SO(10) Unification - Beyond Flat Space-Time

Peter Minkowski

Albert Einstein Center for Fundamental Physics - ITP, University of Bern

Abstract

It is derived through logically distinguished arguments that cosmological Dark Matter consists of the mainly electroweak doublet light neutrino flavors and their antiparticles .

Contribution to the International Conference: The Standard Theory and Beyond in the LHC Era - 2015 , Albufeira, Portugal, 25. - 30. October 2015 .

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1 - Introduction

The present outline goes back 40 years to questions arising from our common paper with Harald Fritzsch, elaborated 1973 - 1975 at Caltech, published as ref. 1 = [1-1975] .

The most pressing question then, was raised by Murray Gell-Mann : ”Now you must extend your unification scheme to at least include rigid supersymmetry.”, and he added ”Maybe this gives a clue as ro small neutrino masses.” I answered : ”I will think about it” .

This thinking is testified, here, with respect to neutrino masses in ref. 2 = [2-2015] , 40 years later, and with intermediate answers in ref. 3 = [3-1997] with reapect to breaking of pure (N=1) superYang-Mills systems, after 22 years, and refs. 4 = [4-2003] and 5 = [5-2003] including classical supergravity backgrounds, after 28 years.

Only within the last months I cpntacted Murray Gell-Mann per e-mail, since he was unable to participate at the ISSP2015 in Erice earlier this year, against his original plan. My answer was : ”Only now I arrived at an answer to your question, Murray, which is no, I do not need to extend the original scope of our paper with Harald Fritzsch (ref. 1 = [1-1975]) to include any form of susy.”

The derivations leading to the above answer were mainly laid out in ref. 2 = [2-2015] as contribution to the Workshop on Neutrinos , Nanyang Technological University, Singapore, 09. - 14. February 2015 .

In section 2 we will pass in short review the derivation added as ’Notes’ in in ref. 2 = [2-2015] , that Dark Matter is composed of the weakly interacting light neutrino flavors ν_e , ν_μ , ν_τ and their antiparticles, in the sense of beeing described by Majorana fields.

2 - Note added in proof: Dark matter is formed by the 3 predominantly active light neutrino-antineutrino flavors, proof by elimination of alternatives

In correcting the original slidefile: singanu2015.pdf , which contains my contribution to the Conference on Massive Neutrinos , 09.-14. February 2015, IAS, Nanyang Techological University, Singapore, entitled 'Massive Neutrinos and SO (10) Unification', I realized that combining its content with related papers of the author, not discussed there, an important consequence for the nature of Dark Matter can be reached by elimination of alternatives. – This Note is devoted to this topic.

The first alternative concerns the 'Strong CP Problem' , which is rejected in its very existence in the paper by the author from 1978, forming ref. 6 = [6-1978] .

The second alternative concerns a rigid supersymmetry structure enriching an exact and broken gauge theory containing the Standard Model , as well as extension of this supersymmetric structure to open and closed superstrings. Hereto let me cite a recent paper by Ashoke Sen , ref. 7 = [7-2015] .

A candidate for Dark Matter in the above framework , beeing the Lowest Supersymmetric Partner Particle (LSP) beeing stable or sufficiently long-lived , is compromised by the trance anomaly in uncurved 3+1 dimensions in ref. 8 = [8-1977] . The latter investigation was prompted by underpinning the universal scale of the slope of Regge trajectories, whence interpreted as oscillatory modes of u,d,s quark- and antiquark flavors in QCD , published for mesons in ref. 9 = [9-1979] and for baryons in ref. 10 = [10-1980] .

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Later we investigated and established in my group of collaborators at the ITP in Bern direct proofs for special cases of supersymmetric gauge theories. I cite here only 3 papers in acending order of time: 1 with Markus Leibundgut ref. 3 = [3-1997] and 2 with Luzi Bergamin ref. 4 = [4-2003] and ref. 5 = [5-2003] . This eliminates Alternative 2 .

Qod erat demonstrandum	What was to be demonstrated
Ibi tertium non datur	In this outline a 3d Alternative is not given

What remains to be done, in the hopefully not too distant future, is to show how the electromagnetic long-range interactions of the mainly active 3 neutrino and antineutrino flavors develop a vacuum condensate and as a consequence their equilibrium density and pressure do *not* correspond to (almost) massless free states at a corresponding temperature of $\sim\ 2\,^\circ\ K$. Here the original framework was laid out in a paper with Raul Horvat and Josip Trampetic: ref. 11 = [11-2008] .

