Higher and partial angular moments B→K*ll

CP³ Origins
Cosmology & Particle Physics

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worked based on J. Gratrex, M. Hopfer and RZ , arXiv:1506.03970
"Generalised helicity formalism, higher moments in $B \to K_{J_K}(\to K\pi)\bar{\ell}_1\ell_2$"

25 Oct - 1 Nov Albufeira

structure

1. introduction

2. sketch computation with H_{eff} = dim 6 operators
   "**lepton factorisation approximation** (LFA)"

   methods: - "Dirac trace technology"
   - "Wigner-Jacob-Wick" using SO(3)-reps

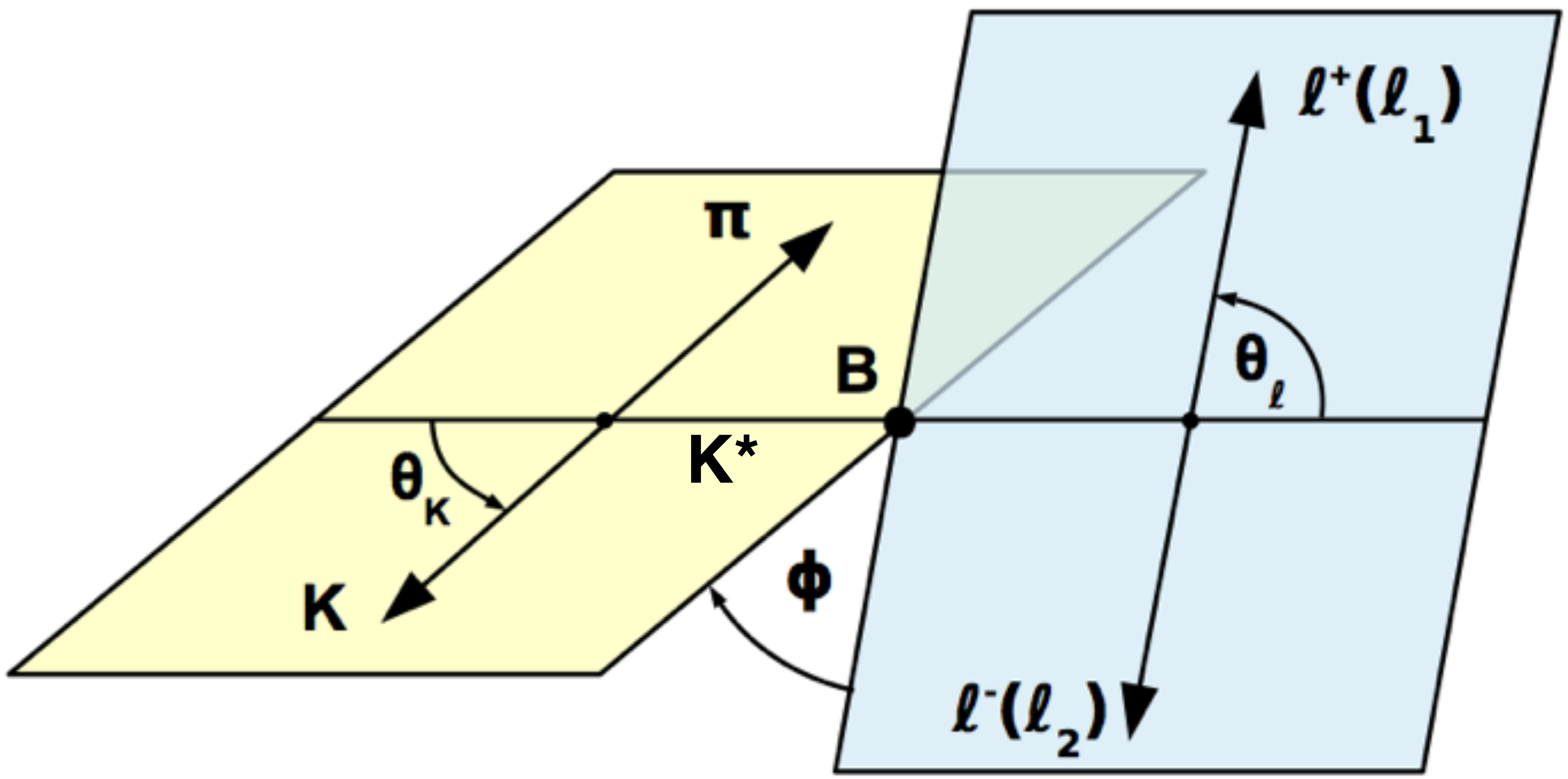
3. **method of (partial) moments** (diagnosing "anomalies")
   beyond LFA - higher moments
   qualitative discussion QED corrections
   diagnosing QED corrections using higher moments

   relevant for lepton universality violation

4. conclusions & summary

The decay topology B-> V(->SS)l1 l2

For $\bar{B} \to \bar{K}^*(\to \bar{K}\pi)\ell^+\ell^-$ in particular



- **Kπ-pair** coming from K* is in **p-wave** (L=1) at **amplitude level**

what about lepton pair?

- principle no restriction - specifying approximation crucial

Lepton factorisation approximation (LFA)

- Heff of dim=6 with 10 operators

$$H^{\rm eff} = -\frac{4G_F}{\sqrt{2}}\frac{\alpha}{4\pi}V_{\rm ts}V^*_{\rm tb}\sum_{i=V,A,S,P,\mathcal{T}}(C_iO_i + C'_iO'_i)\;.$$

$$O_{S(P)} = \bar{s}_L b\,\bar{\ell}(\gamma_5)\ell\;, \qquad O_{V(A)} = \bar{s}_L\gamma^\mu b\,\bar{\ell}\gamma_\mu(\gamma_5)\ell\;,$$

$$O_{\mathcal{T}} = \bar{s}_L\sigma^{\mu\nu}b\,\bar{\ell}\sigma_{\mu\nu}\ell\;, \qquad O' = O|_{s_L\to s_R}$$

standard in literature (clearly dominant)

