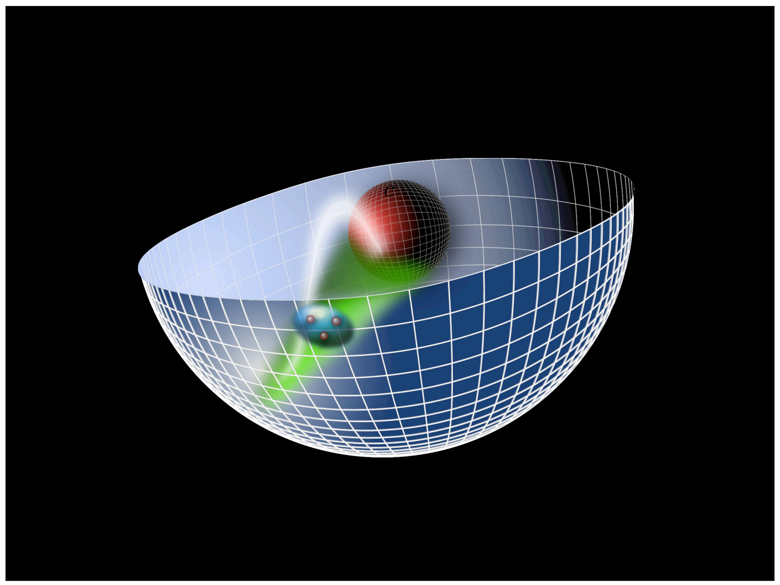
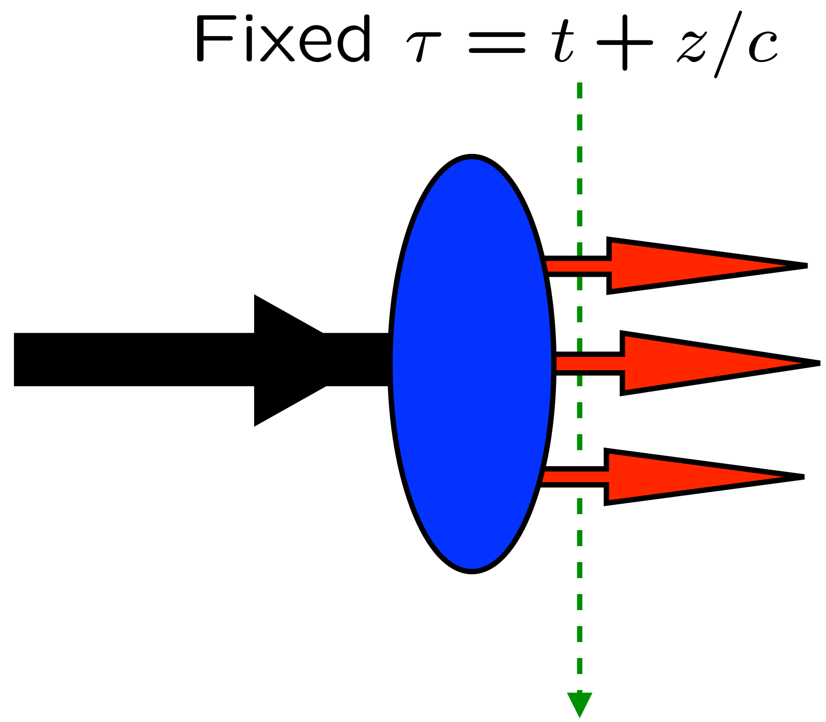
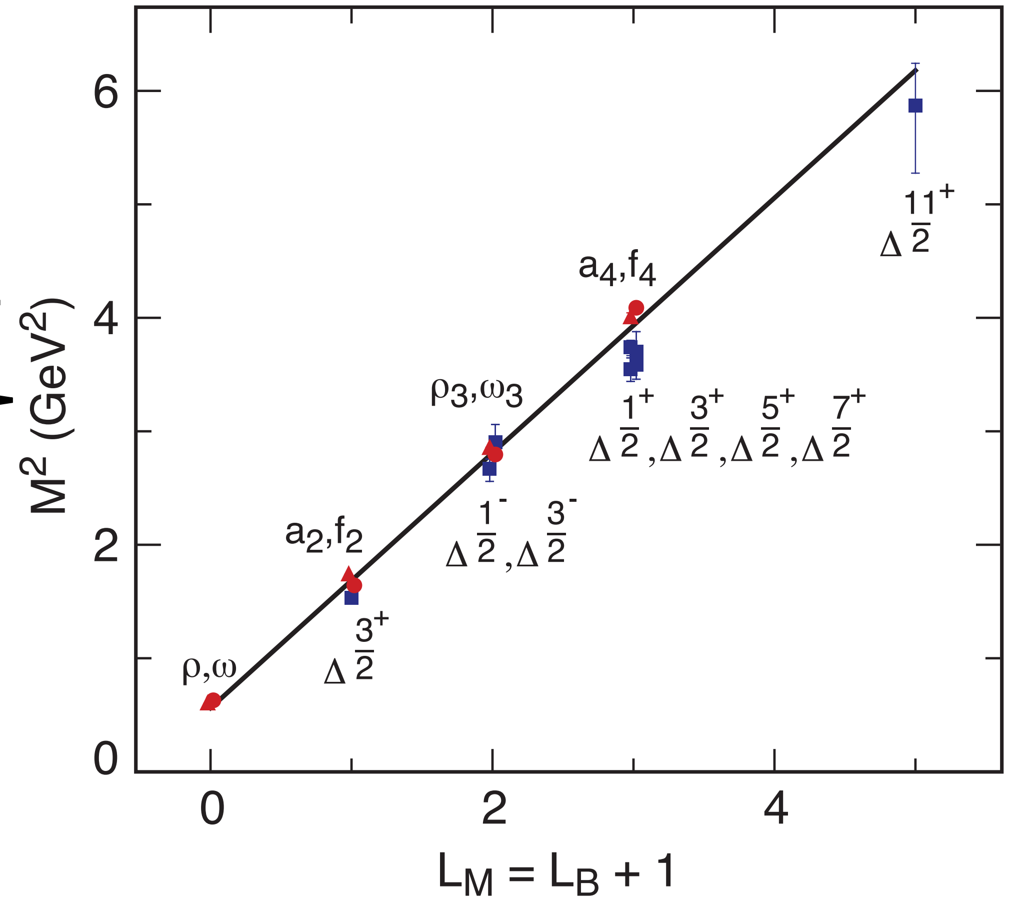
Light-Front Holographic QCD, Color Confinement, and Supersymmetric Features of QCD







Stan Brodsky

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The Standard Theory and Beyond

Albufeira, Portugal October 24-31, 2015

Gell Mann, Fritzsch, Leutwyler

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

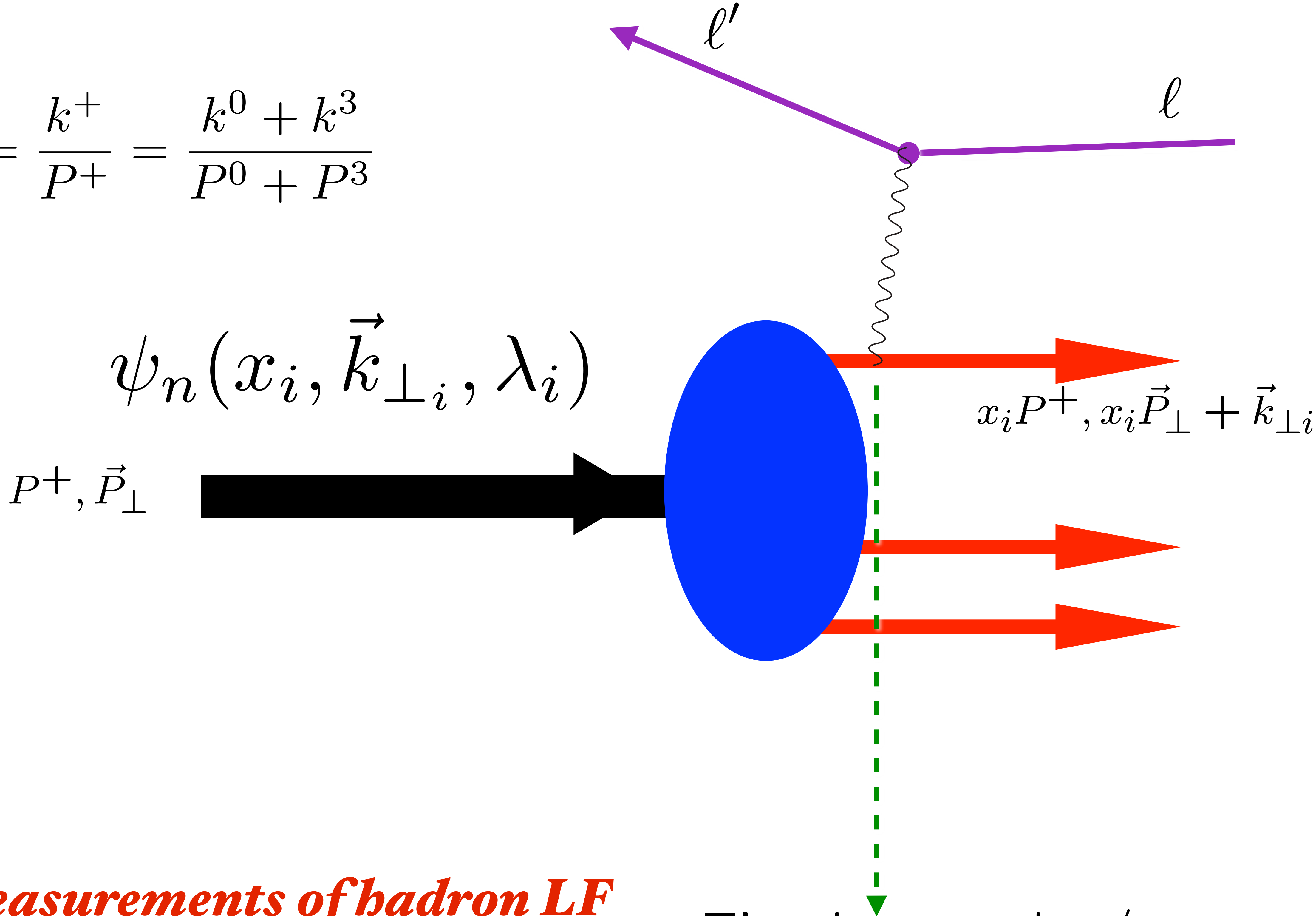
Where does the QCD Mass Scale come from?

How does color confinement arise?

- **de Alfaro, Fubini, Furlan:** *Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!*

Unique confinement potential!

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Fixed $\tau = t + z/c$

Measurements of hadron LF wavefunction are at fixed LF time

$$x_{bj} = x = \frac{k^+}{P^+}$$

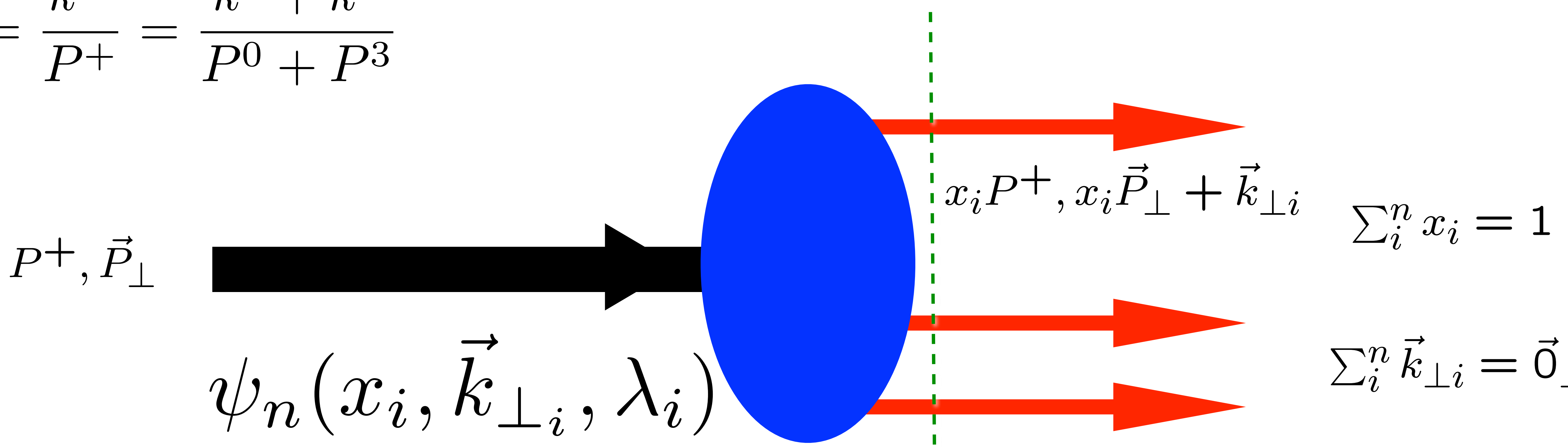
Like a flash photograph

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



$$\int \psi_{BS}(p,k) dk^- \to \psi_{LF}$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

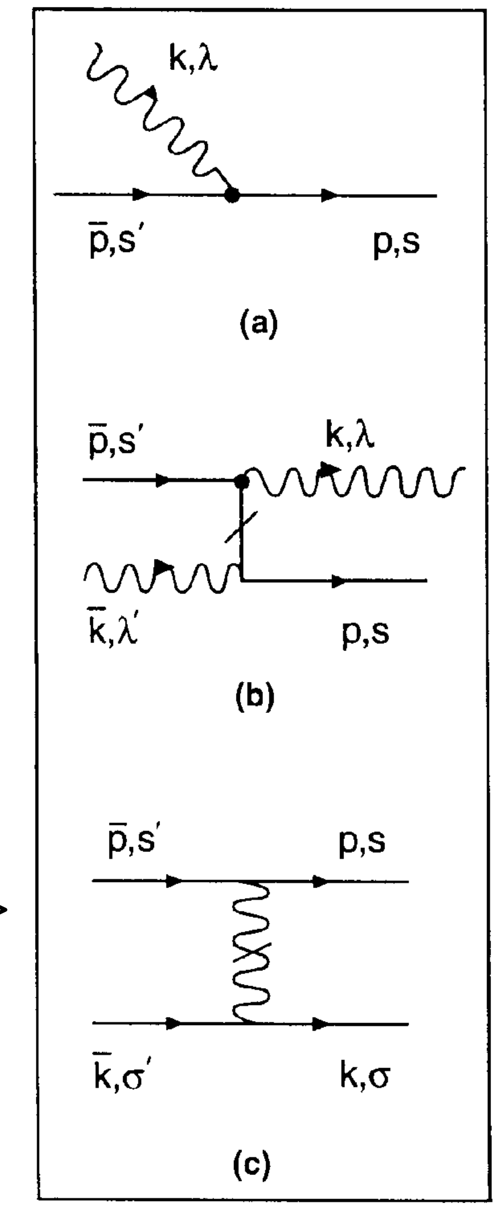
$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

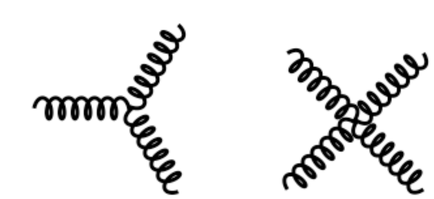
H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$



Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions



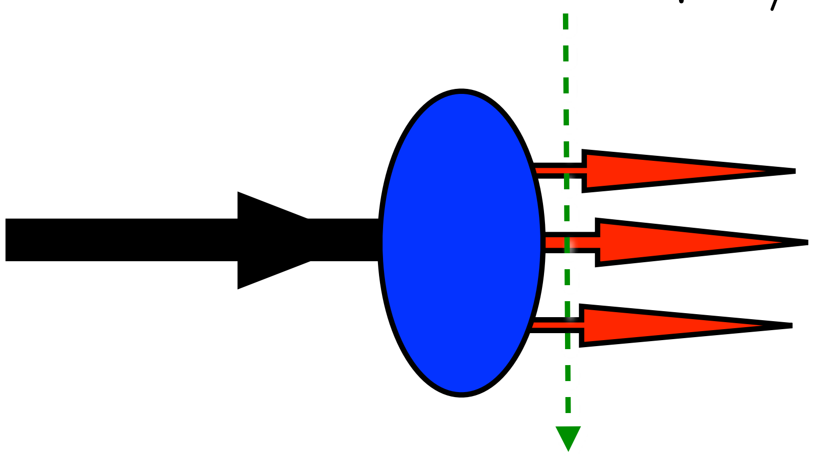
LFWFs: Off-shell in P- and invariant mass

H^{int}_{LF}

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}^{QCD}_{LF}|\psi> = M^2|\psi>$$

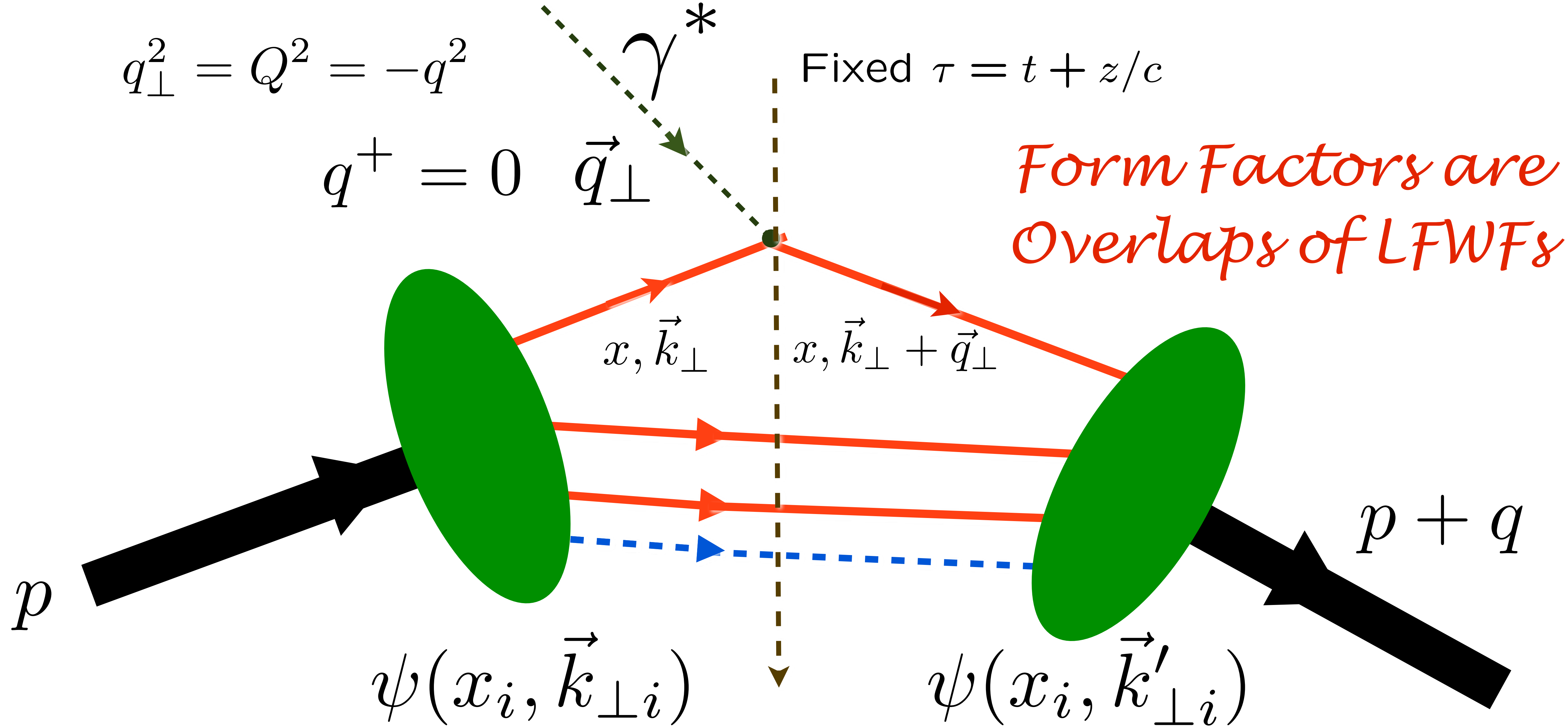
Direct connection to QCD Lagrangian

Off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$< p+q|j^+(0)|p >= 2p^+ F(q^2)$$

Interaction picture



struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Exact LF Formula for Pauli Form Factor

