RG Effects in DM Direct Detection

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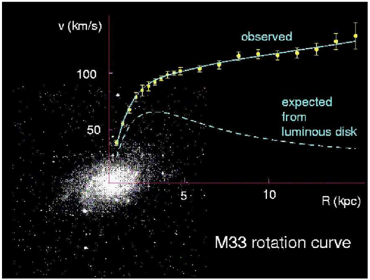
Workshop "The Standard Theory and Beyond"
Albufeira, October 30, 2015

With Fady Bishara, Benjamin Grinstein, Jure Zupan – work in progress

Introduction

Dark Matter Facts

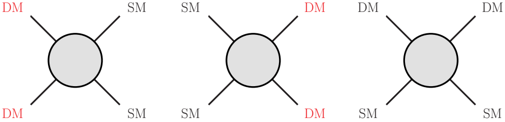
- DM exists
  - All evidence via its gravitation
- Particle nature?
- What we know about DM
  - DM is cold and neutral
  - Relic abundance $\Omega_{\mathrm{DM}} h^2 = 0.1198(26)$
    [PLANCK / PDG 2014]



- Thermal history motivates interaction with SM

Direct Detection Basics

- Three roads to discovery:
  - Indirect detection
  - Production at colliders
  - Direct detection

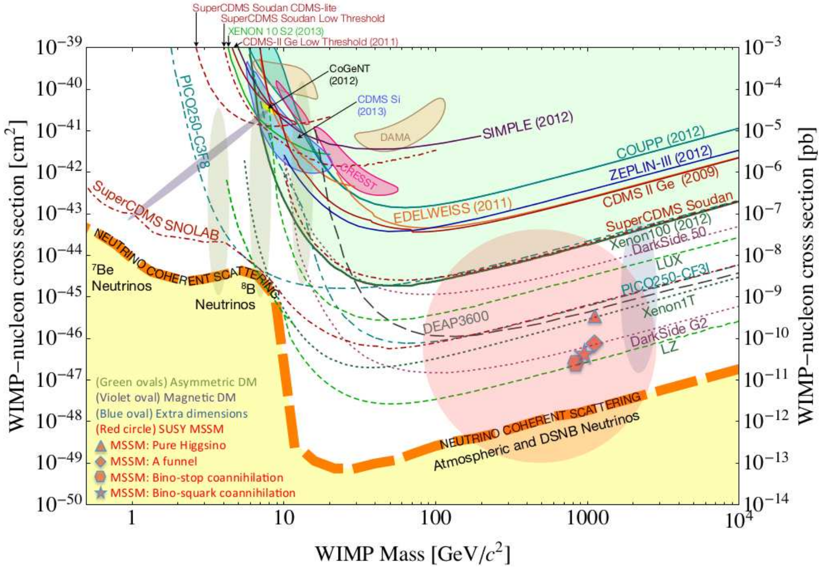


- Focus on direct detection (scattering on nuclei)
  - Maximal momentum transfer is $\lesssim 200\,\text{MeV}$
  - Complementary information, proves cosmological lifetime
  - Assume velocity distribution (Maxwell); $v \sim 10^{-3}$

$$\frac{dR}{dE_R} = \frac{\rho_0}{m_A m_\chi} \int_{v_{min}} dv\, v\, f_1(v) \frac{d\sigma}{dE_R}(v, E_R)\,.$$

[Lewin & Smith, Astropart.Phys.6 (1996)]

Direct Detection Limits



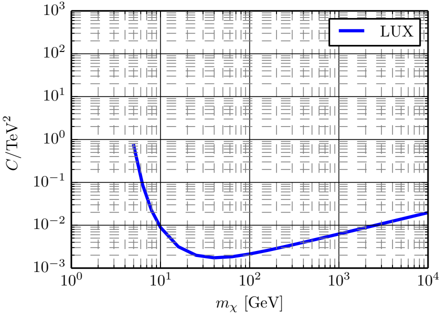
Interactions with Nuclei

- Traditionally "spin independent" (SI) and "spin dependent" (SD)
- However [see Freytsis & Ligeti, 1012.5317]