The cold dark natter energy density is according to the PDG [12-2014]

$$\Omega_{cdm} = 0.265(11)\,\Omega_{crit} \;\; ; \;\; \varrho_{e\,crit} = 0.05375(13) \times 10^{7}\, h^{2}\ \text{meV cm}^{-3}$$

(1) $$\sim 2.434 \times 10^{5}\ \text{meV}\ cm^{-3}$$

$$\text{cm}^{-1} = 0.019732\ \text{meV}\ ,\ \ \text{cm}^{-3} = 7.6835 \times 10^{-6}\ \text{meV}^{3}$$

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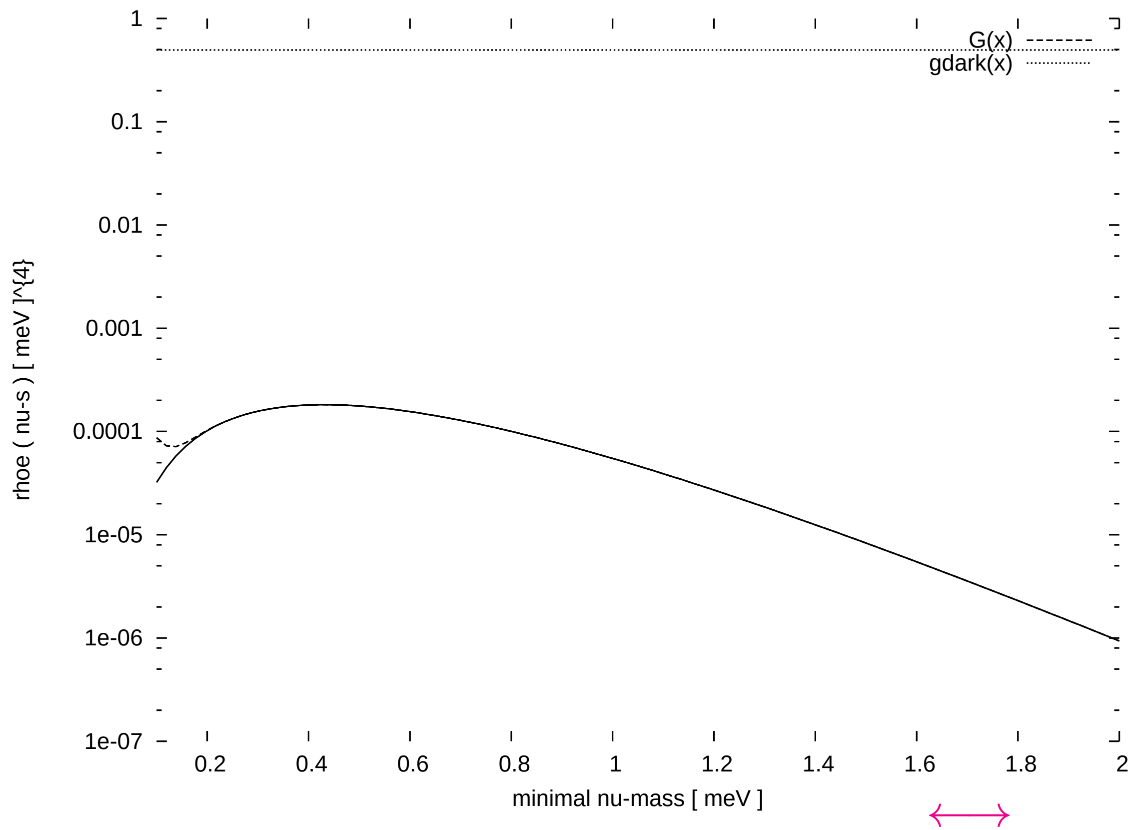
It follows using rational units based on 1 meV

$$\varrho_{\,e\,crit} \sim 1.871\ \textbf{meV}^{\,4} \quad ; \quad \deg\ K \sim 0.862 \times 10^{\,-4}\ \textbf{meV}$$

(2)

$$\longrightarrow \varrho_{\,e\,dark} \sim 0.496\ \textbf{meV}^{\,4} \quad ; \quad \varrho_{\,e\,dark} = \Omega_{\,dark}\,\varrho_{\,e\,crit}\ ;\ \Omega_{\,dark} = 0.265(11)$$



In Fig. 1 we plot the nonrelativistically approximated thermal energy density for 3 neutrino-antineutrino flavors with masses

(3)

$$m_{\nu_1} = m_{\nu} \ , \ m_{\nu_2} = 10 \, m_{nu_1} \ , \ m_{\nu_3} = 50 \, m_{nu_1}$$

$$\varrho_{e\,\nu} \sim \left(\sqrt{2\,\pi} / \pi^{\,2} \right) T^{\,4} \, (m \, / \, T \,)^{\,3/2} \, e^{\,-m/T} \ \ ; \ \ \nu \to \nu_{\,1,2,3`}$$

The ratios

$$r \, (\, m \ [\textbf{meV}] \,) \ = \ \varrho_{e\,\nu\,s} \, / \, \varrho_{\,e\,dkm\,obs}$$

(4) become

$$r \, (\, 0.1 \,) \ = \ (\, 5694 \,)^{\,-1} \, , \, r \, (\, 0.4 \,) \ = \ (\, 2746 \,)^{\,-1} \, , \, r \, (\, 1.0 \,) \ = \ (\, 9030 \,)^{\,-1}$$

3 - Conclusions - Outlook

1) SO10 and grvity

The chargelike unifying gauge group of SO10 exhibits a remarkable stability in the limit of vanishing curvature underlying quantized gravity , once the full range of the gravitational interactions , including the associated nonperturbative domain is shown to be amenable to this limit (of negligible curvature) .

