S_3 Symmetry as the Origin of CKM Matrix

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Outline

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Discrete symmetries in particle physics

- In SM fermions (both leptons and quarks) come in three generations
- There are inter-generational differences, in contrast to their uniformity in gauge interactions
- Two types of hierarchies in the flavor sector:
  - Large hierarchy within the charged fermion sector and enormous hierarchy between charged fermion and neutrino masses
  - Mixing information in quark and lepton sector
- Finite discrete symmetry groups (e.g., S_3, S_4, D_4, A_4 etc.) provide an effective way of explaining some of these flavor issues
- We will consider the case of S_3 symmetric model

Outline

Basics of S_3

- S_3 is the permutation group of three objects
- The order of S_3 is $3! = 6$
- The six elements correspond to the following transformations

$$
\begin{aligned}
e &: (x_1, x_2, x_3) \to (x_1, x_2, x_3), \\
a_1 &: (x_1, x_2, x_3) \to (x_2, x_1, x_3), \\
a_2 &: (x_1, x_2, x_3) \to (x_3, x_2, x_1), \\
a_3 &: (x_1, x_2, x_3) \to (x_1, x_3, x_2), \\
a_4 &: (x_1, x_2, x_3) \to (x_3, x_1, x_2), \\
a_5 &: (x_1, x_2, x_3) \to (x_2, x_3, x_1).
\end{aligned}
$$

- S_3 can also be thought of as the symmetry of an equilateral triangle with a_1 and $a_1 a_2$ being the reflection and the $2\pi/3$ rotation respectively

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Scalar Potential

- The scalar sector consists of *three* $\mathrm{SU}(2)_L$ scalar doublets ϕ_i $(i = 1, 2, 3)$
- $(\phi_1, \phi_2)^T$ transform as an S_3 doublet and ϕ_3 is an S_3 singlet
- Most general S_3-symmetric scalar potential with these fields can be written as,

$$
\begin{aligned}
V(\phi_i) =& \mu_1^2(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) + \mu_3^2\phi_3^\dagger\phi_3 + \lambda_1(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2)^2 \\
&+ \lambda_2(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1)^2 + \lambda_3\left\{(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1)^2 + (\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2)^2\right\} \\
&+ \lambda_4\left\{(\phi_3^\dagger\phi_1)(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1) + (\phi_3^\dagger\phi_2)(\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2) + \mathrm{h.c.}\right\} \\
&+ \lambda_5(\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) + \lambda_6\left\{(\phi_3^\dagger\phi_1)(\phi_1^\dagger\phi_3) + (\phi_3^\dagger\phi_2)(\phi_2^\dagger\phi_3)\right\} \\
&+ \lambda_7\left\{(\phi_3^\dagger\phi_1)(\phi_3^\dagger\phi_1) + (\phi_3^\dagger\phi_2)(\phi_3^\dagger\phi_2) + \mathrm{h.c.}\right\} + \lambda_8(\phi_3^\dagger\phi_3)^2
\end{aligned}
$$

Scalar Potential

- The minimization conditions for the potential are:

$$2\mu_1^2 = -2\lambda_1(v_1^2+v_2^2) - 2\lambda_3(v_1^2+v_2^2) - 6\lambda_4 v_2 v_3 - (\lambda_5+\lambda_6+2\lambda_7)v_3^2\,,$$
$$2\mu_1^2 = -2\lambda_1(v_1^2+v_2^2) - 2\lambda_3(v_1^2+v_2^2) - \frac{3v_3}{v_2}\lambda_4(v_1^2-v_2^2) - (\lambda_5+\lambda_6+2\lambda_7)v_3^2\,,$$
$$2\mu_3^2 = \lambda_4\frac{v_2}{v_3}(v_2^2-3v_1^2) - (\lambda_5+\lambda_6+2\lambda_7)(v_1^2+v_2^2) - 2\lambda_8 v_3^2\,.$$

- There exists three nontrivial conditions for consistency,

$\lambda_4 = 0$ $\Rightarrow$ a massless scalar

$v_2 = \sqrt{3}v_1$ and $v_3 = 0$ $\Rightarrow$ interesting for DM aspect (?)