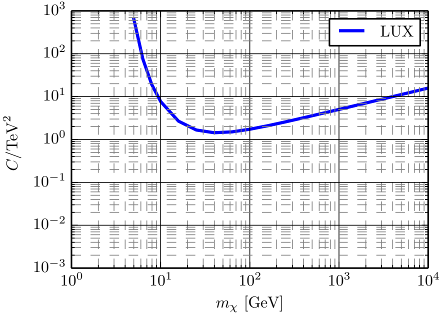
Operator	SI / SD	Suppression
$(\bar{\chi}\chi)\,(\bar{q}q)$	SI	–
$(\bar{\chi}i\gamma_5\chi)\,(\bar{q}q)$	SI	q^2
$(\bar{\chi}\chi)\,(\bar{q}i\gamma_5 q)$	SD	q^2
$(\bar{\chi}\gamma_5\chi)\,(\bar{q}\gamma_5 q)$	SD	q^4
$(\bar{\chi}\gamma_\mu\chi)\,(\bar{q}\gamma^\mu q)$	SI	–
$(\bar{\chi}\gamma_\mu\gamma_5\chi)\,(\bar{q}\gamma^\mu q)$	SI	v^2
$(\bar{\chi}\gamma_\mu\chi)\,(\bar{q}\gamma^\mu\gamma_5 q)$	SD	v^2 or q^2
$(\bar{\chi}\gamma_\mu\gamma_5\chi)\,(\bar{q}\gamma^\mu\gamma_5 q)$	SD	–

Velocity Suppression – $v \sim 10^{-3}$

- $(\bar{\chi}\gamma_\mu\chi)(\bar{Q}_L^i\gamma^\mu Q_L^i)$



- $(\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{Q}_L^i\gamma^\mu Q_L^i)$



[Data from LUX collaboration, 1310.8214]

Interactions with Nuclei

- Momentum-dependent interactions are leading in many UV models
  - Majorana DM interacting with dark photon
  - Composite DM with constitutents charged under ew gauge force
- Can write a full (NR, Galilean-invariant) EFT at the nuclear scale
  - Angular-momentum dependent coupling
  - Spin and angular-momentum dependent coupling
- Nuclear matrix elements quadratic in p^2
  [Fitzpatrick et al. 1203.3542]
- Consistently, should take into account NLO chiral Lagrangian
  [Cirigliano, Graesser, Ovanesyan 1205.2695 – scalar operators]

Why RG Effects?

- Electroweak loops can mix suppressed and unsupressed operators
  [Freytsis & Ligeti, 1012.5317; see also Haisch et al. 1302.4454; Crivellin et al. 1402.1173, 1408.5046; D'Eramo et al. 1409.2893]

- A complete EFT framework for all scales is needed for the consistent interpretation of direct detection data.

The Setup

Interactions with the SM

- Assume DM is an electroweak multiplet χ
  - Isospin J, hypercharge Y
  - One component neutral, χ^0
  - E/W mass splitting: charged components $\chi^\pm$, $\chi^{\pm\pm}$, ... will decay
  - $Y \neq 0$ leads to strong bounds from Z exchange
- Several examples:
  - Neutralinos in the MSSM (bino, higgsino, wino)
  - Minimal Dark Matter [Cirelli et al. hep-ph/0512090, ...]
  - "Technibaryons" [Nussinov, Phys.Lett. B165 (1985) 55, ...]
- Potential mediators ϕ, assume $\Lambda \approx m_\phi \gg m_\chi$

UV Effective Field Theory

- Three possibilities for DM mass:
  - $(m_\chi \gg v_{\text{ew}})$ [Work in progress; see also Hill et al., 1111.0016, 1401.3339, 1409.8290]
  - $m_\chi \sim v_{\text{ew}}$
  - $m_\chi \ll v_{\text{ew}}$
- Assume that mediators are integrated out at $\mu \sim \Lambda$
  - Dim.4 gauge interactions
  - Higher-dimensional effective operators
- Here: Assume DM is a Dirac fermion

Relevant scales

- Mediators: integrated out at $\mu \sim \Lambda \gg v_{\text{ew}}$
- SM: usual tower of EFTs
  - $\mu \gg v_{\text{ew}}$: Unbroken SM
  - $\mu \sim v_{\text{ew}}$: Broken SM
  - $\mu \ll v_{\text{ew}}$: N_f-flavor QCD$\times$QED
  - $\mu \sim \Lambda_{\text{QCD}}$: Chiral Lagrangian (including baryon terms)
  - $\mu \ll \Lambda_{\text{QCD}}$: Non-relativistic interactions with nucleons
- DM fields:
  - $\mu \gg m_\chi$: Relativistic DM
  - $\mu \ll m_\chi$: Use "HQET"-type EFT (as for the b quark)

