Exotic Discoveries in Familiar Places

Theory of the Onia and Exotics

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Outline

1) The crucial importance of quarkonium

2) Discovery of the exotic hadrons X, Y, Z, P_c

3) How are the tetraquarks X, Y, Z and pentaquarks P_c assembled?

4) A new dynamical picture for the X, Y, Z, P_c

5) The present and the future

In 1974…

J.J. Aubert *et al.*, Phys. Rev. Lett. **33** (1974) 1404;
J.-E. Augustin et al., Phys. Rev. Lett. **33** (1974) 1406

- Our mentors still call it the *November Revolution*:
  Resonance in e^+e^- of mass 3.1 GeV and tiny width discovered
- J/ψ, the 1S state of charmonium $(c\bar{c})$

- Before 1974, the idea of confined quarks interacting via QCD—years after the deep-inelastic scattering experiments—was still not universally accepted

- What changed? Largely, the fact that the new charm quark was heavy enough to climb out of the QCD brown muck $(m_c \gg \Lambda_{\mathrm{QCD}})$ to allow for a simple quantum-mechanical rather than complicated quantum field theory picture

Quarkonium Spectroscopy

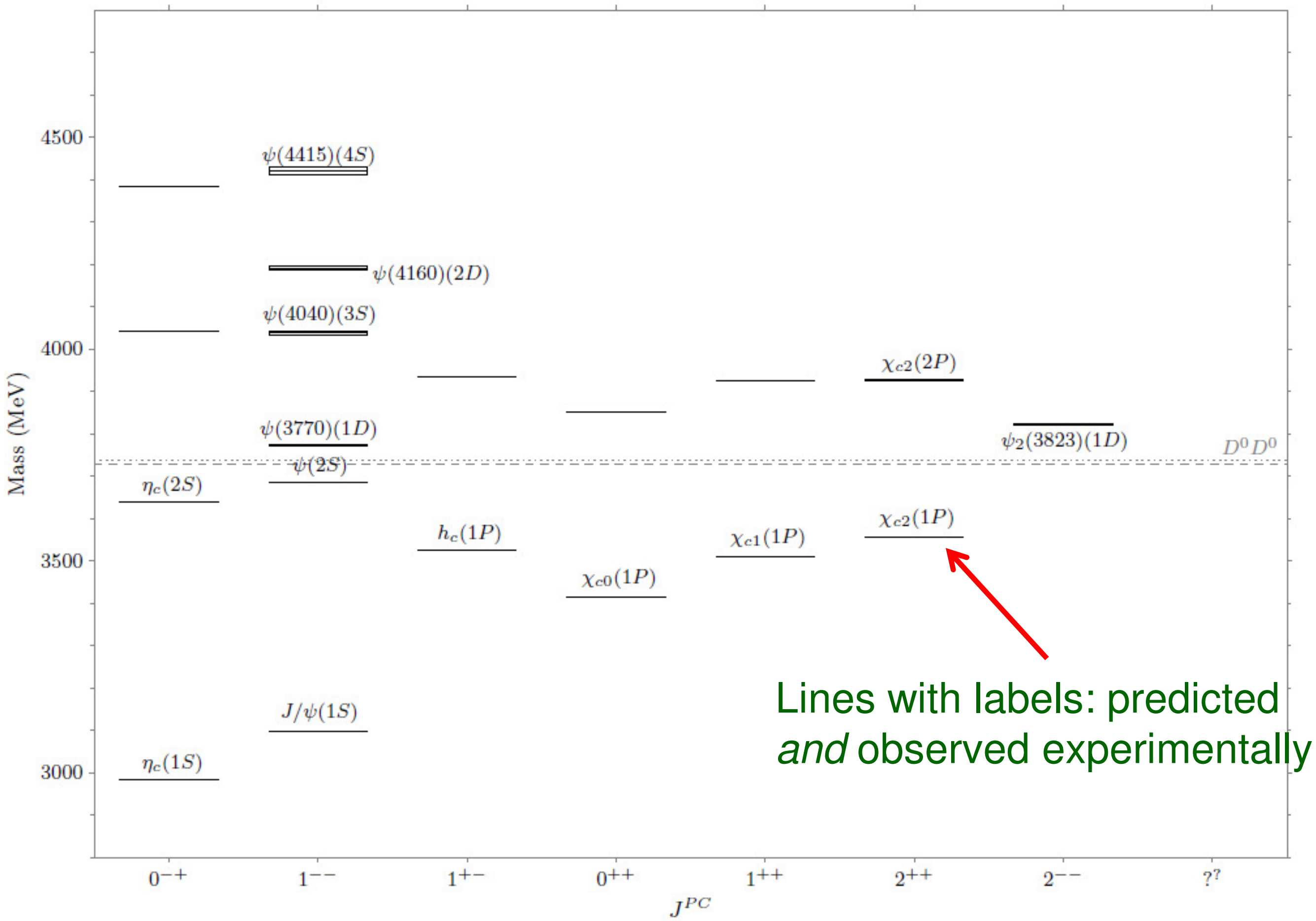
- $m_c \gg \Lambda_{\text{QCD}}$: The *c* quark is heavy enough to use *asymptotic freedom* for understanding the quark binding: $\alpha_s(q^2 = m_c^2) \approx 0.3$, rather than $O(1)$ for *u*, *d*, *s*
- The $c, \bar{c}$ can be treated as nonrelativistic (and of course *a fortiori* for *b* quarks)
- Confinement: represented by monotonically increasing potential *e.g.*, linear dependence to model this *string* or *color flux tube*
- So model the strong "Coulomb" and confining potentials:

$$V(r) = -\frac{k\alpha_s}{r} + br,$$ the "***Cornell potential***"

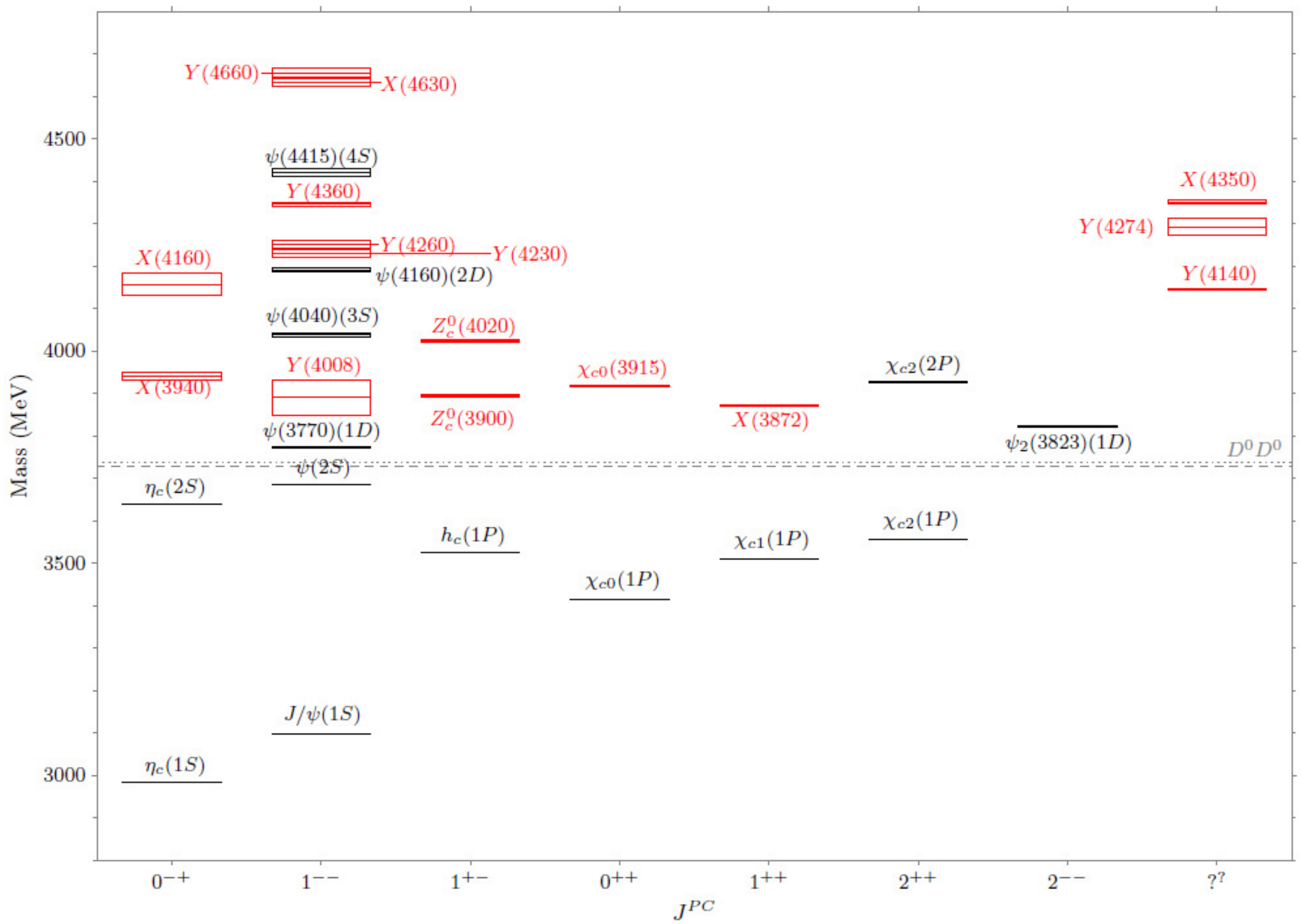
[E. Eichten *et al.*, PRD **17** (1978) 3090; **21** (1980) 203],

and feed it into $-\frac{\hbar^2}{2\mu}\nabla^2\psi(\boldsymbol{r}) + V(\boldsymbol{r})\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r})$

What the Charmonium System Should Look Like



What the Charmonium System Really Looks Like
Neutral Sector, May 2016



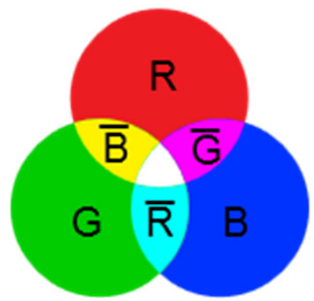
Exotics?

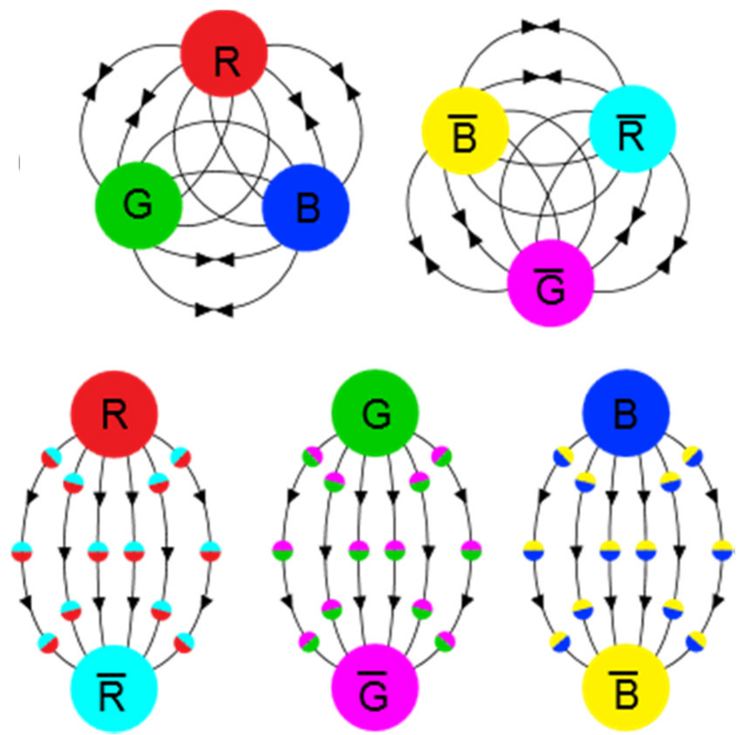
- Yes, unless you decide we *really* don't understand charmonium at all
- But the Cornell potential (and other models) predict not only a spectrum of eigenvalues, but the wave functions as well, which have been rather successful in predicting decay modes and branching fractions
- And, we'll see that *charged* charmoniumlike states have been seen, which cannot be just $c\bar{c}$

What kinds of exotics are possible?

Mathematically, all SU(3) color singlets (required by confinement) can be broken down to the mesonic ($r\bar{r}$+$b\bar{b}$+$g\bar{g}$) and baryonic (rbg) types

gg, ggg, … (*glueball*)
$q\bar{q}g$, $q\bar{q}gg$, … (*hybrid meson*)
$q\bar{q}q\bar{q}$, $q\bar{q}q\bar{q}q\bar{q}$, …
(*tetraquark*, *hexaquark*, …)
$qqqq\bar{q}$, $qqqqqqq\bar{q}$, …
(*pentaquark*, *octoquark*, …)





i.e., (# of q) – (# of $\bar{q}$) = 0 mod 3
& any number of g except one by itself

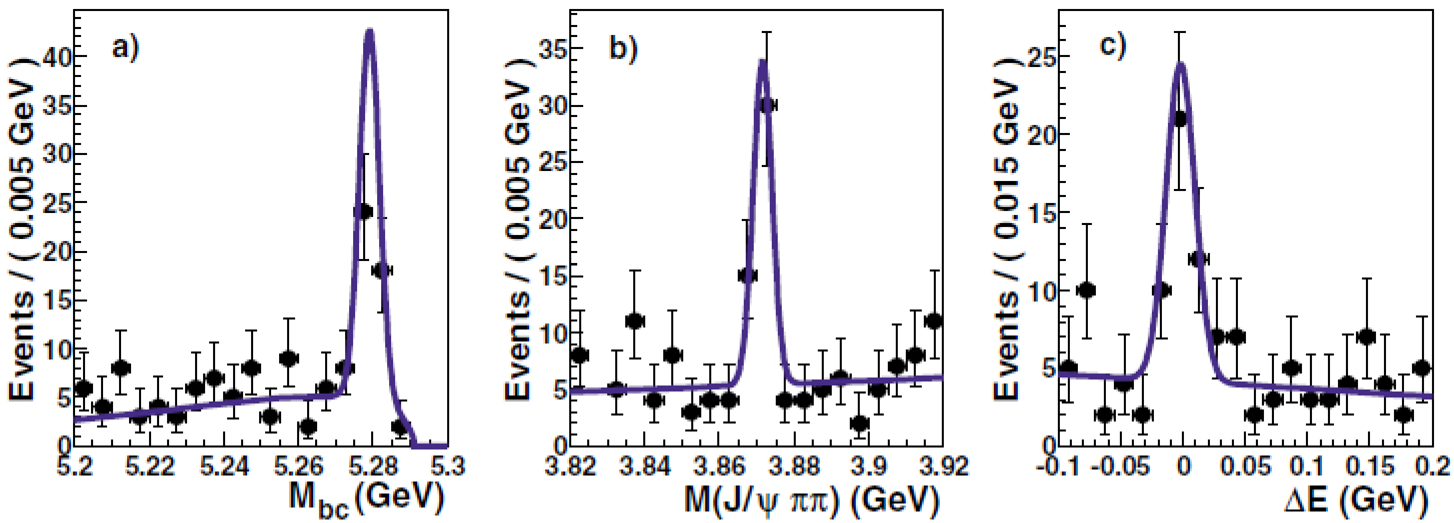
Why No Unambiguous Exotics in the Light Sector?

- Easier for exotics made of *u*, *d*, *s* to "hide'' amongst ordinary hadrons of the same quantum numbers by mixing
- Mesons with $J^{PC} = 1^{-+}$, not allowed for $q\bar{q}$ $[\pi_1(1400), \pi_1(1600)]$ have been observed, but the partial-wave analysis to disentangle from other effects in that energy range is complicated
- A seemingly strong signal for a new particle, even one confirmed by multiple experiments, can turn out to be due to entirely different physics
  - *e.g.*, in the early 2000's, a famous pentaquark candidate $\Theta^+(1540)$ turned out not to be an *s*-channel *K*-*N* compound resonance, but the result of an unfortunate choice of kinematical *cuts* on the data and *t*-channel exchanges
- …So when the breakthrough finally came in **2003**, it was not instantly accepted by everyone

In 2003…

The Belle Collaboration found evidence for a new particle at mass 3872 MeV

S.K. Choi *et al.*, Phys. Rev. Lett. **91** (2003) 262001



Reminder: Their primary physics goal was to find *CP* violation in the neutral *B* sector

X = Unknown

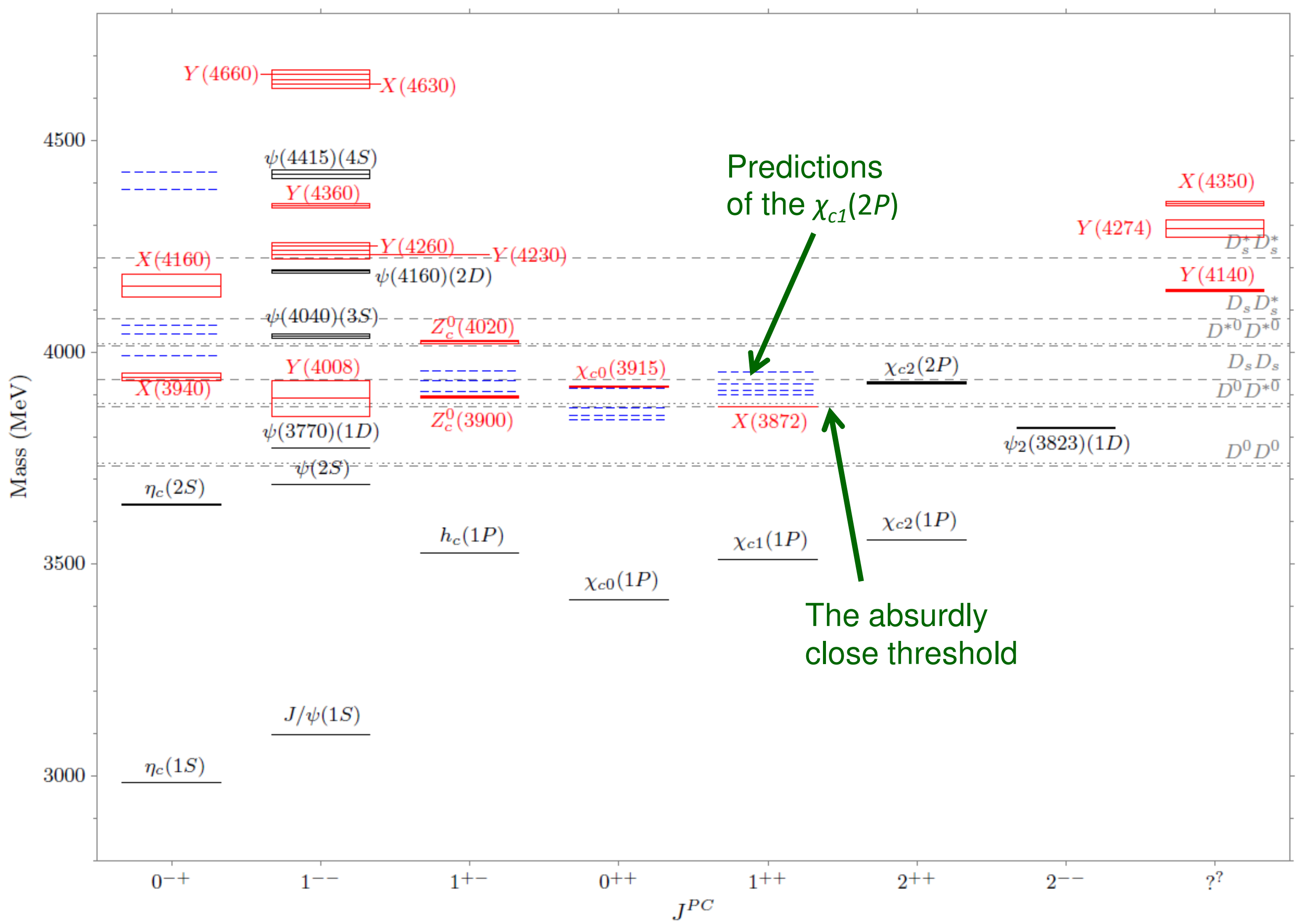
- Belle found a new charmoniumlike resonance appearing in

$$B \to K\,(J/\psi\,\pi^+\pi^-)$$

- Has been confirmed at BABAR, CDF, DØ, LHCb, CMS
- $J^{PC} = 1^{++}$, but not believed to be ordinary $c\bar{c}$: Mass is many 10's of MeV below nearest $\bar{c}c$ candidate with these quantum numbers, $\chi_{c1}(2P)$

