BSM & Higgs @ 100 TeV, CERN,
MARCH, 2015

Seung J. Lee (KAIST)

Precision Higgs Boson Studies at the FCC-hh

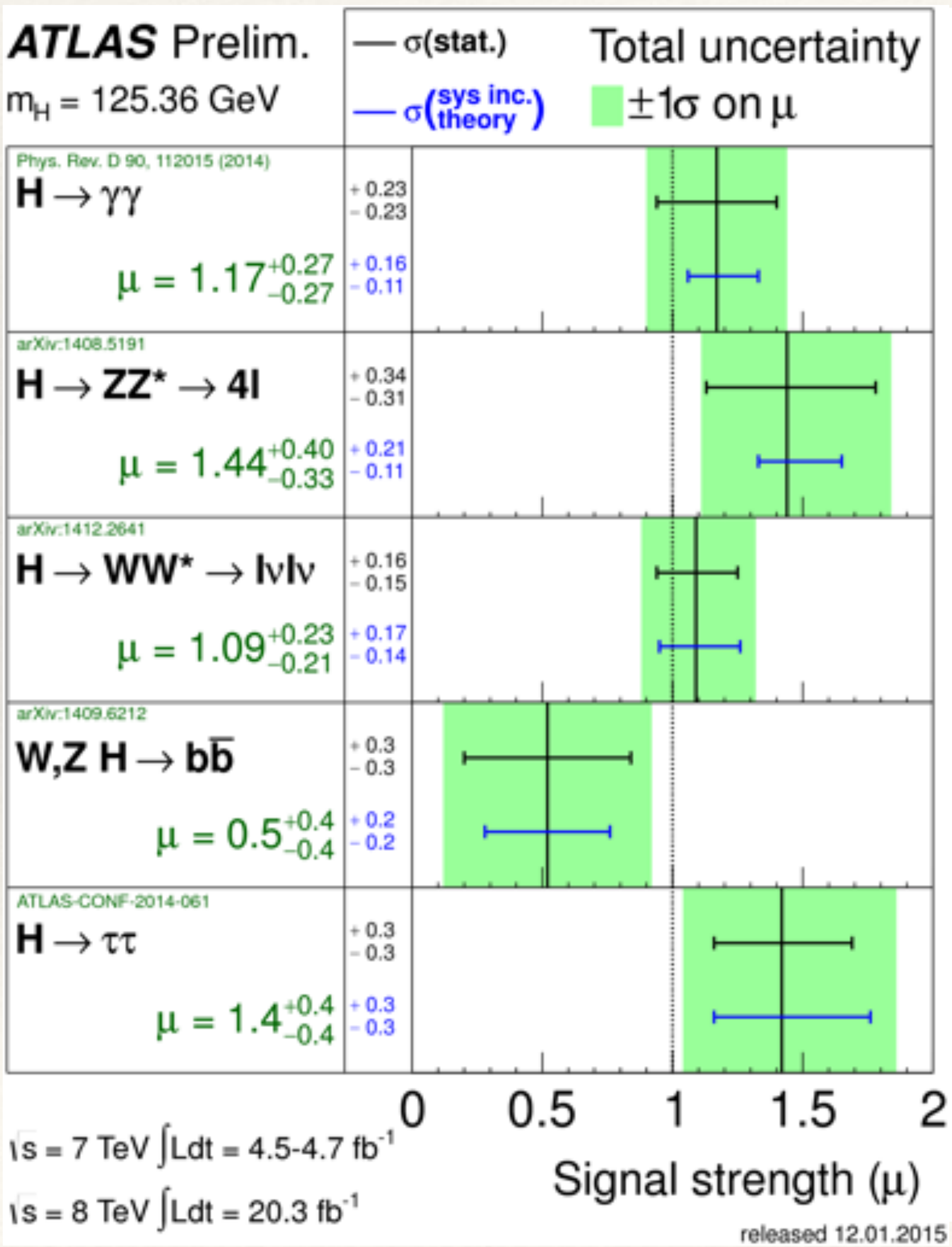
Study of SM Higgs boson partial widths and branching fractions and ratio of them

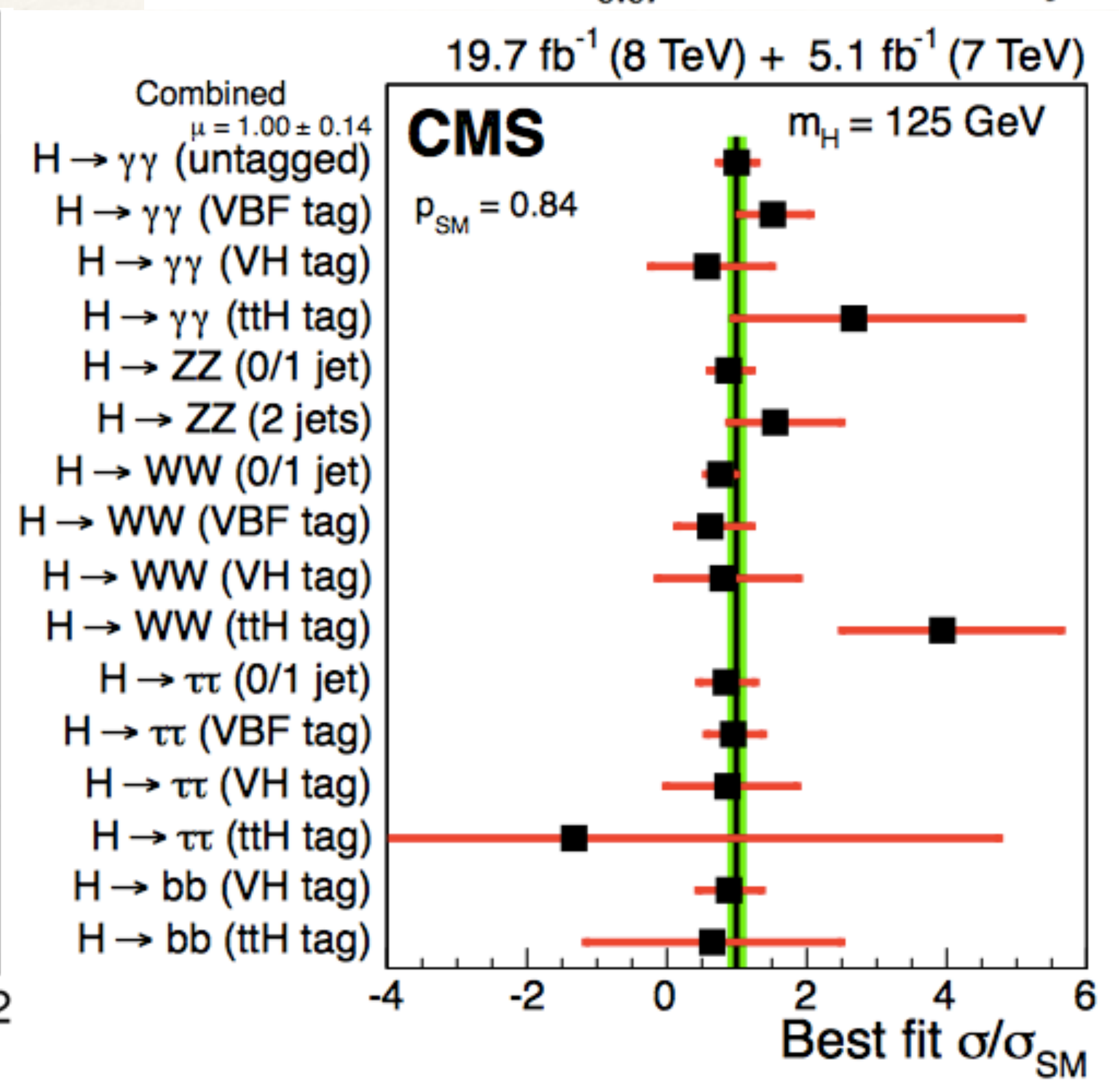
With L. Almeida, S. Porkorski, J. Wells
arXiv:1311.6721v3 Phys. Rev. D 89, 033006 (2014)

Nothing but Higgs, with ~ 10-20% Precision for Higgs couplings so far

$$m_H = 125.02\ ^{+0.26}_{-0.27}\,(\text{stat})\ ^{+0.14}_{-0.15}\,(\text{syst})\ \text{GeV}$$