Above the electroweak scale

Operator Basis

- Construct operators in unbroken e/w phase

$$\mathcal{L}^{\text{eff}} = \mathcal{L}^{\text{SM}} + \mathcal{L}^{\text{DM}} + \sum \frac{C_j^{(5)}}{\Lambda} Q_j^{(5)} + \sum \frac{C_j^{(6)}}{\Lambda^2} Q_j^{(5)} + \dots$$

- Extension of "SM-EFT" [Buchmüller et al. 1986, Grzadkowski et al. 2010]

Operator Basis – Dimension Five

$$Q_1^{(5)} = \frac{g_1}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}\chi)B_{\mu\nu}\,, \qquad Q_2^{(5)} = \frac{g_2}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}\tilde{\tau}^a\chi)W^a_{\mu\nu}\,,$$

$$Q_3^{(5)} = (\bar{\chi}\chi)(H^\dagger H)\,, \qquad Q_4^{(5)} = (\bar{\chi}\tilde{\tau}^a\chi)(H^\dagger\tau^a H)\,,$$

$$Q_5^{(5)} = \frac{g_1}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}\gamma_5\chi)B_{\mu\nu}\,, \qquad Q_6^{(5)} = \frac{g_2}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}\tilde{\tau}^a\gamma_5\chi)W^a_{\mu\nu}\,,$$

$$Q_7^{(5)} = i(\bar{\chi}\gamma_5\chi)(H^\dagger H)\,, \qquad Q_8^{(5)} = i(\bar{\chi}\tilde{\tau}^a\gamma_5\chi)(H^\dagger\tau^a H)\,.$$

Operator Basis – Dimension Six

$$Q_{1,i}^{(6)} = (\bar{\chi}\gamma_\mu\tilde{\tau}^a\chi)(\bar{Q}_L^i\gamma^\mu\tau^a Q_L^i), \qquad Q_{5,i}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\tilde{\tau}^a\chi)(\bar{Q}_L^i\gamma^\mu\tau^a Q_L^i).$$

$$Q_{2,i}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{Q}_L^i\gamma^\mu Q_L^i), \qquad Q_{6,i}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{Q}_L^i\gamma^\mu Q_L^i),$$

$$Q_{3,i}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{u}_R^i\gamma^\mu u_R^i), \qquad Q_{7,i}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{u}_R^i\gamma^\mu u_R^i),$$

$$Q_{4,i}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{d}_R^i\gamma^\mu d_R^i), \qquad Q_{8,i}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{d}_R^i\gamma^\mu d_R^i).$$

$$Q_{9,i}^{(6)} = (\bar{\chi}\gamma_\mu\tilde{\tau}^a\chi)(\bar{L}_L^i\gamma^\mu\tau^a L_L^i), \qquad Q_{12,i}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\tilde{\tau}^a\chi)(\bar{L}_L^i\gamma^\mu\tau^a L_L^i),$$

$$Q_{10,i}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{L}_L^i\gamma^\mu L_L^i), \qquad Q_{13,i}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{L}_L^i\gamma^\mu L_L^i),$$

$$Q_{11,i}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{\ell}_R^i\gamma^\mu \ell_R^i), \qquad Q_{14,i}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{\ell}_R^i\gamma^\mu \ell_R^i).$$

$$Q_{15}^{(6)} = (\bar{\chi}\gamma^\mu\tilde{\tau}^a\chi)(H^\dagger i \overleftrightarrow{D}^a{}_\mu H), \qquad Q_{17}^{(6)} = (\bar{\chi}\gamma^\mu\gamma_5\tilde{\tau}^a\chi)(H^\dagger i \overleftrightarrow{D}^a{}_\mu H),$$

$$Q_{16}^{(6)} = (\bar{\chi}\gamma^\mu\chi)(H^\dagger i \overleftrightarrow{D}_\mu H), \qquad Q_{18}^{(6)} = (\bar{\chi}\gamma^\mu\gamma_5\chi)(H^\dagger i \overleftrightarrow{D}_\mu H).$$