SM: C_V=C_9 + long-distance; C_A=C_{10} are relevant

- lepton pair restricted to **S-** and **P-wave** at amplitude level in LFA

since decay rate square amplitude $\Longrightarrow$

$\sin(\theta_{K,l})^2$ $\cos(\theta_{K,l})^2$ - maximum-powers

Differential decay rate

$$\frac{32\pi}{3}\frac{d^4\Gamma}{dq^2\ d\cos\theta_\ell\ d\cos\theta_K\ d\phi} = \mathrm{Re}\Big[G_0^{0,0}(q^2)\Omega_0^{0,0} + G_0^{0,1}(q^2)\Omega_0^{0,1} + G_0^{0,2}(q^2)\Omega_0^{0,2} + G_0^{2,0}(q^2)\Omega_0^{2,0} + G_0^{2,1}(q^2)\Omega_0^{2,1} + G_1^{2,1}(q^2)\Omega_1^{2,1} + G_0^{2,2}(q^2)\Omega_0^{2,2} + G_1^{2,2}(q^2)\Omega_1^{2,2} + G_2^{2,2}(q^2)\Omega_2^{2,2}\Big],$$

$$\Omega_m^{l_K,l_\ell} \equiv D^{l_K}_{m,0}\left((\phi,\theta_K,-\phi)\right) D^{l_\ell}_{m,0}\left((0,\theta_\ell,0)\right)$$

$$D^l_{m,0}(\phi,\theta,-\phi) = \sqrt{\frac{(l-m)!}{(l+m)!}}P_{lm}(\cos\theta)e^{-im\phi}\ .$$

$$G_2^{2,2} \sim \left(H_+^V\bar{H}_-^V + H_+^A\bar{H}_-^A - 2\left(H_+^T\bar{H}_-^T + 2H_+^{T_t}\bar{H}_-^{T_t}\right)\right)$$

Hadronic helicity amplitudes e.g. $H_\lambda^{V[A]} = \langle\bar{K}^*(\lambda)|\bar{s}\gamma^\mu[\gamma_5]b|\bar{B}\rangle\epsilon^*(\lambda)_\mu$

For completeness: connection standard literature-notation

- standard notation goes back at least to Treiman & Pais '68
  "pion phase shift information from Kl_4 decays"

$$\frac{8\pi}{3}\frac{d^4\Gamma}{dq^2\ d\cos\theta_\ell\ d\cos\theta_K\ d\phi} = (g_{1s} + g_{2s}\cos 2\theta_\ell + g_{6s}\cos\theta_\ell)\sin^2\theta_K + (g_{1c} + g_{2c}\cos 2\theta_\ell + g_{6c}\cos\theta_\ell)\cos^2\theta_K + (g_3\cos 2\phi + g_9\sin 2\phi)\sin^2\theta_K\sin^2\theta_\ell + (g_4\cos\phi + g_8\sin\phi)\sin 2\theta_K\sin 2\theta_\ell + (g_5\cos\phi + g_7\sin\phi)\sin 2\theta_K\sin\theta_\ell$$

$$G_0^{0,0} = \frac{4}{9}\left(3\left(g_{1c} + 2g_{1s}\right) - \left(g_{2c} + 2g_{2s}\right)\right)\ , \quad G_0^{0,1} = \frac{4}{3}\left(g_{6c} + 2g_{6s}\right)\ , \quad G_0^{0,2} = \frac{16}{9}\left(g_{2c} + 2g_{2s}\right)\ ,$$

$$G_0^{2,0} = \frac{4}{9}\left(6\left(g_{1c} - g_{1s}\right) - 2\left(g_{2c} - g_{2s}\right)\right)\ , \quad G_0^{2,1} = \frac{8}{3}\left(g_{6c} - g_{6s}\right)\ , \quad G_0^{2,2} = \frac{32}{9}\left(g_{2c} - g_{2s}\right)\ ,$$

$$G_1^{2,1} = \frac{16}{\sqrt{3}}\underbrace{\left(g_5 + ig_7\right)}_{=\mathcal{G}_5}\ , \quad G_1^{2,2} = \frac{32}{3}\underbrace{\left(g_4 + ig_8\right)}_{=\mathcal{G}_4}\ , \quad G_2^{2,2} = \frac{32}{3}\underbrace{\left(g_3 + ig_9\right)}_{=\mathcal{G}_3}$$

N.B. usually use $g_x \to J_x$ (to emphasise different convention later)

Convenience & illustration of $G_m^{lk,ll'}$'s

1. *endpoint symmetries* Hiller RZ'13

kinematic endpoint K* enhanced symmetry (threshold expansion in effective theory)

helicity amplitudes: $H_+^{V,A} = H_-^{V,A} = -H_0^{V,A}$

angular distribution two (one SM) parameters

$$G_0^{0,0} \neq 0\,, \quad G_0^{2,2} \to \mathrm{Re}[G_0^{2,2}]\,, \quad G_1^{2,2} \to -2\mathrm{Re}[G_0^{2,2}]\,, \quad G_2^{2,2} \to 2\mathrm{Re}[G_0^{2,2}]$$

2. *examples of 12-angular observables in literature*

$$\langle P_2\rangle_{\text{bin}} = \frac{\left\langle 2G_0^{0,1} - G_0^{2,1}\right\rangle_{\text{bin}}}{3\mathcal{N}_{\text{bin}}}\,, \quad \langle P_4'\rangle_{\text{bin}} = \frac{\left\langle \mathrm{Re}\left[G_1^{2,2}\right]\right\rangle_{\text{bin}}}{4\mathcal{N}'_{\text{bin}}}\,, \quad \langle P_5'\rangle_{\text{bin}} = \frac{\left\langle \mathrm{Re}\left[G_1^{2,1}\right]\right\rangle_{\text{bin}}}{2\sqrt{3}\mathcal{N}'_{\text{bin}}}\,,$$

$$\langle P_8'\rangle_{\text{bin}} = \frac{\left\langle \mathrm{Im}\left[G_1^{2,2}\right]\right\rangle_{\text{bin}}}{4\mathcal{N}'_{\text{bin}}}\,, \quad \langle P_6'\rangle_{\text{bin}} = \frac{\left\langle \mathrm{Im}\left[G_1^{2,1}\right]\right\rangle_{\text{bin}}}{2\sqrt{3}\mathcal{N}'_{\text{bin}}}\,, \quad \langle A_{\mathrm{FB}}\rangle_{\text{bin}} = \frac{1}{2}\frac{\left\langle G_0^{0,1}\right\rangle_{\text{bin}}}{\left\langle G_0^{0,0}\right\rangle_{\text{bin}}}\,,$$

forward backward type observables $l_l=1$ (odd in θ_l)

Some references on computation mode

$O_{V,A}$ SM	$m_\ell = 0$	Krüger,Sehgal,Sinha,Sinha	'99
$O_{V,A}$ SM	$m_\ell \neq 0$	Faessler,Gutsche, Ivanov, Körner, Lyubivitskij	'02
idem		Krüger, Matias	'05
add $O_{S,P}$	$m_\ell \neq 0$	Altmanshofer,Ball,Bharucha,Buras,Straub,Wick	'08
add $O_{\mathcal{T}}$	$m_\ell \neq 0$	Gosh et al/Bobeth et al	'10'12
all	$m_{\ell_1} \neq m_{\ell_2} \neq 0$ *	our work	'15

in $G_m^{lk,ll}$ - basis expression relatively compact nevertheless provide mathematica notebook in arxiv-file results in Mathematica notebook