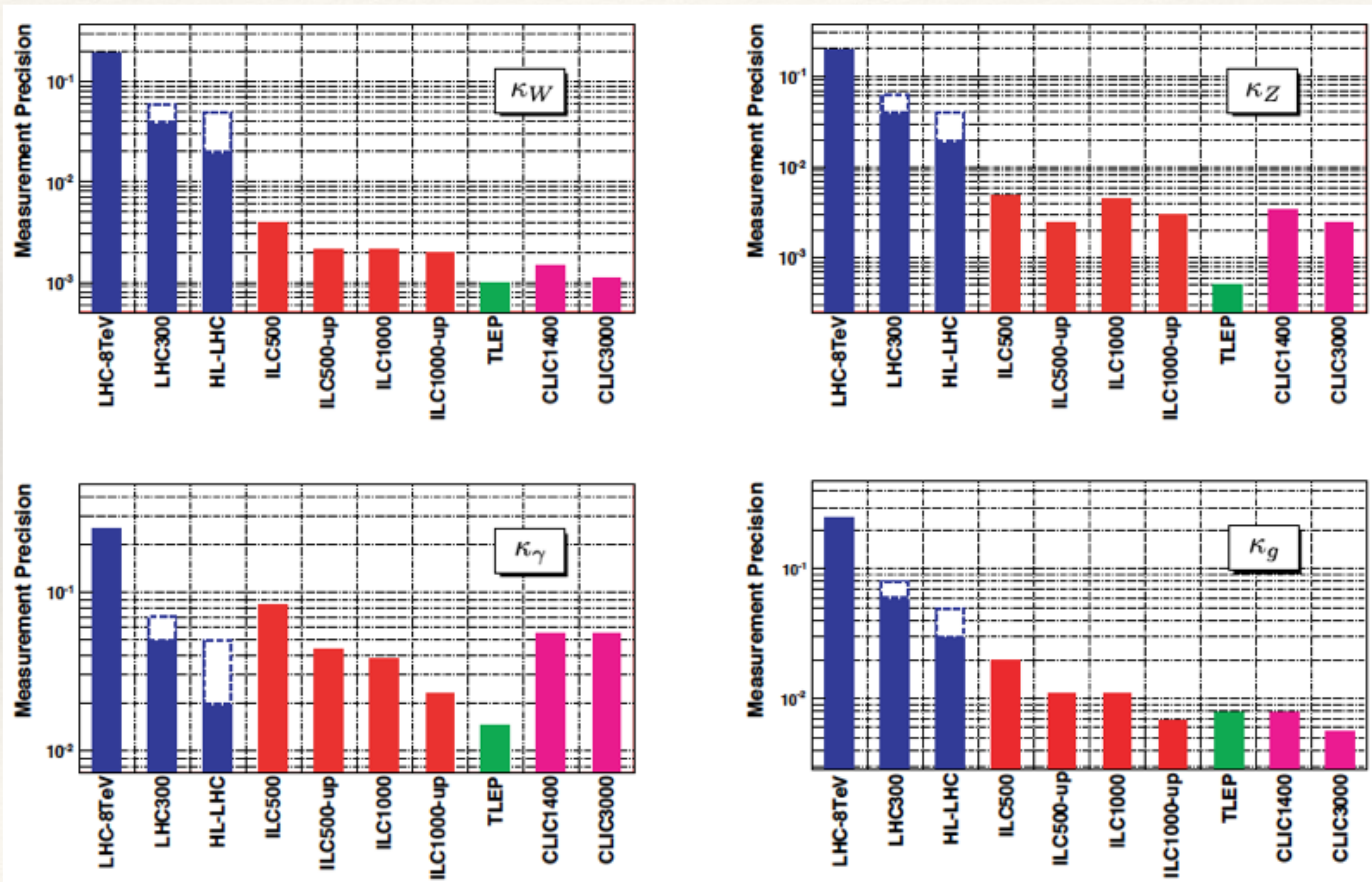
$$\mu = 1.00 \pm 0.09\,(\text{stat})\ ^{+0.08}_{-0.07}\,(\text{theo}) \pm 0.07\,(\text{syst})$$





New era of precision studies of the Higgs sector

(from Snowmass Higgs working group report): Higgs precision will approach that of EWP



@ 100 TeV

4. Higgs cross sections for HE-LHC

SM Higgs production cross sections at $\sqrt{s}$ = 14, 33, 40, 60, 80 and 100 TeV (M_H=125 GeV)

Process	$\sqrt{s}$ = 14 TeV	$\sqrt{s}$ = 33 TeV	$\sqrt{s}$ = 40 TeV	$\sqrt{s}$ = 60 TeV	$\sqrt{s}$ = 80 TeV	$\sqrt{s}$ = 100 TeV
ggF [a]	50.35 pb	178.3 pb (3.5)	231.9 pb (4.6)	394.4 pb (7.8)	565.1 pb (11.2)	740.3 pb (14.7)
VBF [b]	4.40 pb	16.5 pb (3.8)	23.1 pb (5.2)	40.8 pb (9.3)	60.0 pb (13.6)	82.0 pb (18.6)
WH [c]	1.63 pb	4.71 pb (2.9)	5.88 pb (3.6)	9.23 pb (5.7)	12.60 pb (7.7)	15.90 pb (9.7)
ZH [c]	0.904 pb	2.97 pb (3.3)	3.78 pb (4.2)	6.19 pb (6.8)	8.71 pb (9.6)	11.26 pb (12.5)
ttH [d]	0.623 pb	4.56 pb (7.3)	6.79 pb (11)	15.0 pb (24)	25.5 pb (41)	37.9 pb (61)
$gg \to HH^{e}(\lambda=1)$	33.8 fb	207 fb (6.1)	298 fb (8.8)	609 fb (18)	980 fb (29)	1.42 pb (42)

PDF is NNLO(NLO) MSTW2008 set. Numbers in () parentheses are the cross-section ratio wrt 14 TeV.

a) NNLO+NNLL QCD + NLO EW corrections. QCD scale and PDF+α_s uncertainties remain constant about +-8% for both (D. de Florian).

b) NNLO QCD only with VBF@NNLO (M. Zaro).

c) NNLO QCD only with VH@NNLO (R. Harlander).

d) NLO QCD. (M. Spira).

e) NLO QCD with HPAIR. The central scale is the invariant mass of the Higgs pair. The scale is varied by a factor 2 up and down. (M. Spira).

https://twiki.cern.ch/twiki/bin/view/LHCPhysics/HiggsEuropeanStrategy2012

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WH [c]	1.63 pb	4.71 pb (2.9)	5.88 pb (3.6)	9.23 pb (5.7)	12.60 pb (7.7)	15.90 pb (9.7)	x 10 !
ZH [c]	0.904 pb	2.97 pb (3.3)	3.78 pb (4.2)	6.19 pb (6.8)	8.71 pb (9.6)	11.26 pb (12.5)	x 12 !
ttH [d]	0.623 pb	4.56 pb (7.3)	6.79 pb (11)	15.0 pb (24)	25.5 pb (41)	37.9 pb (61)	x 60 !
gg → HH[e]*(λ=1)*	33.8 fb	207 fb (6.1)	298 fb (8.8)	609 fb (18)	980 fb (29)	1.42 pb (42)	x 42 !

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- careful exposition of the decay partial widths and branching fractions of a SM Higgs boson with mass near 125 GeV.
- state-of-the-art formulas that can be used in any precision electroweak analysis to investigate compatibility of the data with the SM predictions in these most fundamental and sensitive observables

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- Other calculations exist in the literature, mostly notably from the computer program HDECAY; however, we wish to provide an independent calculation that includes the latest advances and allows us to vary the renormalization scale in all parts of the computations. This flexibility will be useful in discussions regarding uncertainties

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  uncertainties

- We also aim to detail the errors that each input into the computation propagates to the final answer for each observable

Our Expansion Formalism of Partial Widths and Uncertainties

Taylor expand the full expressions for partial width around the input observables. This expansion is made possible by the fact that with the discovery of the Higgs boson, and knowledge of its mass, all input observables are now known to good enough accuracy to render an expansion of this nature useful and accurate.

We represent the partial width expansion by

$$\Gamma_{H\to X} = \Gamma_X^{(\mathrm{ref})}\left(1 + \sum_i a_{\mathcal{T}_i,X}\overline{\delta\mathcal{T}_i}\right)$$

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$$\overline{\delta\mathcal{T}_i} = \frac{\mathcal{T}_i - \mathcal{T}_{i,ref}}{\mathcal{T}_{i,ref}},$$

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	$\Gamma_X^{(\text{Ref})}$/GeV	$a_{m_t,X}$	$a_{m_H,X}$	$a_{\alpha(M_Z),X}$	$a_{\alpha_S(M_Z),X}$	$a_{m_b,X}$	$a_{M_Z,X}$	$a_{m_c,X}$	$a_{m_\tau,X}$	$a_{G_F,X}$
total	4.17×10^{-3}	-3.3×10^{-2}	4.34	8.35×10^{-1}	-5.05×10^{-1}	1.32	-3.21	7.80×10^{-2}	1.24×10^{-1}	8.49×10^{-1}
gg	3.61×10^{-4}	-1.62×10^{-1}	2.89	0.	2.48	-6.51×10^{-2}	3.76×10^{-1}	0.	0.	1.00
$\gamma\gamma$	1.08×10^{-5}	-2.69×10^{-2}	4.32	2.56	1.80×10^{-2}	8.29×10^{-3}	-1.86	0.	0.	7.24×10^{-1}
$b\bar{b}$	2.35×10^{-3}	8.07×10^{-3}	8.09×10^{-1}	3.76×10^{-2}	-1.12	2.36	-2.72×10^{-1}	0.	0.	9.53×10^{-1}
$c\bar{c}$	1.22×10^{-4}	-4.52×10^{-2}	7.99×10^{-1}	1.02×10^{-2}	-3.10	0.	-4.89×10^{-1}	2.67	0.	9.70×10^{-1}
$\tau^+\tau^-$	2.58×10^{-4}	4.71×10^{-2}	9.95×10^{-1}	-2.09×10^{-2}	-2.14×10^{-3}	0.	-1.61×10^{-2}	0.	2.01	1.02
WW^*	9.43×10^{-4}	-1.13×10^{-1}	1.37×10^{1}	3.66	9.04×10^{-3}	0.	-1.21×10^{1}	0.	0.	2.49×10^{-1}
ZZ^*	1.17×10^{-4}	2.27×10^{-2}	1.53×10^{1}	-7.37×10^{-1}	-1.82×10^{-3}	0.	-1.12×10^{1}	0.	0.	2.53
$Z\gamma$	6.89×10^{-6}	-1.52×10^{-2}	1.11×10^{1}	8.45×10^{-1}	0.	-7.93×10^{-3}	-4.82	0.	0.	2.62
$\mu^+\mu^-$	8.93×10^{-7}	4.82×10^{-2}	9.92×10^{-1}	-4.31×10^{-2}	-2.19×10^{-3}	0.	-1.62×10^{-2}	0.	0.	1.02