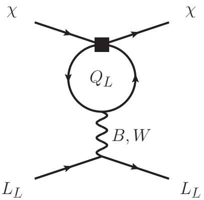
Mixing – General Structure

- RGE (Renormalization Group Equations):

$$\frac{d}{d\log\mu} C(\mu) = \gamma^T C(\mu)$$

- Do we need to sum the logs?
  - $\alpha_1(\mu_{EW}) \approx 0.01$, $\alpha_2(\mu_{EW}) \approx 0.03$, $\alpha_\lambda(\mu_{EW}) \approx 0.04$, $\alpha_t(\mu_{EW}) \approx 0.08$
  - Would need $\Lambda_{NP} \sim 10^4$ TeV

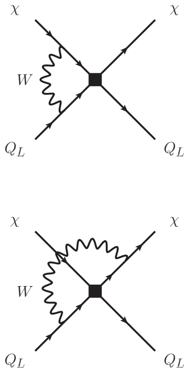
- Importance of RGE:
  - Mixing of suppressed and unsuppressed operators
  - Penguin insertions mix lepton and quark operators

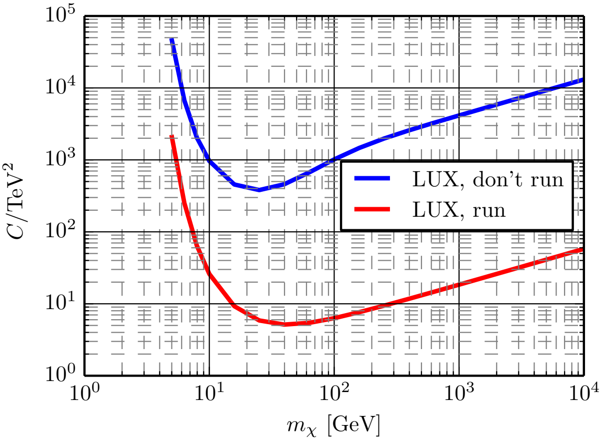


- We calculated the complete mixing of dim.5 and dim.6 operators for fermionic DM in an arbitrary $SU(2)_L \times U(1)_Y$ representation

Mixing Example – W Exchange

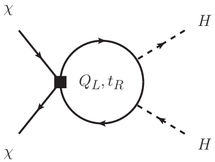
$$(\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{Q}_L^i\gamma^\mu Q_L^i) \qquad \Rightarrow \qquad (\bar{\chi}\gamma_\mu\tilde{\tau}^a\chi)(\bar{Q}_L^i\gamma^\mu\tau^a Q_L^i),$$

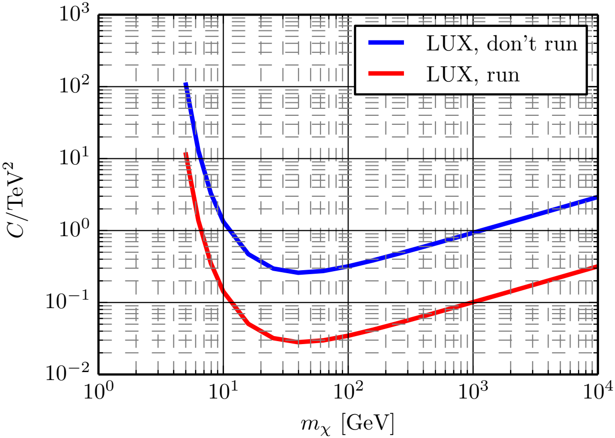




Mixing Example – top Yukawa

$$(\bar{\chi}\gamma_\mu\chi)(\bar{u}^i_R\gamma^\mu u^i_R) - (\bar{\chi}\gamma_\mu\chi)(\bar{Q}^i_L\gamma^\mu Q^i_L) \quad \Rightarrow \quad (\bar{\chi}\gamma^\mu\chi)(H^\dagger i \overleftrightarrow{D}_\mu H)$$





[See also Crivellin et al., 1402.1173]

Below the electroweak scale

N_f-flavor QCD

- Below the electroweak scale, have usual N_f-flavor QCD plus DM field
  - If $m_\chi \ll \mu$, DM is still a relativistic field
  - If $m_\chi \gg \mu$, need to treat DM as non-relativistic ("HχET")

$$\mathcal{L} = \bar{\chi}(iv \cdot D)\chi + \mathcal{O}(1/m_\chi, 1/\Lambda)$$

- Keep operators up to mass dimension seven
- Previously known:
  - Dim.4 in heavy DM limit [Hill et al., 1401.3339]
  - Dim.4 for electroweak-scale DM masses [Cf. Hisano et al., 1104.0228]

