The Standard Model

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Plan of Lectures

1. Symmetries
2. QCD
3. The leptonic SM
4. The Standard Model
5. The SM as an EFT
   - EW precision measurements
   - Flavor physics
   - Neutrino masses
6. Summary

Symmetries

The Lagrangian

$$\mathcal{L}[\phi_i(x), \partial_\mu \phi_i(x)]$$

- A function of the fields and their derivatives only
- Depends on the fields taken at one space-time point x^μ only
- Real
- Invariant under the Poincaré group
- Analytic function in the fields
- Invariant under certain internal symmetry groups
- Natural
- (Renormalizable)

The Lagrangian: Examples

- The most general renormalizable $\mathcal{L}_{\phi,\psi}$:
  $\mathcal{L}(\phi,\psi) = \mathcal{L}_{\rm kin} + \mathcal{L}_\psi + \mathcal{L}_\phi + \mathcal{L}_{\rm Yuk}$
- Real scalar ϕ:
  $\mathcal{L}_S = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{m^2}{2}\phi^2 - \frac{\mu}{2\sqrt{2}}\phi^3 - \frac{\lambda}{4}\phi^4$
- Dirac fermion ψ:
  $\mathcal{L}_F = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi$
- A single Dirac fermion and a single real scalar:
  $\mathcal{L}(\phi,\psi) = \mathcal{L}_S + \mathcal{L}_F + \mathcal{L}_{\rm Yuk}; \quad \mathcal{L}_{\rm Yuk} = -Y\overline{\psi_L}\psi_R\phi + {\rm h.c.}$
- A single fermion charged under a local $U(1)$ symmetry:
  $\mathcal{L}_{\rm kin} = i\bar{\psi}D\!\!\!\!/\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu};$
  $D^\mu = \partial^\mu + ieq_\psi A^\mu, \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

Imposed *vs.* Accidental

- Symmetry = invariance properties of the Lagrangian
- *Imposed symmetries* are the starting point of model building. In these lectures, we will see how imposed symmetries lead to predictions that can be tested in experiments.
- *Accidental symmetries* are a result of (i) the imposed symmetries, (ii) the particle content, (iii) renormalizability. In general they are broken by nonrenormalizable terms and thus expected to approximately hold in low energy experiments.

Symmetries and their consequences

Type	Consequences
Spacetime	Conservation of E, P, L
Discrete	Selection rules
Global (exact)	Conserved charges
Global (spon. broken)	Massless scalars
Local (exact)	Interactions, massless spin-1 mediators
Local (spon. broken)	Interactions, massive spin-1 mediators

Symmetries and fermion masses

- Dirac mass
  - $m_D \overline{\psi_L} \psi_R + \text{h.c.}$
  - Allowed only for fermions in a vector-like representation
    Forbidden for fermions in a chiral representation
  - Dirac fermion has 4 degrees of freedom
- Majorana mass
  - $m_M \overline{\psi^c_R} \psi_R, \quad \psi^c = C\overline{\psi}^T$
  - Allowed only for fermions that are neutral under $U(1)$ or in a real rep of $SU(N)$
    Forbidden for charged [complex rep] fermions under $U(1)$ [$SU(N)$]
  - Majorana fermion has 2 degrees of freedom

Defining a model

- The symmetry;
- The transformation properties of the fermions and scalars;
- The pattern of spontaneous symmetry breaking (SSB)

Analyzing a model

- Write down the most general $\mathcal{L}$
- Extract the spectrum
- Obtain the interactions among the mass eigenstates
- Accidental symmetries
- Count and identify the parameters
- Experimental tests

QCD: Quarks and $SU(3)_C$

Defining the QCD model

- The symmetry is a local $SU(3)_C$
- Fermions: $Q_{Li}(3), \quad Q_{Ri}(3), \quad i = 1, \ldots, 6$
- No scalars, no SSB

$SU(3)_C$

- Eight generators: $L_{1,\ldots,8}$:
  $[L_a, L_b] = i f_{abc} L_c$
- A single coupling constants:
  g_s
- Eight gauge boson degrees of freedom:
  G_a^μ (8)
- Field strengths:
  $G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f_{abc} G_b^\mu G_c^\nu$
- The covariant derivative:
  $D^\mu = \partial^\mu + i g_s G_a^\mu L_a$
- For $SU(3)$-triplets:
  $L_a = \frac{1}{2}\lambda_a$ with λ_a = The 3×3 Gell-Mann matrices

$\mathcal{L}_{\rm kin}$

$$\mathcal{L}_{\rm kin} = -\frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} + i\overline{Q_{Li}} \not{D} Q_{Li} + i\overline{Q_{Ri}} \not{D} Q_{Ri}$$

- $D^\mu Q_L = \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a\right) Q_L$
- $D^\mu Q_R = \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a\right) Q_R$

$\mathcal{L}_\psi.\mathcal{L}_\phi, \mathcal{L}_{\text{Yuk}}$

$$\mathcal{L}_\psi = -\overline{Q_{Li}} M^Q_{ij} Q_{Rj} + \text{h.c.}$$

- $Q_L(3),\ Q_R(3)$ = vector representation;
  Dirac mass allowed
- $Q_L(3),\ Q_R(3)$ = complex representation of $SU(3)$;
  No Majorana mass
- Without loss of generality, can choose a basis where
  $M^Q = \text{diag}(m_u, m_d, m_s, m_c, m_b, m_t)$

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- Without loss of generality, can choose a basis where
  $M^Q = \text{diag}(m_u, m_d, m_s, m_c, m_b, m_t)$
- No scalars: $\boxed{\mathcal{L}_{\text{Yuk}} = 0, \quad \mathcal{L}_\phi = 0}$

$\mathcal{L}_{\rm QCD}$

- Define a Dirac fermion $q = (Q_L, Q_R)^T$
- $q = u, d, s, c, b, t$

$$\boxed{\mathcal{L}_{\rm QCD} = -\frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} + i\bar{q}\not{D}q - m_q \bar{q}q}$$

The spectrum

- A massless gluon (color-octet)
- Six massive Dirac fermions (color-triplets)

The interactions

- Gluon-fermions interactions:
  $-\frac{g_s}{2}\overline{q}\lambda_a\gamma_\mu G^\mu_a q$
- Gluon self-interactions:
  $g_s f_{abc}(\partial^\mu G^\nu_a)G^\mu_b G^\nu_c \ + g_s^2(f_{abc}G^\mu_b G^\nu_c)(f_{ade}G^\mu_d G^\nu_e)$
- Experiment: $\alpha_s(m_Z^2) = 0.1185 \pm 0.0006$
- $g_s \downarrow$ for $E \uparrow$
  - Perturbative QCD successful at $E \gg GeV$
- $g_s \uparrow$ for $E \downarrow$
  - Calculations difficult for $E \lesssim GeV$
  - Confinement: Quarks and gluons are bound in hadrons

The strong interactions

The strong interactions are:

- Vectorial
- Parity-conserving
- Diagonal
- Universal

Hadrons

- We do not observe free quarks in Nature
- All asymptotic states are singlets of $SU(3)_C$
- Hadrons = bound states of quarks and gluons
- Three types of hadrons:
  - Mesons: $M = q\bar{q}$
  - Baryons: $B = qqq$
  - Antibaryons: $\bar{B} = \bar{q}\bar{q}\bar{q}$

Accidental symmetries

- $\mathcal{L}_{\text{kin}}$ has a large accidental symmetry:
  $G^{\text{global}}_{\text{QCD}}(M^Q = 0) = U(6)_{Q_L} \times U(6)_{Q_R}$
- The quark masses break this symmetry to a subgroup:
  $$\boxed{G^{\text{global}}_{\text{QCD}} = U(1)_u \times U(1)_d \times U(1)_s \times U(1)_c \times U(1)_b \times U(1)_t}$$
- All quarks are stable;
  (Of course, quarks are not stable, *e.g.* $b \to c\bar{c}s$
  $\Longrightarrow$ QCD is an incomplete model of quark interactions)
- $u\bar{u} \to t\bar{t}$ allowed; $u\bar{t} \to t\bar{u}$ forbidden