This hurdle mastered , the uncurved space exhibits two persisting dynamical properties :

a) spomtameous and anomalous breaking of scale invariance – the ressidual traceanomaly

b) restoration of CP violation in the limit of pure QCD , independent of quark masses and mixings , a Josephson effect . This was derived by the author in ref. 6 = [6-1978] .

2) Neutrino mass and mixing and Dark Matter

This brings into focus the hypothesis that neutrino flavors intrinsic to SO10 form the observed Dark Matter . This was discussed in the recent paper in ref. 13 = [13-2015] .

3) Main previous results

The main results used are based on R. Horvat, P. Minkowski and J. Trampetic : ref. 11 = [11-2008] .

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3) continued

We present ref. 11 = [11-2008] at the end of this outline .

4) We hope that for the case in hand, the presented material will help to devise improved , adequate strategies in the experimental DM search .

Thank you

4 - Dark consequences from light neutrino condensations

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Dark consequences from light neutrino condensations

Raul Horvat

Physics Division, Rudjer Bošković Institute, Zagreb, Croatia

Peter Minkowski

ITP, University of Bern, CH-3012 Bern, Switzerland

Josip Trampetić

Theoretical Physics Division, Rudjer Bošković Institute, Zagreb, Croatia

Abstract

In this paper we discuss light neutrino dipole moments, computed in the neutrino-mass extended standard model (SM), as a possible source for neutrino condensates which may cause cosmological constant observed today.

In this paper we propose light neutrino long range dipole-dipole forces arising from dipole moments, computed in the neutrino-mass extended standard model (SM), as a possible source for neutrino condensates. These condensates will cause acceleration through the associated cosmological constant provided the vacuum pressure is dominant and negative [1]. The computation depends on the nature of the neutrinos, however, we first discuss the consequences of the neutrino-photon interaction with characteristic electromagnetic properties of Majorana neutrinos: the transition dipole moments [2, 3, 4, 5, 6]. These minuscule transition dipole moments are sensitive probes of fluctuations at scales as small as 10^{-35} cm [7], as seen through electromagnetic interactions at long range. This can also shed more light on the expansion of the universe and the cosmological constant problem [1].

The transition matrix elements relevant for $\nu_i \longrightarrow \nu_j;\ i \neq j$ in neutrino mass extended standard model for Majorana neutrinos are given in [2, 3, 4, 5, 6]. The photon-neutrino effective vertex is basicaly determined from the $\nu_i \longrightarrow \nu_j\ \gamma$ transition, which is generated through electroweak processes that arise from one-loop diagrams via the exchange of $\ell = e, \mu, \tau$ leptons and weak bosons, and is given by

$$
\begin{aligned}
J_\mu^{\mathrm{eff}} \epsilon^\mu(q) &= \left\{ F_1(q^2) \bar{\nu}_j(p')_L \left(\gamma_\mu q^2 - q_\mu \not{q}\right) \nu_i(p)_L \right. \\
&- \mathrm{i} F_2(q^2) \left[m_{\nu_j} \bar{\nu}_j(p')_R\, \sigma_{\mu\nu}\, q^\nu \nu_i(p)_L \right. \\
&+ \left.\left. m_{\nu_i} \bar{\nu}_j(p')_L\, \sigma_{\mu\nu}\, q^\nu \nu_i(p)_R \right] \right\} \epsilon^\mu(q). \qquad (5)
\end{aligned}
$$

The above effective interaction is invariant under electromagnetic gauge transformations. The first term in (5) vanishes for real photon due to the electromagnetic gauge condition.

The general decomposition of the F_2 term of the transition matrix element $T[A, B]$ obtained from (5), leads to the well known expression for the electric and magnetic dipole moments

$$(6)\ d^{\rm el}_{ji} = \frac{-e}{M^{*2}}\left(m_{\nu_i} - m_{\nu_j}\right)\sum_{k=e,\mu,\tau}{\rm U}^{\dagger}_{jk}{\rm U}_{ki}F_2(\frac{m^2_{\ell_k}}{m^2_W}),$$

$$(7)\ \mu_{ji} = \frac{-e}{M^{*2}}\left(m_{\nu_i} + m_{\nu_j}\right)\sum_{k=e,\mu,\tau}{\rm U}^{\dagger}_{jk}{\rm U}_{ki}F_2(\frac{m^2_{\ell_k}}{m^2_W}),$$

where $i, j = 1, 2, 3$ denotes neutrino species, and

$$(8)\quad F_2(\frac{m^2_{\ell_k}}{m^2_W}) \simeq -\frac{3}{2} + \frac{3}{4}\frac{m^2_{\ell_k}}{m^2_W}, \quad \frac{m^2_{\ell_k}}{m^2_W} \ll 1,$$

was obtained after the loop integration. In Eqs. (6) and (7) $M^* = 4\pi\, v = 3.1$ TeV, where $v = (\sqrt{2}\, G_F)^{-1/2} = 246$ GeV represents the vacuum expectation value of the scalar Higgs field [7].

