SUGRA SO(10) and Inflation

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Outline

- Introduction
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- Starobinsky from No-Scale SUGRA
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- No scale SUGRA SO(10) MSGUT and inflation
- Conclusions

- The idea of inflation was introduced to solve the problems of standard big bang: Horizon Problem, Flatness problem, Monopole problem
- Inflation also explains the observed inhomogeneities over homogeneous background of universe.

The inhomogeneities arose from quantum fluctuations during the inflationary period and are mainly of two types:

- Scalar: seed of large scale structure.
- Tensor: results to primordial gravitational waves.

Both the density fluctuations and the gravitational waves have been detected via their effect on the inhomogeneities in the cosmic microwave background.

Cosmic microwave background Radiation (CMBR): Picture of Early universe

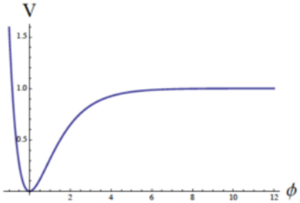
- CMBR is the relics from the Big Bang: Discovered by A. Penzias and R. Wilson in 1965.
- The temperature variation in CMBR measured by WMAP, COBE, PLANCK etc. satellites bears evidence of small density fluctuations in the early universe.
- The B-mode polarization in CMBR (if detected by experiments like BICEP and Keck Array) is the evidence for primordial gravitational waves.

Constraints from observations

- Inflation Data: Scale invariant power spectrum of curvature (density, scalar) perturbations : $P_R = (2.142 \pm 0.049) \times 10^{-9}$, spectral index $n_s = .967 \pm 0.004$ and scale invariance $kdn_s/dk \simeq 0$ (PLANCK, 2015).
- Tensor perturbation(gravity waves) suppressed $P_T/P_R = r < 0.1$.
- $N_{e-folds} \sim$ 50-60.

Starobinsky Inflation Model

- $$L = \sqrt{-g}\left(\frac{1}{2}R + \frac{R^2}{12M^2}\right) \equiv$$
  $$L = \sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{3}{4}M^2(1 - e^{-\sqrt{2/3}\phi})^2\right)$$



- It predicts $n_s - 1 = -2/N$ and $r = 12/N^2$. i.e. $n_s \sim .964$, $r \sim .004$ for N=55.

Starobinsky From No-Scale SUGRA[1]

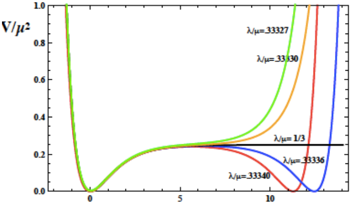
$$K = -3ln(T + T^* - \frac{1}{3}|\phi^2|); \quad W = \frac{\mu^2}{2}\Phi^2 - \frac{\lambda}{3}\Phi^3$$

Fixing $T{=}T^*{=}c/2$ gives

$$L_{eff} = \frac{c}{(c - |\phi|^2/3)^2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{(c - |\phi|^2/3)^2}|\frac{\partial W}{\partial\phi}|^2$$

$\phi = \sqrt{3c}tanh\frac{\chi}{\sqrt{3}}$ and for $\mu = \lambda/3$

$\Rightarrow V = \mu^2/4(1 - e^{-\sqrt{2/3}\phi})^2$



[1] Ellis et. al. PRL,2013

SO(10) GUT

- The minimal supersymmetric grand unified theory [2] based on SO(10) gauge group Contains: **10**(H_i), **210**(Φ_{ijkl}) and **126**$(\Sigma_{ijklm})(\overline{\mathbf{126}}(\overline{\Sigma}_{ijklm}))$ as Higgs supermultiplets.
- The renormalizable superpotential:

$$
\begin{aligned}
W &= \frac{m_\Phi}{4!}\Phi^2 + \frac{\lambda}{4!}\Phi^3 + \frac{m_\Sigma}{5!}\Sigma\overline{\Sigma} + \frac{\eta}{4!}\Phi\Sigma\overline{\Sigma} + m_H H^2 \\
&\quad + \frac{1}{4!}\Phi H(\gamma\Sigma + \bar{\gamma}\overline{\Sigma})
\end{aligned}
$$

- The **10** and $\overline{\mathbf{126}}$ are required to give masses to the fermions while **126**($\overline{\mathbf{126}}$) breaks the SO(10) gauge symmetry to MSSM together with **210**-plet.
- Different intermediate symmetries are possible with **210**-plet.

