Form Factors For $B_s \to K\ell\nu$ Decays From Lattice HQET

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Motivation

Weak decays of heavy mesons are a very important piece in understanding how well the Standard Model describes Nature. Lattice QCD allows non-perturbative computation of low-energy hadronic matrix elements contributing to these processes.

- Significance of precision tests in the beauty sector often limited by the uncertainties on the theory side ⇒ lattice computations with an overall accuracy of a few % are desired.
- The CKM matrix encodes the couplings of flavour-changing weak interactions. Here we concentrate on the matrix element V_{ub} via the decay $B_s \to K\ell\nu$.

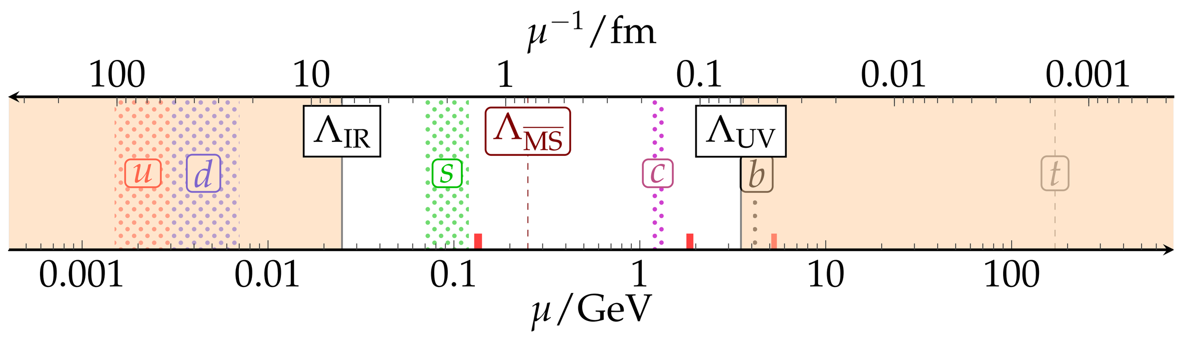
V_{ub} puzzle

- Processes with $b \to u$ transitions, $\Gamma \propto |V_{ub}|^2$:
  1. Inclusive semi-leptonic $B \to X_u \ell\nu$ involves optical theorem, heavy quark expansion and perturbation theory.
  2. Exclusive semi-leptonic $B \to \pi\ell\nu$ and $B_s \to K\ell\nu$ involves hadronic form factor $f_+(q^2)$.
  3. Exclusive leptonic $B \to \tau\nu$ involves hadronic decay constant f_B.
- Taking error bars at face value, there is a $\sim 3\sigma$ *tension* between the inclusive and exclusive determinations of V_{ub}. However, the uncertainties on both sides are largely systematic.
- Both exclusive decays use lattice input for the hadronic elements.

⇒ Precise and reliable lattice calculations with good control of systematics required to resolve the issue whether this tension really hints at New Physics in the B-sector.

Challenge of B-physics on the lattice

Multiple physical scales to be covered: $\Lambda_{IR} = L^{-1} \ll m_\pi, \ldots, m_D, m_B \ll a^{-1} = \Lambda_{UV}$.



- $L \gtrsim 4/m_\pi \approx 6\,\mathrm{fm}$ to suppress finite-size effects for physical light quarks.
- At the same time, a small enough to tame discretization errors in the heavy sector.
- Propagation of the charm quark, $a \lesssim 1/(2m_D) \approx 0.05\,\mathrm{fm}$, still resolvable, but the b-quark scale ($m_b/m_c \sim 4$) has to be separated from the others in a theoretically sound way before simulating the theory – here:

  Heavy Quark Effective Theory formulation for the b-quark in heavy-light systems.
- Use fully non-perturbative renormalization and matching.

Non-perturbative HQET

Lagrangian:

(Continuum) asymptotic expansion of QCD in $\Lambda_{QCD}/m_b \ll 1$, truncation errors of $O(\Lambda^2_{QCD}/m^2_b)$:

lowest (static) order — 1st order correction in $1/m_h$

$$\overline{\psi}_b \{\gamma_\mu D_\mu + m_b\} \psi_b \;\to\; \mathcal{L}_{HQET}(x) = \overline{\psi}_h(x)\, D_0\, \psi_h(x) - \omega_{kin}\mathcal{O}_{kin}(x) - \omega_{spin}\mathcal{O}_{spin}(x)$$

$$\mathcal{O}_{kin}(x) = \overline{\psi}_h(x)\,\mathbf{D}^2\,\psi_h(x), \qquad \mathcal{O}_{spin}(x) = \overline{\psi}_h(x)\,\boldsymbol{\sigma}\cdot\mathbf{B}\,\psi_h(x)$$