- Now called $\boldsymbol{X(3872)}$ [and believed to be a ($c\bar{c}u\bar{u}$) state]
  - $m_{X(3872)} = 3871.69 \pm 0.17$ MeV
  - Width: $\Gamma_{X(3872)} < 1.2$ MeV
  - Note: $m_{X(3872)} - m_{D^{*0}} - m_{D^0} = -0.11 \pm 0.21$ MeV
    Leads to endless speculation that *X*(3872) is a $D\bar{D}^*$ hadronic molecule

The Peculiar *X*(3872), the first *tetraquark*



…And in 2005: *Y*

BABAR Collaboration (B. Aubert *et al.*, PRL **95**, 142001 [2005])

Charmoniumlike states started to show up
in initial-state radiation (ISR) e^+e^- annihilation:

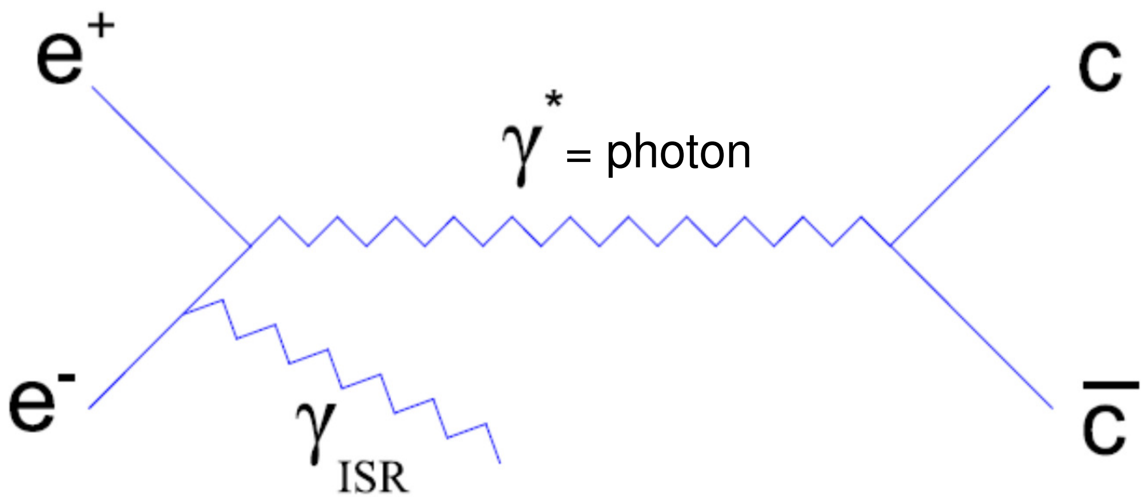


Figure from Nielsen *et al.*,
Phys. Rept. **497** (2010) 41

Such states necessarily have $J^{PC} = 1^{--}$
(same quantum numbers as the photon), and are called "*Y*"

This first-discovered one is named ***Y*(4260)**

…And in 2013: *Z*

BESIII Collaboration [Beijing] (M. Ablikim *et al.*, PRL **110**, 252001 [2013]),
Belle Collaboration (Z. Liu *et al.*, PRL **110**, 252002 [2013])

- A charged charmoniumlike resonance is observed in
$$Y(4260) \to \pi^-(\pi^+ J/\psi)$$
- Minimal possible flavor content: $c\bar{c}u\bar{d}$:
No question that it has four valence quarks
- Now called $\boldsymbol{Z_c^+}$**(3900)**, $J^P = 1^+$
- *The first manifestly exotic state ever confirmed beyond 5σ by two experiments*

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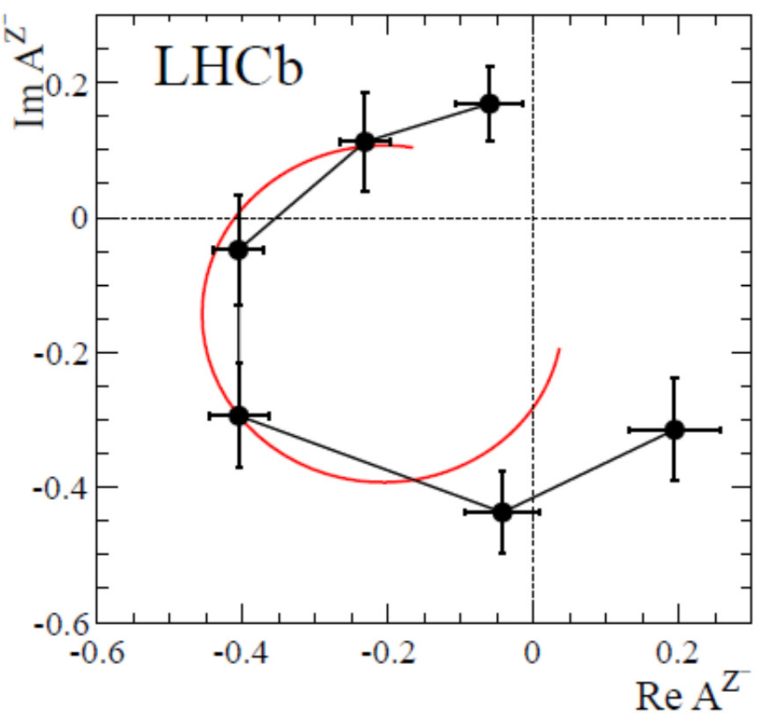
  [not counting the $\Theta^+(1540)$]
- What if all these states are not really states, but rather brilliant forgeries, like the $\Theta^+(1540)$?

…And in 2014: **Resonance**

LHCb Collaboration (R. Aaij *et al.*, PRL **112**, 222002 [2014])

- The first charged charmoniumlike exotic was actually first seen by Belle in 2008 (PRL **100**, 142001 [2008]) and confirmed by them in papers from 2009 and 2013
- LHCb not only confirmed the state at 13.9σ, now called ***Z*⁺(4430)**, $J^P = 1^+$ but for the first time plotted the full complex production amplitude and showed that it obeys the proper phase-shift looping behavior of a Breit-Wigner **resonance**

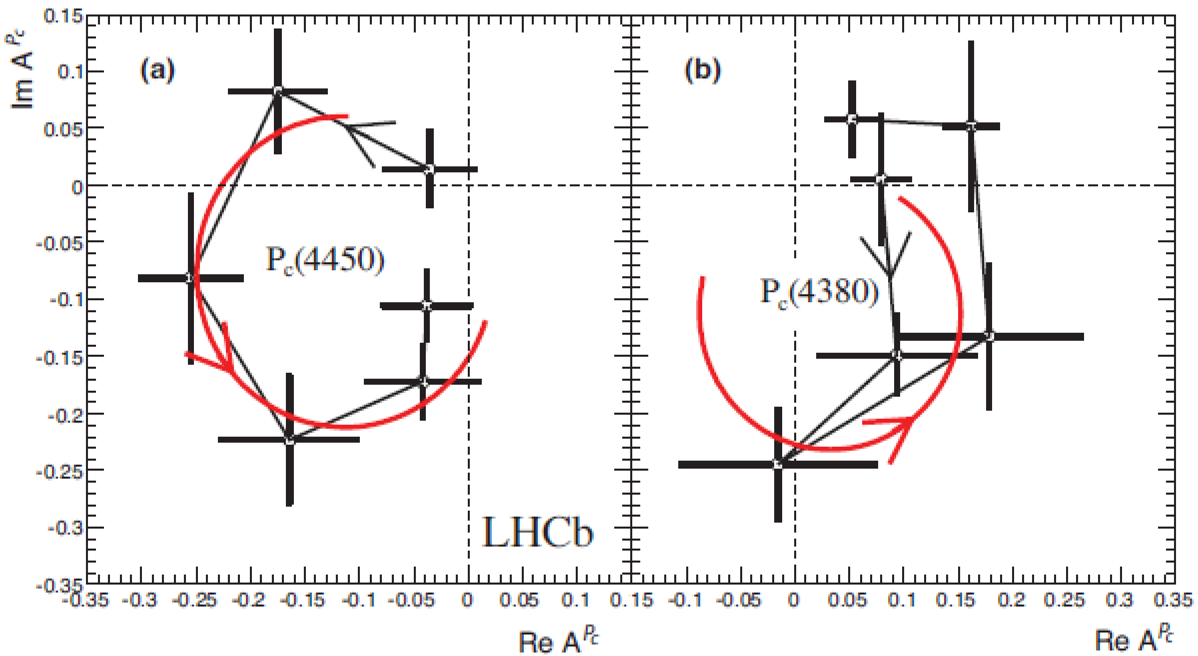


- **Welcome to the Age of the Third Hadron**

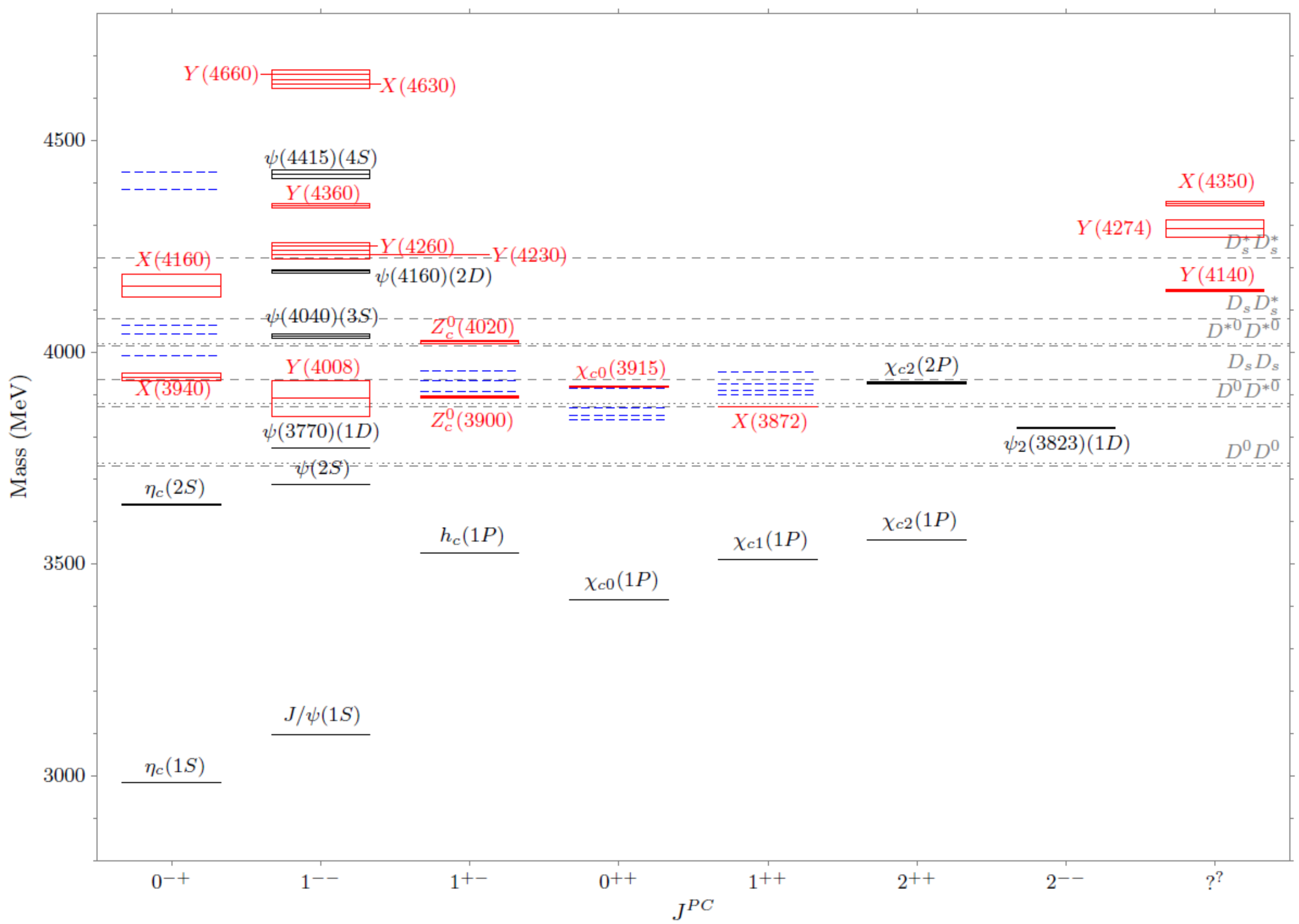
…And in 2015: P_c

LHCb Collaboration [R. Aaij *et al.*, PRL **115** (2015) 072001]

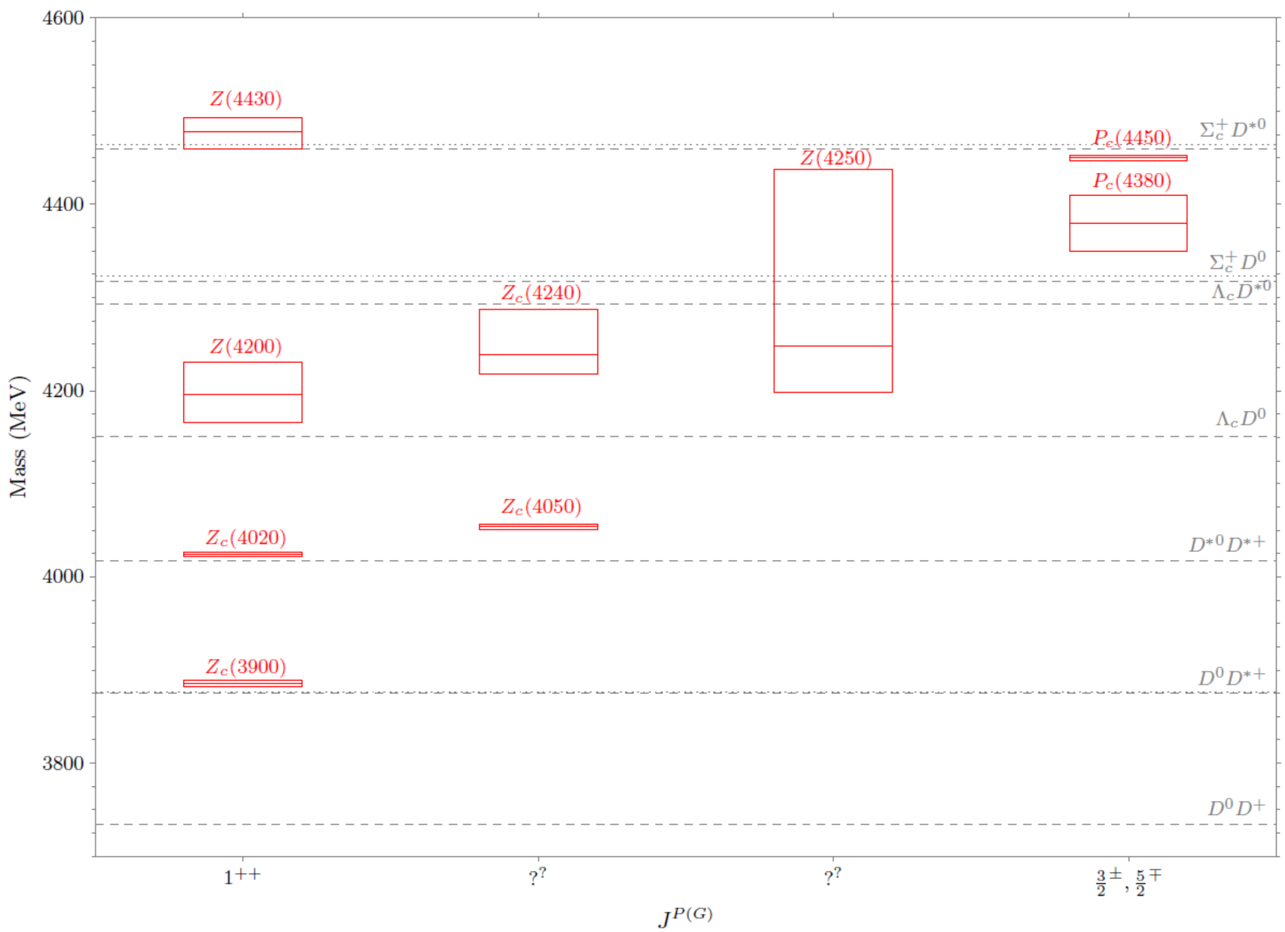
- The first two *baryonic* charmoniumlike exotics, $P_c^+(4450)$, $P_c^+(4380)$
- Decay to $J/\psi + p \longrightarrow$Valence structure $c\bar{c}uud$: **Pentaquarks**!
- $m_1 = 4380 \quad \pm 8 \quad \pm 29$ MeV, $\Gamma_1 = 205 \pm 18 \pm 86$ MeV, **9σ significance**
- $m_2 = 4449.8 \pm 1.7 \pm 2.5$ MeV, $\Gamma_2 = \ 39 \ \pm 5 \pm \ 19$ MeV, **12σ significance**
- Preferred J^P assignments:
  $(3/2^-, 5/2^+) >$
  $(3/2^+, 5/2^-) >$
  $(5/2^+, 3/2^-)$
- **Welcome to the Age of the Fourth Hadron**