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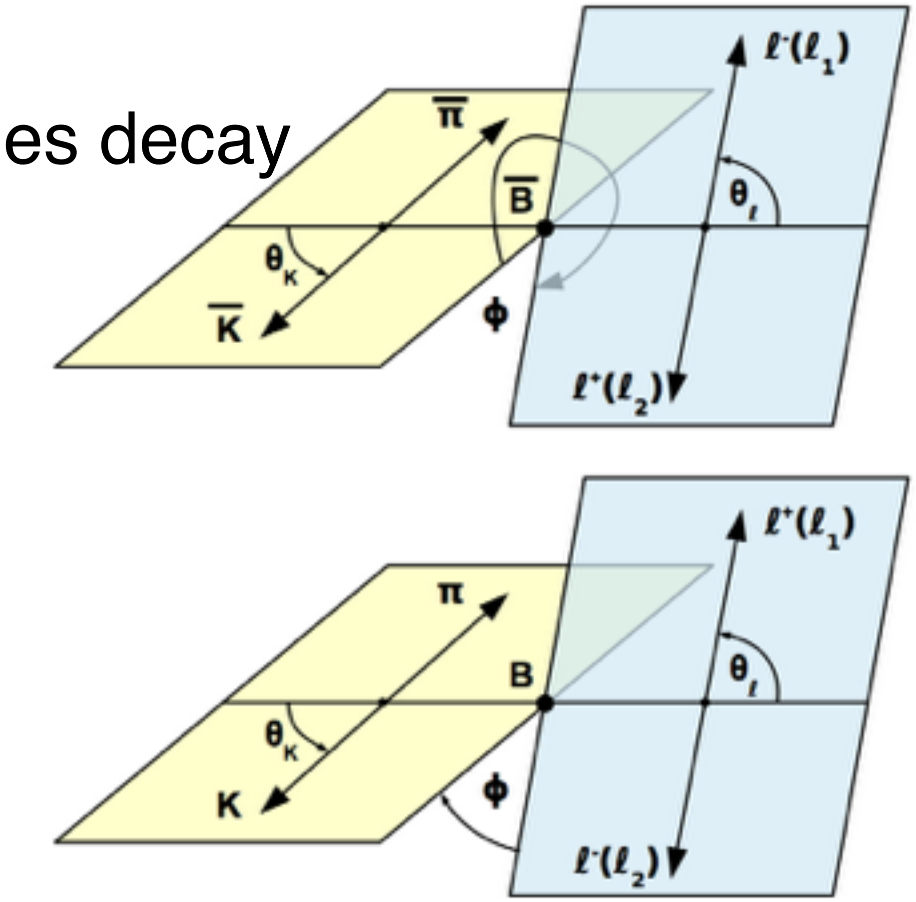
*$m_{l1} \neq m_{l2}$ useful for semileptonic -& interesting for lepton flavour violation $B \to K\mu e$

Note on conventions

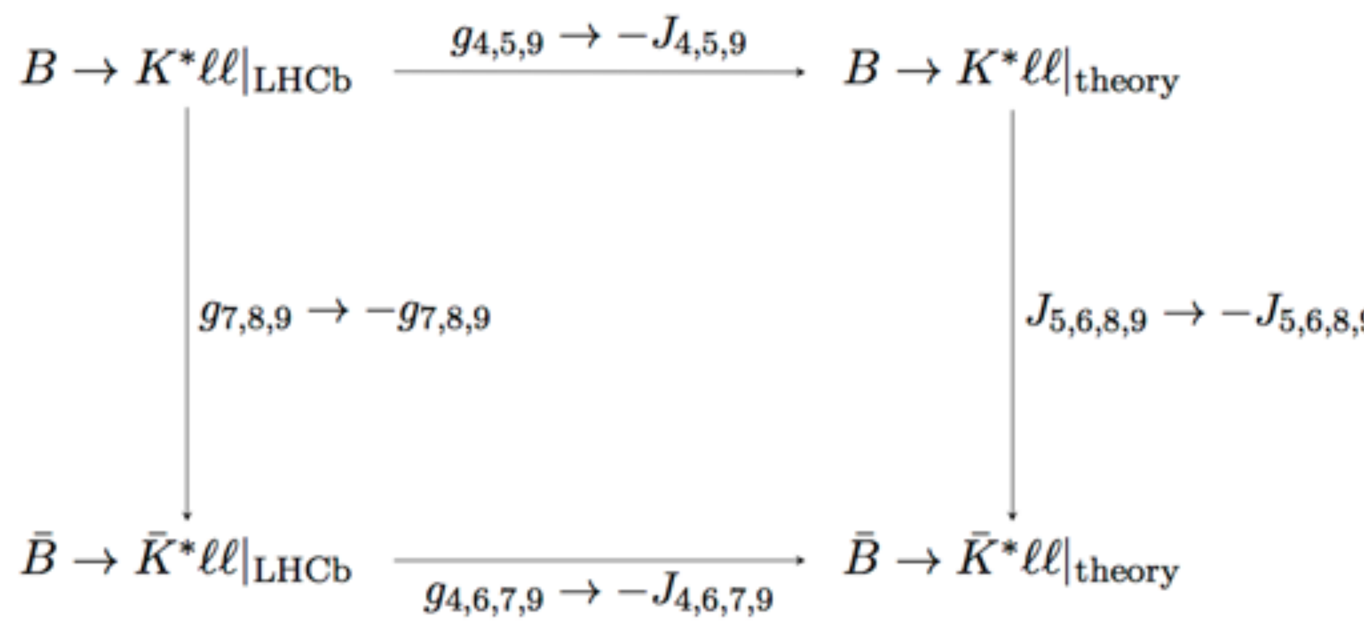
- need to define angles of decay and anti-particles decay
- we follow **LHCb conventions**:



$$\left.\frac{d^4(\Gamma \pm \bar{\Gamma})}{dq^2 d\cos\theta_\ell d\cos\theta_K d\phi}\right|_{\rm LHCb}$$

CP-even (CP-odd) limit of CP-conservation

- **“theorist’s conventions”** differ
  By matching our calculation we get the following diagram:



1) differ in translation in sign in g(J)$_{789}$ from literature i.e. ϕ→-ϕ
2) def. ϕ is subtle
3) not affect current “fits” but important when weak or strong phases included
4) hopefully can be clarified near future

