

V_{cb} AND V_{ub} FROM
SEMILEPTONIC B DECAYS

PAOLO GAMBINO
UNIVERSITÀ DI TORINO & INFN



IMPORTANCE OF $|V_{xb}|$

V_{cb} plays an important role in the determination of UT

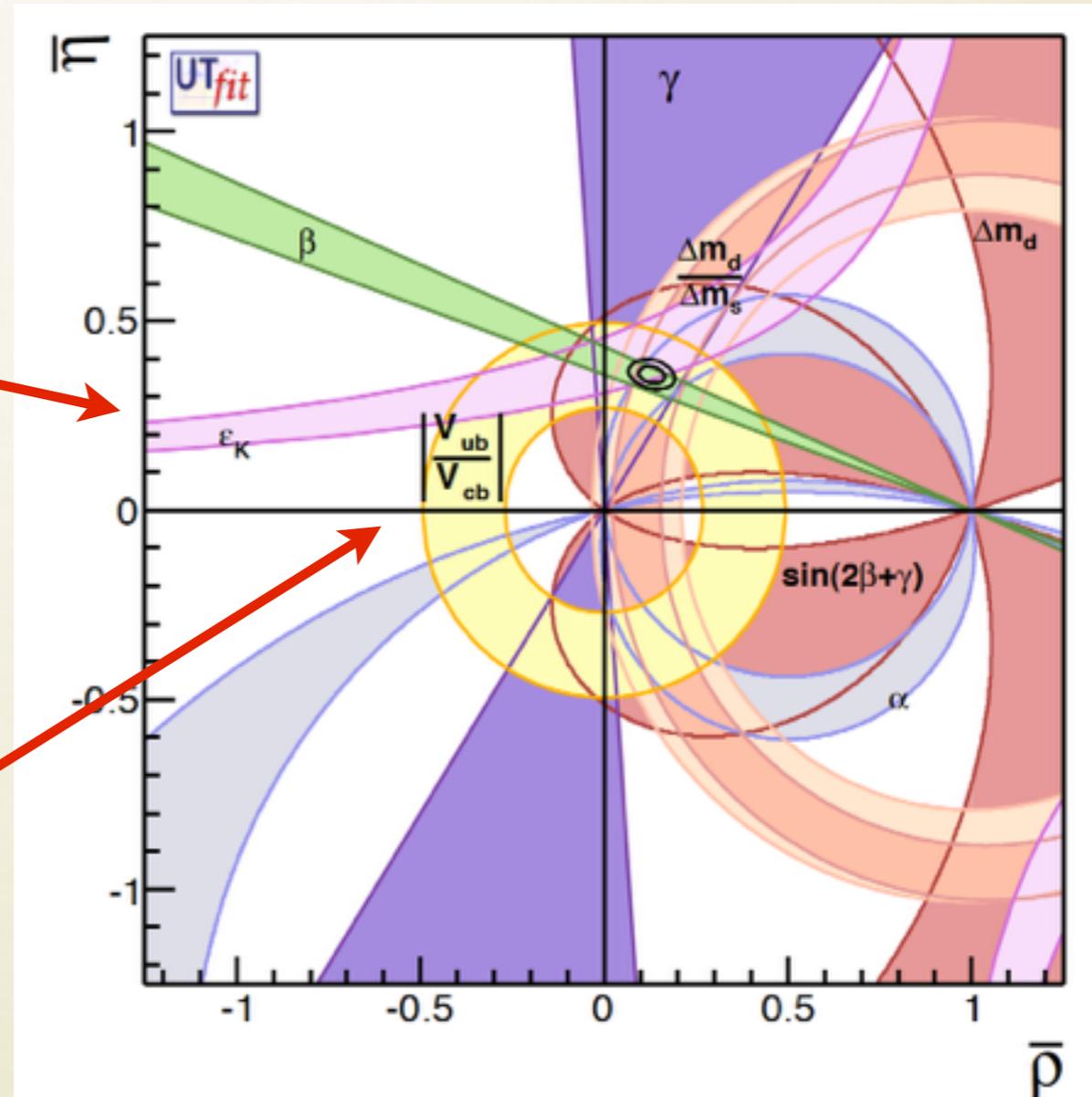
$$\varepsilon_K \approx x|V_{cb}|^4 + \dots$$

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2 \left[1 + O(\lambda^2) \right]$$

where it often dominates the theoretical uncertainty.

V_{ub}/V_{cb} constrains directly the UT



**Since several years, exclusive decays prefer smaller $|V_{ub}|$ and $|V_{cb}|$
Relation to semitauonic anomaly (3.9σ)?**

INCLUSIVE SEMILEPTONIC B DECAYS

OPE allows us to write inclusive observables as double series in Λ/m_b and α_s

$$M_i = M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)}\right) \frac{\mu_\pi^2}{m_b^2} \\ + \left(M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)}\right) \frac{\mu_G^2}{m_b^2} + M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)} \frac{\rho_{LS}^3}{m_b^3} + \dots$$

$$\mu_\pi^2(\mu) = \frac{1}{2M_B} \left\langle B \left| \bar{b} \left(i \vec{D} \right)^2 b \right| B \right\rangle_\mu$$

$$\mu_G^2(\mu) = \frac{1}{2M_B} \left\langle B \left| \bar{b} \frac{i}{\sqrt{2}} \sigma_{\mu\nu} G^{\mu\nu} b \right| B \right\rangle_\mu$$

OPE valid for inclusive enough measurements, away from perturbative singularities \Rightarrow semileptonic width, moments

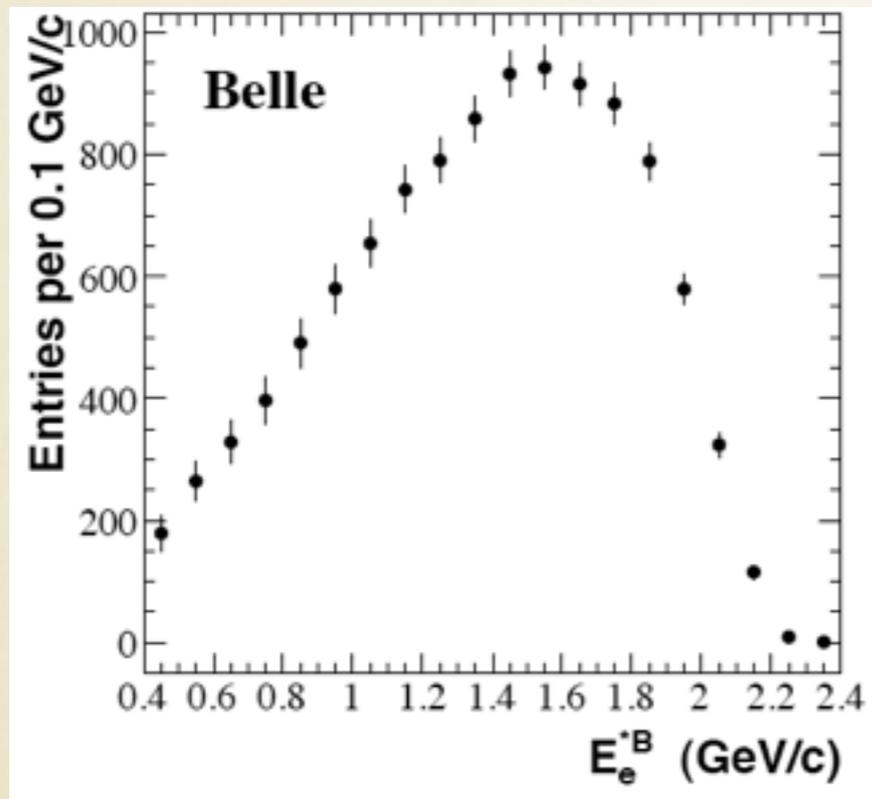
Current fits includes 6 non-pert parameters

$$m_{b,c} \quad \mu_{\pi,G}^2 \quad \rho_{D,LS}^3$$

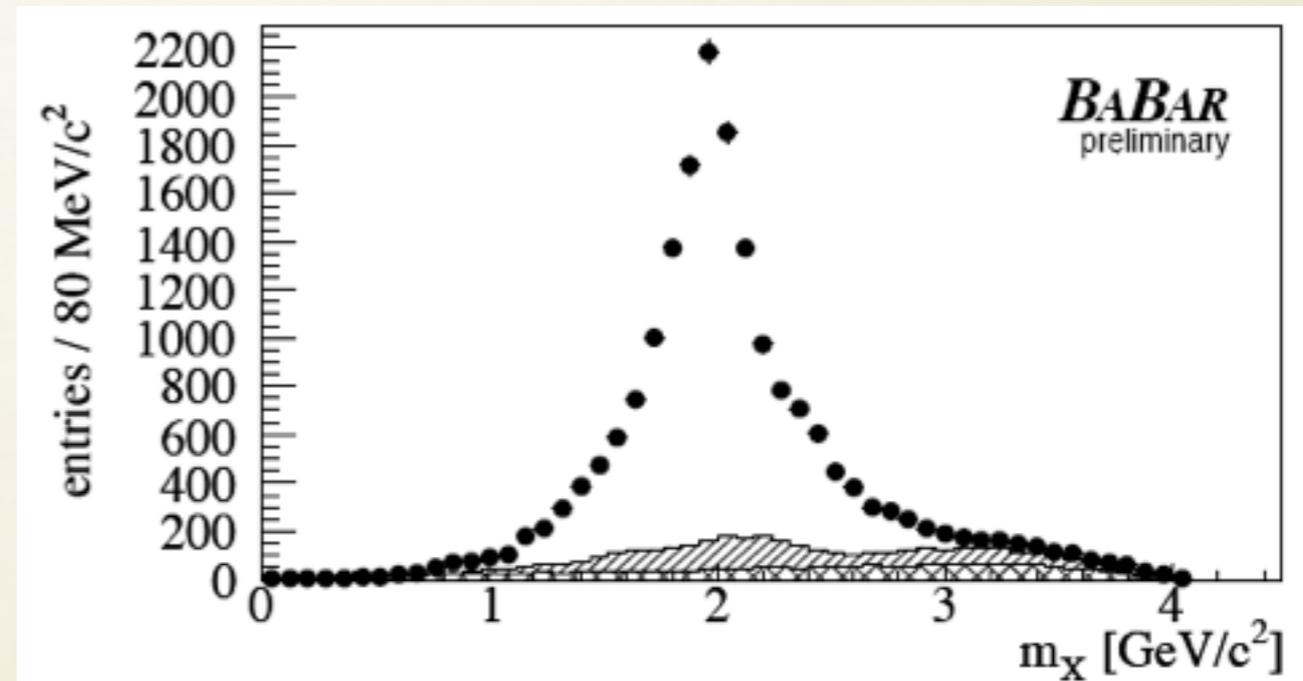
and all known corrections up to $O(\Lambda^3/m_b^3)$

EXTRACTION OF THE OPE PARAMETERS

E_1 spectrum



hadronic mass spectrum



Global **shape** parameters (first moments of the distributions) tell us about m_b , m_c and the B structure, total **rate** about $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the quarks \rightarrow useful in many applications (rare decays, V_{ub} ,...)

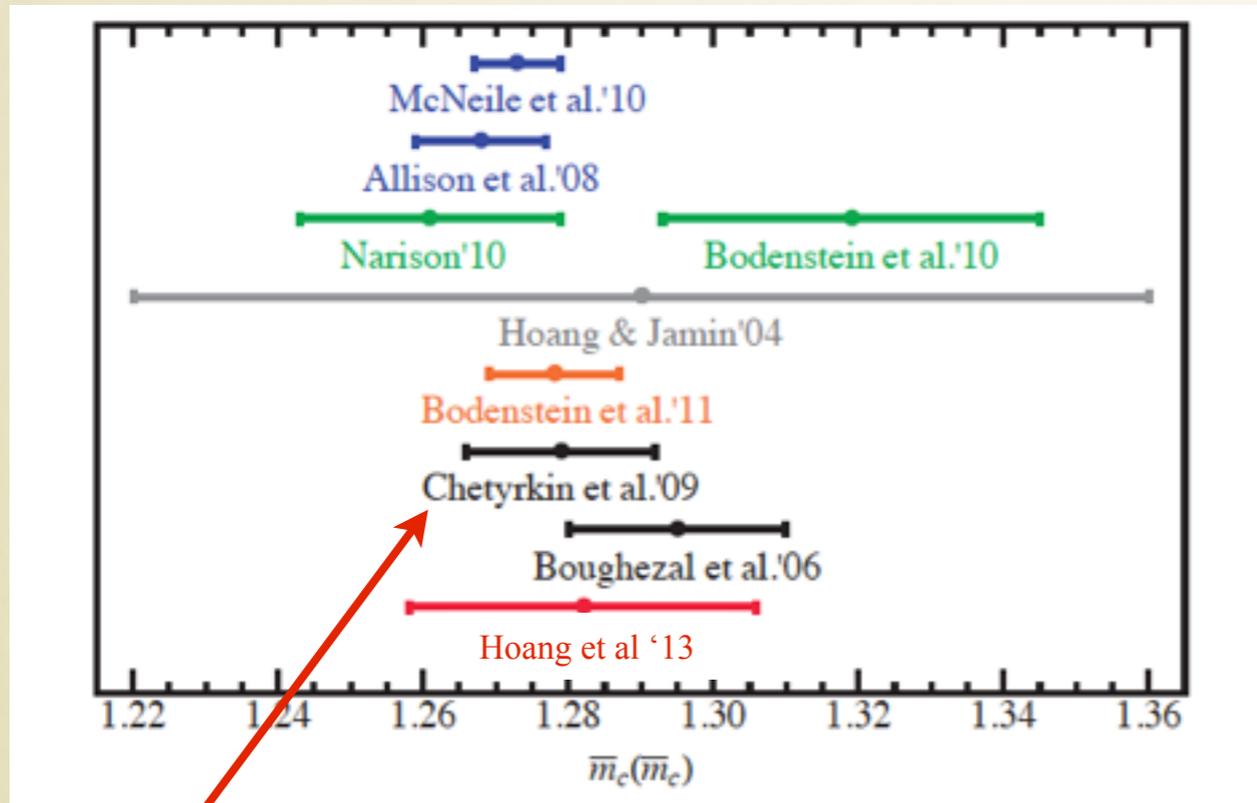
LATEST SEMILEPTONIC FIT

Alberti, Healey, Nandi, PG 1411.6560

- **kinetic scheme** calculation based on 1107.3100; hep-ph/0401063
- includes all $O(a_s^2)$ and $O(a_s/m_b^2)$ corrections
- reassessment of theoretical errors, realistic correlations following Schwanda, PG, 1307.4551
- **external constraints:** precise heavy quark mass determinations, plus mild constraints on μ^2_G from hyperfine splitting and Q^3_{LS} from sum rules

