit{Sing}_R(p_p^{(R)})\right] \otimes \mathrm{d}\phi_n(\{\tilde{p}\}^{(R)})|\mathcal{M}_n^{(0)}(\{\tilde{p}\}_n^{(R)})|^2$$

$$= \underbrace{\left(8\pi\alpha_\mathrm{s}\mu^{2\epsilon}\right)^p \sum_R \left[\int_p [\mathrm{d}p_p^{(R)}]\mathit{Sing}_R(p_p^{(R)})\right]}_{\boldsymbol{I}_p^{(0)}(\{p\}_n;\epsilon)} \otimes \mathrm{d}\sigma_n^{\mathrm{B}}$$

the integrated counter-terms $[X]_R \propto \int_p [\mathrm{d}p_p^{(R)}]\mathit{Sing}_R(p_p^{(R)})$ are

independent of the process & observable

$\Rightarrow$ need to compute only once (admittedly cumbersome, though)

Summation over unresolved flavors

- integrated counter-terms $[X]_{f_i\dots}$ carry flavor indices of unresolved patrons

  ⇒ need to sum over unresolved flavors:

  technically simple, though tedious, result can be summarized in flavor-summed integrated counter-terms

  P. Bolzoni, G. Somogyi, ZT arXiv:0905.4390

- symbolically:

$$\left(X^{(0)}\right)_{f_i\dots}^{(j,l)\dots} = \sum [X^{(0)}]_{f_k\dots}^{(j,l)\dots}$$

- and precisely, for instance, two-flavor sum:

$$\sum_{\{m+2\}} \frac{1}{S_{\{m+2\}}} \sum_t \sum_{k\neq t} [X_{kt}^{(0)}]_{f_k f_t}^{(\dots)} \equiv \sum_{\{m\}} \frac{1}{S_{\{m\}}} \left(X_{kt}^{(0)}\right)^{(\dots)}$$

Integrating out unresolved momenta

two types of singly-unresolved

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \right) \right\}$$

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$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

G. Somogyi, ZT arXiv:0807.0509
U. Aglietti, V. Del Duca, C. Duhr, G. Somogyi, ZT arXiv:0807.0514
P. Bolzoni, S. Moch, G. Somogyi, ZT arXiv:0905.4390

Collinear integrals

convolution of the integral of AP-splitting function over ordinary phase space

$$\int_0^{\alpha_0} \mathrm{d}\alpha\,(1-\alpha)^{2d_0-1}\frac{s_{\widetilde{ir}Q}}{2\pi}\int \mathrm{d}\phi_2(p_i,p_r;p_{(ir)})\frac{1}{s_{ir}^{1+\kappa\epsilon}}P_{f_i f_r}^{(\kappa)}(z_i,z_r;\epsilon)\,, \qquad \kappa=0,\,1$$

$$\mathrm{d}\phi_2(p_i,p_r;p_{(ir)}) = \frac{s_{ir}^{-\epsilon}}{8\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\,\mathrm{d}s_{ir}\,\mathrm{d}v\,\delta\big(s_{ir}-Q^2\alpha\big(\alpha+(1-\alpha)x\big)\big)$$

$$\times\;[v\,(1-v)]^{-\epsilon}\,\Theta(1-v)\Theta(v)$$

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$$\frac{z_r^{k+\delta\epsilon}}{s_{ir}^{1+\kappa\epsilon}}\,g_I^{(\pm)}(z_r)\,, \qquad z_r = \frac{\alpha Q^2 + (1-\alpha) v s_{\widetilde{ir}Q}}{2\alpha Q^2 + (1-\alpha) s_{\widetilde{ir}Q}}$$

δ	Function	$g_I^{(\pm)}(z)$
0	g_A	1
∓ 1	$g_B^{(\pm)}$	$(1-z)^{\pm\epsilon}$
0	$g_C^{(\pm)}$	$(1-z)^{\pm\epsilon}{}_2F_1(\pm\epsilon,\pm\epsilon,1\pm\epsilon,z)$
± 1	$g_D^{(\pm)}$	${}_2F_1(\pm\epsilon,\pm\epsilon,1\pm\epsilon,1-z)$

Soft integrals

convolution of the integral of eikonal factors over ordinary phase space

$$\mathcal{J} \propto -\int_0^{y_0} \mathrm{d}y\,(1-y)^{d_0'-1}\frac{Q^2}{2\pi}\int \mathrm{d}\phi_2(p_r,K;Q)\left(\frac{s_{ik}}{s_{ir}s_{kr}}\right)^{1+\kappa\epsilon}$$

$$\mathrm{d}\phi_2(p_r,K;Q) = \frac{(Q^2)^{-\epsilon}}{16\pi^2}\frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)}\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\mathrm{d}\varepsilon_r\,\varepsilon_r^{1-2\epsilon}\delta(y-\varepsilon_r)$$
$$\times\,\mathrm{d}(\cos\vartheta)\,\mathrm{d}(\cos\varphi)(\sin\vartheta)^{-2\epsilon}(\sin\varphi)^{-1-2\epsilon}$$

G. Somogyi, ZT arXiv:0807.0509
U. Aglietti, V. Del Duca, C. Duhr, G. Somogyi, ZT arXiv:0807.0514
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Computing the integrals

- Use algebraic and symmetry relations to reduce to a basic set $\Rightarrow$ MI's (but no IBP was used), not minimal
- two strategies:

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5. evaluate parametric integrals of pole coefficients in terms of multiple polylogs, or numerically e.g. by SecDec

Status of (287) integrals

Int	status
$\mathcal{I}^{(k)}_{1\mathcal{C},0}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},1}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},2}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},3}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},4}$	✔
$\mathcal{I}^{(k,l)}_{1\mathcal{C},5}$	✔
$\mathcal{I}^{(k,l)}_{1\mathcal{C},6}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},7}$	✔
$\mathcal{I}_{1\mathcal{C},8}$	✔

