The Standard Model

2015 CERN-Fermilab HCP Summer School
CERN, 24-26 June 2015

Yossi Nir (Weizmann Institute of Science)
Plan of Lectures

1. Symmetries
2. QCD
3. The leptonic SM
4. The Standard Model
5. The SM as an EFT
   - EW precision measurements
   - Flavor physics
   - Neutrino masses
6. Summary
Symmetries
The Lagrangian

\[ \mathcal{L}[\phi_i(x), \partial_\mu \phi_i(x)] \]

- A function of the fields and their derivatives only
- Depends on the fields taken at one space-time point \( x^\mu \) only
- Real
- Invariant under the Poincaré group
- Analytic function in the fields
- Invariant under certain internal symmetry groups
- Natural
- (Renormalizable)
The Lagrangian: Examples

- The most general renormalizable $\mathcal{L}_{\phi,\psi}$:
  \[ \mathcal{L}(\phi, \psi) = \mathcal{L}_{\text{kin}} + \mathcal{L}_\psi + \mathcal{L}_\phi + \mathcal{L}_{\text{Yuk}} \]

- Real scalar $\phi$:
  \[ \mathcal{L}_S = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{m^2}{2} \phi^2 - \frac{\mu}{2\sqrt{2}} \phi^3 - \frac{\lambda}{4} \phi^4 \]

- Dirac fermion $\psi$:
  \[ \mathcal{L}_F = i \bar{\psi} \dot{\psi} - m \bar{\psi} \psi \]

- A single Dirac fermion and a single real scalar:
  \[ \mathcal{L}(\phi, \psi) = \mathcal{L}_S + \mathcal{L}_F + \mathcal{L}_{\text{Yuk}}; \quad \mathcal{L}_{\text{Yuk}} = -Y \bar{\psi}_L \psi_R \phi + \text{h.c.} \]

- A single fermion charged under a local $U(1)$ symmetry:
  \[ \mathcal{L}_{\text{kin}} = i \bar{\psi} \partial \psi - \frac{1}{4} F^{\mu \nu} F_{\mu \nu}; \]
  \[ D^\mu = \partial^\mu + ie q_\psi A^\mu, \quad F^{\mu \nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \]
Symmetries

**Imposed vs. Accidental**

- **Symmetry** = invariance properties of the Lagrangian
- *Imposed symmetries* are the starting point of model building. In these lectures, we will see how imposed symmetries lead to predictions that can be tested in experiments.
- *Accidental symmetries* are a result of (i) the imposed symmetries, (ii) the particle content, (iii) renormalizability. In general they are broken by nonrenormalizable terms and thus expected to approximately hold in low energy experiments.
## Symmetries and their consequences

<table>
<thead>
<tr>
<th>Type</th>
<th>Consequences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spacetime</td>
<td>Conservation of $E$, $P$, $L$</td>
</tr>
<tr>
<td>Discrete</td>
<td>Selection rules</td>
</tr>
<tr>
<td>Global (exact)</td>
<td>Conserved charges</td>
</tr>
<tr>
<td>Global (spon. broken)</td>
<td>Massless scalars</td>
</tr>
<tr>
<td>Local (exact)</td>
<td>Interactions, massless spin-1 mediators</td>
</tr>
<tr>
<td>Local (spon. broken)</td>
<td>Interactions, massive spin-1 mediators</td>
</tr>
</tbody>
</table>
Symmetries and fermion masses

- Dirac mass
  - $m_D \overline{\psi}_L \psi_R + h.c.$
  - Allowed only for fermions in a vector-like representation
  - Forbidden for fermions in a chiral representation
  - Dirac fermion has 4 degrees of freedom

- Majorana mass
  - $m_M \overline{\psi}_R^c \psi_R$, $\psi^c = C\psi^T$
  - Allowed only for fermions that are neutral under $U(1)$ or in a real rep of $SU(N)$
  - Forbidden for charged [complex rep] fermions under $U(1)$ [$SU(N)$]
  - Majorana fermion has 2 degrees of freedom
Symmetries

Defining a model

- The symmetry;
- The transformation properties of the fermions and scalars;
- The pattern of spontaneous symmetry breaking (SSB)
Symmetries

Analyzing a model

- Write down the most general $\mathcal{L}$
- Extract the spectrum
- Obtain the interactions among the mass eigenstates
- Accidental symmetries
- Count and identify the parameters
- Experimental tests
QCD: Quarks and $SU(3)C$
Defining the QCD model

- The symmetry is a local $SU(3)_C$
- Fermions: $Q_{Li}(3), \quad Q_{Ri}(3), \quad i = 1, \ldots, 6$
- No scalars, no SSB
\( \text{SU}(3)_C \)

- Eight generators: \( L_1, \ldots, 8 \):
  \[
  [L_a, L_b] = i f_{abc} L_c
  \]
- A single coupling constants:
  \( g_s \)
- Eight gauge boson degrees of freedom:
  \( G_\mu^a \) (8)
- Field strengths:
  \[
  G_\mu^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f_{abc} G_\mu^b G_\nu^c
  \]
- The covariant derivative:
  \[
  D_\mu = \partial_\mu + i g_s G_\mu^a L_a
  \]
- For \( \text{SU}(3) \)-triplets:
  \[
  L_a = \frac{1}{2} \lambda_a \text{ with } \lambda_a = \text{The } 3 \times 3 \text{ Gell-Mann matrices} 
  \]
$\mathcal{L}_{\text{kin}}$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} + i Q_L \bar{\psi} Q_L + i Q_R \bar{\psi} Q_R$$

- $D^\mu Q_L = (\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a) Q_L$
- $D^\mu Q_R = (\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a) Q_R$
\( \mathcal{L}_\psi \cdot \mathcal{L}_\phi, \mathcal{L}_{\text{Yuk}} \)

\[
\mathcal{L}_\psi = -\bar{Q}_L M^Q_{ij} Q_R^j + \text{h.c.}
\]

- \( Q_L(3), Q_R(3) = \) vector representation;
  - Dirac mass allowed

- \( Q_L(3), Q_R(3) = \) complex representation of \( SU(3) \);
  - No Majorana mass

- Without loss of generality, can choose a basis where
  \( M^Q = \text{diag}(m_u, m_d, m_s, m_c, m_b, m_t) \)
\( \mathcal{L}_{\psi} \mathcal{L}_\phi, \mathcal{L}_{\text{Yuk}} \)

\[
\mathcal{L}_\psi = -\overline{Q}_L M^Q_{ij} Q_R + \text{h.c.}
\]

- \( Q_L(3), Q_R(3) = \text{vector representation; Dirac mass allowed} \)
- \( Q_L(3), Q_R(3) = \text{complex representation of } SU(3); \text{ No Majorana mass} \)
- Without loss of generality, can choose a basis where \( M^Q = \text{diag}(m_u, m_d, m_s, m_c, m_b, m_t) \)
- No scalars: \( \mathcal{L}_{\text{Yuk}} = 0, \mathcal{L}_\phi = 0 \)
QCD

\[ \mathcal{L}_{\text{QCD}} \]

- Define a Dirac fermion \( q = (Q_L, Q_R)^T \)
- \( q = u, d, s, c, b, t \)

\[
\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G_\mu^a G_{a\mu
u} + i\bar{q} \slashed{D} q - m_q \bar{q} q
\]
The spectrum

- A massless gluon (color-octet)
- Six massive Dirac fermions (color-triplets)
The interactions

- Gluon-fermions interactions:
  \[ -\frac{g_s}{2} \bar{q} \gamma_\mu \lambda_a G^a_\mu q \]

- Gluon self-interactions:
  \[ g_s f_{abc} (\partial^\mu G_a^\nu) G_b^\mu G_c^\nu + g_s^2 (f_{abc} G_b^\mu G_c^\nu) (f_{ade} G_d^\mu G_e^\nu) \]

- Experiment: \( \alpha_s(m_Z^2) = 0.1185 \pm 0.0006 \)

- \( g_s \downarrow \) for \( E \uparrow \)
  - Perturbative QCD successful at \( E \gg GeV \)

- \( g_s \uparrow \) for \( E \downarrow \)
  - Calculations difficult for \( E \lesssim GeV \)
  - Confinement: Quarks and gluons are bound in hadrons
The strong interactions

The strong interactions are:

- Vectorial
- Parity-conserving
- Diagonal
- Universal
Hadrons

- We do not observe free quarks in Nature
- All asymptotic states are singlets of $SU(3)_C$
- Hadrons = bound states of quarks and gluons
- Three types of hadrons:
  - Mesons: $M = q\bar{q}$
  - Baryons: $B = qqq$
  - Antibaryons: $\bar{B} = q\bar{q}\bar{q}$
Accidental symmetries