How compute: 2 methods

cf. talk Korner standard Jacob-Wick method

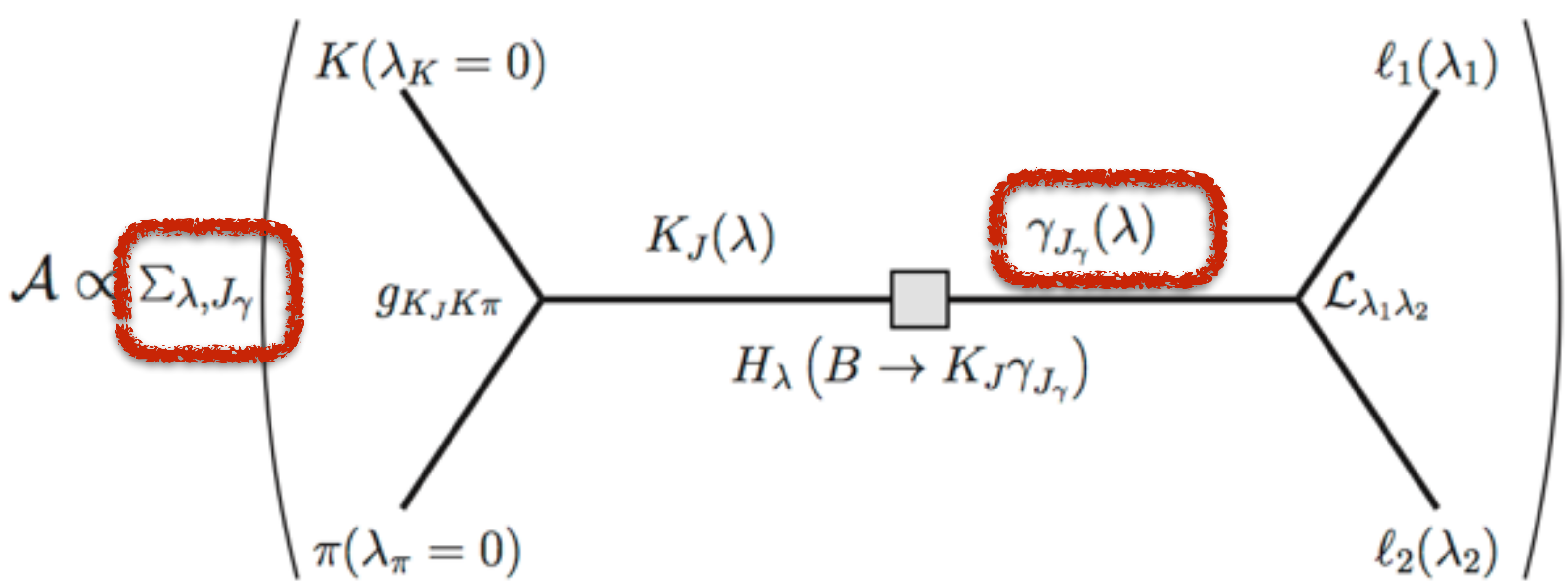
- **Dirac-trace technology** (parameterisation of momenta - say in B-frame)

$$(\ell_2)^\mu = (f_\ell(E_2, q_0, -q_z), -|\vec{p}_\ell| \sin\theta_\ell \cos\phi, +|\vec{p}_\ell| \sin\theta_\ell \sin\phi, f_\ell(E_2, q_z, -q_0)) \,,$$
$$(p_K)^\mu = (f_{K^*}(E_K, p_0, q_z), -|\vec{p}_K| \sin\theta_K, 0, -f_{K^*}(E_K, q_z, p_0)) \,,$$

- **Jacob-Wick-technology** (use SO(3)/Wigner representation matrices)

generalised standard formalism: $B \to K_J(\to K\pi)\,\gamma^*(\to l_1 l_2)$ by decomposing SO(3,1) tensors into SO(3) irreps and summing J_γ (up to spin 2)

$$\underbrace{g_{\mu\nu}}_{SO(3,1)_{(j_1,j_2)=(\frac{1}{2},\frac{1}{2})}} = \underbrace{q_\mu q_\nu}_{\mathrm{SO}(3)_{\mathrm{j}=0}} - \sum_{\lambda\in\{\pm,0\}} \underbrace{\omega_\mu(\lambda)\omega^*_\nu(\lambda')}_{\mathrm{SO}(3)_{\mathrm{j}=1}}$$



Addressing the nature of the anomalies through moments analysis

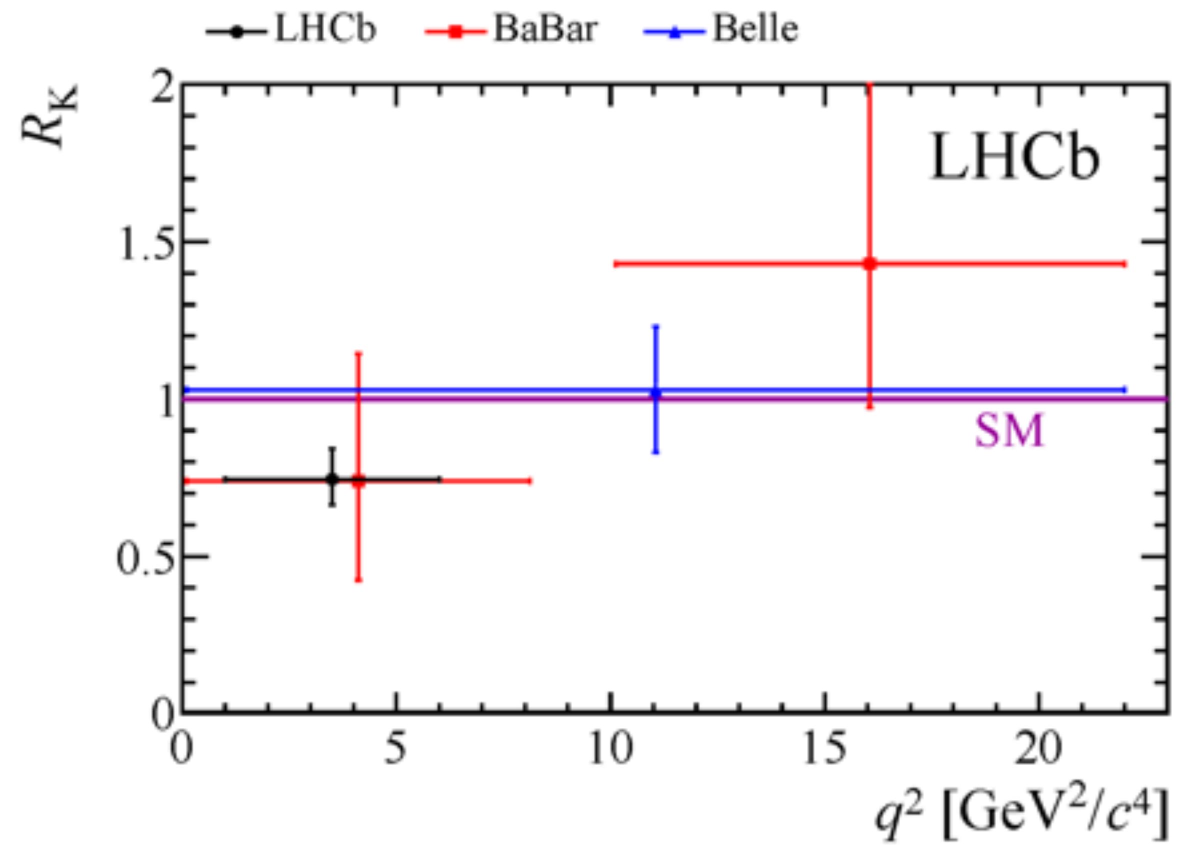
Current interest: R$_K$-anomaly

in combination with $H\to\mu\tau$ "anomaly" is rather interesting

c.f. talks by Crivellin, Celis, de Medeiros Varzielas, Matias, Nisandzic

$$R_K \equiv \frac{\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathcal{B}(B^+ \to K^+e^+e^-)}$$

