Flavor Dependence of Constituent-Quark Masses and Flavor Content in Low-Energy Baryons

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Some Particular Remarks

A few remarks on constituent/valence quarks and QCD:

- 2014: 50th anniversary of the quark model (M. Gell-Mann / G. Zweig)
- 43 years after the invention of QCD in late 1972 (M. Gell-Mann and H. Fritzsch) – not yet solved in the non-perturbative regime
- Problems: Confinement and spontaneous breaking of chiral symmetry (SBχS) towards lower energies
- SBχS: Clue to generation of dynamical masses of constituent quarks and their interactions (mediated by Goldstone bosons rather than by gluons)

Outline

Low-Energy QCD / Relevant Degrees of Freedom

Relativistic Constituent-Quark Model (RCQM)
- Extension to heavy flavors ($\rightsquigarrow$ all known baryons)

Baryon Spectroscopy
- Light, strange, charm, bottom

Dynamical Masses of Constituent Quarks

Baryon Structure
- Nucleon e.m. form factors - Flavor content
- Baryon electromagnetic form factors
- Nucleon and baryon axial form factors / charges
- Nucleon gravitational form factors

Meson-Baryon Interaction Vertices
- Microscopic πNN and $\pi N\Delta$ vertex form factors

Summary and Conclusions

Low-Energy QCD

Low-energy QCD of N_f flavors is characterized by:

- spontaneous breaking of chiral symmetry ($SB\chi S$):

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

→ appearance of $(N_f^2 - 1)$ **Goldstone bosons** $\vec{\phi}$
→ generation of quasiparticles with dynamical mass, i.e. **constituent quarks** ψ

- thus (effective) interaction Lagrangian:

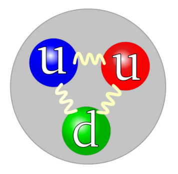
$$\mathcal{L}_{\mathrm{int}} \sim ig\bar{\psi}\gamma_5\vec{\lambda}^f \cdot \vec{\phi}\psi$$

A. Manohar and H. Georgi: Nucl. Phys. B 234 (1984) 189
E.V. Shuryak: Phys. Rep. **115**, 151 (1984)
L.Ya. Glozman and D.O. Riska: Phys. Rep. **268**, 263 (1996)
see also:
S. Weinberg: Phys. Rev. Lett. **105**, 261601 (2010)

Baryons

Baryons are considered as colorless bound states of three constituent quarks.

Here the proton:



- 'Constituent' quarks are quasiparticles with dynamical mass, NOT the original QCD d.o.f. (i.e. 'current' quarks).
- 'Constituent' quarks are confined and interact via hyperfine interactions associated with $SB\chi S$, i.e. Goldstone-boson exchange.

Framework

Relativistic quantum mechanics (RQM)

i.e. **quantum theory** respecting **Poincaré invariance**

(theory on a Hilbert space $\mathcal{H}$ corresponding to a finite number of particles, not a field theory)

Invariant mass operator

$$\hat{M} = \hat{M}_{free} + \hat{M}_{int}$$

Eigenvalue equations

$$\hat{M}\,|P,J,\Sigma\rangle = M\,|P,J,\Sigma\rangle \quad , \qquad \hat{M}^2 = \hat{P}^{\mu}\hat{P}_{\mu}$$

$$\hat{P}^{\mu}\,|P,J,\Sigma\rangle = P^{\mu}\,|P,J,\Sigma\rangle \quad , \qquad \hat{P}^{\mu} = \hat{M}\hat{V}^{\mu}$$

Relativistic Constituent-Quark Model (RCQM)

Interacting mass operator

$$\begin{aligned}
\hat{M} &= \hat{M}_{free} + \hat{M}_{int} \\
\hat{M}_{free} &= \sqrt{\hat{H}^2_{free} - \hat{\vec{P}}^2_{free}} \\
\hat{M}^{rest\ frame}_{int} &= \sum_{i<j}^{3} \hat{V}_{ij} = \sum_{i<j}^{3} [\hat{V}^{conf}_{ij} + \hat{V}^{hf}_{ij}]
\end{aligned}$$

fulfilling the **Poincaré algebra**

$$\begin{array}{lll}
[\hat{P}_i, \hat{P}_j] = 0, & [\hat{J}_i, \hat{H}] = 0, & [\hat{P}_i, \hat{H}] = 0, \\
[\hat{K}_i, \hat{H}] = -i\hat{P}_i & [\hat{J}_i, \hat{J}_j] = i\epsilon_{ijk}\hat{J}_k & [\hat{J}_i, \hat{K}_j] = i\epsilon_{ijk}\hat{K}_k, \\
[\hat{J}_i, \hat{P}_j] = i\epsilon_{ijk}\hat{P}_k, & [\hat{K}_i, \hat{K}_j] = -i\epsilon_{ijk}\hat{J}_k, & [\hat{K}_i, \hat{P}_j] = -i\delta_{ij}\hat{H}
\end{array}$$

$\hat{H}, \hat{P}_i$ ... time and space translations,
$\hat{J}_i$ ... rotations, $\hat{K}_i$ ... Lorentz boosts

Universal GBE RCQM of $SU(5)_V \times U(1)$

Phenomenologically, baryons with 5 flavors: u, d, s, c, b

$$\Rightarrow \quad H_{free} = \sum_{i=1}^{3} \sqrt{m_i^2 + \vec{k}_i^2}$$

$$V^{conf}(\vec{r}_{ij}) = B + C\, r_{ij}$$

$$V^{hf}(\vec{r}_{ij}) = \left[V_{24}(\vec{r}_{ij}) \sum_{f=1}^{24} \lambda_i^f \lambda_j^f + V_0(\vec{r}_{ij}) \lambda_i^0 \lambda_j^0 \right] \vec{\sigma}_i \cdot \vec{\sigma}_j$$

- i.e., for $N_f = 5$, we have the exchange of a **24-plet** plus a **singlet** of Goldstone bosons.

L.Ya. Glozman and D.O. Riska: Nucl. Phys. A **603**, 326 (1996)

J.P. Day, K.-S. Choi, and W. Plessas: arXiv:1205.6918

J.P. Day, K.-S. Choi, and W. Plessas: Few-Body Syst. **54**, 329 (2013)

Universal GBE RCQM Parametrization

$$
\begin{aligned}
V^{conf}(\vec{r}_{ij}) &= B + C\, r_{ij} \\
V_\beta(\vec{r}_{ij}) &= \frac{g_\beta^2}{4\pi}\frac{1}{12m_i m_j}\left\{\mu_\beta^2\frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - 4\pi\delta(\vec{r}_{ij})\right\} \\
&= \frac{g_\beta^2}{4\pi}\frac{1}{12m_i m_j}\left\{\mu_\beta^2\frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - \Lambda_\beta^2\frac{e^{-\Lambda_\beta r_{ij}}}{r_{ij}}\right\}
\end{aligned}
$$

$$B = -402 \text{ MeV}, \quad C = 2.33 \text{ fm}^{-2}$$

$$\beta = 24: \quad \frac{g_{24}^2}{4\pi} = 0.7, \quad \mu_{24} = \mu_\pi = 139 \text{ MeV}, \quad \Lambda_{24} = 700.5 \text{ MeV}$$

$$\beta = 0: \quad \left(\frac{g_0}{g_{24}}\right)^2 = 1.5, \quad \mu_0 = \mu_{\eta'} = 958 \text{ MeV}, \quad \Lambda_0 = 1484 \text{ MeV}$$

$$m_u = m_d = 340 \text{ MeV}, \quad m_s = 480 \text{ MeV},$$

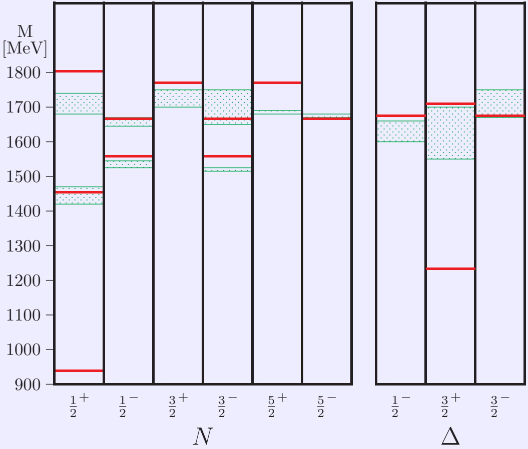
$$m_c = 1675 \text{ MeV}, \quad m_b = 5055 \text{ MeV}$$