Counting parameters

- $M^Q \implies 36_R + 36_I$ parameters
- $[U(6)]^2 \to [U(1)]^6$
  $\implies (2 \times 15)_R + (2 \times 21 - 6)_I$ parameters can be removed
- $6_R + 0_I$ physical parameters;
  6 quark masses
- Experiments:
  - $m_u = 2.3^{+0.7}_{-0.5},\ m_d = 4.8^{+0.5}_{-0.3},\ m_s = 95 \pm 5$ [MeV]
  - $m_c = 1.27 \pm 0.03,\ m_b = 4.18 \pm 0.03,\ m_t = 173.2 \pm 0.9$ [GeV]
- The QCD model is a seven parameter model:
  $\alpha_s, m_u, m_d, m_s, m_c, m_b, m_t$

LSM: Leptons and $SU(2)_L \times U(1)_Y$

Defining the LSM

- The symmetry is a local $SU(2)_L \times U(1)_Y$
- Fermions: $L_{Li}(2)_{-1/2}, \quad E_{Ri}(1)_{-1}, \quad i = 1, 2, 3$
- Scalars: $\phi(2)_{+1/2}$
- SSB: $SU(2)_L \times U(1)_Y \ \to \ U(1)_{\rm EM}$ where $Q_{\rm EM} = T_3 + Y$

$SU(2)_L \times U(1)_Y$

- Four generators: $T_{1,2,3}, \quad Y$:
  $[T_a, T_b] = i\epsilon_{abc} T_c, \quad [T_a, Y] = 0$
- Two coupling constants:
  g for $SU(2)$ couplings; g' for $U(1)$ coupling
- Four gauge boson degrees of freedom:
  $W_a^\mu(3)_0, \ B^\mu(1)_0$
- Field strengths:
  $W_a^{\mu\nu} = \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g\epsilon_{abc} W_b^\mu W_c^\nu, \ B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$
- The covariant derivative:
  $D^\mu = \partial^\mu + igW_a^\mu T_a + ig' Y B^\mu$
- For $SU(2)$-doublets:
  $T_a = \frac{1}{2}\sigma_a$ with $\sigma_a =$ The 2×2 Pauli matrices

$\mathcal{L}_{\text{kin}}$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} W_a^{\mu\nu} W_{a\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + i\overline{L_{Li}} \not{D} L_{Li} + i\overline{E_{Ri}} \not{D} E_{Ri} + (D^\mu \phi)^\dagger (D_\mu \phi)$$

- $D^\mu L_L = \left(\partial^\mu + \frac{i}{2} g W_a^\mu \sigma_a - \frac{i}{2} g' B^\mu\right) L_L$
- $D^\mu E_R = \left(\partial^\mu - i g' B^\mu\right) E_R$
- $D^\mu \phi = \left(\partial^\mu + \frac{i}{2} g W_a^\mu \sigma_a + \frac{i}{2} g' B^\mu\right) \phi$

$\mathcal{L}_\psi$

$$\boxed{\mathcal{L}_\psi = 0}$$

- $L_L(2)_{-1/2},\ E_R(1)_{-1}$ = chiral representation
  No Dirac mass
- $L_L(2)_{-1/2},\ E_R(1)_{-1}$ = charged under $U(1)_Y$
  No Majorana mass

$\mathcal{L}_{\text{Yuk}}$

$$\boxed{\mathcal{L}_{\text{Yuk}} = Y^e_{ij} \overline{L_{Li}} E_{Rj} \phi + \text{h.c.}}$$

- $i, j = 1, 2, 3$ = flavor indices
- Y^e is a general complex 3×3 matrix of dimensionless couplings
- Without loss of generality, can choose a basis where $Y^e = \text{diag}(y_e, y_\mu, y_\tau)$

$\mathcal{L}_\phi$

$$\boxed{\mathcal{L}_\phi = -\mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2}$$

- λ dimensionless and real;
  $\lambda > 0$ for the potential to be bounded from below
- μ^2 is of mass dimension 2 and real;
  $\mu^2 < 0$ required for SSB

$\mathcal{L}_\phi$ and SSB

$$\boxed{\mathcal{L}_\phi = -\mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2}$$

- Define $v^2 \equiv -\mu^2/\lambda$
- $\mathcal{L}_\phi = -\lambda \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2$
- $\Longrightarrow |\langle \phi \rangle| = v/\sqrt{2}$
- $\Longrightarrow$ SSB $SU(2) \times U(1) \to U(1)$
- $\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \Longrightarrow Q_{\mathrm{EM}} = T_3 + Y$ conserved

A technical point

- ϕ has 4 degrees of freedom
- A convenient choice: $\phi(x) = \exp\left[i\frac{\sigma_i}{2}\theta^i(x)\right]\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$
- $\theta^{1,2,3}$ represent the three would-be Goldstone bosons that are eaten by the three gauge bosons that acquire masses as a result of the SSB
- The local $SU(2)_L$ symmetry allows one to rotate away any dependence on the three θ^i
- The unitary gauge: $\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

$\mathcal{L}_{\text{LSM}}$

$$\begin{aligned}\mathcal{L}_{\text{LSM}} = \quad & - \quad \frac{1}{4} W_b^{\mu\nu} W_{b\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + (D^\mu \phi)^\dagger (D_\mu \phi) \\ & + \quad i\overline{L_{Li}} \not{D} L_{Li} + i\overline{E_{Ri}} \not{D} E_{Ri} \\ & + \quad \left(Y^e_{ij} \overline{L_{Li}} E_{Rj}\, \phi + \text{h.c.} \right) \\ & - \quad \lambda \left(\phi^\dagger \phi - v^2/2 \right)^2\end{aligned}$$

The scalar spectrum

- h - a single real massive scalar degree of freedom
- $m_h = \sqrt{2\lambda} v$
- Experiment: $m_h = 125.09 \pm 0.21 \pm 0.11$ GeV

The vector boson spectrum I

- Three broken generators $\Longrightarrow$ Three massive vector bosons
- $(D_\mu \phi)^\dagger (D^\mu \phi)$ contains terms $\propto v^2$:

$$
\begin{aligned}
\mathcal{L}_{VM} &= \frac{1}{8}(0\ v)\begin{pmatrix} gW_3 + g'B & g(W_1 - iW_2) \\ g(W_1 + iW_2) & -gW_3 + g'B \end{pmatrix}^\dagger \\
&\times \begin{pmatrix} gW_3 + g'B & g(W_1 - iW_2) \\ g(W_1 + iW_2) & -gW_3 + g'B \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}
\end{aligned}
$$

- W_1, W_2 do not have a well defined $Q_{\rm EM}$;
  W_3, B are not mass eigenstates

The vector boson spectrum II

- $W^{\pm} = \frac{1}{\sqrt{2}}(W_1 \mp iW_2)$
- Define $\tan\theta_W \equiv g'/g$
  - $Z^0 = \cos\theta_W W_3 - \sin\theta_W B$
  - $A^0 = \sin\theta_W W_3 + \cos\theta_W B$
- $\mathcal{L}_{VM} = \frac{1}{4}g^2v^2W^+W^- + \frac{1}{8}(g^2+g'^2)v^2Z^0Z^0$
- $\boxed{m_W^2 = \frac{1}{4}g^2v^2, \quad m_Z^2 = \frac{1}{4}(g^2+g'^2)v^2, \quad m_A^2 = 0}$
- $m_A = 0$ a result of $U(1)_{\text{EM}}$ gauge invariance;
  A consistency check of our calculation

The $\rho = 1$ relation

- $\tan\theta_W \equiv g'/g$
  $\Longrightarrow \theta_W$ can be extracted from various weak interaction rates
- $\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$
  $\Longrightarrow \theta_W$ can be extracted from the spectrum
- $\rho = 1$ is a consequence of the SSB by scalar doublets
- $m_W = 80.385 \pm 0.015$ GeV; $m_Z = 91.1876 \pm 0.0021$ GeV
  $\Longrightarrow \sin^2\theta_W = 1 - (m_W/m_Z)^2 = 0.2229 \pm 0.0004$