Charmonium: May 2016
Neutral sector

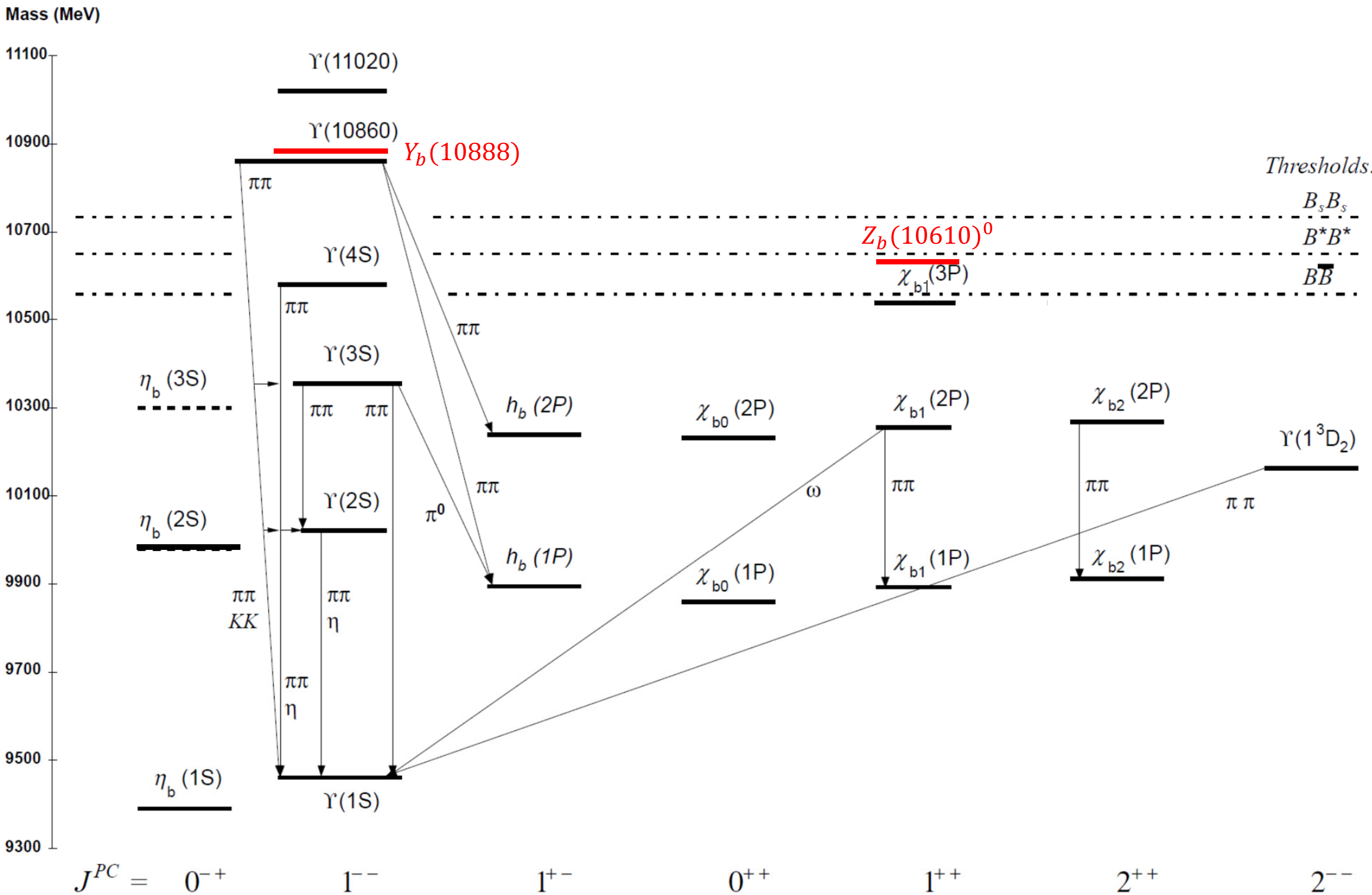


Charmonium: May 2016
Charged sector



THE BOTTOMONIUM SYSTEM

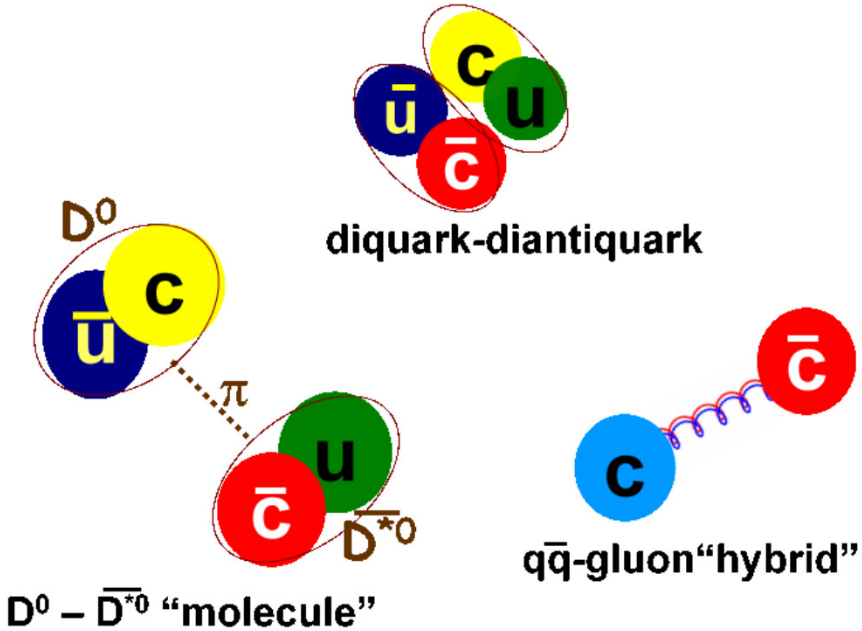
from the Particle Data Group, http://pdglive.lbl.gov/

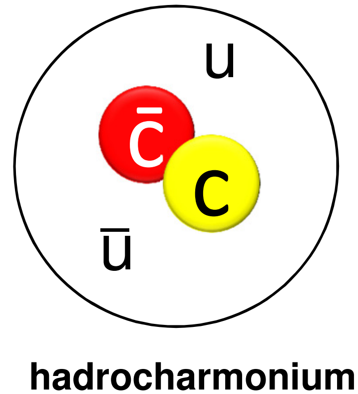


The exotics scorecard: May 2016

- **29** observed exotics
  - 24 in the charmonium sector
  - 4 in the (much less explored) bottomonium sector
  - 1 with a single *b* quark (and an *s*, a *u*, and a *d*)
- **12** confirmed (& at most 1 of the other 17 disproved)

How are tetraquarks assembled?





cusp effect:
Resonance created by rapid opening of meson-meson threshold

Image from Godfrey & Olsen,
Ann. Rev. Nucl. Part. Sci. **58** (2008) 51

Trouble with the dynamical pictures

- Hybrids
  - Only usable for neutral states; then what are the *Z*'s?
  - Only produces certain quantum numbers (like $J^{PC} = 1^{++}$) easily
- Diquark and hadrocharmonium pictures
  - What stabilizes the states against instantly segregating into meson pairs?
  - Diquark models tend to overpredict the number of bound states
  - Why wouldn't hadrocharmonium *always* decay into charmonium, instead of $D\bar{D}$?
- Cusp effect
  - Might be able to generate some resonances on its own, but >20 of them? And certainly not ones as narrow as $X(3872)$ ($\Gamma < 1.2$ MeV)

The hadron molecular picture

- A number of *XYZ* states are *suspiciously* close to hadron thresholds
  - *e.g.*, recall $m_{X(3872)} - m_{D^{*0}} - m_{D^0} = -0.11 \pm 0.21 \text{ MeV}$
- So we theorists have *hundreds* of papers analyzing the *XYZ* states as dimeson molecules
- But not all of them are!
  - *e.g.*, *Z*(4430) is a prime example
- Moreover, some *XYZ* states lie slightly *above* a hadronic threshold
  - *e.g.*, Y(4260) lies about 30 MeV above the $D_s^* \overline{D_s^*}$ threshold
  - How can one have a bound state with *positive* binding energy?

Prompt production

- If hadronic molecules are really formed, they must be very weakly bound, with very low relative momentum between their mesonic components
- They might appear in *B* decays, but would almost always be blown apart in collider experiments
- But CDF & CMS saw many!
  [Prompt *X*(3872) production, $\sigma \approx 30$ nb]
  - CDF Collaboration (A. Abulencia *et al.*), PRL **98**, 132002 (2007)
  - CMS Collaboration (S. Chatrchyan *et al.*), JHEP **1304**, 154 (2013)

➢ Hadronic molecules may exist, but it is not easy to get *X*(3872) to fit this profile

It is entirely possible…

- …that no single structure accommodates all of these exotic states
- Some could be molecules, some could be hybrids, some could be kinematical effects, or a mixture
- But what none of these pictures take into account is the full complexity of QCD dynamics for rather short-lived states
- Here, then, is my suggestion:

Amazing (well-known) fact about color:

- The short-distance color attraction of combining two color-$\mathbf{3}$ quarks ($\mathbf{3}$ = red, blue, green) into a color-$\overline{\mathbf{3}}$ diquark is *fully half as strong* as that of combining a $\mathbf{3}$ and a $\overline{\mathbf{3}}$ into a color-neutral singlet (*i.e.*, **diquark attraction** is nearly as strong as the **confining attraction**)

- Just as one computes a spin-spin coupling,
$$\overrightarrow{s_1} \cdot \overrightarrow{s_2} = \frac{1}{2}\left[(\overrightarrow{s_1} + \overrightarrow{s_2})^2 - \overrightarrow{s_1}^2 - \overrightarrow{s_2}^2\right],$$
from two particles in representations 1 and 2 combined into representation 1+2:

- If s_1, s_2 = spin $\frac{1}{2}$, and $\overrightarrow{s_1} + \overrightarrow{s_2}$ = spin 0, get $-\frac{3}{4}$; if spin 1, get $+\frac{1}{4}$

- The exact analogue formula for color charges gives the result stated above

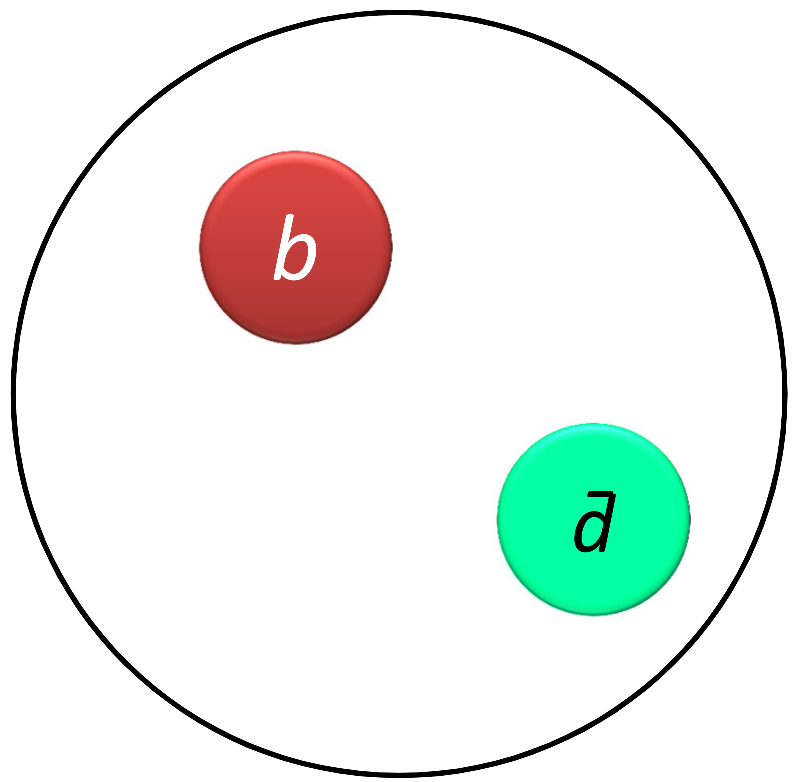
A new tetraquark picture

Stanley J. Brodsky, Dae Sung Hwang, RFL

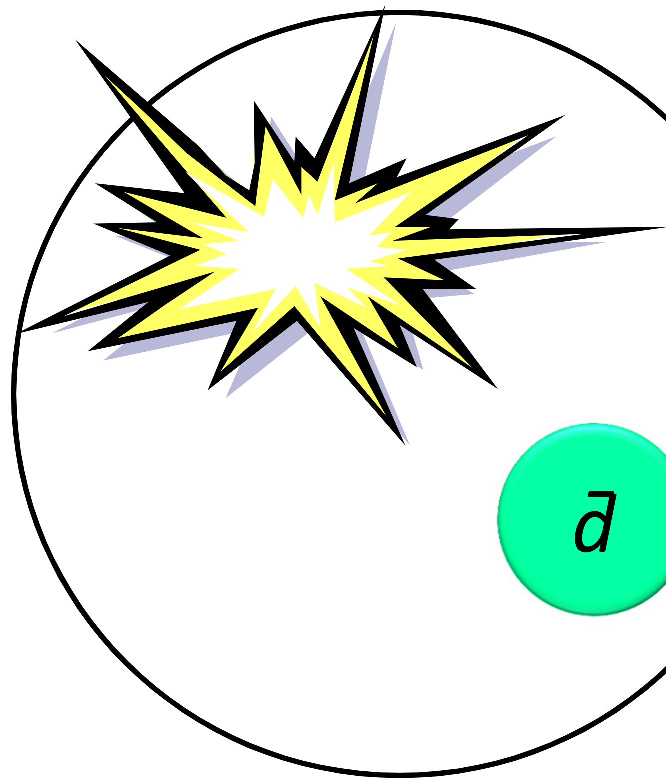
Physical Review Letters **113**, 112001 (2014)

- CLAIM: At least some of the observed tetraquark states are bound states of diquark-antidiquark pairs
- BUT the pairs are not in a static configuration; they are created with a lot of relative energy, and rapidly separate from each other
- Diquarks are not color neutral! They cannot, by confinement, separate asymptotically far
- They must hadronize via large-*r* tails of mesonic wave functions, which suppresses decay widths to make them observably narrow

Nonleptonic $\bar{B}^0$ meson decay



Nonleptonic $\overline{B}^0$ meson decay



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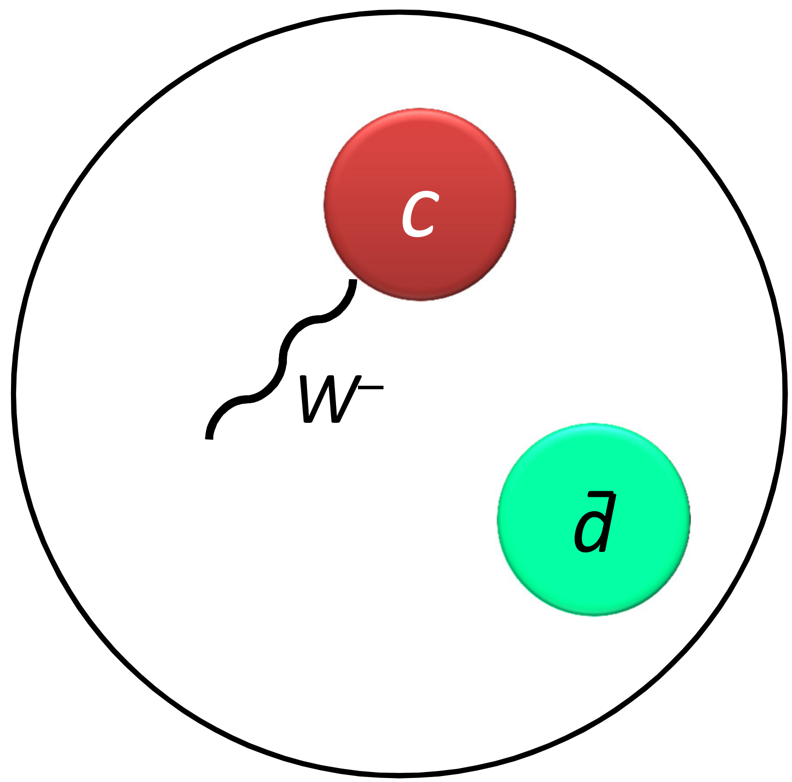

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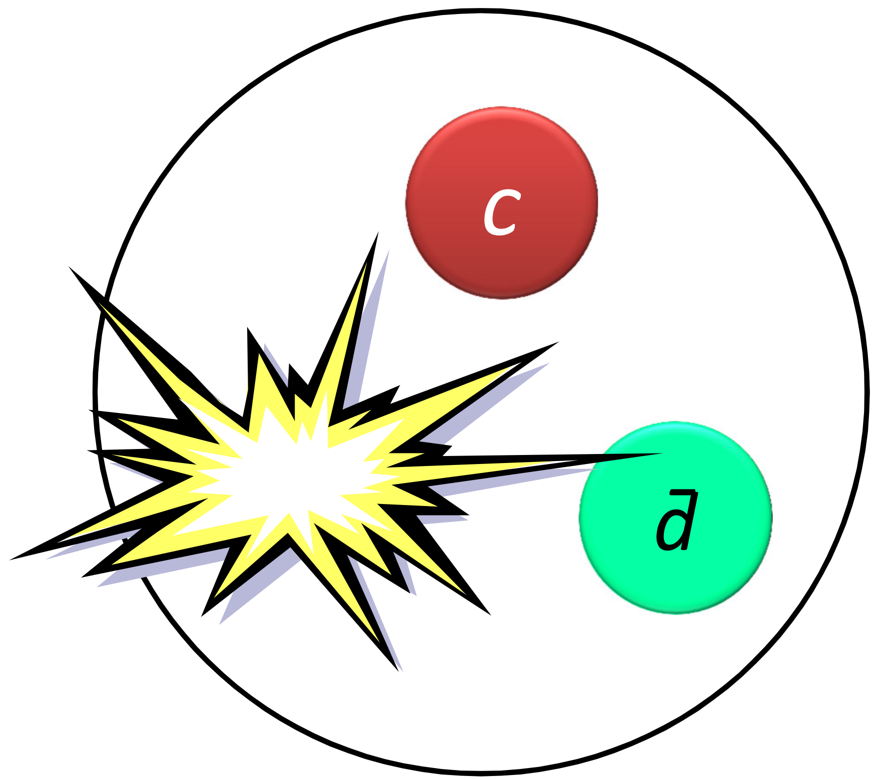
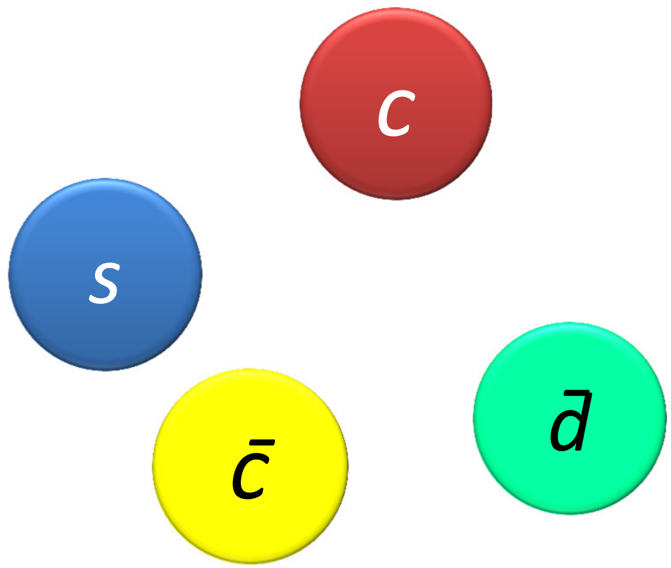


Nonleptonic $\bar{B}^0$ meson decay

B.R.~22%

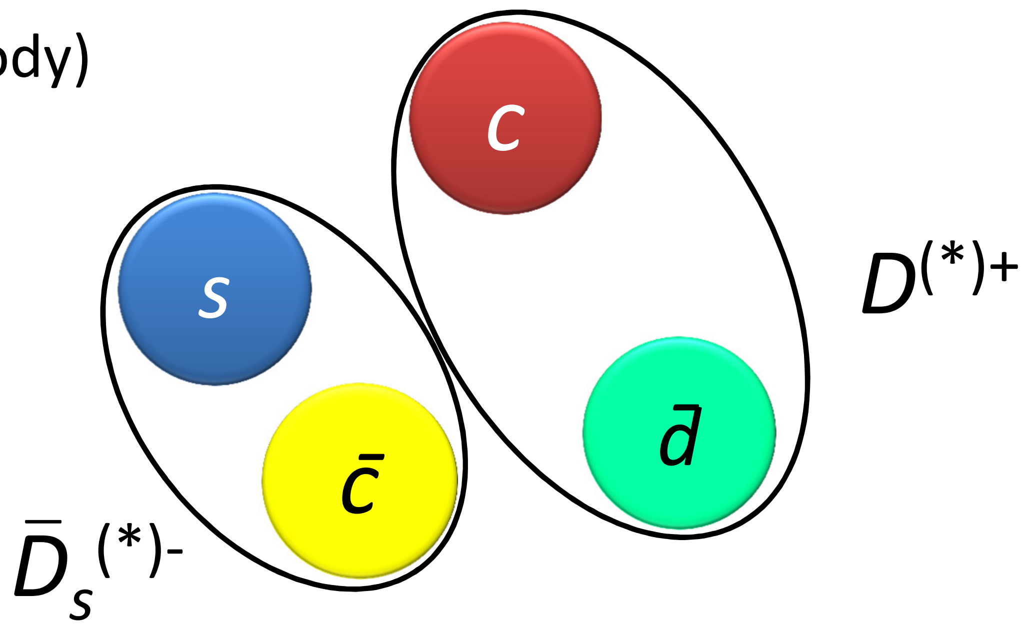
(Branching Ratio = probability)