All Baryon **Excitation Spectra**

of

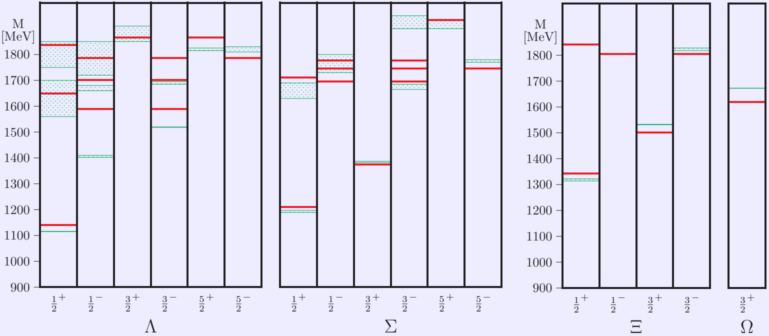
u, d, s, c, b Flavors

Light Baryon Spectra



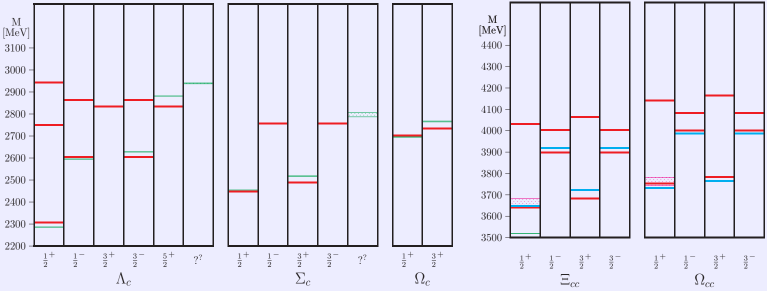
red	Universal GBE RCQM
green	PDG 2013 (experiment)

Strange Baryon Spectra



red	Universal GBE RCQM
green	PDG 2013 (experiment)

Charm Baryon Spectra



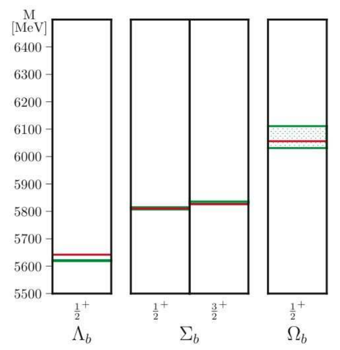
Left panel – single charm:

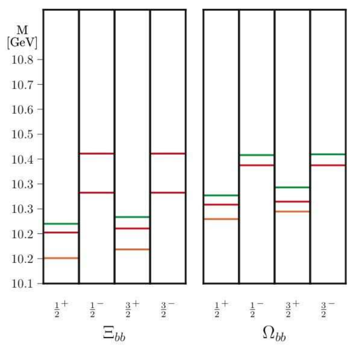
red	Universal GBE RCQM prediction
green	PDG 2013 (experiment)

Right panel – double charm:

green	M. Mattson et al.: Phys. Rev. Lett. 89 (2002) 112001 (SELEX experiment)
cyan	S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)
magenta	L. Liu et al.: Phys. Rev. D 81 (2010) 094505 (Lattice QCD)

Bottom Baryon Spectra





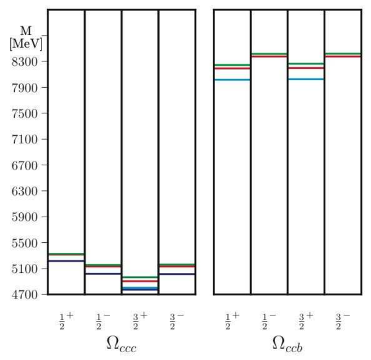
Left panel – single bottom:

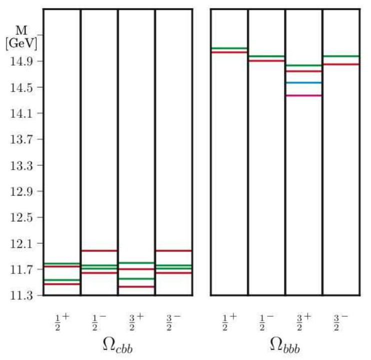
red	Universal GBE RCQM prediction
green	PDG 2013 (experiment)

Right panel – double bottom:

green	W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817 (nonrel. one-gluon-exchange CQM)
orange	D. Ebert, R.N. Faustov, V.O. Galkin, and A.P. Martynenko: Phys. Rev. D 66 (2002) 014008 (RCQM)

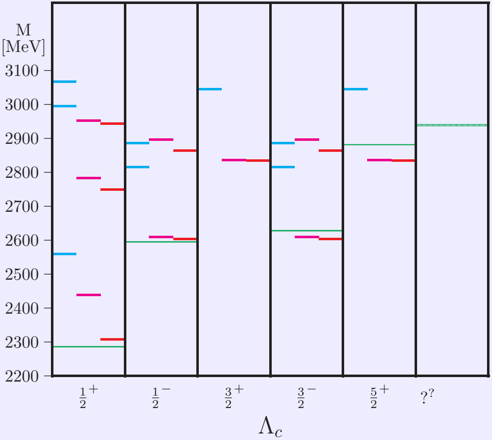
Triple-Heavy Baryon Spectra





red	Universal GBE RCQM
green	W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817 (nonrelativistic one-gluon-exchange CQM)
blue	S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)
cyan	A.P. Martynenko: Phys. Lett. B 663 (2008) 317 (RCQM)
magenta	S. Meinel: Phys. Rev. D 82 (2010) 114502 (lattice QCD)

Influence of Light-Heavy Q-Q Interaction



leftmost cyan levels	confinement only
middle magenta levels	including only light-light GBE
rightmost red levels	including full GBE RCQM

Systematics of Constituent-Quark Masses

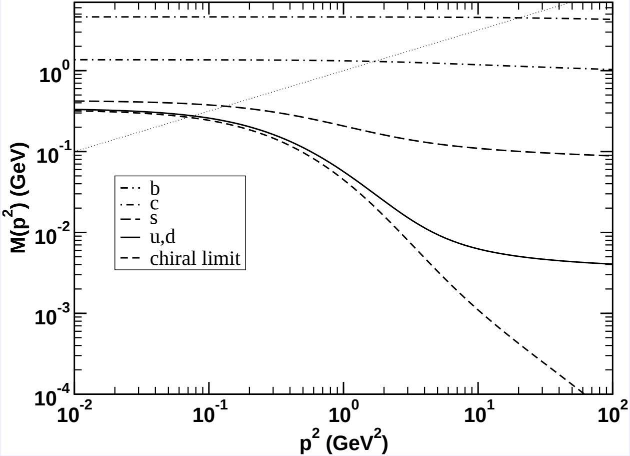
Dynamical mass gain $\Delta m = m_Q - m_q$ due to SBχS is quite similar for all flavors:

Quark	PDG	RCQM		DSE
flavor	m_q	m_Q	Δm	Δm
$\frac{1}{2}(u+d)$	$3.3 - 4.2$	340	~ 336	~ 276
s	95 ± 5	480	~ 385	~ 278
c	1275 ± 25	1675	~ 400	~ 330
b	4660 ± 30	5055	~ 395	~ 400

PDG	Particle Data Group (current-quark masses)
RCQM	Relativistic Constituent-Quark Model
DSE	Dyson-Schwinger Equation

Is Δm a new challenge for flavor physics?