Note that in the case of a mass degenerate pair the electric dipole moment vanishes, while the magnetic one is dominated by the first term in (8). In the case of off-diagonal transition moments, the first term in (8) vanishes in the summation over leptons due to the orthogonality condition of the neutrino mixing matrix U [8] (GIM cancellation).

The mixing matrix U is governing the decomposition of a coherently produced left-handed neutrino $\widetilde{\nu}_{L,\ell}$ associated with charged-lepton-flavor $\ell = e, \mu, \tau$ into the mass eigenstates $\nu_{L,i}$:

$$|\widetilde{\nu}_{L,\ell};\, \vec{p}\rangle = \sum_i \mathrm{U}_{\ell i} |\nu_{L,i};\, \vec{p}, m_i\,\rangle, \tag{9}$$

The characterizing feature of Majorana neutrinos (i.e. 4-component notation the Hermitian, neutrino-flavor antisymmetric, electric and magnetic dipole operators), i.e. fields that do not distinguish particle from anti-particle ($\psi_i = \psi_i^c$), producing a transition matrix element T[A, B] which is a complex anti-symmetric quantity in lepton-flavor space:

$$\begin{aligned} \mathrm{T}_{\mathrm{ji}} &= -\mathrm{i}\epsilon^{\mu}\bar{\nu}_j \left[(A_{ji} - A_{ij}) - (B_{ji} - B_{ij})\gamma_5\right] \sigma_{\mu\nu} q^{\nu} \nu_i \\ &= -\mathrm{i}\epsilon^{\mu}\bar{\nu}_j \left[2\mathrm{i}\,\Im A_{ji} - 2\Re B_{ji}\gamma_5\right] \sigma_{\mu\nu} q^{\nu} \nu_i\;, \end{aligned} \tag{10}$$

i.e. antisymmetric with respect to neutrino mass eigenstates. From this equation it is explicitly clear that for $i = j$, $d^{\mathrm{el}}_{\nu_i} = \mu_{\nu_i} = 0$. Also, considering transition moments, only one of two terms in (10) is non-vanishing if the interaction respects CP invariance: The first term vanishes if the relative CP of ν_i and ν_j is even, and the second term vanishes if it is odd [5]. Dipole moments describing the transition from Majorana neutrino mass eigenstate-flavor ν_j to ν_i in the mass extended standard model are:

$$d^{\rm el}_{\nu_j\nu_i} = \frac{3\,{\rm i\,e}}{2M^{*2}}\left(m_{\nu_i} - m_{\nu_j}\right)\sum_{k=e,\mu,\tau}\frac{m^2_{\ell_k}}{m^2_W}\,\Re({\rm U}^\dagger_{jk}{\rm U}_{ki}), \tag{11}$$

$$\mu_{\nu_j\nu_i} = \frac{3\,{\rm i\,e}}{2M^{*2}}\left(m_{\nu_i} + m_{\nu_j}\right)\sum_{k=e,\mu,\tau}\frac{m^2_{\ell_k}}{m^2_W}\,\Im({\rm U}^\dagger_{jk}{\rm U}_{ki})\,, \tag{12}$$

where the neutrino-flavor mixing matrix U is approximatively unitary, i.e it is necessarily of the following form [7]

$$\sum_{i=1}^{3}{\rm U}^\dagger_{jk}{\rm U}_{ki} = \delta_{ji} - \varepsilon_{ji}, \tag{13}$$

where ε is a hermitian nonnegative matrix (i.e. with all eigenvalues nonnegative) and

$$|\varepsilon| = \sqrt{{\rm Tr}\ \varepsilon^2} = \mathcal{O}\left(m_{\nu_{\rm light}}/m_{\nu_{\rm heavy}}\right) \sim 10^{-22}\ \text{to}\ 10^{-21}. \tag{14}$$

It is important to note that the first term δ_{ji} from (13) in our case does not contribute, and that the case $|\varepsilon| = 0$ is excluded by the very existence of oscillation effects.