- $\mathcal{L}_{\text{kin}}$ has a large accidental symmetry:
  $G_{\text{QCD}}^{\text{global}} (M_Q = 0) = U(6)_Q^L \times U(6)_Q^R$

- The quark masses break this symmetry to a subgroup:
  $G_{\text{QCD}}^{\text{global}} = U(1)_u \times U(1)_d \times U(1)_s \times U(1)_c \times U(1)_b \times U(1)_t$

- All quarks are stable;
  (Of course, quarks are not stable, e.g. $b \rightarrow c\bar{c}s$
  $\implies$ QCD is an incomplete model of quark interactions)

- $u\bar{u} \rightarrow t\bar{t}$ allowed; $u\bar{t} \rightarrow t\bar{u}$ forbidden
### Counting parameters

- \( M^Q \implies 36_R + 36_I \) parameters
- \([U(6)]^2 \rightarrow [U(1)]^6\)
  \(\implies (2 \times 15)_R + (2 \times 21 - 6)_I\) parameters can be removed
- 6\(R\) + 0\(I\) physical parameters;
  6 quark masses
- Experiments:
  - \( m_u = 2.3^{+0.7}_{-0.5}, \; m_d = 4.8^{+0.5}_{-0.3}, \; m_s = 95 \pm 5 \) [MeV]
  - \( m_c = 1.27 \pm 0.03, \; m_b = 4.18 \pm 0.03, \; m_t = 173.2 \pm 0.9 \) [GeV]

- The QCD model is a seven parameter model:
  \( \alpha_s, m_u, m_d, m_s, m_c, m_b, m_t \)
LSM: Leptons and $SU(2)_L \times U(1)_Y$
Defining the LSM

- The symmetry is a local $SU(2)_L \times U(1)_Y$
- Fermions: $L_{Li}(2)_{-1/2}$, $E_{Ri}(1)_{-1}$, $i = 1, 2, 3$
- Scalars: $\phi(2)_{+1/2}$
- SSB: $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$ where $Q_{EM} = T_3 + Y$
\[ SU(2)_L \times U(1)_Y \]

- Four generators: \( T_{1,2,3}, \ Y: \)
  \[
  [T_a, T_b] = i\epsilon_{abc}T_c, \quad [T_a, Y] = 0
  \]
- Two coupling constants:
  \( g \) for \( SU(2) \) couplings; \( g' \) for \( U(1) \) coupling
- Four gauge boson degrees of freedom:
  \( W^\mu_a(3)_0, \ B^\mu(1)_0 \)
- Field strengths:
  \[
  W^{\mu\nu}_a = \partial^\mu W^\nu_a - \partial^\nu W^\mu_a - g\epsilon_{abc}W^\mu_bW^\nu_c, \quad B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu
  \]
- The covariant derivative:
  \[
  D^\mu = \partial^\mu + igW^\mu_aT_a + ig'Y B^\mu
  \]
- For \( SU(2) \)-doublets:
  \( T_a = \frac{1}{2}\sigma_a \) with \( \sigma_a = \) The \( 2 \times 2 \) Pauli matrices
\[ \mathcal{L}_{\text{kin}} = -\frac{1}{4} W_a^{\mu\nu} W_{a\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + i \bar{L}_L \gamma L_L + i \bar{E}_R \gamma E_R + (D^\mu \phi)^\dagger (D_\mu \phi) \]

- \[ D^\mu L_L = (\partial^\mu + \frac{i}{2} g W_a^{\mu \sigma} \sigma_a - \frac{i}{2} g' B^\mu) L_L \]
- \[ D^\mu E_R = (\partial^\mu - ig' B^\mu) E_R \]
- \[ D^\mu \phi = (\partial^\mu + \frac{i}{2} g W_a^{\mu \sigma} \sigma_a + \frac{i}{2} g' B^\mu) \phi \]
\[ \mathcal{L}_\psi = 0 \]

- \( L_L(2)_{-1/2}, E_R(1)_{-1} = \text{chiral representation} \)
  No Dirac mass

- \( L_L(2)_{-1/2}, E_R(1)_{-1} = \text{charged under } U(1)_Y \)
  No Majorana mass
\[ \mathcal{L}_{\text{Yuk}} \]

\[ \mathcal{L}_{\text{Yuk}} = Y^e_{ij} \overline{L}_i E_{Rj} \phi + \text{h.c.} \]

- \( i, j = 1, 2, 3 \) = flavor indices
- \( Y^e \) is a general complex \( 3 \times 3 \) matrix of dimensionless couplings
- Without loss of generality, can choose a basis where \( Y^e = \text{diag}(y_e, y_\mu, y_\tau) \)
\[ \mathcal{L}_\phi = -\mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \]

- \(\lambda\) dimensionless and real;
  \(\lambda > 0\) for the potential to be bounded from below

- \(\mu^2\) is of mass dimension 2 and real;
  \(\mu^2 < 0\) required for SSB
\[ \mathcal{L}_\phi = -\mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \]

- Define \( v^2 \equiv -\mu^2 / \lambda \)
- \( \mathcal{L}_\phi = -\lambda \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2 \)
- \( \implies |\langle \phi \rangle| = v / \sqrt{2} \)
- \( \implies \text{SSB } SU(2) \times U(1) \rightarrow U(1) \)
- \( \langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \implies Q_{EM} = T_3 + Y \text{ conserved} \)
A technical point

- \( \phi \) has 4 degrees of freedom

- A convenient choice: 
  \[
  \phi(x) = \exp \left[ i \frac{\sigma_i}{2} \theta^i(x) \right] \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}
  \]

- \( \theta^{1,2,3} \) represent the three would-be Goldstone bosons that are eaten by the three gauge bosons that acquire masses as a result of the SSB

- The local \( SU(2)_L \) symmetry allows one to rotate away any dependence on the three \( \theta^i \)

- The unitary gauge: 
  \[
  \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}
  \]
\begin{align*}
\mathcal{L}_{\text{LSM}} &= -\frac{1}{4} W^{\mu\nu}_b W_{b\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + (D^{\mu} \phi)^\dagger (D_\mu \phi) \\
&\quad + i \overline{L}_L i \overline{\phi} \overline{L}_L i + i \overline{E}_R i \overline{\phi} \overline{E}_R i \\
&\quad + (Y_{ij}^e \overline{L}_L i \overline{E}_R j \phi + \text{h.c.}) \\
&\quad - \lambda \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2
\end{align*}
The scalar spectrum

- $h$ - a single real massive scalar degree of freedom
- $m_h = \sqrt{2\lambda v}$
- Experiment: $m_h = 125.09 \pm 0.21 \pm 0.11$ GeV
The vector boson spectrum I

- Three broken generators $\Rightarrow$ Three massive vector bosons
- $(D_\mu \phi) \dagger (D^\mu \phi)$ contains terms $\propto v^2$:

$$
\mathcal{L}_{VM} = \frac{1}{8} (0 \ v) \begin{pmatrix}
g W_3 + g' B & g(W_1 - iW_2) \\
g(W_1 + iW_2) & -g W_3 + g' B
\end{pmatrix} \dagger
\times
\begin{pmatrix}
g W_3 + g' B & g(W_1 - iW_2) \\
g(W_1 + iW_2) & -g W_3 + g' B
\end{pmatrix} \begin{pmatrix}0 \\ v\end{pmatrix}
$$

- $W_1, W_2$ do not have a well defined $Q_{EM}$; $W_3, B$ are not mass eigenstates
The vector boson spectrum II

- $W^\pm = \frac{1}{\sqrt{2}} (W_1 \mp iW_2)$
- Define $\tan \theta_W \equiv g'/g$
  - $Z^0 = \cos \theta_W W_3 - \sin \theta_W B$
  - $A^0 = \sin \theta_W W_3 + \cos \theta_W B$
- $\mathcal{L}_{VM} = \frac{1}{4} g^2 v^2 W^+ W^- + \frac{1}{8} (g^2 + g'^2) v^2 Z^0 Z^0$
- $m_{W}^2 = \frac{1}{4} g^2 v^2$, $m_{Z}^2 = \frac{1}{4} (g^2 + g'^2) v^2$, $m_{A}^2 = 0$
- $m_A = 0$ a result of $U(1)_{EM}$ gauge invariance;
  A consistency check of our calculation
The $\rho = 1$ relation

- $\tan \theta_W \equiv g'/g$
  \[ \implies \theta_W \text{ can be extracted from various weak interaction rates} \]