[2] Aulakh, Mohapatra(1982), Clark, Kuo and Nakagawa (1983)

- $$\begin{aligned} p &= <\Phi(1,1,1)>,\ a=<\Phi(15,1,1)>, \\ \omega &= <\Phi(15,1,3)>,\ \sigma=<\Sigma(\overline{10},3,1)>, \\ \bar{\sigma} &= <\bar{\Sigma}(10,3,1)> \end{aligned}$$

- The Superpotential in terms of these *vevs* is,

$$\begin{aligned} W &= m(p^2+3a^2+6\omega^2)+2\lambda(a^3+3p\omega^2+6a\omega^2) \\ &\quad +m_\Sigma\sigma\bar{\sigma}+\eta\sigma\bar{\sigma}(p+3a-6\omega) \end{aligned}$$

- $$SO(10) \xrightarrow{210} \text{Intermediate symmetry} \xrightarrow{126} MSSM$$

  For the first step symmetry breaking one can set $|\sigma| = |\bar{\sigma}|=0$.

The intermediate Symmetries

- If $a \neq 0$ and p=ω=0, it gives $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry.
- If $p \neq 0$ and a=ω=0, this results in $SU(4)_C \times SU(2)_L \times SU(2)_R$ symmetry.
- If $\omega \neq 0$ and p=a=0, it gives $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ symmetry.
- If p=a=-$\omega \neq 0$, this has $SU(5) \times U(1)$ symmetry.
- If p=a=$\omega \neq 0$, $SU(5) \times U(1)$ symmetry but with flipped assignments for particles.

No-Scale SUGRA SO(10)[3]

The superpotential in terms of *vevs* of **210** is given as,

$$W = m(p^2 + 3a^2 + 6\omega^2) + 2\lambda(a^3 + 3p\omega^2 + 6a\omega^2)$$

Here $m = m_\Phi$. Similarly no-scale Kähler potential is,

$$K = -3\ln(T + T^* - \frac{1}{3}(|p|^2 + 3|a|^2 + 6|\omega|^2))$$

The F-term potential has the following form,

$$V = e^G\left[\frac{\partial G}{\partial\phi^i} K^i_{j^*} \frac{\partial G}{\partial\phi_{j^*}} - 3\right]$$

[3] I. Garg, S. Mohanty, PLB,[hep-ph/1504.07725]

- $$K_i^{j^*} = \frac{1}{\Gamma^2}\begin{pmatrix} 3 & -p^* & -3a^* & -6\omega^* \\ -p & \Gamma + \frac{1}{3}|p|^2 & a^* p & 2\omega^* p \\ -3a & ap^* & 3\Gamma + 3|a|^2 & 6a\omega^* \\ -6\omega & 2\omega p^* & 6a^*\omega & 6\Gamma + 12|\omega|^2 \end{pmatrix}$$

  Where $\Gamma = T + T^* - \frac{1}{3}(|p|^2 + 3|a|^2 + 6|\omega|^2)$.

- $$V = \frac{1}{\Gamma^2}\left|\frac{\partial W}{\partial \phi_i}\right|^2$$

- $T = T^* = \frac{1}{2}$.