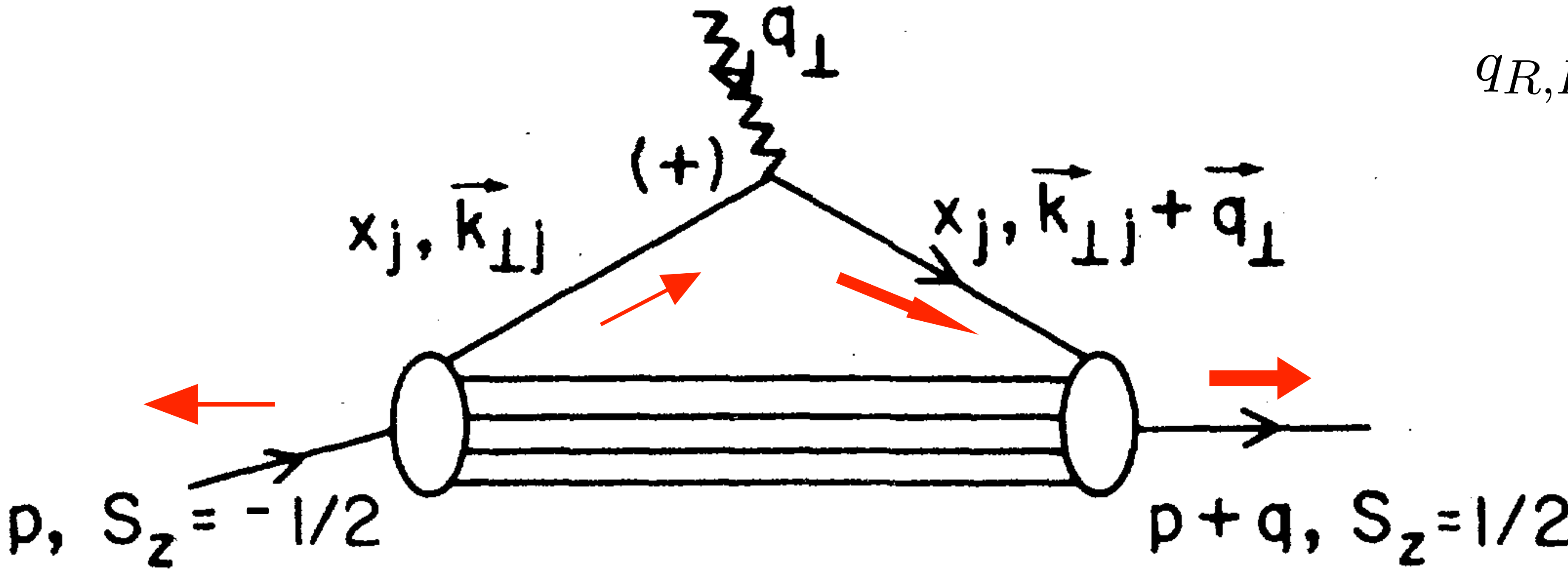
Drell, sjb

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

$$\Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

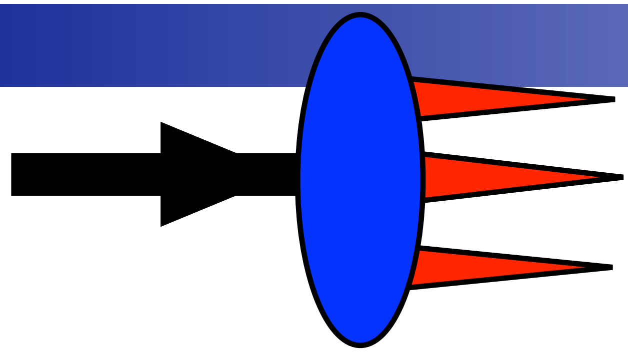
$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

- *Light Front Wavefunctions:*



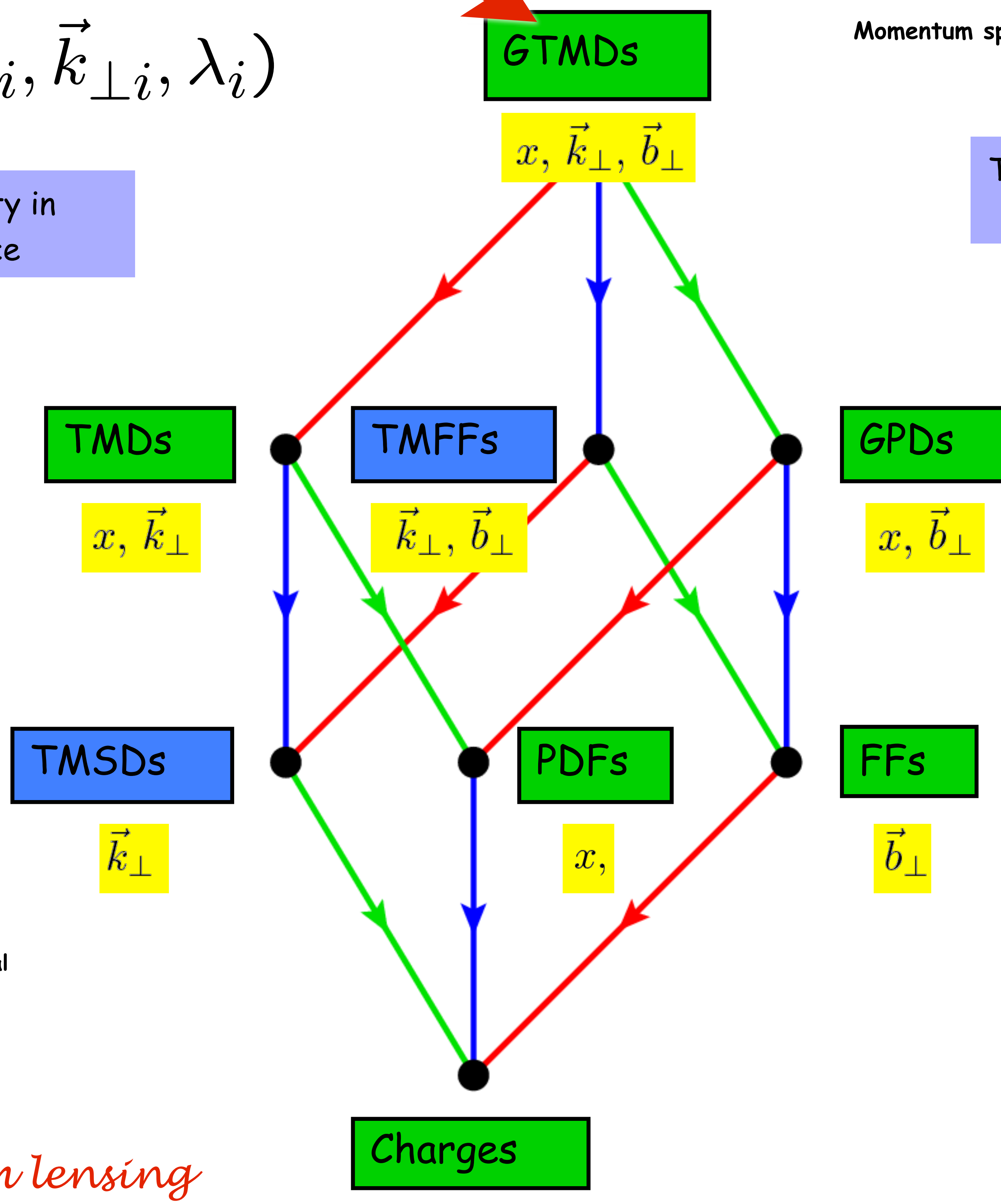
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

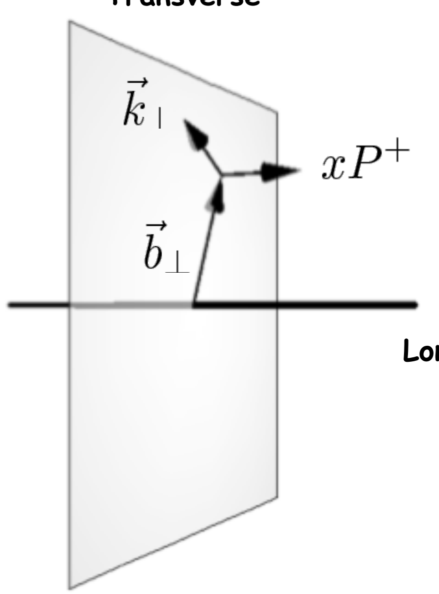
Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

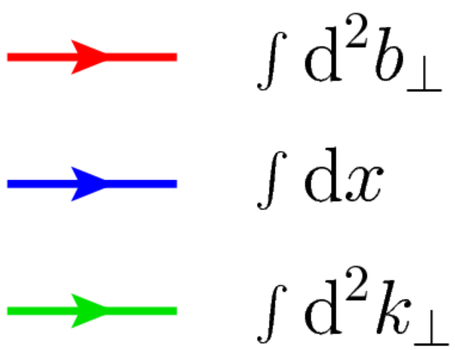
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in position space



Lorce, Pasquini





Sivers, T-odd from lensing

Advantages of the Dirac's Front Form for Hadron Physics

- **Measurements are made at fixed τ**
- **Causality is automatic**
- **Structure Functions are squares of LFWFs**
- **Form Factors are overlap of LFWFs**
- **LFWFs are frame-independent: no boosts, no pancakes!**
- **Same structure function in e p collider and p rest frame**
- **No dependence on observer's frame**
- **LF Holography: Dual to AdS space**
- **LF Vacuum trivial -- no condensates!**
- **Profound implications for Cosmological Constant**

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

Origin of hadronic mass scale if $m_q=0$

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

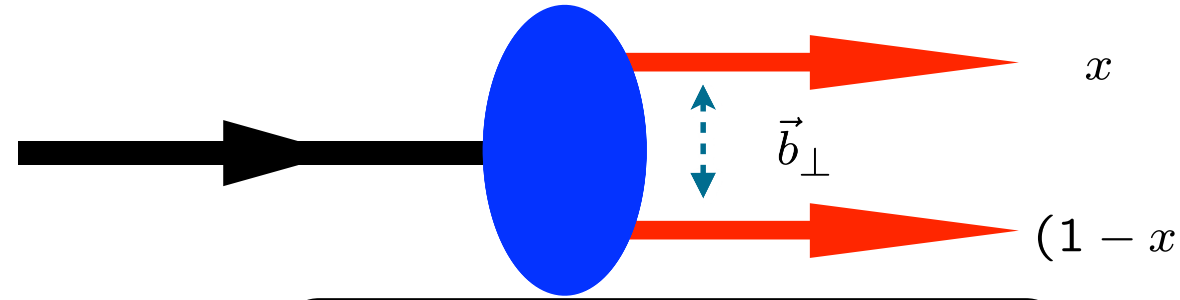
Bohr Spectrum

Semiclassical first approximation to QED

Schrödinger Eq.

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x)b_\perp^2$$

$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states
and retarded interactions

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V^{LF}_{\text{eff}}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \, \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Azimuthal Basis

ζ, ϕ

$m_q = 0$

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD
potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD

• *de Teramond, sjb*

Effective QCD LF Bound-State Equation

- Factor out the longitudinal $X(x)$ and orbital kinematical dependence from LFWF ψ

$$\psi(x,\zeta,\varphi) = e^{iL\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

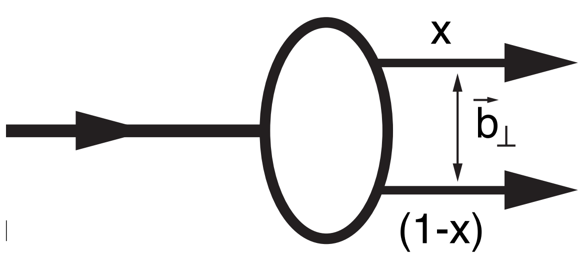
- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple and LF Hamiltonian equation $P_\mu P^\mu|\psi\rangle = M^2|\psi\rangle$ is a LF wave equation for ϕ

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

- Invariant transverse variable in impact space

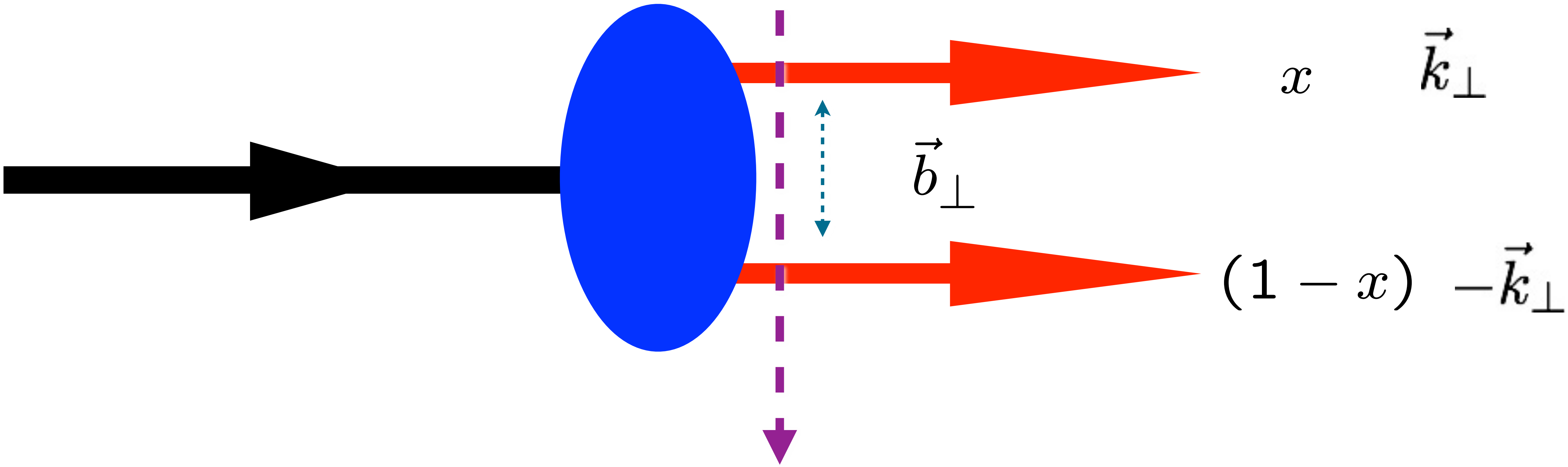
$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2$$

conjugate to invariant mass $\mathcal{M}^2 = \mathbf{k}_\perp^2/x(1-x)$



- Critical value $L = 0$ corresponds to lowest possible stable solution: ground state of the LF Hamiltonian
- Relativistic and frame-independent LF Schrödinger equation: U is instantaneous in LF time and comprises all interactions, including those with higher Fock states.

Fixed $\tau = t + z/c$



$$\zeta^2 \equiv b_\perp^2 x(1-x)$$

Invariant transverse separation

$$\zeta^2 \text{ conjugate to } \frac{k_\perp^2}{x(1-x)} = (p_q + p_{\bar{q}})^2 = \mathcal{M}^2_{q+\bar{q}}$$

$$\int dk^- \Psi_{BS}(P,k) \to \psi_{LF}(x, \vec{k}_\perp)$$

Light-Front Schrödinger Equation

G. de Teramond, sjb

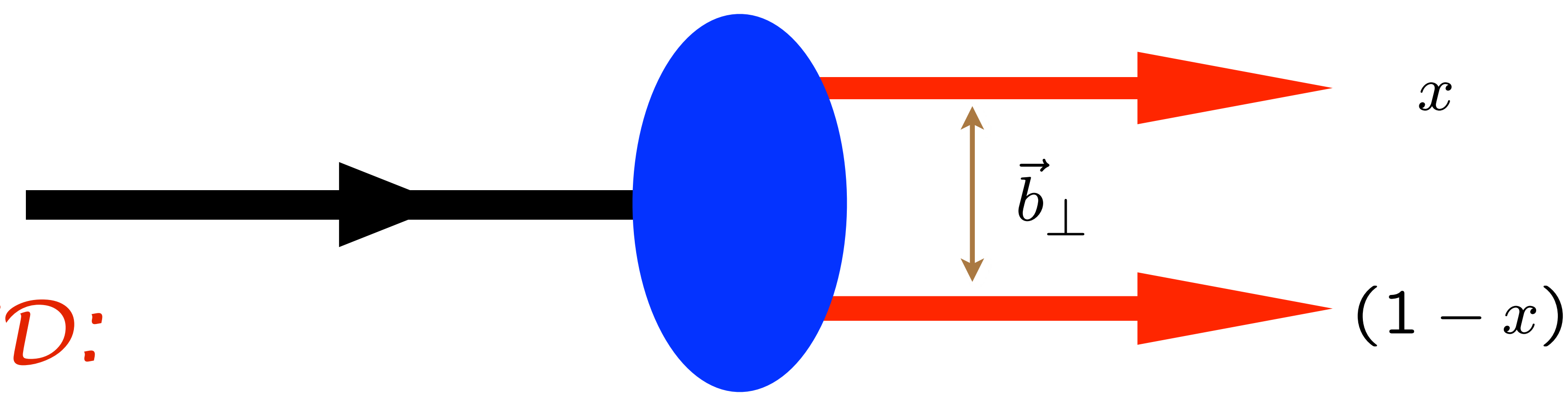
Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \, \psi_{LF}(\zeta)$$

$m_q \sim 0$

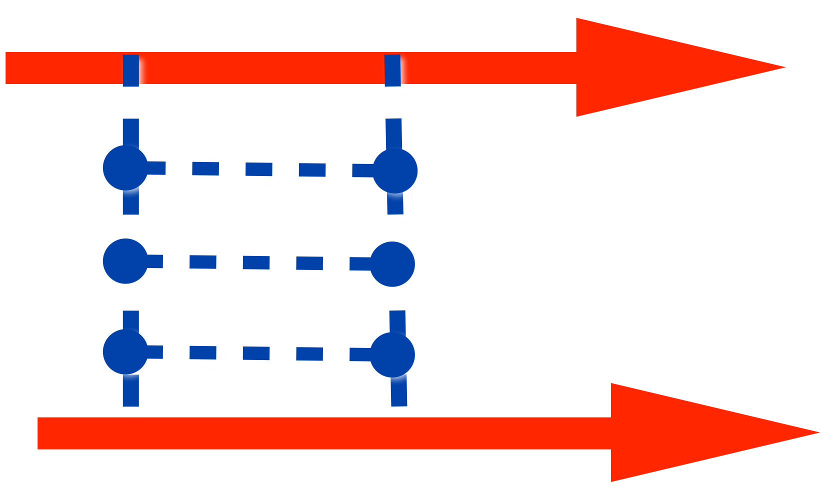
$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$

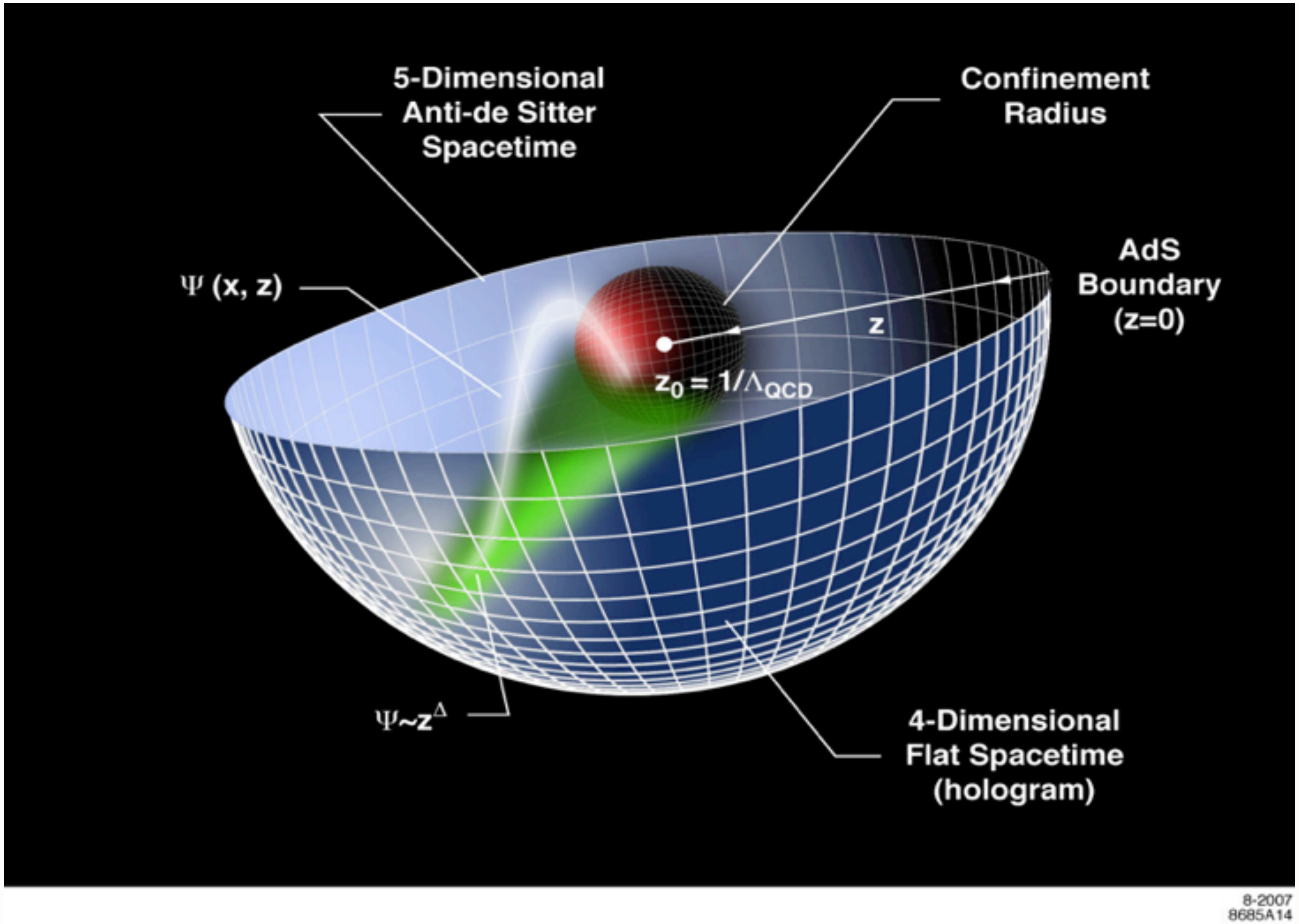


AdS/QCD:

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

U is the exact QCD potential
Conjecture: 'H'-diagrams generate U?