Quark Mass Functions from DSE



A. Höll, A. Krassnigg, C.D. Roberts, and S.V. Wright: Int. J. Mod. Phys. A **20** (2005) 1778

A. Krassnigg, private communication

Rest-Frame Baryon States

Mass operator eigenstates

$$\hat{M}\,|P,J,\Sigma,T,M_T\rangle = M\,|P,J,\Sigma,T,M_T\rangle$$

represented in configuration space

$$\left\langle\vec{\xi},\vec{\eta}\,\middle|\,P,J,\Sigma,T,M_T\right\rangle = \Psi_{PJ\Sigma TM_T}(\vec{\xi},\vec{\eta})$$

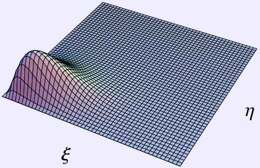
with $\vec{\xi}$ and $\vec{\eta}$ the usual Jacobi coordinates.

Picture the baryon wave functions through
spatial probability density distributions

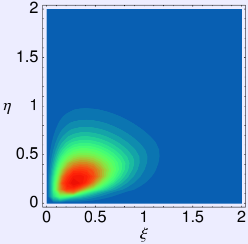
$$\rho(\xi,\eta) = \xi^2\eta^2\int d\Omega_\xi d\Omega_\eta \\ \Psi^{\star}_{PJ\Sigma TM_T}(\xi,\Omega_\xi,\eta,\Omega_\eta)\Psi_{PJ\Sigma TM_T}(\xi,\Omega_\xi,\eta,\Omega_\eta)$$

Pictures of Baryons (rest frame)

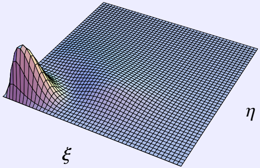
N GBE CQM



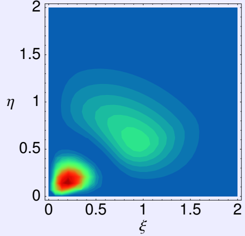
N



N(1440) GBE CQM



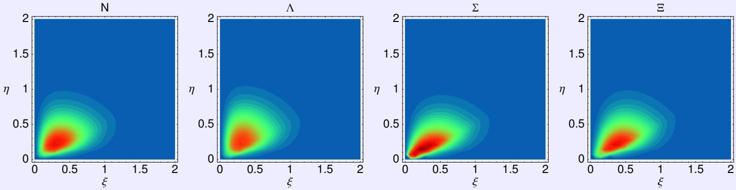
N(1440)



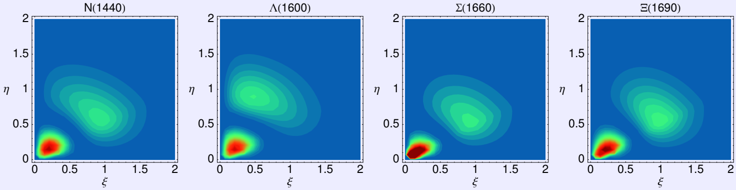
T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

Spatial Probability Density Distributions

$\rho(\xi, \eta)$ for the $\frac{1}{2}^+$ **octet** baryon ground states $N(939)$, $\Lambda(1116)$, $\Sigma(1193)$, $\Xi(1318)$:



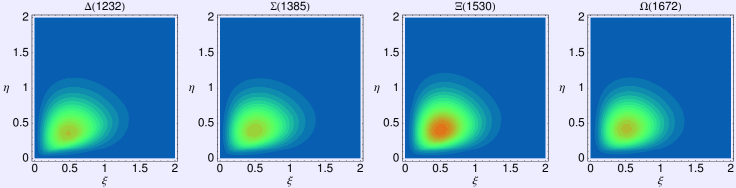
$\rho(\xi, \eta)$ for the $\frac{1}{2}^+$ **octet** baryon states $N(1440)$, $\Lambda(1600)$, $\Sigma(1660)$, $\Xi(1690)$:



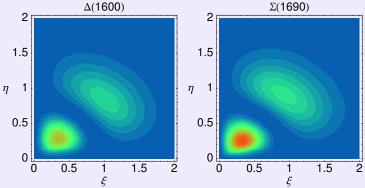
T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

Spatial Probability Density Distributions

$\rho(\xi,\eta)$ for the $\frac{3}{2}^+$ **decuplet** baryon states **Δ(1232), Σ(1385), Ξ(1530), Ω(1672)**:



$\rho(\xi,\eta)$ for the $\frac{3}{2}^+$ **decuplet** baryon states Δ(1600), Σ(1690):



T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

New Quark-Model Classification

multiplet	$(LS)J^P$				
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(939)^{100}$	$\Lambda(1116)^{100}$	$\Sigma(1193)^{100}$	$\Xi(1318)^{100}$
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(1440)^{100}$	$\Lambda(1600)^{96}$	$\Sigma(1660)^{100}$	$\Xi(1690)^{100}$
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(1710)^{100}$		$\Sigma(1880)^{99}$	
octet	$(1\frac{1}{2})\frac{1}{2}^-$	$N(1535)^{100}$	$\Lambda(1670)^{72}$	$\Sigma(1560)^{94}$	
octet	$(1\frac{3}{2})\frac{1}{2}^-$	$N(1650)^{100}$	$\Lambda(1800)^{100}$	$\Sigma(1620)^{100}$	
octet	$(1\frac{1}{2})\frac{3}{2}^-$	$N(1520)^{100}$	$\Lambda(1690)^{72}$	$\Sigma(1670)^{94}$	$\Xi(1820)^{97}$
octet	$(1\frac{3}{2})\frac{3}{2}^-$	$N(1700)^{100}$		$\Sigma(1940)^{100}$	
octet	$(1\frac{3}{2})\frac{5}{2}^-$	$N(1675)^{100}$	$\Lambda(1830)^{100}$	$\Sigma(1775)^{100}$	$\Xi(1950)^{100}$
decuplet	$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1232)^{100}$	$\Sigma(1385)^{100}$	$\Xi(1530)^{100}$	$\Omega(1672)^{100}$
decuplet	$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1600)^{100}$	$\Sigma(1690)^{99}$		
decuplet	$(1\frac{1}{2})\frac{1}{2}^-$	$\Delta(1620)^{100}$	$\Sigma(1750)^{94}$		
decuplet	$(1\frac{1}{2})\frac{3}{2}^-$	$\Delta(1700)^{100}$			
singlet	$(1\frac{1}{2})\frac{1}{2}^-$	$\Lambda(1405)^{71}$			
singlet	$(1\frac{1}{2})\frac{3}{2}^-$	$\Lambda(1520)^{71}$			
singlet	$(0\frac{1}{2})\frac{1}{2}^+$	$\Lambda(1810)^{92}$			

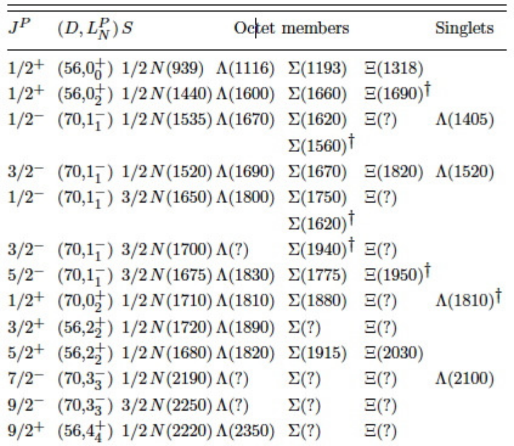
T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77**, 114002 (2008)

See also the PDG: Chin. Phys. C **38**, 090001 (2014)

$SU(3)$ Flavor Multiplets – New

Classification of baryon resonances by the PDG since **2010**

(results from the GBE relativistic CQM marked by asterisks)