Int	status
$\mathcal{I}_{1\mathcal{S},0}$	✔
$\mathcal{I}_{1\mathcal{S},1}$	✔
$\mathcal{I}_{1\mathcal{S},2}$	$(m > 3)$ ✔
$\mathcal{I}^{(k)}_{1\mathcal{S},3}$	✔
$\mathcal{I}_{1\mathcal{S},4}$	✔
$\mathcal{I}_{1\mathcal{S},5}$	✔
$\mathcal{I}_{1\mathcal{S},6}$	✔
$\mathcal{I}_{1\mathcal{S},7}$	✔

Int	status
$\mathcal{I}_{1\mathcal{CS},0}$	✔
$\mathcal{I}_{1\mathcal{CS},1}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{CS},2}$	✔
$\mathcal{I}_{1\mathcal{CS},3}$	✔
$\mathcal{I}_{1\mathcal{CS},4}$	✔

Int	status
$\mathcal{I}^{(k,l)}_{12\mathcal{C},1}$	✔
$\mathcal{I}^{(k,l)}_{12\mathcal{C},2}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},3}$	✔
$\mathcal{I}^{(k,l)}_{12\mathcal{C},4}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},5}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},6}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},7}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},8}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},9}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},10}$	✔

Int	status
$\mathcal{I}_{2\mathcal{S},1}$	✔
$\mathcal{I}_{2\mathcal{S},2}$	✔
$\mathcal{I}_{2\mathcal{S},3}$	✔
$\mathcal{I}_{2\mathcal{S},4}$	✔
$\mathcal{I}_{2\mathcal{S},5}$	✔
$\mathcal{I}_{2\mathcal{S},6}$	✔
$\mathcal{I}_{2\mathcal{S},7}$	✔
$\mathcal{I}_{2\mathcal{S},8}$	✔
$\mathcal{I}_{2\mathcal{S},9}$	✔
$\mathcal{I}_{2\mathcal{S},10}$	✔
$\mathcal{I}_{2\mathcal{S},11}$	✔
$\mathcal{I}_{2\mathcal{S},12}$	✔
$\mathcal{I}_{2\mathcal{S},13}$	✔
$\mathcal{I}_{2\mathcal{S},14}$	✔
$\mathcal{I}_{2\mathcal{S},15}$	✔
$\mathcal{I}_{2\mathcal{S},16}$	✔
$\mathcal{I}_{2\mathcal{S},17}$	✔
$\mathcal{I}_{2\mathcal{S},18}$	✔
$\mathcal{I}_{2\mathcal{S},19}$	✔
$\mathcal{I}_{2\mathcal{S},20}$	✔
$\mathcal{I}_{2\mathcal{S},21}$	✔
$\mathcal{I}_{2\mathcal{S},22}$	✔
$\mathcal{I}_{2\mathcal{S},23}$	✔

Int	status
$\mathcal{I}^{(k)}_{12\mathcal{S},1}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},2}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},3}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},4}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},5}$	✔
$\mathcal{I}_{12\mathcal{S},6}$	✔
$\mathcal{I}_{12\mathcal{S},7}$	✔
$\mathcal{I}_{12\mathcal{S},8}$	✔
$\mathcal{I}_{12\mathcal{S},9}$	✔
$\mathcal{I}_{12\mathcal{S},10}$	✔
$\mathcal{I}_{12\mathcal{S},11}$	✔
$\mathcal{I}_{12\mathcal{S},12}$	✔
$\mathcal{I}_{12\mathcal{S},13}$	✔

Int	status
$\mathcal{I}^{(k)}_{12\mathcal{CS},1}$	✔
$\mathcal{I}_{12\mathcal{CS},2}$	✔
$\mathcal{I}_{12\mathcal{CS},3}$	✔

Int	status	Int	status
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},1}$	✔	$\mathcal{I}^{(k)}_{2\mathcal{CS},1}$	✔
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},2}$	✔	$\mathcal{I}^{(k)}_{2\mathcal{CS},2}$	✔
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},3}$	✔	$\mathcal{I}^{(2)}_{2\mathcal{CS},2}$	✔
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},4}$	✔	$\mathcal{I}^{(k)}_{2\mathcal{CS},3}$	✔
$\mathcal{I}^{(-1,-1,-1,-1)}_{2\mathcal{C},5}$	✔	$\mathcal{I}^{(k)}_{2\mathcal{CS},4}$	✔
$\mathcal{I}^{(k,l)}_{2\mathcal{C},6}$	✔	$\mathcal{I}^{(k)}_{2\mathcal{CS},5}$	✔

✔: pole coefficients are known analytically, finite numerically, in some cases analytically

Structure of insertion operators

recall general form for Born sections

$$\int_p \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_p} = \boldsymbol{I}_p^{(0)}(\{p\}_n;\epsilon) \otimes \mathrm{d}\sigma_n^{\mathrm{B}}$$

Insertion operators involve all possible color connections with given number of unresolved patrons with kinematic coefficients

for 1 unresolved parton on tree SME $|M^{(0)}|^2$:

$$\boldsymbol{I}_1^{(0)}(\{p\}_{m+1};\epsilon) = \frac{\alpha_s}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon \sum_i \left[\mathrm{C}_{1,f_i}^{(0)} \boldsymbol{T}_i^2 + \sum_k \mathrm{S}_1^{(0),(i,k)} \boldsymbol{T}_i \boldsymbol{T}_k\right]$$

kinematic functions contain poles starting from $O(\epsilon^{-2})$ for collinear and from $O(\epsilon^{-1})$ for soft