$$R_K|_{\rm SM} \simeq 1$$



1. charm should not play a *direct* role as coupling to leptons universal

2. QED effects: are they sizeable?

$$\alpha_{\rm QED}\, f(\ln\left(\frac{m_b}{m_\ell}\right))$$

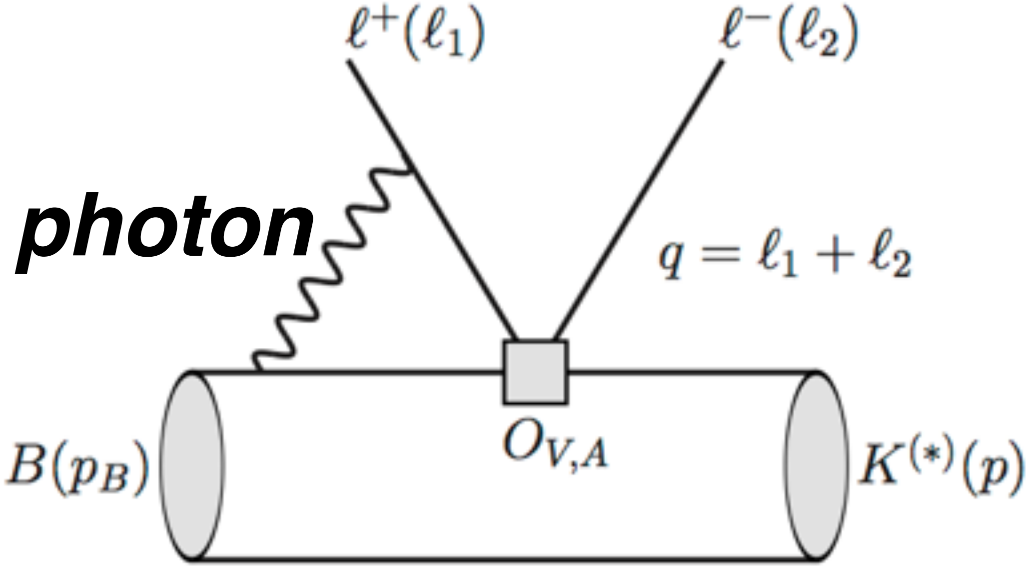
Note: B->Kll; QED effects are not taken into account

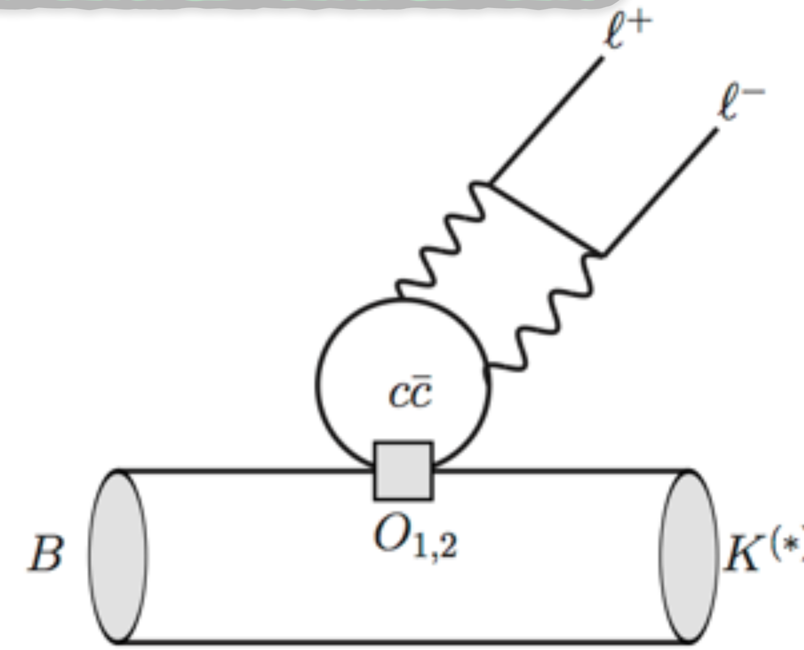
$$R^{\rm sl}_\pi = \frac{\Gamma(\pi\to e^+\nu)}{\Gamma(\pi\to\mu^+\nu)} = \frac{m_e^2}{m_\mu^2}\frac{(m_\pi^2-m_e^2)^2}{(m_\pi^2-m_\mu^2)^2}(1+\delta_{\rm QED})\,, \quad |\delta_{\rm QED}| \simeq 4\%$$

an example

$\Longrightarrow$ suggest a way to diagnose/ bound size

non-factorisable QED corrections





- Becomes a proper 1→3 process and by crossing a 2→2 with Mandelstam variables

$$B(p_B) + \ell^-(-\ell_1) \to K(p) + \ell^-(\ell_2) \; ,$$

$$s[u] = (p \pm \ell_2[\ell_1])^2 = \frac{1}{2}\left[(m_B^2 + m_K^2 + 2m_\ell^2 - q^2) \pm \beta_\ell \sqrt{\lambda} \cos\theta_\ell\right]$$

- ⟹ s[u] enter logs ⟹ **no restriction sin(θ_l),cos(θ_l)-powers;**
  Legendre polynomial [or $\Omega_m^{|k,|l|}$] serves as a complete basis (non-vanishing higher moments)

$$\frac{d^2\Gamma(B \to K\ell^+\ell^-)}{dq^2 \; d\cos\theta_\ell} = \sum_{l_\ell \geq 0} G^{(l_\ell)} P_{l_\ell}(\cos\theta_\ell)$$

diagnosing QED effects $B \to K^{(*)} l^+ l^-$

- **$B \to K l^+ l^-$ moments:**

$$M_{\bar{\ell}\ell}^{(l_\ell)} = \int_{-1}^{1} d\cos\theta_\ell P_{l_\ell}(\cos\theta_\ell) \frac{d^2\Gamma(B \to K\ell^+\ell^-)}{dq^2 \; d\cos\theta_\ell} = \frac{1}{2l_\ell + 1} G_{\bar{\ell}\ell}^{(l_\ell)}$$

1.