$v_1 = \sqrt{3}v_2$ and v_3 arbitrary $\Rightarrow$ present case

- For the case $v_1 = \sqrt{3} v_2$, even after spontaneous symmetry breaking, a $\mathbb{Z}_2$ subgroup of S_3 remains preserved
- To be precise, the vacuum in the $(\phi_1, \phi_2)^T$ basis is $(\sqrt{3}, 1)^T$ and

$$\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} \quad \text{with} \quad \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}^2 = \mathbb{1}_2$$

- This remnant $\mathbb{Z}_2$ will be used to construct the realistic model for quarks

Physical Scalars

- After electroweak symmetry breaking from three $\mathrm{SU}(2)_L$ doublet scalars we will have nine physical scalars

Physical States	Transformation under $\mathbb{Z}_2$
h^0, $H_1^{\pm}$, A_1	Odd
h, H, $H_2^{\pm}$, A_2	Even

- Here h will have the SM-like couplings in the *alignment limit* i.e., $\sin(\beta - \alpha) = 1$ where $\beta = \tan^{-1}(2v_2/v_3)$ and α is the mixing angle in the $h - H$ sector

λs and Masses

The quartic couplings and the physical scalar masses can be connected via the following relations:

$$\lambda_1 = \frac{1}{2v^2 \sin^2\beta} \left\{ \left(m_h^2 \cos^2\alpha + m_H^2 \sin^2\alpha \right) + \left(m_{1+}^2 - m_{2+}^2 \cos^2\beta - \frac{1}{9} m_{h0}^2 \right) \right\} ,$$

$$\lambda_2 = \frac{1}{2v^2 \sin^2\beta} \left\{ (m_{1+}^2 - m_{A1}^2) - (m_{2+}^2 - m_{A2}^2) \cos^2\beta \right\} ,$$

$$\lambda_3 = \frac{1}{2v^2 \sin^2\beta} \left(\frac{4}{9} m_{h0}^2 + m_{2+}^2 \cos^2\beta - m_{1+}^2 \right) ,$$

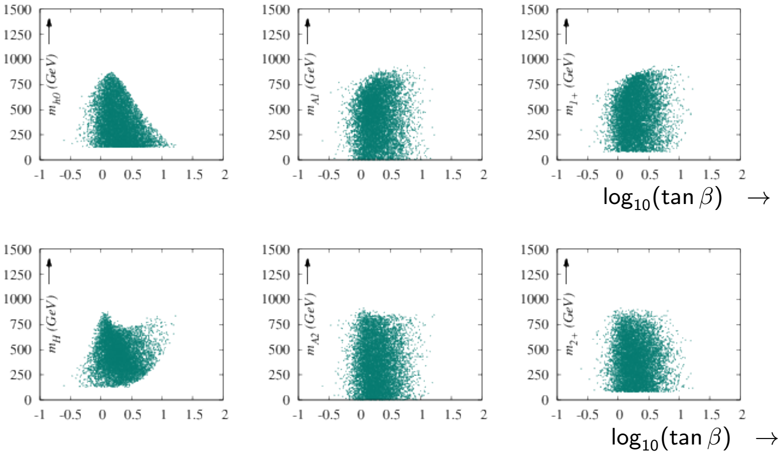
$$\lambda_4 = -\frac{2}{9} \frac{m_{h0}^2}{v^2} \frac{1}{\sin\beta \cos\beta} ,$$

$$\lambda_5 = \frac{1}{v^2} \left\{ \frac{\sin\alpha \cos\alpha}{\sin\beta \cos\beta} \left(m_H^2 - m_h^2 \right) + 2m_{2+}^2 + \frac{1}{9} \frac{m_{h0}^2}{\cos^2\beta} \right\} ,$$

$$\lambda_6 = \frac{1}{v^2} \left(\frac{1}{9} \frac{m_{h0}^2}{\cos^2\beta} + m_{A2}^2 - 2m_{2+}^2 \right) , \quad \lambda_7 = \frac{1}{2v^2} \left(\frac{1}{9} \frac{m_{h0}^2}{\cos^2\beta} - m_{A2}^2 \right)$$

$$\lambda_8 = \frac{1}{2v^2 \cos^2\beta} \left\{ \left(m_h^2 \sin^2\alpha + m_H^2 \cos^2\alpha \right) - \frac{1}{9} m_{h0}^2 \tan^2\beta \right\} .$$

Constraints from unitarity and stability



Unitarity and stability demands that $\tan\beta \in [0.3, 17]$ and the physical scalars are below 1 TeV

Outline

Quark Sector

- In the gauge basis the S_3 transformation properties of the quarks,

$$\mathbf{1}: \quad Q_3,\ u_{3R},\ d_{3R},$$

$$\mathbf{2}: \quad \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}, \begin{pmatrix} u_{1R} \\ u_{2R} \end{pmatrix}, \begin{pmatrix} d_{1R} \\ d_{2R} \end{pmatrix}$$

  where Q_is are the usual left-handed $\mathrm{SU}(2)_L$ quark doublets and u_{iR}s (d_{iR}s) are the right-handed up (down) type $\mathrm{SU}(2)_L$ singlets
- The Yukawa Lagrangian is given by,