The fermion spectrum I

- SSB allows us to tell the $T_3 = \pm 1/2$ components of the doublets:
  $$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \quad \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \quad \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$$
- $\mathcal{L}_{\mathrm{Yuk}}$ contains terms $\propto v$:
  $$\mathcal{L}_{FM} = -\frac{y_e v}{\sqrt{2}}\,\overline{e_L}\,e_R - \frac{y_\mu v}{\sqrt{2}}\,\overline{\mu_L}\,\mu_R - \frac{y_\tau v}{\sqrt{2}}\,\overline{\tau_L}\,\tau_R + \mathrm{h.c.}$$
- $$\boxed{m_e = \frac{y_e v}{\sqrt{2}}, \quad m_\mu = \frac{y_\mu v}{\sqrt{2}}, \quad m_\tau = \frac{y_\tau v}{\sqrt{2}}}$$
- Experiment:
  - $m_e = 0.510998928(11)$ MeV
  - $m_\mu = 105.6583715(35)$ MeV
  - $m_\tau = 1776.82(16)$ MeV

The fermion spectrum II

- The crucial point: While the leptons are in a chiral rep of $SU(2)_L \times U(1)_Y$, the charged leptons – e, μ, τ – are in a vector rep of $U(1)_{\rm EM}$ and thus can acquire Dirac masses
- ν_α are neutral under $U(1)_{\rm EM}$
  $\Longrightarrow$ A-priori, the possibility of Majorana masses is not closed
- $m_\nu \neq 0$ requires VEV carried by a scalar in the $(3)_{+1}$ rep, but there is no such scalar in the SM
- The neutrinos are massless in this model: $\boxed{m_{\nu_\alpha} = 0}$
  (at least at tree level)
- The ν's are degenerate $\Longrightarrow$ Any interaction basis is also a ν mass basis, but only a single interaction basis is an $\ell^\pm$ mass basis;
  $\nu_e, \nu_\mu . \nu_\tau \equiv$ The $SU(2)_L$ partners of e_L, μ_L, τ_L

Summary: The LSM particles

particle	spin	Q	mass (theo) $[v]$
$W^\pm$	1	±1	$\frac{1}{2}g$
Z^0	1	0	$\frac{1}{2}\sqrt{g^2+g'^2}$
A^0	1	0	0
h	0	0	$\sqrt{2\lambda}$
e	$1/2$	-1	$y_e/\sqrt{2}$
μ	$1/2$	-1	$y_\mu/\sqrt{2}$
τ	$1/2$	-1	$y_\tau/\sqrt{2}$
ν_e	$1/2$	0	0
ν_μ	$1/2$	0	0
ν_τ	$1/2$	0	0

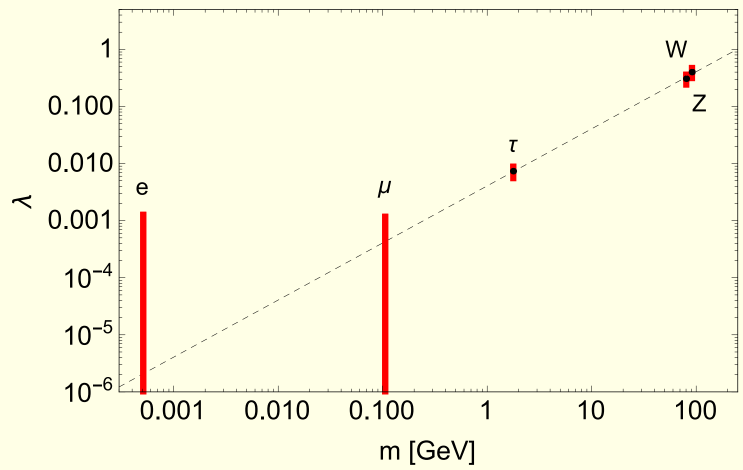
The Higgs boson interactions I

$$
\begin{aligned}
\mathcal{L}_h &= \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2} m_h^2 h^2 - \frac{m_h^2}{2v} h^3 - \frac{m_h^2}{8v^2} h^4 \\
&+ m_W^2 W_\mu^- W^{\mu +} \left(\frac{2h}{v} + \frac{h^2}{v^2} \right) + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \left(\frac{2h}{v} + \frac{h^2}{v^2} \right) \\
&- \frac{h}{v} \left(m_e \, \overline{e_L} \, e_R + m_\mu \, \overline{\mu_L} \, \mu_R + m_\tau \, \overline{\tau_L} \, \tau_R + \text{h.c.} \right)
\end{aligned}
$$

- The dimensionless couplings ($hhhh,\ hhVV,\ h\bar{\ell}\ell$) are unchanged from the symmetry limit
- The dimensionful couplings ($hhh,\ hVV$) arise from the SSB but do not introduce new parameters
- Neither hAA nor $hhAA$ coupling [$\Longleftarrow Q_{\text{EM}}(h) = 0, \quad m_A = 0$]

The Higgs boson interactions II

- All of the Higgs couplings can be written in terms of the masses of the particles to which it couples
- The heavier a particle, the stronger its coupling to h
- Experiment:



A. Efrati

- The Yukawa couplings are diagonal (to be discussed later)

Electromagnetic interactions I

- The coupling of neutral bosons:
  $\propto gW_3T_3 + g'BY$
- Rotate to the mass basis:
  $A(gs_WT_3 + g'c_WY) + Z(gc_WT_3 - g's_WY)$
- The photon field couples to $eQ = e(T_3 + Y)$, so
  $g = e/s_W, \quad g' = e/c_W$
- The electromagnetic interactions are described by
  $\mathcal{L}_{\mathrm{QED}} = eA_\mu\overline{\ell_i}\gamma^\mu\ell_i$
- Experiment ($\alpha \equiv e^2/4\pi$)
  $\alpha^{-1} = 137.035999074 \pm 0.000000044$

Electromagnetic interactions II

The electromagnetic interactions are:

- Vectorial
- Parity-conserving
- Diagonal: A couples to $e^+e^-, \mu^+\mu^-, \tau^+\tau^-$ but not to $e^\pm\mu^\mp, e^\pm\tau^\mp, \mu^\pm\tau^\mp$ pairs; a result of local $U(1)_{\rm EM}$
- Universal: The couplings to the different generations are universal; a result of local $U(1)_{\rm EM}$

NC weak interactions I

- The Z couplings to general fermions:
  $\frac{e}{s_W c_W}(T_3 - s_W^2 Q)\overline{\psi}\not{Z}\psi$
- The Z couplings to the LSM leptons:

$$\begin{aligned}\mathcal{L}_{\rm NC} \;\; = \;\; \frac{e}{s_W c_W}\Bigg[& -\left(\frac{1}{2} - s_W^2\right)\overline{e_L}\not{Z}e_L + s_W^2\overline{e_R}\not{Z}e_R + \frac{1}{2}\overline{\nu_{eL}}\not{Z}\nu_{eL} \\ & -\left(\frac{1}{2} - s_W^2\right)\overline{\mu_L}\not{Z}\mu_L + s_W^2\overline{\mu_R}\not{Z}\mu_R + \frac{1}{2}\overline{\nu_{\mu L}}\not{Z}\nu_{\mu L} \\ & -\left(\frac{1}{2} - s_W^2\right)\overline{\tau_L}\not{Z}\tau_L + s_W^2\overline{\tau_R}\not{Z}\tau_R + \frac{1}{2}\overline{\nu_{\tau L}}\not{Z}\nu_{\tau L}\Bigg]\end{aligned}$$

- Z-exchange gives rise to *neutral current weak interactions*

NC weak interactions II

The neutral current weak interactions are:

- Chiral
- Parity-violating
- Diagonal: a special feature of the LSM
- Universal: a special feature of the LSM

Diagonality and Universality $\Leftrightarrow$ All fermions of a given chirality and a given charge come from the same $SU(2) \times U(1)$ rep

NCWI: experimental tests

- Universality
  - $\Gamma(Z \to \mu^+\mu^-)/\Gamma(Z \to e^+e^-) = 1.0009 \pm 0.0028$
  - $\Gamma(Z \to \tau^+\tau^-)/\Gamma(Z \to e^+e^-) = 1.0019 \pm 0.0032$
- Diagonality
  - $\mathrm{BR}(Z \to e^\pm\mu^\mp) < 1.7 \times 10^{-6}$
  - $\mathrm{BR}(Z \to e^\pm\tau^\mp) < 9.8 \times 10^{-6}$
  - $\mathrm{BR}(Z \to \mu^\pm\tau^\mp) < 1.2 \times 10^{-5}$
- Interactions $\Leftrightarrow$ Spectrum
  - $\frac{\mathrm{BR}(Z \to \ell^+\ell^-)}{\mathrm{BR}(Z \to \nu_\ell\bar{\nu}_\ell)} = 1 - 4\sin^2\theta_W + 8\sin^4\theta_W = 0.505$
    $\Longrightarrow \sin^2\theta_W = 0.226$