- $\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1$
  \[ \implies \theta_W \text{ can be extracted from the spectrum} \]

- $\rho = 1$ is a consequence of the SSB by scalar doublets

- $m_W = 80.385 \pm 0.015 \text{ GeV}; \; m_Z = 91.1876 \pm 0.0021 \text{ GeV}$
  \[ \implies \sin^2 \theta_W = 1 - (m_W/m_Z)^2 = 0.2229 \pm 0.0004 \]
The fermion spectrum I

- SSB allows us to tell the $T_3 = \pm 1/2$ components of the doublets:
  \[
  \begin{pmatrix}
  \nu_{eL} \\
  e_L
  \end{pmatrix}, \quad
  \begin{pmatrix}
  \nu_{\mu L} \\
  \mu L
  \end{pmatrix}, \quad
  \begin{pmatrix}
  \nu_{\tau L} \\
  \tau L
  \end{pmatrix}
  \]

- $\mathcal{L}_{\text{Yuk}}$ contains terms $\propto v$:
  \[
  \mathcal{L}_{FM} = -\frac{y_e v}{\sqrt{2}} \bar{e}_L e_R - \frac{y_{\mu} v}{\sqrt{2}} \bar{\mu}_L \mu_R - \frac{y_{\tau} v}{\sqrt{2}} \bar{\tau}_L \tau_R + \text{h.c.}
  \]

- $m_e = \frac{y_e v}{\sqrt{2}}$, $m_{\mu} = \frac{y_{\mu} v}{\sqrt{2}}$, $m_{\tau} = \frac{y_{\tau} v}{\sqrt{2}}$

- Experiment:
  - $m_e = 0.510998928(11)$ MeV
  - $m_{\mu} = 105.6583715(35)$ MeV
  - $m_{\tau} = 1776.82(16)$ MeV
The fermion spectrum II

- The crucial point: While the leptons are in a chiral rep of $SU(2)_L \times U(1)_Y$, the charged leptons – $e, \mu, \tau$ – are in a vector rep of $U(1)_{EM}$ and thus can acquire Dirac masses

- $\nu_\alpha$ are neutral under $U(1)_{EM}$
  $\implies$ A-priori, the possibility of Majorana masses is not closed

- $m_\nu \neq 0$ requires VEV carried by a scalar in the $(3)_{+1}$ rep, but there is no such scalar in the SM

- The neutrinos are massless in this model: $m_{\nu_\alpha} = 0$ (at least at tree level)

- The $\nu$’s are degenerate $\implies$ Any interaction basis is also a $\nu$ mass basis, but only a single interaction basis is an $\ell^\pm$ mass basis;
  $\nu_e, \nu_\mu, \nu_\tau \equiv$ The $SU(2)_L$ partners of $e_L, \mu_L, \tau_L$
## Summary: The LSM particles

<table>
<thead>
<tr>
<th>particle</th>
<th>spin</th>
<th>$Q$</th>
<th>mass (theo) $[\nu]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^\pm$</td>
<td>1</td>
<td>±1</td>
<td>$\frac{1}{2}g$</td>
</tr>
<tr>
<td>$Z^0$</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{2}\sqrt{g^2 + g'^2}$</td>
</tr>
<tr>
<td>$A^0$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$h$</td>
<td>0</td>
<td>0</td>
<td>$\sqrt{2\lambda}$</td>
</tr>
<tr>
<td>$e$</td>
<td>1/2</td>
<td>−1</td>
<td>$y_e/\sqrt{2}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1/2</td>
<td>−1</td>
<td>$y_\mu/\sqrt{2}$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1/2</td>
<td>−1</td>
<td>$y_\tau/\sqrt{2}$</td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\nu_\mu$</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\nu_\tau$</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
The Higgs boson interactions I

\[
\mathcal{L}_h = \frac{1}{2} \partial_{\mu} h \partial^{\mu} h - \frac{1}{2} m_h^2 h^2 - \frac{m_h^2}{2v} h^3 - \frac{m_h^2}{8v^2} h^4 \\
+ m_W^2 W^- W^\mu \left( \frac{2h}{v} + \frac{h^2}{v^2} \right) + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \left( \frac{2h}{v} + \frac{h^2}{v^2} \right) \\
- \frac{h}{v} \left( m_e \bar{e}_L e_R + m_\mu \bar{\mu}_L \mu_R + m_\tau \bar{\tau}_L \tau_R + h.c. \right)
\]

- The dimensionless couplings (\(hhhh\), \(hhVV\), \(h\ell\ell\)) are unchanged from the symmetry limit
- The dimensionful couplings (\(hhh\), \(hVV\)) arise from the SSB but do not introduce new parameters
- Neither \(hAA\) nor \(hhAA\) coupling \(\leftarrow Q_{EM}(h) = 0, \ m_A = 0\)
The Higgs boson interactions II

- All of the Higgs couplings can be written in terms of the masses of the particles to which it couples.
- The heavier a particle, the stronger its coupling to $h$.
- Experiment:

![Graph showing the relationship between mass (m) and coupling (\lambda) for different particles.]

- The Yukawa couplings are diagonal (to be discussed later).
Electromagnetic interactions I

- The coupling of neutral bosons:
  \[ \propto g W_3 T_3 + g' B Y \]

- Rotate to the mass basis:
  \[ A(g s_W T_3 + g' c_W Y) + Z(g c_W T_3 - g' s_W Y) \]

- The photon field couples to \( e Q = e(T_3 + Y) \), so
  \[ g = e/s_W, \quad g' = e/c_W \]

- The electromagnetic interactions are described by
  \[ \mathcal{L}_{\text{QED}} = e A_\mu \overline{\ell}_i \gamma^\mu \ell_i \]

- Experiment \( (\alpha \equiv e^2/4\pi) \)
  \[ \alpha^{-1} = 137.035999074 \pm 0.000000044 \]
Electromagnetic interactions II

The electromagnetic interactions are:

- Vectorial
- Parity-conserving
- Diagonal: $A$ couples to $e^+e^-, \mu^+\mu^-, \tau^+\tau^-$ but not to $e^{\pm}\mu^{\mp}, e^{\pm}\tau^{\mp}, \mu^{\pm}\tau^{\mp}$ pairs; a result of local $U(1)_{\text{EM}}$
- Universal: The couplings to the different generations are universal; a result of local $U(1)_{\text{EM}}$
NC weak interactions I

- The $Z$ couplings to general fermions:
  \[ \frac{e}{s_Wc_W}(T_3 - s_W^2 Q)\bar{\psi}Z\psi \]

- The $Z$ couplings to the LSM leptons:
  \[
  \mathcal{L}_{NC} = \frac{e}{s_Wc_W} \left[ - \left( \frac{1}{2} - s_W^2 \right) \bar{e_L} Z e_L + s_W^2 \bar{e_R} Z e_R + \frac{1}{2} \bar{\nu_{eL}} Z \nu_{eL} \\
  - \left( \frac{1}{2} - s_W^2 \right) \bar{\mu_L} Z \mu_L + s_W^2 \bar{\mu_R} Z \mu_R + \frac{1}{2} \bar{\nu_{\muL}} Z \nu_{\muL} \\
  - \left( \frac{1}{2} - s_W^2 \right) \bar{\tau_L} Z \tau_L + s_W^2 \bar{\tau_R} Z \tau_R + \frac{1}{2} \bar{\nu_{\tauL}} Z \nu_{\tauL} \right]
  \]

- $Z$-exchange gives rise to neutral current weak interactions
NC weak interactions II

The neutral current weak interactions are:

- Chiral
- Parity-violating
- Diagonal: a special feature of the LSM
- Universal: a special feature of the LSM

Diagonality and Universality ⇔ All fermions of a given chirality and a given charge come from the same $SU(2) \times U(1)$ rep
NCWI: experimental tests

- **Universality**
  \[ \frac{\Gamma(Z \to \mu^+ \mu^-)}{\Gamma(Z \to e^+ e^-)} = 1.0009 \pm 0.0028 \]
  \[ \frac{\Gamma(Z \to \tau^+ \tau^-)}{\Gamma(Z \to e^+ e^-)} = 1.0019 \pm 0.0032 \]

- **Diagonality**
  \[ \text{BR}(Z \to e^\pm \mu^\mp) < 1.7 \times 10^{-6} \]
  \[ \text{BR}(Z \to e^\pm \tau^\mp) < 9.8 \times 10^{-6} \]
  \[ \text{BR}(Z \to \mu^\pm \tau^\mp) < 1.2 \times 10^{-5} \]