$$\mathcal{L}_{\rm Yuk} = \mathcal{L}^U_{\rm Yuk} + \mathcal{L}^D_{\rm Yuk}$$

  where

$$\begin{aligned}\mathcal{L}^U_{\rm Yuk} =& - y_1^u\left(\bar{Q}_1\tilde{\phi}_3 u_{1R} + \bar{Q}_2\tilde{\phi}_3 u_{2R}\right) - y_2^u\Big\{\left(\bar{Q}_1\tilde{\phi}_2 + \bar{Q}_2\tilde{\phi}_1\right)u_{1R} \\ &+ \left(\bar{Q}_1\tilde{\phi}_1 - \bar{Q}_2\tilde{\phi}_2\right)u_{2R}\Big\} - y_3^u\bar{Q}_3\tilde{\phi}_3 u_{3R} - y_4^u\bar{Q}_3\left(\tilde{\phi}_1 u_{1R} + \tilde{\phi}_2 u_{2R}\right) \\ &- y_5^u\left(\bar{Q}_1\tilde{\phi}_1 + \bar{Q}_2\tilde{\phi}_2\right)u_{3R} + \mathrm{h.c.} \\ \mathcal{L}^D_{\rm Yuk} =& \mathcal{L}^U_{\rm Yuk} \text{ with } (u_{iR} \to d_{iR}, y_i^u \to y_i^d, \tilde{\phi}_i \to \phi_i)\end{aligned}$$

Quark Mass Matrices

- The general form of the quark mass matrix (for brevity only up-type case is shown) is given by,

$$\mathcal{M}_u = \begin{pmatrix} y_1^u v_3 + y_2^u v_2 & y_2^u v_1 & y_5^u v_1 \\ y_2^u v_1 & y_1^u v_3 - y_2^u v_2 & y_5^u v_2 \\ y_4^u v_1 & y_4^u v_2 & y_3^u v_3 \end{pmatrix}$$

- This can be block-diagonalized by the unitary matrix,

$$X = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

in the following way,

$$\mathcal{M}_u^{\text{block}} = X^\dagger \mathcal{M}_u X = \begin{pmatrix} y_1^u v_3 - 2y_2^u v_2 & 0 & 0 \\ 0 & y_1^u v_3 + 2y_2^u v_2 & 2y_5^u v_2 \\ 0 & 2y_4^u v_2 & y_3^u v_3 \end{pmatrix}$$

- Recall that the vacuum alignment $v_1 = \sqrt{3} v_2$ implies the breaking $S_3 \to \mathbb{Z}_2$

- The $\mathbb{Z}_2$-odd combination of fermion will not mix with the $\mathbb{Z}_2$-even counterparts in the fermion mass terms
- We can define the *top quark* as the $\mathbb{Z}_2$-odd combination with mass

$$m_t = |y_1^u v_3 - 2y_2^u v_2| = v|y_1^u \cos\beta - y_2^u \sin\beta|$$

- The block-diagonal form of $\mathcal{M}_u^{\rm block}$ implies that one can define the following intermediate basis in the up-sector separating the $\mathbb{Z}_2$-odd and $\mathbb{Z}_2$-even components,

$$\begin{pmatrix} t \\ c' \\ u' \end{pmatrix} = X^\dagger \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

- Rotate further the 2×2 block to get the physical u and c quarks
- Similar treatment can be followed for down sector by identifying the b as the $\mathbb{Z}_2$-odd combination

Getting the CKM

- Now one can get

$$\mathcal{M}_u^{\rm diag} = U_L^\dagger \mathcal{M}_u^{\rm block} U_R = {\rm diag}(m_t, m_c, m_u)$$

where both U_L and U_R are block-diagonal

- One can take U_L to be of the form

$$U_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_u & -\sin\theta_u \\ 0 & \sin\theta_u & \cos\theta_u \end{pmatrix}$$

- Then $\mathcal{M}_u^{\rm diag} = U_L^\dagger \mathcal{M}_u^{\rm block} U_R = U_L^\dagger X^\dagger \mathcal{M}_u X U_R = \mathcal{U}_L^\dagger \mathcal{M}_u \mathcal{U}_R$ (say); where $\mathcal{U}_{L,R} = XU_{L,R}$
- Similarly in the down sector $\mathcal{D}_{L,R} = XD_{L,R}$ where

$$D_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_d & -\sin\theta_d \\ 0 & \sin\theta_d & \cos\theta_d \end{pmatrix}$$