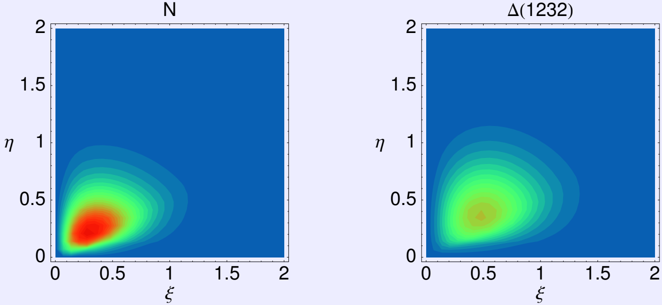
J^P	(D,L^P_N)	S	Octet members				Singlets
$1/2^+$	$(56,0^+_0)$	$1/2$	$N(939)$	$\Lambda(1116)$	$\Sigma(1193)$	$\Xi(1318)$	
$1/2^+$	$(56,0^+_2)$	$1/2$	$N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$	$\Xi(1690)^\dagger$	
$1/2^-$	$(70,1^-_1)$	$1/2$	$N(1535)$	$\Lambda(1670)$	$\Sigma(1620)$ $\Sigma(1560)^\dagger$	$\Xi(?)$	$\Lambda(1405)$
$3/2^-$	$(70,1^-_1)$	$1/2$	$N(1520)$	$\Lambda(1690)$	$\Sigma(1670)$	$\Xi(1820)$	$\Lambda(1520)$
$1/2^-$	$(70,1^-_1)$	$3/2$	$N(1650)$	$\Lambda(1800)$	$\Sigma(1750)$ $\Sigma(1620)^\dagger$	$\Xi(?)$	
$3/2^-$	$(70,1^-_1)$	$3/2$	$N(1700)$	$\Lambda(?)$	$\Sigma(1940)^\dagger$	$\Xi(?)$	
$5/2^-$	$(70,1^-_1)$	$3/2$	$N(1675)$	$\Lambda(1830)$	$\Sigma(1775)$	$\Xi(1950)^\dagger$	
$1/2^+$	$(70,0^+_2)$	$1/2$	$N(1710)$	$\Lambda(1810)$	$\Sigma(1880)$	$\Xi(?)$	$\Lambda(1810)^\dagger$
$3/2^+$	$(56,2^+_2)$	$1/2$	$N(1720)$	$\Lambda(1890)$	$\Sigma(?)$	$\Xi(?)$	
$5/2^+$	$(56,2^+_2)$	$1/2$	$N(1680)$	$\Lambda(1820)$	$\Sigma(1915)$	$\Xi(2030)$	
$7/2^-$	$(70,3^-_3)$	$1/2$	$N(2190)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	$\Lambda(2100)$
$9/2^-$	$(70,3^-_3)$	$3/2$	$N(2250)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	
$9/2^+$	$(56,4^+_4)$	$1/2$	$N(2220)$	$\Lambda(2350)$	$\Sigma(?)$	$\Xi(?)$	

PDG: J. Phys. G **37**, 075021 (2010); Phys. Rev. D **86**, 010001 (2012);
Chin. Phys. C **38**, 090001 (2014)

N and Δ Rest-Frame Wave Functions

Rest-frame **spatial distribution of constituent quarks** in terms of 3-body Jacobi coordinates $\vec{\xi}$ and $\vec{\eta}$:

$$\rho(\xi,\eta) = \xi^2\eta^2 \int d\Omega_\xi d\Omega_\eta \, \Psi^\star_{PJ\Sigma TM_T}(\xi,\Omega_\xi,\eta,\Omega_\eta)\Psi_{PJ\Sigma TM_T}(\xi,\Omega_\xi,\eta,\Omega_\eta)$$



Units on abscissa and ordinates are [*fm*]

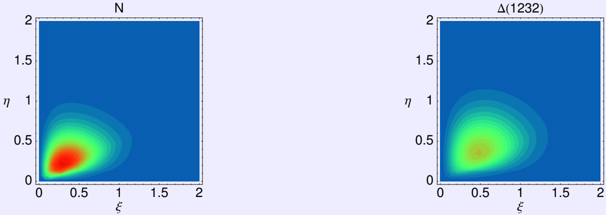
Root-Mean-Square Radii

The **root-mean-square radius** (in the rest frame):

$$r_{\mathrm{rms}} = \sqrt{\langle r_i^2 \rangle} = \left(\int d^3 r_i \left\langle P = 0, J, \Sigma \left| \hat{r}_i^2 \right| P = 0, J, \Sigma \right\rangle \right)^{\frac{1}{2}}$$

Is NOT an **observable**! Is NOT **relativistically invariant**!

→ Idea about the spatial distribution of constituent quarks.



r_{rms}^{N}=0.304 fm

r_E^p=0.905 fm, $(r_E^n)^2$=−0.128 fm^2

$r_{\mathrm{rms}}^{\Delta}$=0.390 fm

$r_E^{\Delta^{++}}$=$r_E^{\Delta^{+}}$=$r_E^{\Delta^{-}}$=0.656 fm, $r_E^{\Delta^{0}}$=0 fm

See: K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Baryon Reactions

Applications of the RCQM

RCQM studies of various baryon reactions:

- Nucleon electromagnetic form factors (including flavor content of the nucleons)
- Nucleon axial form factors
- Δ and hyperon electroweak structures
- Nucleon gravitational form factors
- $NN\pi$ and $N\Delta\pi$ strong vertex form factors

Various Baryon Reactions

Matrix elements of a transition operator $\hat{O}$ between baryon eigenstates $|P, J, \Sigma, T, T_3, Y\rangle$

$$\langle P', J', \Sigma', T', T'_3, Y'|\hat{O}|P, J, \Sigma, T, T_3, Y\rangle$$

$\hat{O}$ ... $\hat{J}^{\mu}_{\rm em}$ $\rightarrow$ electromagnetic FF's

... $\hat{A}^{\mu}_{\rm axial}$ $\rightarrow$ axial FF's

... $\hat{S}$ $\rightarrow$ scalar FF

... $\hat{\Theta}^{\mu\nu}$ $\rightarrow$ gravitational/tensor FF's

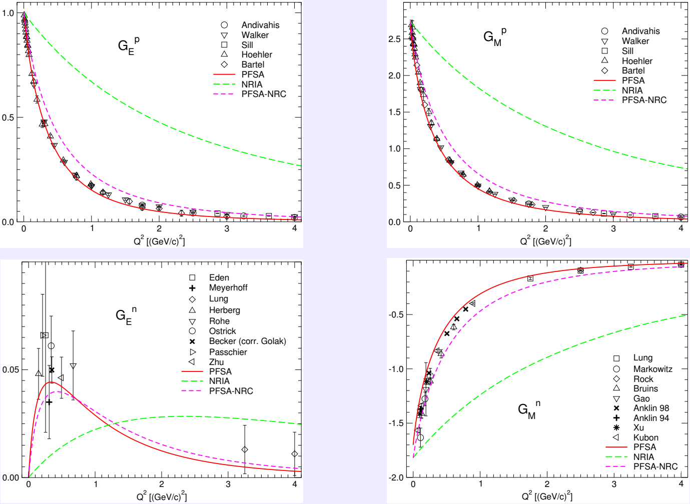
... $\hat{D}^{\mu}_{\lambda}$ $\rightarrow$ hadronic decays

To be calculated from microscopic three-quark ME's

$$\langle p'_1, p'_2, p'_3; \sigma'_1, \underset{\substack{\Uparrow \\ \text{boosted 3-body states}}}{\sigma'_2}, \sigma'_3; f_{i'_1}, f_{i'_2}, f_{i'_3}|\hat{O}|p_1, p_2, p_3; \sigma_1, \underset{\substack{\Uparrow \\ \text{boosted 3-body states}}}{\sigma_2}, \sigma_3; f_{i_1}, f_{i_2}, f_{i_3}\rangle$$

Electromagnetic Nucleon Form Factors

Covariant predictions of the GBE CQM:



R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici: Phys. Lett. **B511** (2001) 33

Nucleon Electric Radii and Magnetic Moments

Electric radii r_E^2 [fm^2]

Baryon	**GBE PFSM**	Experiment
p	0.82	0.7692 ± 0.0123 1) 0.70870 ± 0.00113 2)
n	-0.13	-0.1161 ± 0.0022

1) CODATA value (PDG)

2) Pohl et al.: Nature **466** (2010) 213

Magnetic moments μ [n.m.]