- **Interactions ⇔ Spectrum**
  \[ \frac{\text{BR}(Z \to \ell^+ \ell^-)}{\text{BR}(Z \to \nu_\ell \bar{\nu}_\ell)} = 1 - 4 \sin^2 \theta_W + 8 \sin^4 \theta_W = 0.505 \]
  \[ \implies \sin^2 \theta_W = 0.226 \]
CC weak interactions I

- The $W$ couplings to a leptons:
  \[ \mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \left[ \bar{\nu}_e e_L - \nu_e W^+ e_L + \bar{\nu}_\mu \mu_L - \nu_\mu W^+ \mu_L + \bar{\nu}_\tau \tau_L - \nu_\tau W^+ \tau_L + \text{h.c.} \right] \]

- $W$-exchange gives rise to charged current weak interactions.
CC weak interactions II

The charged current weak interactions are:

- Only left-handed leptons
- Parity-violating
- Diagonal: a special feature of the LSM
- Universal: a special feature of the LSM

Diagonality and Universality $\Leftrightarrow$ The degeneracy of the neutrinos
CCWI: experimental tests

- **Universality**
  - \( \frac{\Gamma(W^+ \to \mu^+\nu_\mu)}{\Gamma(W^+ \to e^+\nu_e)} = 0.98 \pm 0.02 \)
  - \( \frac{\Gamma(W^+ \to \tau^+\nu_\tau)}{\Gamma(W^+ \to e^+\nu_e)} = 1.04 \pm 0.02 \)

- **Interactions ⇔ Spectrum**
  - Define \( G_F \equiv \frac{g^2}{4\sqrt{2}m_W^2} = \frac{\pi\alpha}{\sqrt{2}s_W^2 m_W^2} \)
  - Experiment: \( G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2} \)
  - \( \Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} f \left( \frac{m_e^2}{m_\mu^2} \right) \Rightarrow \sin^2 \theta_W = 0.215 \)
  - \( \nu = (\sqrt{2}G_F)^{-1/2} = 246 \text{ GeV} \)
### Summary: The LSM Interactions

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Force Carrier</th>
<th>Coupling</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electromagnetic</td>
<td>$\gamma$</td>
<td>$eQ$</td>
<td>Long</td>
</tr>
<tr>
<td>NC weak</td>
<td>$Z^0$</td>
<td>$\frac{e(T_3 - s_W^2 Q)}{s_W c_W}$</td>
<td>Short</td>
</tr>
<tr>
<td>CC weak</td>
<td>$W^\pm$</td>
<td>$g$</td>
<td>Short</td>
</tr>
<tr>
<td>Yukawa</td>
<td>$h$</td>
<td>$y_\ell$</td>
<td>Short</td>
</tr>
</tbody>
</table>
Accidental symmetries

- $L_{\text{kin}}$ has an accidental symmetry:
  $$G_{\text{LSM}}^{\text{global}}(Y^e = 0) = U(3)_L \times U(3)_E$$

- The Yukawa couplings break this symmetry to a subgroup:
  $$G_{\text{LSM}}^{\text{global}} = U(1)_e \times U(1)_\mu \times U(1)_\tau$$

- $\mu^- \rightarrow e^- \nu_e \nu_\mu$ allowed; $\mu^- \rightarrow e^- e^+ e^-$ forbidden;
  $e^+ e^- \rightarrow \mu^+ \mu^-$ allowed; $e^+ \mu^- \rightarrow \mu^+ e^-$ forbidden

- $U(1)_L$ forbids Majorana masses to neutrinos;
  $$m_\nu = 0 \text{ to all orders in perturbation theory}$$

- $G_{\text{LSM}}^{\text{global}}$ completely broken by nonrenormalizable terms:
  $$(1/\Lambda)L_i L_j \phi \phi$$ (to be discussed later)
**Counting the lepton sector parameters**

- \( Y^e \implies 9_R + 9_I \) parameters

- \( U(3)_L \times U(3)_E \to U(1)_e \times U(1)_\mu \times U(1)_\tau \)
  \[ \implies (2 \times 3)_R + (2 \times 6 - 3)_I \] parameters can be removed

- \( 3_R + 0_I \) physical parameters:
  3 charged lepton masses

- The LSM is a seven parameter model:
  \( g, g', \nu, m_h, m_e, m_\mu, m_\tau \)
The Standard Model
Defining the SM

- The symmetry is a local $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Quarks: $Q_{Li}(3,2)_{+1/6}$, $U_{Ri}(3,1)_{+2/3}$, $D_{Ri}(3,1)_{-1/3}$
  Leptons: $L_{Li}(1,2)_{-1/2}$, $E_{Ri}(1,1)_{-1}$; $(i = 1, 2, 3)$
- Scalars: $\phi(1,2)_{+1/2}$
- SSB: $SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{EM}$
$SU(3)_C \times SU(2)_L \times U(1)_Y$

• Twelve generators:
  
  eight $L_a$ ($SU(3)_C$), three $T_b$ ($SU(2)_L$), a single $Y$ ($U(1)_Y$)
  
  $[L_a, L_b] = i f_{abc} L_c$,  
  $[T_a, T_b] = i \epsilon_{abc} T_c$,
  $[L_a, T_b] = [L_a, Y] = [T_b, Y] = 0$

• Three coupling constants:
  
  $g_s$ for $SU(3)_C$; $g$ for $SU(2)_L$; $g'$ for $U(1)_Y$

• Twelve gauge bosons:
  
  $G^\mu_a (8,1)_0$,  
  $W^\mu_b (1,3)_0$,  
  $B^\mu (1,1)_0$

• The covariant derivative:
  
  $D^\mu = \partial^\mu + ig_s G^\mu_a L_a + ig W^\mu_a T_a + ig' Y B^\mu$
  
  $- SU(3)_C$: $L_a = \frac{1}{2} \lambda_a(0)$ for triplets (singlets)
  
  $- SU(2)_L$: $T_b = \frac{1}{2} \sigma_b(0)$ for doublets (singlets)
\[ \mathcal{L}_{\text{kin}} = -\frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} - \frac{1}{4} W_b^{\mu\nu} W_{a\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + (D^\mu \phi)^\dagger (D_\mu \phi) \\
+ i\overline{Q}_{Li} \not{\! \partial} Q_{Li} + i\overline{U}_{Ri} \not{\! \partial} U_{Ri} + i\overline{D}_{Ri} \not{\! \partial} D_{Ri} + i\overline{L}_{Li} \not{\! \partial} L_{Li} + i\overline{E}_{Ri} \not{\! \partial} E_{Ri} \]

- \( D^\mu Q_L = (\partial^\mu + \frac{i}{2} g_s G_a^{\mu} \lambda_a + \frac{i}{2} g W_b^{\mu} \sigma_b + \frac{i}{6} g' B^\mu) Q_L \)
- \( D^\mu U_R = (\partial^\mu + \frac{i}{2} g_s G_a^{\mu} \lambda_a + \frac{2i}{3} g' B^\mu) U_R \)
- \( D^\mu D_R = (\partial^\mu + \frac{i}{2} g_s G_a^{\mu} \lambda_a - \frac{i}{3} g' B^\mu) D_R \)
- \( D^\mu L_L = (\partial^\mu + \frac{i}{2} g W_a^{\mu} \sigma_a - \frac{i}{2} g' B^\mu) L_L \)
- \( D^\mu E_R = (\partial^\mu - ig' B^\mu) E_R \)
- \( D^\mu \phi = (\partial^\mu + \frac{i}{2} g W_a^{\mu} \sigma_a + \frac{i}{2} g' B^\mu) \phi \)
$\mathcal{L}_\psi$

$\mathcal{L}_\psi = 0$

- Quarks:
  - $Q_L, U_R, D_R = \text{chiral representation}$
    No Dirac mass
  - $Q_L, U_R, D_R = \text{charged under } U(1)_Y$
    No Majorana mass

- Leptons: same as in the LSM
\[ \mathcal{L}_{\text{Yuk}} = Y_{ij}^u \overline{Q} L_i U_{Rj} \tilde{\phi} + Y_{ij}^d \overline{Q} L_i D_{Rj} \phi + Y_{ij}^e \overline{L} L_i E_{Rj} \phi + \text{h.c.} \]