The transition dipole moments in general receive very small contributions because of the smallness of the neutrino mass, $|m_\nu| \simeq 10^{-2}$ eV [9]. The largest contribution among them is proportional to $\Re$ and $\Im$ parts of $\mathrm{U}^\dagger_{3\tau}\mathrm{U}_{\tau 2}$, which corresponds to the $2 \to 3$ transition. For the sum and difference of neutrino masses we assume hierarchical structure and take $|m_3 + m_2| \simeq |m_3 - m_2| \simeq |\Delta m^2_{32}|^{1/2} = 0.05$ eV [9]. For the mixing matrix elements [8] we set $|\Re(\mathrm{U}^\dagger_{3\tau}\mathrm{U}_{\tau 2})| \simeq |\Im(\mathrm{U}^\dagger_{3\tau}\mathrm{U}_{\tau 2})| \leq 0.5$.

The electric and magnetic transition dipole moments of neutrinos $d^{\mathrm{el}}_{\nu_2\nu_3}$ and $\mu_{\nu_2\nu_3}$ are then denoted as $\left(d^{\mathrm{el}}_{\mathrm{mag}}\right)_{23}$ and are given by

$$\begin{aligned} \left|\left(d^{\mathrm{el}}_{\mathrm{mag}}\right)_{23}\right| &= \frac{3e}{2M^{*2}}\,\frac{m_\tau^2}{m_W^2}\sqrt{|\Delta m^2_{32}|}\begin{pmatrix}|\Re(\mathrm{U}^\dagger_{3\tau}\mathrm{U}_{\tau 2})|\\ |\Im(\mathrm{U}^\dagger_{3\tau}\mathrm{U}_{\tau 2})|\end{pmatrix}, \\ &\lesssim 2.03\times 10^{-30}[\mathrm{e/eV}] = 0.38\times 10^{-34}\,[\mathrm{e\ cm}], \\ &= 2.07\times 10^{-24}\,\mu_B. \qquad (15) \end{aligned}$$

Note that neutrino mass extended standard model, as a consequence of loops (8), produces four orders of magnitude higher moments for a Dirac neutrino versus Majorana neutrino (15), due to an (m_ℓ^2/m_W^2)-suppression of Majorana moments relative to the Dirac ones [10].

Also note that electric transition dipole moments of light neutrinos are smaller than the ones of the d-quark. This is *the* order of magnitude of light neutrino transition dipole moments underlying the see–saw mechanism [11]. It is by orders of magnitude smaller than in lepton flavor unprotected SUSY models. See properties of neutrinos with respect to models which contain flavor mixing, the mass spectrum, dipole moments, electroweak radius, ect. including additional contributions arising from SUSY GUT's, extra dimensions, non-commutativity of space-time, etc., in [2, 12] and refs quoted therein. Of course rigorously established experimental bounds on the dipole moments of neutrinos are by orders of magnitude weaker than implied by our hypotheses (15). The properties of astrophysical neutrinos can be found in the following references [2, 13].

Up to this point our presentation is fully relativistic but valid only for not too large momenta as appropriate for the long range approximation adopted.

The non-relativistic components of electric and magnetic fields, whose coefficients are our electric and magnetic dipole moments, are

$$E_j(\vec{d}|_0) = \frac{1}{4\pi} d_k \, \partial_k \, \partial_j \, \frac{1}{r} \, , \tag{16}$$

$$B_j(\vec{\mu}|_0) = \frac{1}{4\pi} \mu_k \left[\partial_k \, \partial_j \, - \, \delta_{kj} \, \Delta \right] \frac{1}{r} \, , \tag{17}$$

For dipole "$\vec{d}$" at position "0" determined with position vector $\vec{x}_0$

we have the following fields at point $\vec{x}$:

$$
\begin{aligned}
\vec{E}(\vec{x}|_0) &= \frac{1}{4\pi}\left(3\vec{e}(\vec{e}\vec{d}) - \vec{d}\right)\frac{1}{r^3} - \frac{1}{3}\vec{d}\delta^3(\vec{x}-\vec{x}_0)\,, && (18)\\
\vec{B}(\vec{x}|_0) &= \frac{1}{4\pi}\left(3\vec{e}(\vec{e}\vec{\mu}) - \vec{\mu}\right)\frac{1}{r^3} + \frac{2}{3}\vec{\mu}\delta^3(\vec{x}-\vec{x}_0)\,. && (19)\\
r &= |\vec{x} - \vec{x}_0|,\ \vec{e} = (\vec{x} - \vec{x}_0)/r,\ \partial_n = \partial/\partial_{x_n}\,.
\end{aligned}
$$

For neutrinos in the non-relativistic equal dipole-dipole approximations they are of the form represented by hermitian operators whose matrix elements are given in Eqs (6,7,10,11,12).