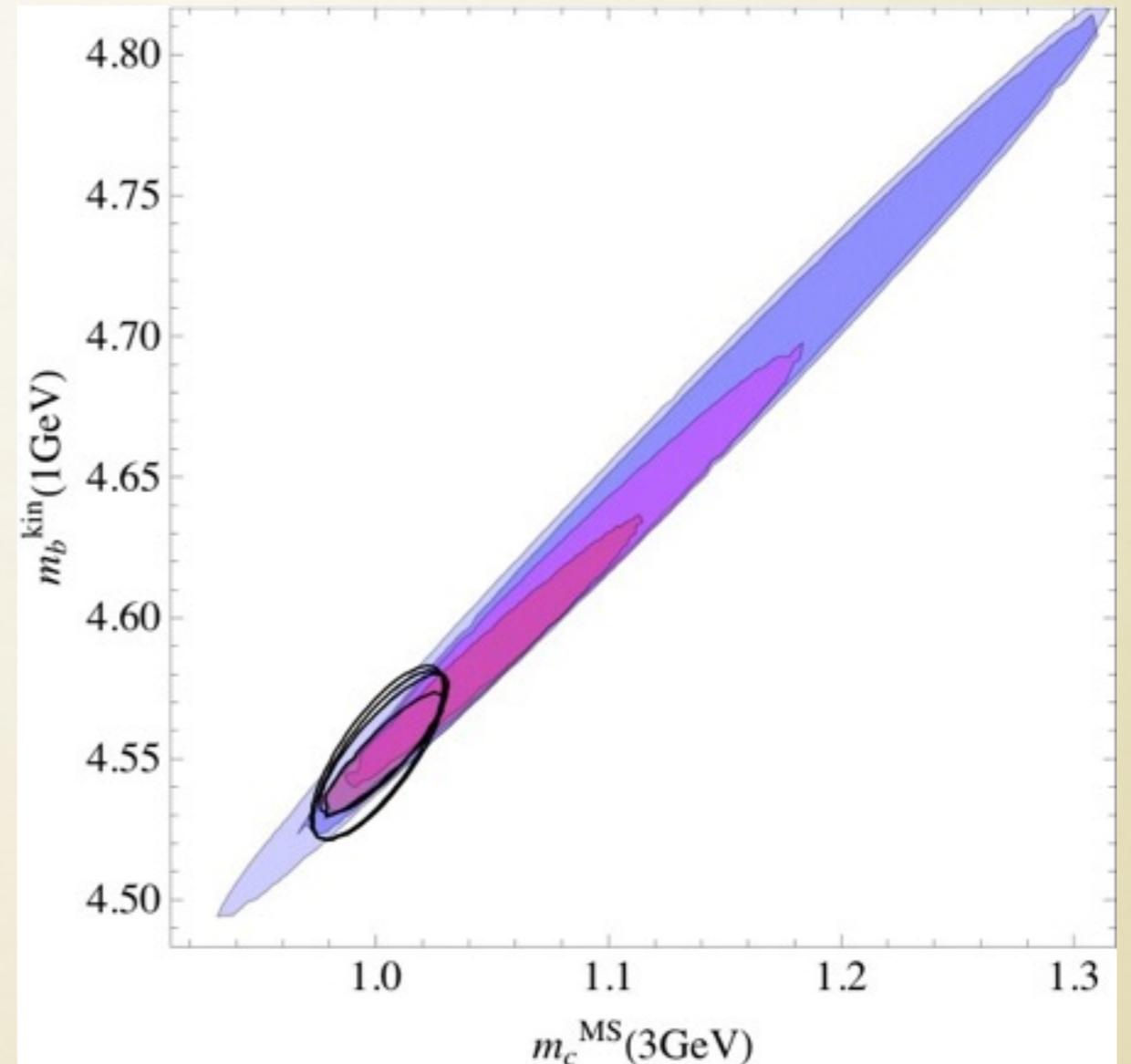
Previous global fits: Buchmuller, Flaecher hep-ph/0507253,
Bauer et al, hep-ph/0408002 (1S scheme)

CHARM MASS DETERMINATIONS



our default choice

sum rules studies of $\sigma(e^+e^- \rightarrow \text{hadrons})$
almost all at NNNLO



Remarkable improvement in recent years.

m_c can be used as precise input to fix m_b instead of radiative moments

FIT RESULTS

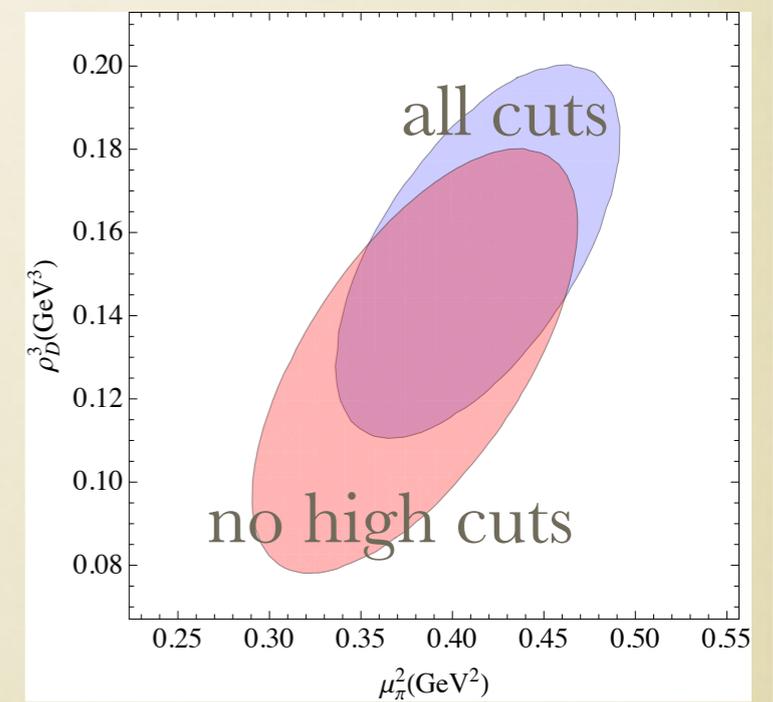
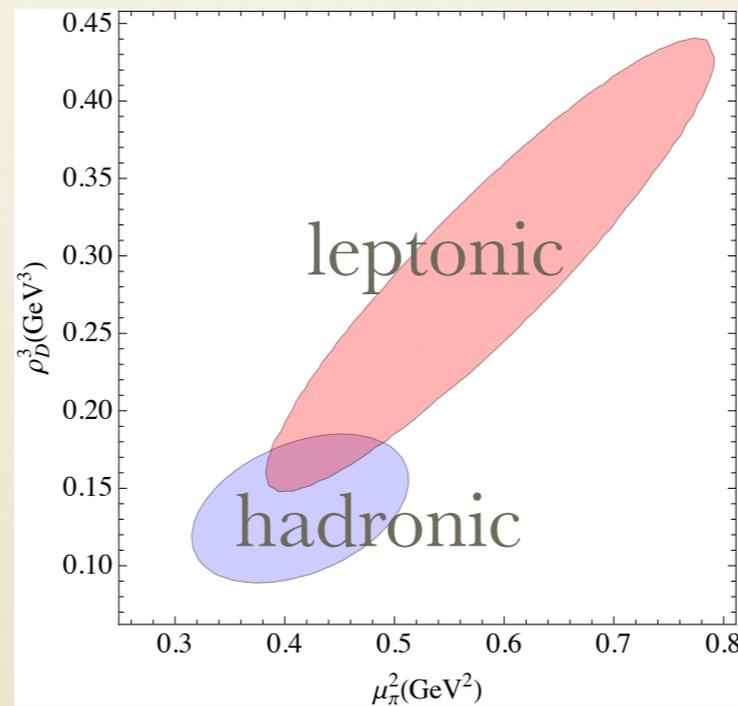
m_b^{kin}	$\overline{m}_c(3\text{ GeV})$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$BR_{cl\nu}$	$10^3 V_{cb} $
4.553	0.987	0.465	0.170	0.332	-0.150	10.65	42.21
0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78

Alberti et al, 1411.6560

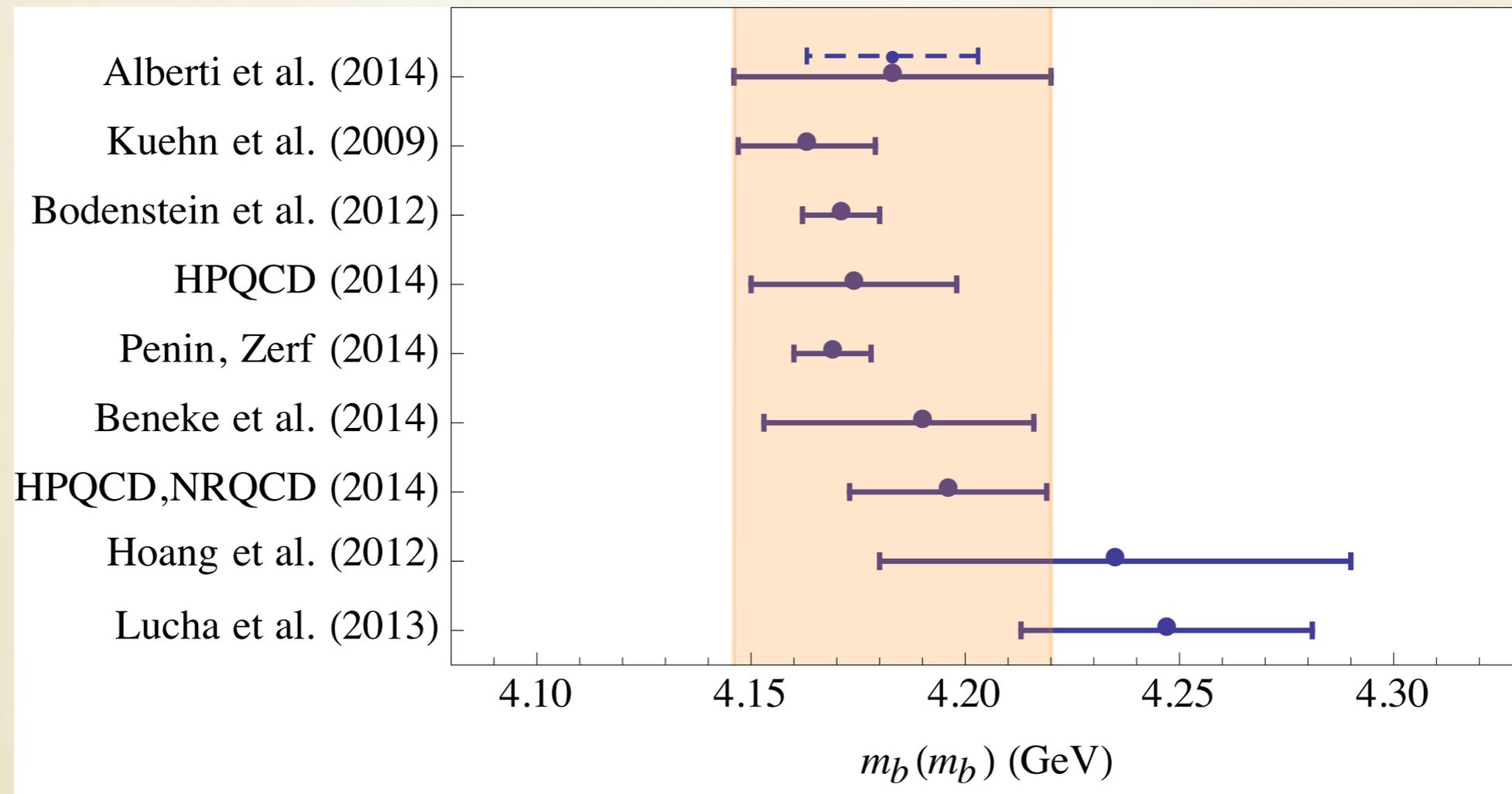
WITHOUT MASS CONSTRAINTS

$$m_b^{kin}(1\text{ GeV}) - 0.85 \overline{m}_c(3\text{ GeV}) = 3.714 \pm 0.018 \text{ GeV}$$

- results depend little on assumption for correlations and choice of inputs, 1.8% determination of V_{cb}
- 20-30% determination of the OPE parameters



RESULTS: BOTTOM MASS



The fit gives $m_b^{kin}(1\text{GeV})=4.553(20)\text{GeV}$

scheme translation error $m_b^{kin}(1\text{GeV})=m_b(m_b)+0.37(3)\text{GeV}$

$$\bar{m}_b(\bar{m}_b)=4.183(37)\text{GeV}$$

HIGHER ORDER EFFECTS

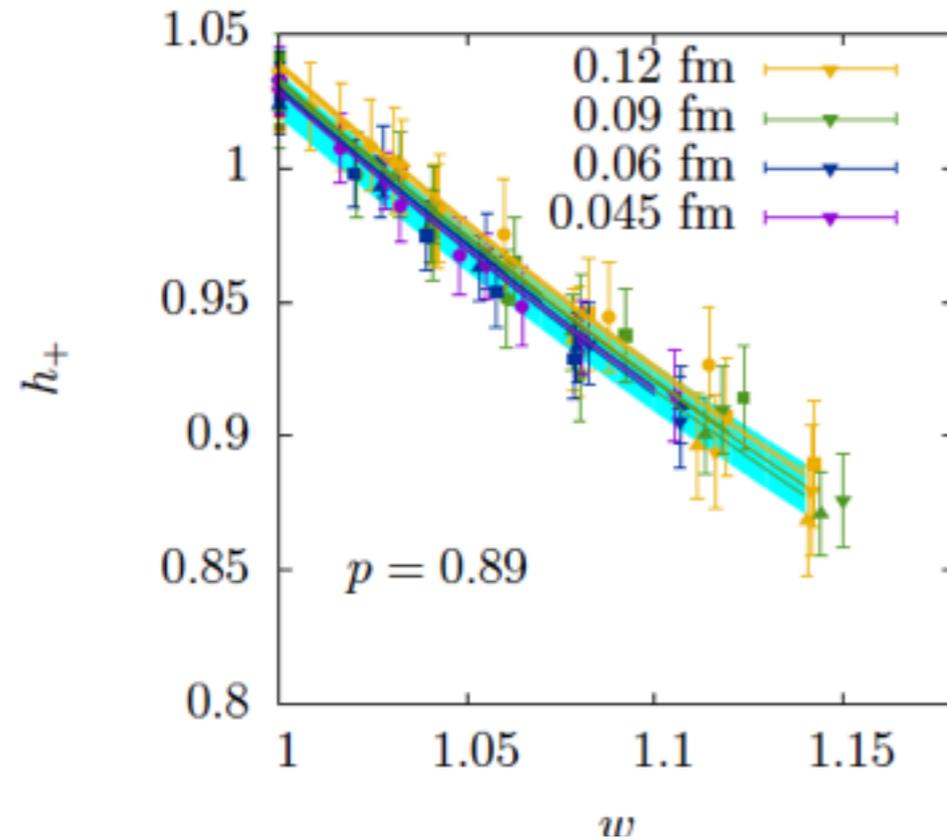
- Reliability of the method depends on our ability to control higher order effects. Quark-hadron duality violation would manifest as inconsistency in the fit.
- **Purely perturbative corrections** complete at NNLO, small residual error (kin scheme)_{Melnikov,Biswas,Czarnecki,Pak,PG}
- **Higher power corrections** $O(1/m_Q^{4,5})$ known
Mannel,Turczyk,Uraltsev 2010 Proliferation of matrix elements, Ground State Saturation Approximation (GSSA) see also Mannel Heinonen 2014
New preliminary fit with m.e. floating around GSSA
 $|V_{cb}| = (42.09 \pm 0.79) \times 10^{-3}$ Healey, Turczyk, PG
- **Mixed corrections** perturbative corrections to power suppressed coefficients completed at $O(\alpha_s/m_b^2)$
Becher, Boos, Lunghi, Alberti, Ewerth, Nandi, PG, Mannel,Pivovarov, Rosenthal