Changes in physical length scale mapped to evolution in the 5th dimension z

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

AdS/CFT

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$ ← *invariant measure*

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale** κ
- **Uses AdS_5 as template for conformal theory**

Albufeira

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Light-Front Bound State Equation!

$$z \leftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

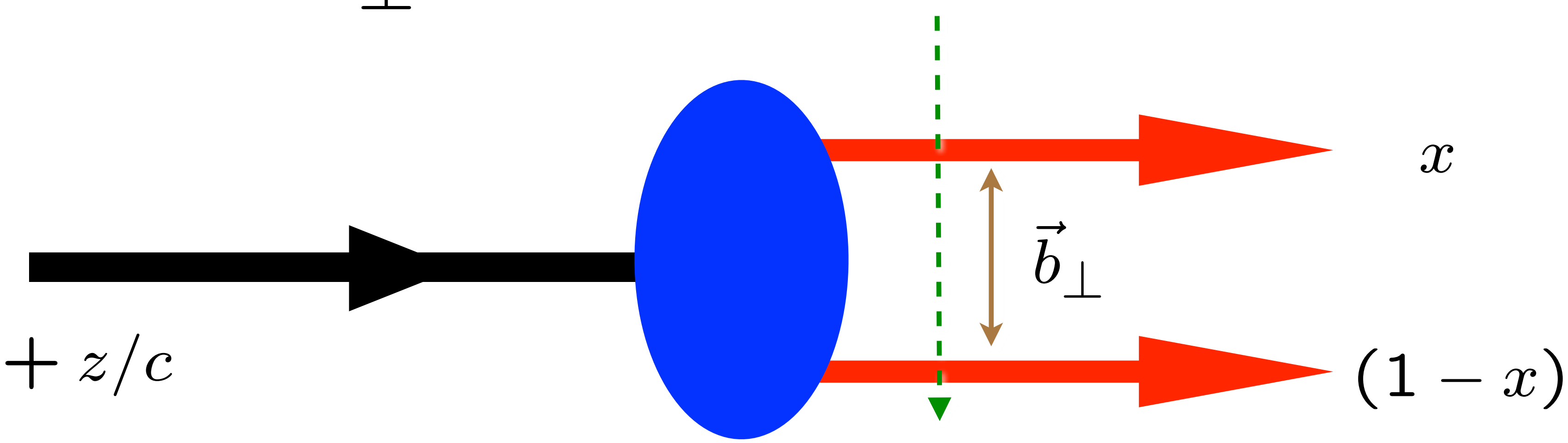
de Teramond, sjb

LF(3+1) ⟷ AdS_5

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



Fixed $\tau = t + z/c$

$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

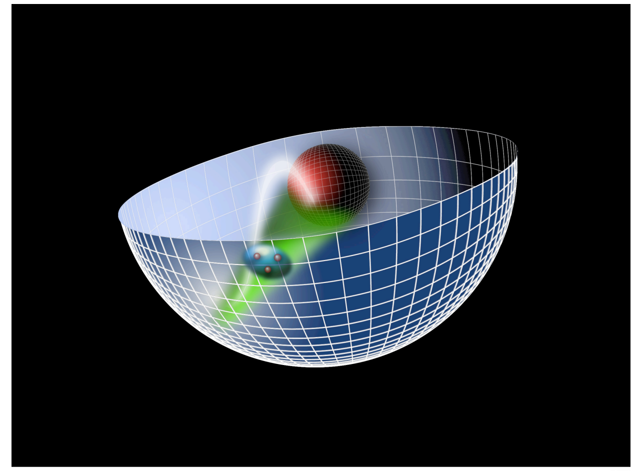
$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique
Confinement Potential!

Conformal Symmetry
of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici**

Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!

Meson Spectrum in Soft Wall Model

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

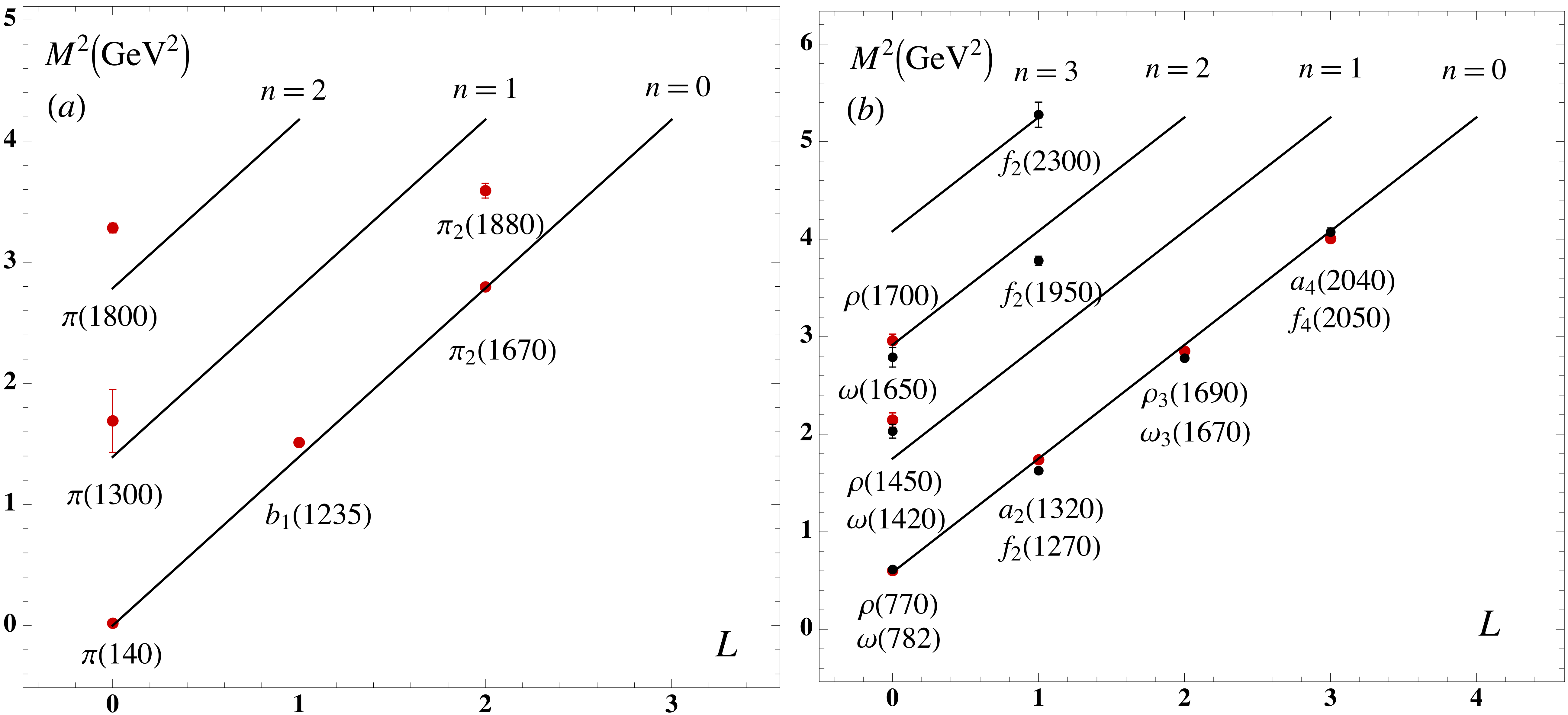
- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

G. de Teramond, H. G. Dosch, sjb

$$m_u = m_d = 0$$

de Tèramond, Dosch, sjb



$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

Albufeira

Light-Front Holography and Supersymmetric Features of QCD

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Light-Front Schrödinger Equation

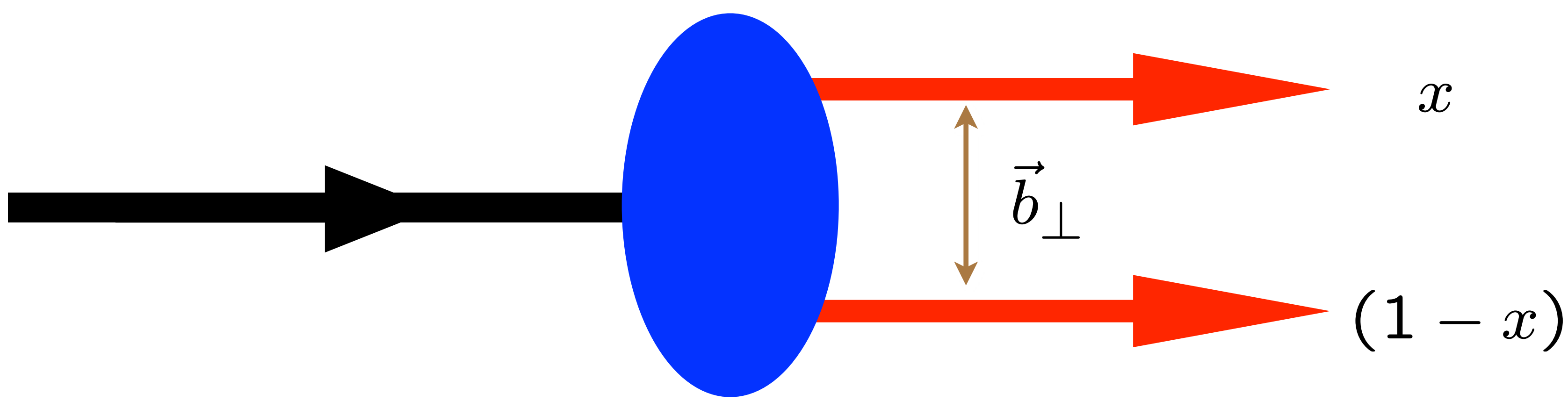
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

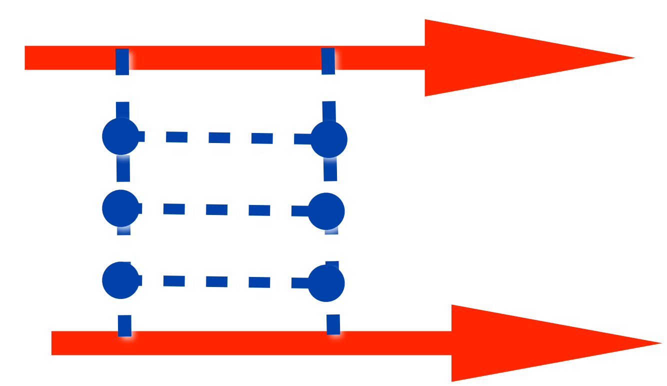
Frame Independent!

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2 .$$



U is the confining QCD potential



Conjecture: 'H'-diagrams generate

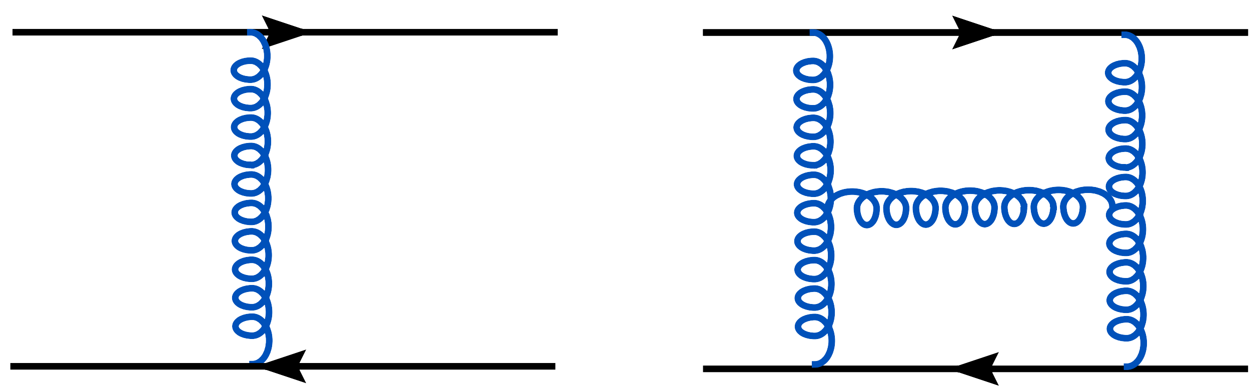
$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Heavy Quark Potential is IR Divergent in QCD

$$V(Q^2) = -\frac{(4\pi)^2 C_F}{Q^2} a(Q^2) \Big[1 + (c_{2,0} + c_{2,1} N_f) a(Q^2) + (c_{3,0} + c_{3,1} N_f + c_{3,2} N_f^2) a(Q^2)^2$$

$$+ (c_{4,0} + c_{4,1} N_f + c_{4,2} N_f^2 + c_{4,3} N_f^3) a(Q^2)^3 + 8\pi^2 C_A^3 \ln \frac{\mu_{IR}^2}{Q^2} a(Q^2)^3 \Big]$$

Smirnov, Smirnov, Steinhauser, 2010



$\log \kappa^2 \zeta^2$

Summation of H graphs: confining potential?

Confinement eliminates IR divergences

Self-consistent mass scale κ

- **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau)>= i\frac{\partial}{\partial\tau}|\psi(\tau)>$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

- **Dosch, de Teramond, sjb**

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

dAFF: New Time Variable

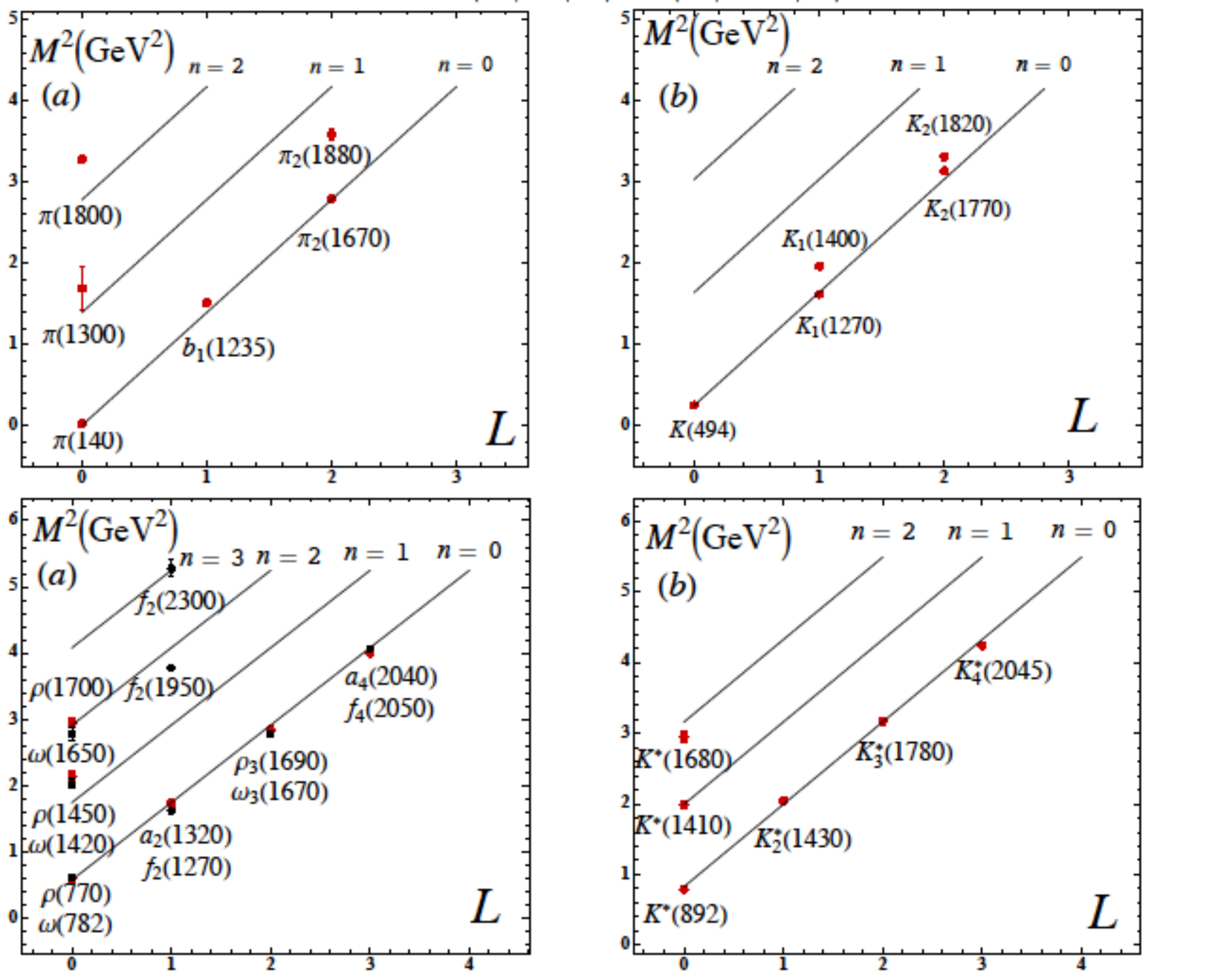
$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**

De Tèramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

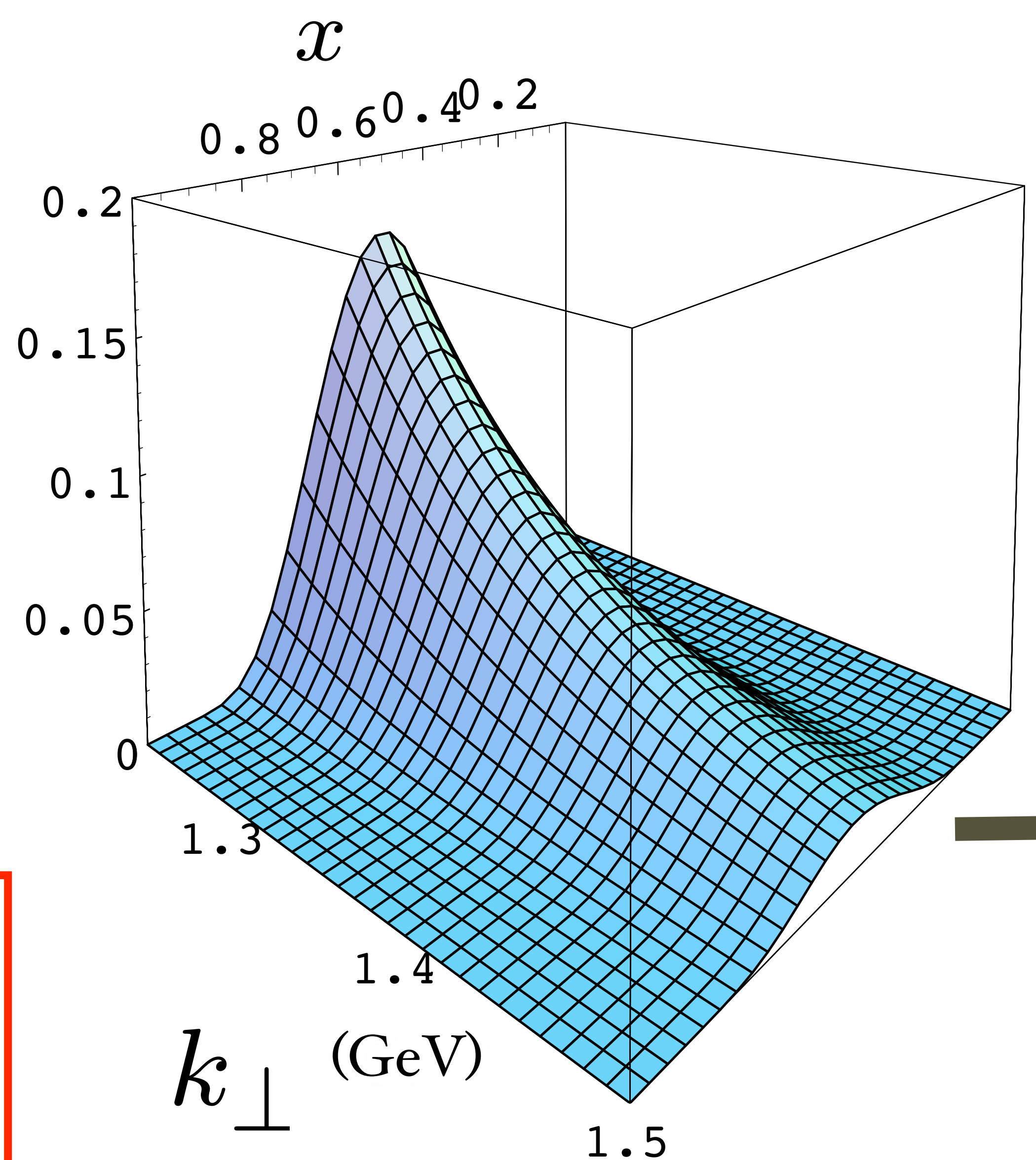
$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$



Prediction from AdS/QCD: Meson LFWF

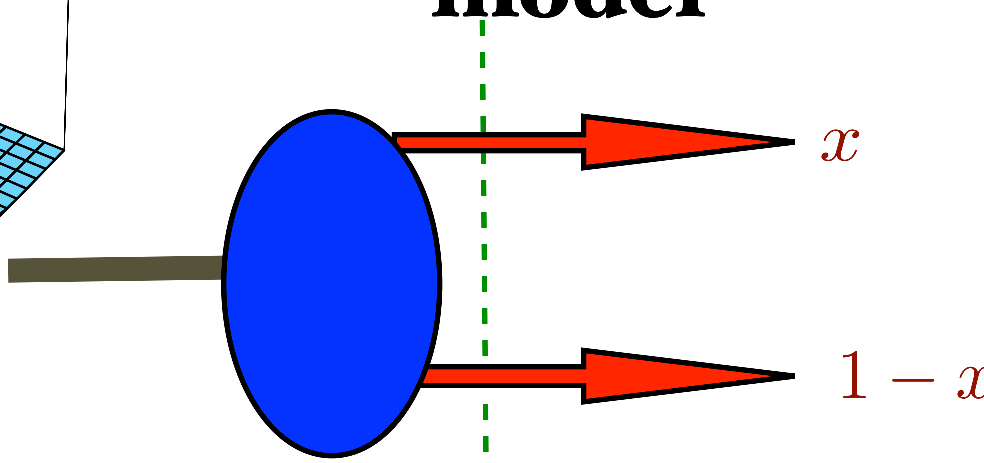
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

Same as DSE!