Operator Basis

$$\mathcal{Q}_1^{(5)} = \frac{e}{8\pi^2}(\bar\chi\sigma^{\mu\nu}\chi)F_{\mu\nu}\,, \qquad \mathcal{Q}_2^{(5)} = \frac{e}{8\pi^2}(\bar\chi\sigma^{\mu\nu}\chi)\widetilde{F}_{\mu\nu}\,,$$

$$\mathcal{Q}_{1,f}^{(6)} = (\bar\chi\gamma_\mu\chi)(\bar f\gamma^\mu f), \qquad \mathcal{Q}_{2,f}^{(6)} = (\bar\chi\gamma_\mu\gamma_5\chi)(\bar f\gamma^\mu f),$$

$$\mathcal{Q}_{3,f}^{(6)} = (\bar\chi\gamma_\mu\chi)(\bar f\gamma^\mu\gamma_5 f)\,, \qquad \mathcal{Q}_{4,f}^{(6)} = (\bar\chi\gamma_\mu\gamma_5\chi)(\bar f\gamma^\mu\gamma_5 f)\,,$$

$$\mathcal{Q}_{5,f}^{(6)} = (\bar\chi\chi)(\bar f f)\,, \qquad \mathcal{Q}_{6,f}^{(6)} = (\bar\chi i\gamma_5\chi)(\bar f f)\,,$$

$$\mathcal{Q}_{7,f}^{(6)} = (\bar\chi\chi)(\bar f i\gamma_5 f)\,, \qquad \mathcal{Q}_{8,f}^{(6)} = (\bar\chi\gamma_5\chi)(\bar f\gamma_5 f)\,.$$

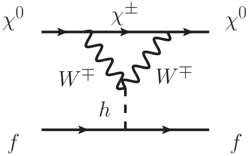
$$\mathcal{Q}_1^{(7)} = \frac{\alpha_s}{12\pi}(\bar\chi\chi)G^{a,\mu\nu}G^a_{\mu\nu}\,, \qquad \mathcal{Q}_2^{(7)} = \frac{\alpha_s}{12\pi}(\bar\chi i\gamma_5\chi)G^{a,\mu\nu}G^a_{\mu\nu}\,,$$

$$\mathcal{Q}_3^{(7)} = \frac{\alpha_s}{8\pi}(\bar\chi\chi)G^{a,\mu\nu}\widetilde{G}^a_{\mu\nu}\,, \qquad \mathcal{Q}_4^{(7)} = \frac{\alpha_s}{8\pi}(\bar\chi i\gamma_5\chi)G^{a,\mu\nu}\widetilde{G}^a_{\mu\nu}\,,$$

Matching – Dim.-4 Gauge Interactions – light DM

$$\mathcal{Q}_{5,f}^{(6)} = (\bar{\chi}\chi)(\bar{f}f)$$

- Consider dim.-4 "Higgs penguin" contribution
  - Recall we have no tree-level Higgs coupling
  - $Y = 0$
  - $m_\chi \ll v_{\rm ew}$: Expand to linear order in m_χ *and momenta*!
  - $\mathcal{Q}_{{\rm eom},f} \equiv [\bar{\chi}(i\partial\!\!\!/ - m_\chi)\chi](\bar{f}f)$ contributes and cancels ξ_W gauge dependence

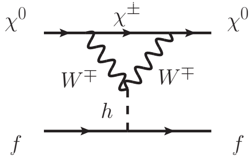


$$\mathcal{C}_{5,f}^{(6)} = -\frac{3}{2}\frac{\alpha^2}{s_w^4}\frac{m_f m_\chi}{M_W^2 M_h^2} J(J+1)$$

Matching – Dim.-4 Gauge Interactions

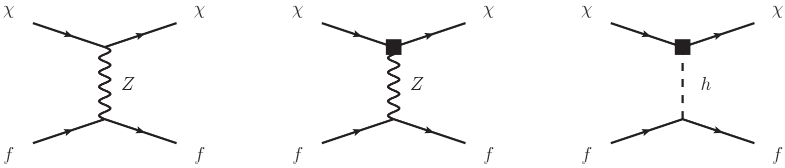
$$\mathcal{Q}_{5,f}^{(6)} = (\bar{\chi}\chi)(\bar{f}f)$$