PROSPECTS

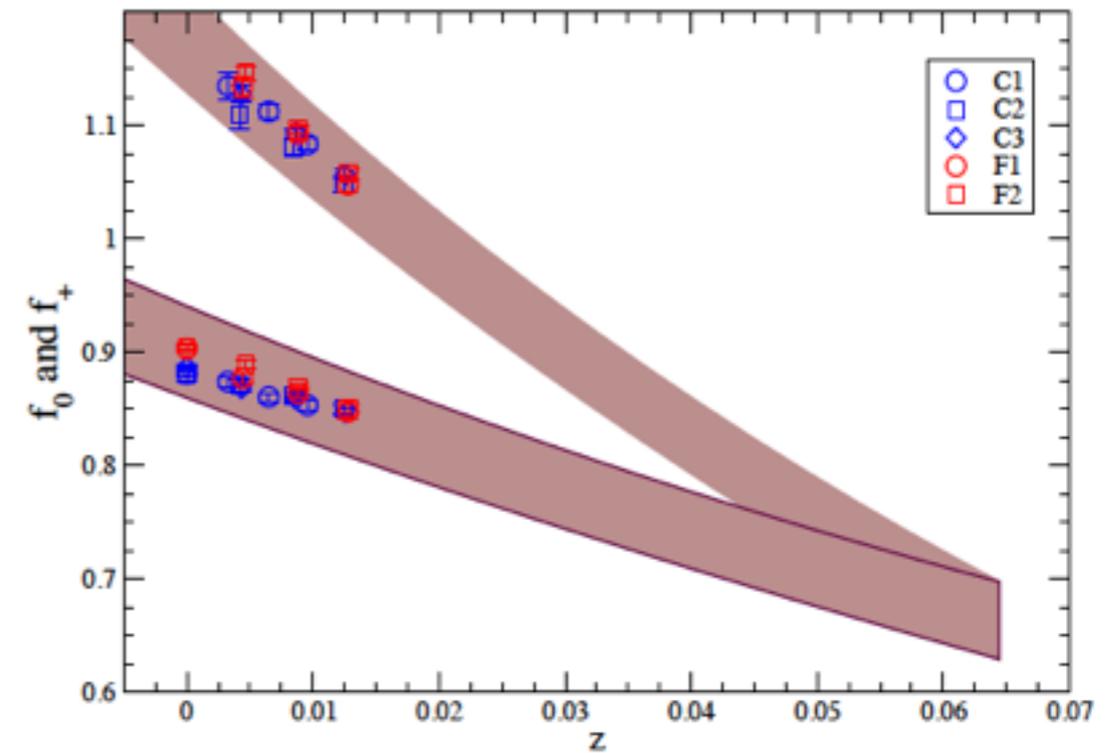
- Theoretical uncertainties already dominant
- $O(a_s/m_b^3)$ calculation under way
- $O(1/m_Q^{4,5})$ effects need further investigation: small effect on V_{cb} , sizeable on HQE matrix elements
- NNNLO corrections to total width feasible
- Electroweak corrections
- New observables in view of Belle-II: FB asymmetry Turczyk
- **Lattice QCD information on local matrix elements is the next frontier**

NEW RESULTS FOR $B \rightarrow D l \nu$ F.F.

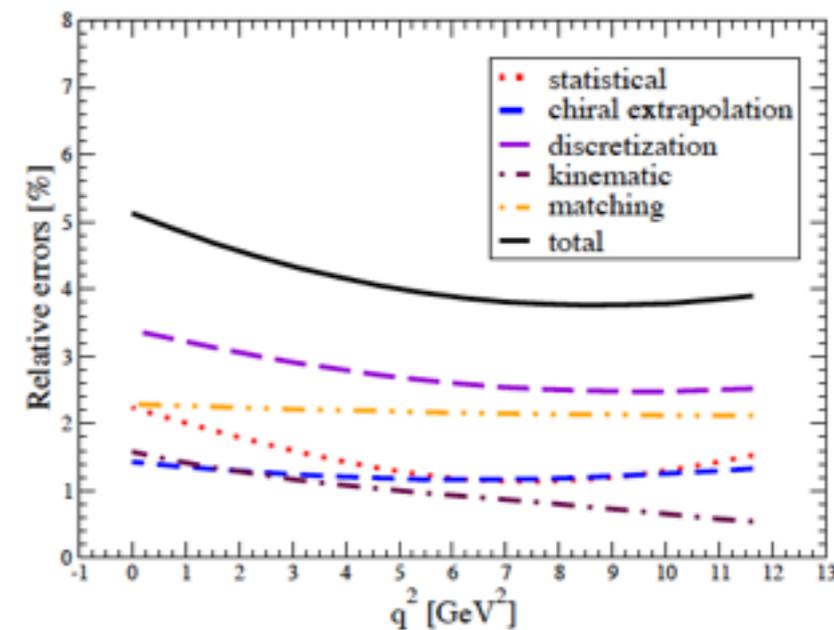
FNAL/MILC 1503.07237



HPQCD 1505.03925

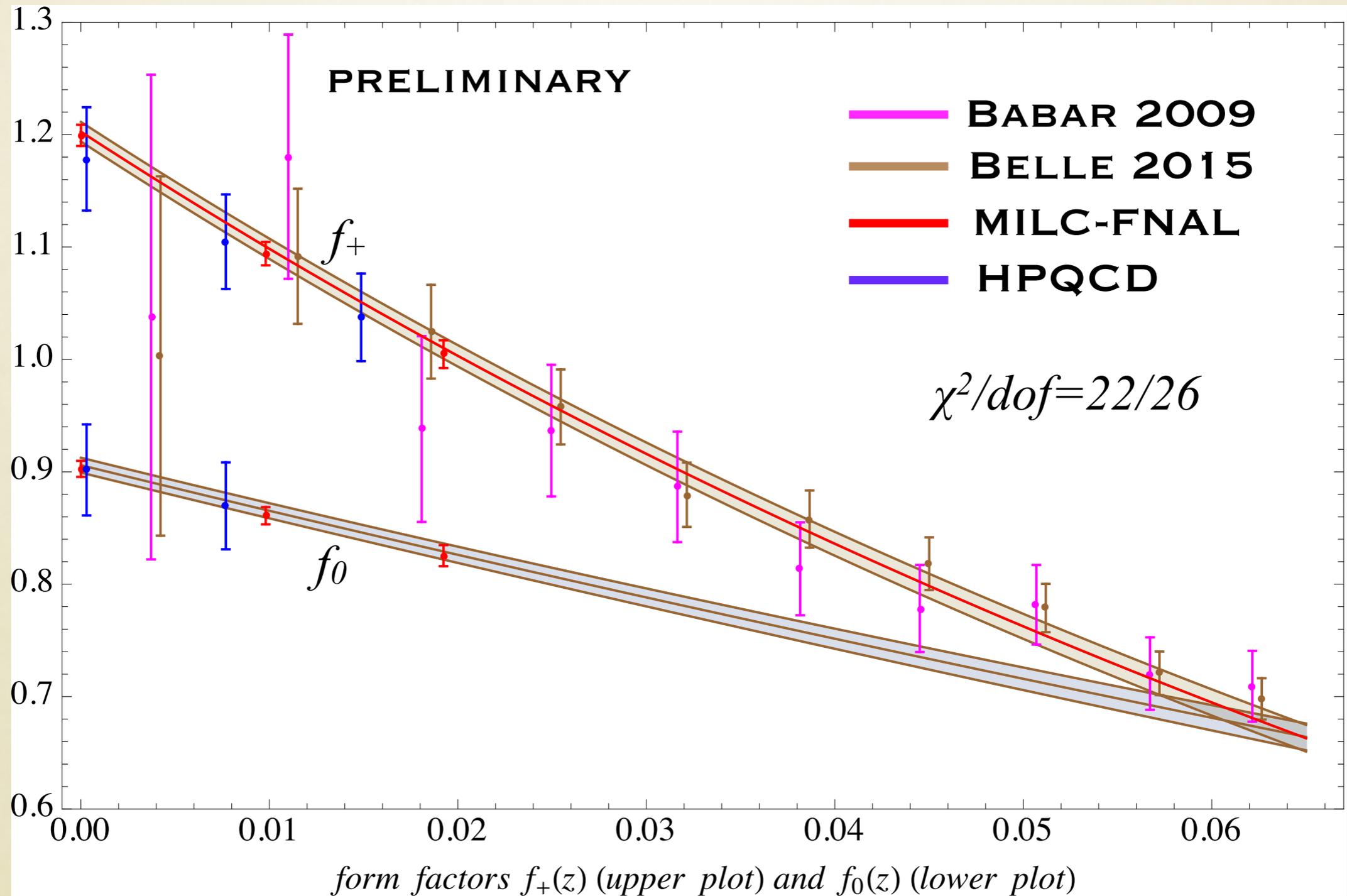


Source	f_+ (%)
Statistics+matching+ χ PT cont. extrap.	1.2
(Statistics)	(0.7)
(Matching)	(0.7)
(χ PT/cont. extrap.)	(0.6)
Heavy-quark discretization	0.4
Lattice scale r_1	0.2
Total error	1.2



Global fit to $B \rightarrow Dlv$

D. Bigi, PG



A global fit to $B \rightarrow D l \nu$

- $|V_{cb}| = 40.62(0.98) 10^{-3}$ preliminary (BGL, N=2)
- **$|V_{cb}| = 40.49(0.99) 10^{-3}$** preliminary (BGL, N=3,4)
- based on z -expansion with unitarity constraints using Boyd, Grinstein, Lebed & Caprini, Lellouch, Neubert 1997
- assumes no correlation between FNAL and HPQCD, 3% syst error on Babar data, correct treatment of last bin, no finite size bin effect, updated Belle results 1510.03657
- CLN parameterization gives $|V_{cb}| = 40.85(95) 10^{-3}$ but terrible fit (p-value $< 10^{-5}$) *when lattice results for f_0 are included.*
We are getting too precise for CLN!!
- Non-zero recoil lattice results are crucial: only zero recoil leads to $|V_{cb}| = 39.6(2.1) 10^{-3}$ (BGL) $40.0(1.1) 10^{-3}$ (CLN)
- Very precise **$R(D) = 0.302(3)$** , 1.9σ from HFAG average

EXCLUSIVE $B \rightarrow D^* \ell \nu$

At zero recoil, where rate vanishes, the ff is

$$\mathcal{F}(1) = \eta_A \left[1 + O\left(\frac{1}{m_c^2}\right) + \dots \right]$$

Thanks to measurement of slopes and shape parameters, **exp error only**
 $\sim 1.3\%$

The ff $F(1)$ cannot be experimentally determined. Lattice QCD is the best hope to compute it. Only one unquenched Lattice calculation:

$$F(1) = 0.906(13) \implies |V_{cb}| = 39.04(49)_{\text{exp}}(53)_{\text{lat}}(19)_{\text{QED}} 10^{-3}$$

Bailey et al 1403.0635 (FNAL/MILC)

1.9% error (adding in quadrature)

$\sim 2.9\sigma$ or $\sim 8\%$ from inclusive determination

PROSPECTS FOR EXCLUSIVE V_{cb}

- Most experimental $B \rightarrow D^{(*)}$ results tied up with CLN don't include CLN error: also $R(D^{*})$ should have larger uncertainty! new exp analyses under way. More at Belle-II.
- Need for alternative calculations and extension of $B \rightarrow D^{*} ff$ to non-zero recoil. Matching at $1/m_Q^3$ for lattice discretization effects under study by FNAL/MILC. Simulations at physical pion mass and $m_b a \approx 1$?
- **Heavy quark sum rules** (with BPS arguments) favor smaller $F(1)=0.86(2)$ leading to agreement with inclusive. Difficult to improve, how good is BPS limit?
- **QED/EW corrections:** SD log OK, SD remainder tiny if G_μ employed, soft/collinear radiation subtracted out by Photos, intermediate photons (IR finite) are structure dependent: lattice calculations? exp cuts? relevance of Coulomb enhancement for B^0 decay rate?
- New channels (B_s, Λ_b) at Belle-II and LHCb, better understanding of D^{**}

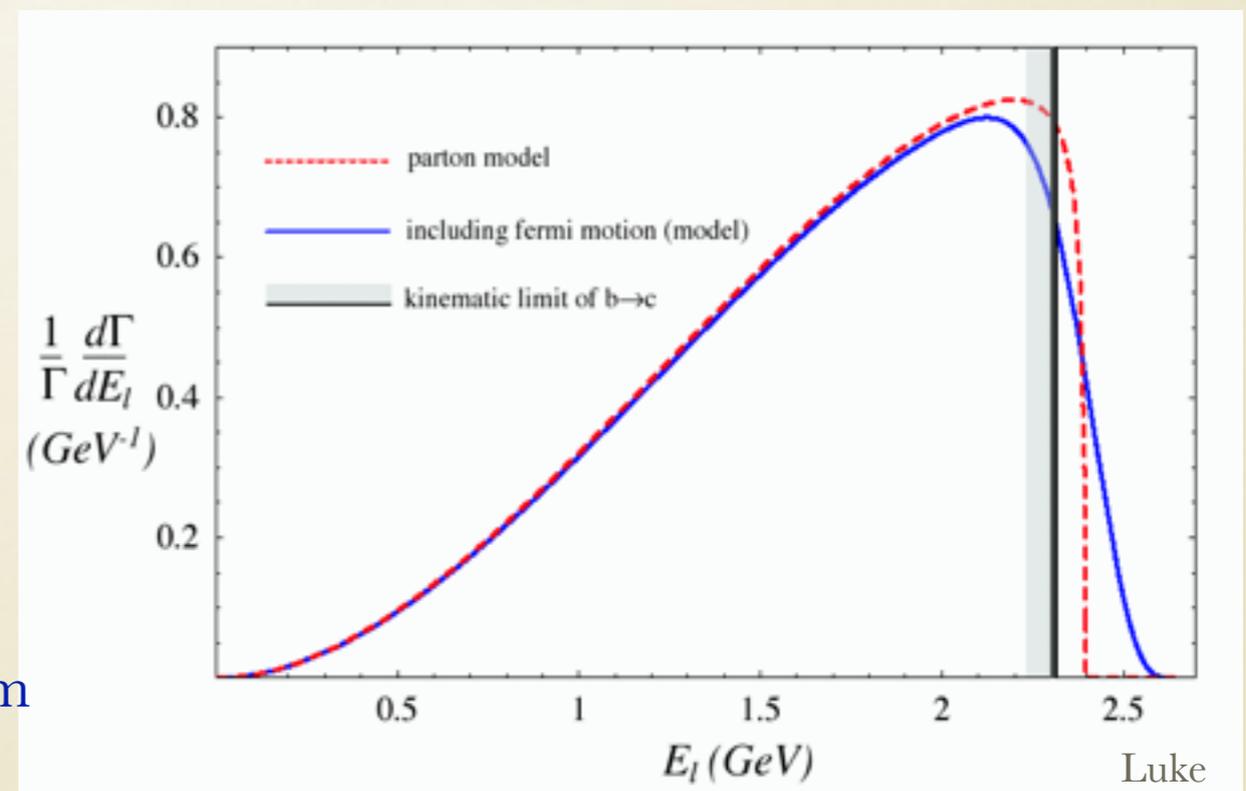
$B \rightarrow X_{ul} \nu$ AND CUTS

Experiments often use kinematic cuts to avoid the $\sim 100x$ larger $b \rightarrow c l \nu$ background:

$$m_X < M_D \quad E_l > (M_B^2 - M_D^2) / 2M_B \quad q^2 > (M_B - M_D)^2 \dots$$

The cuts destroy convergence of the OPE that works so well in $b \rightarrow c$. OPE expected to work only away from pert singularities

Rate becomes sensitive to *local* b-quark wave function properties like Fermi motion. Dominant non-pert contributions can be resummed into a **SHAPE FUNCTION** $f(k^+)$. Equivalently the SF is seen to emerge from soft gluon resummation



HOW TO ACCESS THE SF?

$$\frac{d^3\Gamma}{dp_+ dp_- dE_\ell} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} \int dk C(E_\ell, p_+, p_-, k) F(k) + O\left(\frac{\Lambda}{m_b}\right)$$