Provides Connection of Confinement to Hadron Structure

PRL 109, 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

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(Received 5 April 2012; published 20 August 2012)

We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

See also Ferreira and Dosch

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

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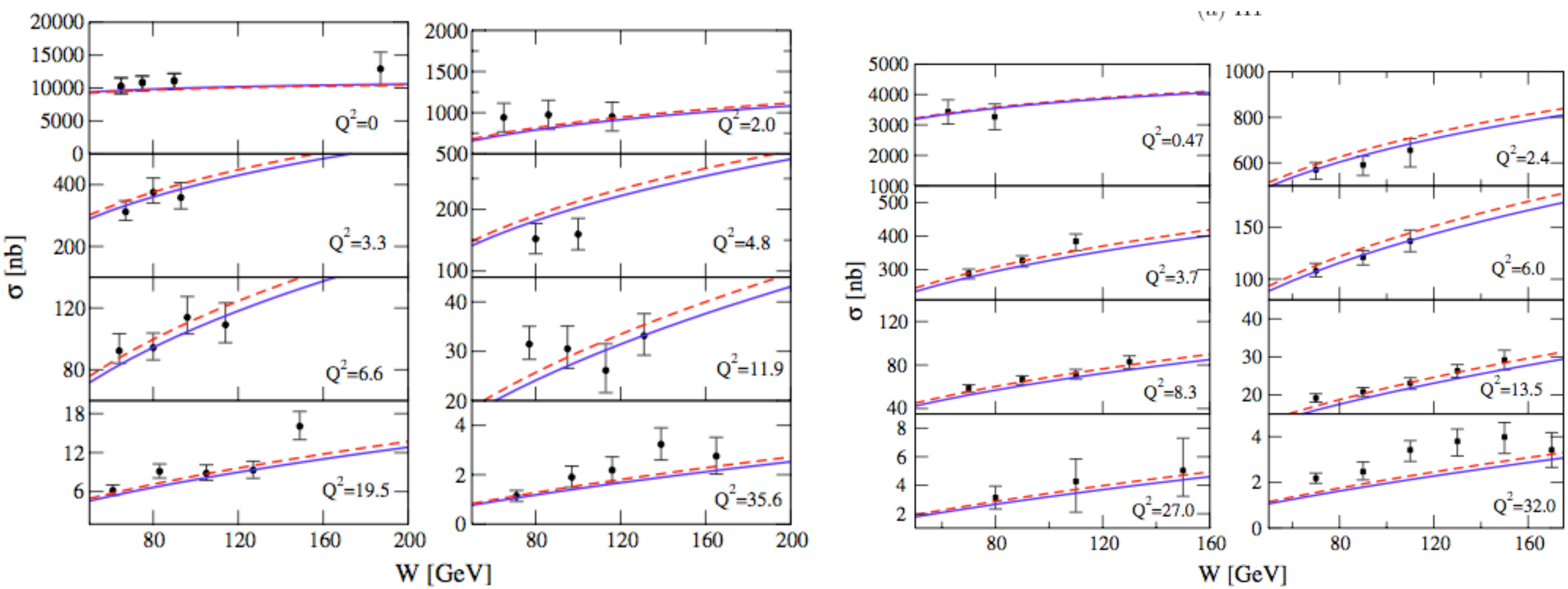
AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

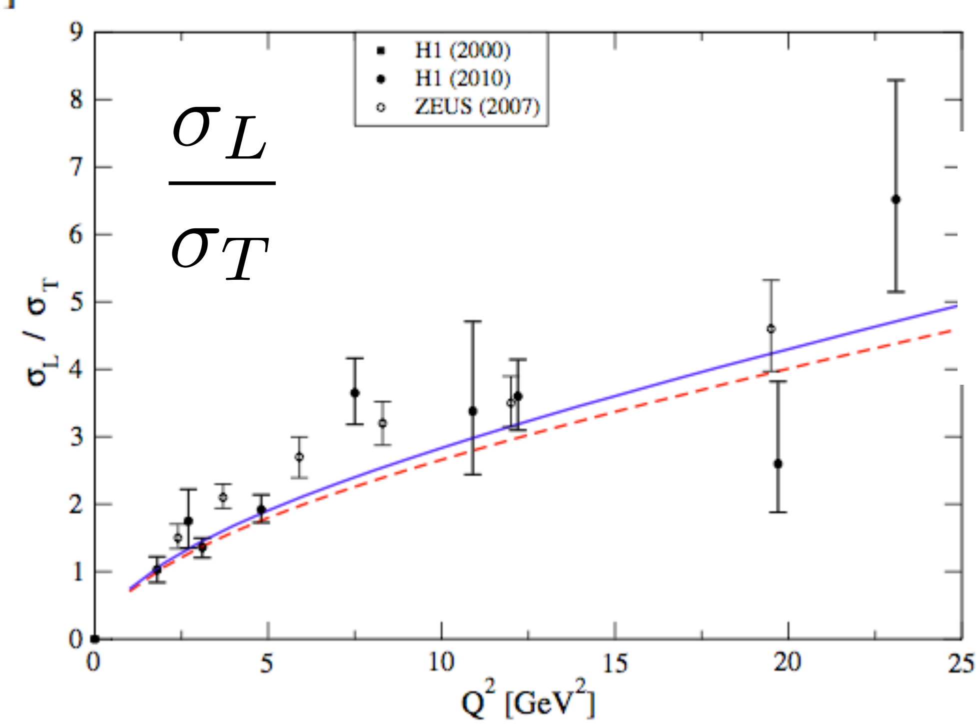


(a) H

(b) ZEUS

J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$



$$\frac{\sigma_L}{\sigma_T}$$

$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

See also Ferreira and Dosch

Haag, Lopuszanski, Sohnius (1974)

Fubini and Rabinovici

Superconformal Algebra

de Teramond Dosch and SJB

1+1

$$\{\psi, \psi^+\} = 1$$

two anti-commuting fermionic operators

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

Realization as Pauli Matrices

$$Q = \psi^+[-\partial_x + W(x)], \quad Q^+ = \psi[\partial_x + W(x)], \qquad W(x) = \frac{f}{x}$$

(Conformal)

$$S = \psi^+ x, \quad S^+ = \psi x$$

Introduce new spinor operators

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

$$\{Q, Q\} = \{Q^+, Q^+\} = 0, \quad [Q, H] = [Q^+, H] = 0$$

Haag, Lopuszanski, Sohnius (1974)

Superconformal Algebra

$$\{\psi, \psi^{+}\} = 1 \qquad B = \frac{1}{2}[\psi^{+}, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^{+} = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^{+}[-\partial_x + \frac{f}{x}], \quad Q^{+} = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^{+}x, \quad S^{+} = \psi x$$

$$\{Q, Q^{+}\} = 2H, \quad \{S, S^{+}\} = 2K$$

$$\{Q, S^{+}\} = f - B + 2iD, \quad \{Q^{+}, S\} = f - B - 2iD$$

generates the conformal algebra

[H,D]= i H, [H, K] =2 i D, [K, D] = - i K

Superconformal Algebra

Baryon Equation

Consider $R_w = Q + wS;$ w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \Big(-\partial_x^2 + w^2x^2 + 2wf - w + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2} \Big)$$

$$G_{22} = \Big(-\partial_x^2 + w^2x^2 + 2wf + w + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2} \Big)$$

Identify $f - \frac{1}{2} = L_B \;, \quad w = \kappa^2$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

Superconformal Algebra

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\psi_J^+$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\right)\phi_J = M^2\phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ !

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M = L_B + 1$

Nucleon Spectrum

- In 2×2 block-matrix form

$$H_{LF} = \begin{pmatrix} -\frac{d^2}{d\zeta^2} - \frac{1-4\nu^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(\nu+1) & 0 \\ 0 & -\frac{d^2}{d\zeta^2} - \frac{1-4(\nu+1)^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda\nu \end{pmatrix}$$

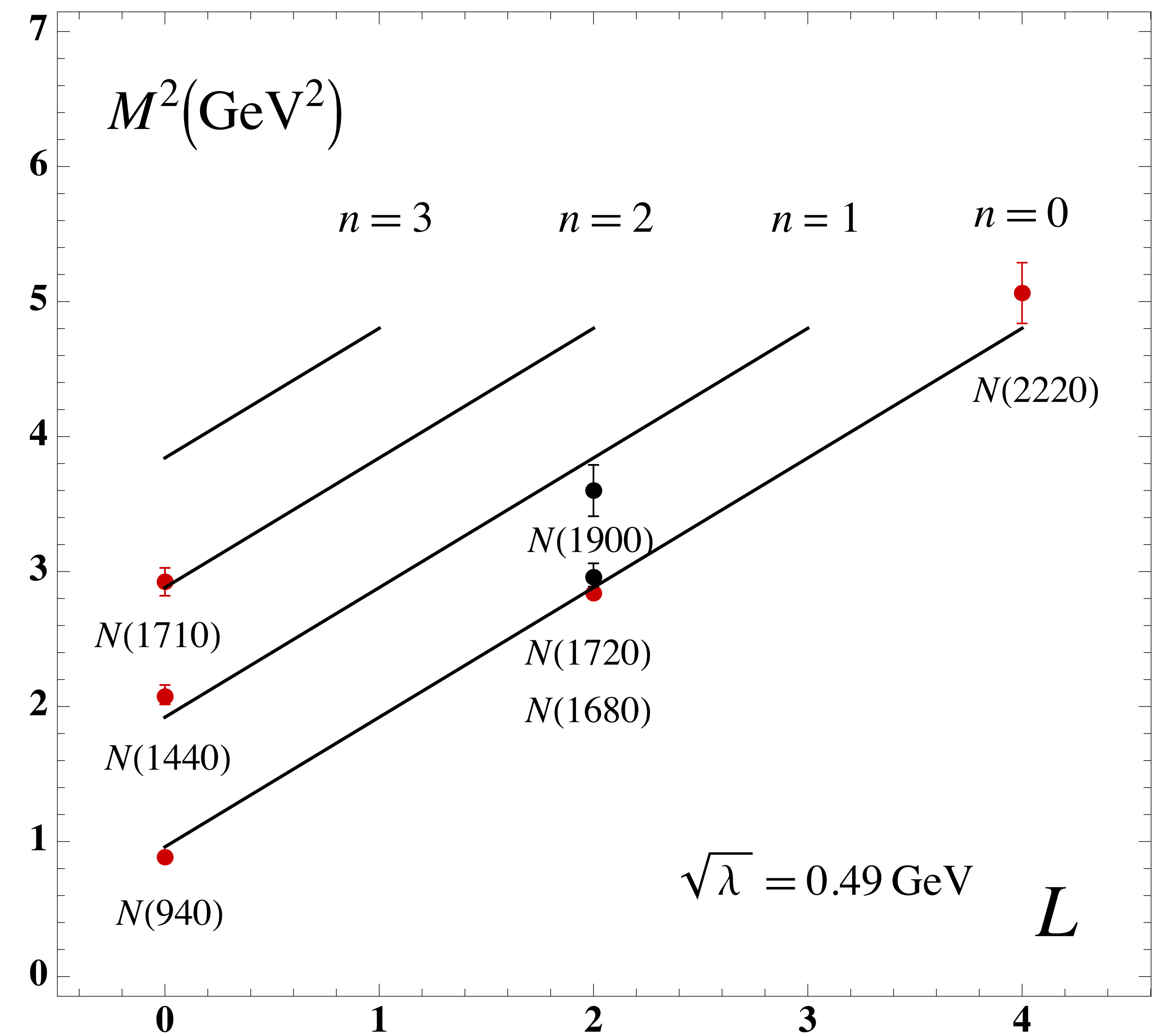
- Eigenfunctions

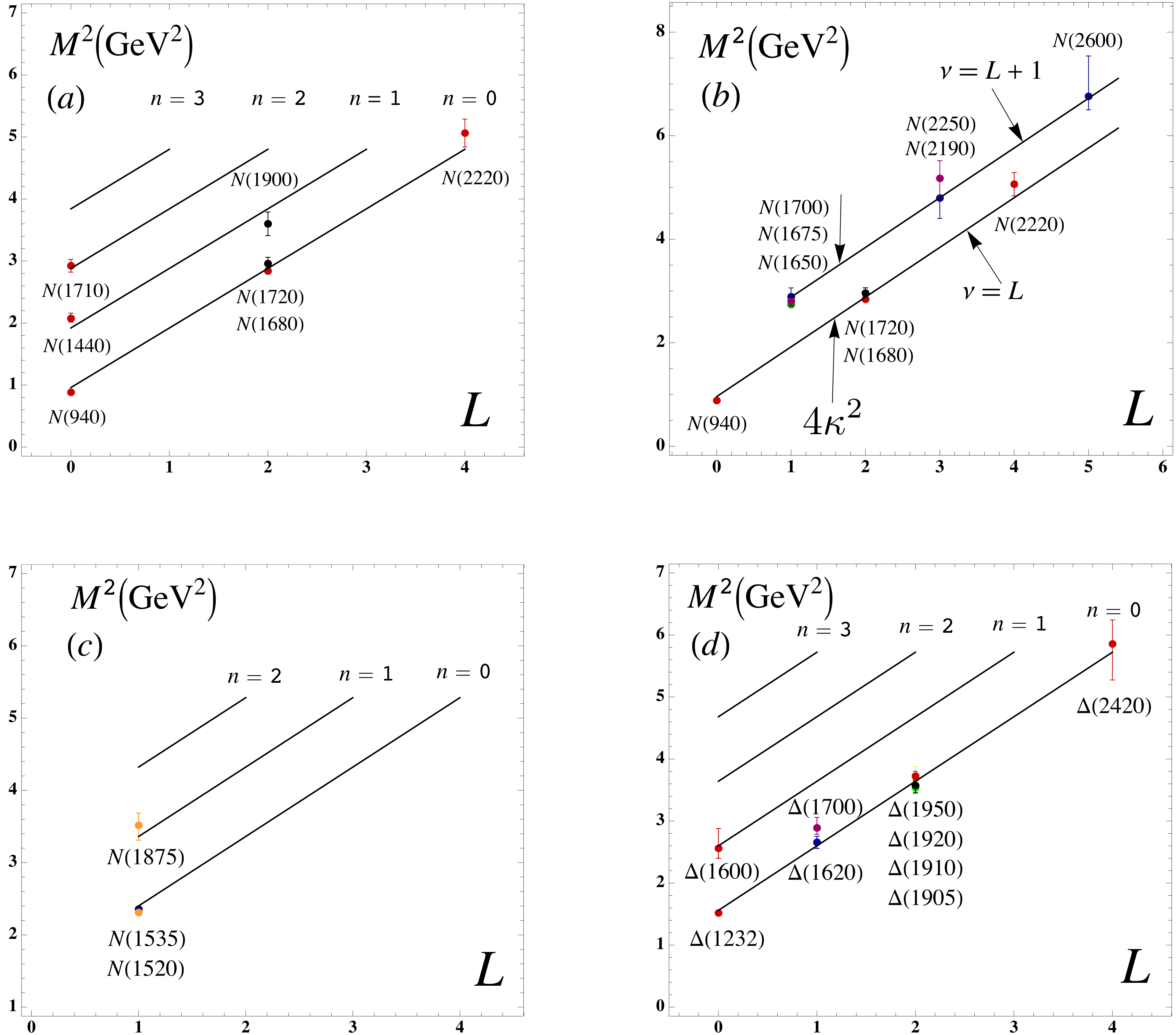
$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^\nu(\lambda\zeta^2)$$
$$\psi_-(\zeta) \sim \zeta^{\frac{3}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^{\nu+1}(\lambda\zeta^2)$$

- Eigenvalues

$$M^2 = 4\lambda(n+\nu+1)$$

- Lowest possible state $n=0$ and $\nu=0$
- Orbital excitations $\nu = 0, 1, 2 \cdots = L$
- L is the relative LF angular momentum between the active quark and spectator cluster

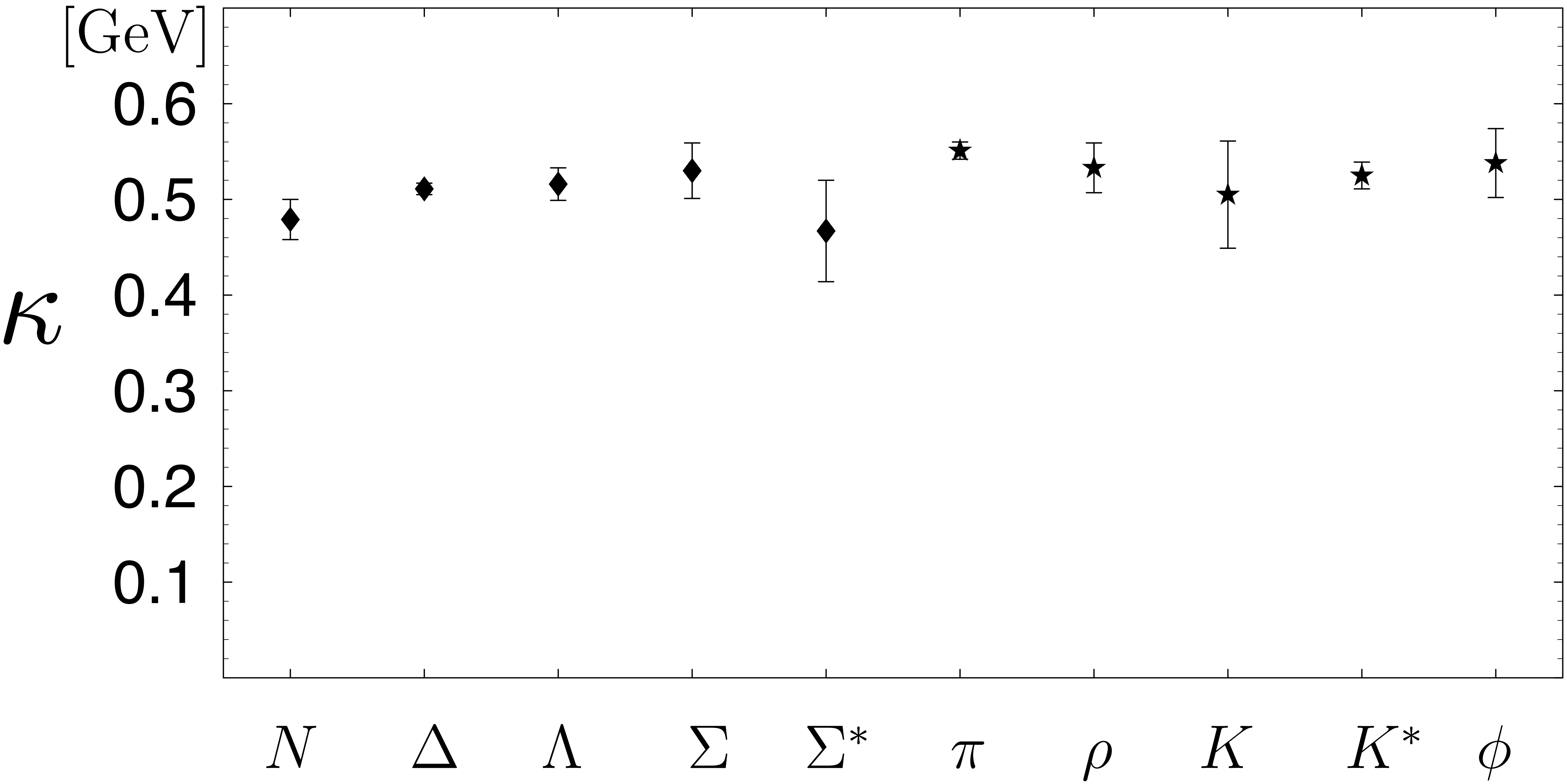




$M^2(\mathrm{GeV}^2)$
(a)
n = 3
n = 2
n = 1
n = 0
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
L
$M^2(\mathrm{GeV}^2)$
(b)
$\nu = L + 1$
N(2600)
N(2250)
N(2190)
N(1700)
N(1675)
N(1650)
N(2220)
$\nu = L$
N(1720)
N(1680)
N(940)
$4\kappa^2$
L
$M^2(\mathrm{GeV}^2)$
(c)
n = 2
n = 1
n = 0
N(1875)
N(1535)
N(1520)
L
$M^2(\mathrm{GeV}^2)$
(d)
n = 3
n = 2
n = 1
n = 0
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
L