CC weak interactions I

- The W couplings to a leptons:
  $\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}}\left[\overline{\nu_{eL}}\not{W}^{+}e_L^{-} + \overline{\nu_{\mu L}}\not{W}^{+}\mu_L^{-} + \overline{\nu_{\tau L}}\not{W}^{+}\tau_L^{-} + \text{h.c.}\right]$
- W-exchange gives rise to *charged current weak interactions*

CC weak interactions II

The charged current weak interactions are:

- Only left-handed leptons
- Parity-violating
- Diagonal: a special feature of the LSM
- Universal: a special feature of the LSM

Diagonality and Universality $\Leftrightarrow$ The degeneracy of the neutrinos

CCWI: experimental tests

- Universality
  - $\Gamma(W^+ \to \mu^+ \nu_\mu)/\Gamma(W^+ \to e^+ \nu_e) = 0.98 \pm 0.02$
  - $\Gamma(W^+ \to \tau^+ \nu_\tau)/\Gamma(W^+ \to e^+ \nu_e) = 1.04 \pm 0.02$
- Interactions $\Leftrightarrow$ Spectrum
  - Define $G_F \equiv \frac{g^2}{4\sqrt{2} m_W^2} = \frac{\pi \alpha}{\sqrt{2} s_W^2 m_W^2}$
  - Experiment: $G_F = 1.1663787(6) \times 10^{-5}\ \mathrm{GeV}^{-2}$
  - $\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right) \;\Rightarrow\; \sin^2\theta_W = 0.215$
  - $v = (\sqrt{2} G_F)^{-1/2} = 246\ \mathrm{GeV}$

Summary: The LSM interactions

interaction	force carrier	coupling	range
electromagneric	γ	eQ	long
NC weak	Z^0	$\frac{e(T_3 - s_W^2 Q)}{s_W c_W}$	short
CC weak	$W^\pm$	g	short
Yukawa	h	y_ℓ	short

Accidental symmetries

- $\mathcal{L}_{\rm kin}$ has an accidental symmetry:
  $G_{\rm LSM}^{\rm global}(Y^e=0)=U(3)_L\times U(3)_E$

- The Yukawa couplings break this symmetry to a subgroup:
  $\boxed{G_{\rm LSM}^{\rm global}=U(1)_e\times U(1)_\mu\times U(1)_\tau}$

- $\mu^-\to e^-\overline{\nu_e}\nu_\mu$ allowed; $\mu^-\to e^-e^+e^-$ forbidden;
  $e^+e^-\to\mu^+\mu^-$ allowed; $e^+\mu^-\to\mu^+e^-$ forbidden

- $U(1)_L$ forbids Majorana masses to neutrinos;
  $\boxed{m_\nu=0 \text{ to all orders in perturbation theory}}$

- $G_{\rm LSM}^{\rm global}$ completely broken by nonrenormalizable terms:
  $(1/\Lambda)L_{Li}L_{Lj}\phi\phi$ (to be discussed later)

Counting the lepton sector parameters

- $Y^e \implies 9_R + 9_I$ parameters
- $U(3)_L \times U(3)_E \to U(1)_e \times U(1)_\mu \times U(1)_\tau$
  $\implies (2 \times 3)_R + (2 \times 6 - 3)_I$ parameters can be removed
- $3_R + 0_I$ physical parameters:
  3 charged lepton masses
- The LSM is a seven parameter model:
  $g, g', v, m_h, m_e, m_\mu, m_\tau$

The Standard Model

Defining the SM

- The symmetry is a local $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Quarks: $Q_{Li}(3,2)_{+1/6},\ U_{Ri}(3,1)_{+2/3},\ D_{Ri}(3,1)_{-1/3}$
  Leptons: $L_{Li}(1,2)_{-1/2},\quad E_{Ri}(1,1)_{-1};\quad (i=1,2,3)$
- Scalars: $\phi(1,2)_{+1/2}$
- SSB: $SU(3)_C \times SU(2)_L \times U(1)_Y \ \to\ SU(3)_C \times U(1)_{\rm EM}$

$SU(3)_C \times SU(2)_L \times U(1)_Y$

- Twelve generators:
  eight L_a ($SU(3)_C$), three T_b ($SU(2)_L$), a single Y ($U(1)_Y$)
  $[L_a, L_b] = if_{abc}L_c, \quad [T_a, T_b] = i\epsilon_{abc}T_c,$
  $[L_a, T_b] = [L_a, Y] = [T_b, Y] = 0$
- Three coupling constants:
  g_s for $SU(3)_C$; g for $SU(2)_L$; g' for $U(1)_Y$
- Twelve gauge bosons:
  $G_a^\mu(8,1)_0, \; W_b^\mu(1,3)_0, \; B^\mu(1,1)_0$
- The covariant derivative:
  $D^\mu = \partial^\mu + ig_sG_a^\mu L_a + igW_a^\mu T_a + ig'YB^\mu$
  - $SU(3)_C$: $L_a = \frac{1}{2}\lambda_a(0)$ for triplets (singlets)
  - $SU(2)_L$: $T_b = \frac{1}{2}\sigma_b(0)$ for doublets (singlets)

$\mathcal{L}_{\text{kin}}$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}G_a^{\mu\nu}G_{a\mu\nu} - \frac{1}{4}W_b^{\mu\nu}W_{a\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + (D^\mu\phi)^\dagger(D_\mu\phi)$$
$$+i\overline{Q_{Li}}\not{D}Q_{Li} + i\overline{U_{Ri}}\not{D}U_{Ri} + i\overline{D_{Ri}}\not{D}D_{Ri} + i\overline{L_{Li}}\not{D}L_{Li} + i\overline{E_{Ri}}\not{D}E_{Ri}$$

- $D^\mu Q_L = \left(\partial^\mu + \frac{i}{2}g_s G_a^\mu \lambda_a + \frac{i}{2}g W_b^\mu \sigma_b + \frac{i}{6}g' B^\mu\right) Q_L$
- $D^\mu U_R = \left(\partial^\mu + \frac{i}{2}g_s G_a^\mu \lambda_a + \frac{2i}{3}g' B^\mu\right) U_R$
- $D^\mu D_R = \left(\partial^\mu + \frac{i}{2}g_s G_a^\mu \lambda_a - \frac{i}{3}g' B^\mu\right) D_R$
- $D^\mu L_L = \left(\partial^\mu + \frac{i}{2}g W_a^\mu \sigma_a - \frac{i}{2}g' B^\mu\right) L_L$
- $D^\mu E_R = \left(\partial^\mu - ig' B^\mu\right) E_R$
- $D^\mu \phi = \left(\partial^\mu + \frac{i}{2}g W_a^\mu \sigma_a + \frac{i}{2}g' B^\mu\right) \phi$

$\mathcal{L}_\psi$

$$\boxed{\mathcal{L}_\psi = 0}$$

- Quarks:
  - $Q_L,\ U_R,\ D_R$ = chiral representation
    No Dirac mass
  - $Q_L,\ U_R,\ D_R$ = charged under $U(1)_Y$
    No Majorana mass
- Leptons: same as in the LSM

$\mathcal{L}_{\text{Yuk}}$

$$\mathcal{L}_{\text{Yuk}} = Y^u_{ij}\overline{Q_{Li}}U_{Rj}\tilde{\phi} + Y^d_{ij}\overline{Q_{Li}}D_{Rj}\phi + Y^e_{ij}\overline{L_{Li}}E_{Rj}\phi + \text{h.c.}$$

- Y^u, Y^d, Y^e: general complex 3×3 matrices of dimensionless couplings
- Without loss of generality, can choose a basis,
  $Y^e \;\to\; V_{eL}Y^eV^\dagger_{eR} = \hat{Y}^e$, where $\hat{Y}^e = \text{diag}(y_e, y_\mu, y_\tau)$
- Without loss of generality, can choose a basis,
  $Y^u \to \hat{Y}_u = V_{uL}Y^uV^\dagger_{uR}$, where $\hat{Y}^u = \text{diag}(y_u, y_c, y_t)$
- Without loss of generality, can choose a basis,
  $Y^d \to \hat{Y}_d = V_{dL}Y^dV^\dagger_{dR}$, where $\hat{Y}^d = \text{diag}(y_d, y_s, y_b)$
- Unless $V_{uL} = V_{dL}$, the basis with $\hat{Y}^u$ is different from the basis with $\hat{Y}^d$.