Bare currents, $V^{stat}_{0,k} = \bar\psi_u \gamma_{0,k}\psi_h + O(a)$, also get multiplicatively renormalized as in QCD.

$$V_0^{stat,RGI} = Z^{stat}_{A,RGI}(g_0) Z^{stat}_{V/A}(g_0) V_0^{stat}, \qquad V_k^{stat,RGI} = Z^{stat}_{A,RGI}(g_0) V_k^{stat}. \tag{1}$$

Benefits of our lattice HQET approach to B-physics:

- $1/m$–terms appear as local operator insertions in correlation functions ⇒ Renormalizability (at each $1/m$–order) & existence of the continuum limit.
- The HQET effective couplings / parameters $\omega_i \in \{m_{bare}, Z_A^{HQET}, c_A^{(1)}, \omega_{kin}, \omega_{spin}\}$ fixed through *non-perturbative matching*, such that no uncancelled power divergences in a^{-1} (induced by operator mixing in the effective theory) remain that would spoil taking the continuum limit.

In this particular example, we:

- Obtain the ground state matrix element $\langle K|V^\mu(0)|B_s\rangle$ mediating the transition.
- Renormalize the HQET currents and relate them to QCD.
- Take the continuum limit.

Lattices & techniques

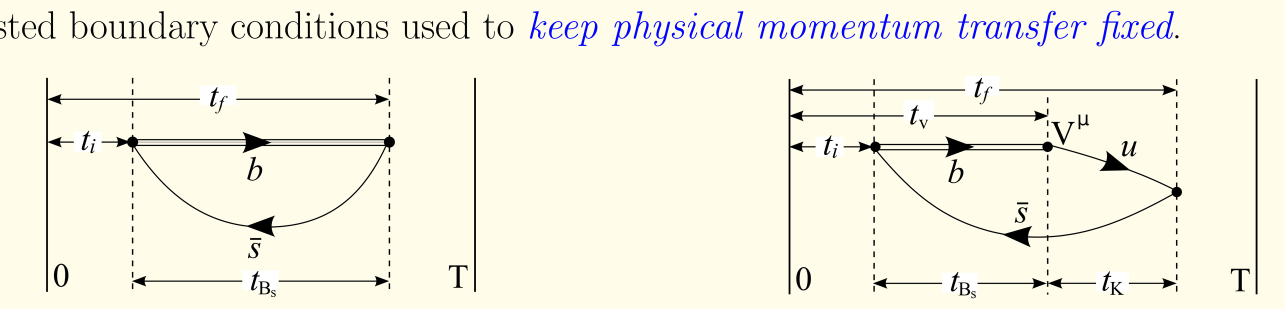
CLS ensembles:

id	β	L/a	a [fm]	m_π [MeV]	N_{cfg}	κ_s	$\theta/(2\pi)$
A5	5.2	32	0.0749(8)	330	1000	0.13535	0.034
F6	5.3	48	0.0652(6)	310	300	0.13579	0.350
N6	5.5	48	0.0483(4)	340	300	0.13631	0

- $N_f = 2$ mass-degenerate non-perturbatively $O(a)$-improved Wilson quarks with $Lm_\pi \gtrsim 4$ and plaquette gauge action.

Computation of static-light and 3-point correlation functions:

- Variant of stochastic all-to-all propagator method for light quarks (*full time-dilution*).
- At the leading order, the b-quark propagator is a product of gauge links.
- Two different HYP-smeared static quark propagators (HYP1 and HYP2) for better statistical precision and control of discretization errors.
- Three levels of light-quark smearing to enhance ground-state dominance.
- Twisted boundary conditions used to *keep physical momentum transfer fixed*.



Schematic set-up of the calculation of the two-point heavy-light (left) and three-point (right) correlation functions.