LFA

$$M_{\bar{\ell}\ell}^{(l_\ell > 2)} = 0$$

beyond LFA (eg. QED effects)

$$M_{\bar{\ell}\ell}^{(l_\ell > 2)} \neq 0$$

2.

likely QED-signature

$$M_{\bar{e}e}^{(l_\ell > 2)} \neq M_{\bar{\mu}\mu}^{(l_\ell > 2)}$$

$$|M_{\bar{e}e}^{(l_\ell > 2)}| > |M_{\bar{\mu}\mu}^{(l_\ell > 2)}|$$

$$\left[\alpha_{\rm QED}\, f(\ln\left(\frac{m_b}{m_\ell}\right))\text{-effects}\right]$$

3.

R_K is $M_{\bar{\ell}\ell}^{(l_\ell = 0)}$-moment - behaviour of moment in l_ℓ crucial
Rough computation suggests moderate fall-off
Amplitude: S-wave : D-wave = 1 : ~0.5(large uncertainty)

refinement: competitor signature

- **higher dimensional** operators (dimension 8,10….) $\delta H^{\text{eff}} = C^{(j)} O^{(j)} + ..$

$$O^{(j)} = \bar{s}_L \Gamma^{(j)}_{\mu_1 \ldots \mu_j} b \; \bar{\ell} \Gamma^{(j)\;\mu_1 \ldots \mu_j} \ell$$

with **higher SO(3)-spin** $\Gamma^{(j)}_{\mu_1 \ldots \mu_j} \equiv \gamma_{\{\mu_1} D^+_{\mu_2} \ldots D^+_{\mu_j\}}, \; D^+ \equiv \overleftarrow{D} + \overrightarrow{D}, \text{ with } \overrightarrow{D}$

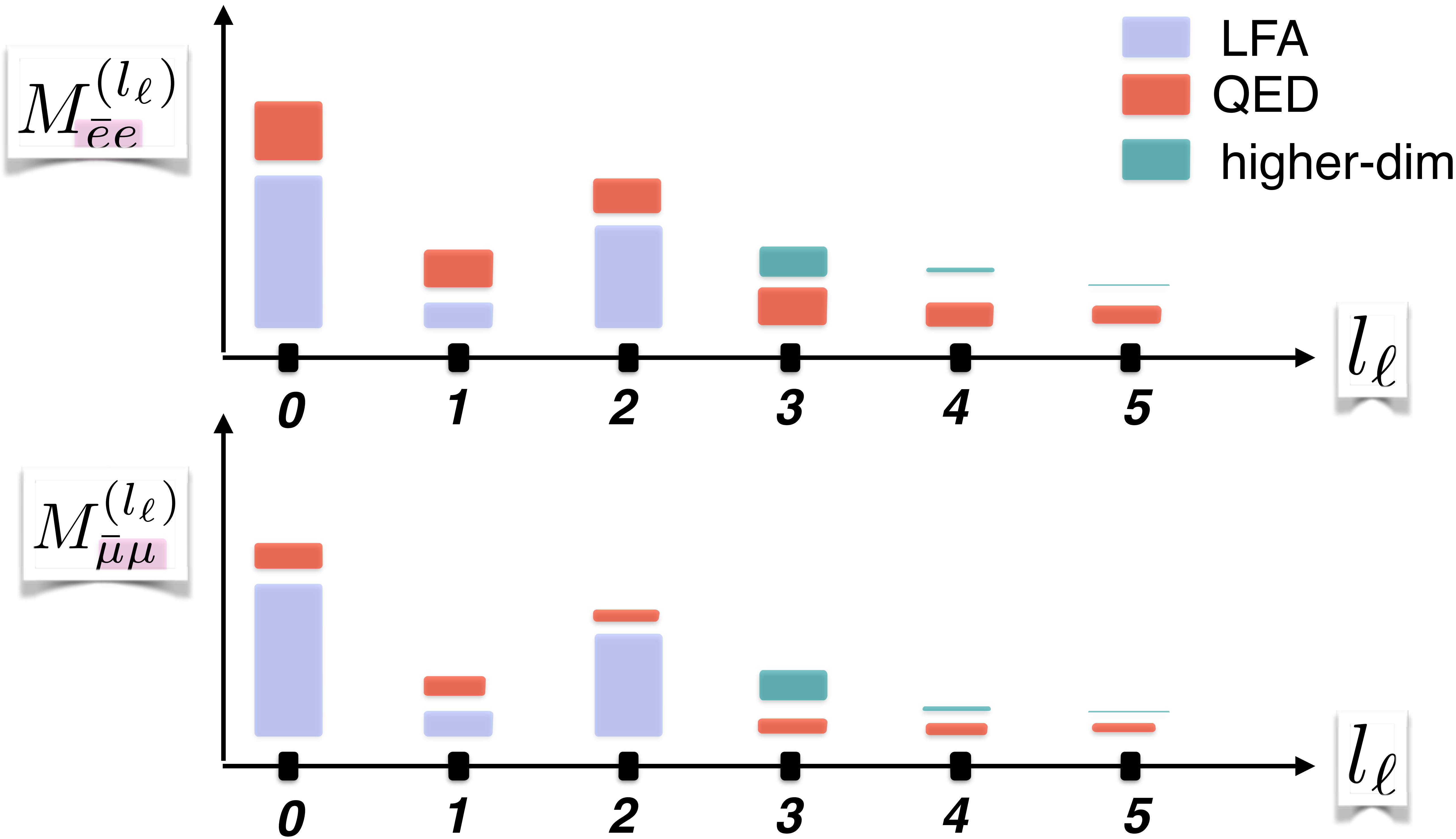
- QED versus higher dimensional operators

$$C^{(j)} = \frac{\mathcal{O}(1)}{(m_W^2)^j} \left[1 + \alpha_{\text{QED}} f_j \cdot \left(\frac{m_W^2}{m_b^2} \right)^{(j-1)} \right] , \quad \text{for } j \geq 1 ,$$

QED wins even without logs for Wilson coefficients

time for a graphical summary ….

qualitative overview of effects*



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* emphasis on qualitative (size of effects are for illustration only)

Method of (partial) moments

- **method of moments** extendable to B->K*ll using orthogonality of Legendre P.
  see also Beaujean, Chraszcz, Serra vanDyk '15

$$M_m^{l_K,l_\ell} \equiv \frac{1}{8\pi}\int_{-1}^{1} d\cos\theta_K \int_{-1}^{1} d\cos\theta_\ell \int_0^{2\pi} d\phi \, (\Omega_m^{l_K,l_\ell})^* \frac{d^4\Gamma}{d(\text{angles})} = \frac{(1+\delta_{m0})G_m^{l_K,l_\ell}}{2(2l_K+1)(2l_\ell+1)}$$

- our proposal is to look for
  1) **partial moments** (or in θ_l-angle)

$$k_m^{l_\ell}(\theta_K) = \frac{1}{4\pi}\int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi \, (\Omega_m^{l_K,l_\ell})^* \frac{d^4\Gamma}{d(\text{angles})} = \frac{1+\delta_{m0}}{2\,(2l_\ell+1)} \sum_{l_K\geq 0} D_{m,0}^{l_K}(\Omega_K)\, G_m^{l_K,l_\ell}$$

for example: $k_0^2(\theta_K) = \frac{1}{5}\left(G_0^{0,2} + \frac{1}{2}\left(3\cos^2\theta_K - 1\right) G_0^{2,2}\right)$ in LFA