G. Somogyi, ZT hep-ph/0609041

Structure of insertion operators

recall general form for Born sections

$$\int_p \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_p} = \boldsymbol{I}_p^{(0)}(\{p\}_n;\epsilon) \otimes \mathrm{d}\sigma_n^{\mathrm{B}}$$

for 2 unresolved patrons on tree SME $|M^{(0)}|^2$:

$$\boldsymbol{I}_2^{(0)}(\{p\}_m;\epsilon) = \left[\frac{\alpha_{\mathrm{s}}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 \Bigg\{ \sum_i \left[\mathrm{C}_{2,f_i}^{(0)} \boldsymbol{T}_i^2 + \sum_k \mathrm{C}_{2,f_i f_k}^{(0)} \boldsymbol{T}_k^2\right] \boldsymbol{T}_i^2$$

$$+ \sum_{j,l} \left[\mathrm{S}_2^{(0),(j,l)} C_{\mathrm{A}} + \sum_i \mathrm{CS}_{2,f_i}^{(0),(j,l)} \boldsymbol{T}_i^2\right] \boldsymbol{T}_j \boldsymbol{T}_l$$

$$+ \sum_{i,k,j,l} \mathrm{S}_2^{(0),(i,k)(j,l)} \{\boldsymbol{T}_i \boldsymbol{T}_k, \boldsymbol{T}_j \boldsymbol{T}_l\} \Bigg\}$$

the iterated doubly-unresolved has the same color structure, kinematic coefficients differ

G. Somogyi et al arXiv:0905.4390, arXiv:1301.3504, arXiv:1301.3919

Structure of insertion operators

general form at one loop

$$\int_1 \mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} = \boldsymbol{I}_1^{(0)}(\{p\}_m;\epsilon) \otimes \mathrm{d}\sigma_m^{\mathrm{V}} + \boldsymbol{I}_1^{(1)}(\{p\}_m;\epsilon) \otimes \mathrm{d}\sigma_m^{\mathrm{B}}$$

for 1 unresolved parton on loop SME $|M^{(1)}|^2$:

$$\boldsymbol{I}_1^{(1)}(\{p\}_m;\epsilon) = \left[\frac{\alpha_{\mathrm{s}}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 \sum_i \left[\mathrm{C}_{1,f_i}^{(1)} C_{\mathrm{A}} \boldsymbol{T}_i^2 + \sum_k \mathrm{S}_1^{(1),(i,k)} C_{\mathrm{A}} \boldsymbol{T}_i \boldsymbol{T}_k + \sum_{\substack{k,l \\ k\neq l}} \mathrm{S}_1^{(1),(i,k,l)} \sum_{a,b,c} f_{abc} T_i^a T_k^b T_l^c\right]$$

present for m > 3 (four or more hard partons)

G. Somogyi, ZT arXiv:0807.0509

Structure of insertion operators

singly-unresolved integrated singly unresolved:

$$\int_1 \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A}_1} \right)^{\mathrm{A}_1} = \left[\frac{1}{2} \left\{ \boldsymbol{I}_1^{(0)}(\{p\}_m;\epsilon), \boldsymbol{I}_1^{(0)}(\{p\}_m;\epsilon) \right\} + \boldsymbol{I}_{1,1}^{(0,0)}(\{p\}_m;\epsilon) \right] \otimes \mathrm{d}\sigma_m^{\mathrm{B}}$$

for 1 unresolved parton contributions on iterated I:

$$\boldsymbol{I}_{1,1}^{(0,0)}(\{p\}_m;\epsilon) = \left[\frac{\alpha_{\mathrm{s}}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2} \right)^\epsilon \right]^2 \sum_i \left[\mathrm{C}_{1,1,f_i}^{(0,0)} C_{\mathrm{A}} \boldsymbol{T}_i^2 + \sum_k \mathrm{S}_{1,2}^{(0,0),(i,k)} C_{\mathrm{A}} \boldsymbol{T}_i \boldsymbol{T}_k \right]$$

kinematic functions contain poles starting from $O(\epsilon^{-3})$ only

G. Somogyi, ZT arXiv:0807.0509

Structure of insertion operators

- the color structures are independent of the precise definition of subtractions (momentum mappings), only subleading coefficients of ϵ-expansion in kinematic functions may depend
- we computed all insertion operators analytically (defined in our subtraction scheme) up to $O(\epsilon^{-2})$ for arbitrary m

Light in the tunnel

Cancellation of poles

- we checked the cancellation of the leading and first subleading poles (defined in our subtraction scheme) for arbitrary m
- for m=2,
  - the insertion operators are independent of the kinematics (momenta are back-to-back, so MI's are needed at the endpoints only)
  - color algebra is trivial:

$$\boldsymbol{T}_1 \boldsymbol{T}_2 = -\boldsymbol{T}_1^2 = -\boldsymbol{T}_2^2 = -C_{\mathrm{F}}$$

- so can demonstrate the cancellation of poles

Example: $H\to b\bar{b}$ at $\mu = m_H$

$$\sigma_m^{\mathrm{NNLO}} = \int_m \left\{ \mathrm{d}\sigma_m^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

$$\mathrm{d}\sigma_{H\to b\bar{b}}^{\mathrm{VV}} = \left(\frac{\alpha_s(\mu^2)}{2\pi} \right)^2 \mathrm{d}\sigma_{H\to b\bar{b}}^{\mathrm{B}} \left\{ + \frac{2C_{\mathrm{F}}^2}{\epsilon^4} + \left(\frac{11 C_{\mathrm{A}} C_{\mathrm{F}}}{4} + 6C_{\mathrm{F}}^2 - \frac{C_{\mathrm{F}} n_{\mathrm{f}}}{2} \right) \frac{1}{\epsilon^3} \right.$$
$$+ \left[\left(\frac{8}{9} + \frac{\pi^2}{12} \right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(\frac{17}{2} - 2\pi^2 \right) C_{\mathrm{F}}^2 - \frac{2 C_{\mathrm{F}} n_{\mathrm{f}}}{9} \right] \frac{1}{\epsilon^2}$$
$$\left. + \left[\left(-\frac{961}{216} + \frac{13\zeta_3}{2} \right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(\frac{109}{8} - 2\pi^2 - 14\zeta_3 \right) C_{\mathrm{F}}^2 + \frac{65 C_{\mathrm{F}} n_{\mathrm{f}}}{108} \right] \frac{1}{\epsilon} \right\}$$