The CKM matrix

- Define $\boxed{V = V_{uL} V_{dL}^{\dagger}}$
- In the basis where $Y^u = \hat{Y}^u$, we have $Y^d = V\hat{Y}^d$
- In the basis where $Y^d = \hat{Y}^d$, we have $Y^u = V^{\dagger}\hat{Y}^u$

- Note: $V_{uL}, V_{uR}, V_{dL}, V_{dR}$ depend on the basis from which we start. V, however, does not
- V plays a crucial role in the charged current weak interactions

$\mathcal{L}_\phi$

$$\boxed{\mathcal{L}_\phi = -\mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2}$$

- Choosing $\mu^2 < 0$ and $\lambda > 0$ leads to SSB with $|\langle \phi \rangle| = v/\sqrt{2}$
- $\phi = SU(3)_C$ singlet $\Longrightarrow SU(3)_C$ remains unbroken
- $SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{\rm EM}$

$\mathcal{L}_{\rm SM}$

$$\begin{aligned}
\mathcal{L}_{\rm SM} = \quad & - \quad \frac{1}{4}G_a^{\mu\nu}G_{a\mu\nu} - \frac{1}{4}W_b^{\mu\nu}W_{b\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + (D^\mu\phi)^\dagger(D_\mu\phi) \\
& + \quad i\overline{Q_{Li}}\not{D}Q_{Li} + i\overline{U_{Ri}}\not{D}U_{Ri} + i\overline{D_{Ri}}\not{D}D_{Ri} + i\overline{L_{Li}}\not{D}L_{Li} + i\overline{E_{Ri}}\not{D}E_{Ri} \\
& + \quad \left(Y_{ij}^u\overline{Q_{Li}}U_{Rj}\,\widetilde{\phi} + Y_{ij}^d\overline{Q_{Li}}D_{Rj}\,\phi + Y_{ij}^e\overline{L_{Li}}E_{Rj}\,\phi + \text{h.c.}\right) \\
& - \quad \lambda\left(\phi^\dagger\phi - v^2/2\right)^2
\end{aligned}$$

The boson spectrum

- Local $SU(3)_C \times U(1)_{\text{EM}}$ symmetry
  $\Longrightarrow$ A massless color-octet gluon, a massless neutral photon
- SSB of $SU(2)_L \times U(1)_Y \to U(1)_{\text{EM}}$
  $\Longrightarrow$ Three massive weak vector bosons $W^{\pm}, Z^0$
- SSB by an $SU(2)$ doublet
  $\Longrightarrow \rho \equiv m_W^2/(m_Z^2 \cos^2\theta_W) = 1$
- Three would be Goldstone bosons eaten by $W^{\pm}, Z^0$
  $\Longrightarrow$ A single massive Higgs boson

The fermion spectrum

- All charged fermions acquire Dirac masses, $m_f = \frac{y_f v}{\sqrt{2}}$;
  While in chiral reps of $SU(2)_L \times U(1)_Y$, they are in vectorial reps of $SU(3)_C \times U(1)_{\rm EM}$:
  - LH and RH e, μ, τ: $(1)_{-1}$
  - LH and RH u, c, t: $(3)_{+2/3}$
  - LH and RH d, s, b: $(3)_{-1/3}$
- Neutrinos are massless in spite of being in the $(1)_0$ rep

Summary: The SM particles

particle	spin	color	Q	mass $[v]$
$W^{\pm}$	1	(1)	± 1	$\frac{1}{2}g$
Z^0	1	(1)	0	$\frac{1}{2}\sqrt{g^2+g'^2}$
A^0	1	(1)	0	0
g	1	(8)	0	0
h	0	(1)	0	$\sqrt{2\lambda}$
e,μ,τ	1/2	(1)	-1	$y_{e,\mu,\tau}/\sqrt{2}$
ν_e,ν_μ,ν_τ	1/2	(1)	0	0
u,c,t	1/2	(3)	$+2/3$	$y_{u,c,t}/\sqrt{2}$
d,s,b	1/2	(3)	$-1/3$	$y_{d,s,b}/\sqrt{2}$

The Higgs boson interactions

$$\begin{aligned}\mathcal{L}_h &= \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2}m_h^2 h^2 - \frac{m_h^2}{2v}h^3 - \frac{m_h^2}{8v^2}h^4 \\ &+ m_W^2 W_\mu^- W^{\mu+}\left(\frac{2h}{v}+\frac{h^2}{v^2}\right) + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \left(\frac{2h}{v}+\frac{h^2}{v^2}\right) \\ &- \frac{h}{v}\left(m_e\,\overline{e_L}\,e_R + m_\mu\,\overline{\mu_L}\,\mu_R + m_\tau\,\overline{\tau_L}\,\tau_R\right. \\ &\quad +m_u\,\overline{u_L}\,u_R + m_c\,\overline{c_L}\,c_R + m_t\,\overline{t_L}\,t_R \\ &\quad \left. +m_d\,\overline{d_L}\,d_R + m_s\,\overline{s_L}\,s_R + m_b\,\overline{b_L}\,b_R + \text{h.c.}\right).\end{aligned}$$

- The Higgs boson couples diagonally also to the quark mass eigenstates
- The Higgs couplings are not universal: $y_f \propto m_f$

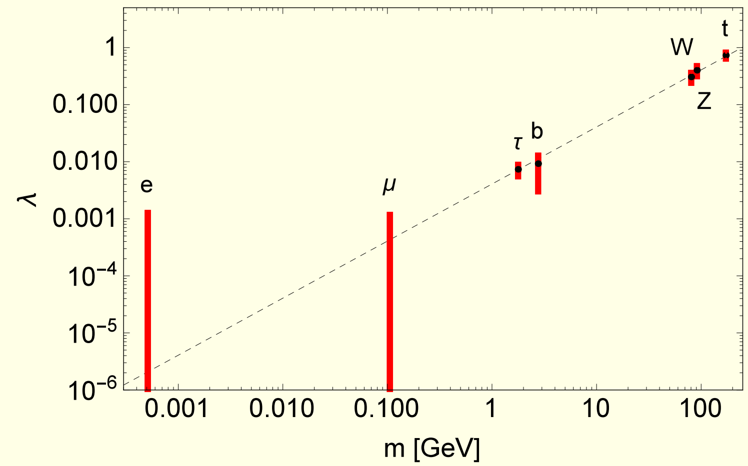
Diagonality of Yukawa interactions

$$
\begin{aligned}
h\overline{D_L}Y^d D_R &= h\overline{D_L}(V_{dL}^\dagger V_{dL})Y^d(V_{dR}^\dagger V_{dR})D_R \\
&= h(\overline{D_L}V_{dL}^\dagger)(V_{dL}Y^d V_{dR}^\dagger)(V_{dR}D_R) \\
&= h(\overline{d_L}\ \overline{s_L}\ \overline{b_L})\hat{Y}^d(d_R\ s_R\ b_R)^T
\end{aligned}
$$

- The diagonality is due to two ingredients of the SM:
  - All SM fermions are chiral $\Longrightarrow$ no bare mass terms
  - The scalar sector has a single Higgs doublet
- Experiment:
  $\mathrm{BR}(t \to qh) < 0.8 \times 10^{-2}$ [ATLAS, JHEP06(2014)008; CMS PAS TOP-13-017]
  $\mathrm{BR}(h \to \tau\mu) < 1.5 \times 10^{-2}$ [CMS, 1502.07400]

$y \propto m$

- All of the Higgs couplings can be written in terms of the masses of the particles to which it couples
- The heavier a particle, the stronger its coupling to h
- Experiment:



A. Efrati

Strong and electromagnetic interactions

- Local $SU(3)_C \times U(1)_{\text{EM}}$
  $\Longrightarrow$ Strong and EM interactions are universal
- Strong interactions: The gluons couple all colored particles
  - Quarks = color-triplets $\Longrightarrow$ have strong interactions
  - Leptons = color-singlets $\Longrightarrow$ do not couple to gluons
  - $\mathcal{L}_{\text{QCD, fermions}} = -\frac{1}{2} g_S \overline{q} \lambda_a \not{G}_a q \quad (q = u, c, t, d, s, b)$
- EM interactions: The photon couples to the EM charge
  - u, d, e are charged $\Longrightarrow$ have EM interactions
  - ν are neutral $\Longrightarrow$ do not couple to the photon
  - $\mathcal{L}_{\text{QED, fermions}} = -e\overline{e_i} \not{A} e_i + \frac{2e}{3} \overline{u_i} \not{A} u_i - \frac{e}{3} \overline{d_i} \not{A} d_i$
    $(e_i = e, \mu, \tau;\ u_i = u, c, t;\ d_i = d, s, b)$

NC weak interactions

- The Z couplings in each generation:

$$\mathcal{L}_{\rm NC} = \frac{e}{s_W c_W}\left[-\left(\frac{1}{2}-s_W^2\right)\,\overline{e_L}\not{Z}e_L + s_W^2\,\overline{e_R}\not{Z}e_R + \frac{1}{2}\,\overline{\nu_L}\not{Z}\nu_L \right.$$
$$+\left(\frac{1}{2}-\frac{2}{3}s_W^2\right)\,\overline{u_L}\not{Z}u_L - \frac{2}{3}s_W^2\,\overline{u_R}\not{Z}u_R$$
$$\left. -\left(\frac{1}{2}-\frac{1}{3}s_W^2\right)\,\overline{d_L}\not{Z}d_L + \frac{1}{3}s_W^2\,\overline{d_R}\not{Z}d_R\right]$$

- Chiral, parity-violating, diagonal, universal
- Universality $\Longleftarrow$ All fermions in the same $SU(3)_C \times U(1)_{\rm EM}$ rep come from the same $SU(3)_C \times SU(2)_L \times U(1)_Y$ rep

NCWI: further experimental tests

- SM:

$$\begin{aligned}\Gamma(Z\to\nu\bar\nu) &\propto 1,\\ \Gamma(Z\to\ell\bar\ell) &\propto 1-4s_W^2+8s_W^4,\\ \Gamma(Z\to u\bar u) &\propto 3\left[1-(8/3)s_W^2+(32/9)s_W^4\right],\\ \Gamma(Z\to d\bar d) &\propto 3\left[1-(4/3)s_W^2+(8/9)s_W^4\right]\end{aligned}$$

- Experiments:

$$\begin{aligned}\mathrm{BR}(Z\to\nu\bar\nu) &= (6.67\pm0.02)\%,\\ \mathrm{BR}(Z\to\ell\bar\ell) &= (3.37\pm0.01)\%,\\ \mathrm{BR}(Z\to u\bar u) &= (11.6\pm0.6)\%,\\ \mathrm{BR}(Z\to d\bar d) &= (15.6\pm0.4)\end{aligned}$$

Fine! (with $s_W^2=0.225$)

CC weak interactions I

- For leptons, things are simple because there exists an interaction basis that is also a mass basis
- Leptonic W interactions are universal in the lepton mass basis: $\mathcal{L}_{\rm CC}^{\rm leptons} = -\frac{g}{\sqrt{2}} \left[\overline{\nu_{eL}} \not{W}^+ e_L^- + \overline{\nu_{\mu L}} \not{W}^+ \mu_L^- + \overline{\nu_{\tau L}} \not{W}^+ \tau_L^- + {\rm h.c.} \right]$
- For quarks, things are more complicated since there is no interaction basis that is also a mass basis
- $\mathcal{L}_{\rm CC}^{\rm quarks} = -\frac{g}{\sqrt{2}} \left(\overline{u_L} \; \overline{c_L} \; \overline{t_L} \right) \; V \; \not{W}^+ \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + {\rm h.c.}$
- V = the CKM matrix: 3×3, unitary, $3_R + 1_I$ parameters

CC weak interactions II

- Only left-handed particles take part in the CC interactions
- Parity is violated
- W couplings to quark mass eigenstates:
  neither universal nor diagonal
- Universality of gauge interactions hidden in the unitarity of V

CCWI: further experimental tests

- SM:
  $\Gamma(W^+ \to \ell^+ \nu_\ell) \propto 1$
  $\Gamma(W^+ \to u_i \bar{d}_j) \propto 3|V_{ij}|^2 \quad (i = 1, 2;\ j = 1, 2, 3)$
- CKM unitarity:
  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$
  $\Longrightarrow\ \Gamma(W \to \text{hadrons}) \approx 2\Gamma(W \to \text{leptons})$
  $\Longrightarrow\ \Gamma(W \to cX)/\Gamma(W \to \text{hadrons}) \approx 0.5$
- Experiments:
  $\Gamma(W \to \text{hadrons})/\Gamma(W \to \text{leptons}) = 2.09 \pm 0.01$
  $\Gamma(W \to cX)/\Gamma(W \to \text{hadrons}) = 0.49 \pm 0.04$
- Flavor physics and the CKM matrix:
  A topic on its own

Summary: The SM quark interactions

interaction	force carrier	coupling	range
electromagnetic	γ	eQ	long
Strong	G	g_s	long
NC weak	Z^0	$\frac{e(T_3 - s_W^2 Q)}{s_W c_W}$	short
CC weak	$W^\pm$	gV	short
Yukawa	h	y_q	short

Accidental symmetries

- $\mathcal{L}_{\rm kin}$ has an accidental symmetry:
  $G^{\rm global}_{\rm SM}(Y^{u,d,e}=0) = U(3)_Q \times U(3)_U \times U(3)_D \times U(3)_L \times U(3)_E$
- The Yukawa couplings break this symmetry to a subgroup:
  $G^{\rm global}_{\rm SM} = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$
- $U(1)_B$ forbids proton decay ($e.g.$ $p \to \pi^0 e^+,\ p \to K^+\nu$);
  ($U(1)_B$ is anomalous, but still $\Delta B = \Delta L = 3n$ is respected)
- $U(1)_{B-L}$ forbids Majorana masses to neutrinos;
  $m_\nu = 0$ to all orders in perturbation theory and non-perturbatively
- LFV forbidden ($e.g.$ $\mu \to e\gamma,\ \tau \to \mu\mu\mu$);
  Neutrino oscillations violate $U(1)_e \times U(1)_\mu \times U(1)_\tau$

Counting the quark sector parameters

- $Y^u, Y^d \implies 18_R + 18_I$ parameters
- $U(3)_Q \times U(3)_U \times U(3)_D \to U(1)_B$
  $\implies (3 \times 3)_R + (3 \times 6 - 1)_I$ parameters can be removed
- $9_R + 1_I$ physical parameters:
  6 quark masses, 3 CKM angles, 1 CKM phase
- Experiment:
  - $|V_{us}| = 0.2253 \pm 0.0008$
  - $|V_{cb}| = 0.041 \pm 0.001$
  - $|V_{ub}| = 0.0041 \pm 0.0005$
  - $\sin 2\beta = 0.68 \pm 0.02$

Comments on CP violation

- The KM phase $\Longrightarrow$ CP is violated in the SM
- If there were only two generations:
  $2(4_R + 4_I) - [3(1_R + 3_I) - 1_I] = 5_R + 0_I$
  4 quark masses, 1 Cabibbo angle
  $\Longrightarrow$ CP is an accidental symmetry of a two generation SM
- An additional allowed term: $\theta_{\rm QCD}\epsilon_{\mu\nu\rho\sigma}G^{\mu\nu}G^{\rho\sigma}$
  CP violation by strong interactions
  Experiment (EDM): $\theta_{\rm QCD} \lesssim 10^{-10}$