- \( Y^u, Y^d, Y^e \): general complex 3 \( \times \) 3 matrices of dimensionless couplings
- Without loss of generality, can choose a basis,
  \( Y^e \rightarrow V_{eL} Y^e V_{eR}^\dagger = \hat{Y}^e \), where \( \hat{Y}^e = \text{diag}(y_e, y_\mu, y_\tau) \)
- Without loss of generality, can choose a basis,
  \( Y^u \rightarrow \hat{Y}^u = V_{uL} Y^u V_{uR}^\dagger \), where \( \hat{Y}^u = \text{diag}(y_u, y_c, y_t) \)
- Without loss of generality, can choose a basis,
  \( Y^d \rightarrow \hat{Y}^d = V_{dL} Y^d V_{dR}^\dagger \), where \( \hat{Y}^d = \text{diag}(y_d, y_s, y_b) \)
- Unless \( V_{uL} = V_{dL} \), the basis with \( \hat{Y}^u \) is different from the basis with \( \hat{Y}^d \).
The CKM matrix

- Define \( V = V_{uL}V_{dL}^\dagger \)
- In the basis where \( Y^u = \hat{Y}^u \), we have \( Y^d = V\hat{Y}^d \)
- In the basis where \( Y^d = \hat{Y}^d \), we have \( Y^u = V^\dagger\hat{Y}^u \)
- Note: \( V_{uL}, V_{uR}, V_{dL}, V_{dR} \) depend on the basis from which we start. \( V \), however, does not
- \( V \) plays a crucial role in the charged current weak interactions
\[ \mathcal{L}_\phi = -\mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \]

- Choosing \( \mu^2 < 0 \) and \( \lambda > 0 \) leads to SSB with \( |\langle \phi \rangle| = v/\sqrt{2} \)
- \( \phi = SU(3)_C \) singlet \( \implies \) \( SU(3)_C \) remains unbroken
- \( SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{EM} \)
\[ \mathcal{L}_{\text{SM}} = - \frac{1}{4} G_{a}^{\mu\nu} G_{a\mu\nu} - \frac{1}{4} W_{b}^{\mu\nu} W_{b\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + (D^{\mu} \phi)^\dagger (D_{\mu} \phi) \\
+ i \bar{Q}_{Li} \not{D} Q_{Li} + i \bar{U}_{Ri} \not{D} U_{Ri} + i \bar{D}_{Ri} \not{D} D_{Ri} + i \bar{L}_{Li} \not{D} L_{Li} + i \bar{E}_{Ri} \not{D} E_{Ri} \\
+ \left( Y_{ij}^{u} \bar{Q}_{Li} U_{Rj} \not{\phi} + Y_{ij}^{d} \bar{Q}_{Li} D_{Rj} \phi + Y_{ij}^{e} \bar{L}_{Li} E_{Rj} \phi + \text{h.c.} \right) \\
- \lambda (\phi^\dagger \phi - v^2 / 2)^2 \]
The boson spectrum

- Local $SU(3)_C \times U(1)_{\text{EM}}$ symmetry
  $\Rightarrow$ A massless color-octet gluon, a massless neutral photon

- SSB of $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{EM}}$
  $\Rightarrow$ Three massive weak vector bosons $W^\pm, Z^0$

- SSB by an $SU(2)$ doublet
  $\Rightarrow \rho \equiv m_W^2/(m_Z^2 \cos^2 \theta_W) = 1$

- Three would be Goldstone bosons eaten by $W^\pm, Z^0$
  $\Rightarrow$ A single massive Higgs boson
The fermion spectrum

- All charged fermions acquire Dirac masses, \( m_f = \frac{y_f v}{\sqrt{2}} \);
  While in chiral reps of \( SU(2)_L \times U(1)_Y \), they are in vectorial reps of \( SU(3)_C \times U(1)_{EM} \):
  - LH and RH \( e, \mu, \tau \): \((1)_{-1}\)
  - LH and RH \( u, c, t \): \((3)_{+2/3}\)
  - LH and RH \( d, s, b \): \((3)_{-1/3}\)

- Neutrinos are massless in spite of being in the \((1)_0\) rep
### Summary: The SM particles

<table>
<thead>
<tr>
<th>Particle</th>
<th>Spin</th>
<th>Color</th>
<th>$Q$</th>
<th>Mass $[\nu]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^\pm$</td>
<td>1</td>
<td>(1)</td>
<td>$\pm 1$</td>
<td>$\frac{1}{2} g$</td>
</tr>
<tr>
<td>$Z^0$</td>
<td>1</td>
<td>(1)</td>
<td>0</td>
<td>$\frac{1}{2} \sqrt{g^2 + g'^2}$</td>
</tr>
<tr>
<td>$A^0$</td>
<td>1</td>
<td>(1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$g$</td>
<td>1</td>
<td>(8)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$h$</td>
<td>0</td>
<td>(1)</td>
<td>0</td>
<td>$\sqrt{2\lambda}$</td>
</tr>
<tr>
<td>$e, \mu, \tau$</td>
<td>$1/2$</td>
<td>(1)</td>
<td>$-1$</td>
<td>$y_{e,\mu,\tau}/\sqrt{2}$</td>
</tr>
<tr>
<td>$\nu_e, \nu_\mu, \nu_\tau$</td>
<td>$1/2$</td>
<td>(1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$u, c, t$</td>
<td>$1/2$</td>
<td>(3)</td>
<td>$+2/3$</td>
<td>$y_{u,c,t}/\sqrt{2}$</td>
</tr>
<tr>
<td>$d, s, b$</td>
<td>$1/2$</td>
<td>(3)</td>
<td>$-1/3$</td>
<td>$y_{d,s,b}/\sqrt{2}$</td>
</tr>
</tbody>
</table>
The Higgs boson interactions

\[
L_h = \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{2} m_h^2 h^2 - \frac{m_h^2}{2v} h^3 - \frac{m_h^2}{8v^2} h^4 \\
+ m_W^2 W^- W^\mu+ \left( \frac{2h}{v} + \frac{h^2}{v^2} \right) + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \left( \frac{2h}{v} + \frac{h^2}{v^2} \right) \\
- \frac{h}{v} (m_e \overline{e}_L e_R + m_\mu \overline{\mu}_L \mu_R + m_\tau \overline{\tau}_L \tau_R \\
+ m_u \overline{u}_L u_R + m_c \overline{c}_L c_R + m_t \overline{t}_L t_R \\
+ m_d \overline{d}_L d_R + m_s \overline{s}_L s_R + m_b \overline{b}_L b_R + \text{h.c.}) .
\]

- The Higgs boson couples diagonally also to the quark mass eigenstates
- The Higgs couplings are not universal: \( y_f \propto m_f \)
Diagonality of Yukawa interactions

\[
h \overline{D}_L Y^d D_R = h \overline{D}_L (V^\dagger_{dL} V_{dL}) Y^d (V^\dagger_{dR} V_{dR}) D_R \\
= h (D_L V^\dagger_{dL}) (V_{dL} Y^d V^\dagger_{dR}) (V_{dR} D_R) \\
= h (\overline{d}_L \overline{s}_L \overline{b}_L) \hat{Y}^d (d_R s_R b_R)^T
\]

- The diagonality is due to two ingredients of the SM:
  - All SM fermions are chiral \( \implies \) no bare mass terms
  - The scalar sector has a single Higgs doublet

- Experiment:
  \[
  \text{BR}(t \rightarrow qh) < 0.8 \times 10^{-2} \quad [\text{ATLAS, JHEP06(2014)008; CMS PAS TOP-13-017}]
  \]
  \[
  \text{BR}(h \rightarrow \tau\mu) < 1.5 \times 10^{-2} \quad [\text{CMS, 1502.07400}]
  \]
\[ y \propto m \]

- All of the Higgs couplings can be written in terms of the masses of the particles to which it couples
- The heavier a particle, the stronger its coupling to \( h \)
- Experiment:

![Graph showing the relationship between \( \lambda \) and \( m \) in GeV](image-url)
Strong and electromagnetic interactions

- Local $SU(3)_C \times U(1)_{EM}$
  $\Rightarrow$ Strong and EM interactions are universal

- Strong interactions: The gluons couple all colored particles
  - Quarks = color-triplets $\Rightarrow$ have strong interactions
  - Leptons = color-singlets $\Rightarrow$ do not couple to gluons
  - $\mathcal{L}_{QCD, \text{fermions}} = -\frac{1}{2} g_S \bar{q} \lambda_a \not{G}_a q \quad (q = u, c, t, d, s, b)$

- EM interactions: The photon couples to the EM charge
  - $u, d, e$ are charged $\Rightarrow$ have EM interactions
  - $\nu$ are neutral $\Rightarrow$ do not couple to the photon
  - $\mathcal{L}_{QED, \text{fermions}} = -e \bar{e}_i A e_i + \frac{2e}{3} \bar{u}_i A u_i - \frac{e}{3} \bar{d}_i A d_i$
    $(e_i = e, \mu, \tau; \; u_i = u, c, t; \; d_i = d, s, b)$
NC weak interactions