- $m_\chi \sim v_{\rm ew}$: DM is "HQET" field in EFT
- requires on-shell momentum configuration in full theory



$$\mathcal{C}_{5,f}^{(6)} = \frac{\alpha^2}{s_w^4}\frac{m_f}{M_W M_h^2}J(J+1)\left[\frac{(2x^2-1)\left[5x + 2\sqrt{\frac{1+4x}{x^2}}\log\left(\frac{1}{2x}+\frac{1}{2}\sqrt{\frac{1+4x}{x^2}}\right)\right]}{4x^2-1} + \frac{2\log x}{x}\right]$$

Matching – Higher-Dimensional Operators



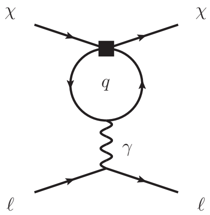
$$\mathcal{C}_{1,u_i}^{(6)}\big|_{n_f=5} = -\frac{Y_\chi}{8} C_{1,i}^{(6)} + \frac{C_{2,i}^{(6)}}{2} + \frac{C_{3,i}^{(6)}}{2} - \frac{\pi\alpha\Lambda^2}{6 s_w^2 c_w^2 m_Z^2} \left(3 - 8 s_w^2\right) \left[\frac{v_{\rm EW}^2}{\Lambda^2} \left(\frac{Y_\chi}{4} C_{15}^{(6)} + C_{16}^{(6)} \right) - Y_\chi \right],$$

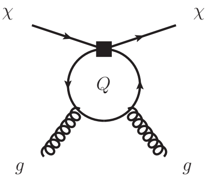
$$\mathcal{C}_{5,f}^{(6)}\big|_{n_f=5} = \frac{m_f \Lambda}{m_h^2} \left(C_3^{(5)} + \frac{Y_\chi}{4} C_4^{(5)} \right), \qquad \mathcal{C}_{6,f}^{(6)}\big|_{n_f=5} = \frac{m_f \Lambda}{m_h^2} \left(C_7^{(5)} + \frac{Y_\chi}{4} C_8^{(5)} \right),$$

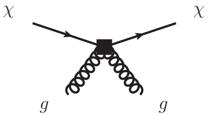
$$\mathcal{C}_{7,f}^{(6)}\big|_{n_f=5} = 0, \qquad \mathcal{C}_{8,f}^{(6)}\big|_{n_f=5} = 0.$$

Running and matching at flavor thresholds

- Below e/w scale it is sufficient to consider QCD$\times$QED singlet DM
- QCD / QED running is well-known [E.g. Hill et al., 1409.8290]
- Penguin insertions will mix lepton and quark operators
- Matching at flavor thresholds







Transition to the nucleon picture

Chiral Lagrangian

- Treat DM operators as $SU(3)_L \times SU(3)_R$ flavor-symmetry spurions

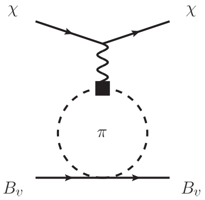
$$\mathcal{L} = \mathcal{L}^0_{\rm QCD} + \frac{s_G(x)}{12\pi} G^a_{\mu\nu} G^{a\mu\nu} - \frac{\theta(x)}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} + \bar{q}(x)\gamma^\mu \big[v_\mu(x) + \gamma_5 a_\mu(x)\big] q(x) - \bar{q}(x)\big[s(x) - i\gamma_5 p(x)\big] q(x),$$

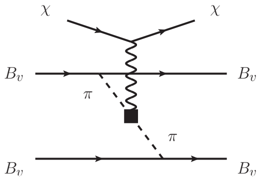
- Construct chiral Lagrangian using $U(x) = \exp\left(i\sqrt{2}\Pi/f_\pi\right)$
  [Gasser and Leutwyler, Nucl.Phys. B250 (1985) 465]
- Include baryon fields (proton, neutron) – $B_v(x) = \exp(i m_B \not{v}_\mu x^\mu) B(x)$
  [Jenkins and Manohar, Phys.Lett. B255 (1991) 558]

$$\Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}, \qquad B_v = \begin{pmatrix} 0 & 0 & p_v \\ 0 & 0 & n_v \\ 0 & 0 & 0 \end{pmatrix}.$$