The SM as an EFT

SM = low energy effective theory

- The SM is not a full theory of Nature
- The SM is a low energy effective theory;
  Valid below some scale $\Lambda (\gg m_Z)$
- $\mathcal{L}_{\rm SM}$ should be extended:
  $\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda} O_{d=5} + \frac{1}{\Lambda^2} O_{d=6} + \cdots$
- $O_{d=n}$ = operators that are
  - Products of SM fields
  - Transforming as singlets under $SU(3)_C \times SU(2)_L \times U(1)_Y$
  - Of dimension n in the fields
- For physics at $E \ll \Lambda$, the effects of $O_{d=n(>4)}$ suppressed by $(E/\Lambda)^{n-4}$
- The larger $n(>4)$, the smaller the effect at low energy

Nonrenormalizable terms, loops

- Tree level processes: Often tree level processes in a particular sector depend on a small subset of the SM parameters $\Longrightarrow$ Relations among various processes that are violated by loop effects and nonrenormalizable terms
  - Example: Electroweak precision measurements (EWPM)
- Rare processes: Processes not allowed at tree level, often related to accidental symmetries of a particular sector. Nonrenormalizable terms and loops can contribute.
  - Example: Flavor changing neutral currents (FCNC)
- Forbidden processes: Nonrenormalizable terms (but not loop corrections!) can break accidental symmetries and allow forbidden processes
  - Example: Neutrino masses

Before and after

- Rare processes and tree level processes:
  - Before all the SM particles have been directly discovered and all the SM parameters measured:
    Assume the validity of the renormalizable SM and indirectly measure the properties of the yet unobserved particles;
    m_c, m_t, m_h predicted in this way
  - Once all the SM particles observed and the parameters measured directly:
    The loop corrections can be quantitatively determined;
    Effects of nonrenormalizable terms unambiguously probed
- All three categories are used to search for new physics

EWPM - before and after

- At tree level, all EW processes depend on only 3 parameters: g, g', v ($\Leftrightarrow \alpha, m_Z, G_F$)
- Of the other 15 parameters, 11 are small and have negligible effects on EWPM
- Of the remaining four, $\delta_{\rm KM}$ and α_S have negligible effects
- Only m_t/v and m_h/v have significant quantum effects
- In the past: EWPM used to predict m_t and m_h
- At present: EWPM probe nonrenormalizable operators (= BSM physics)

EWPM - theory

- In a large class of models, only four dim=6 operators contribute significantly to EWPM:
  $\mathcal{L}_{\text{o.c.}} = \frac{1}{\Lambda^2}\left(c_{WB}\mathcal{O}_{WB} + c_{HH}\mathcal{O}_{HH} + c_{BB}\mathcal{O}_{BB} + c_{WW}\mathcal{O}_{WW}\right)$

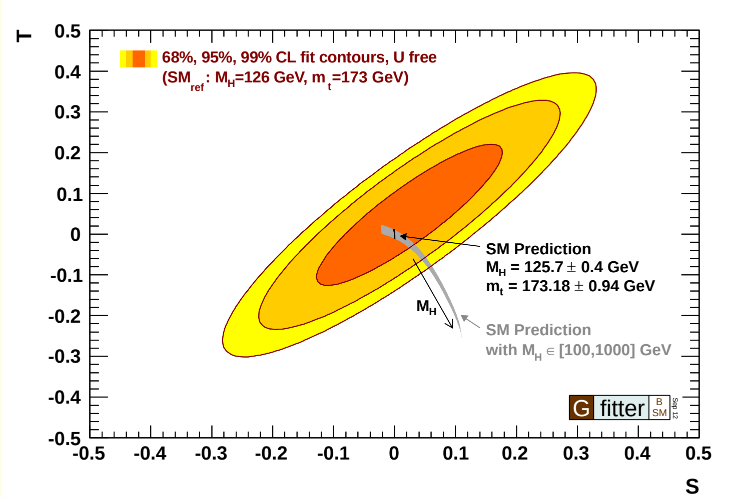
  $\mathcal{O}_{WB} = (H^\dagger \tau^a H) W^a_{\mu\nu} B_{\mu\nu} \to \frac{1}{2} v^2 W^3_{\mu\nu} B_{\mu\nu};$
  $\mathcal{O}_{HH} = |H^\dagger D_\mu H|^2 \to \frac{1}{16} v^4 (g W^3_\mu - g' B_\mu)^2;$
  $\mathcal{O}_{BB} = (\partial_\rho B_{\mu\nu})^2;$
  $\mathcal{O}_{WW} = (D_\rho W^a_{\mu\nu})^2$

EWPM - experiment

- Low energy observables: G_F, α, neutrino scattering, DIS, APV, low-energy e^+e^- scattering
- High energy observables: masses, total widths and partial decay rates of the W and Z bosons

S, T

- $S = \frac{2\sin 2\theta_W}{\alpha}\ \frac{v^2}{\Lambda^2}\ c_{WB}$
  $T = -\frac{1}{2\alpha}\ \frac{v^2}{\Lambda^2}\ c_{HH}$



GFitter

- $\frac{\Lambda}{\sqrt{c_{WB}}} > 9.7\left(\frac{0.14}{S}\right)$ TeV
  $\frac{\Lambda}{\sqrt{c_{HH}}} > 4.4\left(\frac{0.20}{T}\right)$ TeV

FCNC

Flavors = Copies of the same $SU(3)_C \times U(1)_{EM}$ representation:

Up-type quarks	$(3)_{+2/3}$	u, c, t
Down-type quarks	$(3)_{-1/3}$	d, s, b
Charged leptons	$(1)_{-1}$	e, μ, τ
Neutrinos	$(1)_0$	ν_1, ν_2, ν_3

Flavor changing neutral current (FCNC) processes:

- Flavor changing processes that involve either U or D but not both and/or either ℓ^- or ν but not both
- $\mu \to e\gamma$; $K \to \pi\nu\bar{\nu}$ $(s \to d\nu\bar{\nu})$; $D^0 - \overline{D}^0$ mixing $(c\bar{u} \to u\bar{c})$...

FCNC: Loop suppression I

- The $W-$boson cannot mediate FCNC process at tree level since it couples to up-down pairs;
  Only neutral bosons can potentially mediate FCNC at tree level

- Massless gauge bosons have flavor-universal and, in particular, flavor diagonal couplings;
  The gluons and the photon do not mediate FCNC at tree level

What about Z? h?

FCNC: Loop suppression II

- Within the SM, the $Z-$boson does not mediate FCNC at tree level because all fermions with the same chirality, color and charge originate in the same $SU(2)_L \times U(1)_Y$ representation

- Within the SM, the $h-$boson does not mediate FCNC at tree level because
  - All SM fermions are chiral $\Longrightarrow$ no bare mass terms
  - The scalar sector has a single Higgs doublet

Within the SM, all FCNC processes are loop suppressed

FCNC: CKM- and GIM-suppression

- All FC processes $\propto$ off-diagonal entries in the CKM matrix
  - $\Gamma(b \to s\gamma) \propto |V_{tb}V_{ts}|^2 \sim 3 \times 10^{-3}$
  - $\Delta m_B \propto |V_{tb}V_{td}|^2 \sim 10^{-4}$
- If all quarks in a given sector were degenerate
  $\Longrightarrow$ No FC W-couplings
- FCNC in the down (up) sector
  $\propto$ Δm^2 between the quarks of the up (down) sector
- The GIM-suppression effective for processes involving the first two generations
  - $\Delta m_K \propto (m_c^2 - m_u^2)/m_W^2$ ($\Longrightarrow$ was used to predict m_c)
  - $\Delta m_B \propto (m_t^2 - m_c^2)/m_W^2$ ($\Longrightarrow$ was used to predict m_t)

FCNC - experiment

$\Delta m_K/m_K$	7.0×10^{-15}
$\Delta m_D/m_D$	8.7×10^{-15}
$\Delta m_B/m_B$	6.3×10^{-14}
$\Delta m_{B_s}/m_{B_s}$	2.1×10^{-12}
ϵ_K	2.3×10^{-3}
$A_\Gamma/y_{\rm CP}$	≤ 0.2
$S_{\psi K_S}$	0.67 ± 0.02
$S_{\psi\phi}$	-0.04 ± 0.09

High Scale? Degeneracy and Alignment?