What happens next?
Option: Color-allowed

B.R.~5%

(& similar 2-body)



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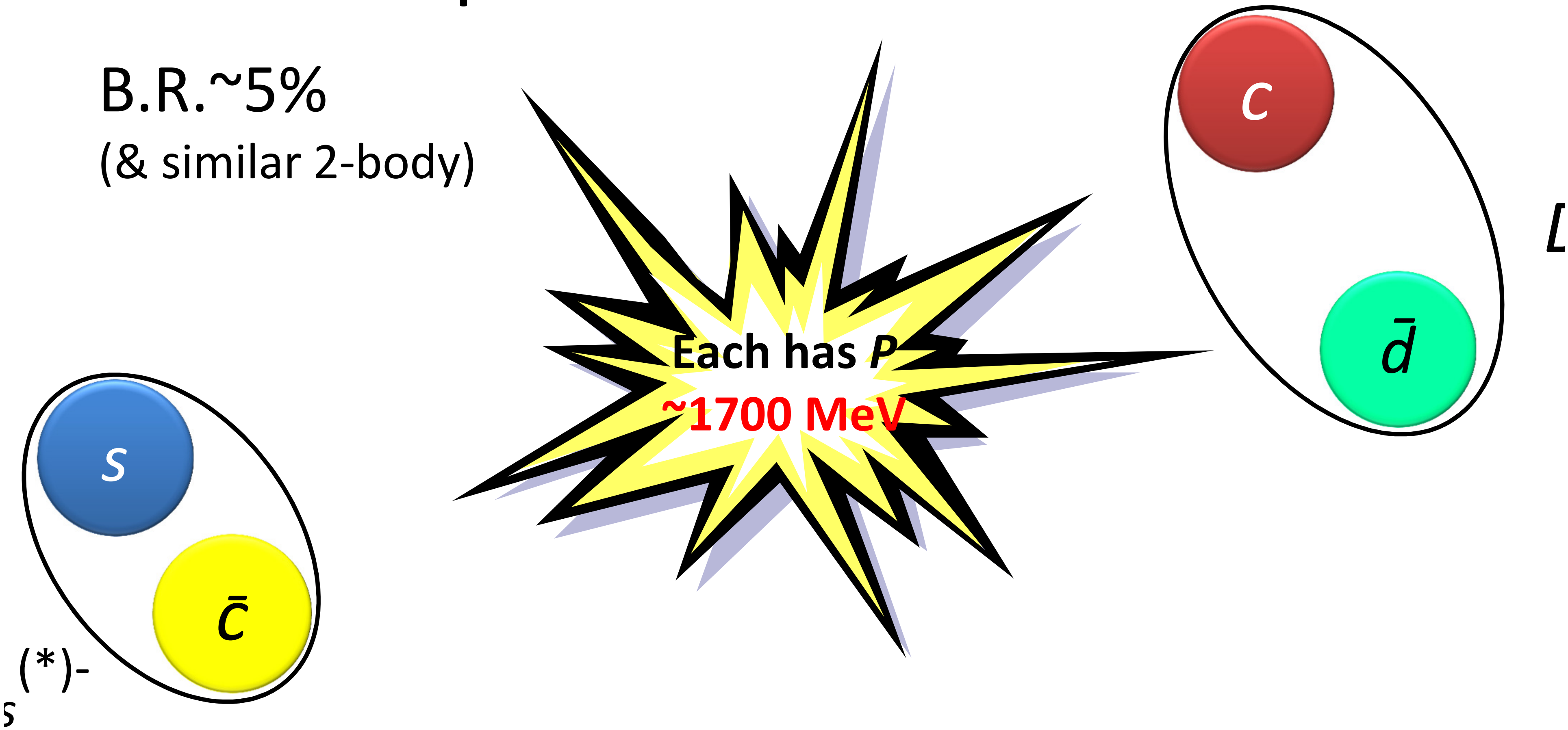
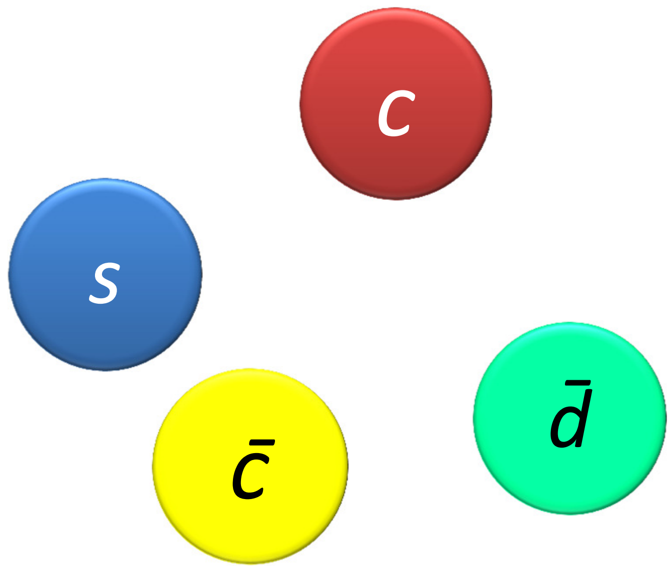


What happens next?
Option: Diquark formation



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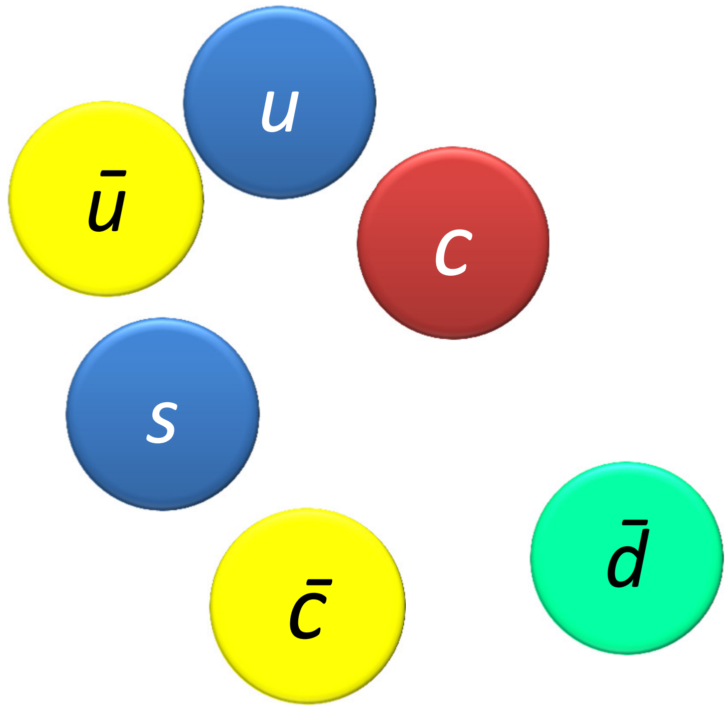

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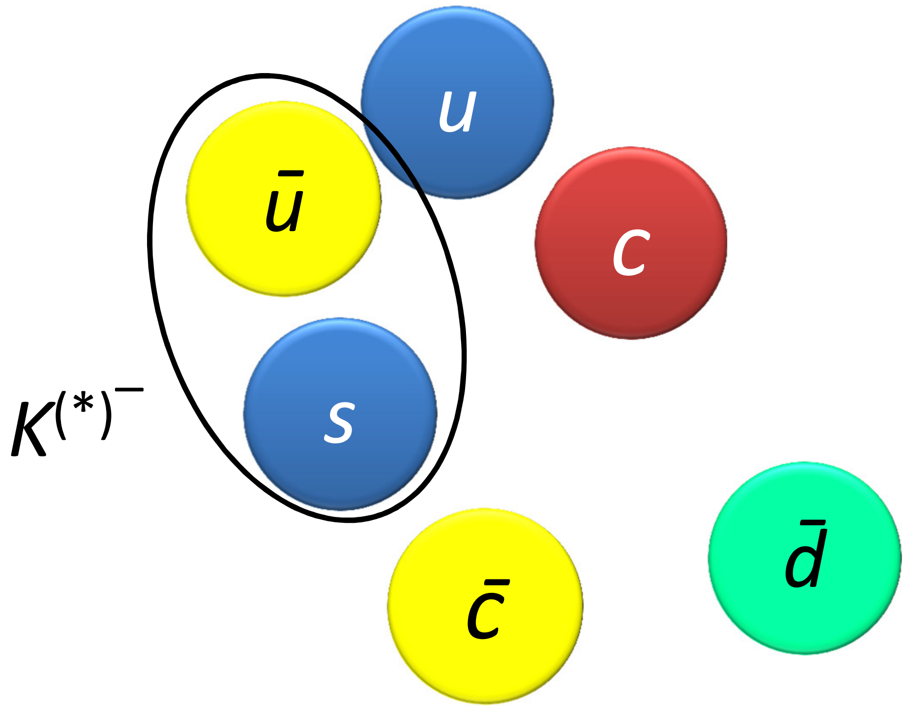

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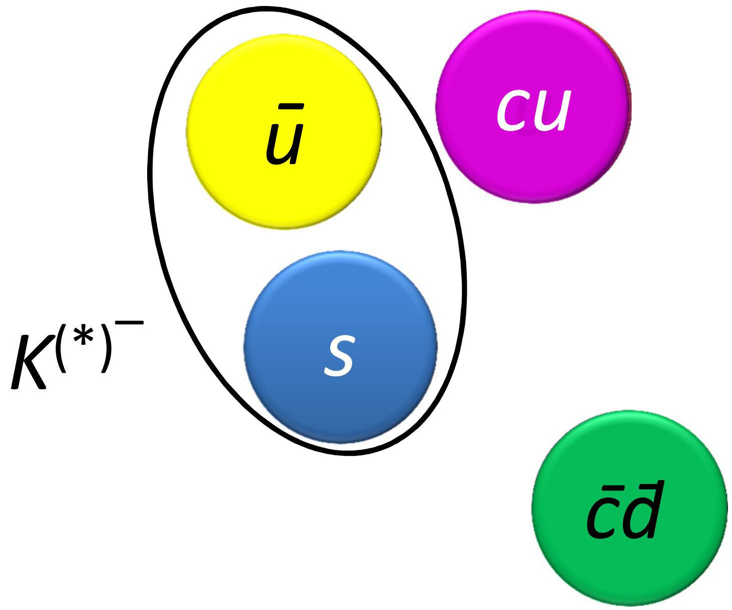

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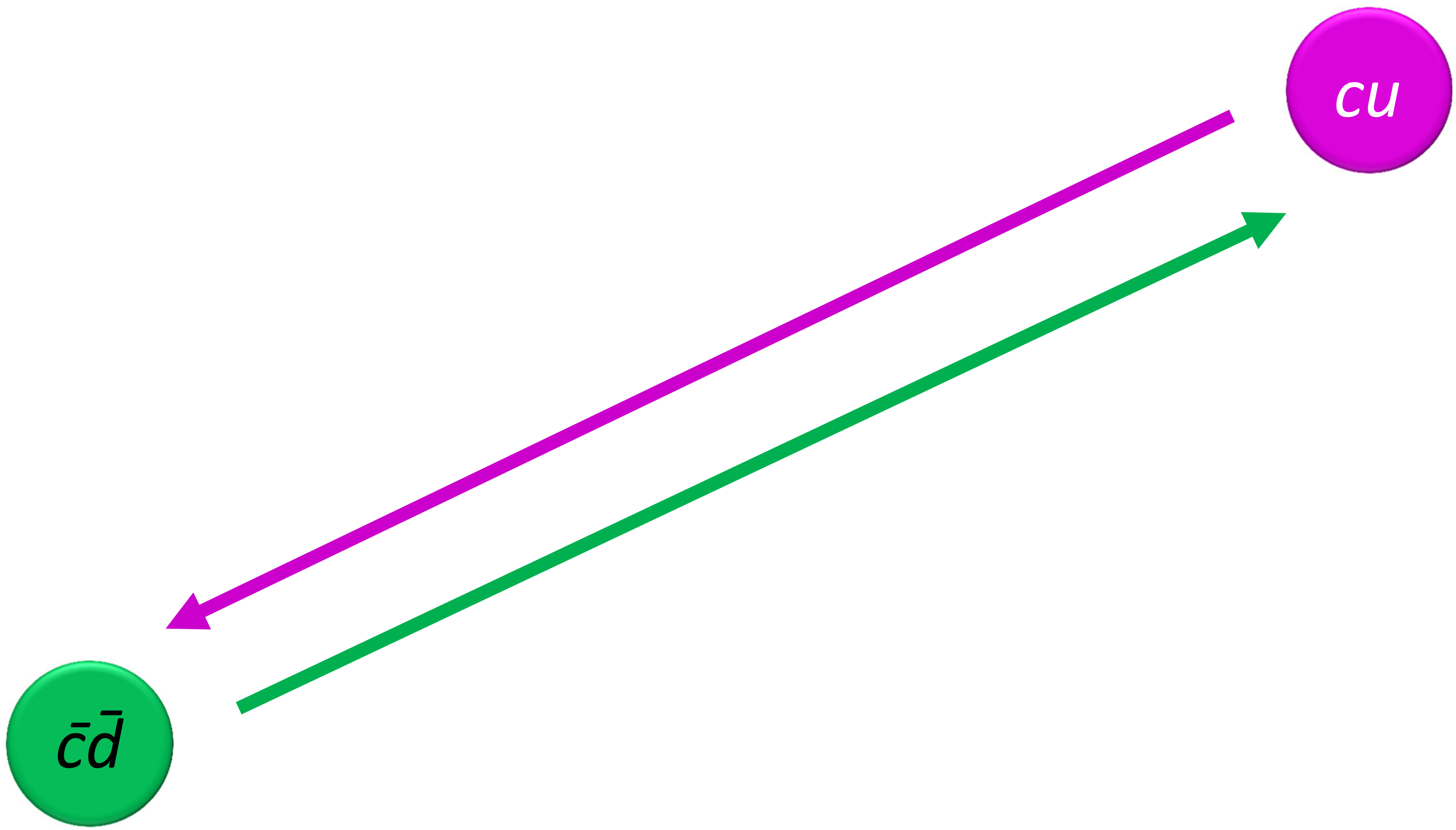
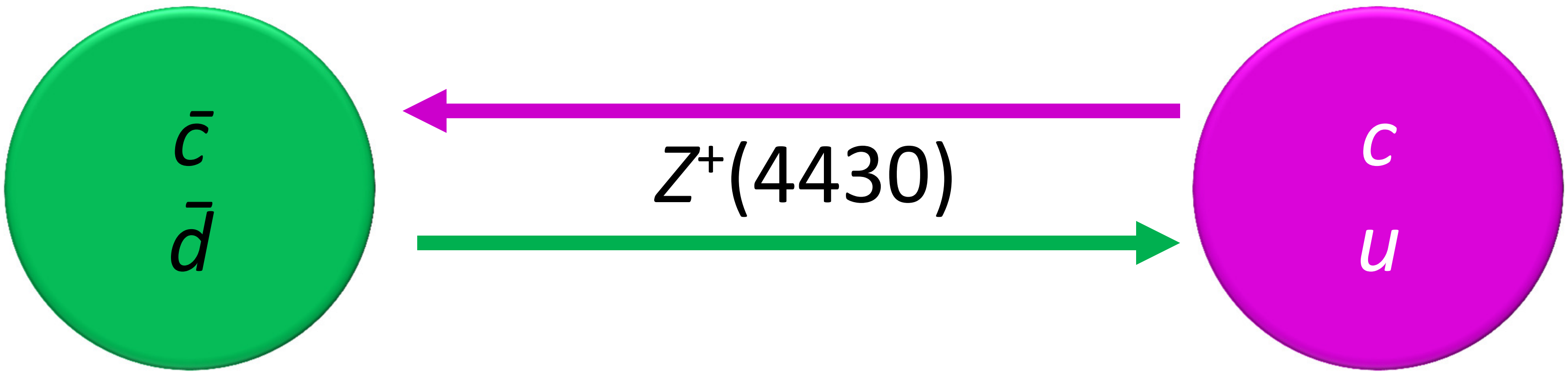


The diquarks are then forced to hadronize by the stretching of meson wave functions from one side to the other



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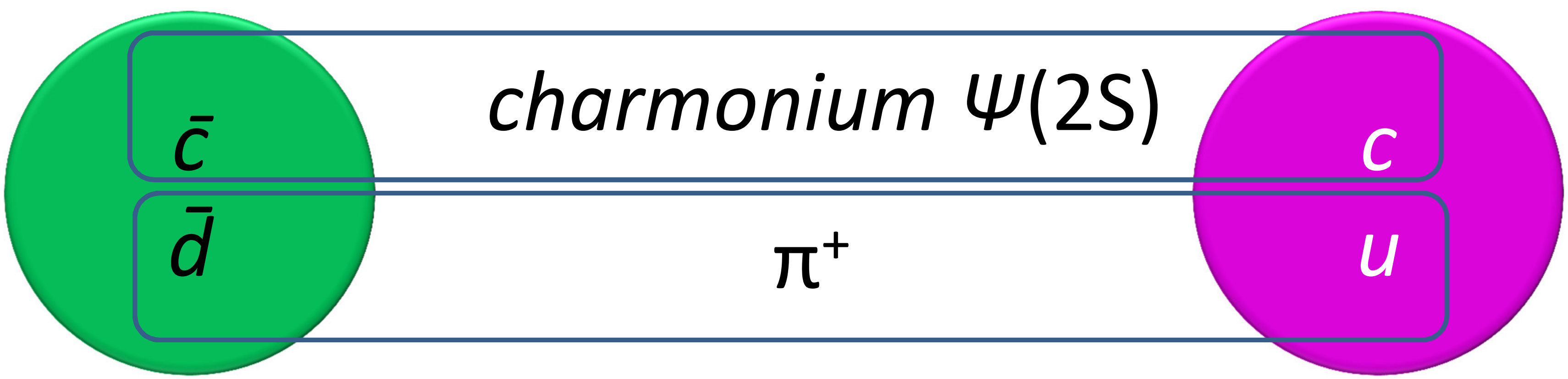
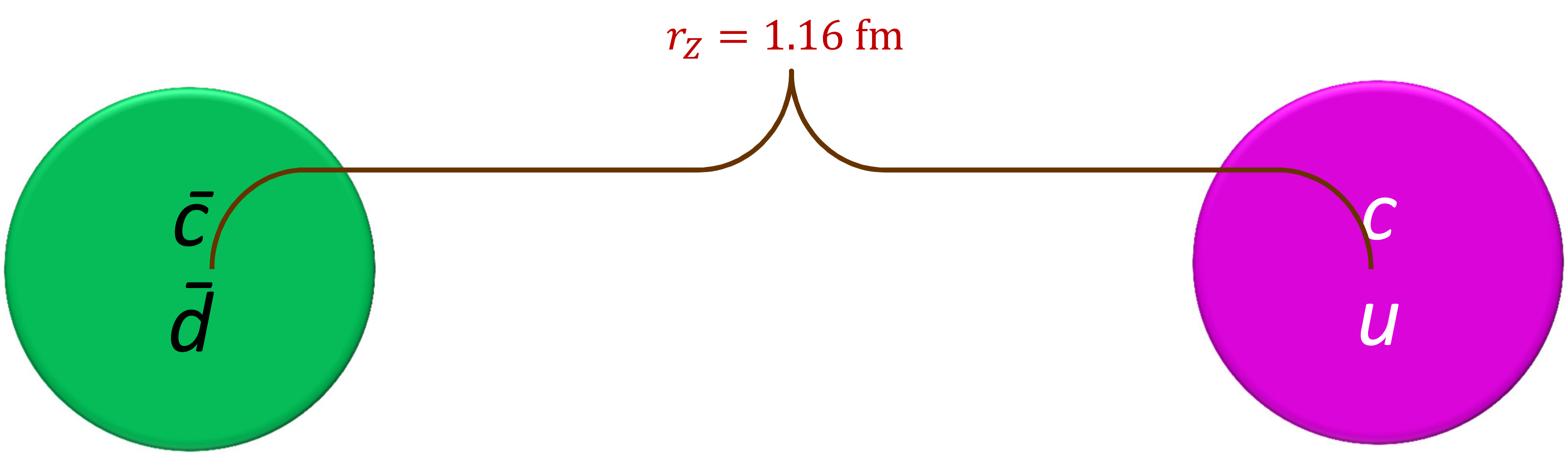