Subleading SFs

Prediction *based on*
resummed pQCD

DGE, ADFR

OPE constraints +
parameterization
without/with resummation

GGOU, BLNP

Fit semileptonic (and radiative) data

SIMBA, NNVub

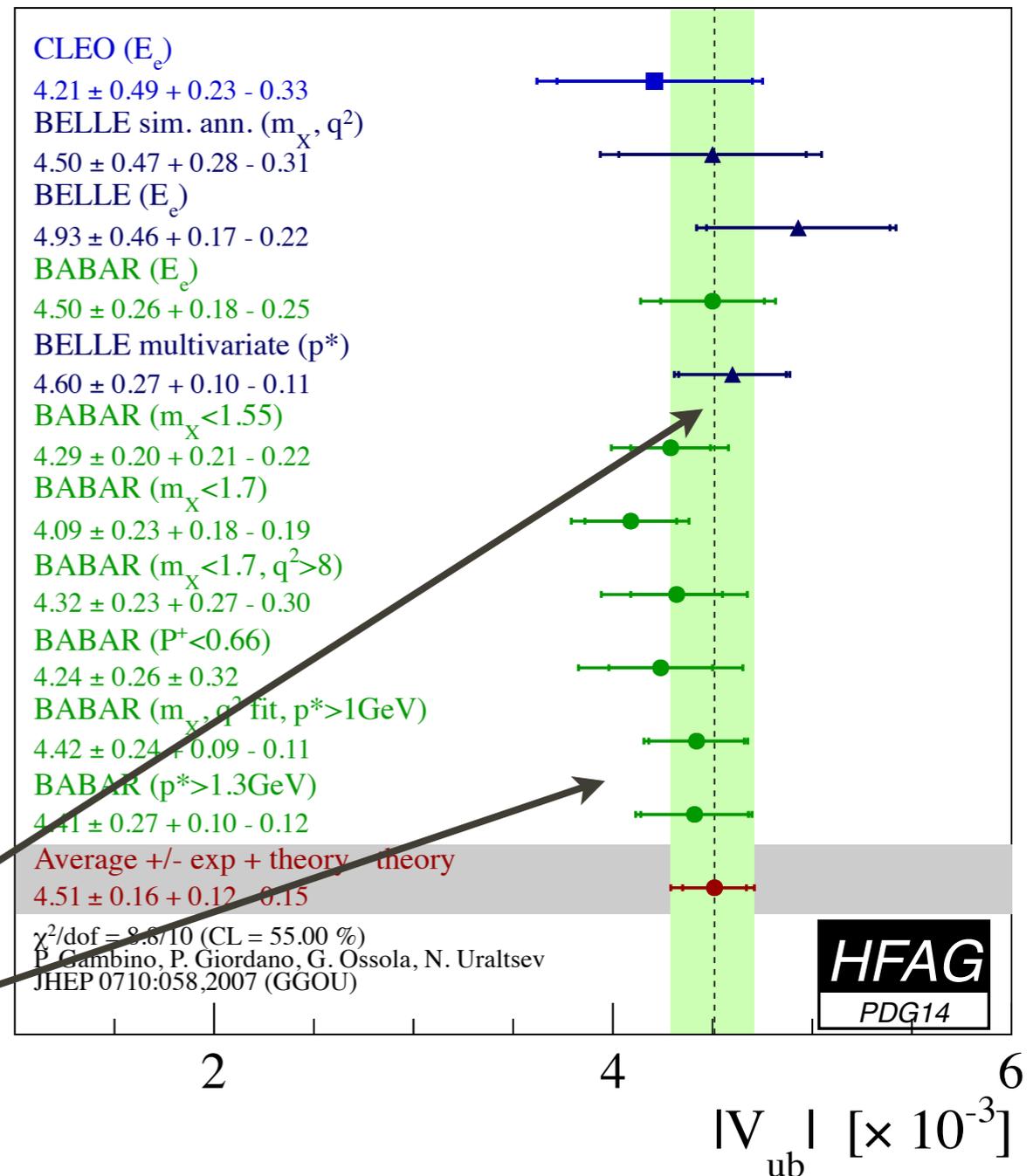
$|V_{ub}|$ DETERMINATIONS

Inclusive: 5% total error

HFAG 2014	Average IV
DGE	4.52(16)(16)
BLNP	4.45(16)(22)
GGOU	4.51(16)(15)

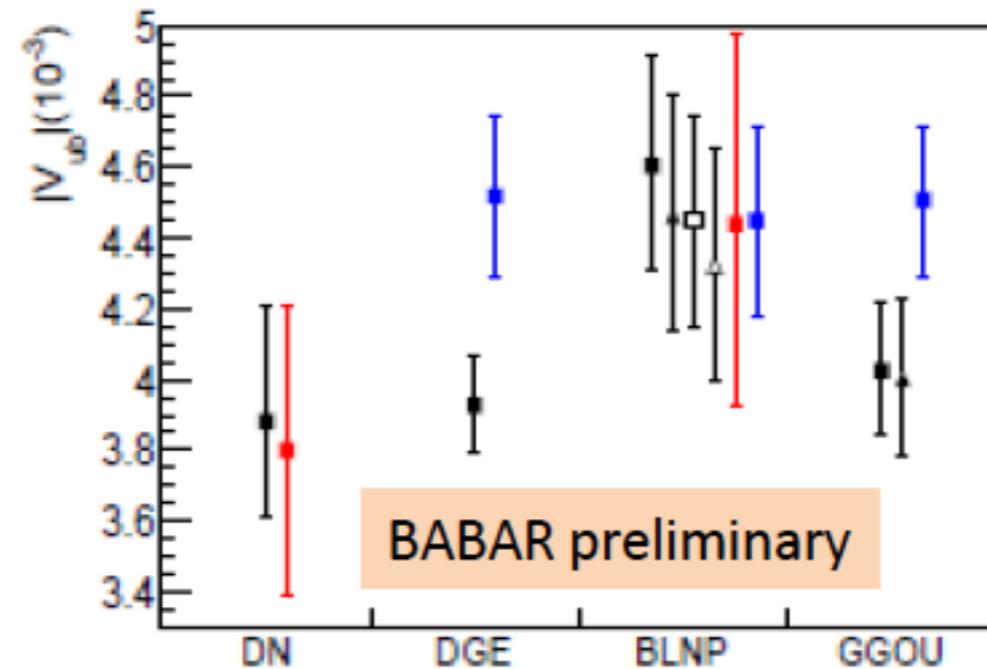
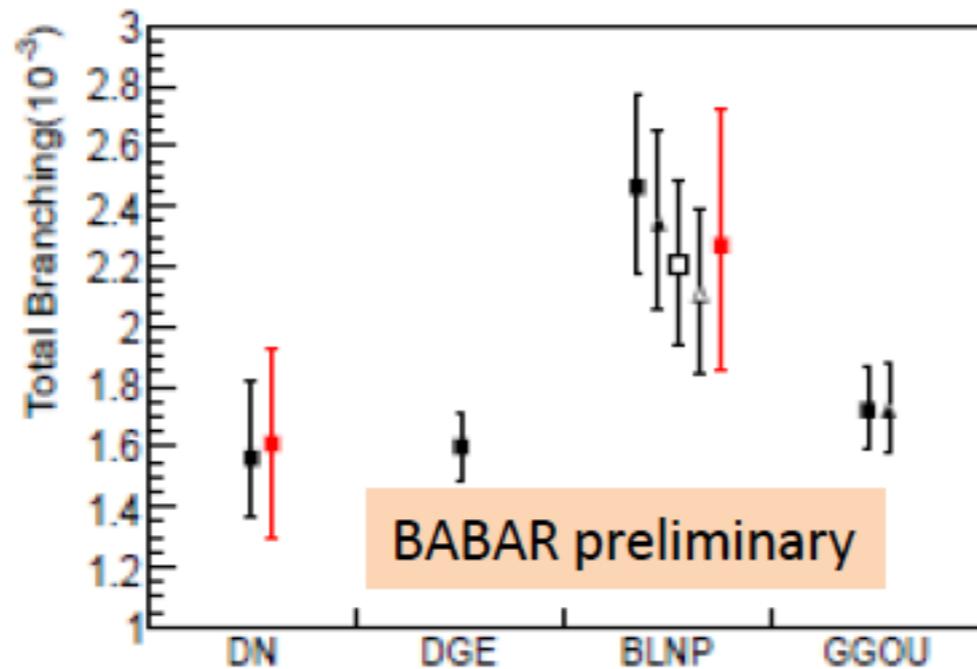
UT fit (without direct V_{ub}):
 $V_{ub} = 3.62(12) \cdot 10^{-3}$

Recent experimental results are theoretically cleanest (2%) but based on background modelling. Signal simulation also relies on theoretical models...



NEW preliminary Babar endpoint analysis

High sensitivity of the BR on the shape of the signal in the endpoint region. GGOU: $|V_{ub}| = 4.03^{+0.20}_{-0.22} \times 10^{-3}$



solid squares and triangles – X_c with mc constraint fit and $X_c+X_s\gamma$ fit of SF parameters (BLNP and GGOU)

solid and open - translation “kinetic” to “shape-function” with $\mu = 2.0\text{GeV}$ and $\mu = 1.5\text{GeV}$ (BLNP), respectively

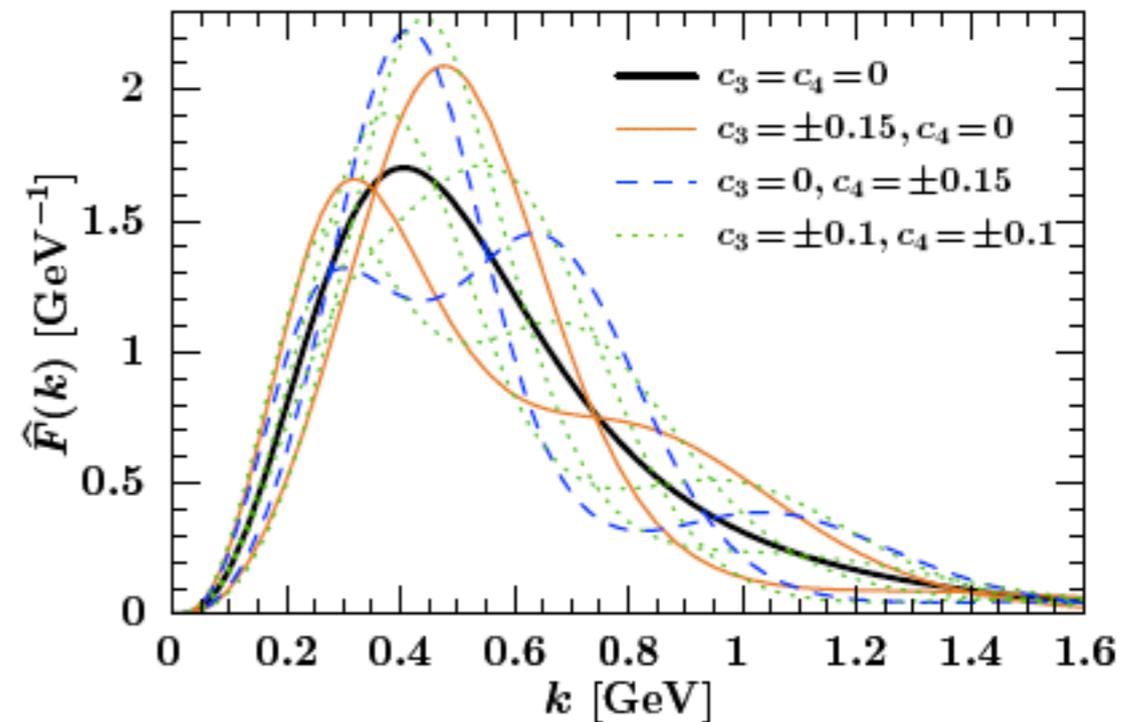
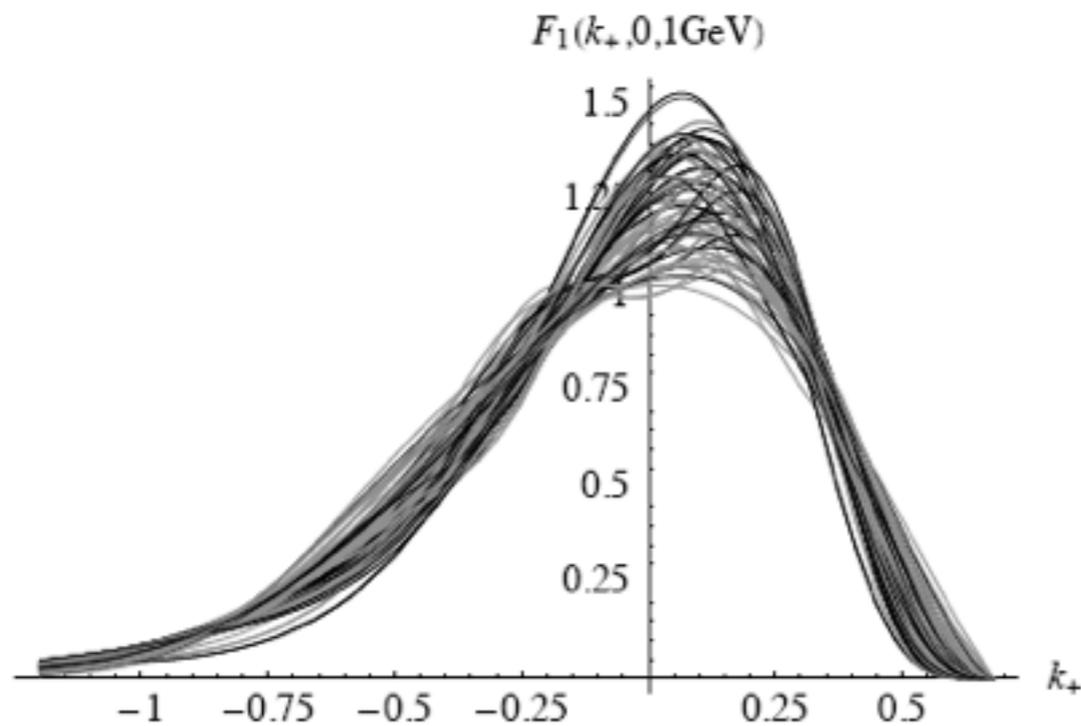
results based on 0.8-2.6GeV/c momentum range

HFAG 2014 average based on tagged and untagged measurements

Consistent with and more precise than our previous result:

BaBar, Phys.Rev. D73(2006)012006 ($p_e > 2 \text{ GeV}/c$)