preliminary

de Tèramond, Dosch, sjb

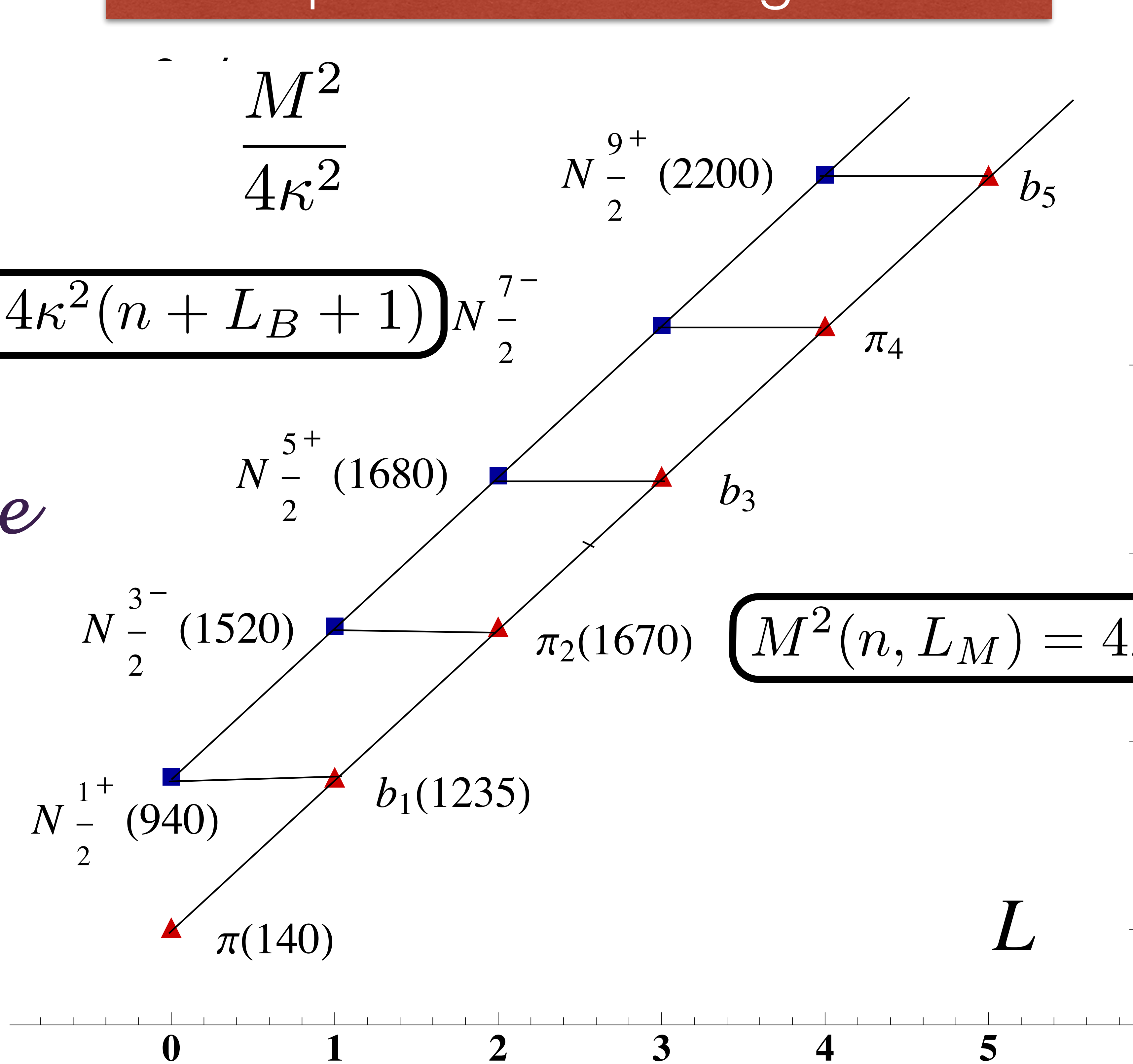


Fit to the slope of Regge trajectories, including radial excitations

Superconformal Algebra

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope



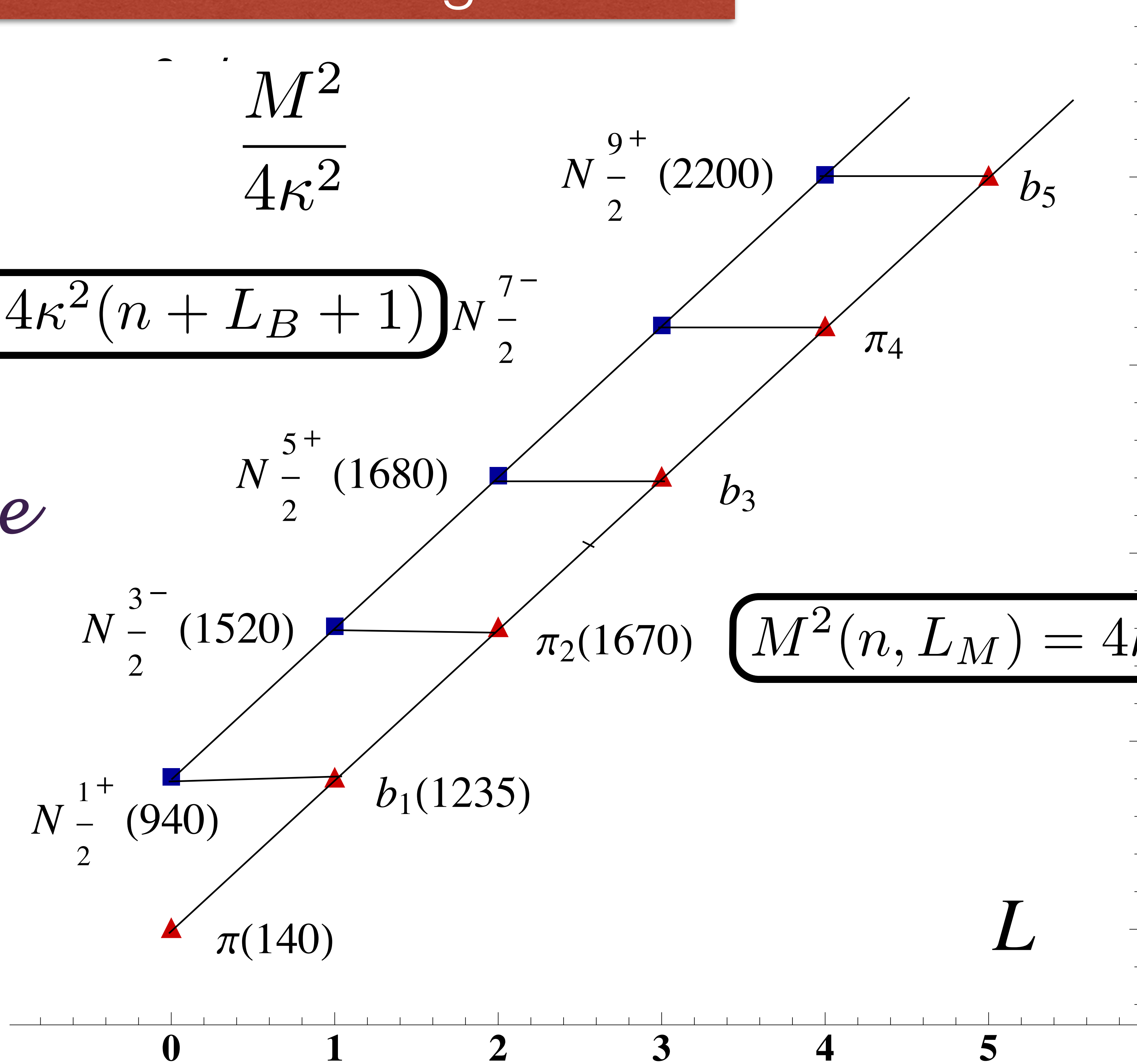
$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

$$\lambda_M^2 = \lambda_B^2 = \kappa^4$$

Superconformal Algebra

de Tèramond, Dosch, sjb



$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

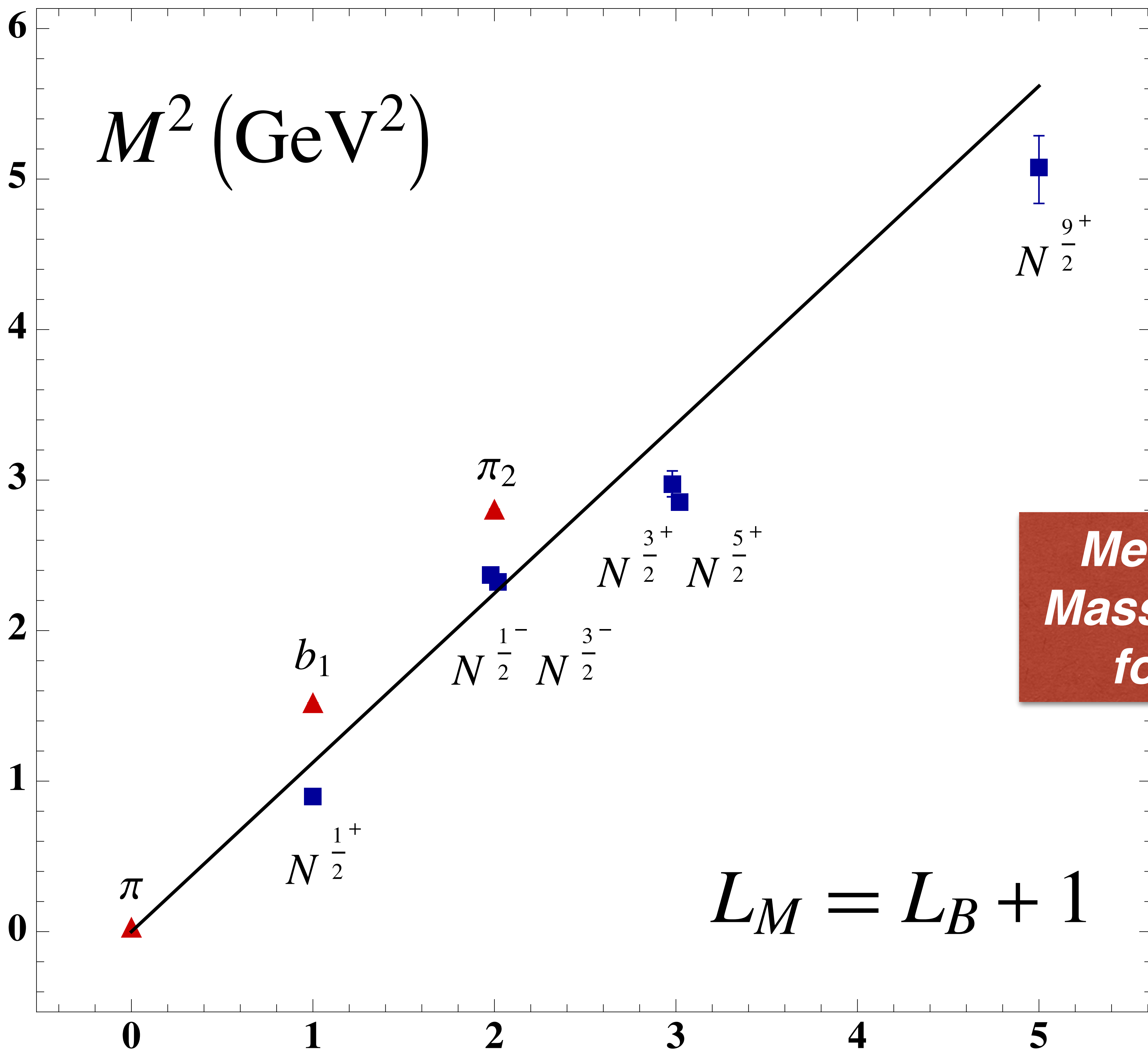
$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same slope

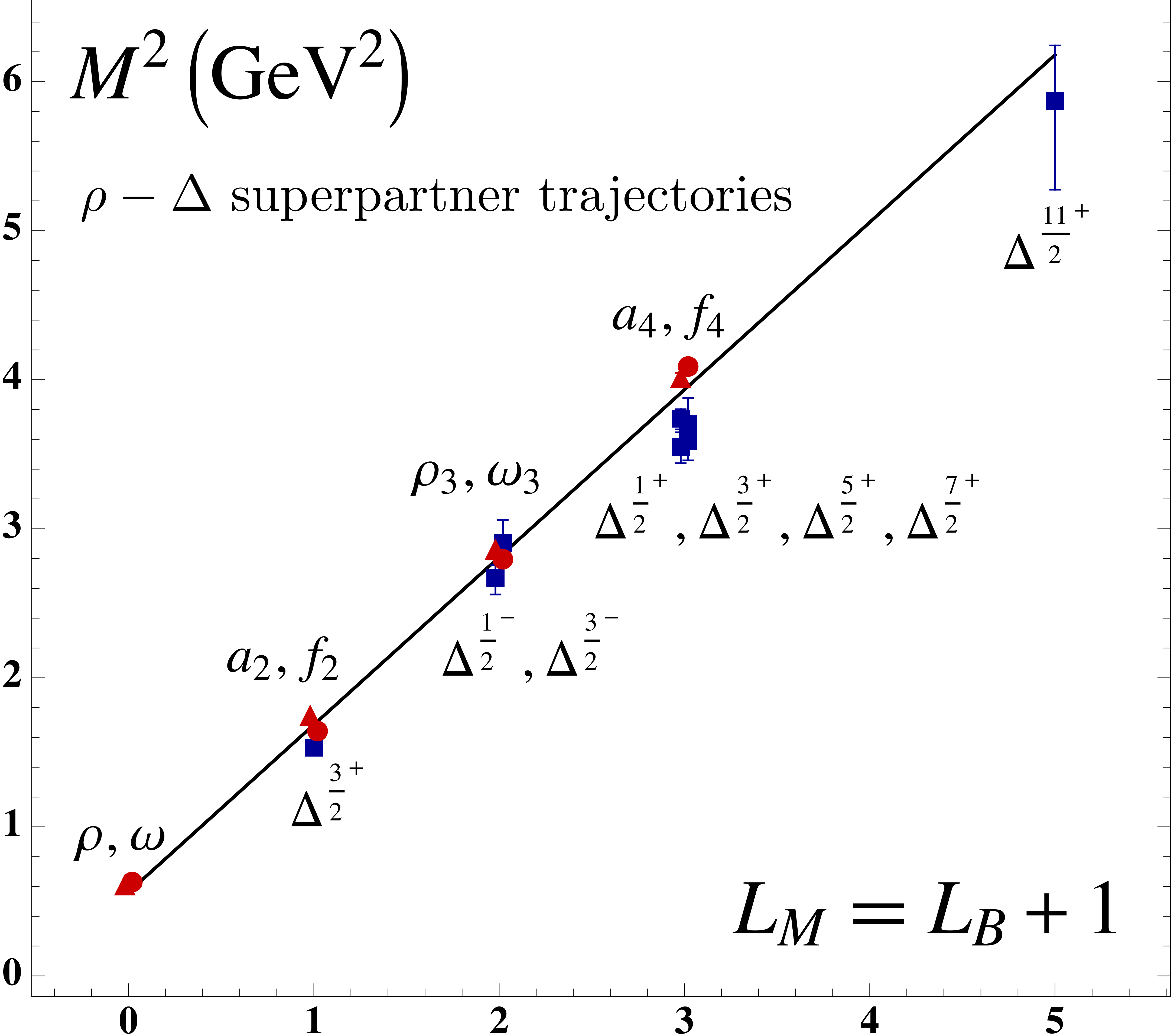
$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for L_M=L_B+1

Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!

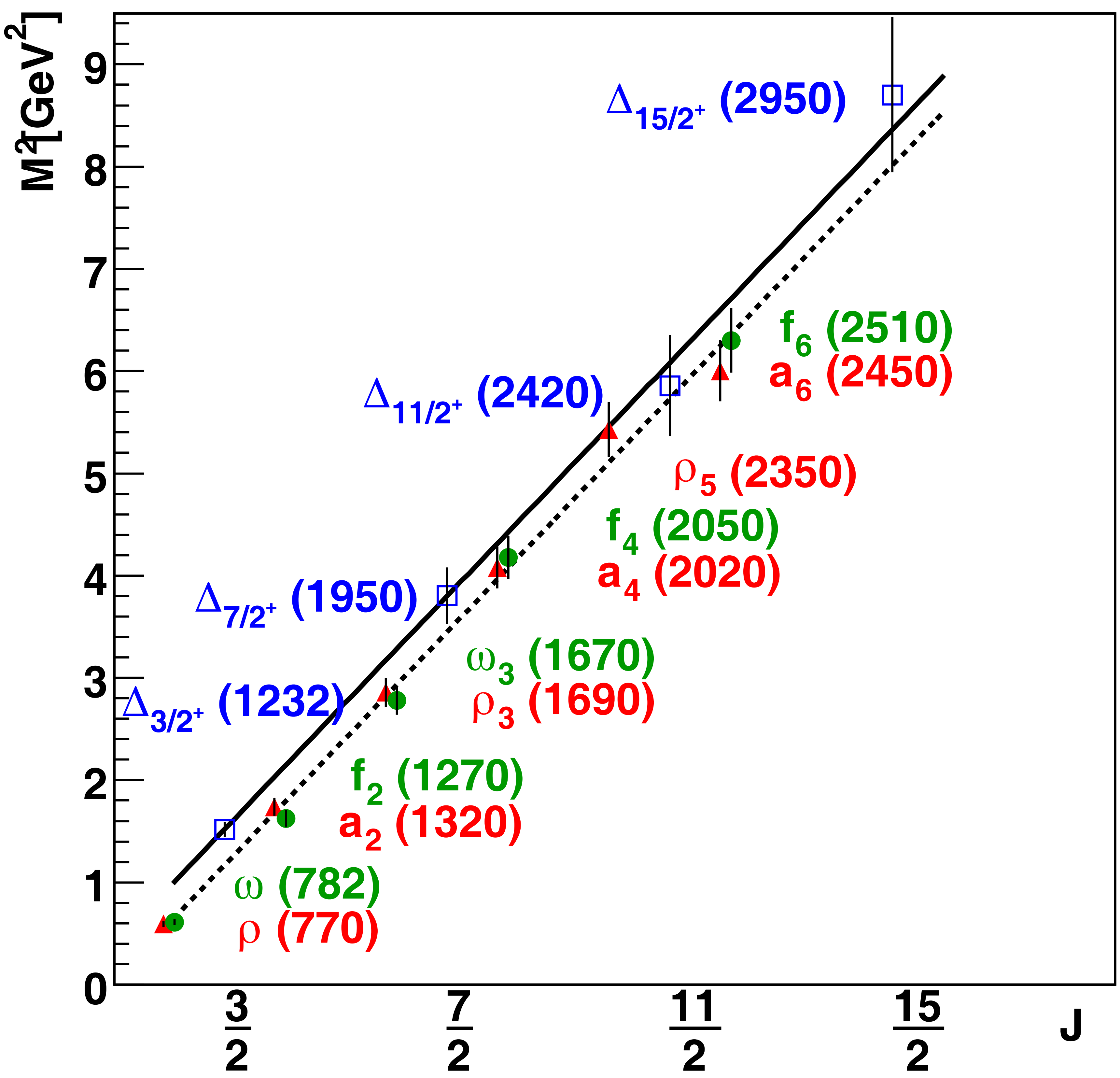


Meson-Baryon Mass Degeneracy for L_M=L_B+1



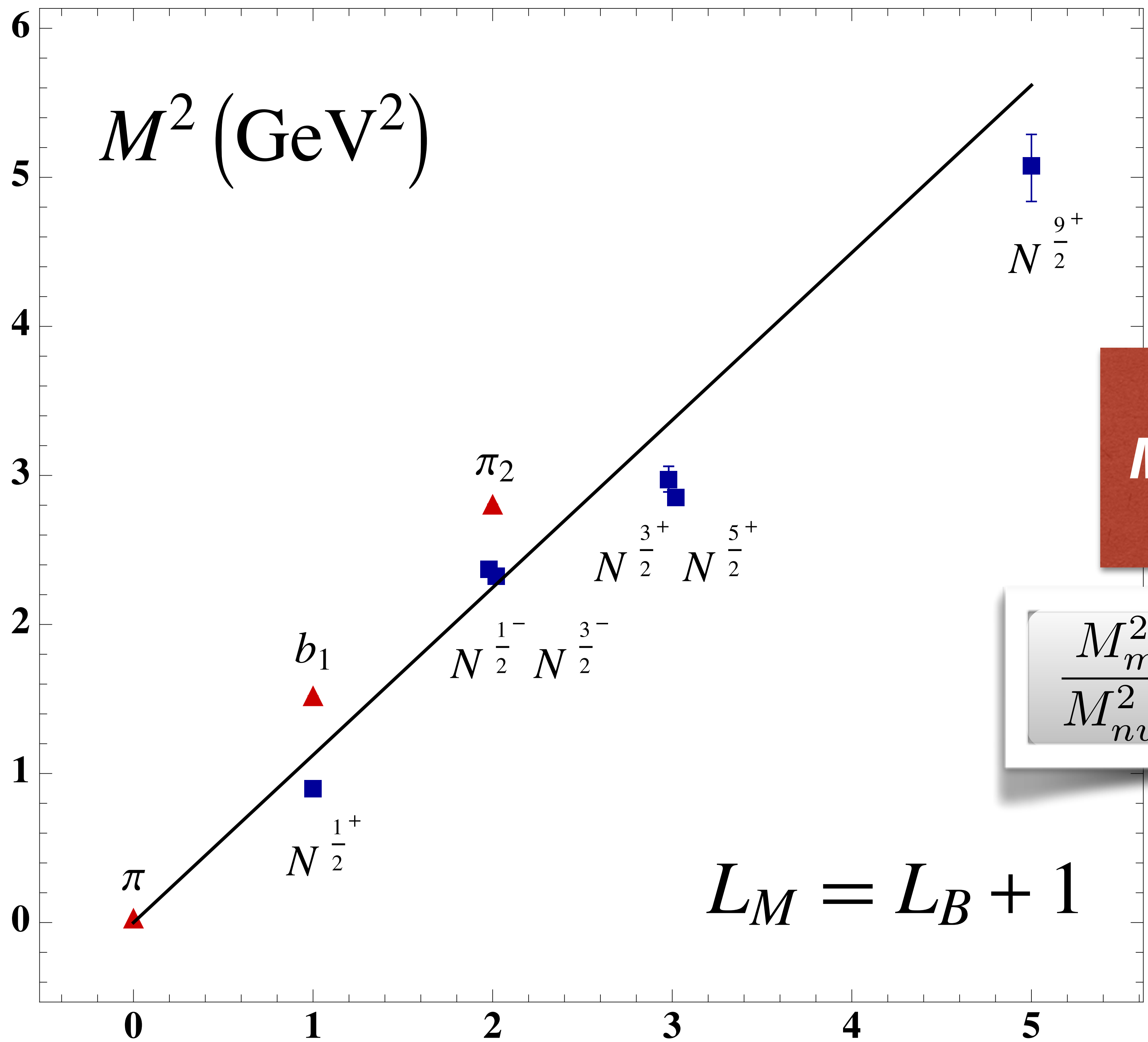
Dosch, de Teramond, sjb

E. Klempt and B. Ch. Metsch



The leading Regge trajectory: Δ resonances with maximal J in a given mass range.
Also shown is the Regge trajectory for mesons with J = L+S.

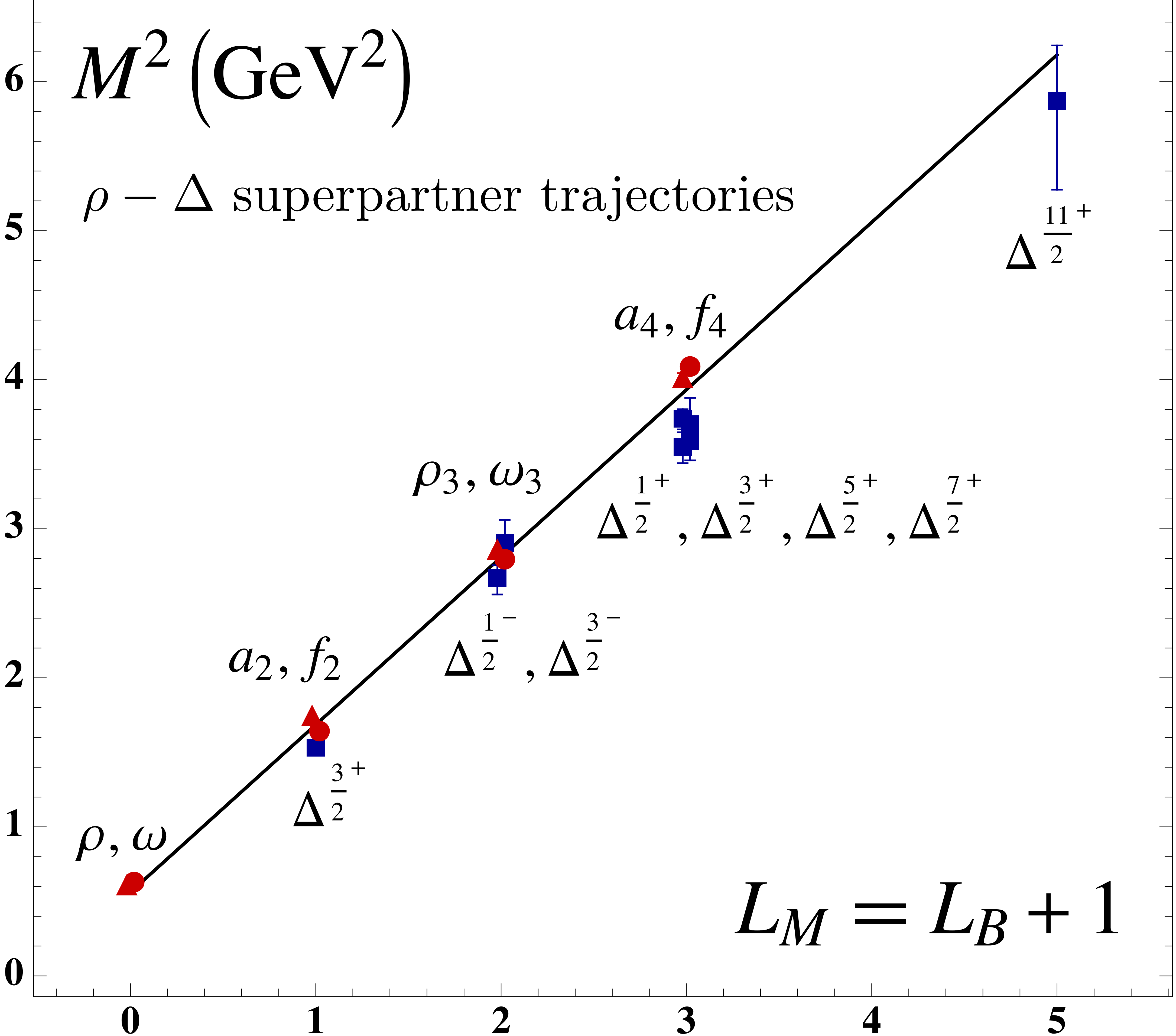
Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!



Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

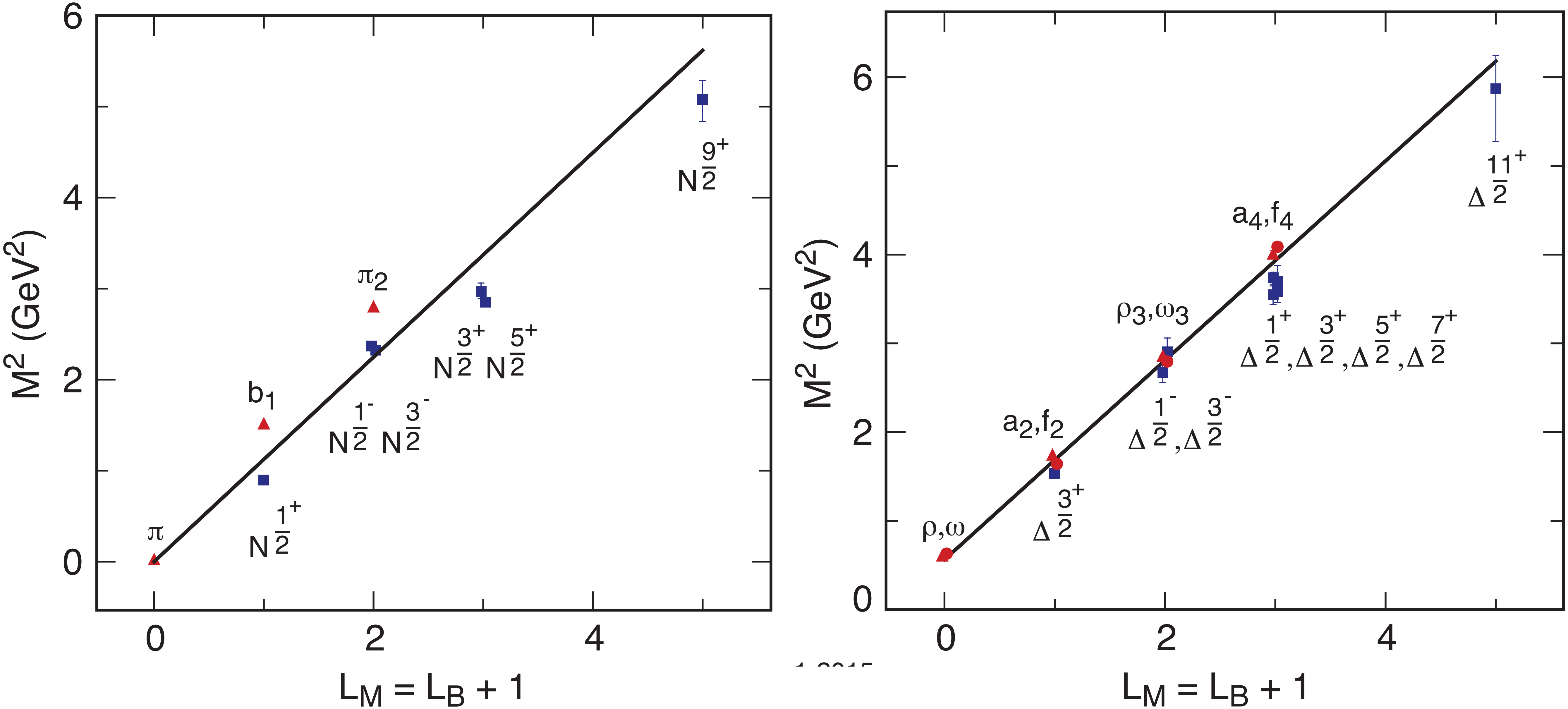
$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon



Dosch, de Teramond, sjb

Solid line: κ = 0.53 GeV



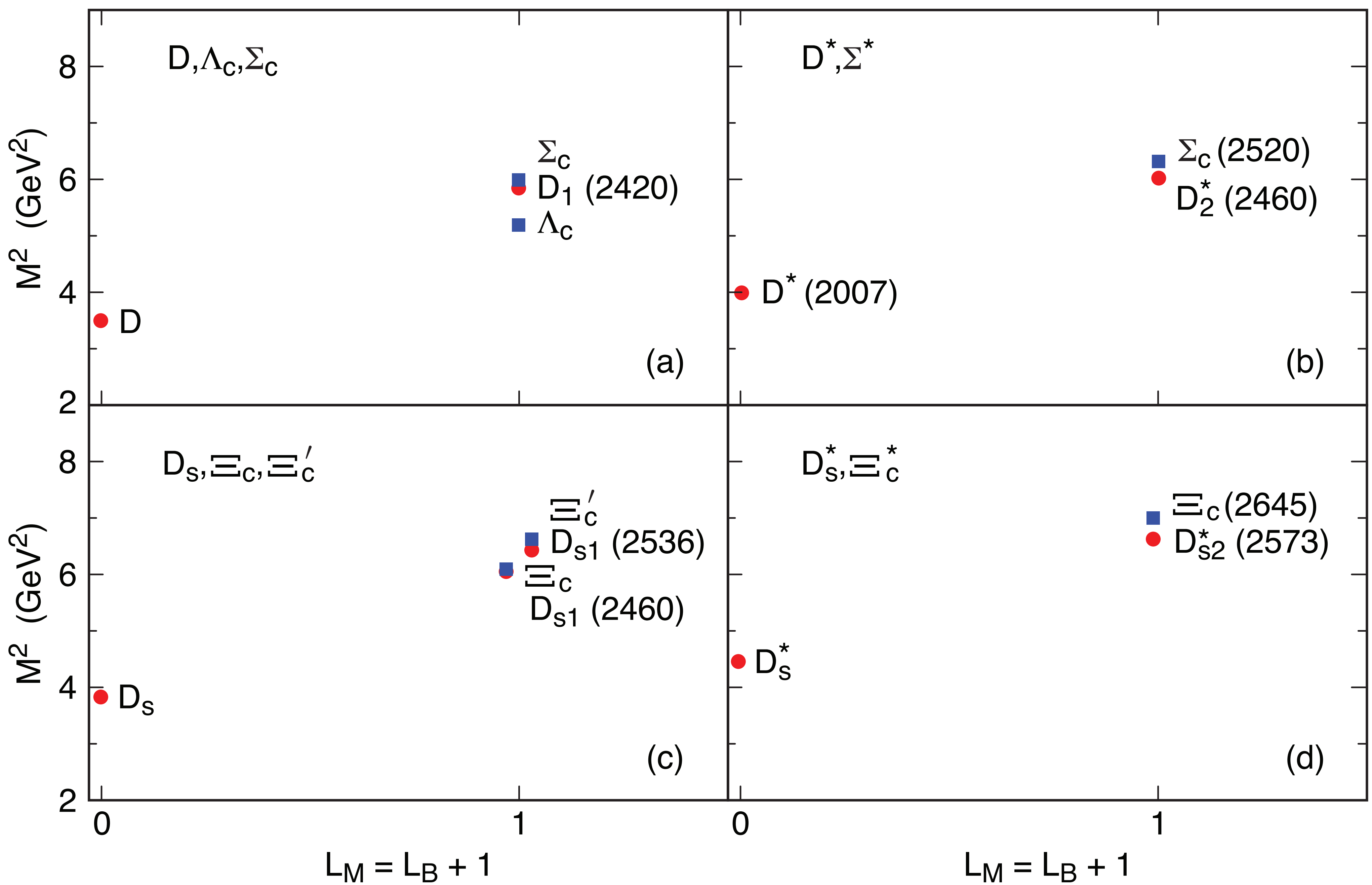
Superconformal meson-nucleon partners

de Tèramond, Dosch, sjb

Dosch, de Teramond, sjb

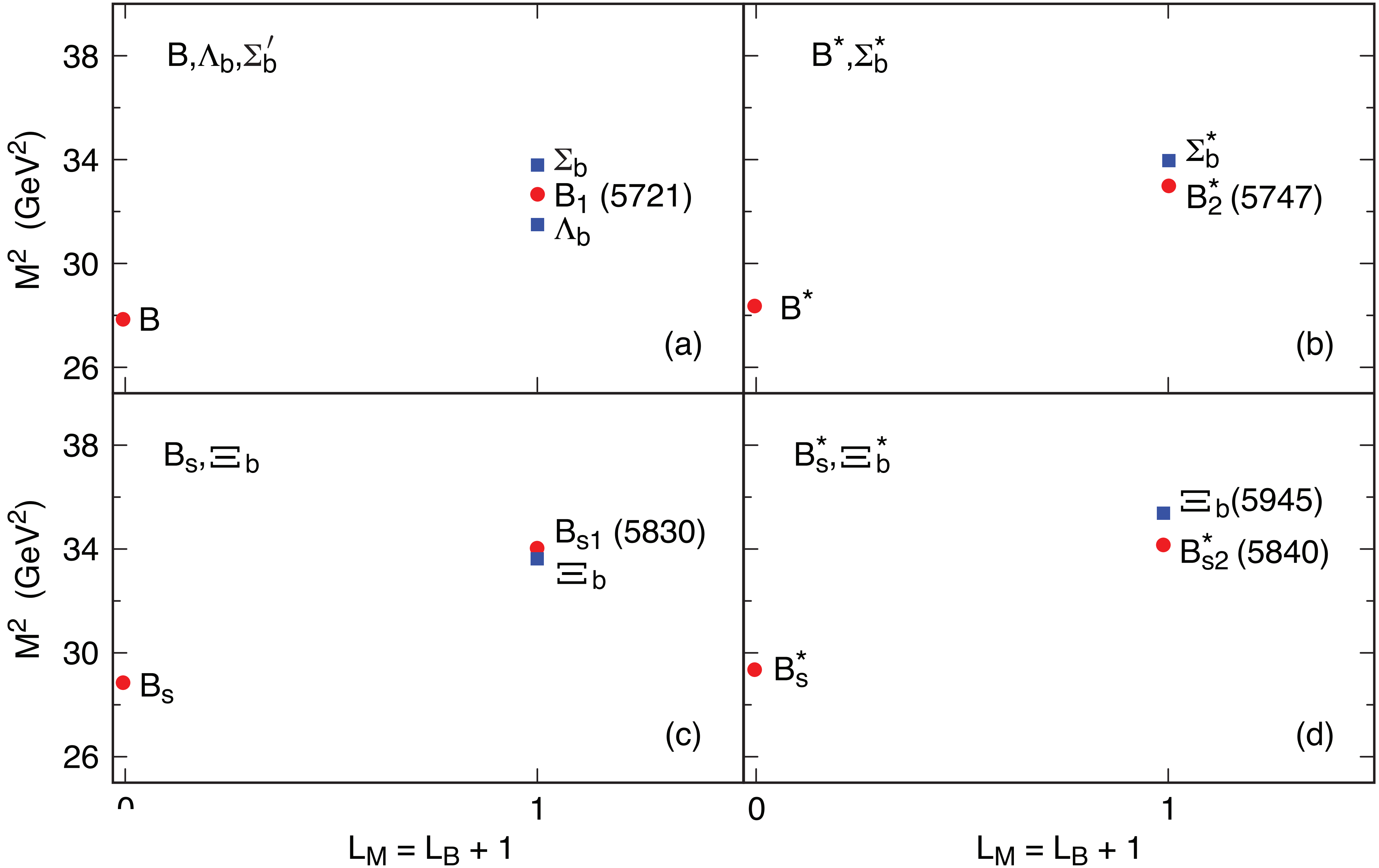
Supersymmetry across the light and heavy-light spectrum

- Introduction of quark masses breaks conformal symmetry without violating supersymmetry



Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Supersymmetric relations for mesons and baryons with b quarks

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S^z_q >= 0$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

No mass -degenerate parity partners!

Universal Hadronic Features

- **Universal quark light-front kinetic energy**

$$\Delta \mathcal{M}^2_{LFKE} = \kappa^2(1 + 2n + L)$$

Equal: Virial Theorem!

- **Universal quark light-front potential energy**

$$\Delta \mathcal{M}^2_{LFPE} = \kappa^2(1 + 2n + L)$$

- **Universal Constant Term**

$$\mathcal{M}^2_{spin} = 2\kappa^2(S + L - 1 + 2n_{diquark})$$

$$M^2 = \Delta \mathcal{M}^2_{LFKE} + \Delta \mathcal{M}^2_{LFPE} + \Delta \mathcal{M}^2_{spin}$$

Some Features of AdS/QCD

- ***Regge spectroscopy—same slope in n,L for mesons,***
- ***Chiral features for m_q=0: m_π=0, chiral-invariant proton***
- ***Hadronic LFWFs***
- ***Counting Rules***
- ***Connection between hadron masses and*** $\Lambda_{\overline{MS}}$

Superconformal AdS Light-Front Holographic QCD (LFHQCD)

Meson-Baryon Mass Degeneracy for L_M=L_B+1

Albufeira

Remarkable Features of Light-Front Schrödinger Equation

Dynamics + Spectroscopy!

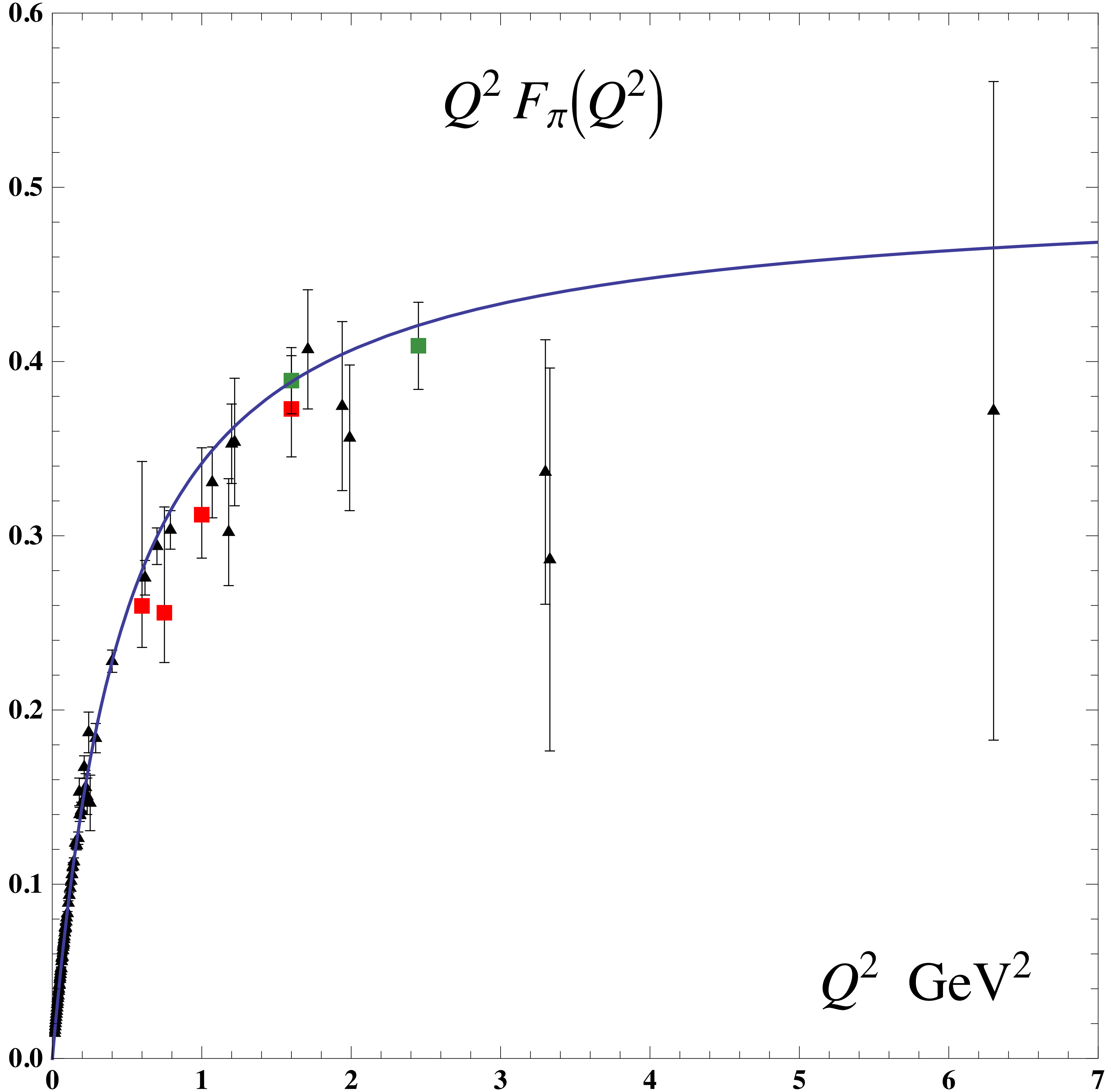
- *Relativistic, frame-independent*
- *QCD scale appears - unique LF potential*
- *Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter*
- *Zero-mass pion for zero mass quarks!*
- *Regge slope same for n and L -- not usual HO*
- *Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry*
- *Phenomenology: LFWFs, Form factors, electroproduction*
- *Extension to heavy quarks*

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Albufeira

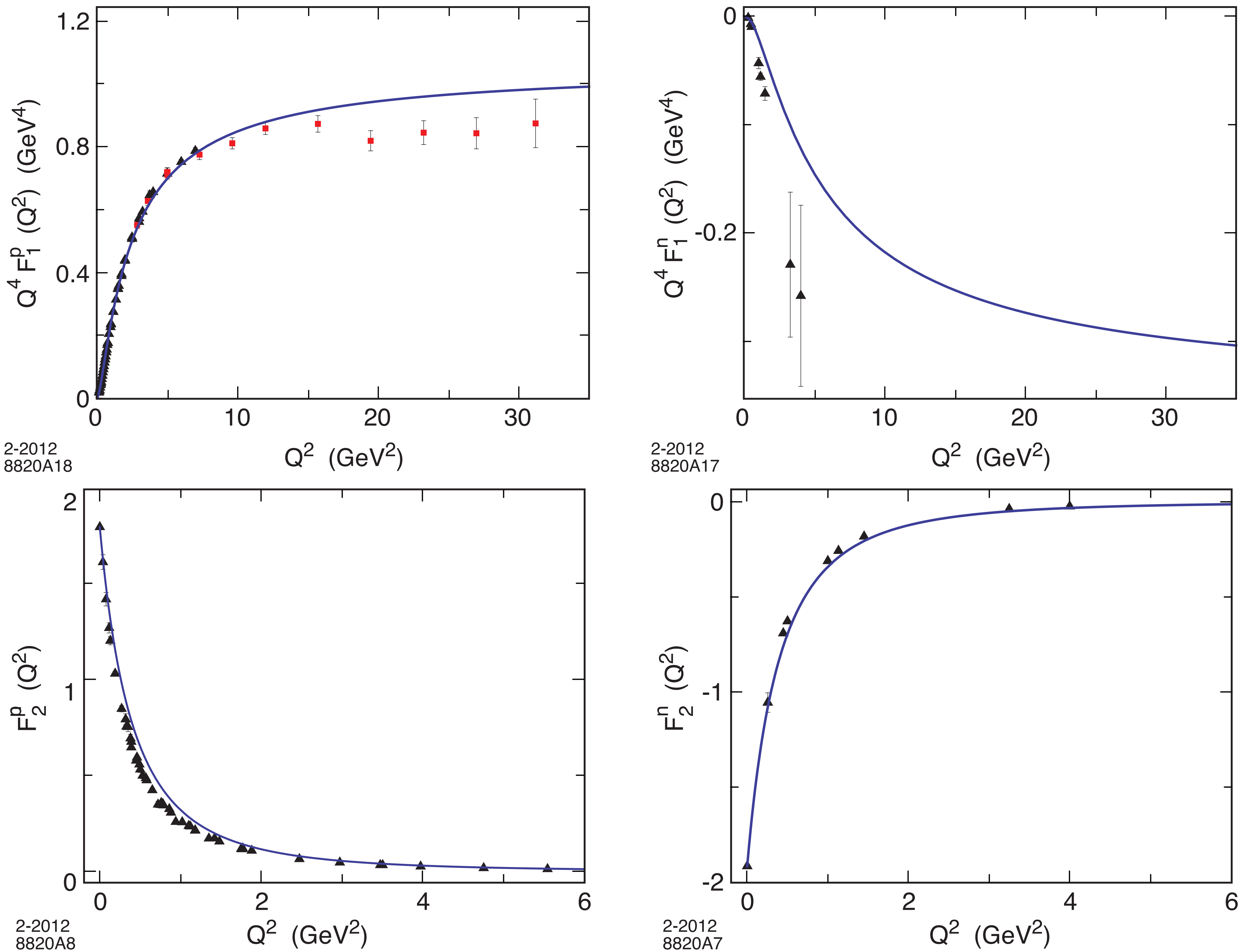
Light-Front Holography and Supersymmetric Features of QCD