Restricting to equal dipole-dipole interactions only in the case of transition $1 \to 2$ we define relative distance vector as $\vec{e} = (\vec{x} - \vec{x}')/r$ where $\vec{x}$ and $\vec{x}'$ are position vectors of dipole 1 and 2 respectively, and obtain well known dipole-dipole potential

$$
\begin{aligned}
V(d\,,d') &= -\frac{1}{4\pi}\left(3(\vec{d}\,\vec{e})(\vec{d}'\,\vec{e}) - (\vec{d}\,\vec{d}')\right)\frac{1}{r^3}\\
&\quad + \frac{1}{3}(\vec{d}\,\vec{d}')\,\delta^3\,(\vec{x}' - \vec{x}), && (20)\\
V(\mu\,,\mu') &= -\frac{1}{4\pi}\left(3(\vec{\mu}\,\vec{e})(\vec{\mu}'\,\vec{e}) - (\vec{\mu}\,\vec{\mu}')\right)\frac{1}{r^3}\\
&\quad - \frac{2}{3}(\vec{\mu}\,\vec{\mu}')\,\delta^3\,(\vec{x}' - \vec{x}). && (21)
\end{aligned}
$$

The discussed above dipole moments give rise to electric and magnetic long range dipole-dipole forces, which are the only ones in the non-relativistic setting. Hence only the nonlocal terms in the potentials $V(d, d')$, $V(\mu, \mu')$ are of concern to us here.

Note that by introducing the gravitational potential for any neutrino pair

$$V_{\rm gravity} = -G_N\,\delta_{j_1 i_1}\,\delta_{j_2 i_2}\frac{m_{\nu_{i_1}}\,m_{\nu_{i_2}}}{|\,r\,|}, \quad i_1 < i_2\,. \tag{22}$$

and equating the generic absolute values of gravitational and dipole-dipole potentials, (21) and (22), at $r = R \neq 0$, together with Eq. (15), we obtain the interesting characteristic distance

$$\begin{aligned} R &= \sqrt{\frac{\alpha_{\rm em}}{m_{\nu_{i_2}}\,m_{\nu_{i_3}}}}\left|\frac{\left(d^{\rm el}_{\rm mag}\right)_{23}}{e}M_{Pl}\right|, \\ &= \sqrt{\frac{\alpha_{\rm em}}{500}}\times 0.38\times 10^{-34}\left(\frac{1{\rm cm}}{L_{Pl}}\right)\times 0.0197\,[{\rm cm}], \\ &= 1.77\times 10^{-6}\,[{\rm cm}], \end{aligned} \tag{23}$$

where the above unique long-range potentials are comparable.

We assume that light neutrino condensates, due to neutrino transition dipole moments interaction energy, are also responsible for formation of dark energy. To estimate the dark energy density due to ν-dipole potentials, ρ^{ν}_{DED}, we first find the absolute value of the caracteristic energy due to dipole-dipole interaction $< \epsilon_\nu >_{vac}$:

$$(24) < \epsilon_\nu >_{vac} \quad \simeq \quad \frac{|\int d^3r\, V|}{v} = \frac{1}{v}\Big|\frac{2}{3}\sum_{i,j=1}^{N_\nu} \vec{\mu}_i\vec{\mu}_j\Big|,$$

where v is an intrinsic volume and N_ν is number neutrino pairs. Next we define $|\mu|^2$ as caracteristic measure of quadratic dipole strenght:

$$(25) \qquad |\mu|^2 \quad = \quad \Big|\frac{2}{3}\sum_{i,j=1}^{N_\nu} \vec{\mu}_i\vec{\mu}_j\Big|,$$

and then the dark energy density due to ν-dipoles is

$$(26) \qquad \rho^\nu_{DED} \quad = \quad \frac{< \epsilon_\nu >_{vac}}{v} = \Big(\frac{|\mu|}{v}\Big)^2.$$

This is maximal for the case for $\mu \times \mu' = \mu_{21} \times \mu_{12} = -(\mu_{12})^2 = |\mu_{12}|^2 \simeq |\mu|^2$, etc. Namely in the two neutrino channel (both spins, i.e. ν and/or $\bar{\nu}$) the dipole-dipole interactions do not change the total energy $\sim\ m_1 + m_2$, provided that the pair is composed of two different mass-flavors, i.e. $1 \neq 2$ (in the s-channel). Antisymmetric type of interactions just changes the flavor ordering $\nu_{1,m_1}\nu_{2,m_2} \to \nu_{2,m_2}\nu_{1,m_1}$, (i.e. $m_1 \longleftrightarrow m_2$ at fixed $1, 2$). This gives the overall contribution, for $d_{ij} \to d_{12}$ with ij mass-eigenstate-flavors, which is, for example, $d \times d' = d_{21} \times d_{12} = -(d_{12})^2 = |d_{12}|^2$ because of the factor i^2 coming from $-(d_{12})^2$. Thus the attraction or repulsion is within one mass-pair-channel and thus fully active without changing the mass of the pair provided of course the *mass-flavors in the pair are distinct*.

In this way identifing, by hypotesis, (26) with the measured dark energy density today ρ_{DED} we have found:

$$v = \Big(\frac{|\mu|^2}{\rho_{DED}}\Big)^{1/2} = \frac{4\pi}{3} R_\nu^3 \,,$$

$$R_\nu = \Big(\frac{9}{16\pi^2}\frac{|\mu|^2}{\rho_{DED}}\Big)^{1/6} \,, \tag{27}$$

where R_ν is linear size of intrinsic volume v.