FUNCTIONAL FORMS

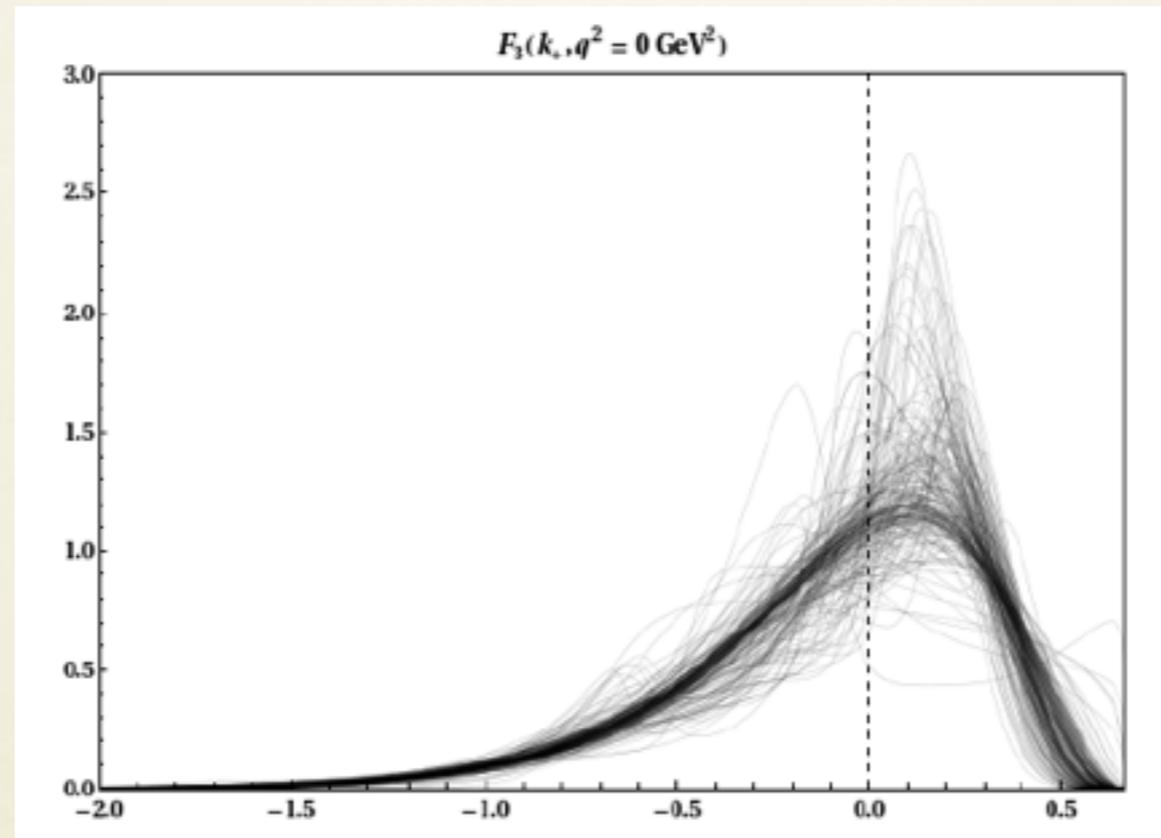


About 100 forms considered in GGOU, large variety, double max discarded. Small uncertainty (1-2%) on V_{ub}

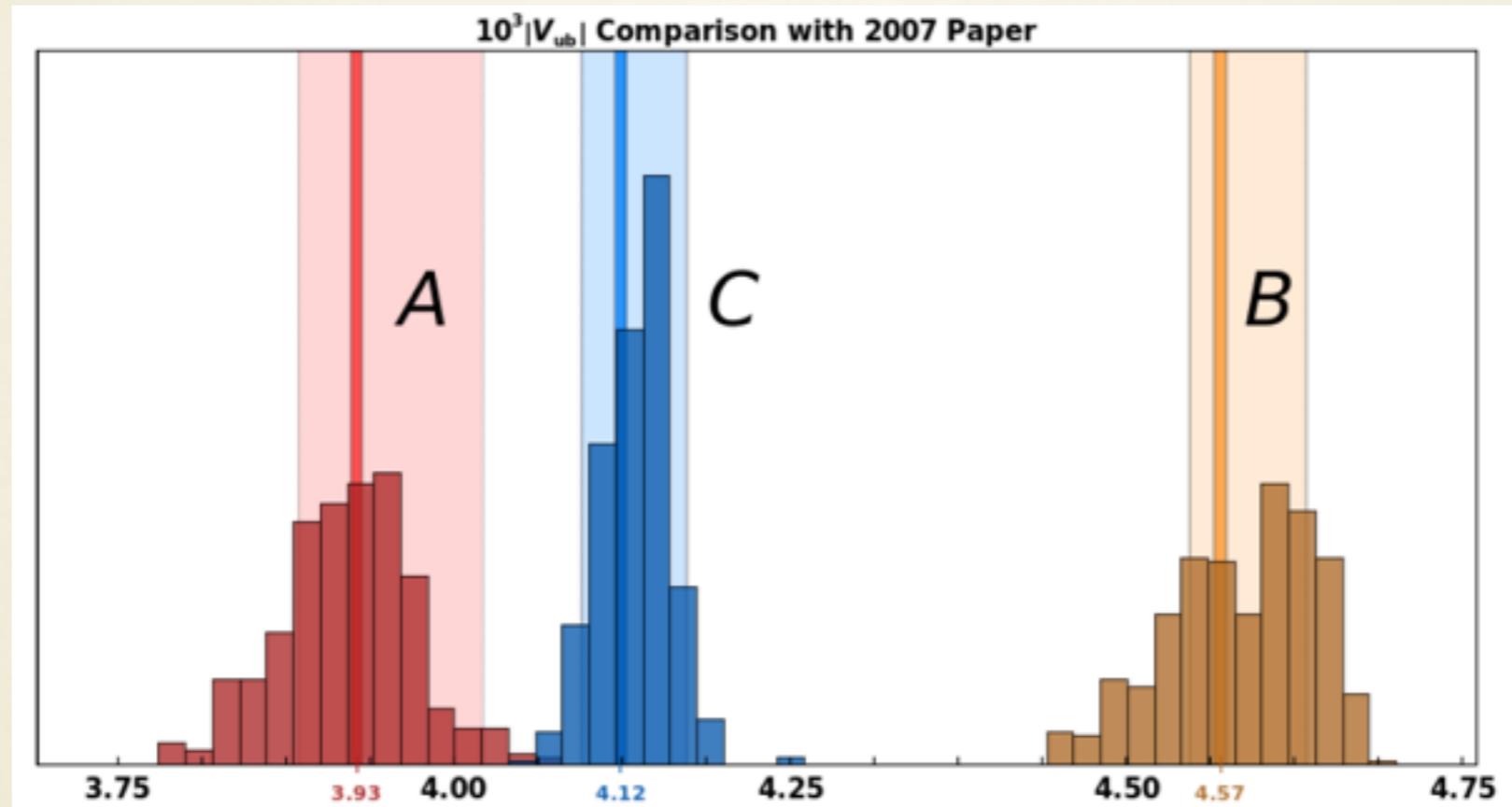
A more systematic method by Ligeti et al. arXiv:0807.1926
Plot shows 9 SFs that satisfy all the first three moments

The NNVub Project

K.Healey, C. Mondino, PG, 1604.07598

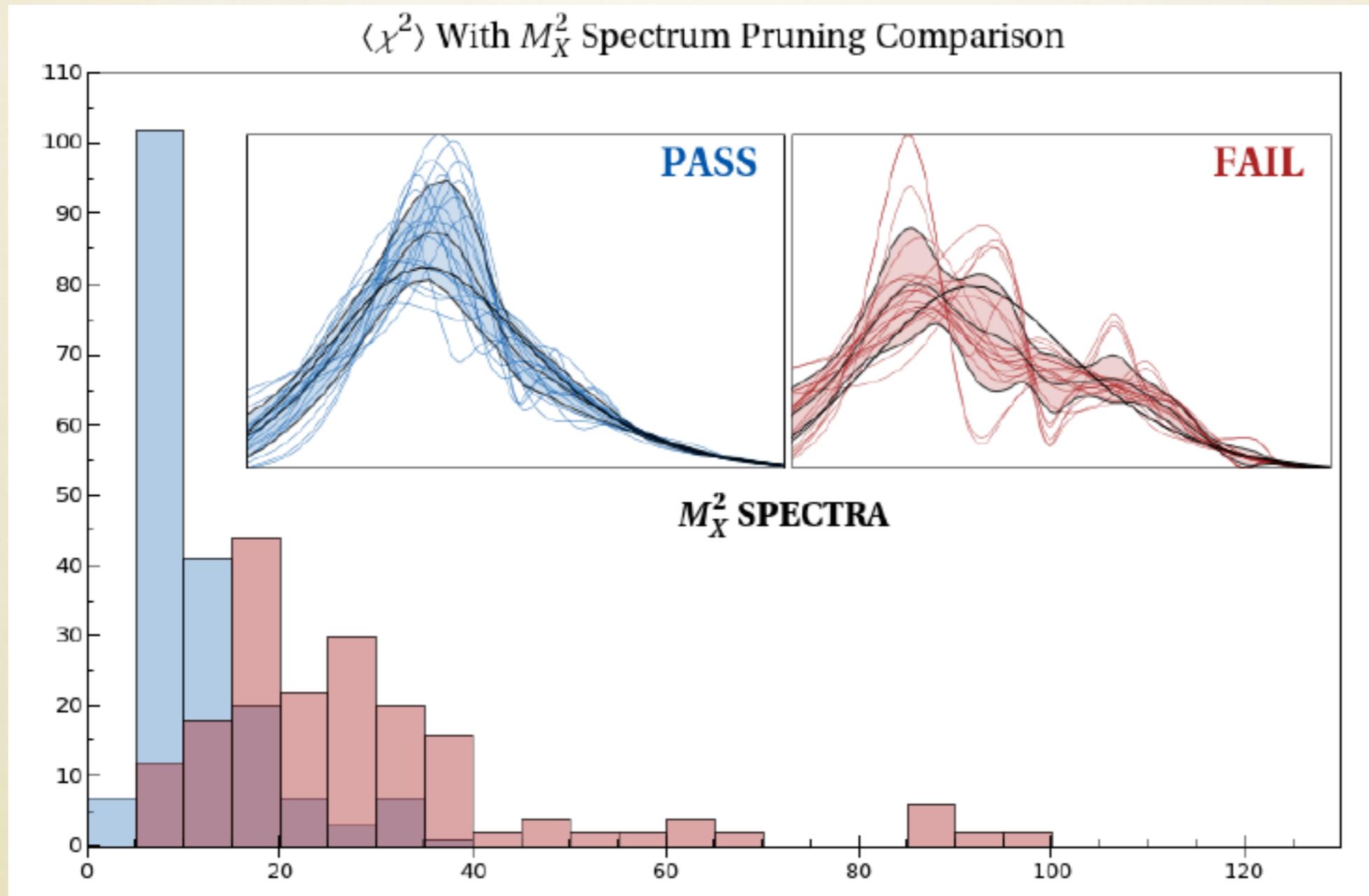


- Use **Artificial Neural Networks** to parameterize shape functions without bias and extract V_{ub} from theoretical constraints and data, together with HQE parameters in a model independent way (without assumptions on functional form).
- Belle-II will be able to measure some kinematic distributions, thus constraining directly the shape functions. NNVub will provide a flexible tool to analyse data.



Experimental cuts (in GeV or GeV ²)	$ V_{ub} \times 10^3$	$ V_{ub} \times 10^3$ [15]
$M_X < 1.55, E_\ell > 1.0$ Babar [44]	4.30(20) ⁽²⁶⁾ ₍₂₇₎	4.29(20) ⁽²¹⁾ ₍₂₂₎
$M_X < 1.7, E_\ell > 1.0$ Babar [44]	4.05(23) ⁽¹⁹⁾ ₍₂₀₎	4.09(23) ⁽¹⁸⁾ ₍₁₉₎
$M_X \leq 1.7, q^2 > 8, E_\ell > 1.0$ Babar [44]	4.23(23) ⁽²³⁾ ₍₂₈₎	4.32(23) ⁽²⁷⁾ ₍₃₀₎
$E_\ell > 2.0$ Babar [41]	4.47(26) ⁽²²⁾ ₍₂₇₎	4.50(26) ⁽¹⁸⁾ ₍₂₅₎
$E_\ell > 1.0$ Belle [45]	4.58(27) ⁽¹⁰⁾ ₍₁₁₎	4.60(27) ⁽¹⁰⁾ ₍₁₁₎

PROSPECTS: LEARNING @ BELLE-II FROM KINEMATIC DISTRIBUTIONS



LQCD calculations for $|V_{ub}|$: recent progress

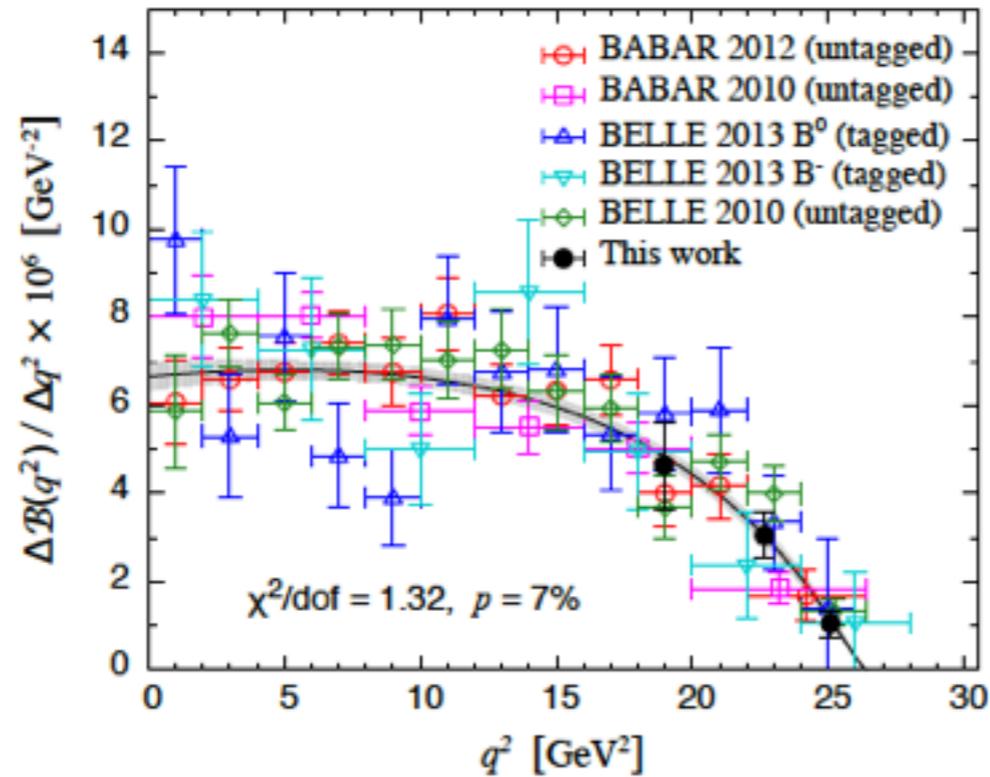
- Disclaimer: the list is not meant to be inclusive. I am focusing on the publicized results.