Stan Brodsky
SLAC NATIONAL ACCELERATOR LABORATORY



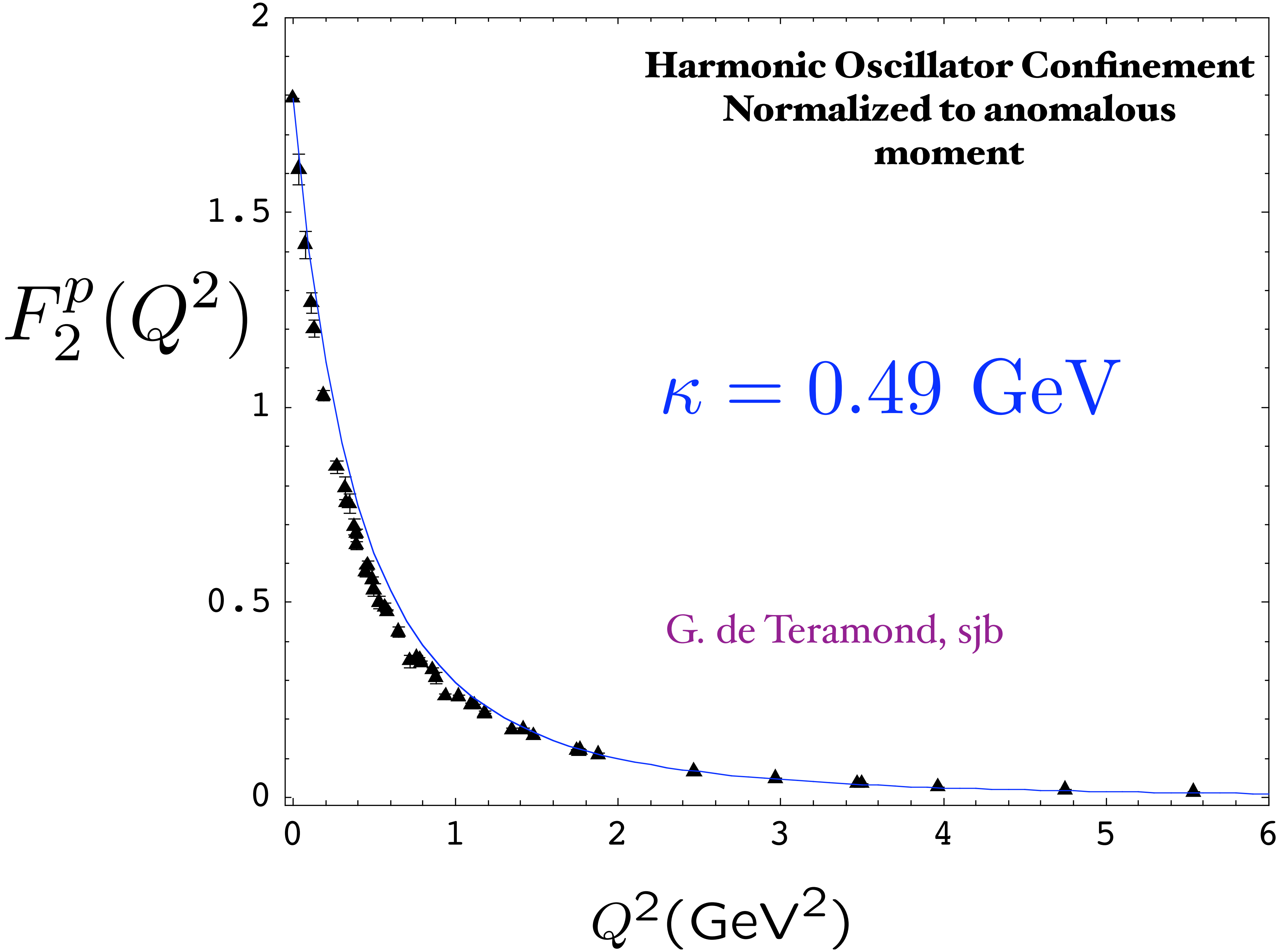
$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Using $SU(6)$ flavor symmetry and normalization to static quantities



Spacelike Pauli Form Factor

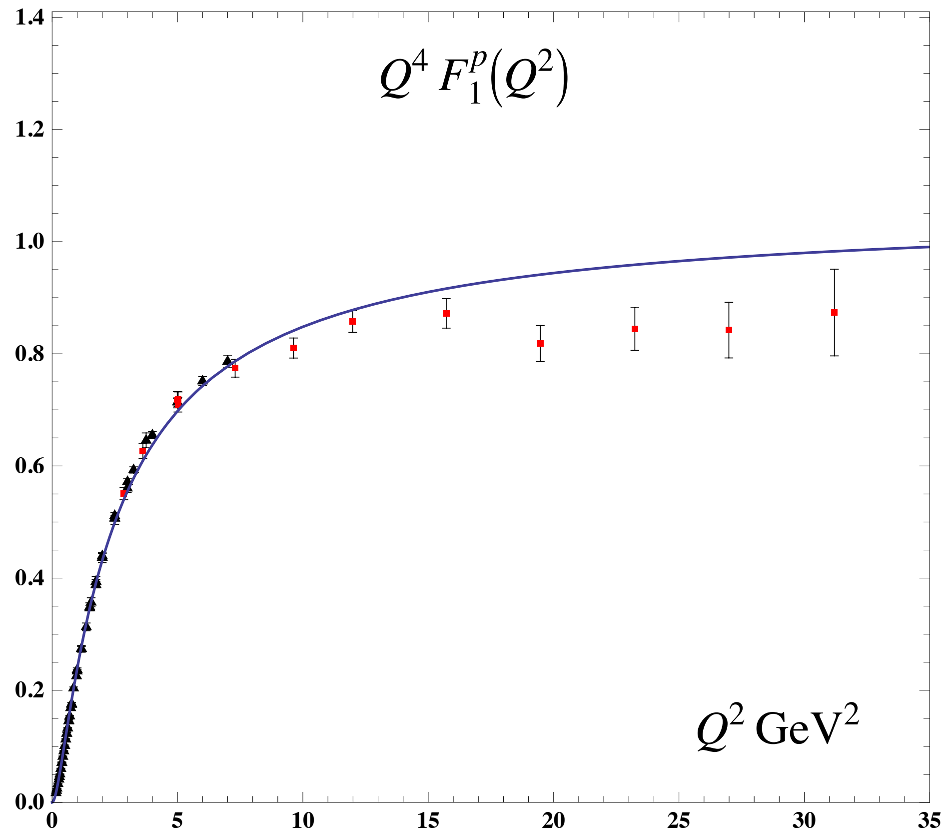
From overlap of $L = 1$ and $L = 0$ LFWFs

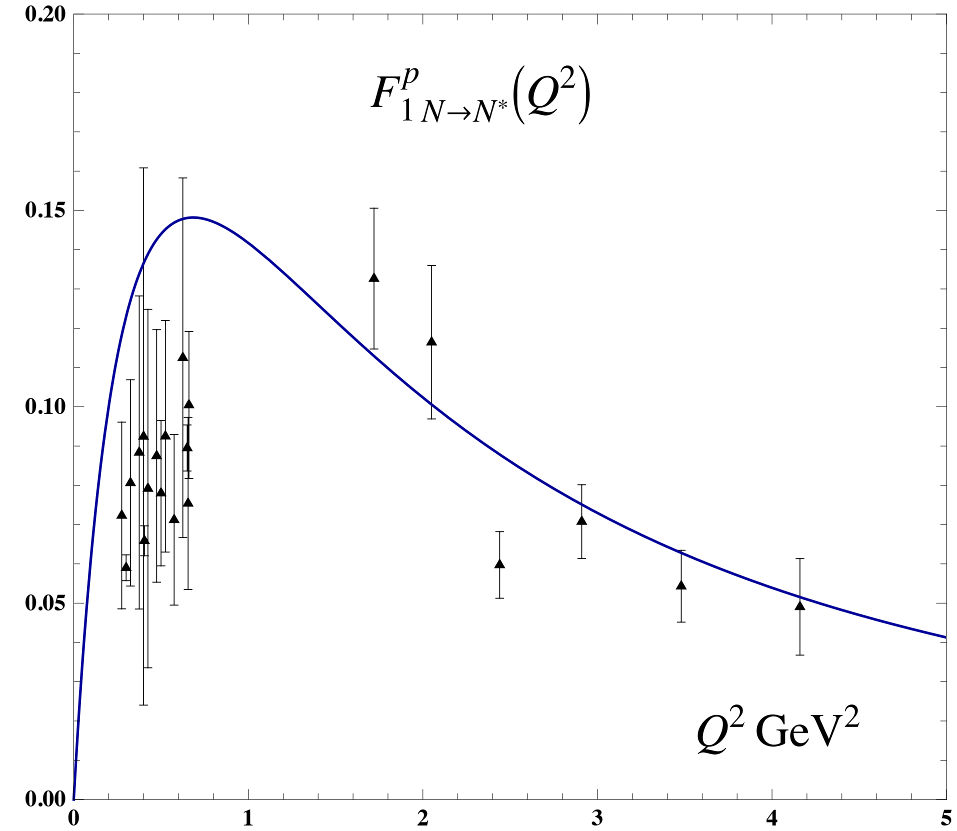


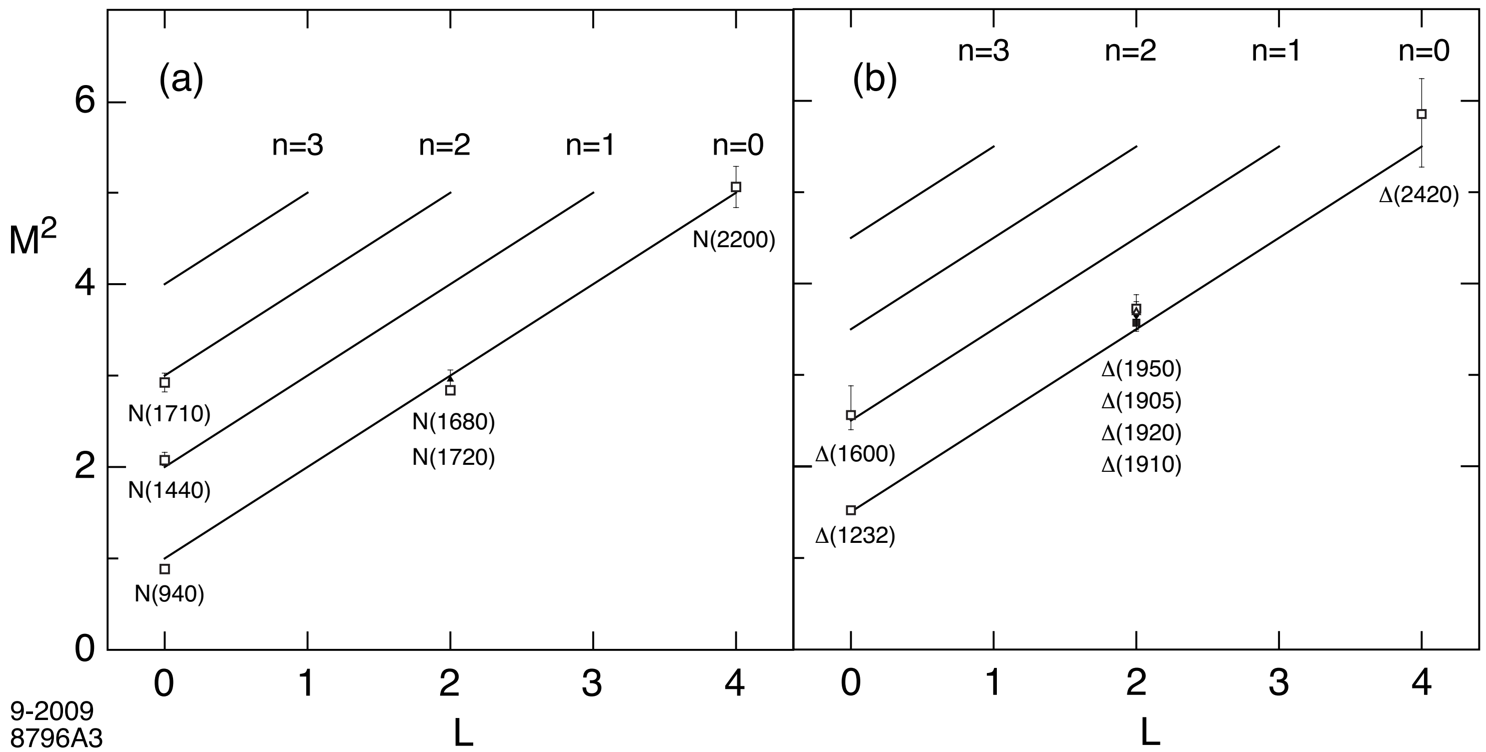
Predict hadron spectroscopy and dynamics

Excited Baryons in Holographic QCD

G. de Teramond & sjb







Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\quad \Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_{1\,N\to N^*}^{p}(Q^2) = R^4 \int \frac{dz}{z^4}\, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\quad \left(F_{1\,N\to N^*}^{p}(0) = 0, \quad V(Q=0,z)=1\right)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_{1\,N\to N^*}^{p}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

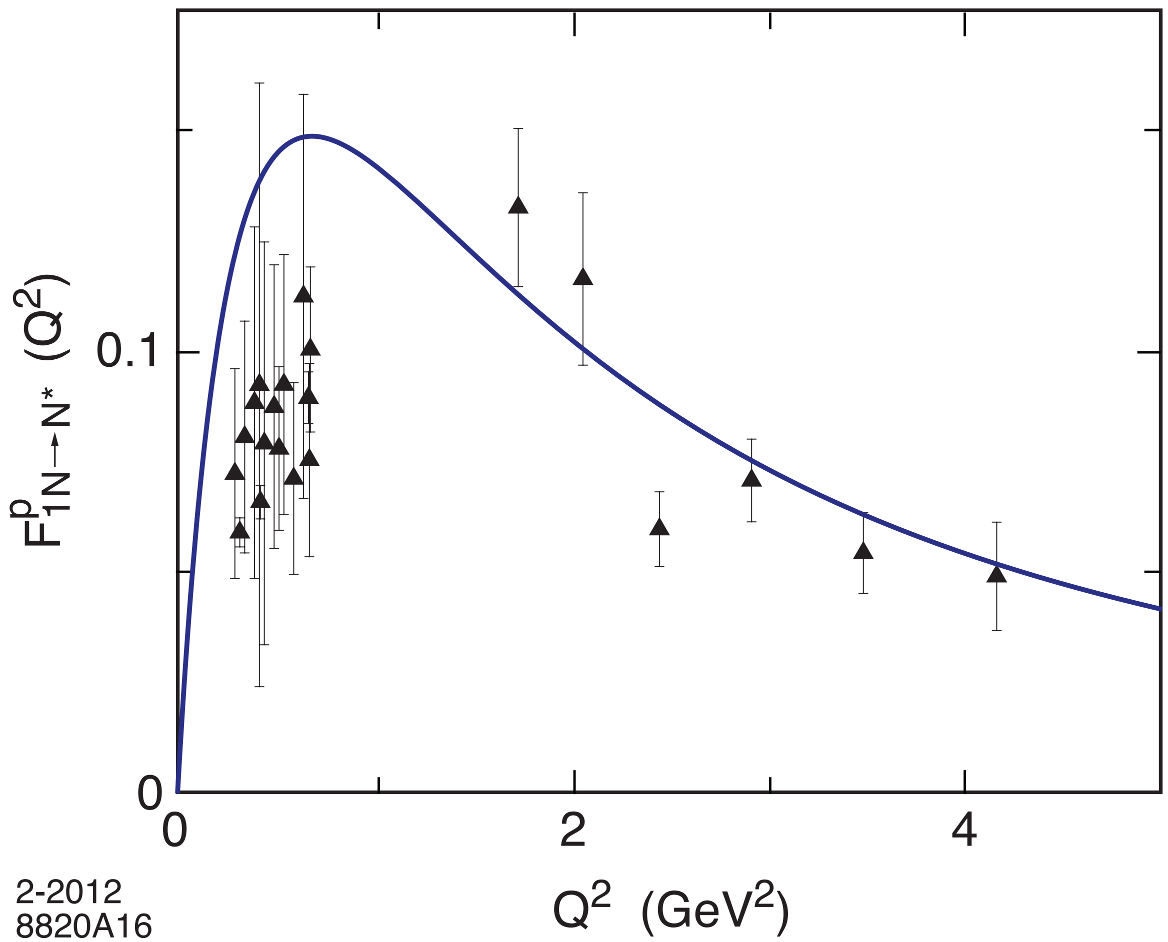
with $\mathcal{M}_{\rho_n}^{\;2} \to 4\kappa^2(n+1/2)$

de Teramond, sjb

Consistent with counting rule, twist 3

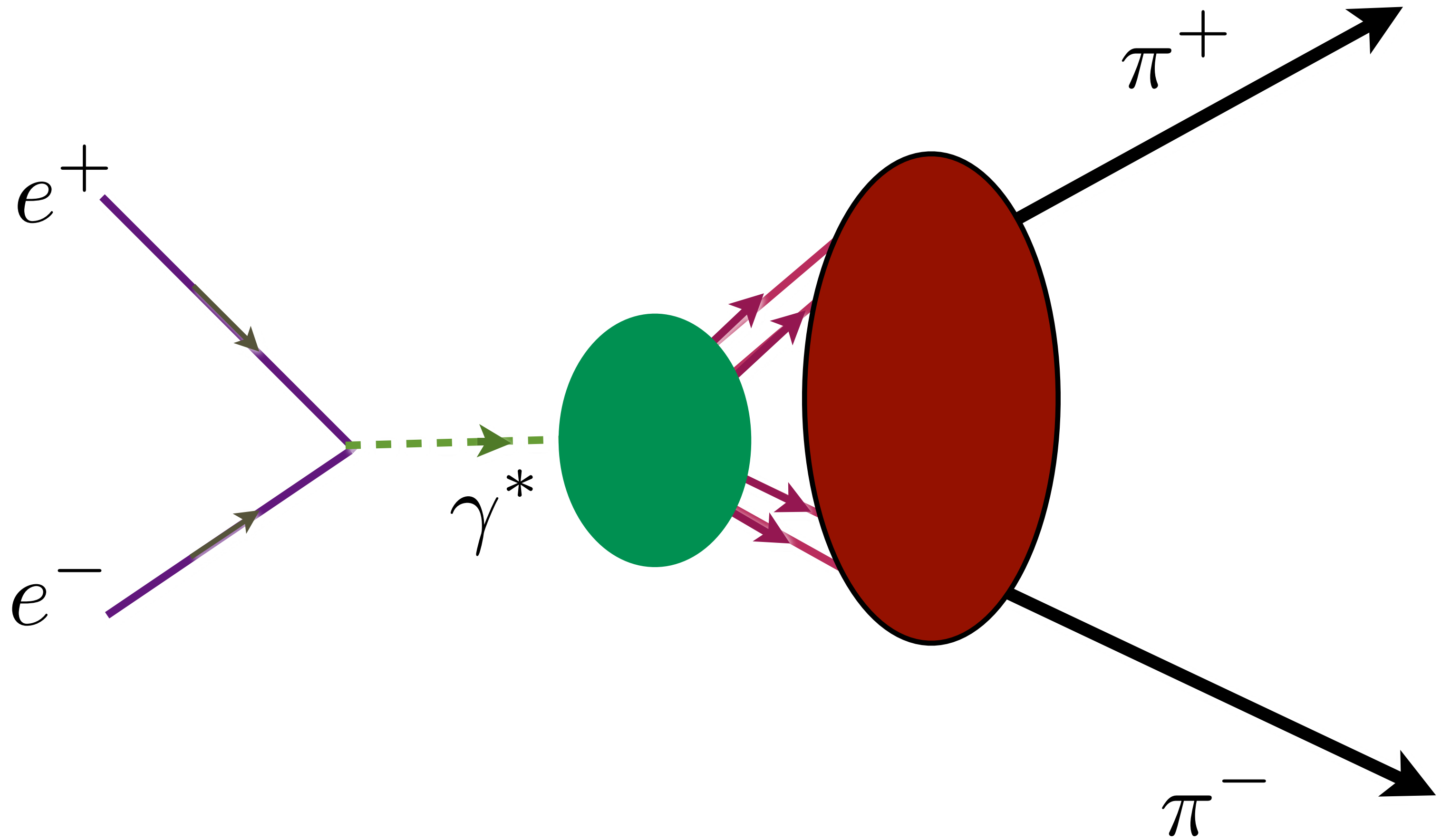
Nucleon Transition Form Factors

$$F^{p}_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\,\frac{\frac{Q^2}{\mathcal{M}_\rho^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}.$$