Input Parameters for our expansion

- Input: $\left\{m_H, M_Z, \Delta^{(5)}_{had}, \alpha_S(M_Z), G_F, m_f\right\}$

$\delta\tau \equiv (\tau - \tau_{ref})/\tau_{ref}$

- Now that we have established our convention that M_W is an output observable, when the W mass appears in formulas below, we should view it as a short-hand notation for the full computation of the W mass within the theory in terms of our agreed-upon inputs. In the SM this substitution is

$$M_W \xrightarrow{SM} (80.368\,\mathrm{GeV})\,(1 + 1.42\,\delta M_Z + 0.21\,\delta G_F - 0.43\,\delta\alpha + 0.013\,\delta M_t - 0.0011\,\delta\alpha_S - 0.00075\,\delta M_H)\,.$$

-

m_H	125.7(4)	pole mass m_t	173.07(89)
$\overline{\mathrm{MS}}$ mass m_c	1.275(25)	$\overline{\mathrm{MS}}$ mass m_b	4.18(3)
pole mass m_τ	1.77682(16)	$\alpha_S(M_Z)$	0.1184(7)
$\alpha(M_Z)$	1/128.96(2)	$\Delta\alpha^{(5)}_{had}$	0.0275(1)
pole mass M_Z	91.1535(21)	G_F	1.1663787(6) $\times 10^{-5}$

$$\alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha_{e,\mu,\tau} - \Delta\alpha_t - \Delta\alpha^{(5)}_{had}}$$

Expansion of BR and ratio of BRs

$$\mathrm{B}(H \to \mathrm{X}) = \mathrm{B}(\mathrm{X})^{(\mathrm{ref})} \left(1 + \sum_{\mathrm{i}} \mathrm{b}_{\tau_\mathrm{i},\mathrm{X}} \overline{\delta\tau_\mathrm{i}}\right),$$

where τ_i represents the $\left\{m_H, M_Z, \Delta\alpha^{(5)}_{had}, \alpha_S(M_Z), m_f\right\}$. Expansion parameters $b_{\tau_i,X}$ are related to $a_{\tau_i,X}$ by

$$b_{\tau_i,X} = a_{\tau_i,X} - a_{\tau_i,tot}.$$

$$\frac{\mathrm{B}(H \to \mathrm{X})}{\mathrm{B}(H \to \mathrm{Y})} = \frac{\mathrm{B}(\mathrm{X})^{(\mathrm{ref})}}{\mathrm{B}(\mathrm{Y})^{(\mathrm{ref})}} \left(1 + \sum_{i} r_{\tau_i,X,Y} \overline{\delta\tau_i}\right),$$

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$$\mathrm{B}(H \to \mathrm{X}) = \mathrm{B}(\mathrm{X})^{(\mathrm{ref})} \left(1 + \sum_{\mathrm{i}} \mathrm{b}_{\tau_{\mathrm{i}},\mathrm{X}} \overline{\delta\tau_{\mathrm{i}}}\right),$$

The table of expansion coefficients enables us to compute the uncertainty in a final state branching ratio due to each input parameter. The percent uncertainty Δ_i^X on branching fraction $B(X)$ due to input parameter τ_i is

$$\Delta_i^X = (100\%) \times |b_{\tau_i,X}| \frac{\Delta\tau_i}{\tau_i^{ref}}$$

where $\Delta\tau_i$ are the current experimental uncertainties in input parameter τ_i. For example, the percentage uncertainty in the $H \to gg$ branching fraction is

$$\Delta_b^{gg} = (100\%)(1.389)\frac{0.03\,\mathrm{GeV}}{4.18\,\mathrm{GeV}} = 1.00\%.$$

Expansion of BR and ratio of BRs

$$\mathrm{B}(H \to \mathrm{X}) = \mathrm{B}(\mathrm{X})^{(\mathrm{ref})}\left(1 + \sum_{\mathrm{i}} \mathrm{b}_{\tau_{\mathrm{i}},\mathrm{X}} \overline{\delta\tau_{\mathrm{i}}}\right),$$

The table of

the uncertaint

input paramet

fraction $B(X)$

the uncertainty in the b-quark mass input observable constitutes the largest uncertainty in the branching ratio computations.

The large uncertainty of the charm quark mass is the decisive contributor to H ->cc uncertainty as well

where $\Delta\tau_i$ are

parameter τ_i.

$H \to gg$ bran

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	$B(X)^{(\mathrm{Ref})}$	b_{m_t}	b_{m_H}	$b_{\alpha(M_Z)}$	$b_{\alpha_S(M_Z)}$	b_{m_b}	b_{M_Z}	b_{m_c}	b_{m_τ}	b_{G_F}
gg	8.68×10^{-2}	-1.29×10^{-1}	-1.46	-8.35×10^{-1}	2.99	-1.39	3.58	-7.8×10^{-2}	-1.24×10^{-1}	1.51×10^{-1}
$\gamma\gamma$	2.58×10^{-3}	6.09×10^{-3}	-2.12×10^{-2}	1.73	5.23×10^{-1}	-1.32	1.35	-7.80×10^{-2}	-1.24×10^{-1}	-1.25×10^{-1}
$b\bar{b}$	5.63×10^{-1}	4.10×10^{-2}	-3.54	-7.98×10^{-1}	-6.16×10^{-1}	1.04	2.93	-7.8×10^{-2}	-1.24×10^{-1}	1.04×10^{-1}
$c\bar{c}$	2.92×10^{-2}	-1.23×10^{-2}	-3.55	-8.25×10^{-1}	-2.59	-1.32	2.72	2.59	-1.24×10^{-1}	1.21×10^{-1}
$\tau^+\tau^-$	6.18×10^{-2}	8.01×10^{-2}	-3.35	-8.56×10^{-1}	5.03×10^{-1}	-1.32	3.19	-7.80×10^{-2}	1.88	1.67×10^{-1}
WW^*	2.26×10^{-1}	-7.99×10^{-2}	9.32	2.82	5.14×10^{-1}	-1.32	-8.91	-7.8×10^{-2}	-1.24×10^{-1}	-5.99×10^{-1}
ZZ^*	2.81×10^{-2}	5.57×10^{-2}	1.10×10^{1}	-1.57	5.03×10^{-1}	-1.32	-7.98	-7.80×10^{-2}	-1.24×10^{-1}	1.68
$Z\gamma$	1.65×10^{-3}	1.78×10^{-2}	6.71	9.89×10^{-3}	5.05×10^{-1}	-1.33	-1.61	-7.80×10^{-2}	-1.24×10^{-1}	1.77
$\mu^+\mu^-$	2.14×10^{-4}	8.11×10^{-2}	-3.35	-8.79×10^{-1}	5.03×10^{-1}	-1.32	3.19	-7.80×10^{-2}	-1.24×10^{-1}	1.67×10^{-1}

Table 14: The reference value and expansion coefficients for Higgs boson decay branching fractions according to Eq. (12). The input parameters for this computation are from Table 1. VV^* partial decay widths are calculated by *Prophecy4f*. These results were computed using $\overline{MS}$ m_b and m_c inputs (see Table 10) rather than their pole mass inputs (see Table 1). Compare results with the pole mass input results of Table 5.

	$B(X)/B(Y)_{Ref}$	r_{m_t}	r_{m_H}	$r_{\alpha(M_Z)}$	$r_{\alpha_S(M_Z)}$	r_{m_b}	r_{M_Z}	r_{m_c}	r_{m_τ}	r_{G_F}
$\gamma\gamma/WW^*$	1.14×10^{-2}	8.60×10^{-2}	-9.35	-1.10	8.99×10^{-3}	8.29×10^{-3}	1.03×10^{1}	0.	0.	4.75×10^{-1}
$b\bar{b}/c\bar{c}$	1.93×10^{1}	5.33×10^{-2}	1.01×10^{-2}	2.74×10^{-2}	1.98	2.36	2.17×10^{-1}	-2.67	0.	-1.67×10^{-2}
$\tau^+\tau^-/\mu^+\mu^-$	2.89×10^{2}	-1.02×10^{-3}	2.55×10^{-3}	2.22×10^{-2}	4.63×10^{-5}	0.	1.09×10^{-4}	0.	2.01	-3.36×10^{-4}
$c\bar{c}/\mu^+\mu^-$	1.36×10^{2}	-9.34×10^{-2}	-1.93×10^{-1}	5.33×10^{-2}	-3.10	0.	-4.73×10^{-1}	2.67	0.	-4.62×10^{-2}
WW^*/ZZ^*	8.05	-1.36×10^{-1}	-1.63	4.40	1.09×10^{-2}	0.	-9.38×10^{-1}	0.	0.	-2.28
$\gamma\gamma/ZZ^*$	9.19×10^{-2}	-4.96×10^{-2}	-1.10×10^{1}	3.30	1.99×10^{-2}	8.29×10^{-3}	9.33	0.	0.	-1.81
$b\bar{b}/ZZ^*$	2.00×10^{1}	-1.47×10^{-2}	-1.45×10^{1}	7.74×10^{-1}	-1.12	2.36	1.09×10^{1}	0.	0.	-1.58
$\tau^+\tau^-/ZZ^*$	2.20	2.44×10^{-2}	-1.43×10^{1}	7.16×10^{-1}	-3.19×10^{-4}	0.	1.12×10^{1}	0.	2.01	-1.52
$Z\gamma/ZZ^*$	5.88×10^{-2}	-3.79×10^{-2}	-4.24	1.58	1.82×10^{-3}	-7.93×10^{-3}	6.37	0.	0.	$9.\times10^{-2}$
$b\bar{b}/\tau^+\tau^-$	9.11	-3.91×10^{-2}	-1.85×10^{-1}	5.85×10^{-2}	-1.12	2.36	-2.56×10^{-1}	0.	0.	-6.25×10^{-2}
$\tau^+\tau^-/cc$	2.12	9.24×10^{-2}	1.96×10^{-1}	-3.11×10^{-2}	3.10	0.	4.73×10^{-1}	-2.67	2.01	4.58×10^{-2}
$\gamma\gamma/Z\gamma$	1.56	-1.17×10^{-2}	-6.74	1.72	1.80×10^{-2}	1.62×10^{-2}	2.96	0.	0.	-1.90
$gg/Z\gamma$	3.31×10^{1}	-1.47×10^{-1}	-8.17	-8.45×10^{-1}	2.48	-5.72×10^{-2}	5.19	0.	0.	-1.62

Table 17: The reference value and expansion coefficients for ratios of Higgs boson decay branching fractions according to Eq. (16). The input parameters for this computation are from Table 1. VV^* partial decay widths are calculated by *Prophecy4f* in this table. These results were computed using $\overline{MS}$ m_b and m_c inputs (see Table 10) rather than their pole mass inputs (see Table 1). Compare results with the pole mass input results of Table 8.