- The $Z$ couplings in each generation:

$$\mathcal{L}_{\text{NC}} = \frac{e}{s_W c_W} \left[ - \left( \frac{1}{2} - s_W^2 \right) \bar{e}_L \gamma^\mu e_L + s_W^2 \bar{e}_R \gamma^\mu e_R + \frac{1}{2} \bar{\nu}_L \gamma^\mu \nu_L ight]$$

$$+ \left( \frac{1}{2} - \frac{2}{3} s_W^2 \right) \bar{u}_L \gamma^\mu u_L - \frac{2}{3} s_W^2 \bar{u}_R \gamma^\mu u_R$$

$$- \left( \frac{1}{2} - \frac{1}{3} s_W^2 \right) \bar{d}_L \gamma^\mu d_L + \frac{1}{3} s_W^2 \bar{d}_R \gamma^\mu d_R \right]$$

- Chiral, parity-violating, diagonal, universal

- Universality $\iff$ All fermions in the same $SU(3)_C \times U(1)_{\text{EM}}$ rep come from the same $SU(3)_C \times SU(2)_L \times U(1)_Y$ rep
NCWI: further experimental tests

- SM:

\[ \Gamma(Z \to \nu\bar{\nu}) \propto 1, \]
\[ \Gamma(Z \to \ell\bar{\ell}) \propto 1 - 4s_W^2 + 8s_W^4, \]
\[ \Gamma(Z \to u\bar{u}) \propto 3 \left[ 1 - \left(\frac{8}{3}\right)s_W^2 + \left(\frac{32}{9}\right)s_W^4 \right], \]
\[ \Gamma(Z \to d\bar{d}) \propto 3 \left[ 1 - \left(\frac{4}{3}\right)s_W^2 + \left(\frac{8}{9}\right)s_W^4 \right], \]

- Experiments:

\[ \text{BR}(Z \to \nu\bar{\nu}) = (6.67 \pm 0.02)\%, \]
\[ \text{BR}(Z \to \ell\bar{\ell}) = (3.37 \pm 0.01)\%, \]
\[ \text{BR}(Z \to u\bar{u}) = (11.6 \pm 0.6)\%, \]
\[ \text{BR}(Z \to d\bar{d}) = (15.6 \pm 0.4)\% \]

Fine! (with \( s_W^2 = 0.225 \))
CC weak interactions I

- For leptons, things are simple because there exists an interaction basis that is also a mass basis

- Leptonic $W$ interactions are universal in the lepton mass basis:
  \[ \mathcal{L}_{CC}^{\text{leptons}} = -\frac{g}{\sqrt{2}} \left[ \nu_{eL} W^+ e^-_L + \nu_{\mu L} W^+ \mu^-_L + \nu_{\tau L} W^+ \tau^-_L + \text{h.c.} \right] \]

- For quarks, things are more complicated since there is no interaction basis that is also a mass basis

- Quark $W$ interactions (using the CKM matrix):
  \[ \mathcal{L}_{CC}^{\text{quarks}} = -\frac{g}{\sqrt{2}} \left( \bar{u}_L \, \bar{c}_L \, \bar{t}_L \right) V \, W^+ \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.} \]

- $V =$ the CKM matrix: $3 \times 3$, unitary, $3_R + 1_I$ parameters
CC weak interactions II

- Only left-handed particles take part in the CC interactions
- Parity is violated
- $W$ couplings to quark mass eigenstates:
  - neither universal nor diagonal
- Universality of gauge interactions hidden in the unitarity of $V$
CCWI: further experimental tests

- SM:
  \[ \Gamma(W^+ \to \ell^+ \nu_\ell) \propto 1 \]
  \[ \Gamma(W^+ \to u_i \bar{d}_j) \propto 3|V_{ij}|^2 \quad (i = 1, 2; \ j = 1, 2, 3) \]

- CKM unitarity:
  \[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1 \]
  \[ \Rightarrow \quad \Gamma(W \to \text{hadrons}) \approx 2\Gamma(W \to \text{leptons}) \]
  \[ \Rightarrow \quad \Gamma(W \to cX)/\Gamma(W \to \text{hadrons}) \approx 0.5 \]

- Experiments:
  \[ \Gamma(W \to \text{hadrons})/\Gamma(W \to \text{leptons}) = 2.09 \pm 0.01 \]
  \[ \Gamma(W \to cX)/\Gamma(W \to \text{hadrons}) = 0.49 \pm 0.04 \]

- Flavor physics and the CKM matrix:
  A topic on its own
Summary: The SM quark interactions

<table>
<thead>
<tr>
<th>interaction</th>
<th>force carrier</th>
<th>coupling</th>
<th>range</th>
</tr>
</thead>
<tbody>
<tr>
<td>electromagnetic</td>
<td>$\gamma$</td>
<td>$eQ$</td>
<td>long</td>
</tr>
<tr>
<td>Strong</td>
<td>$G$</td>
<td>$g_s$</td>
<td>long</td>
</tr>
<tr>
<td>NC weak</td>
<td>$Z^0$</td>
<td>$\frac{e(T_3^s - s_W^2 Q)}{s_W c_W}$</td>
<td>short</td>
</tr>
<tr>
<td>CC weak</td>
<td>$W^\pm$</td>
<td>$gV$</td>
<td>short</td>
</tr>
<tr>
<td>Yukawa</td>
<td>$h$</td>
<td>$y_q$</td>
<td>short</td>
</tr>
</tbody>
</table>
Accidental symmetries

- $\mathcal{L}_{\text{kin}}$ has an accidental symmetry:
  $G_{\text{SM}}^{\text{global}}(Y_{u,d,e} = 0) = U(3)_Q \times U(3)_U \times U(3)_D \times U(3)_L \times U(3)_E$

- The Yukawa couplings break this symmetry to a subgroup:
  $G_{\text{SM}}^{\text{global}} = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$

- $U(1)_B$ forbids proton decay (*e.g.* $p \to \pi^0 e^+, \ p \to K^+ \nu$);
  ($U(1)_B$ is anomalous, but still $\Delta B = \Delta L = 3n$ is respected)

- $U(1)_{B-L}$ forbids Majorana masses to neutrinos;
  $m_\nu = 0$ to all orders in perturbation theory and non-perturbatively

- LFV forbidden (*e.g.* $\mu \to e\gamma$, $\tau \to \mu\mu\mu$);
  Neutrino oscillations violate $U(1)_e \times U(1)_\mu \times U(1)_\tau$
Counting the quark sector parameters

- $Y^u, Y^d \implies 18_R + 18_I$ parameters

- $U(3)_Q \times U(3)_U \times U(3)_D \rightarrow U(1)_B$
  $\implies (3 \times 3)_R + (3 \times 6 - 1)_I$ parameters can be removed

- $9_R + 1_I$ physical parameters:
  6 quark masses, 3 CKM angles, 1 CKM phase

- Experiment:
  - $|V_{us}| = 0.2253 \pm 0.0008$
  - $|V_{cb}| = 0.041 \pm 0.001$
  - $|V_{ub}| = 0.0041 \pm 0.0005$
  - $\sin 2\beta = 0.68 \pm 0.02$
Comments on CP violation

- The KM phase $\Rightarrow$ CP is violated in the SM

- If there were only two generations:
  \[2(4R + 4I) - [3(1R + 3I) - 1I] = 5R + 0I\]
  4 quark masses, 1 Cabibbo angle
  $\Rightarrow$ CP is an accidental symmetry of a two generation SM

- An additional allowed term: $\theta_{QCD} \epsilon_{\mu\nu\rho\sigma} G^{\mu\nu} G^{\rho\sigma}$
  CP violation by strong interactions
  Experiment (EDM): $\theta_{QCD} \lesssim 10^{-10}$
The SM as an EFT
SM = low energy effective theory

- The SM is not a full theory of Nature
- The SM is a low energy effective theory;
  Valid below some scale $\Lambda(\gg m_Z)$
- $\mathcal{L}_{\text{SM}}$ should be extended:
  \[
  \mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} O_{d=5} + \frac{1}{\Lambda^2} O_{d=6} + \cdots
  \]
- $O_{d=n}$ = operators that are
  - Products of SM fields
  - Transforming as singlets under $SU(3)_C \times SU(2)_L \times U(1)_Y$
  - Of dimension $n$ in the fields
- For physics at $E \ll \Lambda$, the effects of $O_{d=n(>4)}$ suppressed by $(E/\Lambda)^{n-4}$
- The larger $n(>4)$, the smaller the effect at low energy
Nonrenormalizable terms, loops