Correlation Functions, Matrix Elements and Form Factors

- At leading order in the weak interactions, the transition amplitude for $B_s \to K\ell\nu$ is:

$$\langle K(p_K)|V^\mu(0)|B_s(p_{B_s})\rangle = \left(p_{B_s} + p_K - \frac{m^2_{B_s} - m^2_K}{q^2}q\right)^\mu \cdot f_+(q^2) + \frac{m^2_{B_s} - m^2_K}{q^2} q^\mu \cdot f_0(q^2) = \sqrt{2m_{B_s}}\left[v^\mu \cdot h_\parallel(p_K\cdot v) + p^\mu_\perp \cdot h_\perp(p_K \cdot v)\right].$$

- In the rest-frame of the B_s-meson, the form factors $h_{\parallel,\perp}$ are obtained from the (QCD) matrix elements and are related to the corresponding renormalized HQET parameters as:

$$(2m_{B_s})^{-1/2}\langle K(p_K)|V^0(0)|B_s\rangle = h_\parallel(E_K) = C_{V_0}(M_b/\Lambda_{\overline{MS}}) h_\parallel^{stat,RGI}(E_K)\cdot[1 + O(1/m_b)],$$

$$(2m_{B_s})^{-1/2}\langle K(p_K)|V^k(0)|B_s\rangle = p_K^k h_\perp(E_K) = p_K^k C_{V_k}(M_b/\Lambda_{\overline{MS}}) h_\perp^{stat,RGI}(E_K)\cdot[1 + O(1/m_b)].$$

The conversion factors C_x connect the matrix elements between HQET and QCD.

- On the lattice, the 2-point as well as the 3-point correlators needed to extract $h^{stat,bare}_{\parallel,\perp}$:

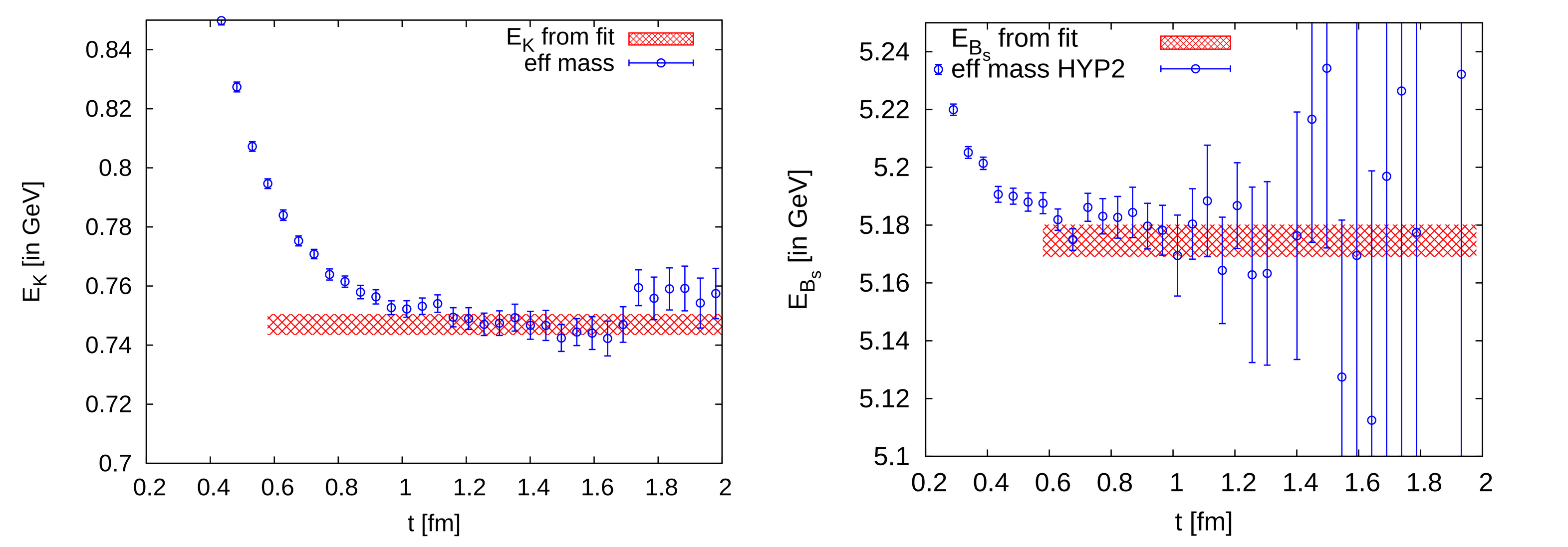
$$\mathcal{C}^K(t_K) \sim \left(\kappa^{(0)}\right)^2 e^{-E_K^{(0)}t_K}, \quad \mathcal{C}^{B_s}_{ij}(t_{B_s}) \sim \sum_{n=0}^{N_{B_s}} \beta_i^{(n)}\beta_j^{(n)} e^{-E^{(n)}_{B_s}t_{B_s}},$$
$$\mathcal{C}^{B_s\to K}_{\mu,i}(t_K, t_{B_s}) \sim \sum_{n=0}^{N_{B_s}} \kappa^{(0)}\varphi_\mu^{(0,n)}\beta_i^{(n)} e^{-E_K^{(0)}t_K} e^{-E^{(n)}_{B_s}t_{B_s}}, \tag{2}$$