If we choose for $\big|\mu\big|$ the value of the dipole moments in Eq. (15) and from observation $\rho_{DED} = (2.3 \text{ meV})^4 \times \frac{\mathrm{h}^2}{0.5}$, with $h = 0.73$ being present day normalized Hubble constant [14] we obtain:

$$R_\nu = 0.84 \times 10^{-13}\text{cm} \simeq (200\,\text{MeV})^{-1} \,. \tag{28}$$

Note that $R_\nu^{-1} \simeq 200$ MeV relates intrinsic volume v to a cosmological period corresponding to $T \sim R_\nu^{-1}$ which represents a distant past of cosmological evolution.

From (23) and (28) it follows $R_\nu \ll R$ which is consistent with the dipole moment interaction dominating gravitational ones.

The elementary 4-neutrino interaction energy density is obviously very small, but it has a collective $(number\ of\ neutrinos)^2$ growth. In addition it definitely will have, for arbitrary moments otherwise, an attractive sub-channel, depending on neutrino spins. The attraction will generate condensation phenomena, i.e. *neutrino-condensates*, by the Fermi-criterion, and since gravity is always attractive those two facts together lead to neutrino condensation phenomena relative to a free fermion gas.

Here we only consider condensates giving rise to a cosmological term or equivalently to vacuum energy-density. Assuming nonvanishing neutrino condensates due to dipole and gravitational long range interactions giving rise to a cosmological term, (and/or to vacuum energy density and pressure), i.e. not canceled by a readjustment of gravitational effects, it follows that these condensates are a specific source of dark energy. The condensate will correspond eventually to some 'vacuum-energy density' and may not be canceled as all other larger condensates, e.g. of QCD, electroweak [15], etc.

The condensate will then alter the neutrino energy-momentum dependence as compared with free massive neutrino motion and thus the mean energy density in neutrinos will be larger for a given thermal ensemble and the same temperature. This temperature is approximately $2^\circ K$ today in 'the universe' and it corresponds to ν-number density n_{ν_0}, i.e. $n_{\nu_0} \simeq 300$ free neutrinos per cm^3 at present.

From known cosmological parameters we have dark energy density today, while energy density of free light neutrinos (above vacuum), at temperature $2^\circ K$ and assuming neutrino mean mass $m_\nu \simeq 20$ meV, is $\rho_{ED} = (0.4634 \text{ meV})^4$. Ratio of those two facts

$$\frac{(dark\, energy\, density)}{(\nu - number\, density) \times (\nu - mean\, mass)},$$

produces an interesting experimental number:

$$\frac{\rho_{DED}}{\rho_{ED}} = (\frac{2.3369}{0.4634})^4 = 5.043^4 \simeq 647. \tag{29}$$

Neutrino mean mass of 20 meV was used due to the assumption of normal neutrino family hierarchy. Of course this number is larger in the case of inverted hierarchy.

If our analysis can overcome the factor 647 and furthermore, since we are comparing two very different types of energy densities, this could be transfered to neutrino condensates.

The experimental ratio in Eq. (29) has no direct bearing on the size of the neutrino condensates, which represent vacuum energy density. It is used only here in order to emphasize that we cannot exclude the possibility that the sum of neutrino condensates equals the observed dark energy density ρ_{DED} in value $(2.34 \text{ meV})^4$ and sign (positive), causing acceleration of the universe expansion today (and tomorrow) and beeing de Sitter-like. Our entire approach also ilustrates the sign of dark energy density which is inconsistent with stability in the framework of local field theory in uncurved space-time.

This could be related to another inconsistency arising from large, but finite, lifetimes of not only light neutrions and 'baryons'. Our estimate of unstable neutrino lifetime from the decay rate in the neutrino-mass extended standard model (SM)

$$\Gamma(\nu_h \to \nu_\ell \, \gamma) \;=\; \frac{m_{\nu_h}^5}{16\pi} \left(\frac{G_F}{\sqrt{2}} \frac{e}{4\pi^2} \, \mathrm{U}^\dagger \mathrm{U} \, F_2 \right)^2$$

$$\simeq \; 1.6 \times 10^{-63} \, \mathrm{meV} \,, \tag{30}$$

gives

$$\tau_{\nu_h} \simeq 4 \times 10^{51} \, \mathrm{s} \,. \tag{31}$$

This value was obtained from ν-dipole moment interaction (5)-(15) with neutrino mass: $m_{\nu_h} = 50$ meV.