Baryon	**GBE PFSM**	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Nucleon r_E^2 and μ – Nonrelativistic !!!

Electric radii r_E^2 [fm^2]

Baryon	**GBE PFSM**	GBE NRIA	Experiment
p	0.82	0.10	$0.7692 \pm 0.0123^{1)}$ $0.70870 \pm 0.00113^{2)}$
n	-0.13	-0.01	-0.1161 ± 0.0022

1) CODATA value (PDG)

2) Pohl et al.: Nature **466** (2010) 213

Magnetic moments μ [n.m.]

Baryon	**GBE PFSM**	GBE NRIA	Experiment
p	2.70	2.74	2.792847356
n	-1.70	-1.82	-1.9130427

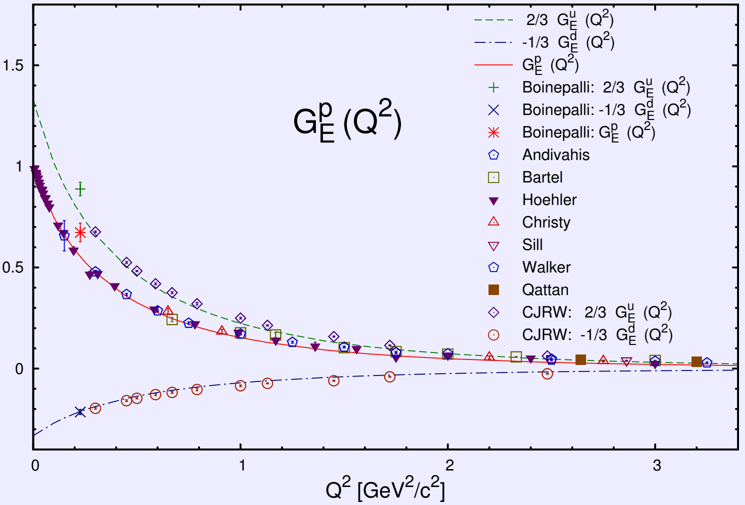
K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Flavor Analysis of Nucleon E.m. FFs

Nucleons N

Proton Electric Form Factor

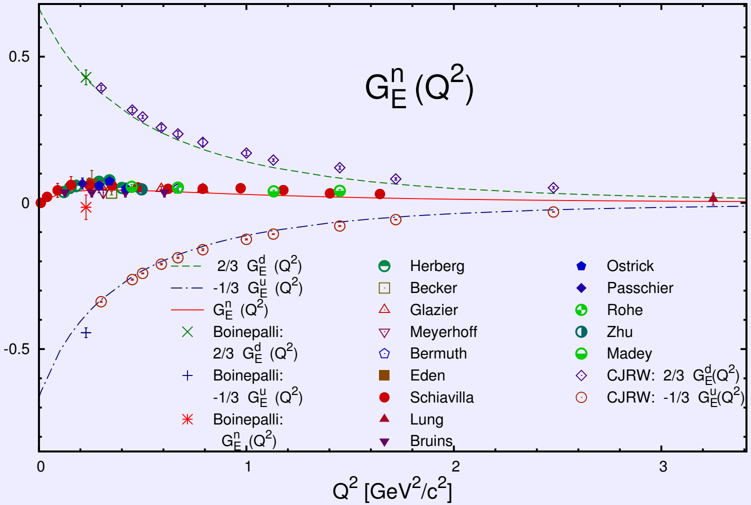
$$G_E^p = \frac{2}{3} G_E^u - \frac{1}{3} G_E^d$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Neutron Electric Form Factor

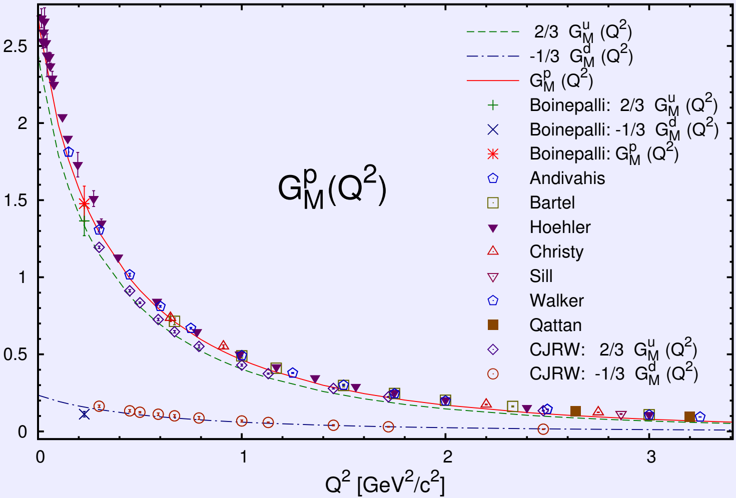
$$G_E^n = \frac{2}{3} G_E^d - \frac{1}{3} G_E^u$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Proton Magnetic Form Factor

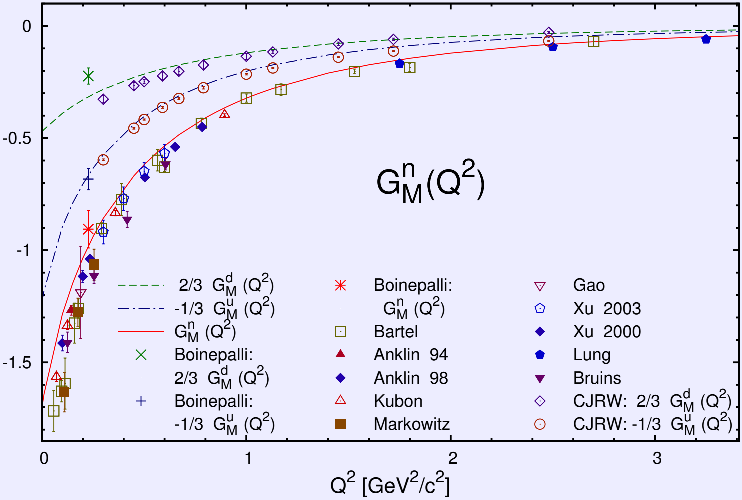
$$G_M^p = \frac{2}{3} G_M^u - \frac{1}{3} G_M^d$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Neutron Magnetic Form Factor

$$G_M^n = \frac{2}{3} G_M^d - \frac{1}{3} G_M^u$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