C. Anastasiou, F. Herzog, A. Lazopoulos, arXiv:0111.2368

$$\sum \int \mathrm{d}\sigma^{\mathrm{A}} = \left(\frac{\alpha_s(\mu^2)}{2\pi} \right)^2 \mathrm{d}\sigma_{H\to b\bar{b}}^{\mathrm{B}} \left\{ - \frac{2C_{\mathrm{F}}^2}{\epsilon^4} - \left(\frac{11 C_{\mathrm{A}} C_{\mathrm{F}}}{4} + 6C_{\mathrm{F}}^2 + \frac{C_{\mathrm{F}} n_{\mathrm{f}}}{2} \right) \frac{1}{\epsilon^3} \right.$$
$$- \left[\left(\frac{8}{9} + \frac{\pi^2}{12} \right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(\frac{17}{2} - 2\pi^2 \right) C_{\mathrm{F}}^2 - \frac{2 C_{\mathrm{F}} n_{\mathrm{f}}}{9} \right] \frac{1}{\epsilon^2}$$
$$\left. - \left[\left(-\frac{961}{216} + \frac{13\zeta_3}{2} \right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(\frac{109}{8} - 2\pi^2 - 14\zeta_3 \right) C_{\mathrm{F}}^2 + \frac{65 C_{\mathrm{F}} n_{\mathrm{f}}}{108} \right] \frac{1}{\epsilon} \right\}$$

V. Del Duca, C. Duhr, G. Somogyi, F. Tramontano, Z. Trócsányi, arXiv:to appear soon

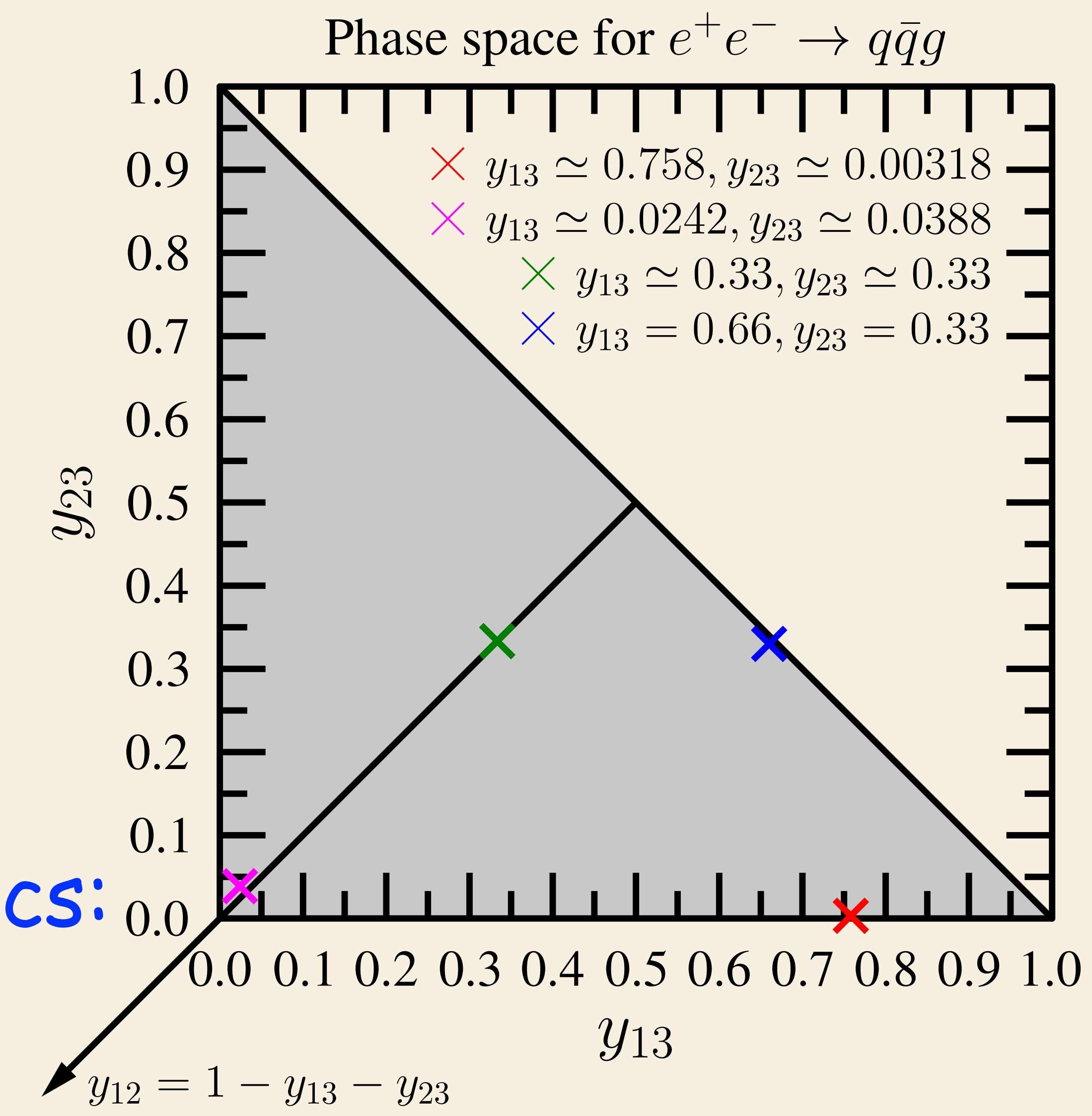
Cancellation of poles

- we checked the cancellation of the leading and first subleading poles (defined in our subtraction scheme) for arbitrary m
- for m=2
- for m=3,
  - color algebra can be performed explicitly:

$$\boldsymbol{T}_1\boldsymbol{T}_2 = \frac{1}{2}C_{\mathrm{A}} - C_{\mathrm{F}}$$

$$\boldsymbol{T}_1\boldsymbol{T}_3 = \boldsymbol{T}_2\boldsymbol{T}_3 = -\frac{1}{2}C_{\mathrm{A}}$$

  - the insertion operators depend on 3-jet kinematics:



Example: $e^+e^- \to$ m(=3) jets at $\mu^2 = s$

$$\sigma_m^{\rm NNLO} = \int_m \left\{ {\rm d}\sigma_m^{\rm VV} + \int_2 \left[{\rm d}\sigma_{m+2}^{\rm RR,A_2} - {\rm d}\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[{\rm d}\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 {\rm d}\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] \right\} J_m$$

$${\rm d}\sigma_3^{\rm VV} = \mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right) + \mathcal{F}inite\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right)$$

$$\begin{aligned}
&\mathcal{P}oles\left(A_3^{(2\times 0)}(1_q,3_g,2_{\bar q}) + A_3^{(1\times 1)}(1_q,3_g,2_{\bar q})\right) \\
&= 2\Bigg[-\left(\boldsymbol{I}^{(1)}_{q\bar q g}(\epsilon)\right)^2 - \frac{\beta_0}{\epsilon}\boldsymbol{I}^{(1)}_{q\bar q g}(\epsilon) \\
&\quad + e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon}+K\right)\boldsymbol{I}^{(1)}_{q\bar q g}(2\epsilon) + \boldsymbol{H}^{(2)}_{q\bar q g}\Bigg] A_3^0(1_q,3_g,2_{\bar q}) \\
&\quad + 2\,\boldsymbol{I}^{(1)}_{q\bar q g}(\epsilon)\,A_3^{(1\times 0)}(1_q,3_g,2_{\bar q})\ . \qquad (4.59)
\end{aligned}$$

$$\boldsymbol{I}^{(1)}_{q\bar q g}(\epsilon) = \mathcal{R}e\,\boldsymbol{I}^{(1)}_0(p_q,p_{\bar q},p_g;\epsilon)$$

$$\begin{aligned}
\boldsymbol{H}^{(2)}_{q\bar q g} &= \frac{e^{\epsilon\gamma}}{4\,\epsilon\,\Gamma(1-\epsilon)}\Bigg[\left(4\zeta_3+\frac{589}{432}-\frac{11\pi^2}{72}\right)N^2 + \left(-\frac{1}{2}\zeta_3-\frac{41}{54}-\frac{\pi^2}{48}\right) \\
&\quad + \left(-3\zeta_3-\frac{3}{16}+\frac{\pi^2}{4}\right)\frac{1}{N^2} + \left(-\frac{19}{18}+\frac{\pi^2}{36}\right)NN_F + \left(-\frac{1}{54}-\frac{\pi^2}{24}\right)\frac{N_F}{N} + \frac{5}{27}N_F^2.\Bigg]\ . \qquad (4.61)
\end{aligned}$$

A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, G. Heinrich arXiv:0710.0346

Example: $e^+e^- \to$ m(=3) jets at $\mu^2 = s$

$$\sigma_m^{\rm NNLO} = \int_m \left\{ {\rm d}\sigma_m^{\rm VV} + \int_2 \left[{\rm d}\sigma_{m+2}^{\rm RR,A_2} - {\rm d}\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[{\rm d}\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 {\rm d}\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] \right\} J_m$$

$${\rm d}\sigma_3^{\rm VV} = \mathcal{P}oles\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right) + \mathcal{F}inite\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right)$$

$$\mathcal{P}oles\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right) + \mathcal{P}oles \sum \int {\rm d}\sigma^{\rm A} =$$ 200k Mathematica lines

= zero numerically in any phase space point:

```
                   0.             2    0. nf
           0. + --- + 0. Nc   + ----- + 0. Nc nf
                  2                  Nc
                Nc
Out[1]= -------------------------------------- +
                                   2
                                  e

                 0.             2   0. nf
          0. + --- + 0. Nc   + ----- + 0. Nc nf
                 2                 Nc
               Nc                                             0
------------------------------------------------- + O[e]
                         e
```

Example: $e^+e^- \to$ m(=3) jets at $\mu^2 = s$

$$\sigma_m^{\rm NNLO} = \int_m \left\{ d\sigma_m^{\rm VV} + \int_2 \left[d\sigma_{m+2}^{\rm RR,A_2} - d\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[d\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 d\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] \right\} J_m$$

$$d\sigma_3^{\rm VV} = \mathcal{P}oles\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right) + \mathcal{F}inite\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right)$$

$\mathcal{P}oles\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right) + \mathcal{P}oles \sum \int d\sigma^{\rm A} =$ 200k Mathematica lines

= zero analytically according to C. Duhr

Message:

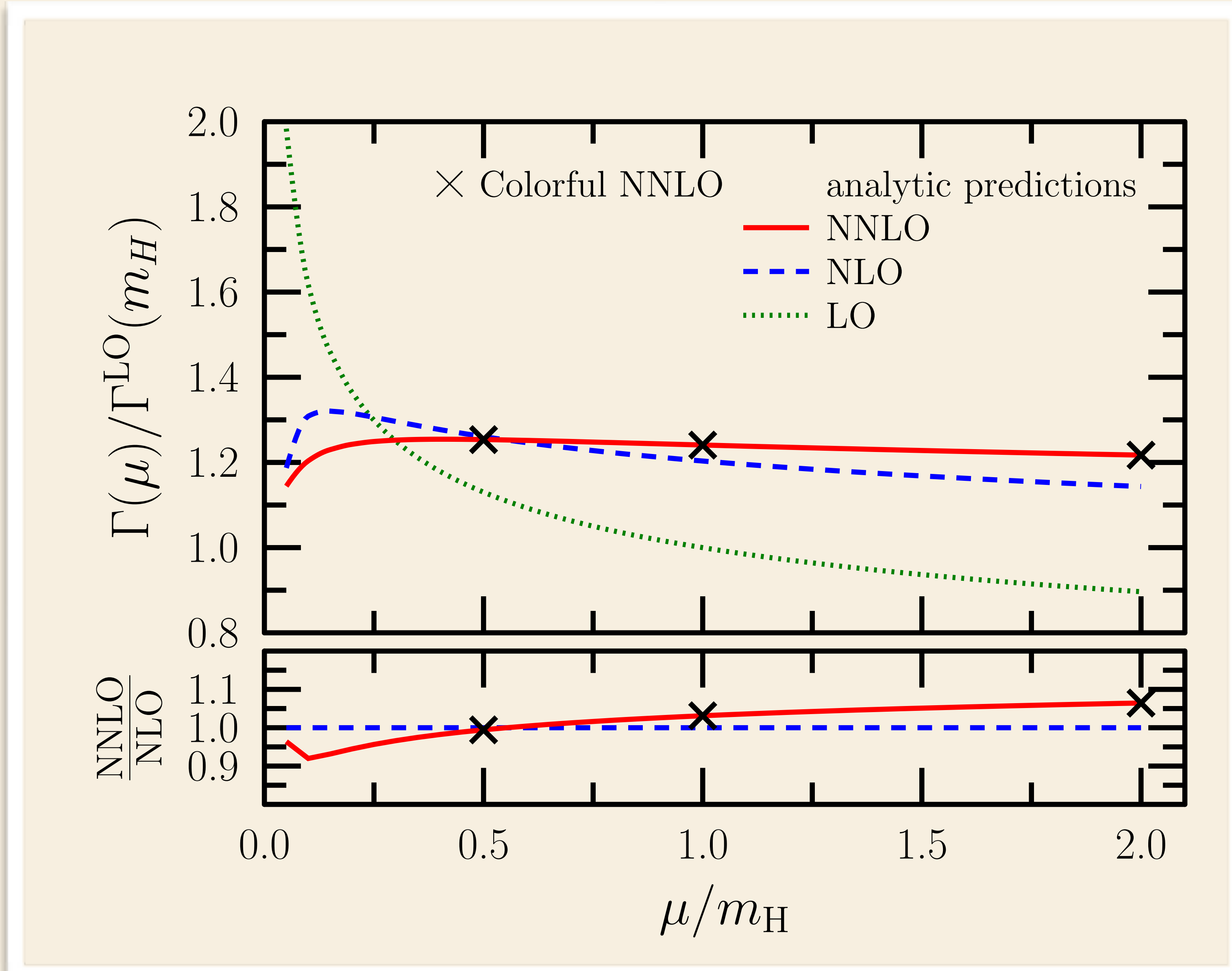
$$\sigma_3^{\rm NNLO} = \int_3 \left\{ d\sigma_3^{\rm VV} + \sum \int d\sigma^{\rm A} \right\}_{\epsilon=0} J_3$$