How well can we predict SM observables?

Percent relative uncertainty, P_Q: $Q = Q_0\,(1 + 0.01\,P_Q)$.

	$P_\Gamma^\pm$(par.add.)	$P_\Gamma^\pm$(par.quad.)	$(P_\Gamma^+, P_\Gamma^-)(\mu)$
total	2.82 (1.79)	1.71 (1.07)	(0.08,0.10)
gg	2.52 (1.83)	1.74 (1.49)	(0.05,0.03)
$\gamma\gamma$	1.45 (0.42)	1.38 (0.35)	(1.31,0.60)
$b\bar{b}$	2.62 (2.43)	1.84 (1.82)	(0.29,0.01)
$c\bar{c}$	7.34 (7.15)	5.55 (5.54)	(0.45,0.35)
$\tau^+\tau^-$	0.36 (0.12)	0.32 (0.08)	(0.01,0.01)
WW^*	4.41 (1.17)	4.97 (1.25)	(0.25,0.31)
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	$P^{\pm}_{\Gamma}$(par.add.)	$P^{\pm}_{\Gamma}$(par.quad.)	$(P^{+}_{\Gamma}, P^{-}_{\Gamma})(\mu)$
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quark	at $\mu = m_H$ ($m_H/2$, $2m_H$)	$P_m(\Delta m)$
$m_c(\mu)$	0.638 (0.675, 0.603) GeV	2.62%
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$m_\pm(m_H)$ is computed using $m_{\rm pole} = m_{\rm ref} \pm \sigma_m$

() => if the Higgs mass uncertainty were 0.1 GeV (instead of 0.4 GeV)

The meaning of "$P_\Gamma^\pm$(par.add.)" is that all input parameters have been allowed to range over their 1σ errors and the maximum percent relative errors are recorded. The meaning of "$P_\Gamma^\pm$(par.quad.)" is that the uncertainties of each parameter are added in Gaussian quadrature. In other words, $P_{\Gamma_i}^\pm(\text{par.quad.}) = 100\,\Delta\Gamma_i/\Gamma_i$, where

$$(\Delta\Gamma_i)^2 = \left(\frac{\partial\Gamma_i}{\partial m_t}\right)^2 (\Delta m_t)^2 + \left(\frac{\partial\Gamma_i}{\partial\alpha_s}\right)^2 (\Delta\alpha_s)^2 + \cdots . \quad (11)$$

The uncertainties in varying the scale parameter μ in the calculation, attempts to capture the uncertainty in not knowing higher order corrections. A full calculation at all orders would give a result that does not depend on μ but a finite-order calculation does, and the uncertainty of dropping the higher order calculations are assumed to be approximated reasonably well by noting how much the result changes by varying μ by a factor of two upward and downward: $m_H/2 < \mu < 2m_H$. The meaning of "$P_\Gamma^\pm(\mu)$" in Table 4 concerns the relative percent uncertainties associated with this scale dependence algorithm.

SM vs. New Physics? Uncertainties in BRs

	Δ_{m_t}	Δ_{m_H}	$\Delta_{\alpha(M_Z)}$	$\Delta_{\alpha_S(M_Z)}$	Δ_{m_b}	Δ_{M_Z}	Δ_{m_c}	Δ_{m_τ}	Δ_{G_F}
gg	0.07	0.46 (0.12)	0.01	1.77	1.00	0.01	0.15	-	-
$\gamma\gamma$	-	0.01 (-)	0.03	0.31	0.94	-	0.15	-	-
$b\bar{b}$	0.02	1.13 (0.28)	0.01	0.36	0.74	0.01	0.15	-	-
$c\bar{c}$	0.01	1.13 (0.28)	0.01	1.53	0.95	0.01	5.08	-	-
$\tau^+\tau^-$	0.04	1.07 (0.27)	0.01	0.30	0.95	0.01	0.15	0.02	-
WW^*	0.04	2.97 (0.74)	0.04	0.30	0.95	0.02	0.15	-	-
ZZ^*	0.03	3.48 (0.87)	0.02	0.30	0.95	0.02	0.15	-	-
$Z\gamma$	0.01	2.14 (0.53)	-	0.30	0.96	-	0.15	-	-
$\mu^+\mu^-$	0.04	1.07 (0.27)	0.01	0.30	0.95	0.01	0.15	-	-

$$\Delta_i^X = (100\%) \times |b_{\tau_i,X}| \frac{\Delta\tau_i}{\tau_i^{ref}}$$

	$P^{\pm}_{\rm BR}$(par.-add.)	$P^{\pm}_{\rm BR}$(par.-quad.)	$(P^{+}_{\rm BR}, P^{-}_{\rm BR})(\mu)$
gg	3.47 (3.12)	2.09 (2.04)	(0.03,1.38)
$\gamma\gamma$	1.45 (1.44)	1.01 (1.01)	(1.81,1.83)
$b\bar{b}$	2.43 (1.58)	1.41 (0.89)	(0.21,0.)
$c\bar{c}$	8.72 (7.87)	5.51 (5.40)	(0.54,0.44)
$\tau^+\tau^-$	2.55 (1.75)	1.47 (1.04)	(0.09,0.07)
WW^*	4.48 (2.26)	3.13 (1.25)	(0.10,0.08)
ZZ^*	4.96 (2.34)	3.63 (1.33)	(0.10,0.08)
$Z\gamma$	3.56 (1.96)	2.36 (1.15)	(0.83,0.80)
$\mu^+\mu^-$	2.53 (1.73)	1.47 (1.04)	(0.07,0.06)

() => if the Higgs mass uncertainty were 0.1 GeV (instead of 0.4 GeV)

SM vs. New Physics? Uncertainties in BRs

	Δ_{m_t}	Δ_{m_H}	$\Delta_{\alpha(M_Z)}$	$\Delta_{\alpha_S(M_Z)}$	Δ_{m_b}	Δ_{M_Z}	Δ_{m_c}	Δ_{m_τ}	Δ_{G_F}
gg	0.07	0.46 (0.12)	0.01	1.77	1.00	0.01	0.15	-	-
$\gamma\gamma$	-	0.01 (-)	0.03	0.31	0.94	-	0.15	-	-
$b\bar{b}$	0.02	1.13 (0.28)	0.01	0.36	0.74	0.01	0.15	-	-
$c\bar{c}$	0.01	1.13 (0.28)	0.01	1.53	0.95	0.01	5.08	-	-
$\tau^+\tau^-$	0.04	1.07 (0.27)	0.01	0.30	0.95	0.01	0.15	0.02	-
WW^*	0.04	2.97 (0.74)	0.04	0.30	0.95	0.02	0.15	-	-
ZZ^*	0.03	3.48 (0.87)	0.02	0.30	0.95	0.02	0.15	-	-
$Z\gamma$	0.01	2.14 (0.53)	-	0.30	0.96	-	0.15	-	-
$\mu^+\mu^-$	0.04	1.07 (0.27)	0.01	0.30	0.95	0.01	0.15	-	-

$$\Delta_i^X = (100\%) \times |b_{\tau_i,X}| \frac{\Delta\tau_i}{\tau_i^{ref}}$$

For example, if the data at a later stage of the LHC, or ILC, or CLIC suggests that the branching fraction into b quarks can be determined to better than 1%, this does not mean that we are sensitive to new physics contributions of 1% to $H \to b\bar{b}$. The reason can be seen from Tables 6 and 7 that the SM uncertainty in computing $B(H \to b\bar{b})$ is presently 2.4% (sum of absolute values of all errors) and expected to not get better than 1.6%, with most of that coming from uncertainty of the bottom Yukawa coupling determination stemming from the uncertainty of the measured bottom quark $\overline{\text{MS}}$ mass

	$P_{\text{BR}}^{\pm}$(par.-add.)	$P_{\text{BR}}^{\pm}$(par.-quad.)	$(P_{\text{BR}}^+, P_{\text{BR}}^-)(\mu)$
gg	3.47 (3.12)	2.09 (2.04)	(0.03,1.38)
$\gamma\gamma$	1.45 (1.44)	1.01 (1.01)	(1.81,1.83)
$b\bar{b}$	2.43 (1.58)	1.41 (0.89)	(0.21,0.)
$c\bar{c}$	8.72 (7.87)	5.51 (5.40)	(0.54,0.44)
$\tau^+\tau^-$	2.55 (1.75)	1.47 (1.04)	(0.09,0.07)
WW^*	4.48 (2.26)	3.13 (1.25)	(0.10,0.08)
ZZ^*	4.96 (2.34)	3.63 (1.33)	(0.10,0.08)
$Z\gamma$	3.56 (1.96)	2.36 (1.15)	(0.83,0.80)
$\mu^+\mu^-$	2.53 (1.73)	1.47 (1.04)	(0.07,0.06)

() => if the Higgs mass uncertainty were 0.1 GeV (instead of 0.4 GeV)

SM vs. New Physics: Uncertainties in BRs

	Δ_{m_t}
gg	0.07
$\gamma\gamma$	-
$b\bar{b}$	0.02
$c\bar{c}$	0.01
$\tau^+\tau^-$	0.04
WW^*	0.04
ZZ^*	0.03
$Z\gamma$	0.01
$\mu^+\mu^-$	0.04

Thus, without reducing this error, any new physics contribution to the $b\bar{b}$ branching fraction that is not at least a factor of three or four larger than 1% cannot be discerned from SM. Thus, a deviation of at least 3% is required of detectable new physics.