Inflation favourable cases

Case I: $a \neq 0$ and p=ω=0, $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

$$L_{K.E.} = \frac{(1-a^2)(\partial_\mu p)^2 + 3(\partial_\mu a)^2 + 6(1-a^2)(\partial_\mu \omega)^2}{(1-a^2)^2},$$

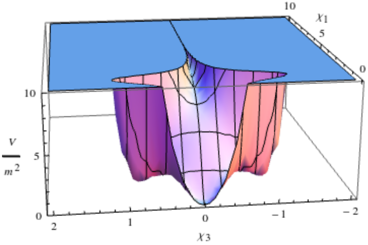
$$V = \frac{36a^4\lambda^2 + 72a^3\lambda m + 36a^2m^2}{(1-a^2)^2}$$

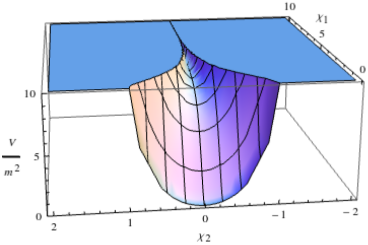
$$a = tanh[\frac{\chi_1}{\sqrt{3}}],\ p = sech[\frac{\chi_1}{\sqrt{3}}]\chi_2,\ \omega = \frac{1}{\sqrt{6}}sech[\frac{\chi_1}{\sqrt{3}}]\chi_3$$

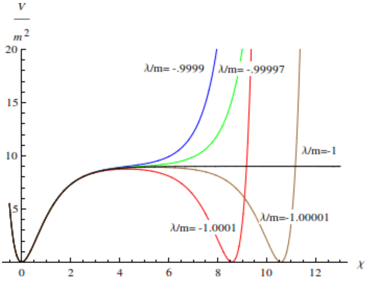
- The potential in the limit $\chi_1 \neq 0$, $\chi_2 = \chi_3$=0 is,

$$V = 36m^2(1 - e^{-\frac{2\chi_1}{\sqrt{3}}})^2$$

for λ= -m.


10
χ_1
5
0
10
$\frac{V}{m^2}$
5
0
2
1
0
−1
−2
χ_3


10
χ_1
5
0
10
$\frac{V}{m^2}$
5
0
2
1
0
−1
−2
χ_2



- $P_R = (1.610 \pm 0.01) \times 10^{-9}$ given by PLANCK5 requires value of $m = 1.311 \times 10^{-6}$ in Planck units.
- n_s = .964 and r = .002 for $N_{e-folds}$=55.
- Varying λ/m in the range (-1.0001 – -0.9999) gives n_s in the range (0.92–1.0) and r in range (0.002 –0.008).
- $SU(5) \times U(1)$ and flipped $SU(5) \times U(1)$ also give Starobinsky inflation potential but for different relation for λ and m.

Inflation unfavourable cases

- $p \neq 0$ and $a = \omega = 0$, $SU(4)_C \times SU(2)_L \times SU(2)_R$ symmetry. The kinetic and potential energy term are given by,

$$L_{K.E.} = \frac{(\partial_\mu p)^2 + 3(1-\frac{p^2}{3})(\partial_\mu a)^2 + 6(1-\frac{p^2}{3})(\partial_\mu \omega)^2}{(1-\frac{p^2}{3})^2}; \;\; V = \frac{4m^2 p^2}{(1-\frac{p^2}{3})^2}$$

$$p = \sqrt{3}\tanh[\frac{\chi_1}{\sqrt{3}}],\; a = \mathrm{sech}[\frac{\chi_1}{\sqrt{3}}]\frac{\chi_2}{\sqrt{3}},\; \omega = \mathrm{sech}[\frac{\chi_1}{\sqrt{3}}]\frac{\chi_3}{\sqrt{6}}$$

$$V(\chi_1 \neq 0, \chi_2 = \chi_3 = 0) = 3m^2 \sinh[\frac{2\chi_1}{\sqrt{3}}]^2$$

- This type of potential increases exponentially with χ_1 and is too steep to obey the slow roll conditions.
- $\omega \neq 0$ and $p = a = 0$, $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ symmetry also gives similar results.