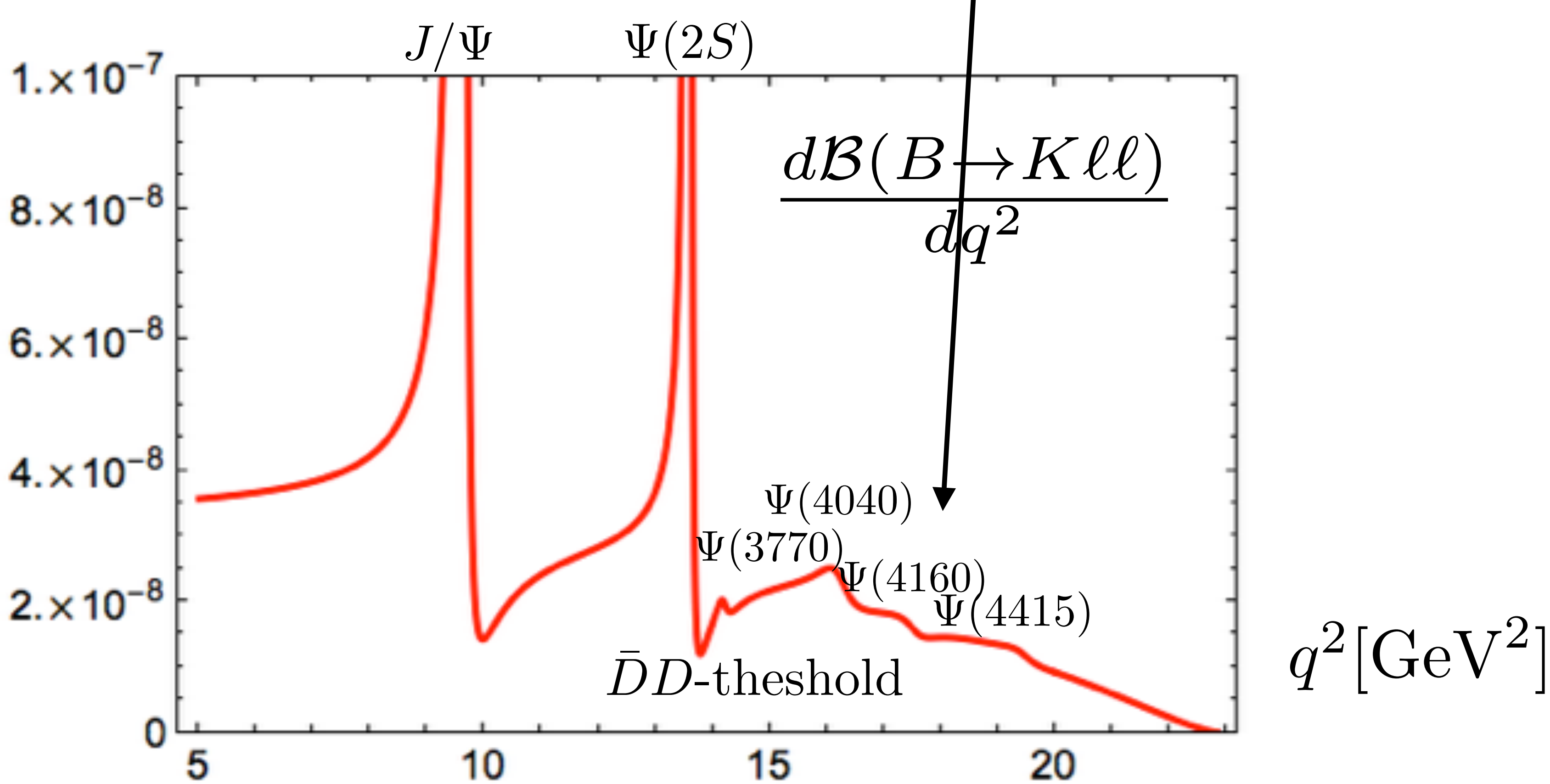
2) **higher moments** i.e. do not assume $M_m^{j,j'} = 0 \,, \quad \forall m \text{ and } j \geq 3 \text{ or } j' \geq 3$
but measure/bound them!

- expect the full program of 1) and 2) to be equivalent in some statistical sense

conclusions and summary

- **angler distribution** computed can be used for **any** $\boldsymbol{S \to V(\to S_1 S_2) l_1 l_2}$-decay

- **comment:** to be sure δC_9 is not charm: (cf. backup slides) resonance residues and phases have to be measured (begun in $B \to K\mu\mu$ Lyon RZ'14 for broad resonances and is pursued by LHCb for rest)



conclusions and summary II

- **method of moments** can help to **clarify nature** of some $b \to s$ "**anomalies**"

Note: standard likelihood-fit assumes assumes distribution of LFA
if no higher moments then ok but else bias

1. **diagnose** $B \to K^{(*)} ll$ **QED-effects** *(previous slide - LHCb analysis under way)*
comment: "of course this does not replace a real calculation"

2. may also use higher moments below to check for possible J/Ψ-backgrounds
in $B \to K^*(\to K \pi)\, \mu\mu$ angular analysis ("home of anomalie*s")*

e.g. $B \to K\mu\mu\gamma$ with 1) γ undetected 2) π from underlying event
$\Rightarrow$ *may result in* $B \to K^*(\to K \pi)\, \mu\mu$ *signal window with downward shift in* q^2
impacts, in particular, below J/Ψ (10^3-enhancement could compensate for "above"