However, the lattice QCD calculation could improve it to match the experimental improvement on time. (arXiv:1404.0319v1, Lepege, Mechenzie, Peskin)

For example, if the data at a later stage of the ILC, or CLIC suggests that the branching fraction into ks can be determined to better than 1%, this does not m hat we are sensitive to new physics contributions of 1% t $I \to b\bar{b}$. The reason can be seen from Tables 6 and 7 tha the SM uncertainty in computing $B(H \to b\bar{b})$ is presently 2.4% (sum of absolute values of all errors) and expected to not get better than 1.6%, with most of that coming from uncertainty of the bottom Yukawa coupling determination stemming from the uncertainty of the measured bottom quark $\overline{\text{MS}}$ mass

	$P^{\pm}_{\text{BR}}$(par.-add.)	$P^{\pm}_{\text{BR}}$(par.-quad.)	$(P^{+}_{\text{BR}}, P^{-}_{\text{BR}})(\mu)$
gg	3.47 (3.12)	2.09 (2.04)	(0.03,1.38)
$\gamma\gamma$	1.45 (1.44)	1.01 (1.01)	(1.81,1.83)
$b\bar{b}$	2.43 (1.58)	1.41 (0.89)	(0.21,0.)
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$\tau^+\tau^-$	2.55 (1.75)	1.47 (1.04)	(0.09,0.07)
WW^*	4.48 (2.26)	3.13 (1.25)	(0.10,0.08)
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$\mu^+\mu^-$	2.53 (1.73)	1.47 (1.04)	(0.07,0.06)

() => if the Higgs mass uncertainty were 0.1 GeV (instead of 0.4 GeV)

SM vs. New Physics: Uncertainties in BRs

	Δ_{m_t}	
gg	0.07	0.4
$\gamma\gamma$	-	0.0
$b\bar{b}$	0.02	1.1
$c\bar{c}$	0.01	1.1
$\tau^+\tau^-$	0.04	1.0
WW^*	0.04	2.9
ZZ^*	0.03	3.4
$Z\gamma$	0.01	2.1
$\mu^+\mu^-$	0.04	1.0

Thus, without reducing this error, any new physics contribution to the $b\bar{b}$ branching fraction that is not at least a factor of three or four larger than 1% cannot be discerned from SM. Thus, a deviation of at least 3% is required of detectable new physics.

	Δ_{m_t}	Δ_{m_H}	$\Delta_{\alpha(M_Z)}$	$\Delta_{\alpha_S(M_Z)}$	Δ_{m_b}	Δ_{M_Z}	Δ_{m_c}	Δ_{m_τ}	Δ_{G_F}
bb	0.02	1.13 (0.28)	0.01	0.36	0.74	0.01	0.15	-	-

However, the lattice QCD calculation could improve it to match the experimental improvement on time. (arXiv:1404.0319v1, Lepege, Mechenzie, Peskin)

For example, if the data at a later stage of the LHC or ILC, or CLIC suggests that the branching fraction into b quarks can be determined to better than 1%, this does not mean that we are sensitive to new physics contributions of 1% to $H \to b\bar{b}$. The reason can be seen from Tables 6 and 7 that the SM uncertainty in computing $B(H \to b\bar{b})$ is presently 2.4% (sum of absolute values of all errors) and expected to not get better than 1.6%, with most of that coming from uncertainty of the bottom Yukawa coupling determination stemming from the uncertainty of the measured bottom quark $\overline{\text{MS}}$ mass

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() => if the Higgs mass uncertainty were 0.1 GeV (instead of 0.4 GeV)

SM vs. New Physics? in BRs

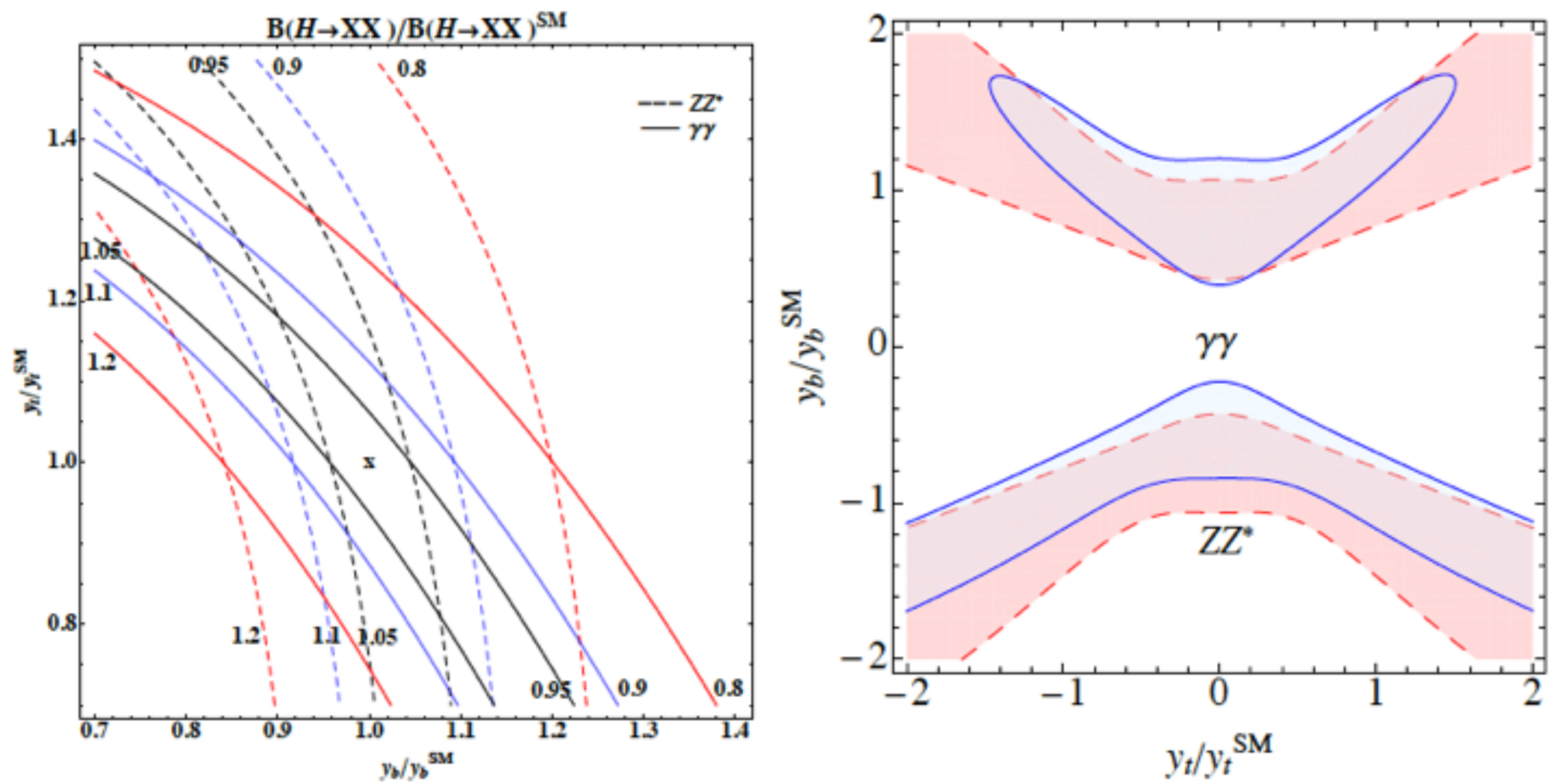
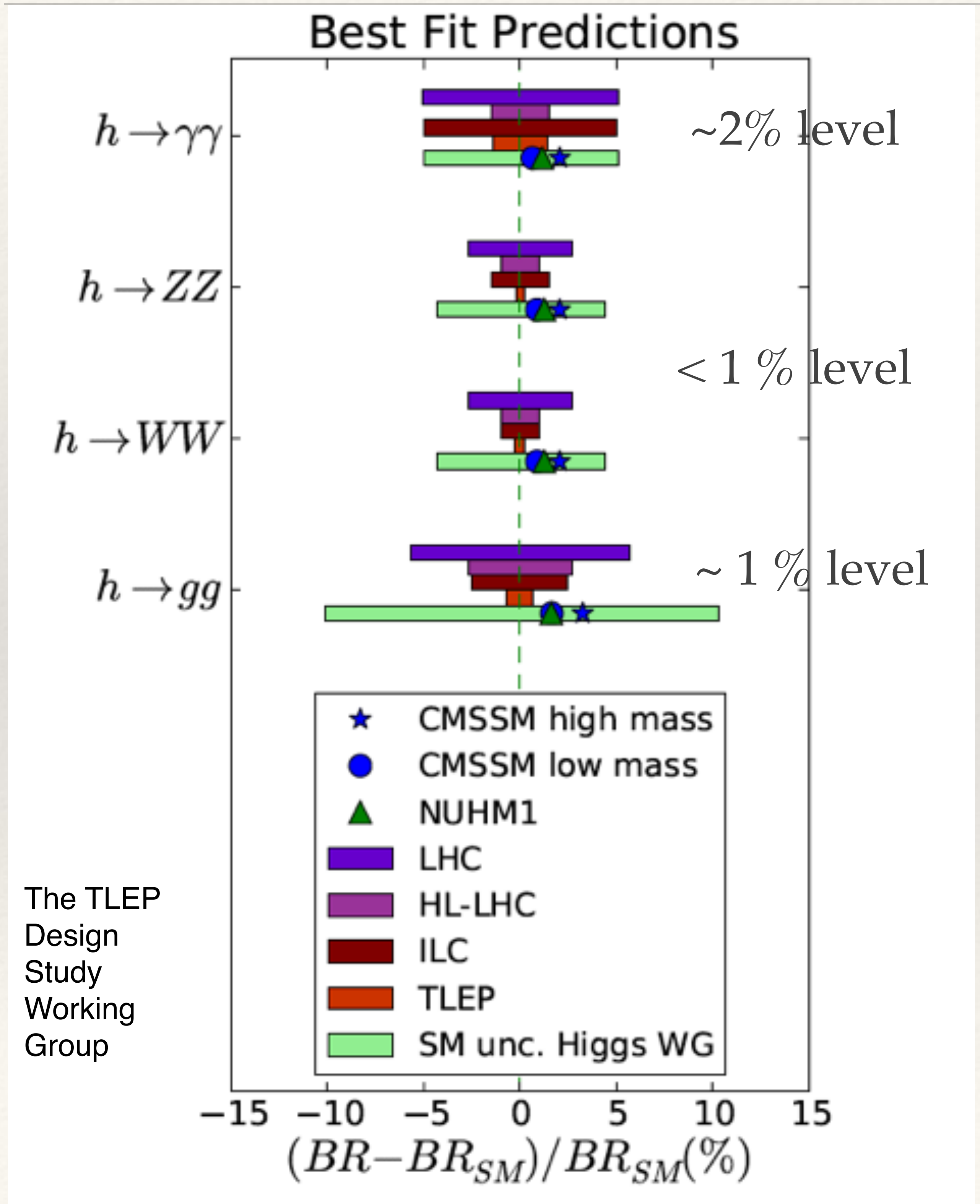