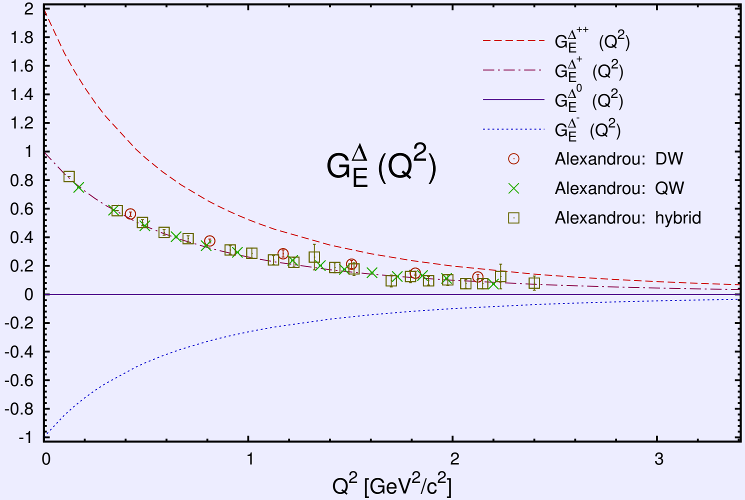
Δ and Hyperon E.m. Form Factors

$$\Delta$$

$$\Lambda, \Sigma, \Xi$$

$$\Sigma^*, \Xi^*, \Omega$$

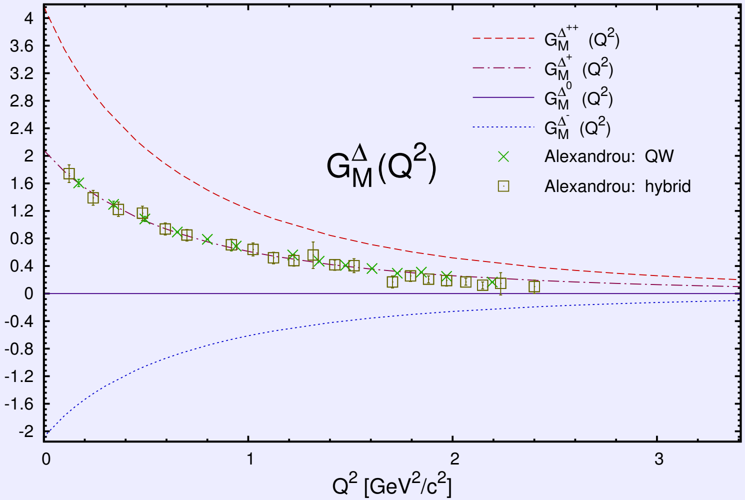
Electric Δ Form Factors



GBE RCQM: Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

Lattice QCD: C. Alexandrou et al. Phys. Rev. D **79** (2009) 014507

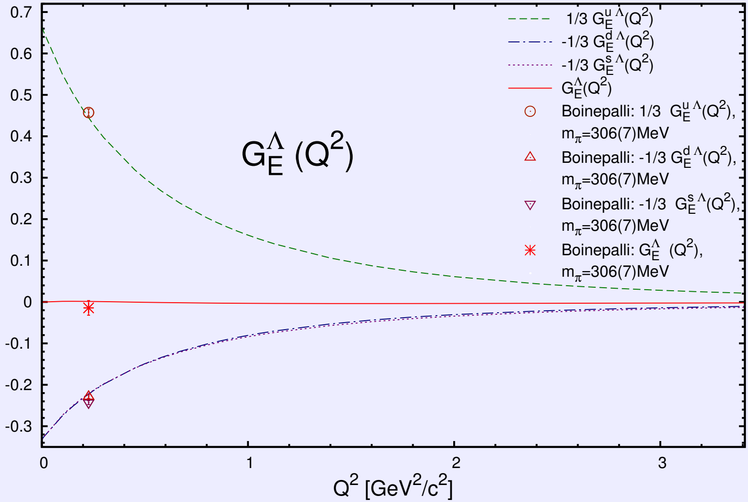
Magnetic Δ Form Factors



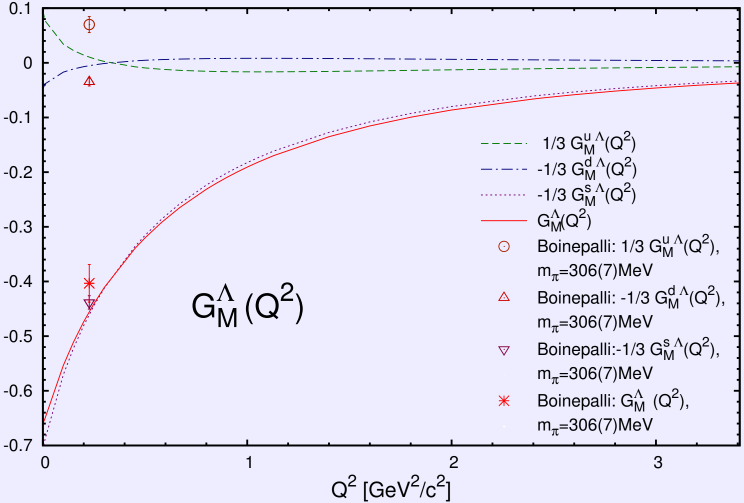
<u>GBE RCQM:</u> Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

<u>Lattice QCD:</u> C. Alexandrou et al. Phys. Rev. D **79** (2009) 014507

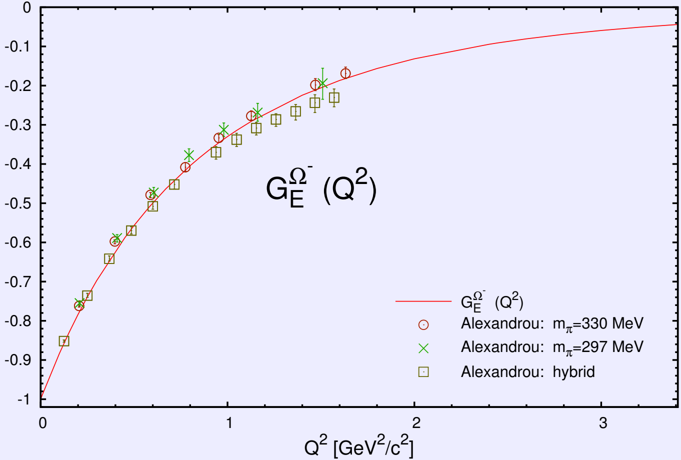
Octet $\Lambda(uds)$ Electric Form Factor



Octet $\Lambda(uds)$ Magnetic Form Factor

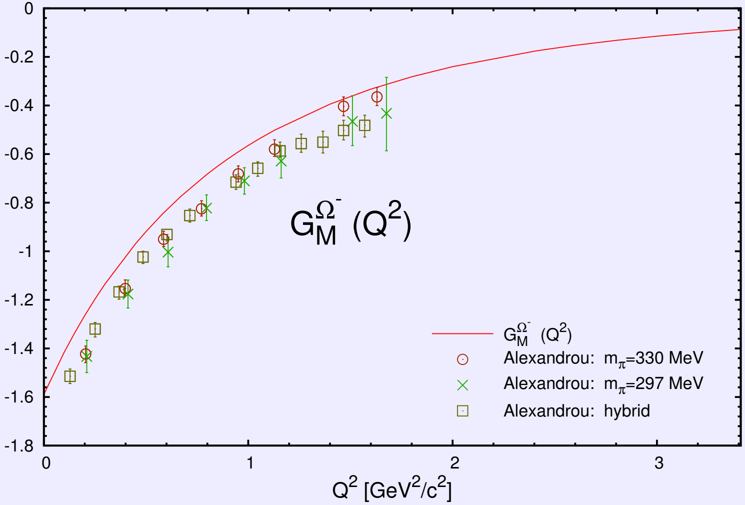


Decuplet $\Omega^-(sss)$ Electric Form Factor



Lattice-QCD: C. Alexandrou et al.: Phys. Rev. D**82** (2010) 034504

Decuplet $\Omega^-(sss)$ Magnetic Form Factor

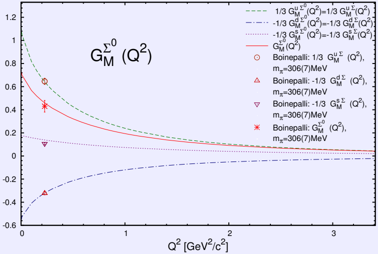


Lattice-QCD: C. Alexandrou et al.: Phys. Rev. D**82** (2010) 034504

Octet $\Sigma^0(uds)$ vs. Decuplet $\Sigma^{*0}(uds)$

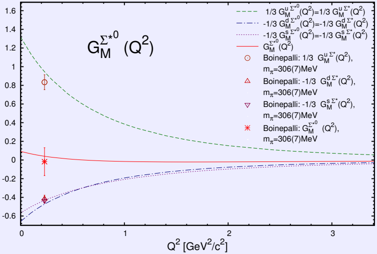
Octet

$$G_M^{\Sigma^0} = \frac{1}{3}G_M^{u,\Sigma} - \frac{1}{3}G_M^{d,\Sigma} - \frac{1}{3}G_M^{s,\Sigma}$$