Lattice Group	Fermilab/MILC	HPQCD	RBC/UKQCD	Alpha	Detmold et al.
Process	$B \rightarrow \pi l \nu$ ($B_s \rightarrow K l \nu$)	$B_s \rightarrow K l \nu$ ($B \rightarrow \pi l \nu$)	$B \rightarrow \pi l \nu$ $B_s \rightarrow K l \nu$	($B_s \rightarrow K l \nu$)	$\Lambda_b \rightarrow p l \nu$
Gauge ensembles	MILC asqtad	MILC asqtad	Domain-Wall	CLS	Domain-Wall
Sea flavors	2+1	2+1	2+1	2	2+1
a (fm)	0.045–0.12	0.09–12	0.086–0.11	0.049–0.076	0.086–0.11
M_π	≥ 177 MeV	≥ 354 MeV	≥ 289 MeV	≥ 310 MeV	≥ 295 MeV
l -quark action	asqtad	HISQ	Domain-Wall	Imprv. Wilson	Domain-Wall
b -quark action	Fermilab Clover	NRQCD	RHQ	Lat. HQET	RHQ
χ PT	NNLO, SU(2), hard- π	HP χ PT+	NLO, SU(2), hard- π		
q^2 -extrapolation	functional BCL	modified z	synthetic BCL		modified- z
Ref.	arXiv:1503.07839 arXiv:1312.3197	arXiv:1406.2279	arXiv:1501.05373v2	arXiv:1411.3916	arXiv:1306.0446 arXiv:1503.01421v2 arXiv:1504.01568

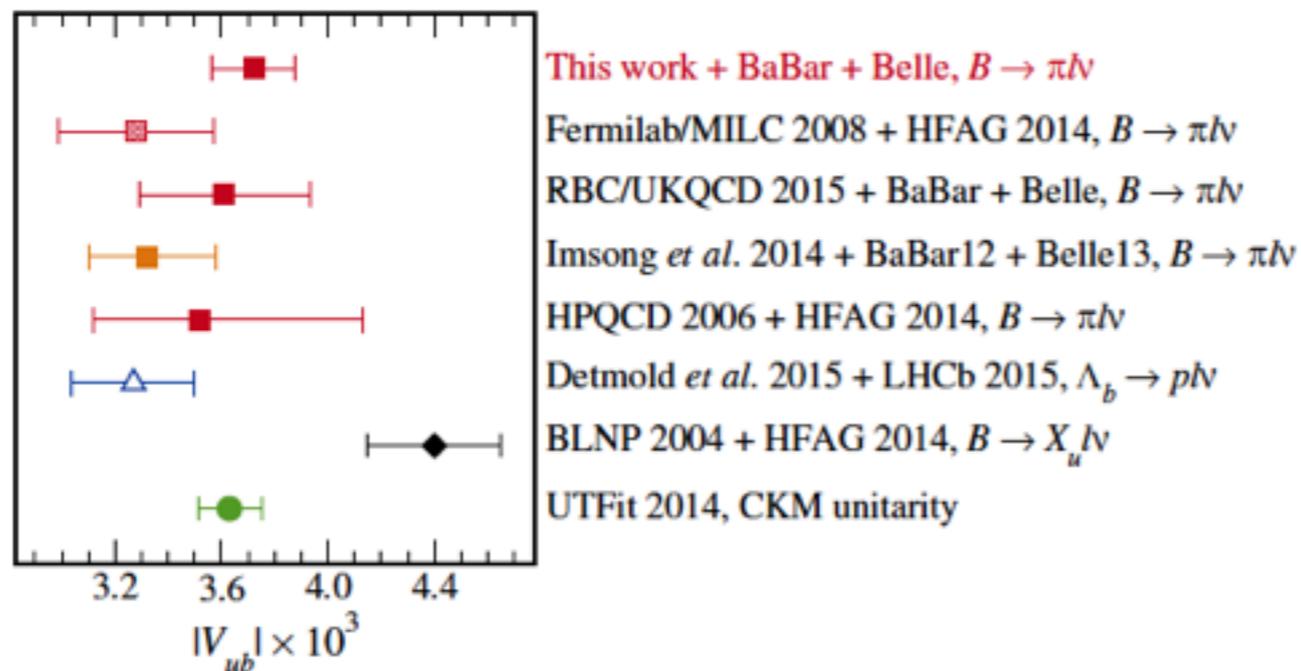
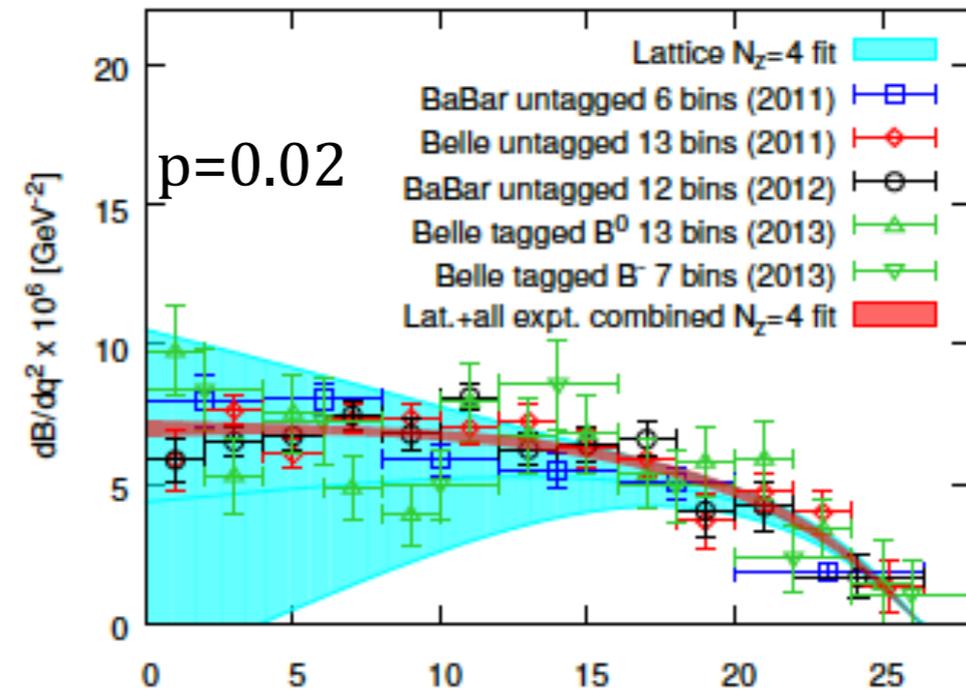
- (): work in progress

RECENT LATTICE $B \rightarrow \pi$

RBC/UKQCD 1501.05373



FNAL/MILC 1503.07839



FNAL $3.72(16) \cdot 10^{-3}$
only 4.3% error

2.2 σ from inclusive

RBC/UKQCD $3.61(32) \cdot 10^{-3}$

1.9 σ from inclusive

LCSR $3.32(26) \cdot 10^{-3}$

2.9 σ from inclusive

LHCb depends
on V_{cb} employed but low

RECENT LATTICE RESULTS

FROM 1503.07839

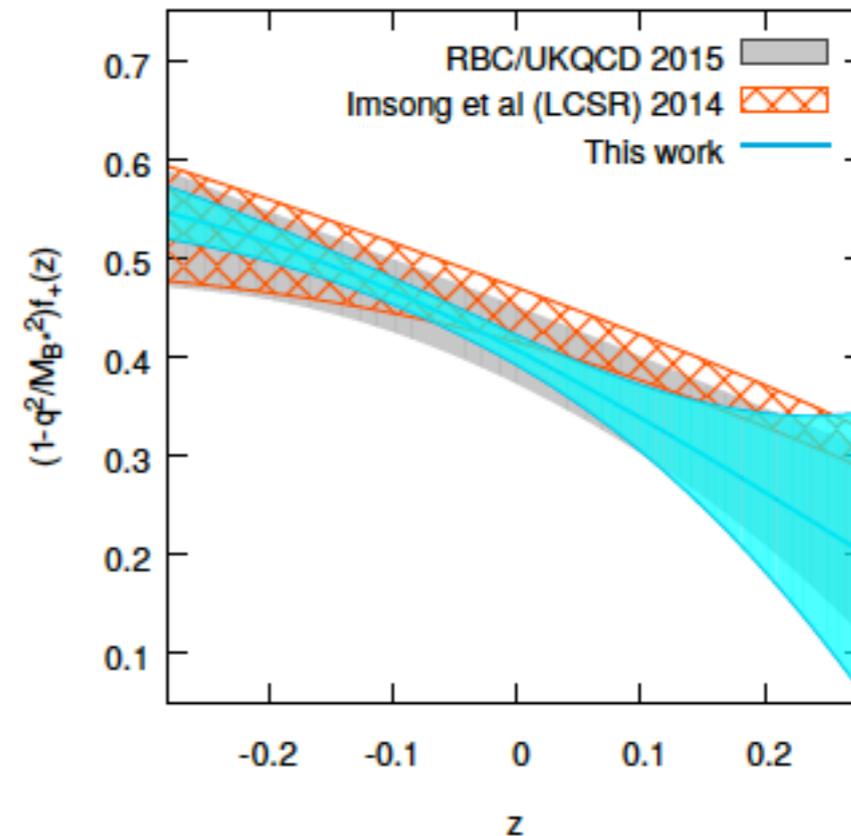
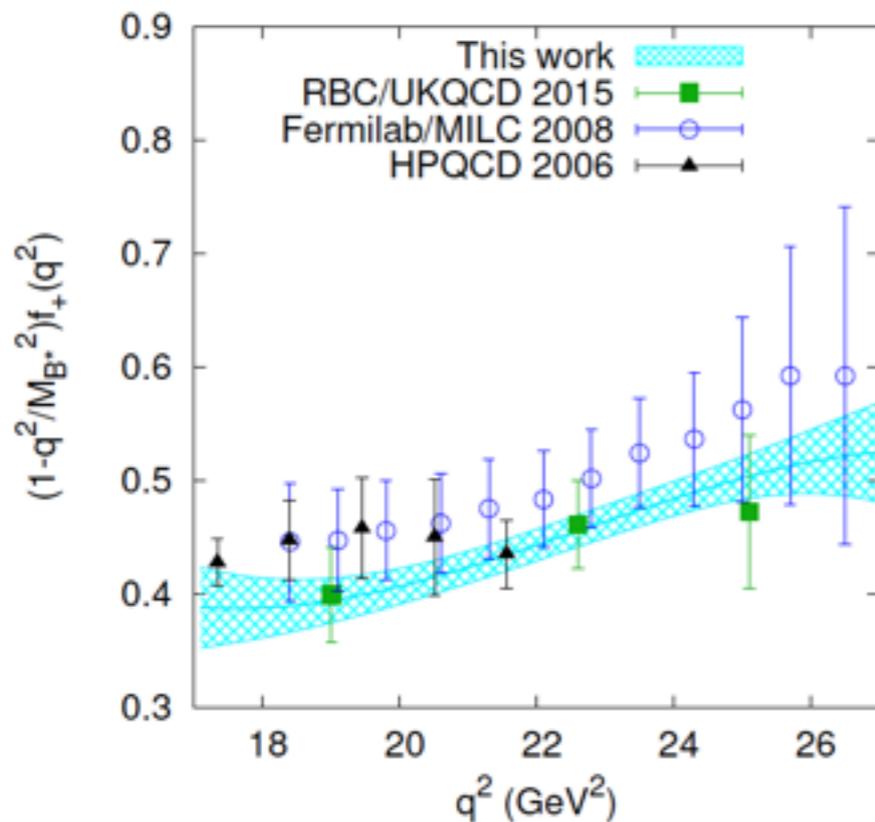
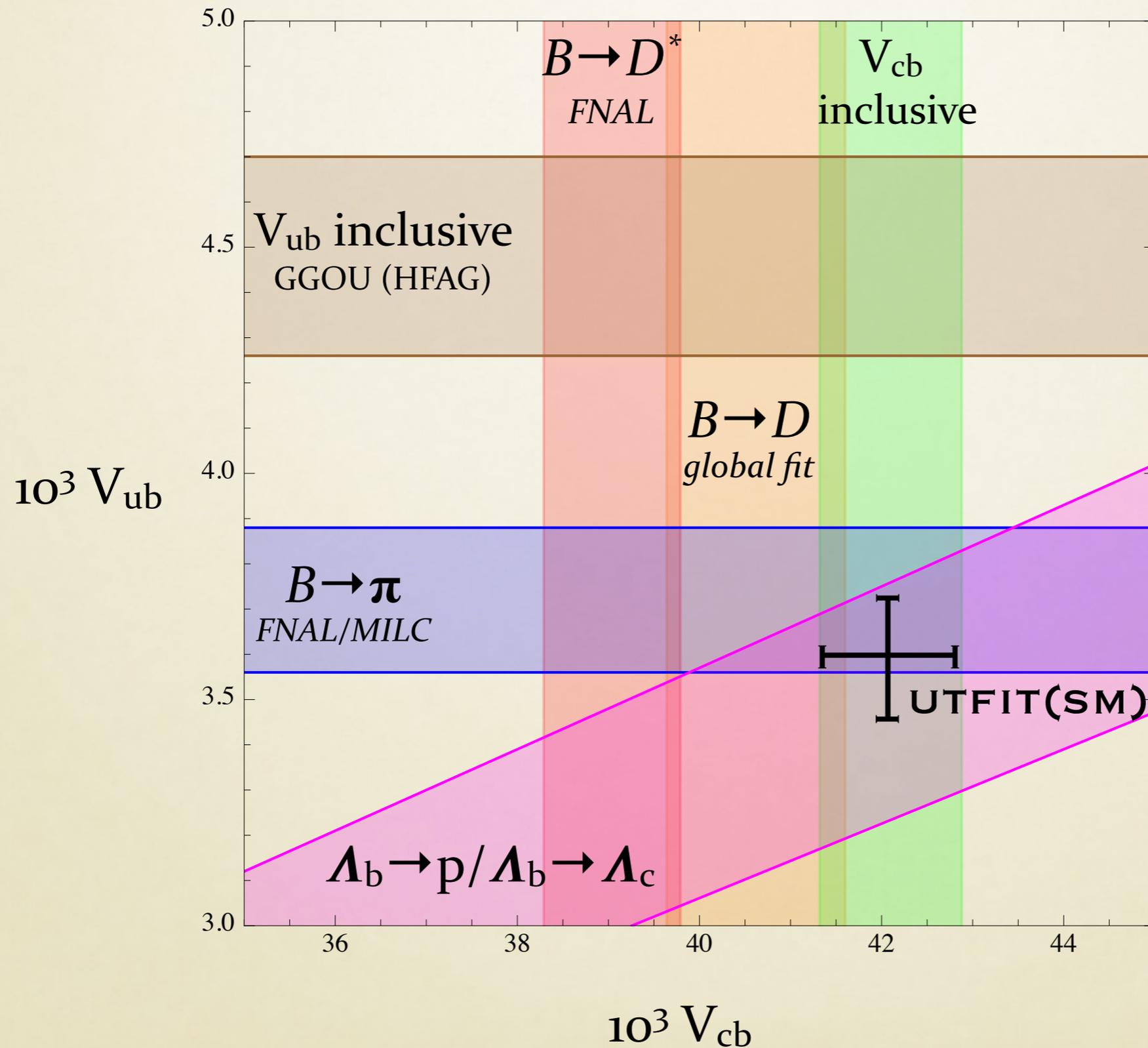


Table XVI. Results of the combined lattice+experiment fits with $N_z = 4$;

Fit	χ^2/dof	dof	p value	b_0^+	b_1^+	b_2^+	b_3^+	$ V_{ub} (\times 10^3)$
Lattice+exp.(all)	1.4	5	0.02	0.419(13)	-0.495(55)	-0.43(14)	0.22(31)	3.72(16)
Lattice+BaBar11 [7]	1.1	9	0.38	0.414(14)	-0.488(73)	-0.24(22)	1.33(44)	3.36(21)
Lattice+BaBar12 [8]	1.1	15	0.34	0.415(14)	-0.551(72)	-0.45(18)	0.27(41)	3.97(22)
Lattice+Belle11 [9]	0.9	16	0.55	0.412(13)	-0.574(65)	-0.40(16)	0.38(36)	4.03(21)
Lattice+Belle13 [10]	1.0	23	0.42	0.406(14)	-0.623(73)	-0.13(22)	0.92(45)	3.81(25)

VISUAL SUMMARY



reasonable consistency among exclusive channels

not all results on the same footing

NEW PHYSICS?

The difference in V_{cb} incl vs excl D^* with FNAL/MILC form factor is **large**: 3σ or about 8%. The perturbative corrections to inclusive V_{cb} total 5%, the power corrections about 4%.