It is interesting to notice that due to the sign of (24-26), the total energy density, ρ, of relic neutrinos,

$$\rho = m_\nu n_\nu - \rho^\nu_{DED} (= n_\nu^2 |\mu|^2) \,, \tag{32}$$

may have an extremum during the cosmological evolution. Namely $n_\nu = n_{\nu_0} \, a^{-3}$, with a being the scale factor of the universe. This, in turn, may entail consequences for an accelerating phase of the expansion of the universe, since near extremum a_{ext}, the EOS for relic neutrino gas,

$$w_\nu + 1 = -\frac{1}{3}\frac{\partial}{\partial(\log a)}(\log \rho) = -\frac{1}{3}\frac{a}{\rho}\frac{d\rho}{da} \ , \tag{33}$$

switches to ≈ -1. Indeed, from (32) we find

$$a_{ext} = \Big(\frac{2n_{\nu_0}|\mu|^2}{m_\nu}\Big)^{1/3} \ . \tag{34}$$

If we are to explore effects for a late-time accelerating phase, then we should set $a_{ext} \sim 1$. However, even for magnetic moments as large as $10^{-10}\mu_B$, the neutrino mass would be hopelessly tiny to induce any observable effect on present acceleration of the universe.

As a way out of the above inconsistency one can recall a recent model proposed by Fardon, Nelson and Weiner (FNW) [16] and developed later by Kaplan, Nelson and Weiner [17], and Peccei [18], in which relic neutrinos are tied together with the sector of ‘standard’ dark energy (represented by a canonically normalized scalar field). The model is very appealing with regard to the ‘cosmic coincidence problem’ [19], since from the known behavior of dark matter, ordinary matter and radiation one finds that any reasonable tracking of these components by dark energy always goes at the expense of the late time transition of its equation of state, thus creating a new problem called the ”why now?” problem. On the other hand, if relic neutrinos can be kept tightly coupled to the original dark energy fluid for most of the history of the universe, the near coincidence at present, $\rho_\Lambda \sim \rho_\nu$, will cease to be perceptive as a coincidence at all. This was possible if the mass of the neutrino was promoted to a dynamical quantity, being a function of the acceleron field (canonically normalized scalar field similar to quintessence). The main feature of the scenario [16] is that although the number density of neutrinos dilutes canonically $(\sim a^{-3})$, the masses of neutrinos change almost inversely $(\sim a^{-3w})$, thereby promoting their energy density to an almost undilutable substance. Hence relic neutrinos become tightly coupled to the original dark energy fluid.

In addition, by applying the FNW scenario to our model, in which the energy density for relic neutrinos is supplemented with a term due to nonvanishing electro-magnetic moments, we can draw some conclusions about intrinsic properties of neutrinos if also $|\mu|$ is considered as a dynamical field (some function of m_ν). In this case one can show that in the FNW scenario the EOS for the coupled dark energy fluid obeys

$$w + 1 = \frac{m_\nu n_\nu - 2m_\nu^2|\mu|^2}{\rho_{totaldark}}. \tag{35}$$

Since the neutrino contribution gives only a small fraction of the total energy density, we have $w \simeq -1$, in accordance with what cosmological data imply. Also, the data imply very slow variation of w with a, which, taken in a literal sense, means that both terms in the numerator of (35) should scale as $\rho_{totaldark} \sim a^{-3(1+w)}$. This entails, $m_\nu \sim a^{-3w}$, $|\mu|^2 \sim a^{-3(1-w)}$. It is interesting to note that although the scaling of m_ν and $|\mu|$ with a are formally different, they become the same in the limit $w \to -1$. This complies with the prediction of the minimally extended SM $|\mu|_R \sim |\mu|$, where to each generation of fermions of the SM a right-handed neutrino field is added, in contrast with more complicated models where the neutrino magnetic moment is disentangled from the neutrino mass.

In conclusion, we have considered the cosmological consequences of long-range interactions in a non-relativistic setting and arising from various electromagnetic form factors of a neutrino. We have emphasised the possibility that the responsible interaction itself has an attractive channel, leading neutrino condensation phenomena to occur. This would entail a sort of dark energy, responsible for the late-time acceleration in the expansion of the universe. In addition, the energy density due to neutrino electromagnetic moments, when superimposed on the standard contribution of a neutrino background, may be responsible for acceleration phases during the history of the universe. When implemented in a recently suggested dark energy scenario with mass varying neutrinos, the electromagnetic neutrino interaction may also shed some light on intrinsic neutrino properties.

The work of R.H. and J.T. is supported by the Croatian Ministry of Science, Education and Sport under Contract No. 0980982930-2872 and No. 0980982930-2900.

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$$
\begin{aligned}
|\mu_{\nu_i}| &= \frac{3e}{2M^{*2}}\, m_{\nu_i} \left[1 - \frac{1}{2}\sum_{\ell=e,\mu,\tau} \frac{m_{\ell_k}^2}{m_W^2}|U_{\ell i}|^2\right] \\
&\lesssim 1.56 \times 10^{-26}[\mathrm{e/eV}] = 0.29 \times 10^{-30}\,[\mathrm{e\ cm}] \\
&= 1.60 \times 10^{-20}\,\mu_B .
\end{aligned}
$$

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