indeed finite in d=4 dimensions

Application

Example: H→b$\bar{b}$

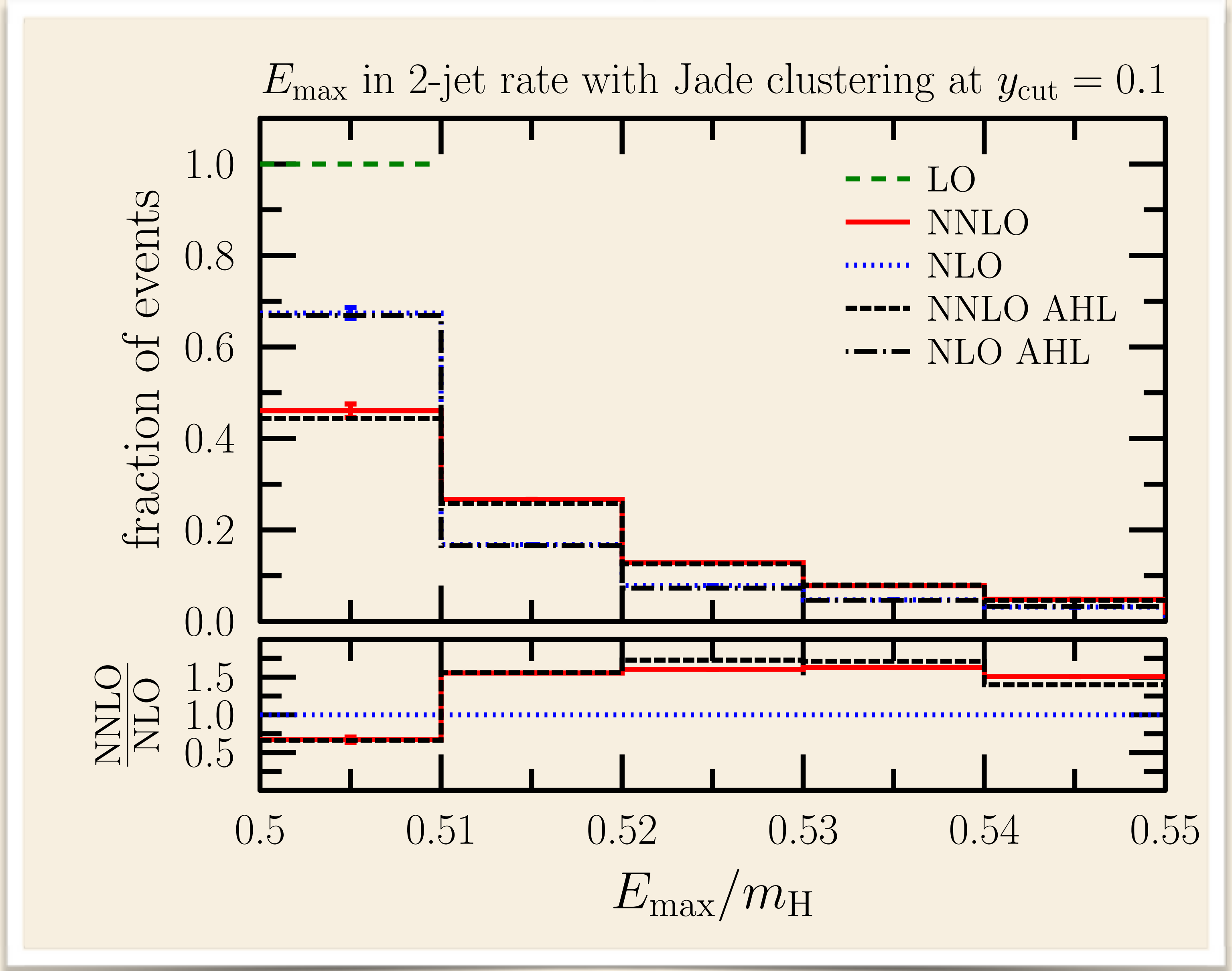
$$\Gamma^{\mathrm{NNLO}}_{H\to b\bar{b}}(\mu = m_H) = \Gamma^{\mathrm{LO}}_{H\to b\bar{b}}(\mu = m_H)\left[1 - \left(\frac{\alpha_s}{\pi}\right)5.666667 - \left(\frac{\alpha_s}{\pi}\right)^2 29.149 + \mathcal{O}(\alpha_s^3)\right]$$



Scale dependence of the inclusive decay rate Γ(H -> b$\bar{b}$)

analytic: K.G. Chetyrkin hep-ph/9608318

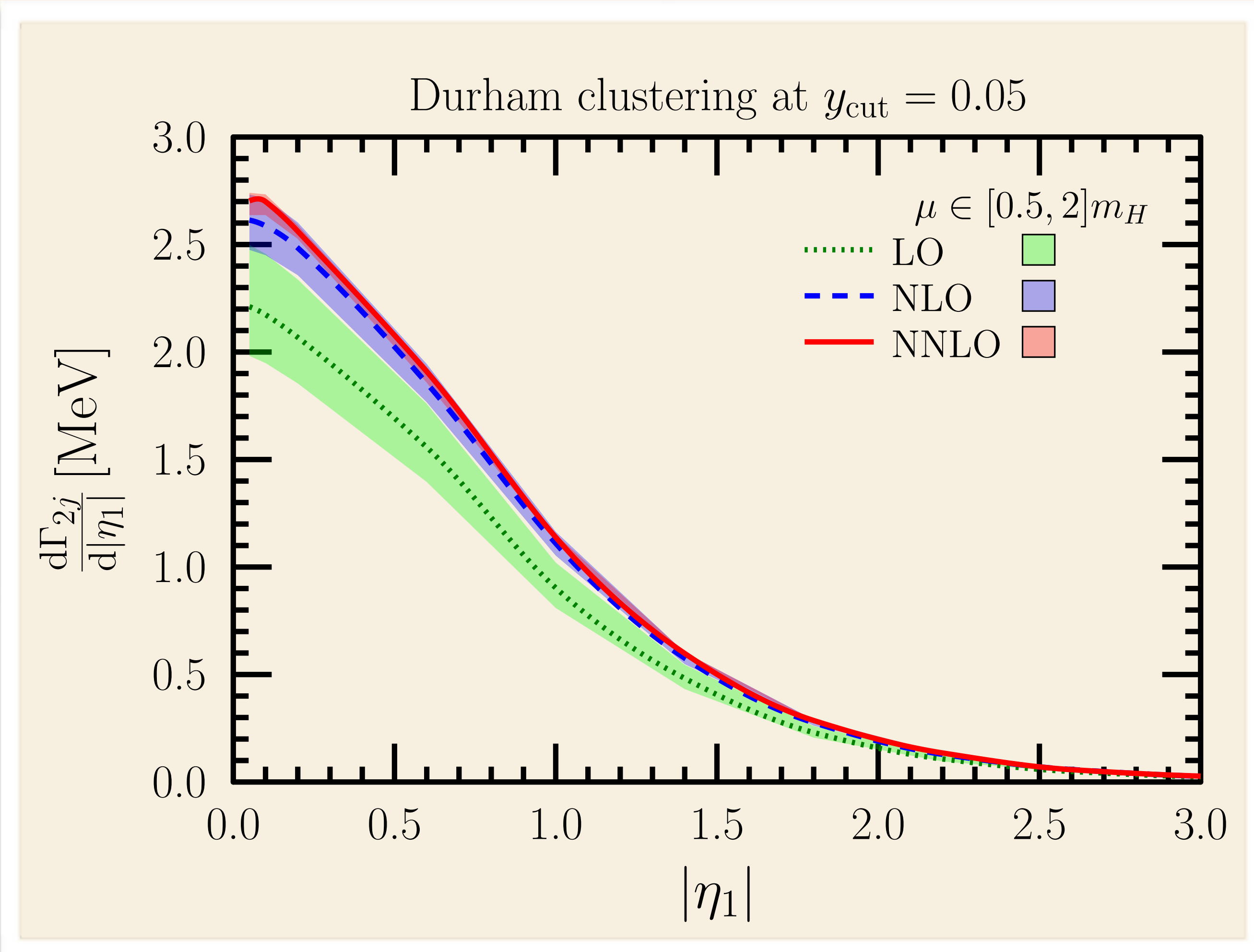
Example: $H \to b\bar{b}$ at $\mu = m_H$