Right Handed currents now excluded since

$$|V_{cb}|_{incl} \simeq |V_{cb}| \left(1 + \frac{1}{2} |\delta|^2 \right)$$

$$|V_{cb}|_{B \rightarrow D^*} \simeq |V_{cb}| \left(1 - \delta \right)$$

$$|V_{cb}|_{B \rightarrow D} \simeq |V_{cb}| \left(1 + \delta \right)$$

Chen, Nam, Crivellin, Buras, Gemmler, Isidori, ...

$$\delta = \epsilon_R \frac{\tilde{V}_{cb}}{V_{cb}} \approx 0.08$$

Most general SU(2) invariant dim 6 NP (without RH neutrino) can explain results, but it is incompatible with $Z \rightarrow b\bar{b}$ data

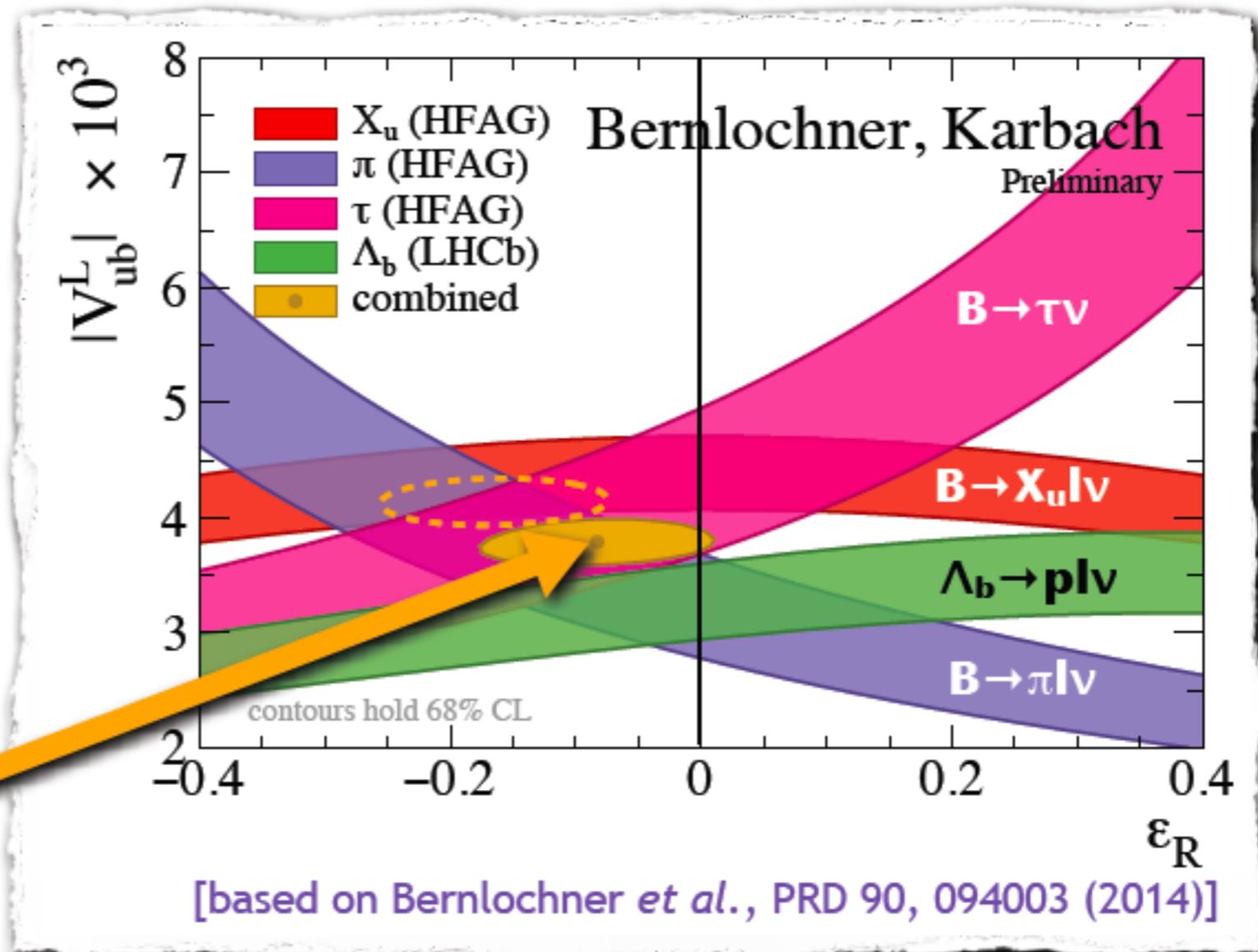
Explaining V_{ub} tension is easier

Crivellin, Pokorski 1407.1320
see Crivellin's talk

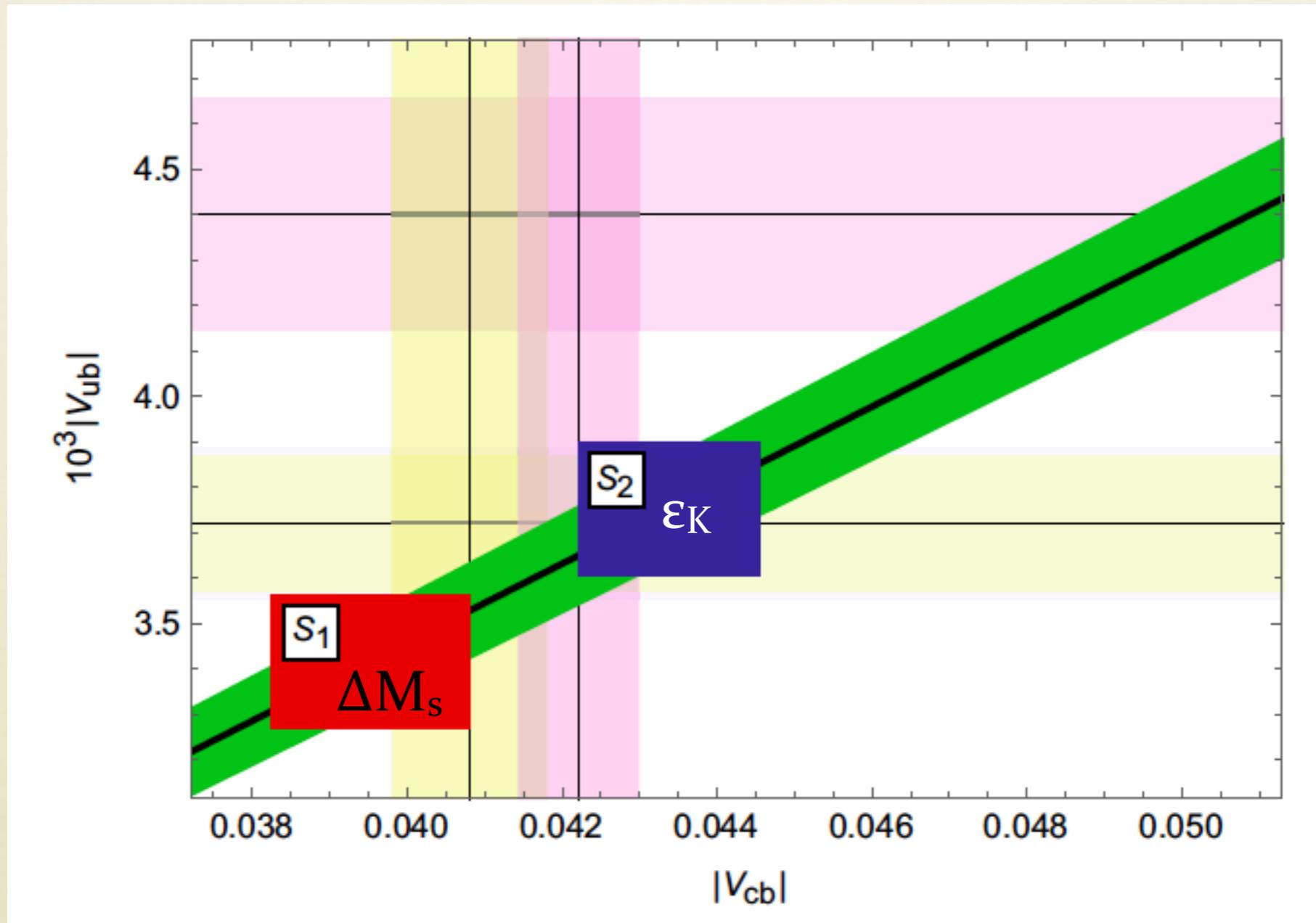
RH CURRENTS DON'T HELP

- ◆ Can ease $|V_{ub}|$ tension by allowing small right-handed contribution to Standard-Model weak current [Crivellin, PRD81 (2010) 031301]
- ◆ RH currents disfavored by Λ_b decays (taking $|V_{cb}|$ from $B \rightarrow D^* l \nu$ + HFAG to obtain $|V_{ub}|$)

$p=0.03$



UUT analysis in CMFV models



$$\left| \frac{V_{ub}}{V_{cb}} \right|_{CMFV} = 0.864 \pm 0.0025$$

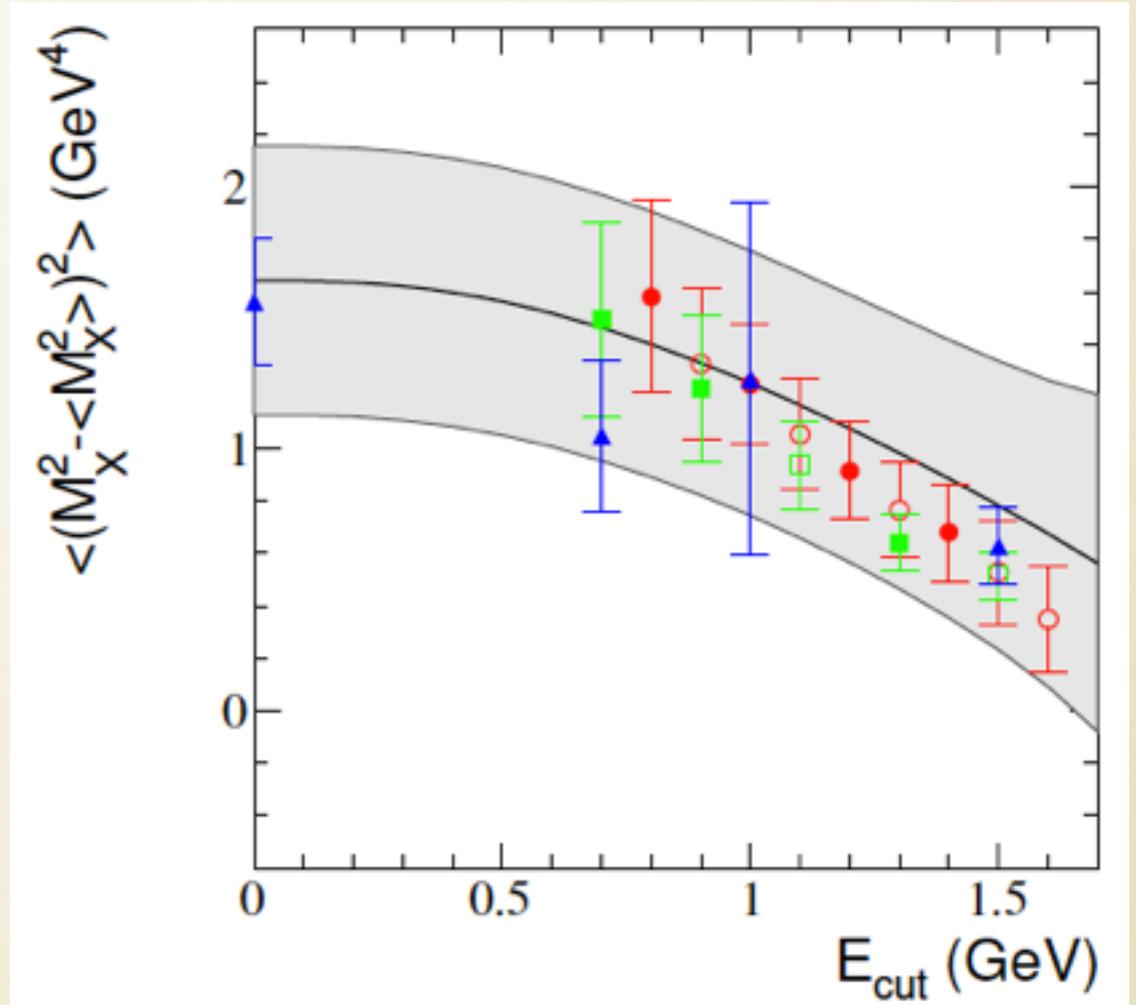
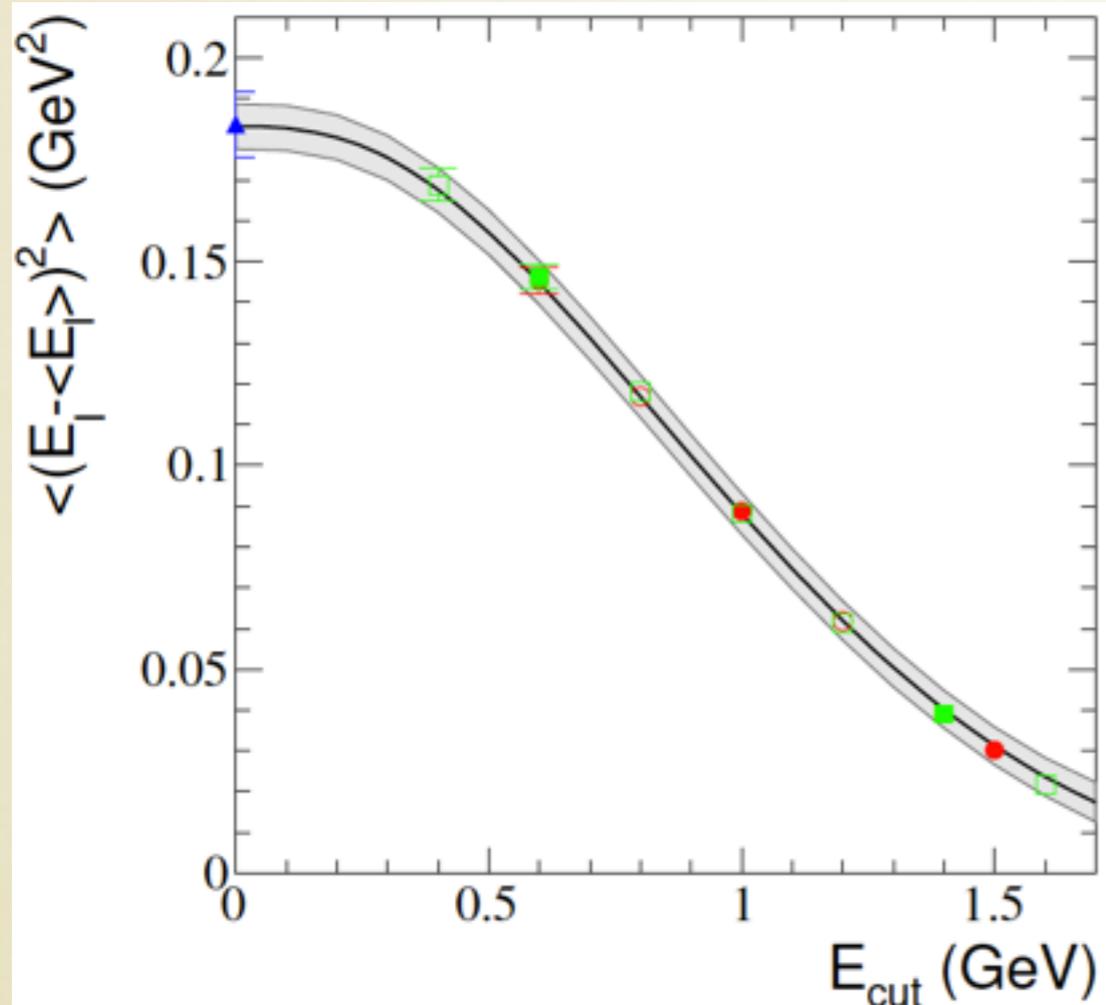
Blanke, Buras 1602.04020
see Buras' talk

SUMMARY

- Improvements of OPE approach to s.l. decays continue. $O(\alpha_s \Lambda^2/m_b^2)$ effects implemented. **No sign of inconsistency in this approach so far, competitive m_b determination.**
- Exclusive/incl. tension in V_{cb} remains (3σ , 8%) only in the D^* channel. **The D channel is becoming competitive and is compatible with both.** The remaining tension calls for new lattice analyses and new data (ongoing Belle analysis, Belle-II)
- Exclusive/incl tension in V_{ub} appears receding because of new FNAL/MILC and HPQCD results and of preliminary Babar results.
- New physics explanations less constrained for V_{ub} than for V_{cb} , but right handed current disfavoured. RH currents don't help.
- Belle-II will improve precision and allow for consistency checks of our methods, especially for inclusive V_{ub} . LHCb potential (for exclusives) greater than expected.