- **Tree level processes**: Often tree level processes in a particular sector depend on a small subset of the SM parameters $\Rightarrow$ Relations among various processes that are violated by loop effects and nonrenormalizable terms
  - Example: Electroweak precision measurements (EWPM)

- **Rare processes**: Processes not allowed at tree level, often related to accidental symmetries of a particular sector. Nonrenormalizable terms and loops can contribute.
  - Example: Flavor changing neutral currents (FCNC)

- **Forbidden processes**: Nonrenormalizable terms (but not loop corrections!) can break accidental symmetries and allow forbidden processes
  - Example: Neutrino masses
Before and after

- Rare processes and tree level processes:
  - Before all the SM particles have been directly discovered and all the SM parameters measured:
    Assume the validity of the renormalizable SM and indirectly measure the properties of the yet unobserved particles; \( m_c, m_t, m_h \) predicted in this way
  - Once all the SM particles observed and the parameters measured directly:
    The loop corrections can be quantitatively determined; Effects of nonrenormalizable terms unambiguously probed
- All three categories are used to search for new physics
**EWPM - before and after**

- At tree level, all EW processes depend on only 3 parameters: $g, g', v \Leftrightarrow \alpha, m_Z, G_F$
- Of the other 15 parameters, 11 are small and have negligible effects on EWPM
- Of the remaining four, $\delta_{KM}$ and $\alpha_S$ have negligible effects
- Only $m_t/v$ and $m_h/v$ have significant quantum effects
- In the past: EWPM used to predict $m_t$ and $m_h$
- At present: EWPM probe nonrenormalizable operators (= BSM physics)
In a large class of models, only four dim=6 operators contribute significantly to EWPM:

\[ \mathcal{L}_{\text{o.c.}} = \frac{1}{\Lambda^2} \left( c_{WB} \mathcal{O}_{WB} + c_{HH} \mathcal{O}_{HH} + c_{BB} \mathcal{O}_{BB} + c_{WW} \mathcal{O}_{WW} \right) \]

\[ \mathcal{O}_{WB} = (H^\dagger \tau^a H) W^a_{\mu\nu} B_{\mu\nu} \rightarrow \frac{1}{2} v^2 W^3_{\mu\nu} B_{\mu\nu}; \]
\[ \mathcal{O}_{HH} = |H^\dagger D_\mu H|^2 \rightarrow \frac{1}{16} v^4 (g W^3_\mu - g' B_\mu)^2; \]
\[ \mathcal{O}_{BB} = (\partial_\rho B_{\mu\nu})^2; \]
\[ \mathcal{O}_{WW} = (D_\rho W^a_{\mu\nu})^2 \]
EWPM - experiment

- Low energy observables: $G_F$, $\alpha$, neutrino scattering, DIS, APV, low-energy $e^+e^-$ scattering
- High energy observables: masses, total widths and partial decay rates of the $W$ and $Z$ bosons
\[ S, T \]

\[ S = \frac{2 \sin 2\theta_W}{\alpha} \frac{v^2}{\Lambda^2} c_{WB} \]
\[ T = -\frac{1}{2\alpha} \frac{v^2}{\Lambda^2} c_{HH} \]

\[ \frac{\Lambda}{\sqrt{c_{WB}}} > 9.7 \left( \frac{0.14}{S} \right) \text{ TeV} \]
\[ \frac{\Lambda}{\sqrt{c_{HH}}} > 4.4 \left( \frac{0.20}{T} \right) \text{ TeV} \]
Flavors = Copies of the same $SU(3)_C \times U(1)_{EM}$ representation:

- **Up-type quarks** $(3)_{+2/3}$: $u, c, t$
- **Down-type quarks** $(3)_{-1/3}$: $d, s, b$
- **Charged leptons** $(1)_{-1}$: $e, \mu, \tau$
- **Neutrinos** $(1)_0$: $\nu_1, \nu_2, \nu_3$

**Flavor changing neutral current (FCNC) processes:**

- Flavor changing processes that involve either $U$ or $D$ but not both and/or either $\ell^-$ or $\nu$ but not both

- $\mu \rightarrow e\gamma$; $K \rightarrow \pi \nu\bar{\nu}$ ($s \rightarrow d\nu\bar{\nu}$); $D^0 - \bar{D}^0$ mixing ($c\bar{u} \rightarrow u\bar{c}$)...
FCNC: Loop suppression I

- The $W$-boson cannot mediate FCNC process at tree level since it couples to up-down pairs; Only neutral bosons can potentially mediate FCNC at tree level
- Massless gauge bosons have flavor-universal and, in particular, flavor diagonal couplings; The gluons and the photon do not mediate FCNC at tree level

What about $Z$? $h$?
FCNC: Loop suppression II

- Within the SM, the $Z$-boson does not mediate FCNC at tree level because all fermions with the same chirality, color and charge originate in the same $SU(2)_L \times U(1)_Y$ representation.

- Within the SM, the $h$-boson does not mediate FCNC at tree level because
  - All SM fermions are chiral $\implies$ no bare mass terms
  - The scalar sector has a single Higgs doublet

Within the SM, all FCNC processes are loop suppressed
**FCNC: CKM- and GIM-suppression**

- All FC processes \( \propto \) off-diagonal entries in the CKM matrix
  - \( \Gamma(b \to s\gamma) \propto |V_{tb}V_{ts}|^2 \sim 3 \times 10^{-3} \)
  - \( \Delta m_B \propto |V_{tb}V_{td}|^2 \sim 10^{-4} \)

- If all quarks in a given sector were degenerate
  \[ \implies \text{No FC } W\text{-couplings} \]

- FCNC in the down (up) sector
  \( \propto \Delta m^2 \) between the quarks of the up (down) sector

- The GIM-suppression effective for processes involving the first two generations
  - \( \Delta m_K \propto (m_c^2 - m_u^2)/m_W^2 \) (\( \implies \) was used to predict \( m_c \))
  - \( \Delta m_B \propto (m_t^2 - m_c^2)/m_W^2 \) (\( \implies \) was used to predict \( m_t \))
FCNC - experiment

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m_K/m_K$</td>
<td>$7.0 \times 10^{-15}$</td>
</tr>
<tr>
<td>$\Delta m_D/m_D$</td>
<td>$8.7 \times 10^{-15}$</td>
</tr>
<tr>
<td>$\Delta m_B/m_B$</td>
<td>$6.3 \times 10^{-14}$</td>
</tr>
<tr>
<td>$\Delta m_{B_s}/m_{B_s}$</td>
<td>$2.1 \times 10^{-12}$</td>
</tr>
<tr>
<td>$\epsilon_K$</td>
<td>$2.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>$A_\Gamma/y_{CP}$</td>
<td>$\leq 0.2$</td>
</tr>
<tr>
<td>$S_{\psi K_S}$</td>
<td>$0.67 \pm 0.02$</td>
</tr>
<tr>
<td>$S_{\psi \phi}$</td>
<td>$-0.04 \pm 0.09$</td>
</tr>
</tbody>
</table>
High Scale? Degeneracy and Alignment?

- \( \frac{z_{sd}}{\Lambda_{NP}^2} (\overline{d}_L \gamma_\mu s_L)^2 + \frac{z_{cu}}{\Lambda_{NP}^2} (\overline{c}_L \gamma_\mu u_L)^2 + \frac{z_{bd}}{\Lambda_{NP}^2} (\overline{d}_L \gamma_\mu b_L)^2 + \frac{z_{bs}}{\Lambda_{NP}^2} (\overline{s}_L \gamma_\mu b_L)^2 \)

- For \( |z_{ij}| \sim 1, \quad \Im(z_{ij}) \sim 1: \)

| Mixing | \( \Lambda_{NP}^{CPC} \gtrsim \) | \( \Lambda_{NP}^{CPV} \gtrsim \) | | Mixing | \( \Lambda_{NP}^{CPC} \gtrsim \) | \( \Lambda_{NP}^{CPV} \gtrsim \) |
|--------|-----------------|-----------------| |-----------------|-----------------|-----------------|
| \( K - \overline{K} \) | 1000 TeV | 200000 TeV | | \( D - \overline{D} \) | 1000 TeV | 3000 TeV |
| \( B - \overline{B} \) | 400 TeV | 800 TeV | | \( B_s - \overline{B_s} \) | 70 TeV | 200 TeV |