- The CKM matrix is thus given by,

$$V_{\rm CKM} = \mathcal{U}_L^\dagger \mathcal{D}_L = U_L^\dagger D_L = \begin{matrix} & \mathbf{b} & \mathbf{s} & \mathbf{d} \\ \mathbf{t} & 1 & 0 & 0 \\ \mathbf{c} & 0 & \cos\theta_C & -\sin\theta_C \\ \mathbf{u} & 0 & \sin\theta_C & \cos\theta_C \end{matrix}$$

where $\theta_C = \theta_d - \theta_u$

- By choosing $\sin\theta_C = \sin(\theta_d - \theta_u) \approx \lambda$, one can reproduce the Cabibbo block of the CKM matrix
- The near block-diagonal structure of CKM matrix can be thought of as a direct consequence of the remnant $\mathbb{Z}_2$ symmetry
- Only a mild breaking of this $\mathbb{Z}_2$ can result in the exact structure of the CKM matrix

Mild Breaking of $\mathbb{Z}_2$ Gives the Exact Form of CKM Matrix

- Introduce a soft S_3-breaking term (say, $\mu_{13}^2(\phi_1^\dagger\phi_3 + \phi_3^\dagger\phi_1)$)
- This will slightly modify the VEV alignment, $v_1 = \sqrt{3}v_2 + \Delta$ with $\Delta \ll v_{1,2}$
- Thus the mass matrix in the up sector (for example) will be,

$$\widetilde{\mathcal{M}}_u = \mathcal{M}_u + \begin{pmatrix} 0 & y_2^u\Delta & y_5^u\Delta \\ y_2^u\Delta & 0 & 0 \\ y_4^u\Delta & 0 & 0 \end{pmatrix} \equiv \mathcal{M}_u + \begin{pmatrix} 0 & \Delta_2^u & \Delta_5^u \\ \Delta_2^u & 0 & 0 \\ \Delta_4^u & 0 & 0 \end{pmatrix}$$

- Clearly, now we will have

$$\mathcal{M}_u^{\rm diag} = \mathscr{U}_L^\dagger \widetilde{\mathcal{M}}_u \mathscr{U}_R$$

where $\mathscr{U}_L$ (and similarly $\mathscr{U}_R$) can be defined as $\mathscr{U}_L = \mathcal{U}_L\mathfrak{U}_L$ and $\mathfrak{U}_L$ is close to unit matrix which takes care of the very small Δ,

$$\mathfrak{U}_L = \begin{pmatrix} 1 & A\lambda^2 & C\lambda^3 \\ -A\lambda^2 & 1 & 0 \\ -C^*\lambda^3 & 0 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Note that $\mathfrak{D}_L$ can also be approximated as $\mathbb{1}_3$

- Thus choosing the parameter $C = A(\rho + i\eta)$, one can write,

$$V_{\rm CKM} = \mathfrak{U}_L^\dagger \mathcal{U}_L^\dagger \mathcal{D}_L \mathfrak{D}_L$$

$$= \begin{matrix} & \mathbf{b} & \mathbf{s} & \mathbf{d} \\ \mathbf{t} & 1 & -A\lambda^2 & A\lambda^3(1-\rho-i\eta) \\ \mathbf{c} & A\lambda^2 & 1-\frac{\lambda^2}{2} & -\lambda \\ \mathbf{u} & A\lambda^3(\rho-i\eta) & \lambda & 1-\frac{\lambda^2}{2} \end{matrix} + \mathcal{O}(\lambda^4)$$

- Even if we consider small departure of $\mathfrak{D}_L$ from $\mathbb{1}_3$ the general form of $V_{\rm CKM}$ will not change

Outline

- The $\mathbb{Z}_2$-odd particles (e.g., $h^0, A_1, H_1^{\pm}$) will only have off-diagonal couplings with quarks involving 3rd generation physical quarks (t or b). But the mild breaking of $\mathbb{Z}_2$ will lead to tiny diagonal couplings
- For example, if h^0 is light enough, it can be looked for in the channel $t \to ch^0$ followed by $h^0 \to \mu\tau$ or $e\tau$. But for heavy h^0 ILC is the better option to produce h^0 via the coupling with SM-like Higgs h, $(h^0 h^0 h)$
- The precise measurement of the decay width $h \to \gamma\gamma$ can also put stringent constraints (see arXiv:1408.6133)
- Like most of the extended scalar sector models here also the FCNC related issues are to be dealt with much care

Outline

Summary and Conclusion

Starting from an S_3 symmetric theory and its spontaneous breaking to an approximate $\mathbb{Z}_2$ answers the following puzzles

- Why the third generation of quarks are so different (massive) from the other two?
  As the third generation quarks are $\mathbb{Z}_2$-odd
- Why the CKM matrix is nearly block-diagonal?
  Because of the misalignment of the mixing between the first two generations in the up and down sectors, both of which are $\mathbb{Z}_2$-even

The study of the lepton sector and a detailed exploration of the flavour constraints are yet to be done (future project)