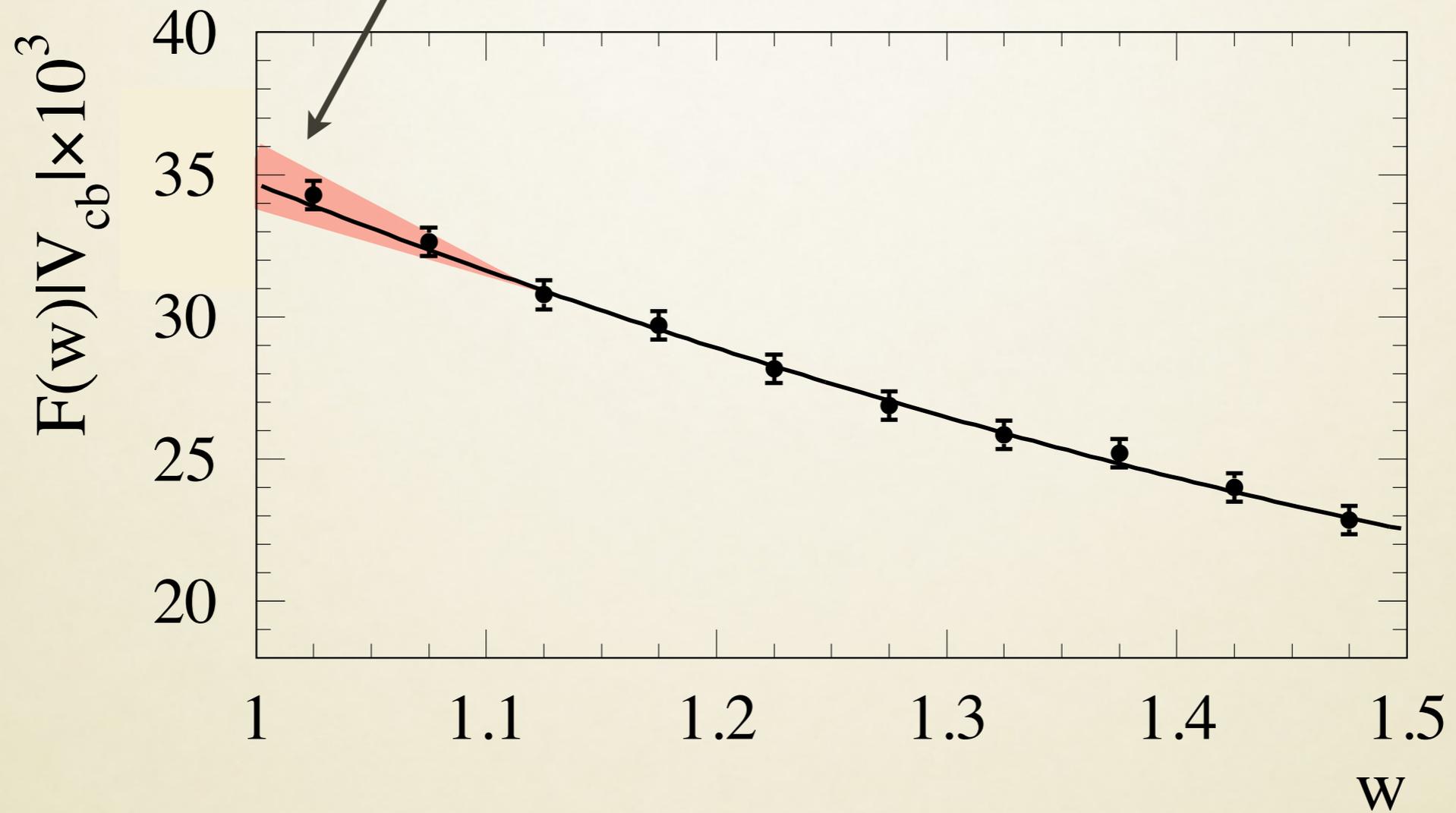
BACK-UP SLIDES

THEORETICAL ERRORS



Theoretical errors are generally the **dominant** ones in the fits. We estimate them in a **conservative** way, mimicking higher orders by varying the parameters by fixed amounts: $m_{c,b}$ 8MeV, $\alpha_s(m_b)$ 0.018, 7% in $1/m^2$ parameters, 30% in $1/m^3$ parameters
New corrections have been within theor. uncertainties so far.

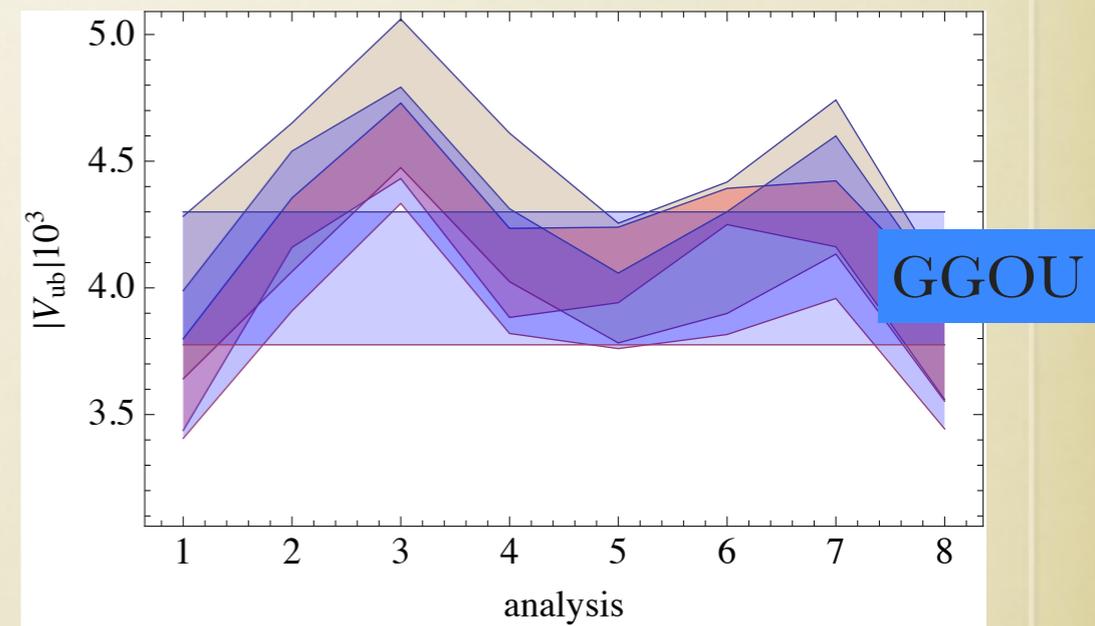
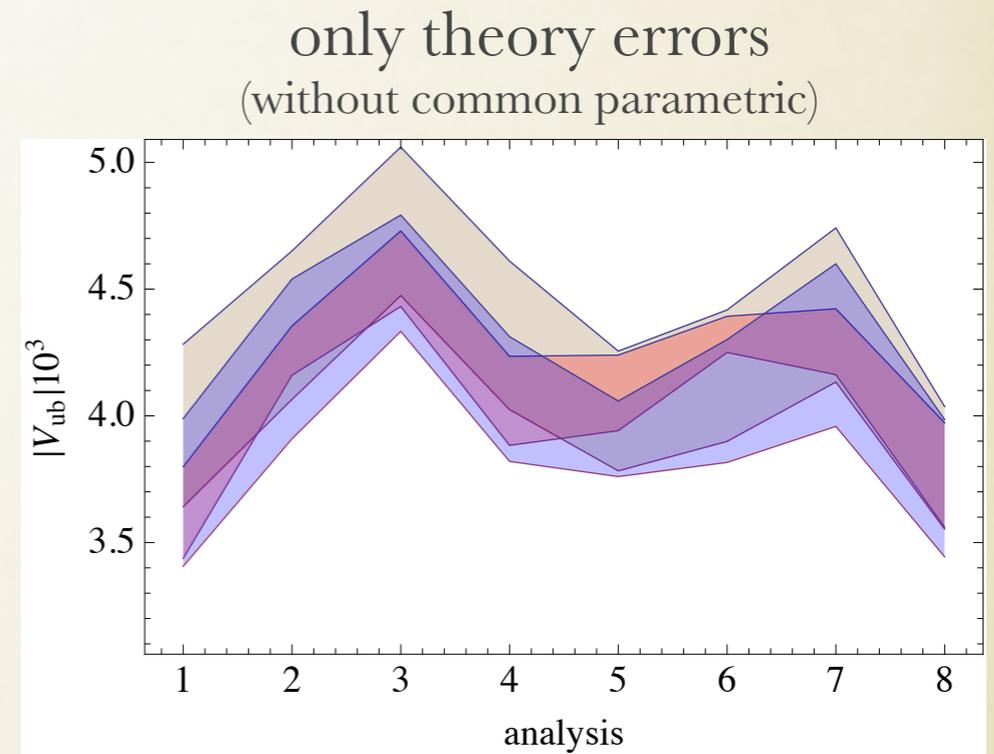
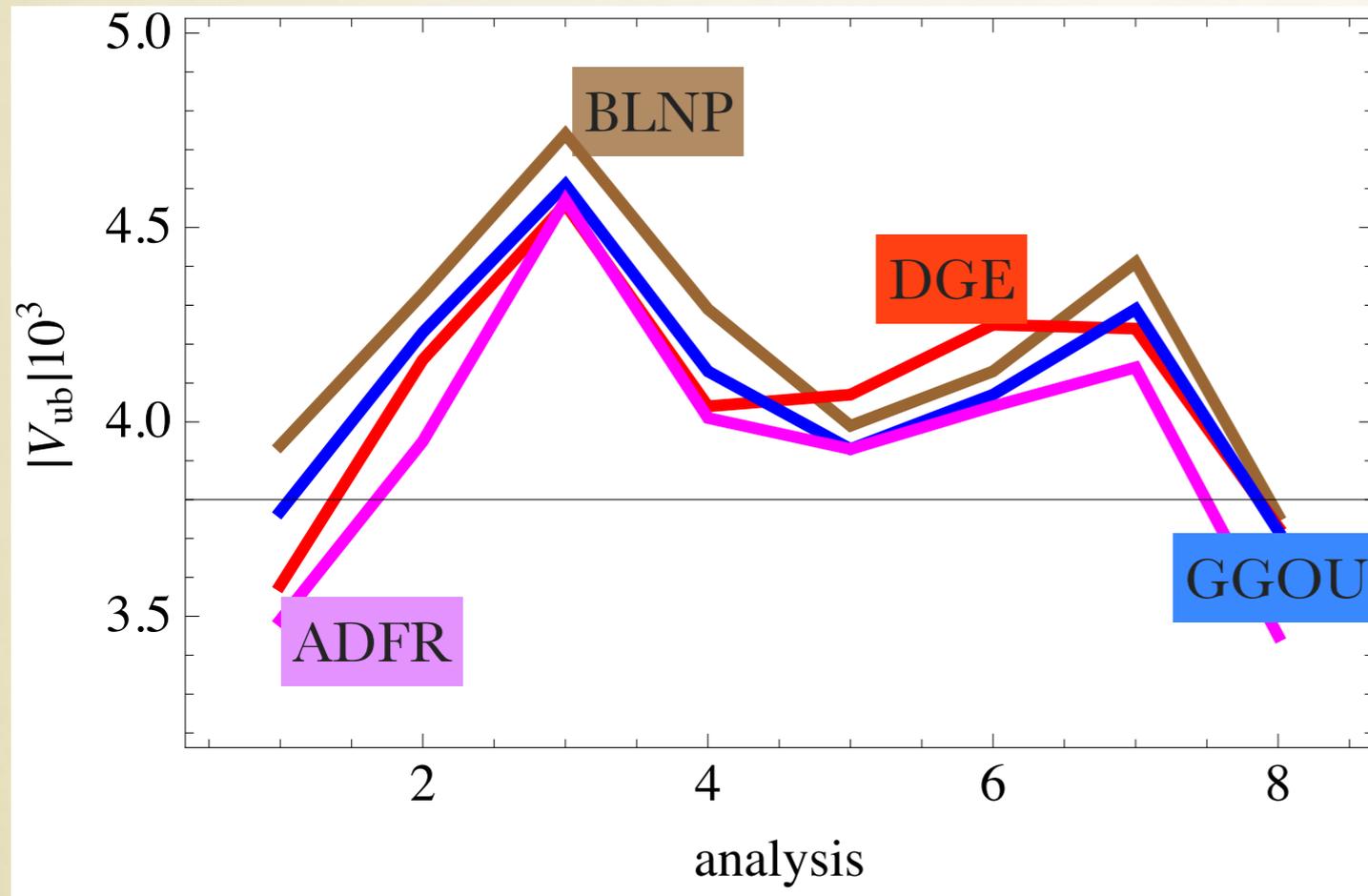
Extrapolation to zero recoil,
possible parameterization effect (qualitative picture)



Babar form factor shape from 0705.4008

A GLOBAL COMPARISON

0907.5386, Phys Rept



- * common inputs (except ADFR)
- * Overall good agreement **SPREAD WITHIN THEORY ERRORS**
- * NNLO BLNP still missing: will push it up a bit
- * Systematic offset of central values: normalization? to be investigated

HIGHER POWER CORRECTIONS

Mannel, Turczyk, Uraltsev 1009.4622

Proliferation of non-pert parameters and powers of $1/m_c$ starting $1/m^5$. At $1/m_b^4$

$$2M_B m_1 = \langle ((\vec{p})^2)^2 \rangle$$

$$2M_B m_2 = g^2 \langle \vec{E}^2 \rangle$$

$$2M_B m_3 = g^2 \langle \vec{B}^2 \rangle$$

$$2M_B m_4 = g \langle \vec{p} \cdot \text{rot } \vec{B} \rangle$$

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$

$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$

$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$

$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$

$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

can be estimated by **Lowest Lying State Saturation** approx by truncating

$$\langle B | O_1 O_2 | B \rangle = \sum_n \langle B | O_1 | n \rangle \langle n | O_2 | B \rangle$$

In LLSA *good convergence* of the HQE. First fit with $1/m^{4,5}$:

$$\frac{\delta V_{cb}}{V_{cb}} \simeq -0.35\% \quad \text{Healey, Turczyk, PG preliminary}$$

Heinonen, Mannel 1407.4384 have more systematic approach

LLSA might set the scale of effect, not yet clear *how much it depends on assumptions on expectation values*. Large corrections to LLSA have been found.

Mannel, Uraltsev, PG, 2012

Allowing 80% gaussian deviations from LLSA seem to leave V_{cb} unaffected.