Reheating via Instant Preheating

- At the end of inflation the inflaton χ_1 can decay to scalar bosons which have a trilinear term with Φ in superpotential e.g. $\Phi H(\gamma\Sigma + \bar{\gamma}\bar{\Sigma})$. Then the $K_{\Sigma}^{\Sigma^*}\,|W_{\Sigma}|^2$ and $K_{\bar{\Sigma}}^{\bar{\Sigma}^*}\,|W_{\bar{\Sigma}}|^2$ type of terms gives,

$$V \supset ((|\gamma|^2 + |\bar{\gamma}|^2)|H|^2 + |\gamma|^2|\Sigma|^2 + |\bar{\gamma}|^2|\bar{\Sigma}|^2)|\sinh[\frac{\chi_1}{\sqrt{3}}]|^2$$

Near the origin $\sinh[\frac{\chi_1}{\sqrt{3}}] \approx \frac{\chi_1}{\sqrt{3}}$,

$$V \supset ((|\gamma|^2 + |\bar{\gamma}|^2)|H|^2 + |\gamma|^2|\Sigma|^2 + |\bar{\gamma}|^2|\bar{\Sigma}|^2)|\frac{\chi_1}{\sqrt{3}}|^2$$

- Perturbative decay of inflaton to scalars is not efficient for typical values of $\gamma,\bar{\gamma} \sim$ O(.1-1.0).
- Non-perturbative decay to scalar bosons (H, Σ) leading to preheating.
- Scalar bosons produced at $\chi_1 = 0$ decay further when $\chi_1 = \chi_{1max}$, to the modes which are not directly coupled to inflaton, e.g. SM fermions and the right-handed neutrinos through Yukawa couplings.
- $$T_R \sim V_0^{1/4} \sim (m^2\chi_1^2)^{1/4} \sim (10^{-18}M_{pl}^4)^{1/4} \sim 10^{14}GeV$$

- At temperature $<< T_R$, we assume that universe settles to the minimum of potential corresponding to MSSM symmetry.
- zero cosmological constant $\Rightarrow a, p, \omega, \sigma(\bar{\sigma})$ have values such that $V = \frac{|W_{\phi_i}|^2}{\Gamma^2} = 0$.
- This can be satisfied if

$$a = \frac{m}{\lambda}\frac{x^2 + 2x - 1}{1 - x};\ p = \frac{m}{\lambda}\frac{x(5x^2 - 1)}{(1 - x)^2};$$
$$\sigma\overline{\sigma} = \frac{2m^2}{\eta\lambda}\frac{x(1 - 3x)(1 + x^2)}{\eta(1 - x)^2};\ \omega = -\frac{m}{\lambda}x$$

where x is the solution of following cubic equation,

$$8x^3 - 15x^2 + 14x - 3 = -\frac{\lambda m_\Sigma}{\eta m}(1 - x)^2$$

- $$m_{3/2}^2 = e^G = e^K |W|^2 \,.$$
- Visible sector contribution to gravitino mass can be made zero or negligible with field values of a, p, ω, $\sigma(\bar{\sigma})$ and tuning $|W| \approx 0$.
- $$\begin{pmatrix} -m_H & \bar{\gamma}\sqrt{3}(\omega - a) & -\gamma\sqrt{3}(\omega + a) & -\bar{\gamma}\bar{\sigma} \\ -\bar{\gamma}\sqrt{3}(\omega + a) & 0 & -(2m_\Sigma + 4\eta(a+\omega)) & 0 \\ \gamma\sqrt{3}(\omega - a) & -(2m_\Sigma + 4\eta(a-\omega)) & 0 & -2\eta\bar{\sigma}\sqrt{3} \\ -\sigma\gamma & -2\eta\sigma\sqrt{3} & 0 & -2m + 6\lambda(\omega - a) \end{pmatrix}$$

  One out of the four Higgs doublets can be made light with the fine tuning condition of $Det\mathcal{H} = 0$.

Conclusions and Discussion

- Starobinsky model of inflation can be derived from no-scale SUGRA SO(10) GUT for the specific intermediate symmetries of $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, $SU(5) \times U(1)$ and flipped $SU(5) \times U(1)$ gauge groups.
- The other intermediate symmetries $SU(4)_C \times SU(2)_L \times SU(2)_R$ or $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ do not give the slow-roll potential required for inflation.
- After reheating when intermediate symmetry breaks to MSSM topological defects may form. Out of favourable cases for inflation $SU(5) \times U(1)$ gives rise to monopoles after inflation and this case therefore can be ruled out from the consideration of topological defects in the cosmological evolution.

THANKS