Figure 1: **Left Panel:** Contours of $B(H \to \gamma\gamma)/B(H \to \gamma\gamma)_{\rm SM}$ (solid lines) and $B(H \to ZZ)/B(H \to ZZ)_{SM}$ (dashed lines) in the $y_t - y_b$ plane. The SM position at $(1, 1)$ is marked with an x. **Right Panel:** The red shaded region is the 1σ allowed region for y_t/y_t^{sm} and y_b/y_b^{sm} given current data limits on $\sigma(H) \times B(H \to ZZ^*)$. The blue shaded region is the current 1σ allowed region from current data limits on $\sigma(H) \times B(H \to \gamma\gamma)$.

FCC-ee: Before 100 TeV



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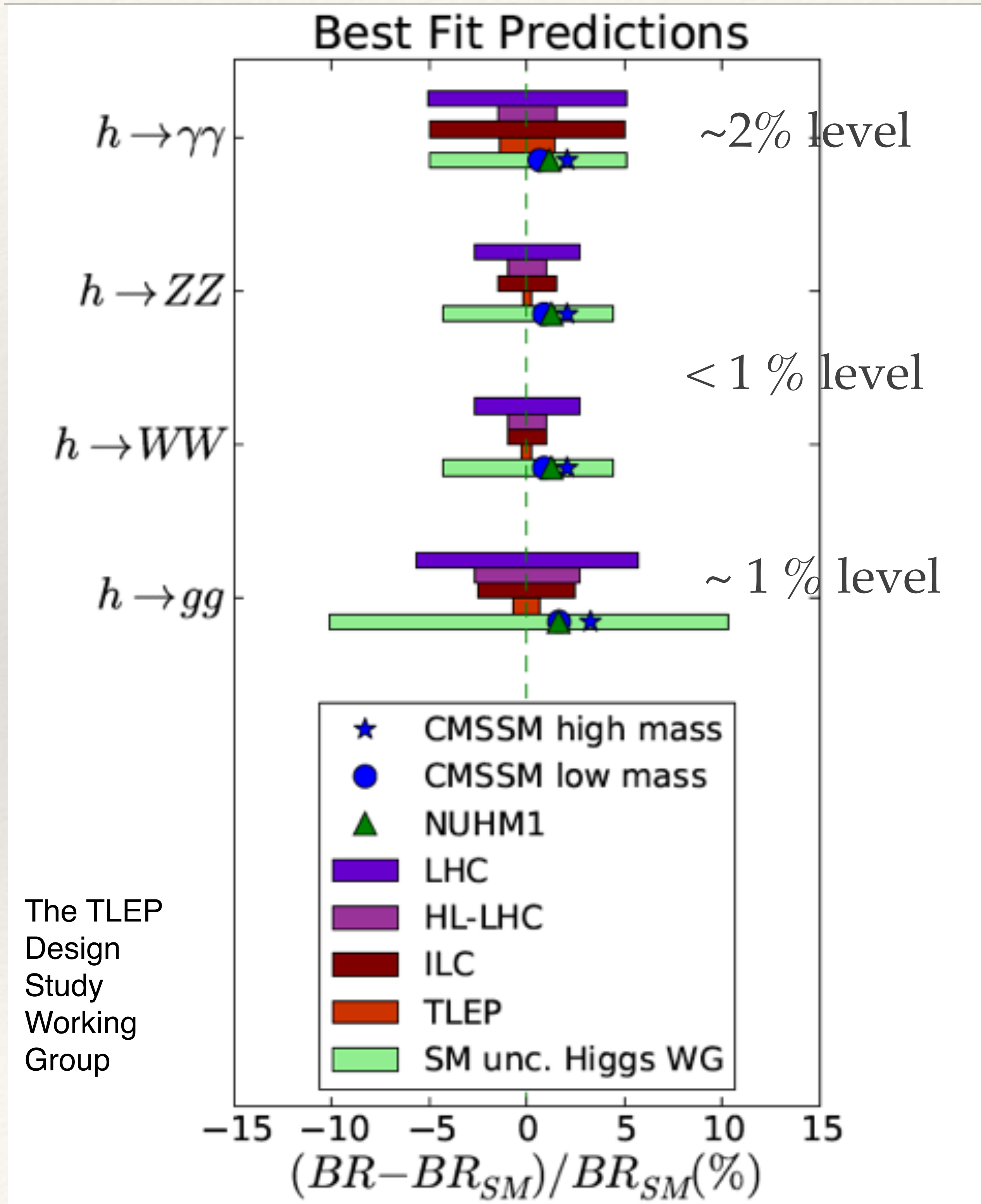
FCC-ee: Before 100 TeV



	$P^{\pm}_{\rm BR}$(par.-add.)	$P^{\pm}_{\rm BR}$(par.-quad.)	$(P^{+}_{\rm BR}, P^{-}_{\rm BR})(\mu)$
gg	3.47 (3.12)	2.09 (2.04)	(0.03,1.38)
$\gamma\gamma$	1.45 (1.44)	1.01 (1.01)	(1.81,1.83)
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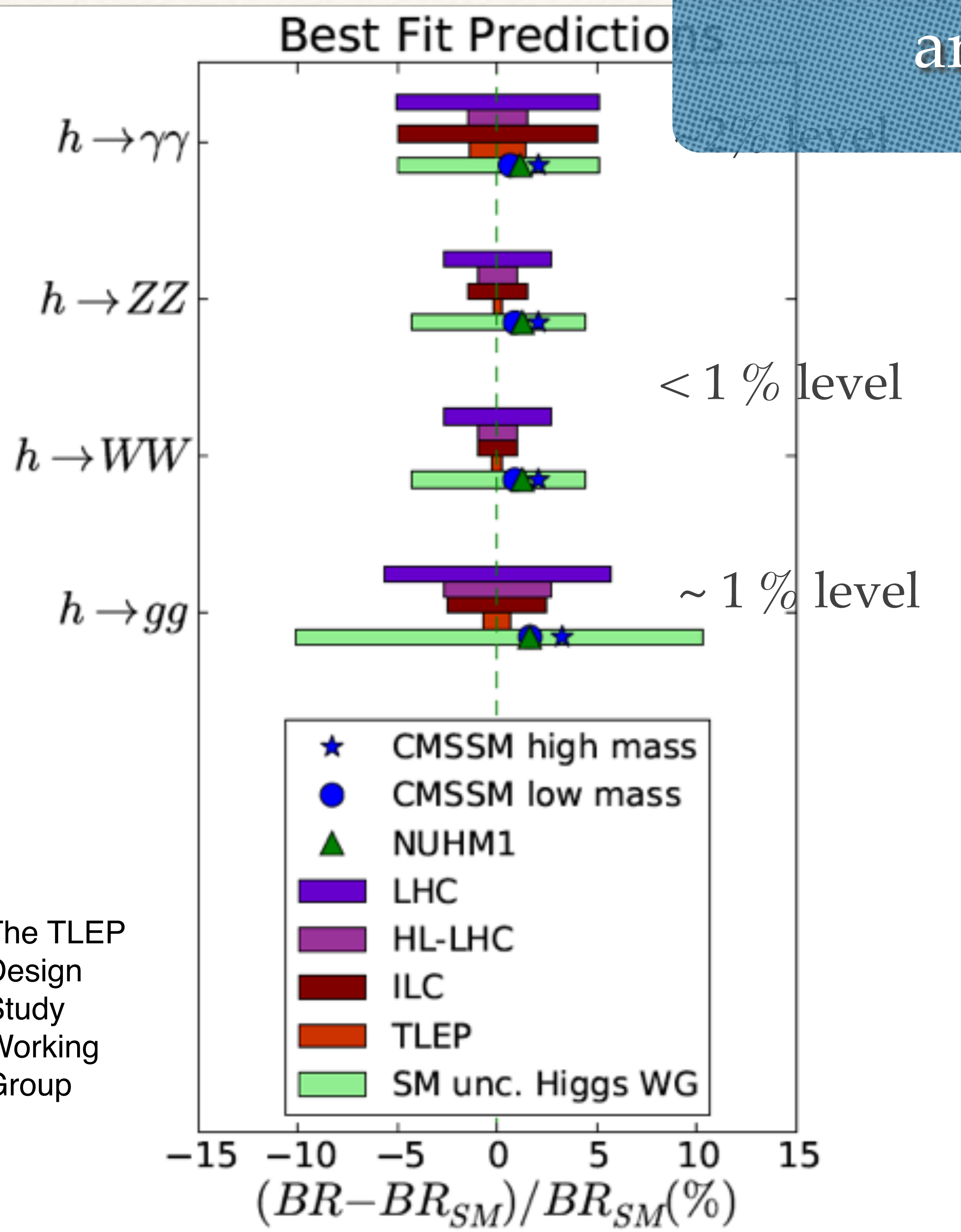
$\mu^+\mu^-$	2.53 (1.73)	1.47 (1.04)	(0.07,0.06)

FCC-ee: Before 100 TeV

Theoretical uncertainties will limit the interpretation of experimentalu measurement!
uncertainties from input parameters are major sources here.