Decuplet

$$G_M^{\Sigma^{*0}} = \frac{1}{3}G_M^{u,\Sigma^*} - \frac{1}{3}G_M^{d,\Sigma^*} - \frac{1}{3}G_M^{s,\Sigma^*}$$



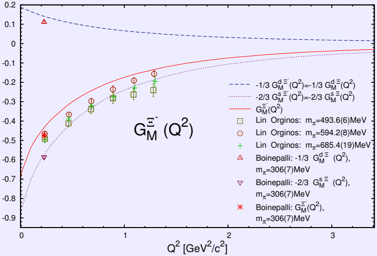
Lattice-QCD: S. Boinepalli et al.: Phys. Rev. D **74**, 093005 (2006)

S. Boinepalli et al.: Phys. Rev. D **80**, 054505 (2009)

Octet $\Xi^-(dss)$ vs. Decuplet Octet $\Xi^{*-}(dss)$

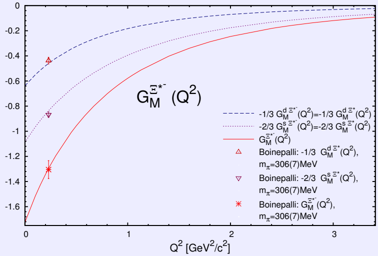
Octet

$$G_M^{\Xi^-} = -\frac{1}{3} G_M^{d,\Xi} - \frac{2}{3} G_M^{s,\Xi}$$



Decuplet

$$G_M^{\Xi^{*-}} = -\frac{1}{3} G_M^{d,\Xi^*} - \frac{2}{3} G_M^{s,\Xi^*}$$



Lattice-QCD: S. Boinepalli et al.: Phys. Rev. D **74**, 093005 (2006)

S. Boinepalli et al.: Phys. Rev. D **80**, 054505 (2009)

Baryon Electric Radii and Magnetic Moments

Electric radii r_E^2 [fm^2]

Baryon	**GBE PFSM**	Experiment
p	0.82	0.7692 ± 0.0123
n	-0.13	-0.1161 ± 0.0022
Σ^-	0.72	$0.61 \pm 0.12 \pm 0.09$

Magnetic moments μ [n.m.]

Baryon	**GBE PFSM**	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427
Λ	-0.64	-0.613 ± 0.004
Σ^+	2.38	2.458 ± 0.010
Σ^-	-0.93	-1.160 ± 0.025
Ξ^0	-1.25	-1.250 ± 0.014
Ξ^-	-0.70	-0.6507 ± 0.0025
Δ^+	2.08	$2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3$
Δ^{++}	4.17	$3.7 - 7.5$
Ω^-	-1.59	-2.020 ± 0.05

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Axial Charges and Axial Form Factors

Axial **Charges** and Axial **Form Factors**

of

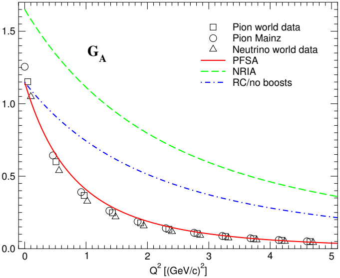
N Ground State and N^* Resonances

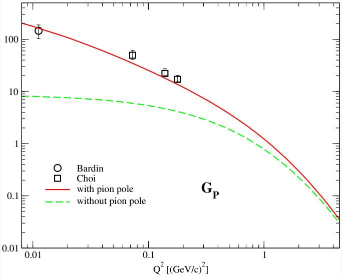
as well as

$\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$

Axial Nucleon Form Factors

Covariant predictions of the GBE RCQM:





$$g_A^{GBE} = 1.15 \qquad \text{vs.} \qquad g_A^{exp} = 1.2695 \pm 0.0029$$

L.Ya. Glozman, M. Radici, R.F. Wagenbrunn, S. Boffi, W. Klink, and W. Plessas: Phys. Lett. B **516**, 183 (2001)

Axial Charges of N and N^* Resonances

State	J^P	EGBE	Lattice QCD	GN	NR
N(939)	$\frac{1}{2}^+$	1.15	1.23∼1.26	1.66	1.65
N(1440)	$\frac{1}{2}^+$	1.16	?	1.66	1.61
N(1535)	$\frac{1}{2}^-$	0.02	∼0.00	-0.11	-0.20
N(1710)	$\frac{1}{2}^+$	0.35	?	0.33	0.42
N(1650)	$\frac{1}{2}^-$	0.51	∼0.55	0.55	0.64

EGBE	**E**xtended **GBE** RCQM covariant result
Lattice	**Lattice QCD** calculations by LHPC Collaboration and Takahashi-Kunihiro (Kyoto)
GN	**G**lozman-**N**efediev $SU(6) \times O(3)$ nonrelativistic QM
NR	**N**on-**R**elativistic EGBE result

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. C **81**, 028201 (2010)

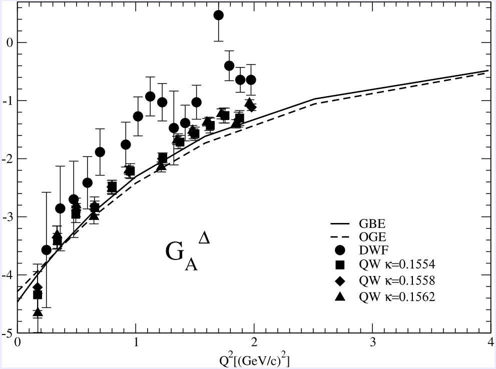
Axial Charges of N and N^* Resonances

		EGBE		psGBE		OGE	
State	J^P	Mass	g_A	Mass	g_A	Mass	g_A
N(939)	$\frac{1}{2}^+$	939	1.15	939	1.15	939	1.11
N(1520)	$\frac{3}{2}^-$	1524	-0.64	1519	-0.21	1520	-0.15
N(1440)	$\frac{1}{2}^+$	1464	1.16	1459	1.13	1578	1.10
N(1535)	$\frac{1}{2}^-$	1498	0.02	1519	0.09	1520	0.13
N(1680)	$\frac{5}{2}^+$	1689	0.89	1728	0.83	1858	0.70
N(1675)	$\frac{5}{2}^-$	1676	0.84	1647	0.83	1690	0.80
N(1710)	$\frac{1}{2}^+$	1757	0.35	1776	0.37	1860	0.32
N(1650)	$\frac{1}{2}^-$	1581	0.51	1647	0.46	1690	0.44
N(1720)	$\frac{3}{2}^+$	1746	0.35	1728	0.34	1858	0.25
N(1700)	$\frac{3}{2}^-$	1608	-0.10	1647	-0.50	1690	-0.47

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. C **81**, 028201 (2010)

Axial Form Factor of the Δ

Covariant predictions of the GBE and OGE RCQMs:



Ki-Seok Choi: PhD Thesis, Graz, 2011

(Lattice QCD data from C. Alexandrou et al., PoS LATTICE2010, 141 (2010))

Axial Charges of $\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$

	J^P	Exp	EGBE	LO	EOT	JT	NR
N	$\frac{1}{2}^+$	1.2695	1.15	1.18	1.314	1.18	1.65
Σ	$\frac{1}{2}^+$	-	0.65	0.636	0.686	0.73	0.93
Ξ	$\frac{1}{2}^+$	-	-0.21	-0.277	-0.299	-0.23	-0.32
Δ	$\frac{3}{2}^+$	-	-4.48	-	-	$\sim$-4.5	-6.00
Σ^*	$\frac{3}{2}^+$	-	-1.06	-	-	-	-1.41
Ξ^*	$\frac{3}{2}^+$	-	-0.75	-	-	-	-1.00