Fascinating $Z(4430)$ fact:

Belle [K. Chilikin *et al.*, PRD **90**, 112009 (2014)] says:

$$\frac{\mathrm{B.R.}\,[Z^-(4430) \to \psi(2S)\pi^-]}{\mathrm{B.R.}\,[Z^-(4430) \to J/\psi\pi^-]} > \mathbf{10}$$

and LHCb has not reported seeing the J/ψ (1S) mode



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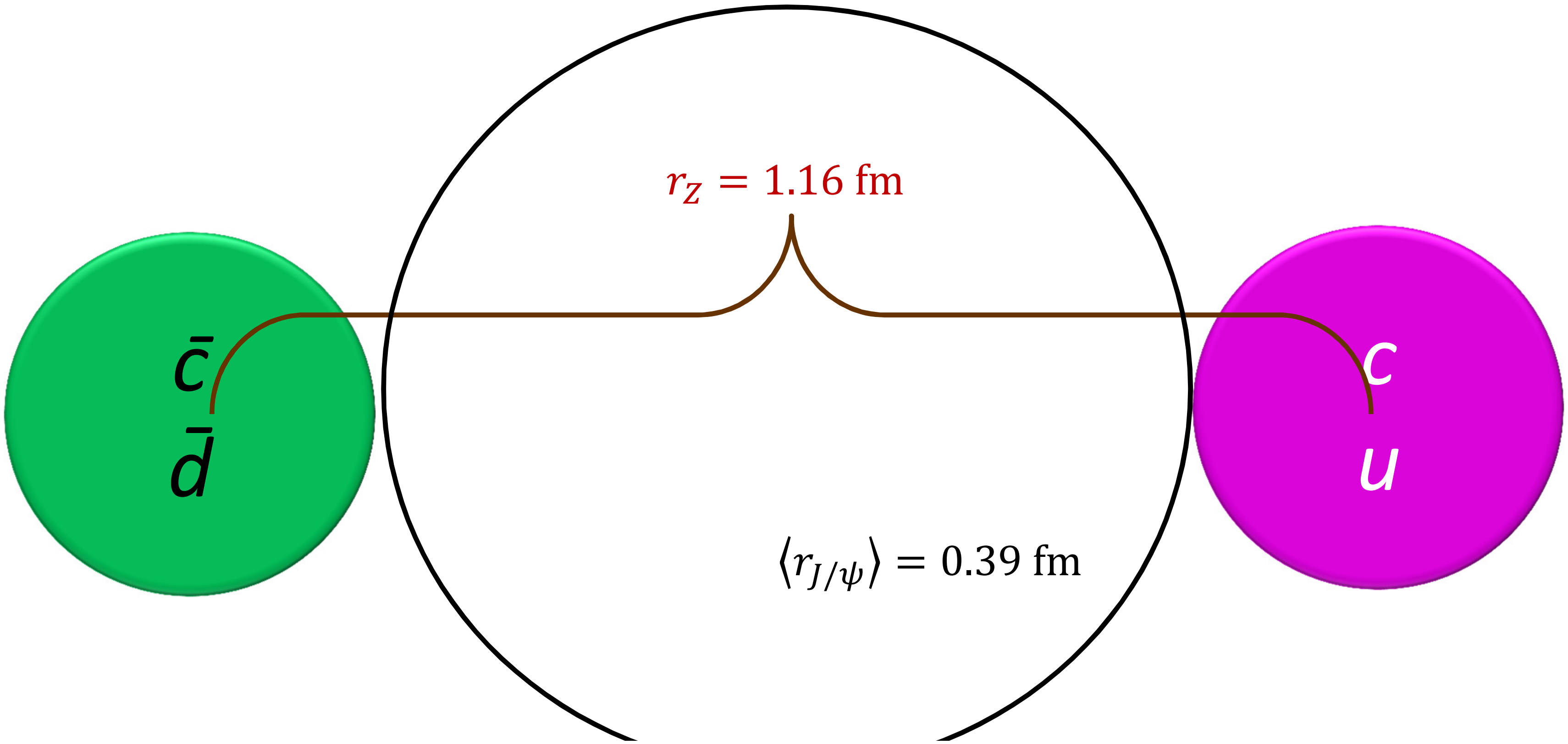

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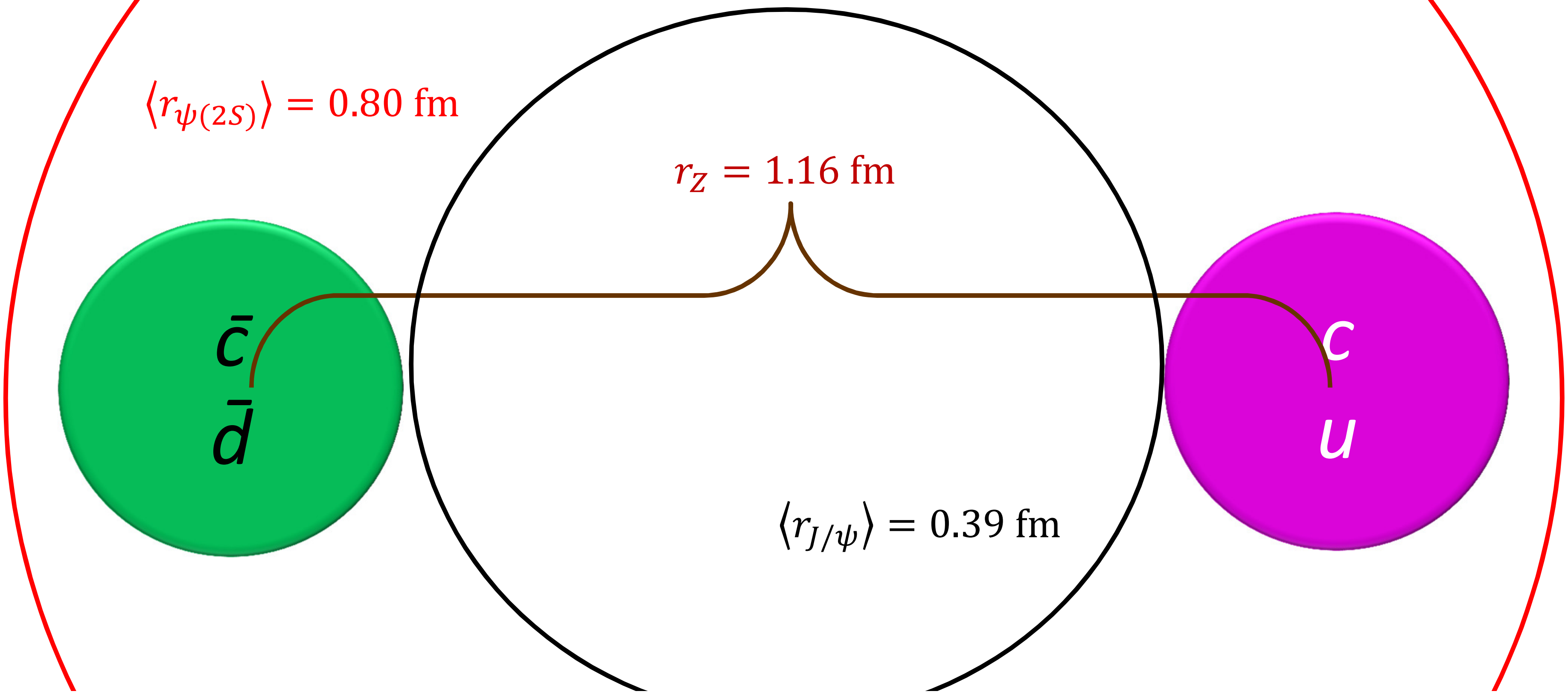


Does the dynamical diquark picture have anything to say about the P_c states?

- Yes. RFL, Phys. Lett. B **749** (2015) 454

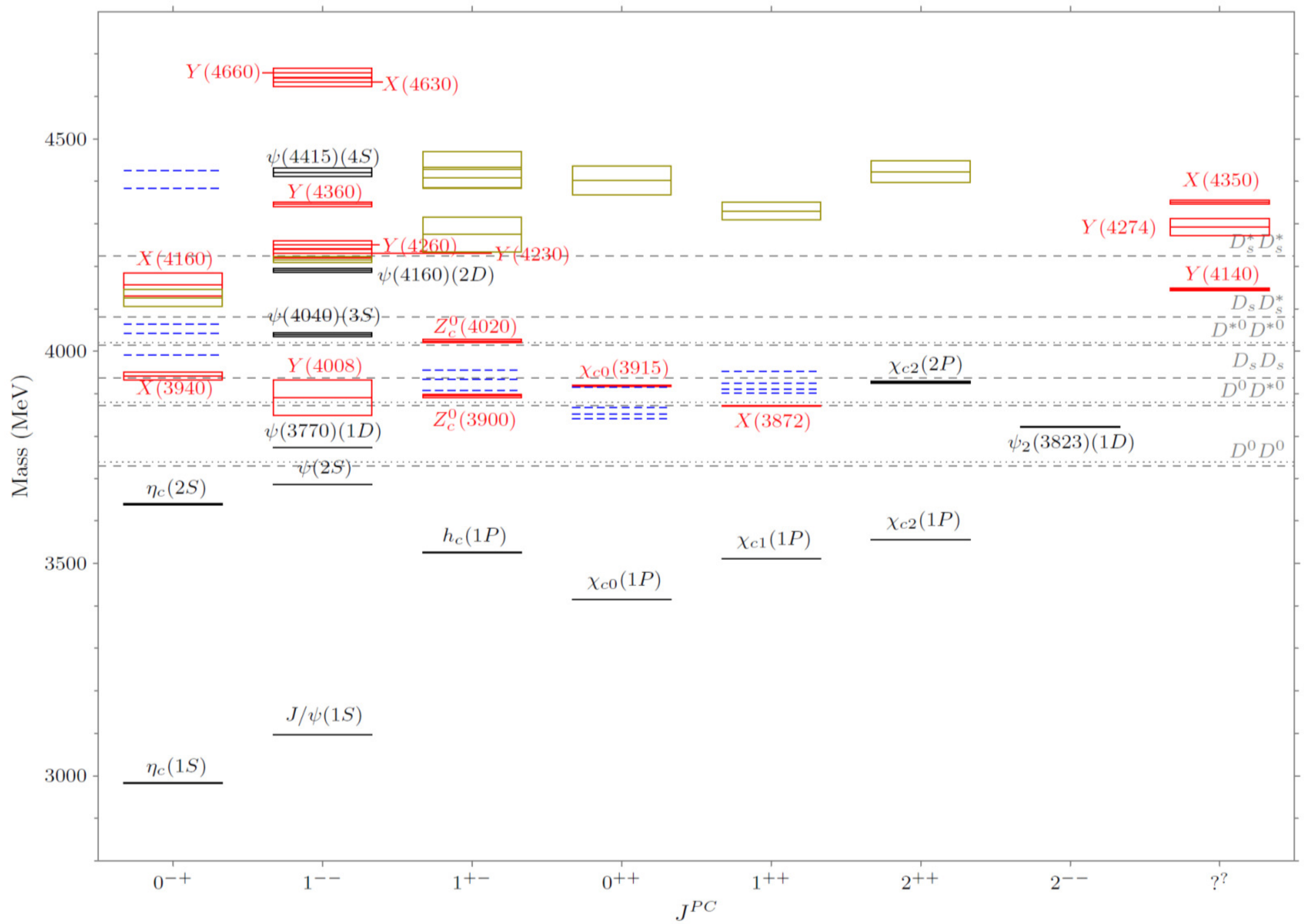
Compounding the diquark attraction into triquarks—
see backup slides for animation

The Present and the Future

- The past two years have provided confirmation of the existence of the *tetraquark* and observation of the *pentaquark*, the third and fourth classes of hadron
- Almost 30 such states (X, Y, Z, P_C) have thus far been observed
- All of the popular physical pictures for describing their structure seem to suffer some difficulty
- We propose an entirely new dynamical picture based on a diquark-antidiquark (or triquark) pair rapidly separating until forced to hadronize due to confinement
- Exotics is a **data-driven** field. Many more exotics remain to be discovered, **especially in the beauty sector**

Backup slides

Charmonium, Including Expected Hybrids



Prompt production

- If hadronic molecules are really formed, they must be very weakly bound, with very low relative momentum between their mesonic components
- They might appear in *B* decays, but would almost always be blown apart in collider experiments
- But CDF & CMS (CERN) saw many! [Prompt *X*(3872) production, σ≈30 nb]
  - CDF Collaboration (A. Abulencia *et al*.), PRL **98**, 132002 (2007)
  - CMS Collaboration (S. Chatrchyan *et al*.), JHEP **1304**, 154 (2013)
- Perhaps strong final-state interactions, π exchanges between D^0 and $\overline{D^{*0}}$?
  - P. Artoisenet and E. Braaten, Phys. Rev. D **81**, 114018 (2010); D **83**, 014019 (2011)
- Such effects can be significant, but do not appear to be sufficient to explain the size of the prompt production
  - C. Bignamini *et al*., Phys.Lett. B **228** (2010); A. Esposito *et al.*, J. Mod. Phys. **4**, 1569 (2013); A. Guerrieri *et al*., Phys. Rev. D **90**, 034003 (2014)

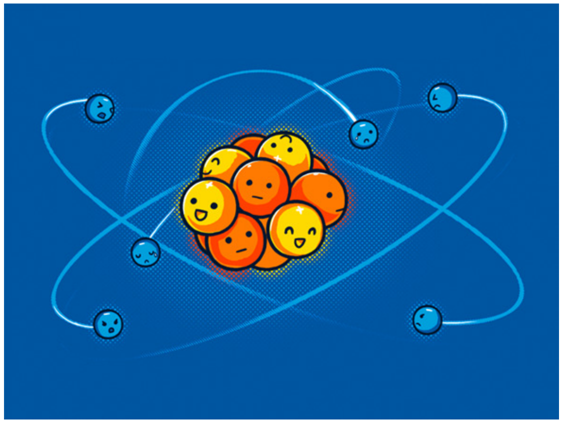
➢ Hadronic molecules may exist, but *X*(3872) does not seem to fit the profile

A new tetraquark picture

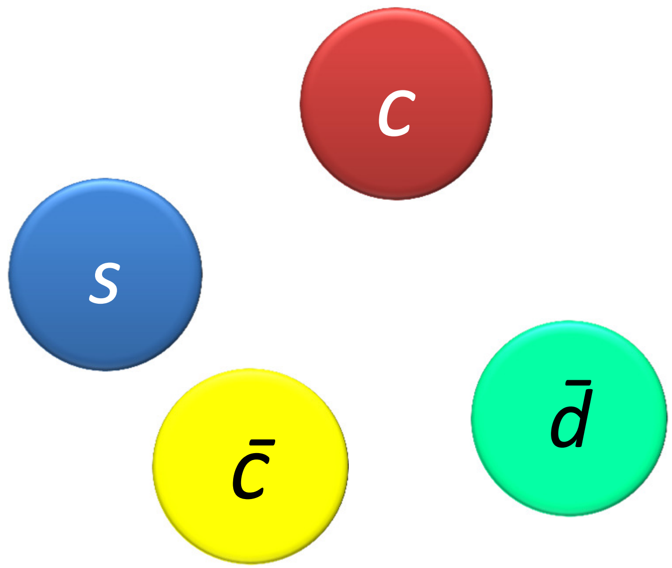
Stanley J. Brodsky, Dae Sung Hwang, RFL

Physical Review Letters **113**, 112001 (2014)

- CLAIM: At least some of the observed tetraquark states are bound states of diquark-antidiquark pairs
- BUT the pairs are not in a static configuration; they are created with a lot of relative energy, and rapidly separate from each other
- Diquarks are not color neutral! They cannot, by confinement, separate asymptotically far
- They must hadronize via large-*r* tails of mesonic wave functions, which suppresses decay widths
- Want to see this in action? Time for some cartoons!