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	$P^{\pm}_{\mathrm{BR}}$(par.-add.)	$P^{\pm}_{\mathrm{BR}}$(par.-quad.)	$(P^{+}_{\mathrm{BR}}, P^{-}_{\mathrm{BR}})(\mu)$
gg	3.47 (3.12)	2.09 (2.04)	(0.03,1.38)
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@ FCC-hh (100 TeV)

4. Higgs cross sections for HE-LHC

SM Higgs production cross sections at √s = 14, 33, 40, 60, 80 and 100 TeV (M_H=125 GeV)

Process	√s = 14 TeV	√s = 33 TeV	√s = 40 TeV	√s = 60 TeV	√s = 80 TeV	√s = 100 TeV
ggF [a]	50.35 pb	178.3 pb (3.5)	231.9 pb (4.6)	394.4 pb (7.8)	565.1 pb (11.2)	740.3 pb (14.7)
VBF [b]	4.40 pb	16.5 pb (3.8)	23.1 pb (5.2)	40.8 pb (9.3)	60.0 pb (13.6)	82.0 pb (18.6)
WH [c]	1.63 pb	4.71 pb (2.9)	5.88 pb (3.6)	9.23 pb (5.7)	12.60 pb (7.7)	15.90 pb (9.7)
ZH [c]	0.904 pb	2.97 pb (3.3)	3.78 pb (4.2)	6.19 pb (6.8)	8.71 pb (9.6)	11.26 pb (12.5)
ttH [d]	0.623 pb	4.56 pb (7.3)	6.79 pb (11)	15.0 pb (24)	25.5 pb (41)	37.9 pb (61)
gg → HH[e]*(λ=1)*	33.8 fb	207 fb (6.1)	298 fb (8.8)	609 fb (18)	980 fb (29)	1.42 pb (42)

PDF is NNLO(NLO) MSTW2008 set. Numbers in () parentheses are the cross-section ratio wrt 14 TeV.

a) NNLO+NNLL QCD + NLO EW corrections. QCD scale and PDF+α_s uncertainties remain constant about +-8% for both (D. de Florian).

b) NNLO QCD only with VBF@NNLO (M. Zaro).

c) NNLO QCD only with VH@NNLO (R. Harlander).

d) NLO QCD. (M. Spira).

e) NLO QCD with HPAIR. The central scale is the invariant mass of the Higgs pair. The scale is varied by a factor 2 up and down. (M. Spira).

c.f.

	TLEP 350	ILC 350
Total Integrated Luminosity (ab^{-1})	**2.6**	0.35
Number of Higgs bosons from $e^+e^- \to HZ$	**340,000**	65,000
Number of Higgs bosons from boson fusion	**70,000**	22,000

https://twiki.cern.ch/twiki/bin/view/LHCPhysics/HiggsEuropeanStrategy2012

@ FCC-hh (100 TeV)

4. Higgs cross sections for HE-LHC

SM Higgs production cross sections at $\sqrt{s}$ = 14, 33, 40, 60, 80 and 100 TeV (M_H=125 GeV)

Process	$\sqrt{s}$ = 14 TeV	$\sqrt{s}$ = 33 TeV	$\sqrt{s}$ = 40 TeV	$\sqrt{s}$ = 60 TeV	$\sqrt{s}$ = 80 TeV	$\sqrt{s}$ = 100 TeV	
ggF [a]	50.35 pb	178.3 pb (3.5)	231.9 pb (4.6)	394.4 pb (7.8)	565.1 pb (11.2)	740.3 pb (14.7)	x 15 !
VBF [b]	4.40 pb	16.5 pb (3.8)	23.1 pb (5.2)	40.8 pb (9.3)	60.0 pb (13.6)	82.0 pb (18.6)	x 20 !
WH [c]	1.63 pb	4.71 pb (2.9)	5.88 pb (3.6)	9.23 pb (5.7)	12.60 pb (7.7)	15.90 pb (9.7)	x 10 !
ZH [c]	0.904 pb	2.97 pb (3.3)	3.78 pb (4.2)	6.19 pb (6.8)	8.71 pb (9.6)	11.26 pb (12.5)	x 12 !
ttH [d]	0.623 pb	4.56 pb (7.3)	6.79 pb (11)	15.0 pb (24)	25.5 pb (41)	37.9 pb (61)	x 60 !
gg → HH[e]*(λ=1)*	33.8 fb	207 fb (6.1)	298 fb (8.8)	609 fb (18)	980 fb (29)	1.42 pb (42)	x 42 !

PDF is NNLO(NLO) MSTW2008 set. Numbers in () parentheses are the cross-section ratio wrt 14 TeV.

a) NNLO+NNLL QCD + NLO EW corrections. QCD scale and PDF+α_s uncertainties remain constant about +-8% for both (D. de Florian).

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@ FCC-hh (100 TeV)

4. Higgs cross sections for HE-LHC

SM Higgs production cross sections at √s = 14, 33, 40, 60, 80 and 100 TeV (M_H=125 GeV)

FCC-hh can measure ratio of BF very precisely (already at the end of HL LHC, some ratios can be measured at ~5% level) . And FCC-hh will have much larger number of events:
(c.f.., for a given integrated luminosity, 2 order of magnitude larger number of events compared to TLEP 350 (FCC-ee) for ZH channel!)

Pro	√s = 100 TeV		
ggF [a]	740.3 pb	(14.7)	x 15 !
VBF [b]	82.0 pb	(18.6)	x 20 !
WH [c]	15.90 pb	(9.7)	x 10 !
ZH [c]	11.26 pb	(12.5)	x 12 !
ttH [d]	37.9 pb	(61)	x 60 !
gg →	1.42 pb	(42)	x 42 !

PDF is ... TeV.

a) NNLO+NNLL QCD + NLO EW corrections. QCD scale and PDF+α_s uncertainties remain constant about +-8% for both (D. de Florian).
b) NNLO QCD only with VBF@NNLO (M. Zaro).
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c.f.

	TLEP 350	ILC 350
Total Integrated Luminosity (ab^{-1})	**2.6**	0.35
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Number of Higgs bosons from boson fusion	**70,000**	22,000

https://twiki.cern.ch/twiki/bin/view/LHCPhysics/HiggsEuropeanStrategy2012

@ FCC-hh (100 TeV)

Percent relative uncertainty, P_Q:

$$Q = Q_0\,(1 + 0.01\,P_Q).$$

	$P^{\pm}$(par.-add.)	$P^{\pm}$(par.-quad.)	$(P^+, P^-)(\mu)$
$\gamma\gamma/WW^*$	3.71 (1.48)	3.04 (0.99)	(1.71,1.75)
$b\bar{b}/c\bar{c}$	8.13 (8.12)	5.62 (5.62)	(0.65,0.42)
$\tau^+\tau^-/\mu^+\mu^-$	0.02 (0.02)	0.02 (0.02)	(0.02,0.02)
$c\bar{c}/\mu^+\mu^-$	7.17 (7.13)	5.54 (5.54)	(0.47,0.38)
WW^*/ZZ^*	0.66 (0.28)	0.53 (0.16)	(0.,0.)
$\gamma\gamma/ZZ^*$	3.61 (0.99)	3.49 (0.88)	(1.71,1.75)
$b\bar{b}/ZZ^*$	7.01 (3.55)	4.96 (2.15)	(0.29,0.01)
$\tau^+\tau^-/ZZ^*$	4.62 (1.21)	4.55 (1.14)	(0.01,0.01)
$Z\gamma/ZZ^*$	1.41 (0.40)	1.35 (0.34)	(0.73,0.71)
$b\bar{b}/\tau^+\tau^-$	2.44 (2.39)	1.82 (1.82)	(0.28,0.01)
$\tau^+\tau^-/c\bar{c}$	7.19 (7.14)	5.54 (5.54)	(0.36,0.45)
$\gamma\gamma/Z\gamma$	2.21 (0.60)	2.14 (0.54)	(0.97,1.04)
$gg/Z\gamma$	4.21 (2.26)	2.99 (1.61)	(0.99,3.11)

() => if the Higgs mass uncertainty were 0.1 GeV (instead of 0.4 GeV)

@ FCC-hh (100 TeV)