EGBE	**E**xtended **GBE** RCQM covariant result
LO	**L**in and **O**rginos lattice-QCD calculation
EOT	**E**rkol, **O**ka, and **T**akahashi lattice-QCD calculation
JT	**J**iang and **T**iburzi χPT calculation
NR	**N**on-**R**elativistic EGBE result

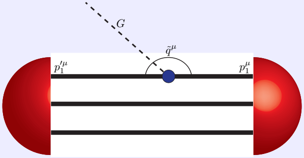
K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. D **82**, 014007 (2010)

Gravitational Form Factors

of

the Nucleon

Gravitational Form Factors

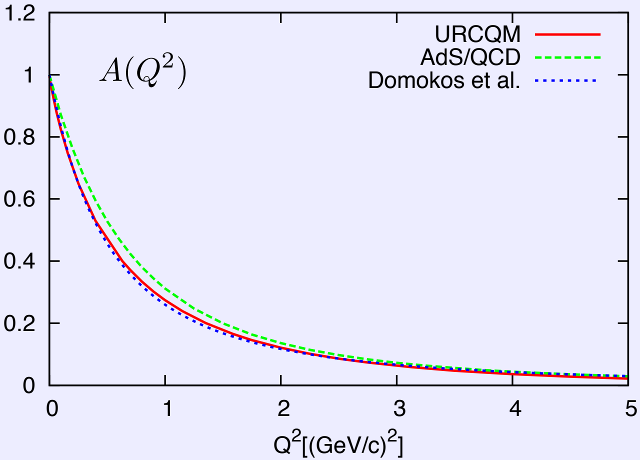


Invariant ME of **energy-momentum tensor** $\hat{\Theta}^{\mu\nu}$:

$$\langle P'J\Sigma'|\hat{\Theta}^{\mu\nu}|PJ\Sigma\rangle = \bar{U}(P')\left[\gamma^{(\mu}\bar{P}^{\nu)}A(Q^2) + \frac{i}{2M}\bar{P}^{(\mu}\sigma^{\nu)}B(Q^2) + \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M}C(Q^2)\right]U(P)$$

$$A(Q^2) \sim \langle P'J\Sigma'|\Theta^{00}|PJ\Sigma\rangle$$

Nucleon Gravitational Form Factor $A(Q^2)$



J.P. Day: PhD Thesis, Univ. Graz, August 2013

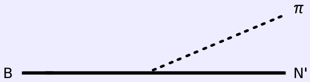
Microscopic Description

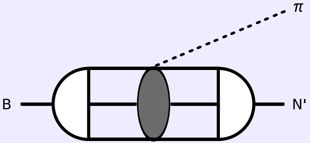
of

Meson-Baryon Interaction Vertices

Meson-Baryon Interaction Vertices

Interaction vertices





$$F_{i\to f}=(2\pi)^4\left\langle f\right|\mathcal{L}_I(0)\left|i\right\rangle \qquad \equiv \qquad \left\langle V',M',J',\Sigma'\right|\hat{D}^{\pi}_{\rm rd}\left|V,M,J,\Sigma\right\rangle$$

where

$$\left\langle p_1',p_2',p_3';\sigma_1',\sigma_2',\sigma_3'\left|\hat{D}^{\pi}_{\rm rd}\right|p_1,p_2,p_3;\sigma_1,\sigma_2,\sigma_3\right\rangle =$$

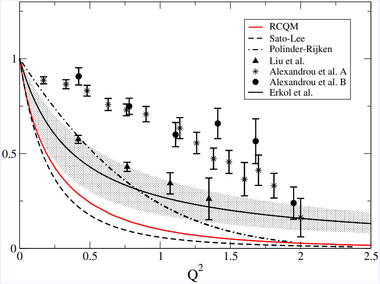
$$3\mathcal{N}_{\rm S}\frac{ig_{qqm}}{2m_1\left(2\pi\right)^{\frac{3}{2}}}\bar{u}(p_1',\sigma_1')\gamma_5\gamma_\mu\lambda_m u(p_1,\sigma_1)\tilde{q}^\mu 2p_{20}\delta(\vec{p}_2-\vec{p}_2')2p_{30}\delta(\vec{p}_3-\vec{p}_3')\delta_{\sigma_2\sigma_2'}\delta_{\sigma_3\sigma_3'}$$

and

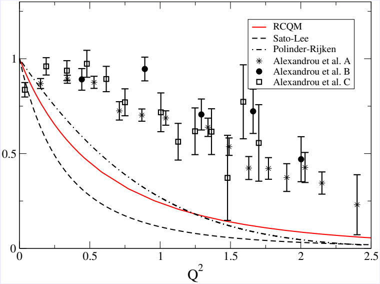
$$G_{\pi NN}\left(Q^2\right) = \frac{1}{f_{\pi NN}}\frac{m_\pi\sqrt{2\pi}}{\sqrt{2M_N}}\frac{\sqrt{E_N'+M_N'}}{E_N'+M_N'+\omega}\frac{F_{i\to f}}{Q_z}$$

$$G_{\pi N\Delta}\left(Q^2\right) = -\frac{1}{f_{\pi N\Delta}}\frac{3\sqrt{2\pi}}{2}\frac{m_\pi}{\sqrt{E_N'+M_N'}\sqrt{2M_\Delta}}\frac{F_{i\to f}}{Q_z}$$

πNN and $\pi N\Delta$ Interaction Vertices



$G_{\pi NN}$



$G_{\pi N\Delta}$

T. Melde, L. Canton, and W. Plessas: Phys. Rev. Lett. **102**, 132002 (2009)

Form-Factor Parametrizations

$$G\left(\vec{q}^{\,2}\right) = \frac{1}{1+\left(\frac{\vec{q}}{\Lambda_1}\right)^2+\left(\frac{\vec{q}}{\Lambda_2}\right)^4} \qquad G\left(Q^2\right) = \frac{1}{1+\left(\frac{Q}{\Lambda}\right)^2}$$

		RCQM	SL	PR		LIU	ERK	ALX
N	$\frac{f_N^2}{4\pi}$	0.0691	0.08	0.075	Λ	0.0649	0.0481	0.0412
	Λ_1	0.451	0.453	0.940		0.747	0.614	1.65
	Λ_2	0.931	0.641	1.102		-	-	-
Δ	$\frac{f_\Delta^2}{4\pi}$	0.188	0.334	0.478				
	Λ_1	0.594	0.458	0.853				
	Λ_2	0.998	0.648	1.014				

T. Melde, L. Canton, and W. Plessas: Phys. Rev. Lett. **102**, 132002 (2009)

Summary and Conclusions

- **Remarkable success** of the CQM – but **relativistic**!
- The **non-relativistic** quark model **does not work** in any instance!
- Surprisingly **good agreement** of predictions by the GBE RCQM with experimental data (wherever such data are available).
- **Small deviations** left in some observables, such as electric radii and magnetic moments.
- Surprisingly **good agreement** of predictions by the GBE RCQM with lattice-QCD results.
- The **valence-quark picture** provides a universal description of all known baryons (with all flavors) at low energies.

Conclusions and Outlook

Lesson:

- Most important symmetries to be included (in the GBE RCQM):
    - **SBχS**
    - **Lorentz invariance**
    - **time-reversal invariance**
    - **current conservation**

Open issues:

- The generation of **dynamical constituent-quark masses** is not yet definitely understood.
- Not discussed here: **Strong resonance decays** are not quantitatively described in the present approach. A **coupled-channels theory** with explicit coupling to decay channels appears to be necessary.

Collaborators

Graz

K. Berger, J.P. Day, K.-S. Choi, L. Glozman, T. Melde, M. Rohrmoser, R.C. Schardmüller, B. Sengl, R.F. Wagenbrunn

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(INFN, Sezione di Pavia)

Padova

L. Canton

(INFN, Sezione di Padova)

Iowa City

W. Klink

(Department of Physics, University of Iowa, USA

Thank you very much

for

your attention!