What happens next?
Option 2: Color-suppressed



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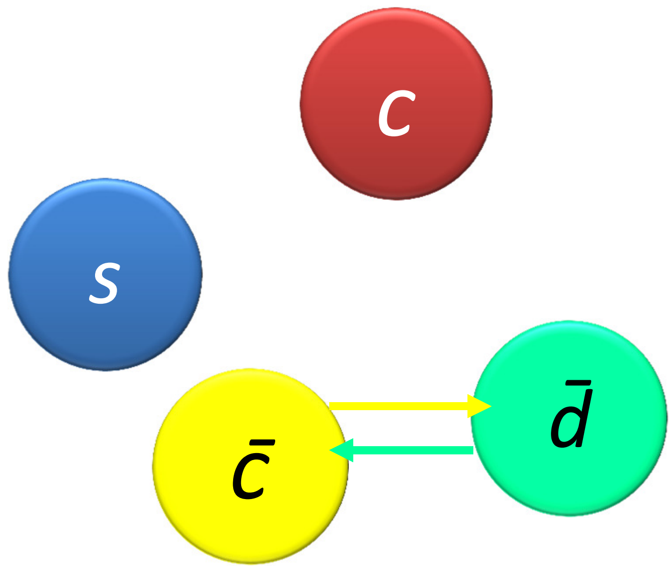

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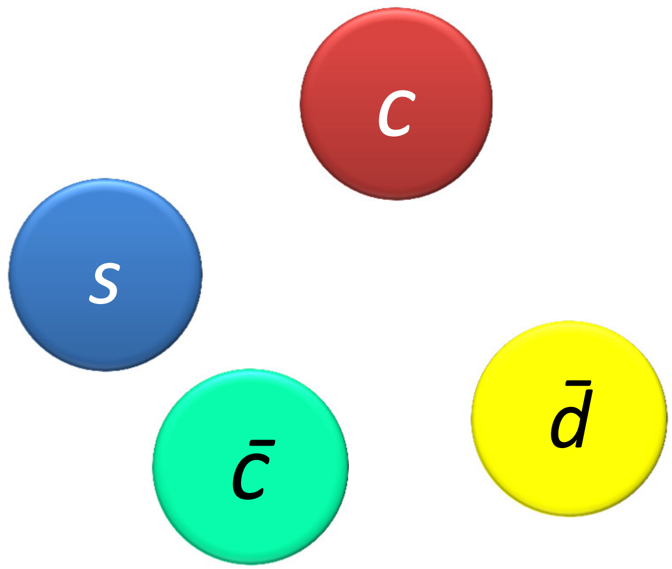

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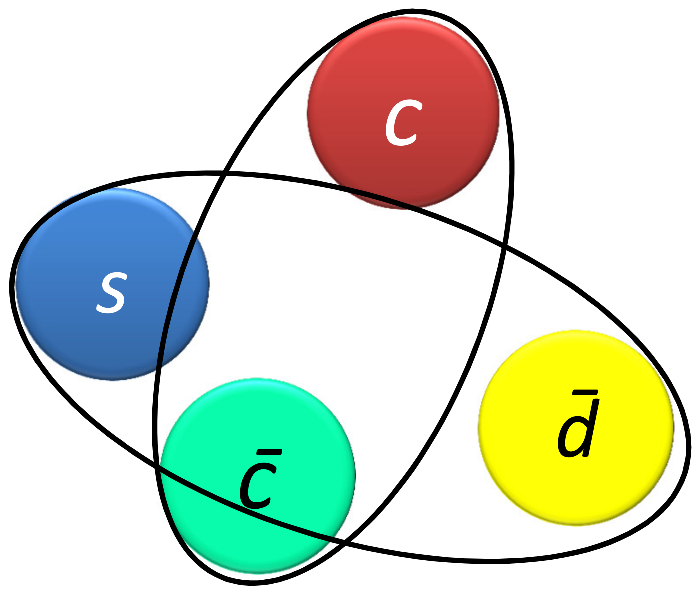
B.R.~2.3%



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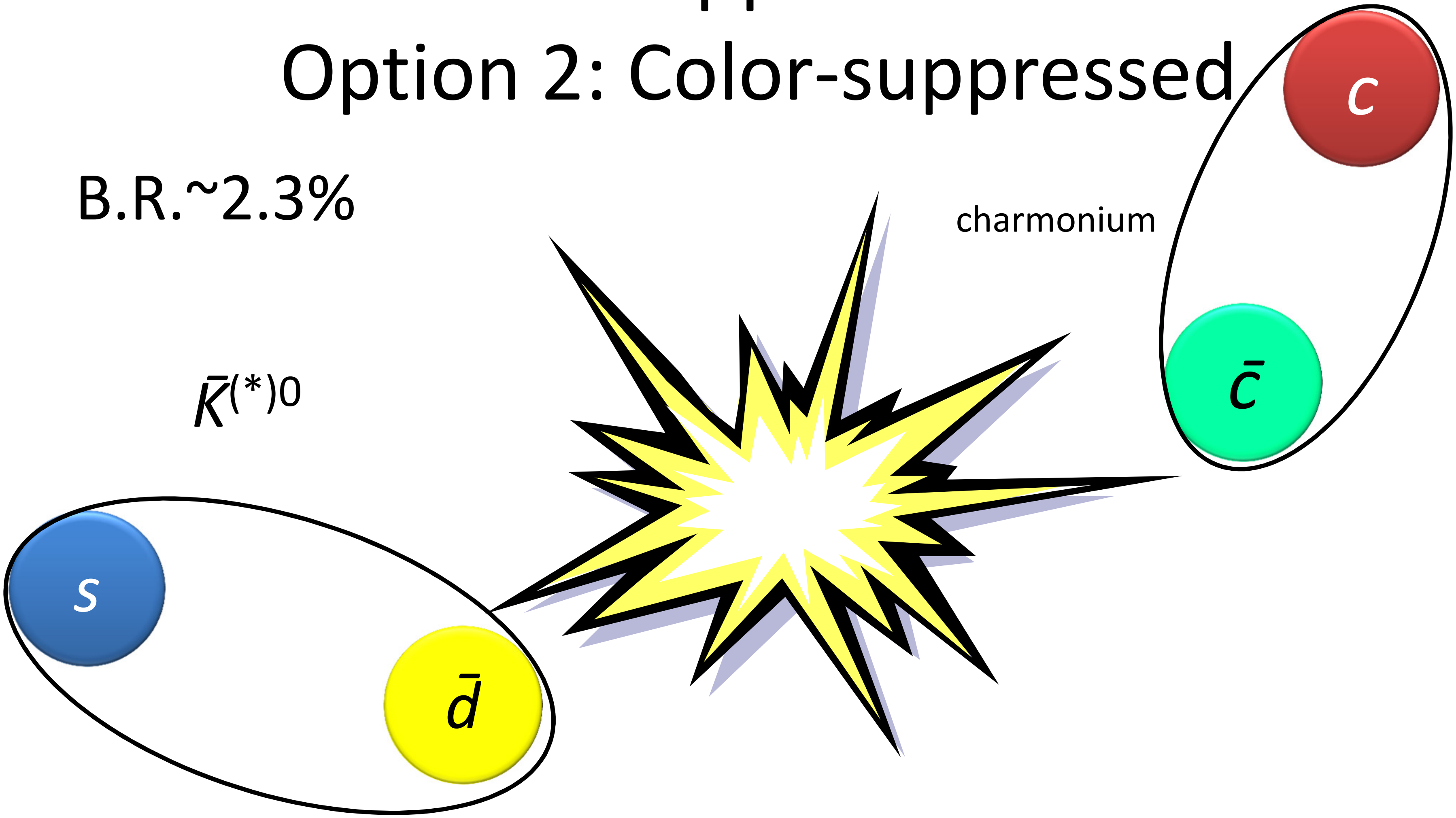

Why doesn't this just happen?
It's called *baryonium*

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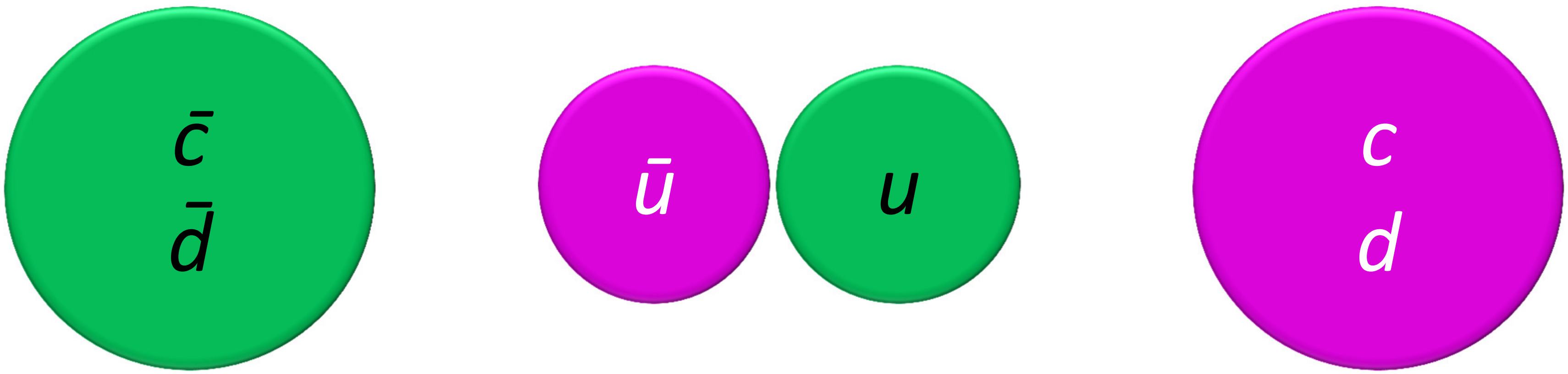


Why doesn't this just happen?
It's called *baryonium*

It *does* happen, as soon as the threshold $2M_{\Lambda_c} = 4573$ MeV is passed

The lightest exotic above this threshold, *X*(4632) , decays into $\Lambda_c + \overline{\Lambda}_c$

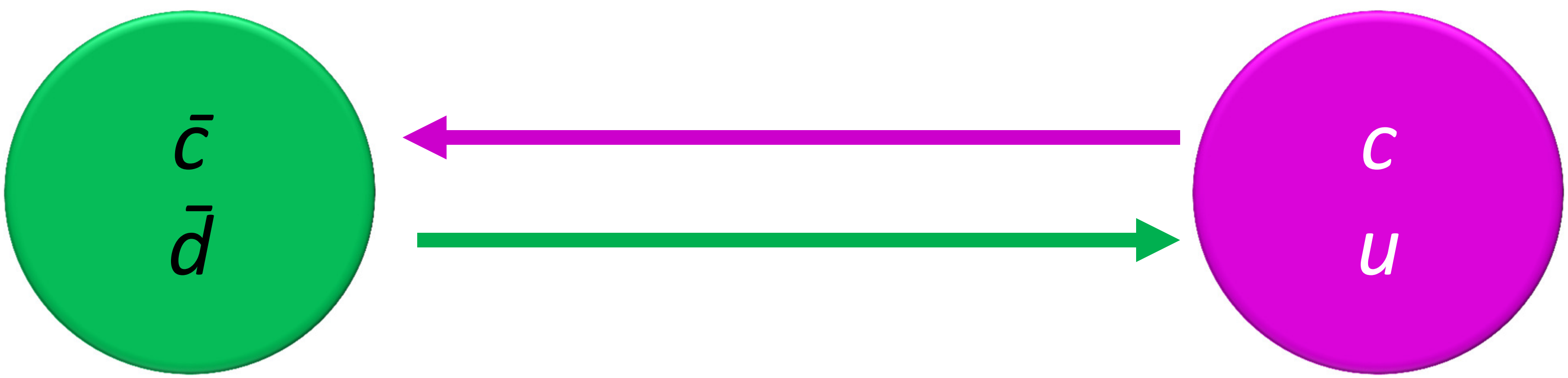
How far apart do the diquarks actually get?

- Since this is still a $\mathbf{3} \leftrightarrow \overline{\mathbf{3}}$ color interaction, just use the Cornell potential:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m_{cq}^2}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2}\mathbf{S}_{cq}\cdot\mathbf{S}_{\overline{cq}},$$

[This variant: Barnes et al., PRD **72**, 054026 (2005)]

- Use that the kinetic energy released in $\overline{B}^0 \longrightarrow K^- + Z^+(4430)$ converts into potential energy until the diquarks come to rest
- Decay transition most effective at this point (WKB turning point)



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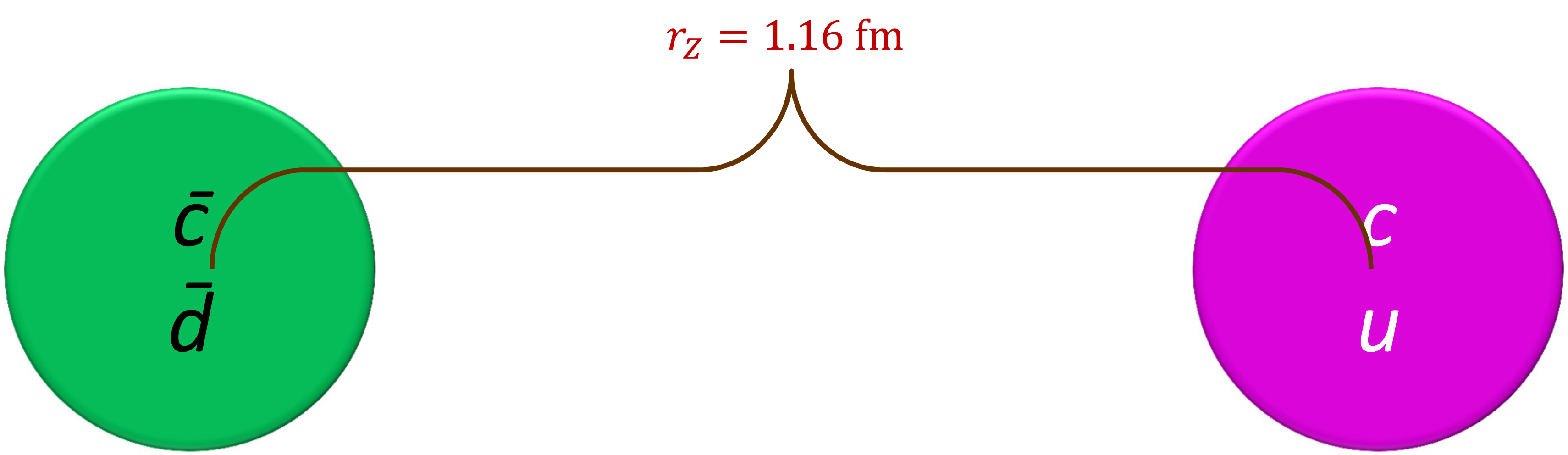
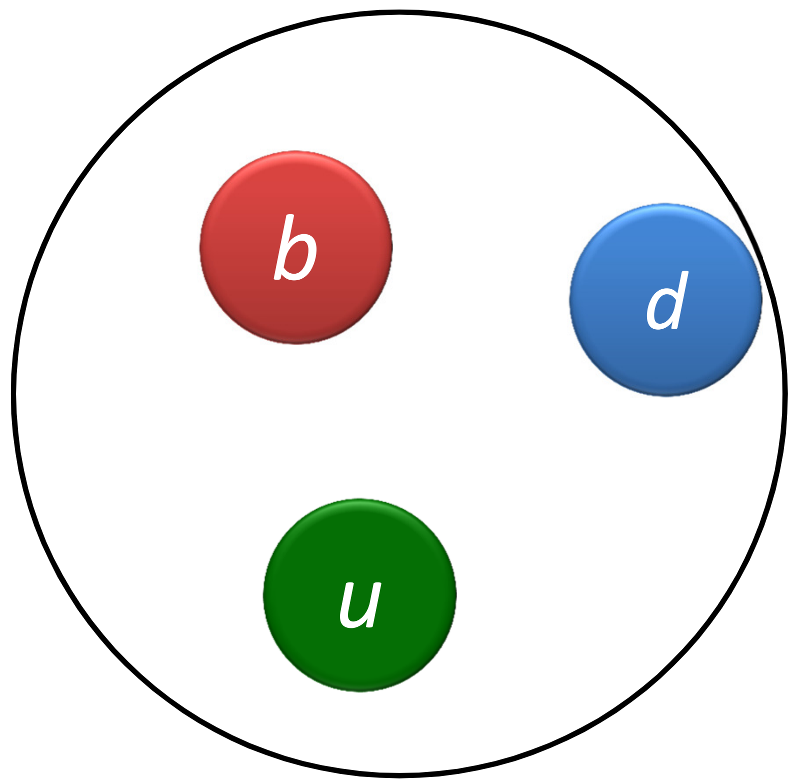


The large-*r* wave function tails and resonance widths

- The simple fact that the diquark-antidiquark pair is capable of separating further than the typical mean size of ordinary hadrons before coming to rest implies:
  - The decay transition overlap matrix elements are suppressed, SO
  - The decay transition rate is suppressed, SO
  - The width is smaller than predicted by generic dimensional analysis (*i.e.*, by phase space alone)
- *e.g.*, $\Gamma[Z(4430)] = 180 \pm 31 \text{ MeV}$
  (*cf.* $\Gamma[\rho(770)] = 150 \text{ MeV}$)
- But why would these diquark-antidiquark states behave like resonances at all?

Nonleptonic Λ_b baryon decay



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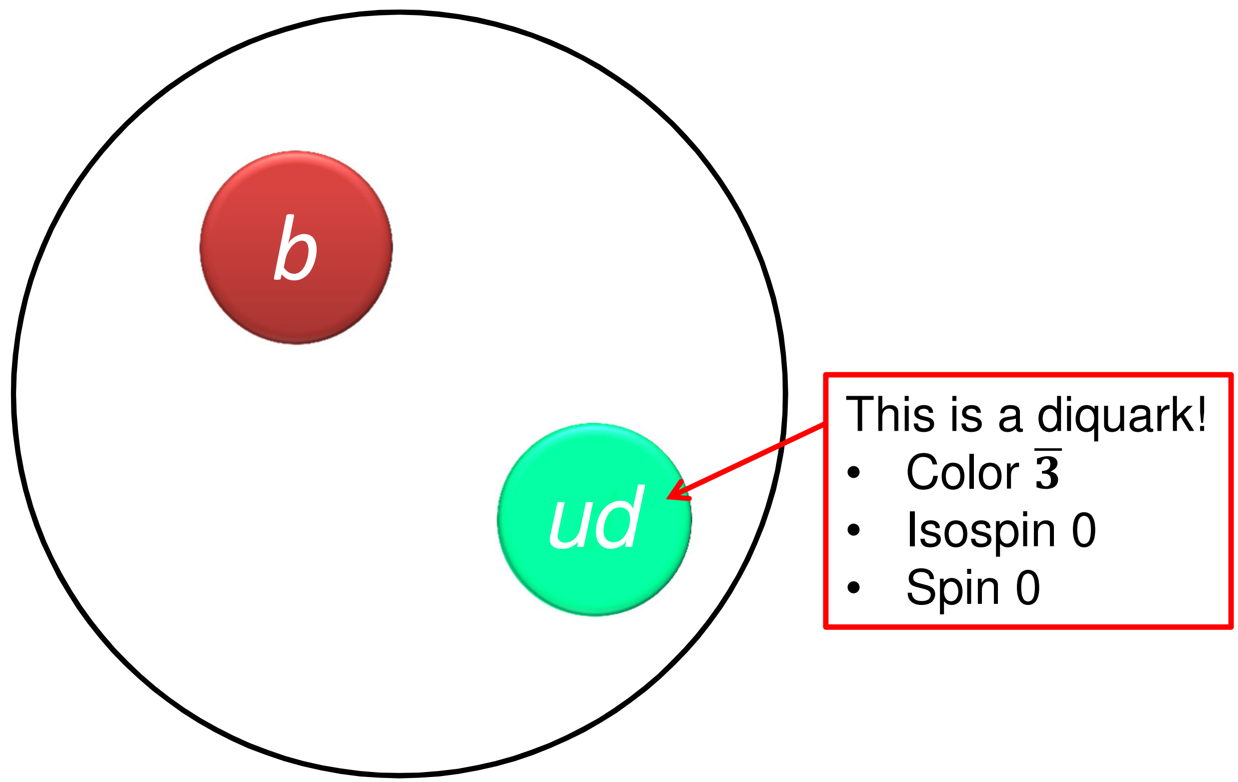

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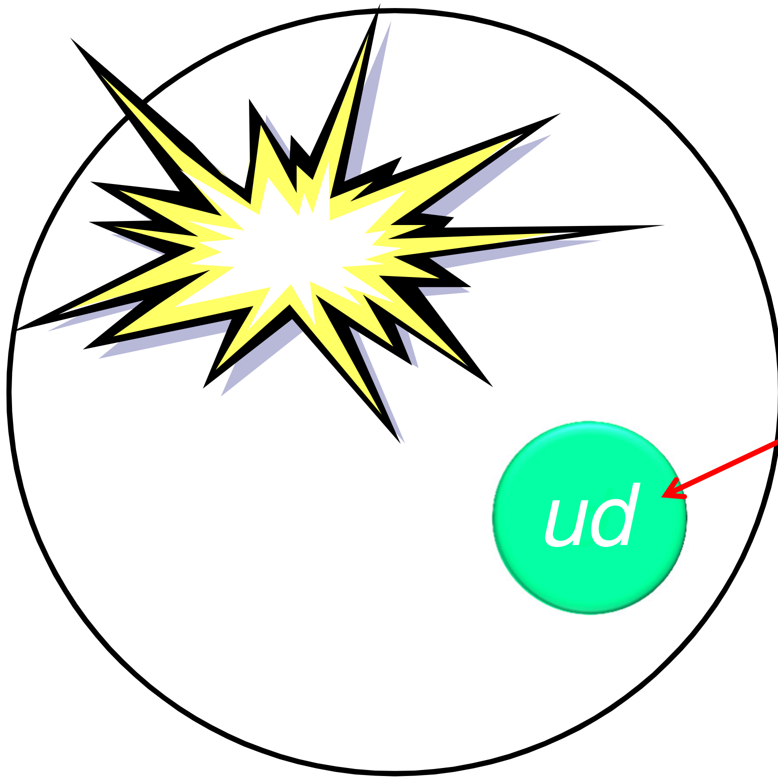

This is a diquark!