for t_K large enough to obtain ground-state dominance in the Kaon sector. The desired form factors are given by the ground-state matrix elements $h_\parallel^{stat,bare} = \varphi_0^{(0,0)}\sqrt{2E_K}$ and $h_\perp^{stat,bare} = \varphi_k^{(0,0)}\sqrt{2E_K}/p_K^k$.

Results

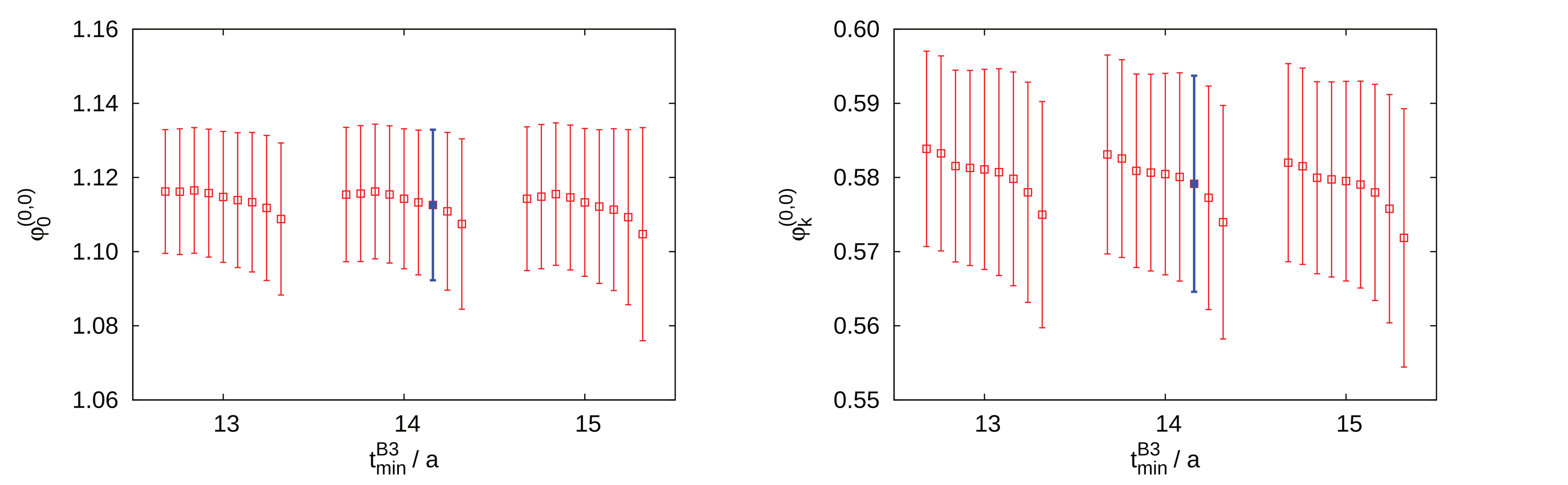
Effective mass plateaus for $\mathcal{C}^K$ and $\mathcal{C}^{B_s}$

In the figures below, we show the results for E_K, E_{B_s} as a function of the source-sink separations of the 2-point functions. The results are for the finest lattice spacing.