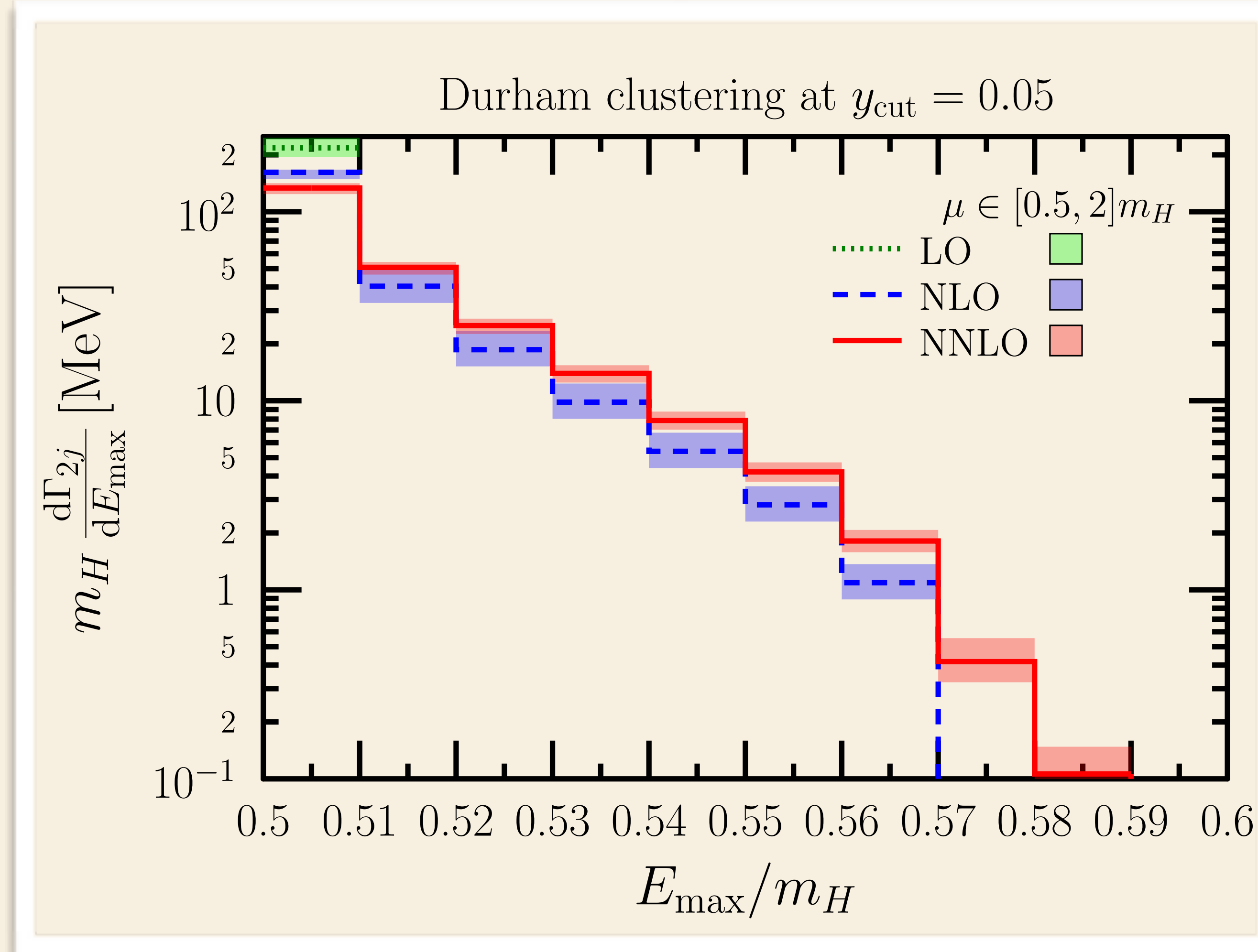


Energy spectrum of the leading jet in the rest frame of the Higgs boson. Jets are clustered using the JADE algorithm with $y_{cut} = 0.1$

AHL = C. Anastasiou, F. Herzog, A. Lazopoulos arXiv:0111.2368

Example: $H \to b\bar{b}$





rapidity distribution energy spectrum

of the leading jet in the rest frame of the Higgs boson.

jets are clustered using the Durham algorithm with y_{cut} = 0.05

Conclusions

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✓ Defined a general subtraction scheme for computing NNLO fully differential jet cross sections (presently only for processes with no colored particles in the initial state)

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- ✓ Defined a general subtraction scheme for computing NNLO fully differential jet cross sections (presently only for processes with no colored particles in the initial state)
- ✓ Subtractions are
  - ✓ fully local
  - ✓ exact and explicit in color (using color state formalism)

Conclusions

- ✓ Defined a general subtraction scheme for computing NNLO fully differential jet cross sections (presently only for processes with no colored particles in the initial state)
- ✓ Subtractions are
  - ✓ fully local
  - ✓ exact and explicit in color (using color state formalism)
- ✓ Demonstrated the cancellation of ϵ-poles for m=2 and 3
- ✓ First application: Higgs-boson decay into a b-quark pair