- $\frac{z_{sd}}{\Lambda_{\rm NP}^2}(\overline{d_L}\gamma_\mu s_L)^2 + \frac{z_{cu}}{\Lambda_{\rm NP}^2}(\overline{c_L}\gamma_\mu u_L)^2 + \frac{z_{bd}}{\Lambda_{\rm NP}^2}(\overline{d_L}\gamma_\mu b_L)^2 + \frac{z_{bs}}{\Lambda_{\rm NP}^2}(\overline{s_L}\gamma_\mu b_L)^2$
- For $|z_{ij}| \sim 1$, $\mathcal{I}m(z_{ij}) \sim 1$:

Mixing	$\Lambda_{\rm NP}^{CPC} \gtrsim$	$\Lambda_{\rm NP}^{CPV} \gtrsim$	Mixing	$\Lambda_{\rm NP}^{CPC} \gtrsim$	$\Lambda_{\rm NP}^{CPV} \gtrsim$
$K-\overline{K}$	1000 TeV	20000 TeV	$D-\overline{D}$	1000 TeV	3000 TeV
$B-\overline{B}$	400 TeV	800 TeV	$B_s-\overline{B_s}$	70 TeV	200 TeV

- For $\Lambda_{\rm NP} \sim 1\ TeV$:

Mixing	$\lvert z_{ij}\rvert \lesssim$	$\mathcal{I}m(z_{ij}) \lesssim$	Mixing	$\lvert z_{ij}\rvert \lesssim$	$\mathcal{I}m(z_{ij}) \lesssim$
$K-\overline{K}$	8×10^{-7}	6×10^{-9}	$D-\overline{D}$	5×10^{-7}	1×10^{-7}
$B-\overline{B}$	5×10^{-6}	1×10^{-6}	$B_s-\overline{B_s}$	2×10^{-4}	2×10^{-5}

m_ν - theory

- SM: $m_\nu = 0$ to all orders in perturbation theory and non-perturbatively
- Guaranteed by the accidental $U(1)_{B-L}$ symmetry
- d=5 terms $\frac{Z^\nu_{ij}}{\Lambda}\phi\phi L_i L_j$ break $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau \to U(1)_B$

The νSM

$$\mathcal{L}_{\nu\text{SM}} = \mathcal{L}_{\text{SM}} + \frac{Z^{\nu}_{ij}}{\Lambda}\phi\phi L_i L_j$$

- $\langle\phi^0\rangle = v/\sqrt{2} \Longrightarrow$ Majorana mass matrix for neutrinos: $m_\nu = \frac{v^2}{\Lambda}\frac{Z^\nu}{2}$
- m_ν can be diagonalized by a unitary transformation: $V_{\nu L} m_\nu V^T_{\nu L} = \hat{m}_\nu = \text{diag}(m_1, m_2, m_3)$

Predictions:

- $m_\nu \neq 0$
- $m_\nu / m_{q,\ell^\pm} \sim v/\Lambda \ll 1$
- $\mathcal{L}^{\ell}_{\text{CC}} = -\frac{g}{\sqrt{2}}\left(\overline{\ell_{L\alpha}} \not{W}^- U_{\alpha i}\nu_i + \text{h.c.}\right)$; $U \neq \mathbf{1}$

The νSM parameters

$\mathcal{L}_{\nu\mathrm{SM}} = \mathcal{L}_{\mathrm{SM}} + \frac{Z^{\nu}_{ij}}{\Lambda}\phi\phi L_i L_j$

- We added to the SM $6_R + 6_I$ parameters (Z^{ν} is symmetric)
- We "lost" $[U(1)]^3$ symmetry $\Longrightarrow$ Can remove 3_I parameters;
- Conclusion: $6_R + 3_I$ new parameters
- 3 neutrino masses;
  3 angles and 3 phases in the leptonic mixing matrix
- Experiment: Gonzalez-Garcia et al., 1409.5439
- $\Delta m^2_{21} = (7.5 \pm 0.2) \times 10^{-5}\ \mathrm{eV}^2, \quad |\Delta m^2_{32}| = (2.5 \pm 0.1) \times 10^{-3}\ \mathrm{eV}^2$
- $|U_{e2}| = 0.55 \pm 0.01, \quad |U_{\mu 3}| = 0.67 \pm 0.03, \quad |U_{e3}| = 0.148 \pm 0.003$

The νSM - summary

$$\mathcal{L}^{\nu}_{\nu\mathrm{SM}} = i\overline{\nu_i}\not{\partial}\nu_i + \frac{g}{2c_W}\overline{\nu_i}\not{Z}\nu_i - \frac{g}{\sqrt{2}}\left(\overline{\ell_{L\alpha}}\not{W}^- U_{\alpha i}\nu_i + \mathrm{h.c.}\right)$$
$$+m_i\nu_i\nu_i + \frac{2m_i}{v}h\nu_i\nu_i + \frac{m_i}{v^2}hh\nu_i\nu_i$$

particle	spin	color	Q	mass $[v]$
ν_1, ν_1, ν_3	$1/2$	(1)	0	$z_i v/(2\Lambda)$

interaction	force carrier	coupling
NC weak	Z^0	$e/(2s_W c_W)$
CC weak	$W^{\pm}$	$gU/\sqrt{2}$
Yukawa	h	$2m/v$

- Accidental symmetry: $U(1)_B$

Summary

Summary I: definition

- The symmetry is a local $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Quarks: $Q_{Li}(3,2)_{+1/6},\ U_{Ri}(3,1)_{+2/3},\ D_{Ri}(3,1)_{-1/3}$
  Leptons: $L_{Li}(1,2)_{-1/2}, \quad E_{Ri}(1,1)_{-1}; \quad (i=1,2,3)$
- Scalars: $\phi(1,2)_{+1/2}$
- SSB: $SU(3)_C \times SU(2)_L \times U(1)_Y \ \to\ SU(3)_C \times U(1)_{\rm EM}$

Summary II: spectrum

particle	spin	color	Q	mass $[v]$
$W^{\pm}$	1	(1)	± 1	$\frac{1}{2}g$
Z^0	1	(1)	0	$\frac{1}{2}\sqrt{g^2+g'^2}$
A^0	1	(1)	0	0
g	1	(8)	0	0
h	0	(1)	0	$\sqrt{2\lambda}$
e, μ, τ	1/2	(1)	-1	$y_{e,\mu,\tau}/\sqrt{2}$
ν_e, ν_μ, ν_τ	1/2	(1)	0	0
u, c, t	1/2	(3)	$+2/3$	$y_{u,c,t}/\sqrt{2}$
d, s, b	1/2	(3)	$-1/3$	$y_{d,s,b}/\sqrt{2}$

Summary III: interactions

interaction	force carrier	coupling	range
electromagneric	γ	eQ	long
Strong	g	g_s	long
NC weak	Z^0	$\frac{e(T_3 - s_W^2 Q)}{s_W c_W}$	short
CC weak	$W^\pm$	gV	short
Yukawa	h	y_q	short

Summary IV: parameters

There are eighteen independent parameters:

- $g_s,\ g,\ g',\ v,\ \lambda$ ($\Longrightarrow \alpha_s,\ \alpha,\ m_Z,\ G_F,\ m_h$)
- $y_e,\ y_\mu,\ y_\tau$ ($\Longrightarrow m_e,\ m_\mu,\ m_\tau$)
- $y_u,\ y_c,\ y_t,\ y_d,\ y_s,\ y_b$ ($\Longrightarrow m_u,\ m_c,\ m_t,\ m_d,\ m_s,\ m_b$)
- $|V_{us}|,\ |V_{cb}|,\ |V_{ub}|,\ \delta_{\rm KM}$ ($\Longrightarrow \lambda,\ A,\ \rho,\ \eta$)

Summary V: accidental symmetries

$U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$

- Explains the non-observation of proton decay
- Explains the non-observation of FCNC charged lepton decays
- Violated in neutrino oscillations
  $\Longrightarrow$ The SM is a low energy effective theory, $\Lambda_{\rm NP} \lesssim 10^{15}$ GeV

Summary VI: successes

- Review of particle physics (Particle Data Group), Chin. Phys. C38 (2014) 090001;
  1676 pages of experimental results
  Almost all consistent with the SM predictions

Summary VII: problems

- Neutrino masses
- Dark matter
- Baryon asymmetry
- Fine tuning
  - m_h^2
  - $\theta_{\rm QCD}$