Nuclear Matrix Elements

- Match to nonrelativistic quantum mechanics
- Calculate matrix elements of nucleons in the nucleus
- Calculation for F, Na, Ge, I, Xe and different form factors available
  [Fitzpatrick et al. 1203.3542]
- At NLO ChPT, need also two-nucleon currents
  [Cirigliano, Graesser, Ovanesyan 1205.2695]





Conclusion and Outlook

- Complete EFT framework is important for consistent interpretation of direct detection data
- We will provide public code for running from UV to Λ_{QCD}
- Many future directions:
  - Scalar and vector DM
  - Non-trivial flavor structure
  - Several multiplets and Higgs interactions
  - Dimension-seven operators in the UV
  - Heavy DM
  - ChPT for non-scalar operators
  - Nuclear matrix elements

Appendix

Spurions

$$v_\mu(x) = \mathrm{diag}\left(\frac{\mathcal{C}^{(6)}_{1,q}|_{n_f=3}}{\Lambda^2} (\bar\chi\gamma_\mu\chi) + \frac{\mathcal{C}^{(6)}_{2,q}|_{n_f=3}}{\Lambda^2} (\bar\chi\gamma_\mu\gamma_5\chi) \right) ,$$

$$a_\mu(x) = \mathrm{diag}\left(\frac{\mathcal{C}^{(6)}_{3,q}|_{n_f=3}}{\Lambda^2} (\bar\chi\gamma_\mu\chi) + \frac{\mathcal{C}^{(6)}_{4,q}|_{n_f=3}}{\Lambda^2} (\bar\chi\gamma_\mu\gamma_5\chi) \right) ,$$

$$s(x) = \mathcal{M}_q + s_\chi = \mathcal{M}_q - \mathrm{diag}\left(\frac{\mathcal{C}^{(6)}_{5,q}|_{n_f=3}}{\Lambda^2} (\bar\chi\chi) + i \frac{\mathcal{C}^{(6)}_{6,q}|_{n_f=3}}{\Lambda^2} (\bar\chi\gamma_5\chi) \right) ,$$

$$p(x) = -\,\mathrm{diag}\left(\frac{\mathcal{C}^{(6)}_{7,q}|_{n_f=3}}{\Lambda^2} (\bar\chi\chi) + i \frac{\mathcal{C}^{(6)}_{8,q}|_{n_f=3}}{\Lambda^2} (\bar\chi\gamma_5\chi) \right) ,$$

$$s_G(x) = \alpha_S\, \mathrm{diag}\left(\frac{\mathcal{C}^{(7)}_{1}|_{n_f=3}}{\Lambda^3} (\bar\chi\chi) + i \frac{\mathcal{C}^{(7)}_{2}|_{n_f=3}}{\Lambda^3} (\bar\chi\gamma_5\chi) \right) ,$$

$$\theta(x) = -4\pi\alpha_S\, \mathrm{diag}\left(\frac{\mathcal{C}^{(7)}_{3}|_{n_f=3}}{\Lambda^3} (\bar\chi\chi) + i \frac{\mathcal{C}^{(7)}_{4}|_{n_f=3}}{\Lambda^3} (\bar\chi\gamma_5\chi) \right) .$$

Chiral Lagrangian

$$\mathcal{L}_{\rm ChPT}^{(2),\rm DM} = - i\frac{f^2}{2}\,\mathsf{Tr}\Big[\big(U\partial_\mu U^\dagger + U^\dagger\partial_\mu U\big)v_\mu + \big(U\partial_\mu U^\dagger - U^\dagger\partial_\mu U\big)a_\mu\Big] + \frac{B_0 f^2}{2}\,\mathsf{Tr}\Big[s_\chi(U+U^\dagger) - ip(U-U^\dagger)\Big] + S_G(x)s_G\,.$$

$$S_G(x) = a_1\frac{f^2}{4}\,\mathsf{Tr}\left(\partial_\mu U^\dagger\partial^\mu U\right) + a_2\frac{B_0 f^2}{2}\,\mathsf{Tr}\left[\mathcal{M}_q(U+U^\dagger)\right],$$