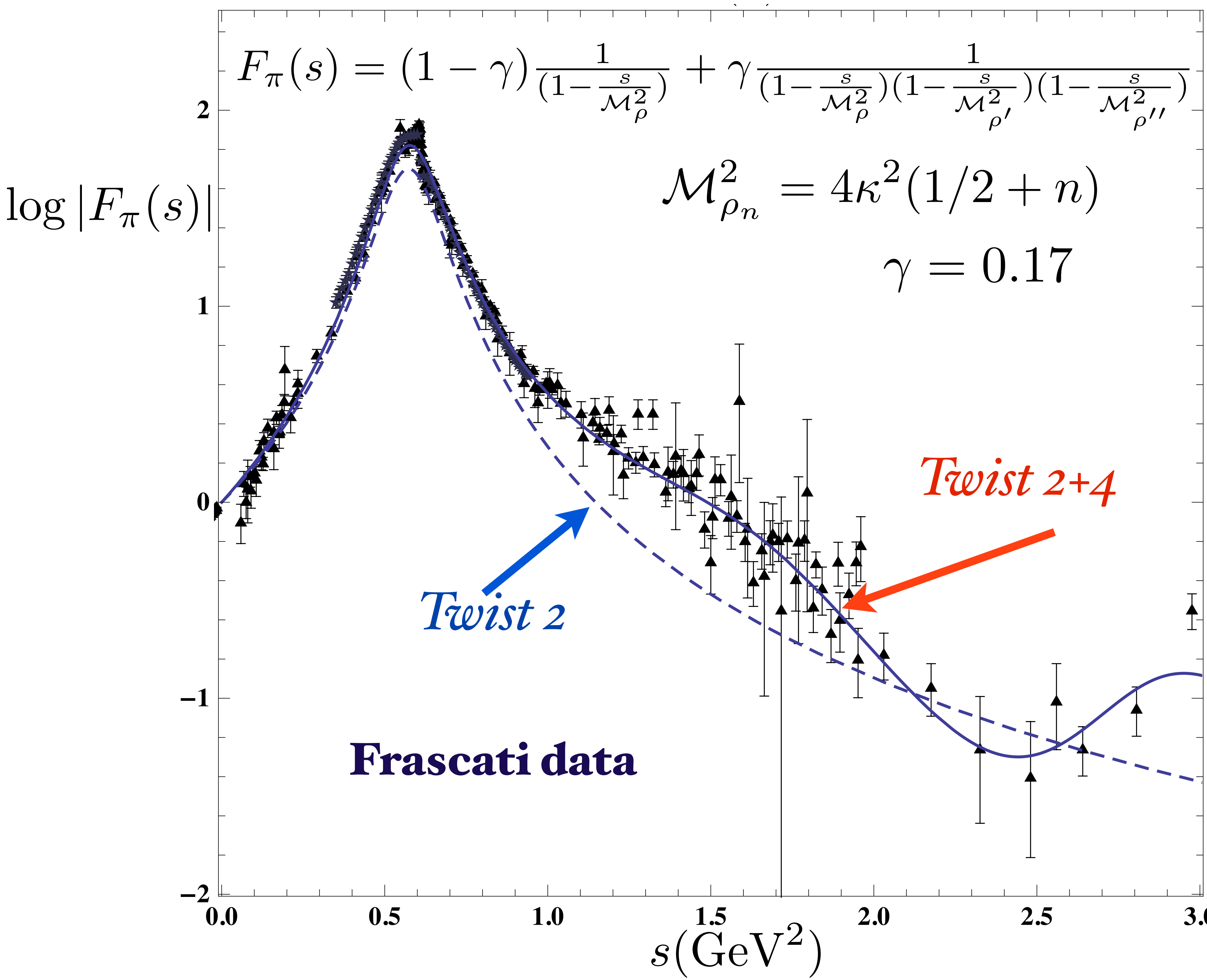


Proton transition form factor to the first radial excited state. Data from JLab

Dressed soft-wall current brings in higher Fock states and more vector meson poles



Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



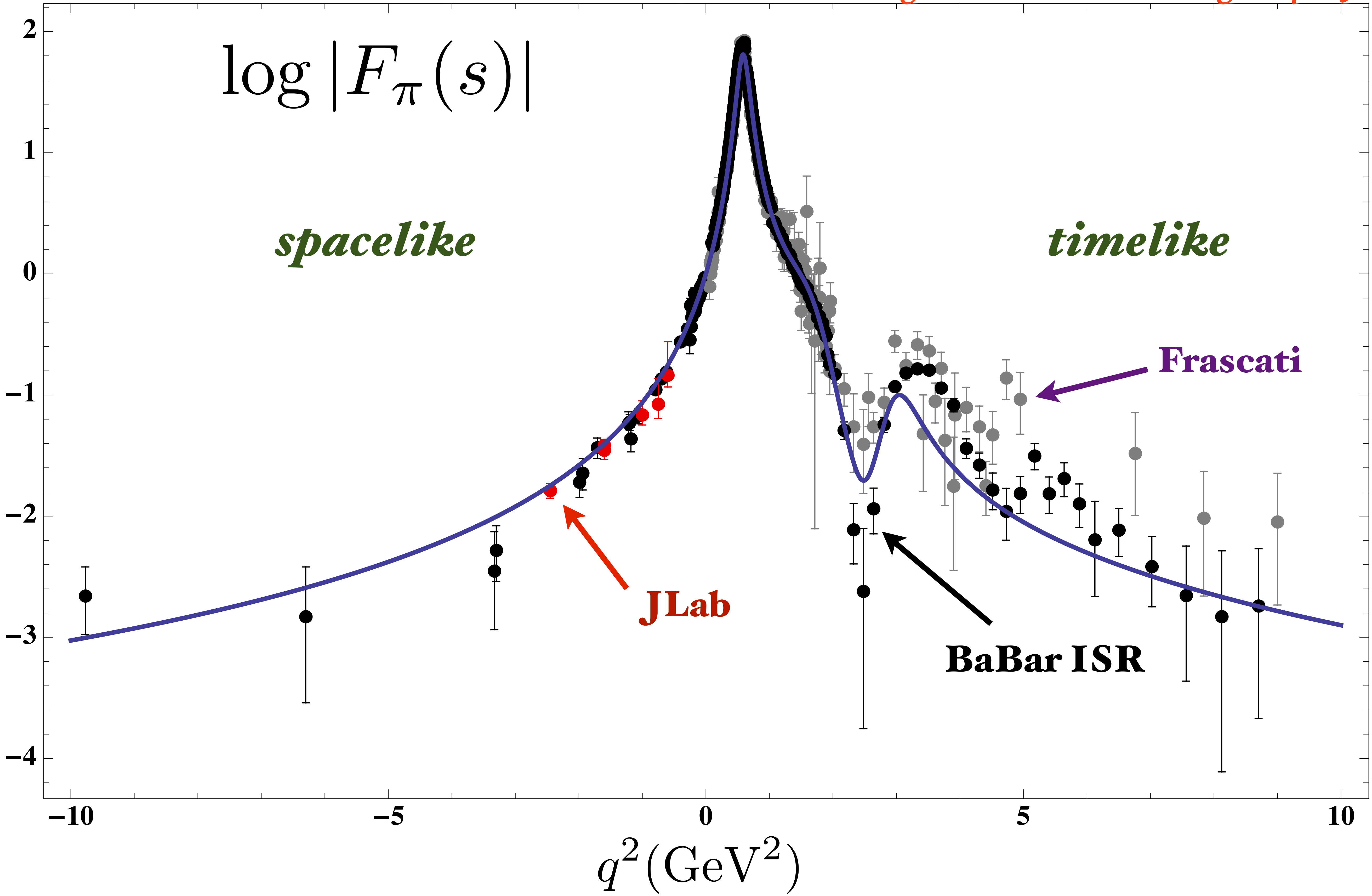
Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

G. de Teramond & sjb

Pion Form Factor from AdS/QCD and Light-Front Holography



Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta\, J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

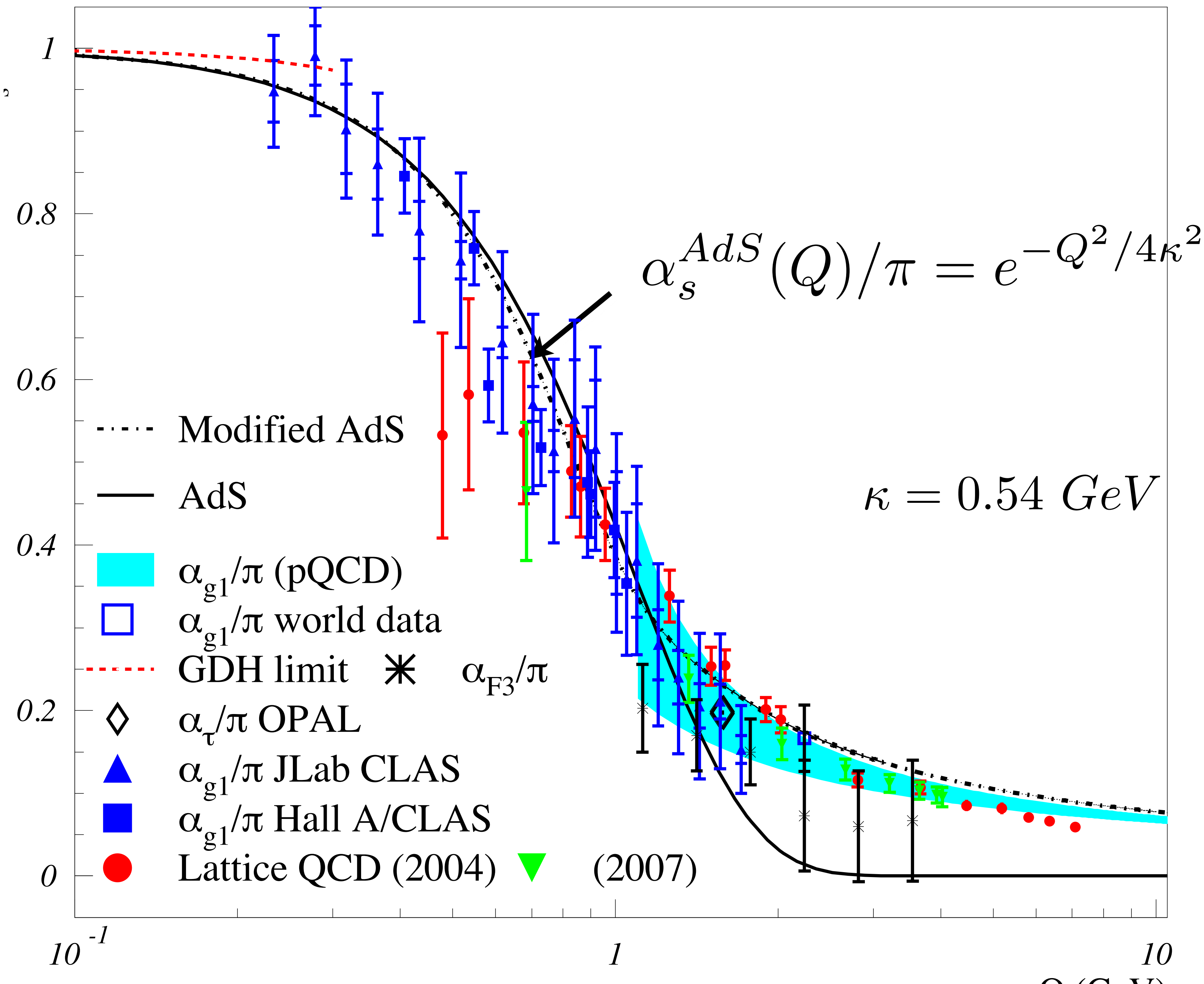
- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Analytic, defined at all scales, IR Fixed Point

$$\frac{\alpha_s(Q)}{\pi}$$



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

Deur, de Tèramond, sjb

All-Scale QCD Coupling

$$m_\rho = \sqrt{2}\kappa$$

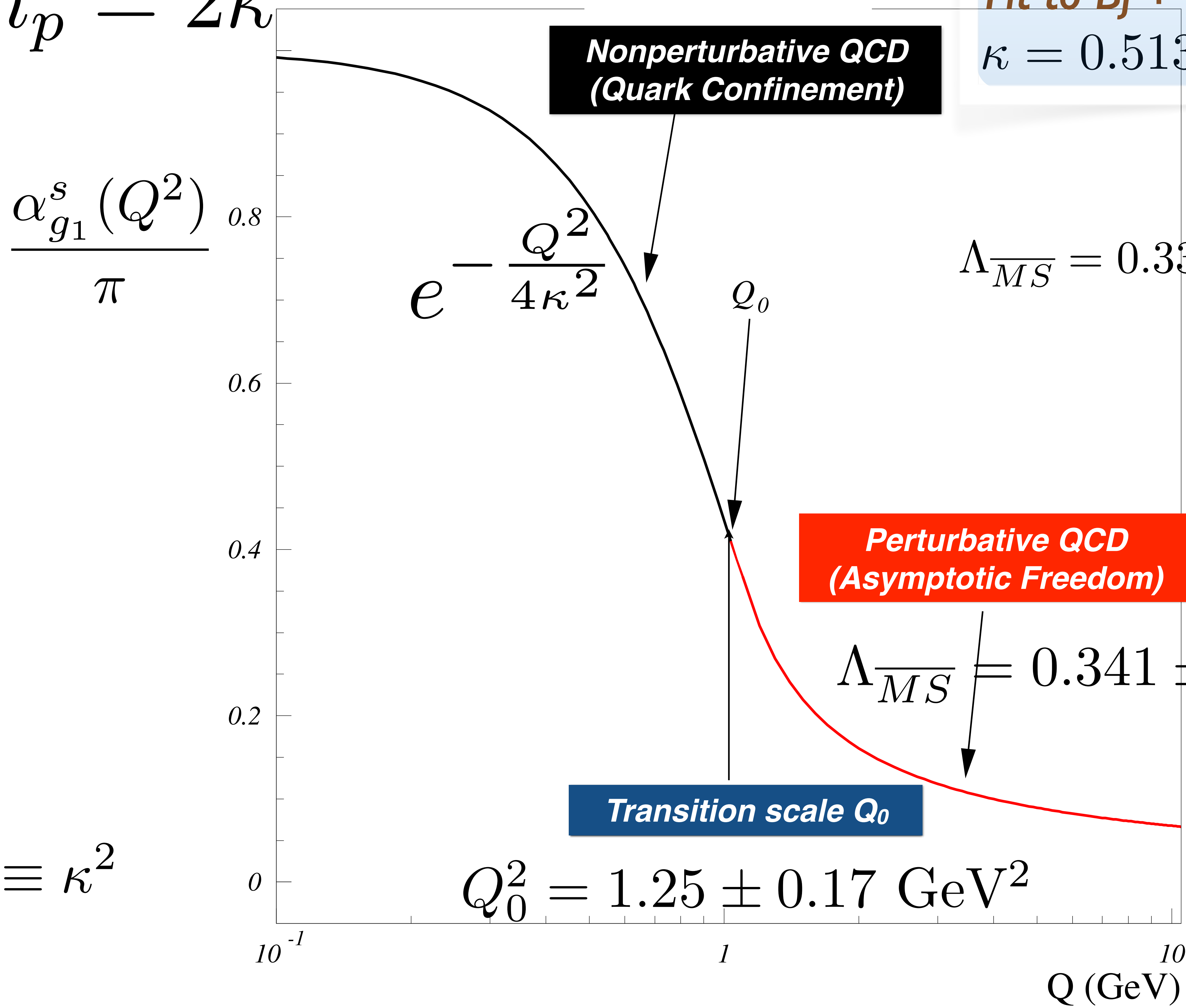
$$m_p = 2\kappa$$

Fit to Bj + DHG Sum Rules:

$$\kappa = 0.513 \pm 0.007\ GeV$$

Expt:

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016\ GeV$$



$$\lambda \equiv \kappa^2$$

de Tèramond, Dosch, sjb

Tony Zee

"Quantum Field Theory in a Nutshell"

Dreams of Exact Solvability

"In other words, if you manage to calculate m_P it better come out proportional to Λ_{QCD} since Λ_{QCD} is the only quantity with dimension of mass around.

Light-Front Holography:

Similarly for m_ρ.

$$m_p \simeq 3.21\ \Lambda_{\overline{MS}}$$

$$m_\rho \simeq 2.2\ \Lambda_{\overline{MS}}$$

Put in precise terms, if you publish a paper with a formula giving m_ρ/m_P in terms of pure numbers such as 2 and π, the field theory community will hail you as a conquering hero who has solved QCD exactly."

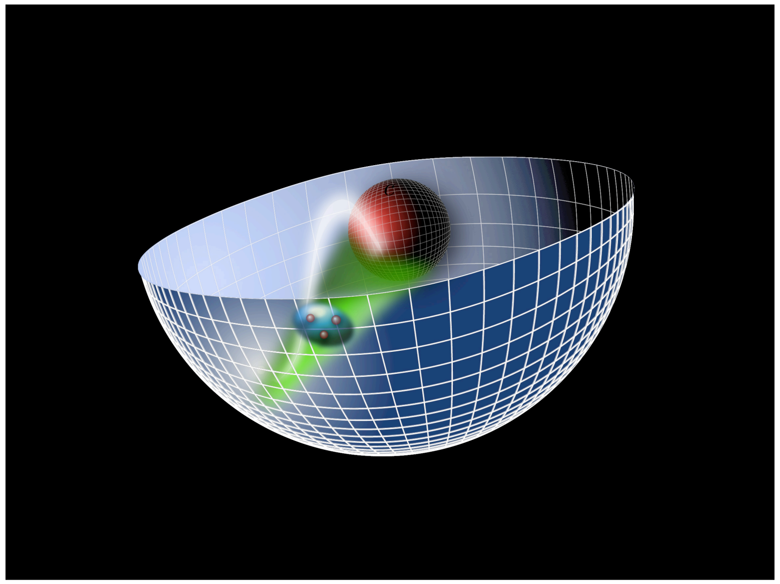
$$(m_q = 0)$$

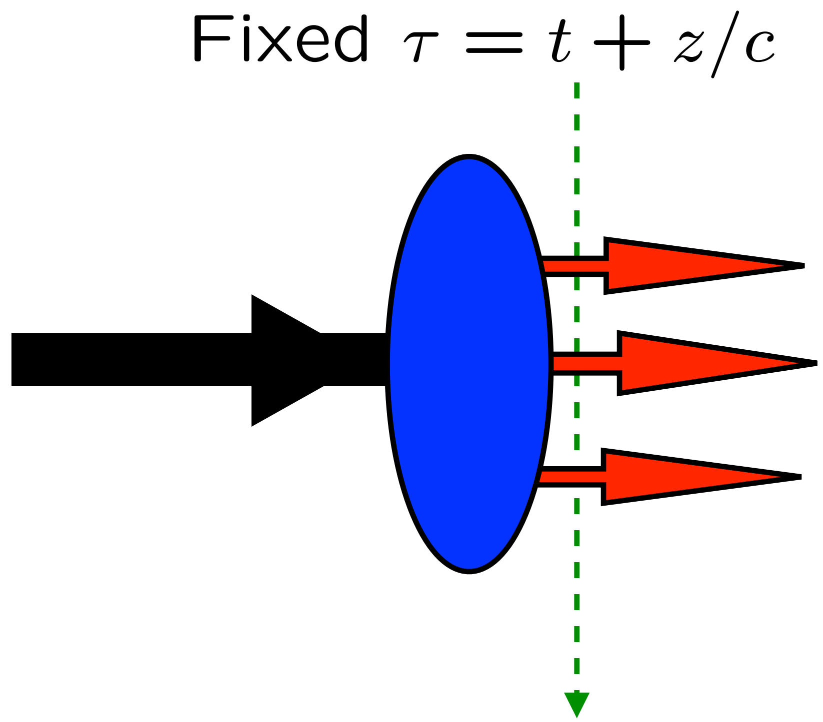
$$m_\pi = 0$$

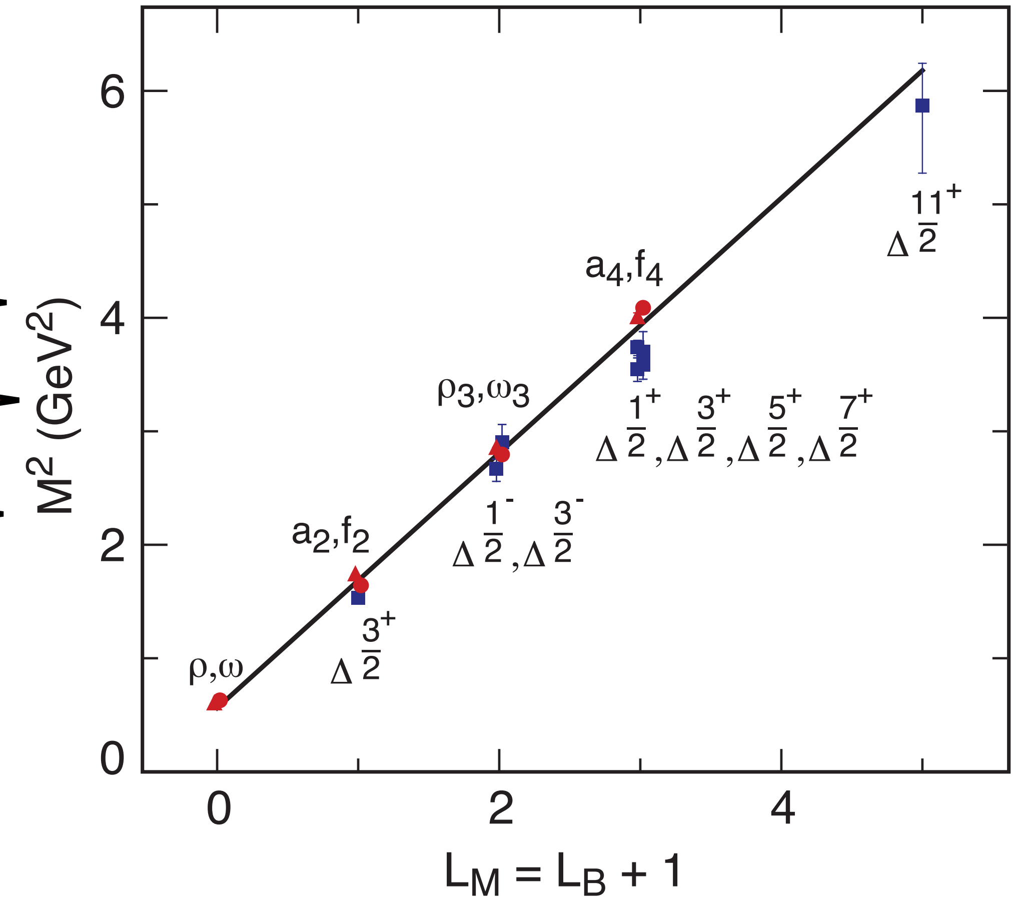
$$\frac{m_\rho}{m_P} = \frac{1}{\sqrt{2}}$$

$$\frac{\Lambda_{\overline{MS}}}{m_\rho} = 0.455 \pm 0.031$$

Light-Front Holographic QCD, Color Confinement, and Supersymmetric Features of QCD







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The Standard Theory and Beyond

Albufeira, Portugal October 24-31, 2015