- Color $\overline{\mathbf{3}}$
- Isospin 0
- Spin 0

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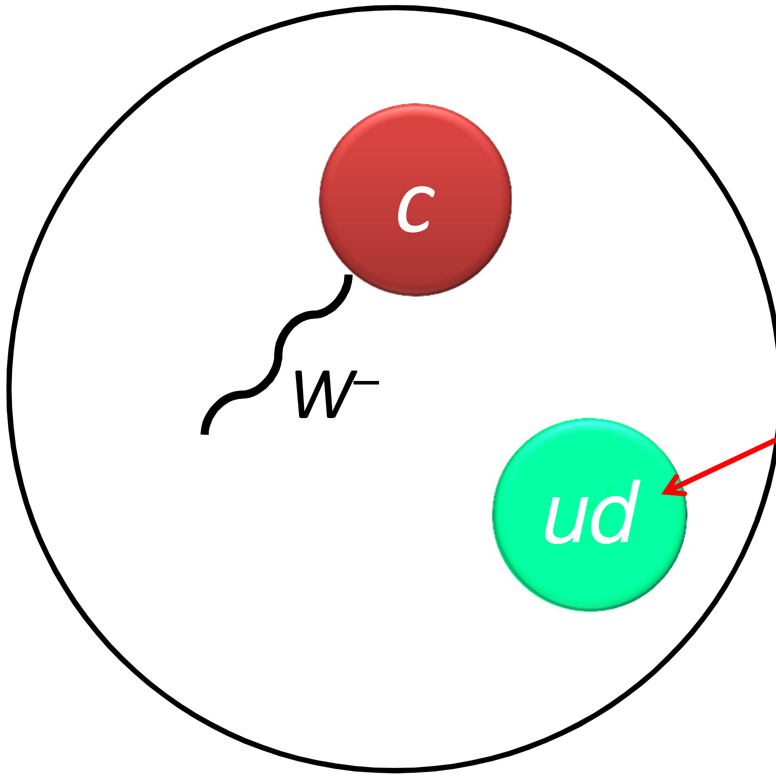

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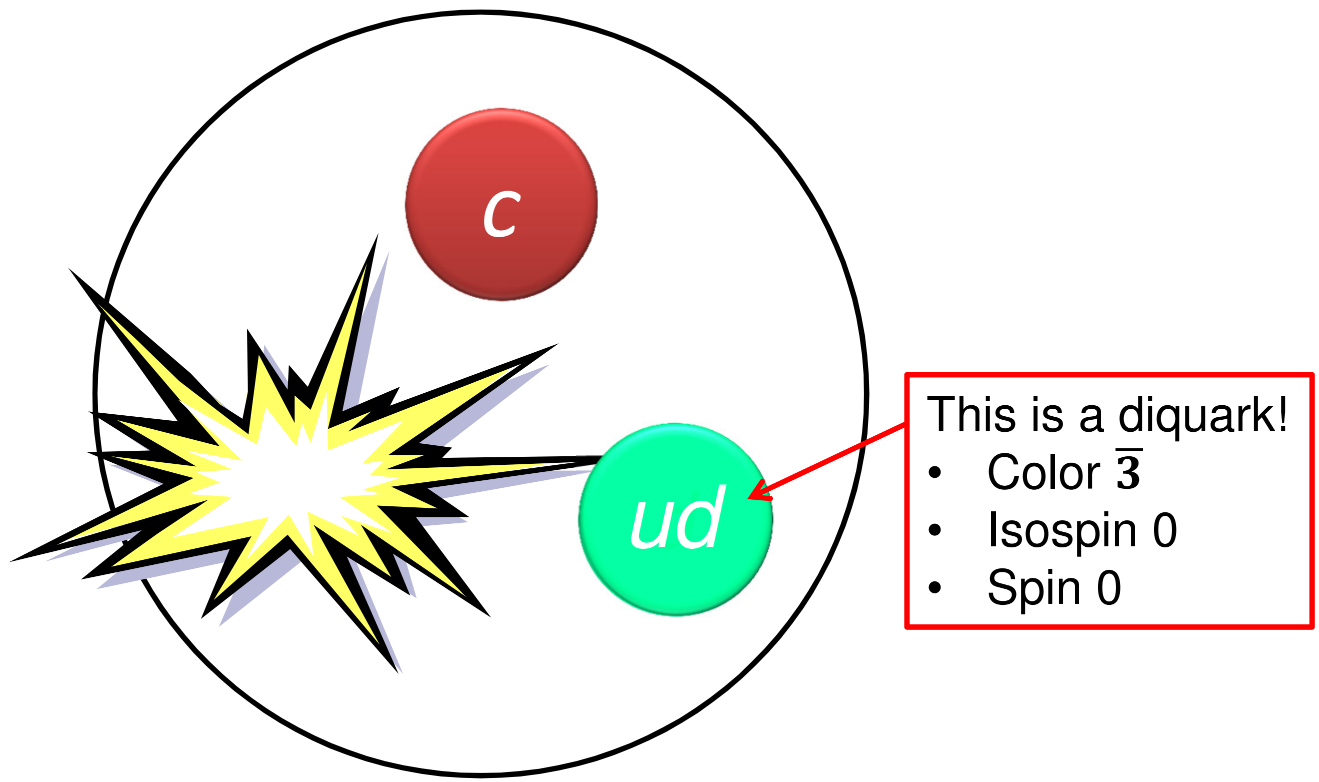

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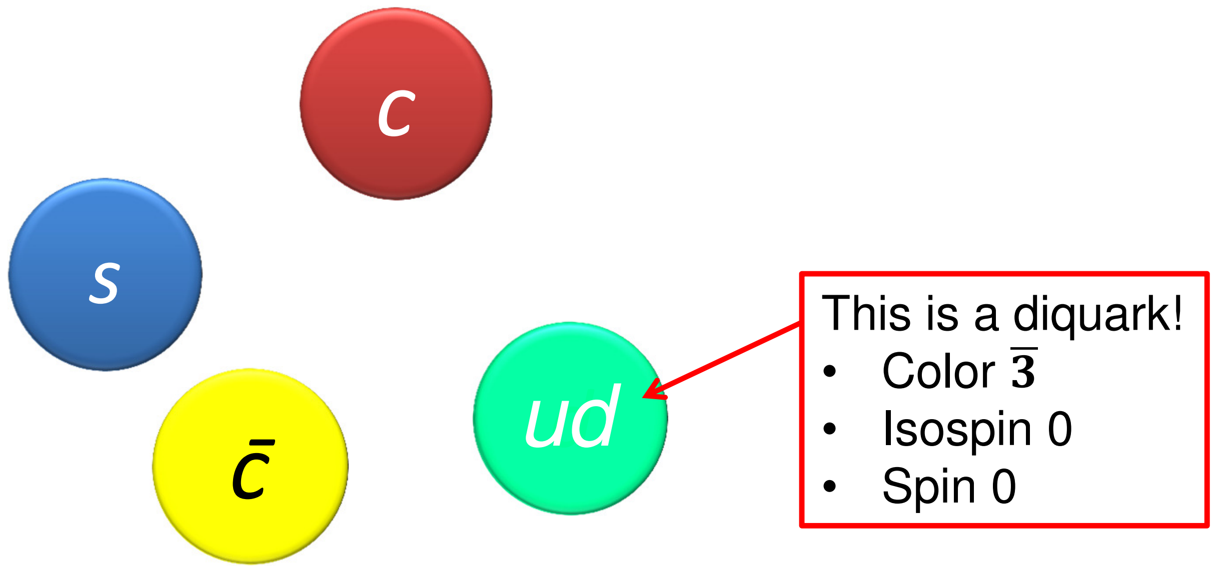
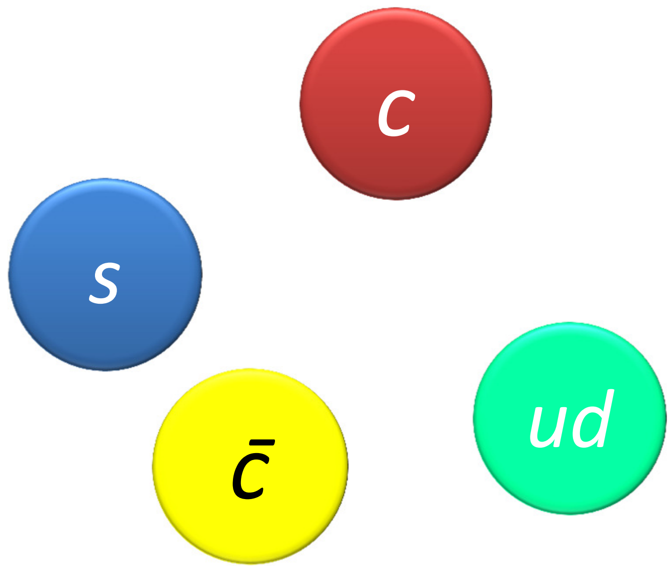


What happens next?
Diquark *and triquark* formation



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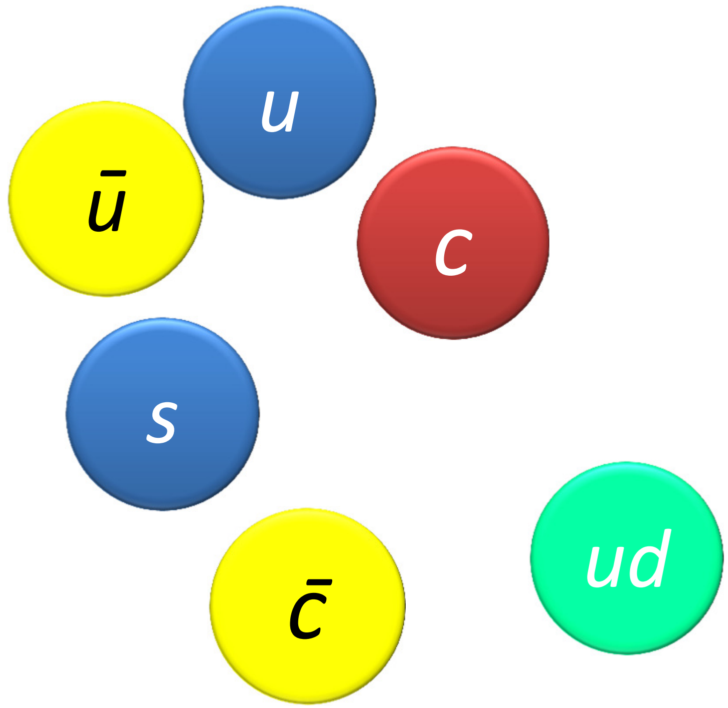

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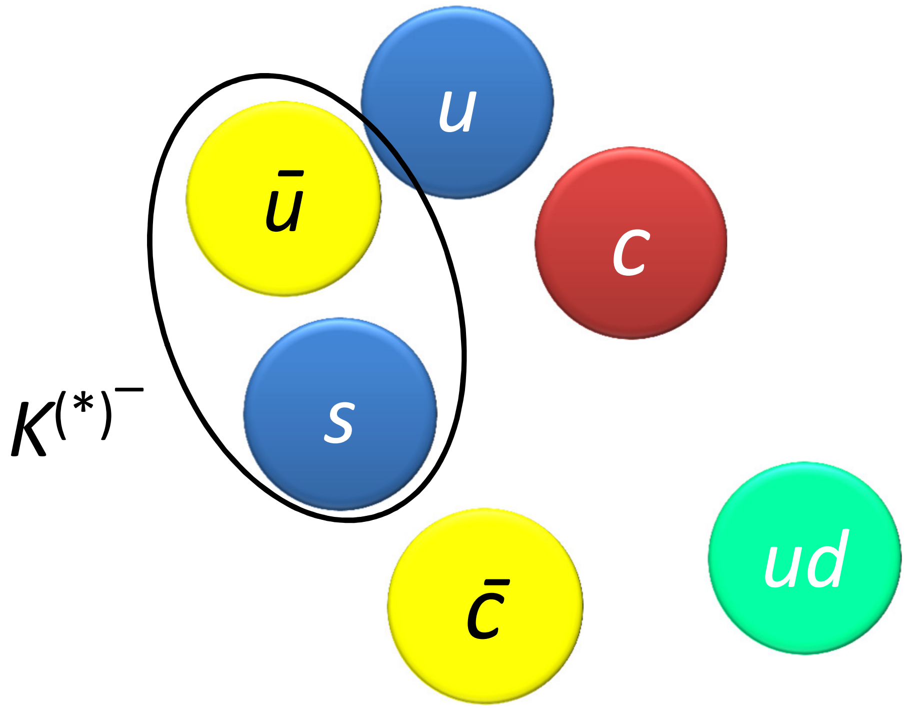

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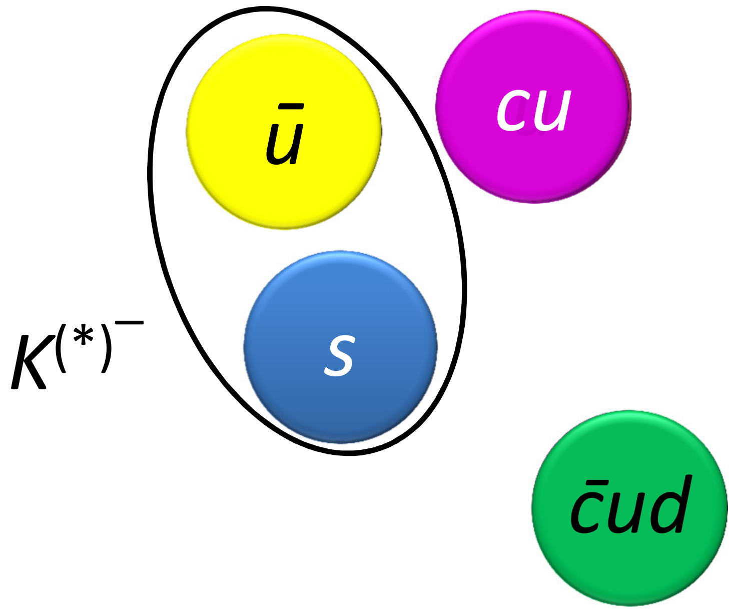

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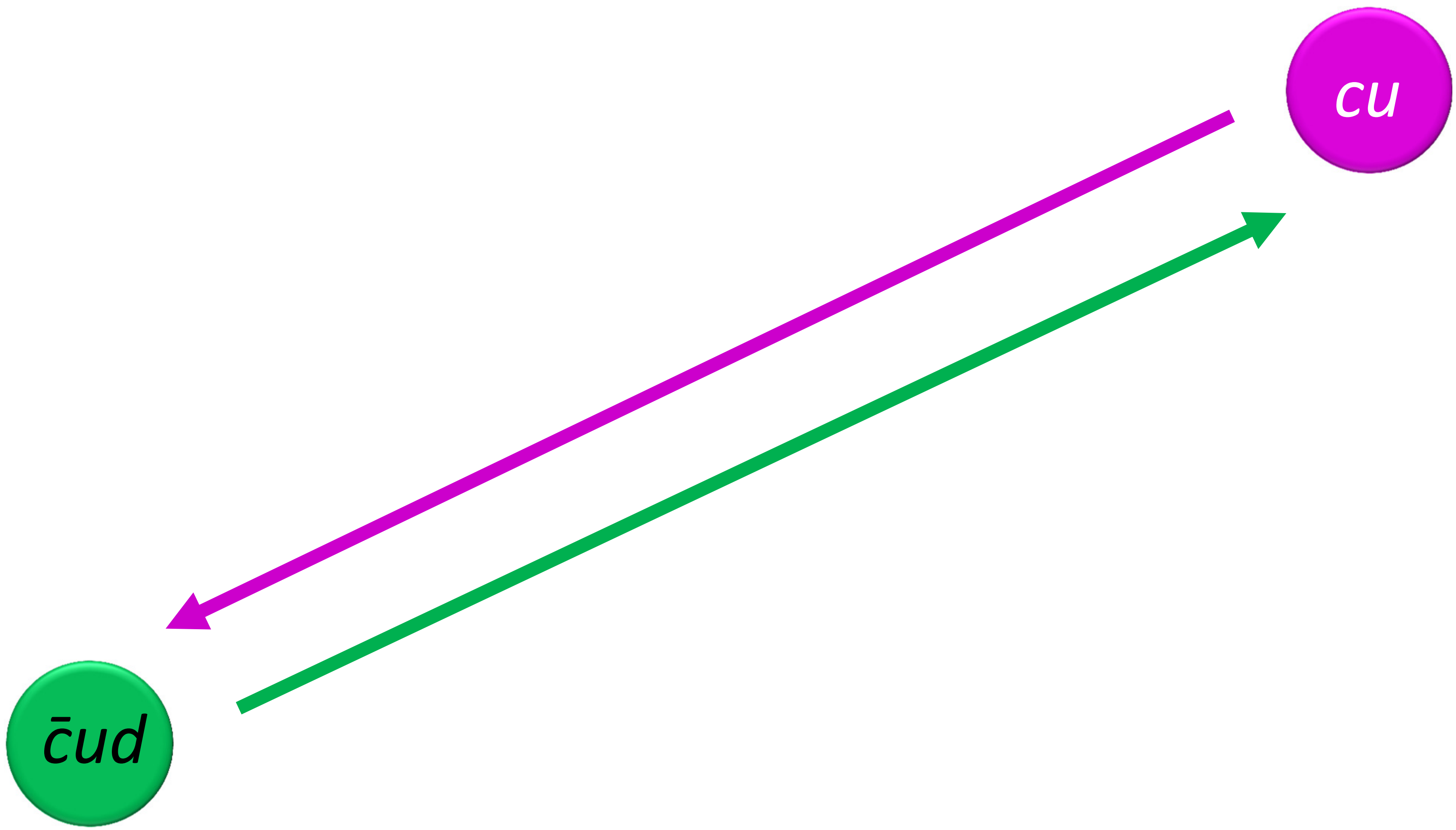
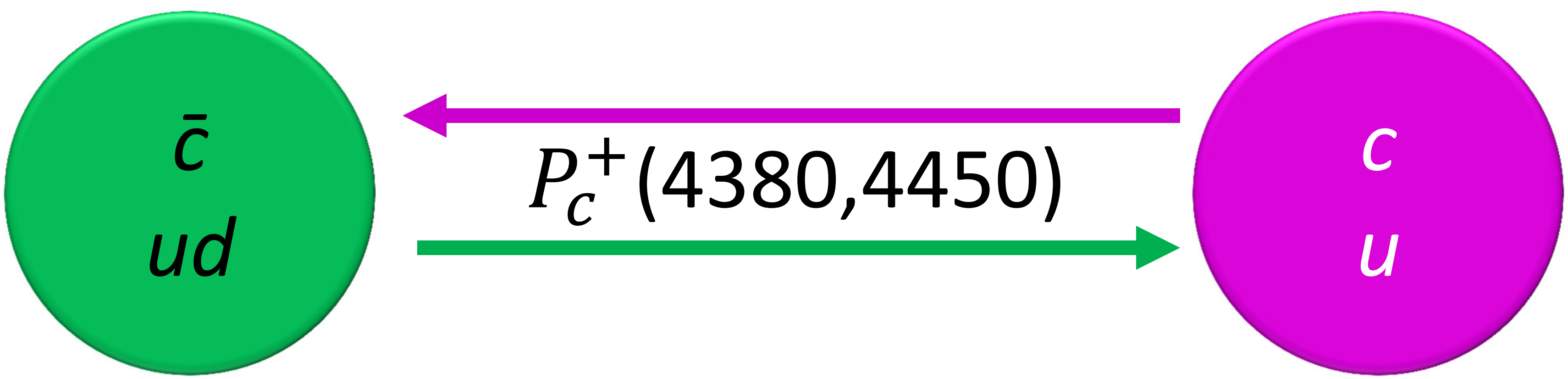