Percent relative uncertainty, P_Q:

$$Q = Q_0\,(1 + 0.01\,P_Q).$$

	$P^{\pm}$(par.-add.)	$P^{\pm}$(par.-quad.)	$(P^+,\ P^-)(\mu)$
$\gamma\gamma/WW^*$	3.71 (1.48)	3.04 (0.99)	(1.71,1.75)
$b\bar{b}/c\bar{c}$	8.13 (8.12)	5.62 (5.62)	(0.65,0.42)
$\tau^+\tau^-/\mu^+\mu^-$	0.02 (0.02)	0.02 (0.02)	(0.02,0.02)
$c\bar{c}/\mu^+\mu^-$	7.17 (7.13)	5.54 (5.54)	(0.47,0.38)
WW^*/ZZ^*	0.66 (0.28)	0.53 (0.16)	(0.,0.)
$\gamma\gamma/ZZ^*$	3.61 (0.99)	3.49 (0.88)	(1.71,1.75)
$b\bar{b}/ZZ^*$	7.01 (3.55)	4.96 (2.15)	(0.29,0.01)
$\tau^+\tau^-/ZZ^*$	4.62 (1.21)	4.55 (1.14)	(0.01,0.01)
$Z\gamma/ZZ^*$	1.41 (0.40)	1.35 (0.34)	(0.73,0.71)
$b\bar{b}/\tau^+\tau^-$	2.44 (2.39)	1.82 (1.82)	(0.28,0.01)
$\tau^+\tau^-/c\bar{c}$	7.19 (7.14)	5.54 (5.54)	(0.36,0.45)
$\gamma\gamma/Z\gamma$	2.21 (0.60)	2.14 (0.54)	(0.97,1.04)
$gg/Z\gamma$	4.21 (2.26)	2.99 (1.61)	(0.99,3.11)

c.f. Δ_i^X for Branching Fractions:

	$P^{\pm}_{\rm BR}$(par.-add.)	$P^{\pm}_{\rm BR}$(par.-quad.)	$(P^+_{\rm BR},\ P^-_{\rm BR})(\mu)$
gg	3.47 (3.12)	2.09 (2.04)	(0.03,1.38)
$\gamma\gamma$	1.45 (1.44)	1.01 (1.01)	(1.81,1.83)
$b\bar{b}$	2.43 (1.58)	1.41 (0.89)	(0.21,0.)
$c\bar{c}$	8.72 (7.87)	5.51 (5.40)	(0.54,0.44)
$\tau^+\tau^-$	2.55 (1.75)	1.47 (1.04)	(0.09,0.07)
WW^*	4.48 (2.26)	3.13 (1.25)	(0.10,0.08)
ZZ^*	4.96 (2.34)	3.63 (1.33)	(0.10,0.08)
$Z\gamma$	3.56 (1.96)	2.36 (1.15)	(0.83,0.80)
$\mu^+\mu^-$	2.53 (1.73)	1.47 (1.04)	(0.07,0.06)

() => if the Higgs mass uncertainty were 0.1 GeV (instead of 0.4 GeV)

@ FCC-hh (100 TeV)

- The percent uncertainty due to the input parameter (with Δm_h=0.1 GeV):

$$\boxed{\Delta_i^{X,Y} = (100\%) \times \left| r_{\tau_i, X, Y} \right| \frac{\Delta\tau_i}{\tau_i^{ref}}}$$

	$P^{\pm}$(par.-add.)	$P^{\pm}$(par.-quad.)	$(P^+, P^-)(\mu)$
$\gamma\gamma/WW^*$	3.71 (1.48)	3.04 (0.99)	(1.71,1.75)
$b\bar{b}/c\bar{c}$	8.13 (8.12)	5.62 (5.62)	(0.65,0.42)
$\tau^+\tau^-/\mu^+\mu^-$	0.02 (0.02)	0.02 (0.02)	(0.02,0.02)
$c\bar{c}/\mu^+\mu^-$	7.17 (7.13)	5.54 (5.54)	(0.47,0.38)
WW^*/ZZ^*	0.66 (0.28)	0.53 (0.16)	(0.,0.)
$\gamma\gamma/ZZ^*$	3.61 (0.99)	3.49 (0.88)	(1.71,1.75)
$b\bar{b}/ZZ^*$	7.01 (3.55)	4.96 (2.15)	(0.29,0.01)
$\tau^+\tau^-/ZZ^*$	4.62 (1.21)	4.55 (1.14)	(0.01,0.01)
$Z\gamma/ZZ^*$	1.41 (0.40)	1.35 (0.34)	(0.73,0.71)
$b\bar{b}/\tau^+\tau^-$	2.44 (2.39)	1.82 (1.82)	(0.28,0.01)
$\tau^+\tau^-/c\bar{c}$	7.19 (7.14)	5.54 (5.54)	(0.36,0.45)
$\gamma\gamma/Z\gamma$	2.21 (0.60)	2.14 (0.54)	(0.97,1.04)
$gg/Z\gamma$	4.21 (2.26)	2.99 (1.61)	(0.99,3.11)

	$100\,\frac{\Delta mt}{mt}$rmt	$100\,\frac{\Delta mh}{mh}$rmh	$100\,\frac{\Delta a(MZ)}{a(MZ)}$ra(Mz)	$100\,\frac{\Delta as(MZ)}{as(MZ)}$ras(Mz)	$100\,\frac{\Delta mb}{mb}$rmb	$100\,\frac{\Delta mZ}{mZ}$rmz	$100\,\frac{\Delta mc}{mc}$rmc	$100\,\frac{\Delta mtau}{mtau}$rmtau	$100\,\frac{\Delta GF}{GF}$rGF
$\gamma\gamma$/WW	0.04	0.74	0.02	0.01	0.01	0.02	0.	0.	0.
bb/cc	0.03	0.	0.	1.17	1.69	0.	5.23	0.	0.
$\tau\tau/\mu\mu$	0.	0.	0.	0.	0.	0.	0.	0.02	0.
WW/ZZ	0.07	0.13	0.07	0.01	0.	0.00	0.	0.	0.
$\gamma\gamma$/ZZ	0.03	0.87	0.05	0.01	0.01	0.02	0.	0.	0.
Zγ/ZZ	0.02	0.34	0.02	0.00	0.01	0.01	0.	0.	0.
$\gamma\gamma$/Zγ	0.01	0.54	0.03	0.01	0.01	0.01	0.	0.	0.
gg/Zγ	0.08	0.65	0.01	1.47	0.04	0.01	0.	0.	0.
$\tau\tau$/ZZ	0.01	1.14	0.01	0.	0.	0.03	0.	0.02	0.

() => if the Higgs mass uncertainty were 0.1 GeV (instead of 0.4 GeV)

@ FCC-hh (100 TeV)

Perturbative Uncertainties become the major source of uncertainty:

cry for higher order perturbative calculations!!

- The percent uncertainty due to the input parameter

$$\Delta_i^{X,Y} = (100\%) \times |r_{x_i,X,Y}| \frac{\Delta x_i}{x_i^{ref}}$$

	$P^{\pm}$(par.-add.)	$P^{\pm}$(par.-quad.)	$(P^+, P^-)(\mu)$
$\gamma\gamma/WW^*$	3.71 (1.48)	3.04 (0.99)	(1.71,1.75)
$b\bar{b}/c\bar{c}$	8.13 (8.12)	5.62 (5.62)	(0.65,0.42)
$\tau^+\tau^-/\mu^+\mu^-$	0.02 (0.02)	0.02 (0.02)	(0.02,0.02)
$c\bar{c}/\mu^+\mu^-$	7.17 (7.13)	5.54 (5.54)	(0.47,0.38)
WW^*/ZZ^*	0.66 (0.28)	0.53 (0.16)	(0.,0.)
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$gg/Z\gamma$	4.21 (2.26)	2.99 (1.61)	(0.99,3.11)

	$100\frac{\Delta mt}{mt}$rmt	$100\frac{\Delta mh}{mh}$rmh	$100\frac{\Delta a(MZ)}{a(MZ)}$ra(Mz)	$100\frac{\Delta as(MZ)}{as(MZ)}$ras(Mz)	$100\frac{\Delta mb}{mb}$rmb	$100\frac{\Delta mZ}{mZ}$rmz	$100\frac{\Delta mc}{mc}$rmc	$100\frac{\Delta mtau}{mtau}$rmtau	$100\frac{\Delta GF}{GF}$rGF
$\gamma\gamma$/WW	0.04	0.74	0.02	0.01	0.01	0.02	0.	0.	0.
bb/cc	0.03	0.	0.	1.17	1.69	0.	5.23	0.	0.
$\tau\tau/\mu\mu$	0.	0.	0.	0.	0.	0.	0.	0.02	0.
WW/ZZ	0.07	0.13	0.07	0.01	0.	0.00	0.	0.	0.
$\gamma\gamma$/ZZ	0.03	0.87	0.05	0.01	0.01	0.02	0.	0.	0.
Zγ/ZZ	0.02	0.34	0.02	0.00	0.01	0.01	0.	0.	0.
$\gamma\gamma$/Zγ	0.01	0.54	0.03	0.01	0.01	0.01	0.	0.	0.
gg/Zγ	0.08	0.65	0.01	1.47	0.04	0.01	0.	0.	0.
$\tau\tau$/ZZ	0.01	1.14	0.01	0.	0.	0.03	0.	0.02	0.

() => if the Higgs mass uncertainty were 0.1 GeV (instead of 0.4 GeV)

Summary

- Higgs Precision can be reaching at the level of EWP
- With improved theoretical tools (e.g. expansion formalism), SM will be tested at per mille level
- SM Higgs vs. BSM Higgs can be tested @ FCC (and ILC) beyond the typical direct search limit
- @ FCC-hh (100 TeV) can measure the ratio of Branching Fractions very precisely, and potentially provide the most precise test of the SM (therefore probing the BSM in higgs observables)
- Lattice QCD may reduce the parametric uncertainties due to heavy quark masses. Also recently a further study on low energy observables were done (see arXiv:1501.02803v1 by Petrov, Porkoski, Wells, Zhang)