- For \( \Lambda_{NP} \sim 1 \text{ TeV} \):

| Mixing | \( |z_{ij}| \lesssim \) | \( \Im(z_{ij}) \lesssim \) | | Mixing | \( |z_{ij}| \lesssim \) | \( \Im(z_{ij}) \lesssim \) |
|--------|-----------------|-----------------| |-----------------|-----------------|-----------------|
| \( K - \overline{K} \) | \( 8 \times 10^{-7} \) | \( 6 \times 10^{-9} \) | | \( D - \overline{D} \) | \( 5 \times 10^{-7} \) | \( 1 \times 10^{-7} \) |
| \( B - \overline{B} \) | \( 5 \times 10^{-6} \) | \( 1 \times 10^{-6} \) | | \( B_s - \overline{B_s} \) | \( 2 \times 10^{-4} \) | \( 2 \times 10^{-5} \) |
$m_\nu$ - theory

- SM: $m_\nu = 0$ to all orders in perturbation theory and non-perturbatively
- Guaranteed by the accidental $U(1)_{B-L}$ symmetry
- d=5 terms $Z_{ij}^\nu \frac{\phi\phi L_i L_j}{\Lambda}$ break $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau \rightarrow U(1)_B$
The $\nu$SM

$$\mathcal{L}_{\nu\text{SM}} = \mathcal{L}_{\text{SM}} + \frac{Z^\nu_{ij}}{\Lambda} \phi \phi L_i L_j$$

- $\langle \phi^0 \rangle = v/\sqrt{2} \implies$ Majorana mass matrix for neutrinos:
  $$m_\nu = \frac{v^2}{\Lambda} \frac{Z^\nu}{2}$$

- $m_\nu$ can be diagonalized by a unitary transformation:
  $$V_{\nu L} m_\nu V_{\nu L}^T = \hat{m}_\nu = \text{diag}(m_1, m_2, m_3)$$

Predictions:

- $m_\nu \neq 0$
- $m_\nu/m_{q,\ell}\pm \sim v/\Lambda \ll 1$
- $\mathcal{L}_{\text{CC}}^\ell = -\frac{g}{\sqrt{2}} (\bar{\ell}_{L\alpha} W^- U_{\alpha i} \nu_i + \text{h.c.}); U \neq 1$
The $\nu$SM parameters

$$\mathcal{L}_{\nu SM} = \mathcal{L}_{SM} + \frac{Z_{ij}^\nu}{\Lambda} \phi \phi L_i L_j$$

- We added to the SM $6_R + 6_I$ parameters ($Z^\nu$ is symmetric)
- We “lost” $[U(1)]^3$ symmetry $\implies$ Can remove $3_I$ parameters;
- Conclusion: $6_R + 3_I$ new parameters
- 3 neutrino masses;
  3 angles and 3 phases in the leptonic mixing matrix
- Experiment: Gonzalez-Garcia et al., 1409.5439
- $\Delta m^2_{21} = (7.5 \pm 0.2) \times 10^{-5}$ eV$^2$, $|\Delta m^2_{32}| = (2.5 \pm 0.1) \times 10^{-3}$ eV$^2$
- $|U_{e2}| = 0.55 \pm 0.01$, $|U_{\mu 3}| = 0.67 \pm 0.03$, $|U_{e3}| = 0.148 \pm 0.003$
The $\nu$SM - summary

\[ \mathcal{L}^{\nu}_{\nu SM} = i\bar{\nu}_i \partial \nu_i + \frac{g}{2c_W} \bar{\nu}_i Z \nu_i \rightleftharpoons \frac{g}{\sqrt{2}} \left( \ell L\alpha W^- U_{\alpha i} \nu_i + \text{h.c.} \right) \\
+ m_i \nu_i \nu_i + \frac{2m_i}{v} h \nu_i \nu_i + \frac{m_i}{v^2} hh \nu_i \nu_i \]

<table>
<thead>
<tr>
<th>particle</th>
<th>spin</th>
<th>color</th>
<th>$Q$</th>
<th>mass [v]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_1, \nu_1, \nu_3$</td>
<td>1/2</td>
<td>(1)</td>
<td>0</td>
<td>$z_i v/(2\Lambda)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>interaction</th>
<th>force carrier</th>
<th>coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC weak</td>
<td>$Z^0$</td>
<td>$e/(2s_W c_W)$</td>
</tr>
<tr>
<td>CC weak</td>
<td>$W^\pm$</td>
<td>$gU/\sqrt{2}$</td>
</tr>
<tr>
<td>Yukawa</td>
<td>$h$</td>
<td>$2m/v$</td>
</tr>
</tbody>
</table>

- Accidental symmetry: $U(1)_B$
Summary
Summary I: definition

- The symmetry is a local $SU(3)_C \times SU(2)_L \times U(1)_Y$

- Quarks: $Q_{Li}(3,2)_{+1/6}, U_{Ri}(3,1)_{+2/3}, D_{Ri}(3,1)_{-1/3}$
  Leptons: $L_{Li}(1,2)_{-1/2}, E_{Ri}(1,1)_{-1}; \ (i = 1, 2, 3)$

- Scalars: $\phi(1,2)_{+1/2}$

- SSB: $SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{EM}$
Summary II: spectrum

<table>
<thead>
<tr>
<th>particle</th>
<th>spin</th>
<th>color</th>
<th>$Q$</th>
<th>mass $[\nu]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^\pm$</td>
<td>1</td>
<td>(1)</td>
<td>±1</td>
<td>$\frac{1}{2}g$</td>
</tr>
<tr>
<td>$Z^0$</td>
<td>1</td>
<td>(1)</td>
<td>0</td>
<td>$\frac{1}{2}\sqrt{g^2 + g'^2}$</td>
</tr>
<tr>
<td>$A^0$</td>
<td>1</td>
<td>(1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$g$</td>
<td>1</td>
<td>(8)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$h$</td>
<td>0</td>
<td>(1)</td>
<td>0</td>
<td>$\sqrt{2\lambda}$</td>
</tr>
<tr>
<td>$e, \mu, \tau$</td>
<td>1/2</td>
<td>(1)</td>
<td>−1</td>
<td>$y_{e,\mu,\tau}/\sqrt{2}$</td>
</tr>
<tr>
<td>$\nu_e, \nu_\mu, \nu_\tau$</td>
<td>1/2</td>
<td>(1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$u, c, t$</td>
<td>1/2</td>
<td>(3)</td>
<td>+2/3</td>
<td>$y_{u,c,t}/\sqrt{2}$</td>
</tr>
<tr>
<td>$d, s, b$</td>
<td>1/2</td>
<td>(3)</td>
<td>−1/3</td>
<td>$y_{d,s,b}/\sqrt{2}$</td>
</tr>
</tbody>
</table>
### Summary III: interactions

<table>
<thead>
<tr>
<th>interaction</th>
<th>force carrier</th>
<th>coupling</th>
<th>range</th>
</tr>
</thead>
<tbody>
<tr>
<td>electromagnetic</td>
<td>$\gamma$</td>
<td>$eQ$</td>
<td>long</td>
</tr>
<tr>
<td>Strong</td>
<td>$g$</td>
<td>$g_s$</td>
<td>long</td>
</tr>
<tr>
<td>NC weak</td>
<td>$Z^0$</td>
<td>$\frac{e(T_3-s_w^2 Q)}{s_w c_w}$</td>
<td>short</td>
</tr>
<tr>
<td>CC weak</td>
<td>$W^{\pm}$</td>
<td>$gV$</td>
<td>short</td>
</tr>
<tr>
<td>Yukawa</td>
<td>$h$</td>
<td>$y_q$</td>
<td>short</td>
</tr>
</tbody>
</table>
Summary IV: parameters

There are eighteen independent parameters:

- $g_s, g, g', \nu, \lambda \mapsto \alpha_s, \alpha, m_Z, G_F, m_h$
- $y_e, y_\mu, y_\tau \mapsto m_e, m_\mu, m_\tau$
- $y_u, y_c, y_t, y_d, y_s, y_b \mapsto m_u, m_c, m_t, m_d, m_s, m_b$
- $|V_{us}|, |V_{cb}|, |V_{ub}|, \delta_{KM} \mapsto \lambda, A, \rho, \eta$
Summary V: accidental symmetries

$U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$

- Explains the non-observation of proton decay
- Explains the non-observation of FCNC charged lepton decays
- Violated in neutrino oscillations
  \[ \Rightarrow \text{The SM is a low energy effective theory, } \Lambda_{NP} \lesssim 10^{15} \text{ GeV} \]
Summary VI: successes

  1676 pages of experimental results
  Almost all consistent with the SM predictions
Summary VII: problems

- Neutrino masses
- Dark matter
- Baryon asymmetry
- Fine tuning
  - $m_h^2$
  - $\theta_{QCD}$