The same color triplet mechanism, supplemented with the fact that the *ud* in *Λ* baryons themselves act as diquarks, predicts a rich spectrum of *pentaquarks*



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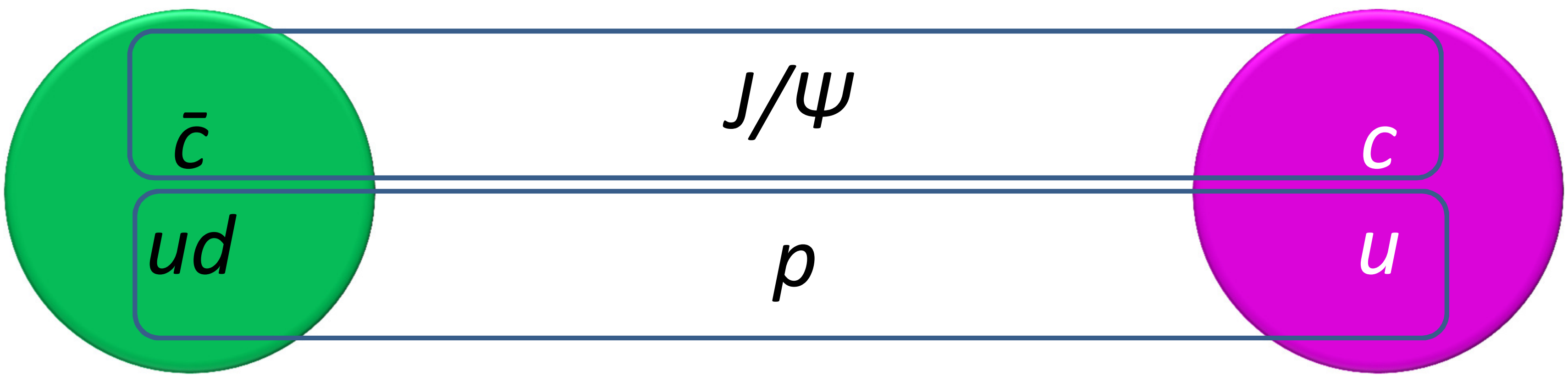


$J^P = \frac{1}{2}^-$ $\quad$ $J^P = L^{(-1)^L}$ $\quad$ $J^P = 0^+, 1^+$

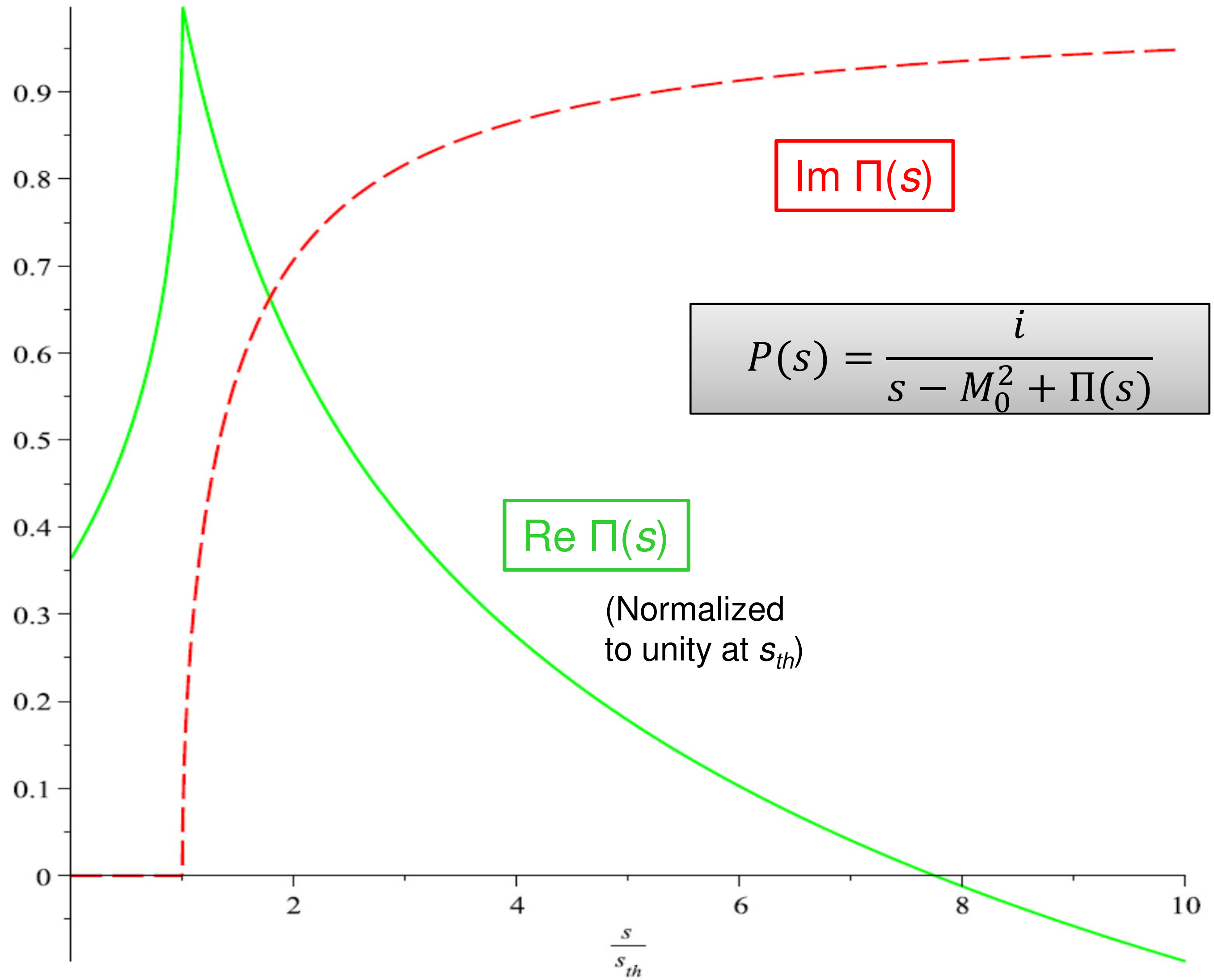
For one thing,

- Diquark-antidiquark pairs create their own bound-state spectroscopy [L. Maiani *et al.*, PRD **71** (2005) 014028]
  - Simple Hamiltonian with spin-spin interactions among the four quarks
  - Once one bound state is found, a whole multiplet arises
  - Then compare predicted spectrum to experiment
- Original version predicts states with quantum numbers and multiplicities not found to exist (*XYZ* phenomenology not very well developed then), but a new version of the model [L. Maiani *et al.*, PRD **89** (2014) 114010] appears to be much more successful
  - Crucial revision: Dominant spin-spin couplings are *within* each diquark
  - *e.g.*, *Z*(4430) is radial excitation of *Z*(3900);
    Y states are *L*=1 color flux tube excitations

And furthermore,

- The presence of nearby hadronic thresholds can attract nearby diquark resonances: *Cusp effect*
  - The complex amplitude $\Pi(s)$ that is a source for the tetraquarks in terms of total energy $\sqrt{s}$ develops a branch point at the threshold to produce on-shell hadrons (due to *unitarity*: the *optical theorem*)
  - But the full amplitude is analytic everywhere, except for resonant poles and cuts that start at the branch points (due to *causality*)
  - This fact allows for a *dispersion relation* (like Kramers-Kronig) that expresses $\mathrm{Re}\,\Pi(s)$ as an integral over $\mathrm{Im}\,\Pi(s)$
  - If $\mathrm{Im}\,\Pi(s)$ at suddenly shoots up from zero, then $\mathrm{Re}\,\Pi(s)$ must develop a sharp peak, or *cusp*
  - Since the self-energy $\Pi(s)$ appears in the resonance propagator Green's function, the cusp in $\mathrm{Re}\,\Pi(s)$ acts as a shift in the mass, effectively dragging the resonant pole toward threshold

The Cusp



$$P(s) = \frac{i}{s - M_0^2 + \Pi(s)}$$

How closely can cusps attract thresholds?

- Consider the $X(3872)$, with $\Gamma < 1.2 \text{ MeV}$
  - We saw that $m_{X(3872)} - m_{D^{*0}} - m_{D^0} = -0.11 \pm 0.21 \text{ MeV}$
  - But also that $X(3872)$ is almost certainly not a $\overline{D^{*0}}D^0$molecule
  - Moreover,

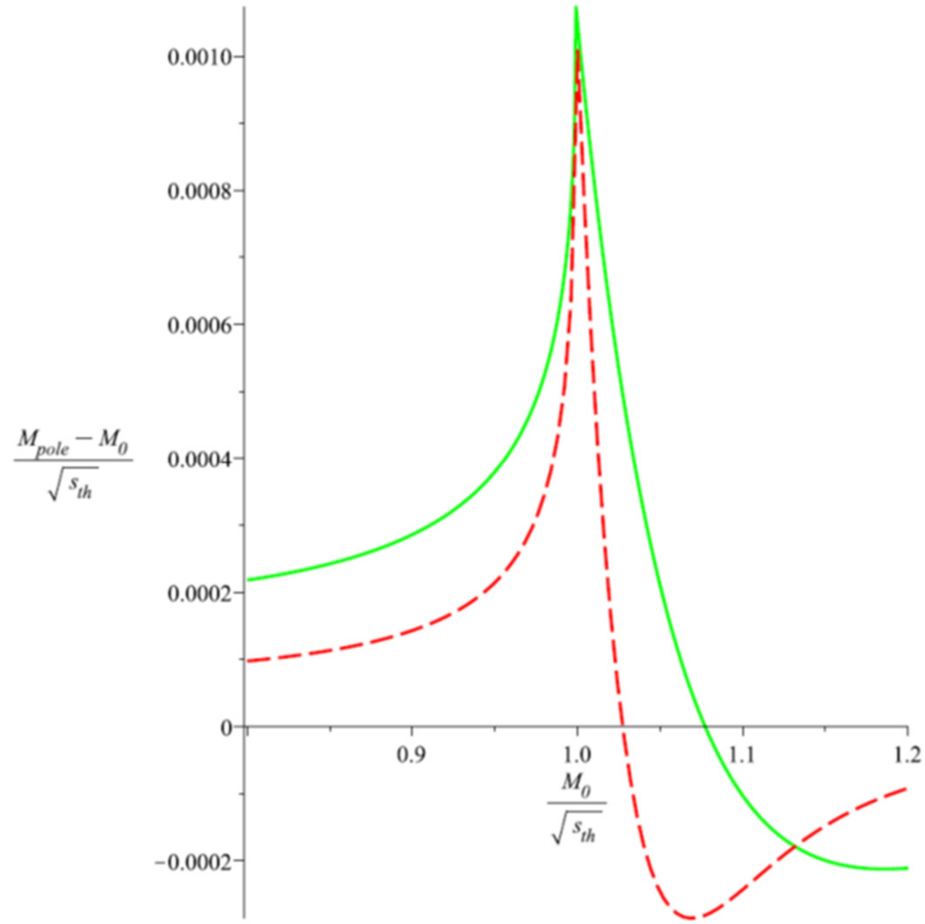
$$m_{X(3872)} - m_{J/\psi} - m_{\rho^0_{peak}} = -0.50 \text{ MeV}$$
$$m_{X(3872)} - m_{J/\psi} - m_{\omega_{peak}} = -7.89 \text{ MeV}$$

  - Bugg [J. Phys. G **35** (2008) 075005] showed that the $X(3872)$ is far too narrow to be a cusp alone—Some sort of resonance must be present
  - But since several channels all open up very near 3.872 GeV, they all contribute to a big cusp that can drag, say, a diquark-antidiquark resonance from perhaps 10's of MeV away to become the $X(3872)$

Example cusp effects

S. Blitz & RFL, Phys. Rev. D **91** (2015) 094025

M_0: Bare resonant pole mass
S_{th}: Threshold s value [here (3.872 GeV)^2]
M_{pole}: Shifted pole mass



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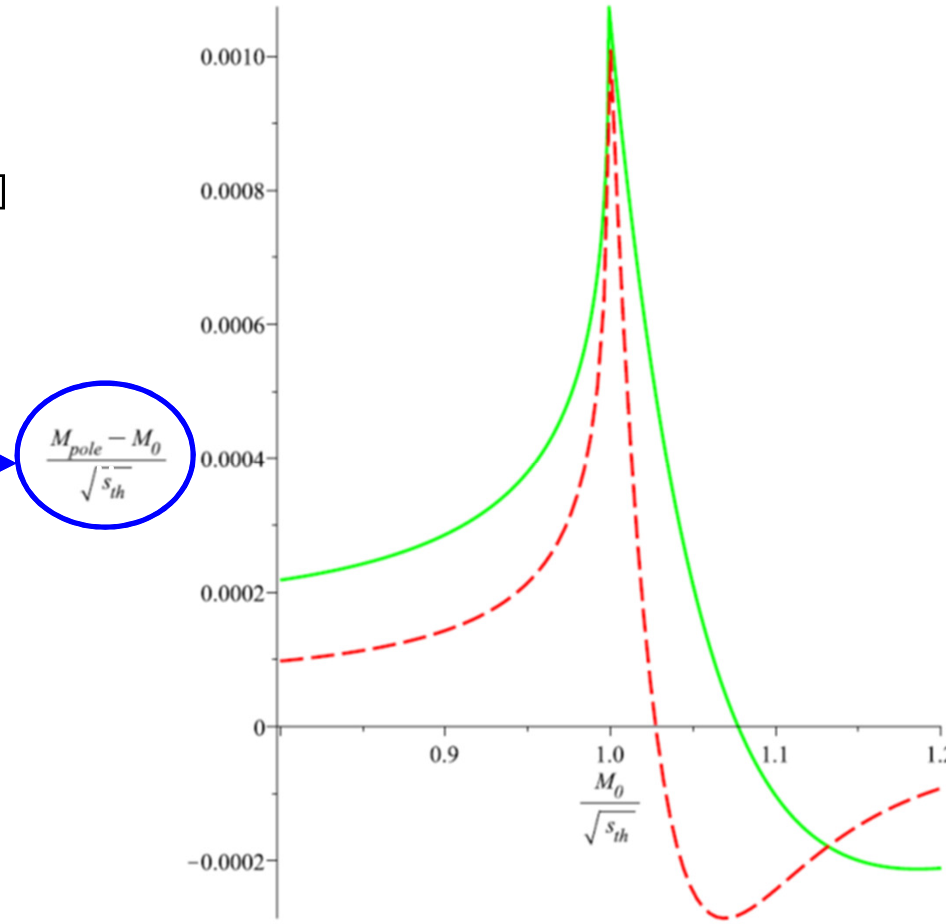
Relative size of pole shift (about 0.12% near S_{th}, or 5 MeV)



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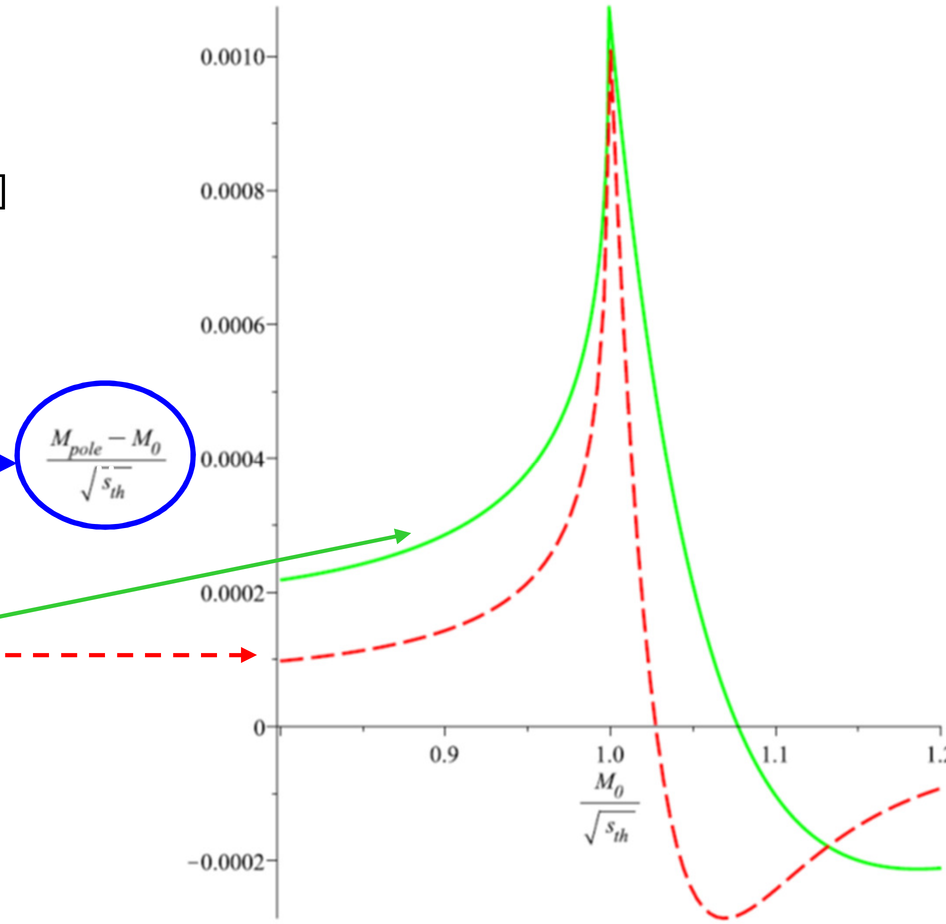

At the charm scale, a cusp from an opening diquark pair threshold is more effective than one from a meson pair!

What determines cusp shapes?

- Traditionally, a phenomenologically-based exponential form factor is used in the case of meson pair production:

$$F^2_{\rm mes}(s) = \exp\left(-\frac{s-s_{th}}{\beta^2}\right),$$

  where β is a typical hadronic scale (~0.5-1.0 GeV)
- For processes at high energy (s), or when the high-s tails of form factors are important (as in dispersion relations), use *constituent counting rules*
  [Matveev *et al*., Lett. Nuovo Cim. **7**, 719 (1973); Brodsky & Farrar, PRL **31**, 1153 (1973)]
- In any hard process in which a constituent is diverted through a finite angle, there will be a factor of $1/s$ (or $1/t$) coming from a propagator of the virtual particle redirecting it
- Using this logic, the form factor $F(s)$ of a particle with 4 quark constituents can quickly be shown to scale as

$$F_{\rm diq}(s) \sim \left(\frac{\alpha_s}{s}\right)^3 \rightarrow F_{\rm diq}(s) = \left(\frac{s_{th}}{s}\right)^3$$

Can the counting rules be used for cross sections as well?

- With *ease*: S. Brodsky and RFL, Phys. Rev. D **91** (2015) 114025
- Exotic states can be produced in threshold regions in e^+e^- (BES, Belle), electroproduction (JLab 12), hadronic beam facilities ($\bar{\text{P}}$ANDA at FAIR, AFTER@LHC) and are best characterized by cross section ratios
- Two examples:

$$1)\ \frac{\sigma(e^+e^-\to Z^+(c\bar{c}u\bar{d})+\pi^-(\bar{u}d))}{\sigma(e^+e^-\to\mu^+\mu^-)} \propto \frac{1}{s^4} \text{ as } s\to\infty$$

$$2)\ \frac{\sigma(e^+e^-\to Z^+(c\bar{c}u\bar{d})+\pi^-(\bar{u}d))}{\sigma(e^+e^-\to\Lambda_c(cud)+\bar{\Lambda}_c(\bar{c}\,\bar{u}\bar{d}))} \to const \text{ as } s\to\infty$$

Ratio numerically smaller if Z_c behaves like weakly-bound dimeson molecule instead of diquark-antidiquark bound state due to weaker meson color van der Waals forces