Form factor extraction via combined fits

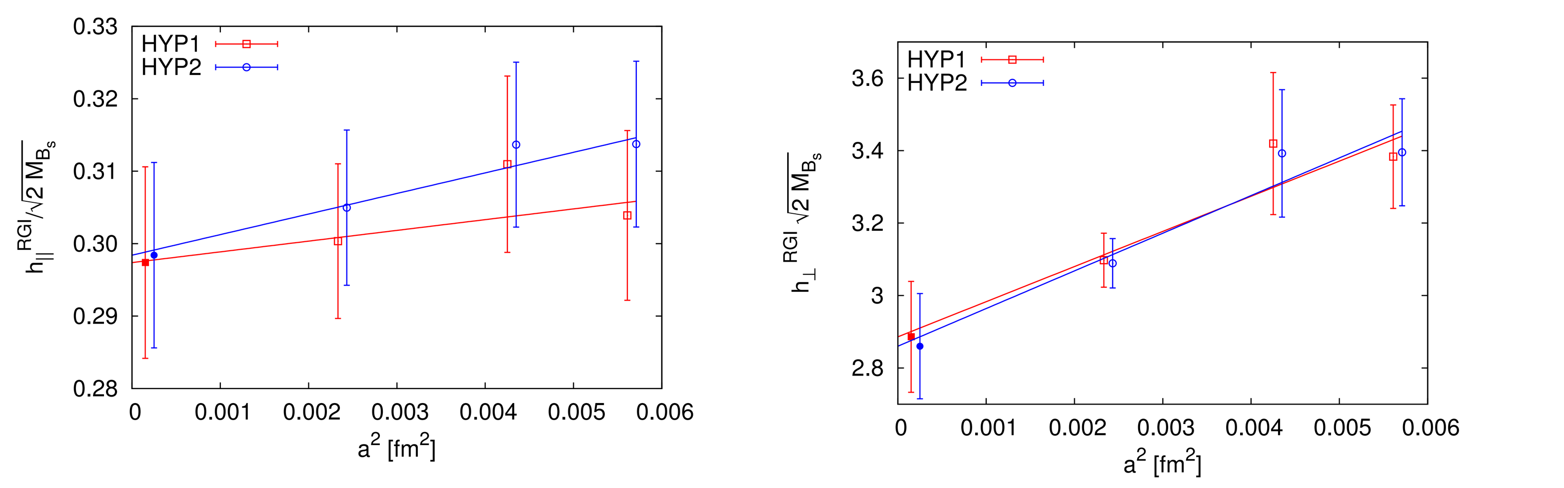
- Using the parameters of the 2-point functions as input, we perform combined fits to the $\mathcal{C}^K$, $\mathcal{C}^{B_s}$ and $\mathcal{C}^{B_s\to K}_\mu$, eq. (2). We need $N_{B_s} = 2$ excited states to obtain a good description of the data and safely extract the form factors $\varphi^{(0,0)}_{0,k}$.
- We fit $\mathcal{C}^{B_s\to K}_\mu$ in rectangles of $t^{K3}_{min} \le t_K \le t^{K3}_{max,\mu}$ and $t^{B3}_{min} \le t_{B_s} \le t^{B3}_{max,\mu}$. The maximum times are chosen to suppress noise and finite-T effects, and we analyze the stability of the fit parameters with respect to t^{B3}_{min} and t^{K3}_{min}. The results are for the the finest lattice spacing.



The x-axis shows different values of t^{B3}_{min}/a (different groups) and t^{K3}_{min}/a (within the group).

Continuum Limit

The renormalized form factors $h^{stat,RGI}_{\parallel,\perp}$ are obtained using eq. (1) at fixed q^2 for different lattice spacings. The continuum limit can be now be taken.



Combining the continuum limits for two lattice discretizations (HYP1 and HYP2), we obtain:

$$h_\parallel^{stat,RGI} = 0.976(41)\mathrm{GeV}^{1/2}, \quad h_\perp^{stat,RGI} = 0.876(43)\mathrm{GeV}^{-1/2}.$$

Conclusions

- Translating to more conventional form factor $f_+(q^2 = 21.22(5)\ \mathrm{GeV}^2) = 1.63(8)(6)$, where the second error is the perturbative uncertainty in C_x.
- There is an additional $\sim 15\%$ uncertainty/ambiguity coming from LO treatment in HQET which will be reduced to 1 – 2% when we include the $O(1/m)$ terms, yielding a result of direct phenomenological interest.
- Within errors, our numbers *confirm previous lattice estimates* of the form factors, despite entirely different source of systematic errors, and the V_{ub} *puzzle seems to remain*.

References

This poster is based on the work by the authors available in arXiv:1601.04277, accepted in *Phys. Lett.* **B**. Current status of HQET can be found in R. Sommer *Non-perturbative Heavy Quark Effective Theory: Introduction and Status*, *Nucl. Part. Phys. Proc.* **261-262**(2015) 338-367. Details about the configurations used are available in P. Fritzsch et al. *The strange quark mass and Lambda parameter of two flavor QCD*, *Nucl. Phys.* **B865** (2012) 397-429.