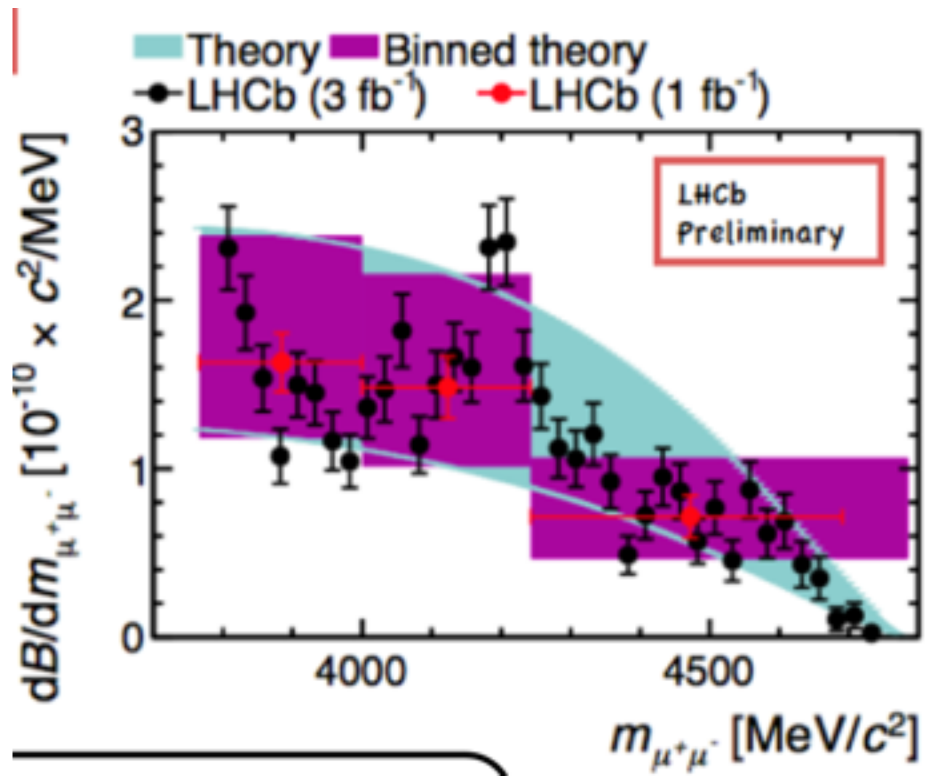
Now, LHCb estimates this event to be negligible but general lesson
is that checking higher moments could help to clarify matters

thanks for your attention

BACKUP

II.C comment charm resonances in B→K (*)ll

$$\mathrm{BF}(B \to K\ell\ell)$$

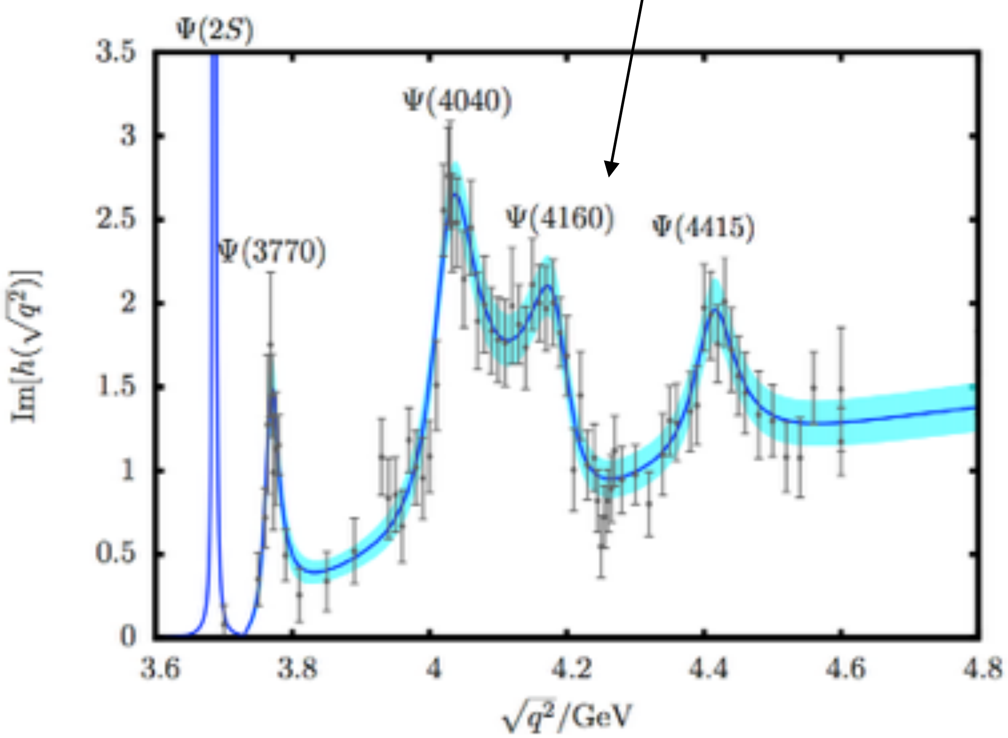


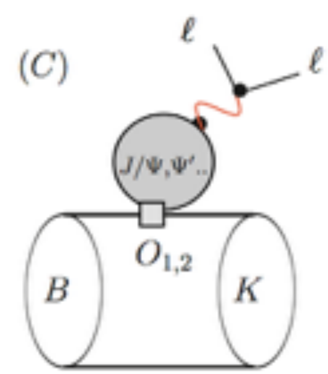
LHCb PRL 111 (2013)

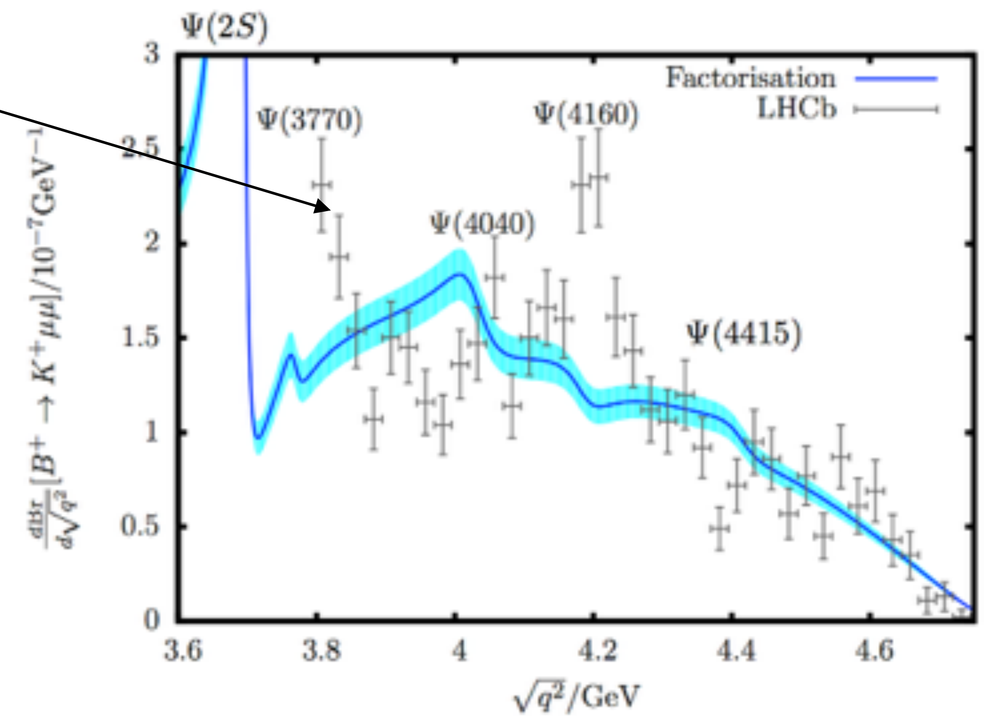
pronounced $J^{PC} = 1^{--}$ charm resonance structure

Lyon RZ 1406.0566

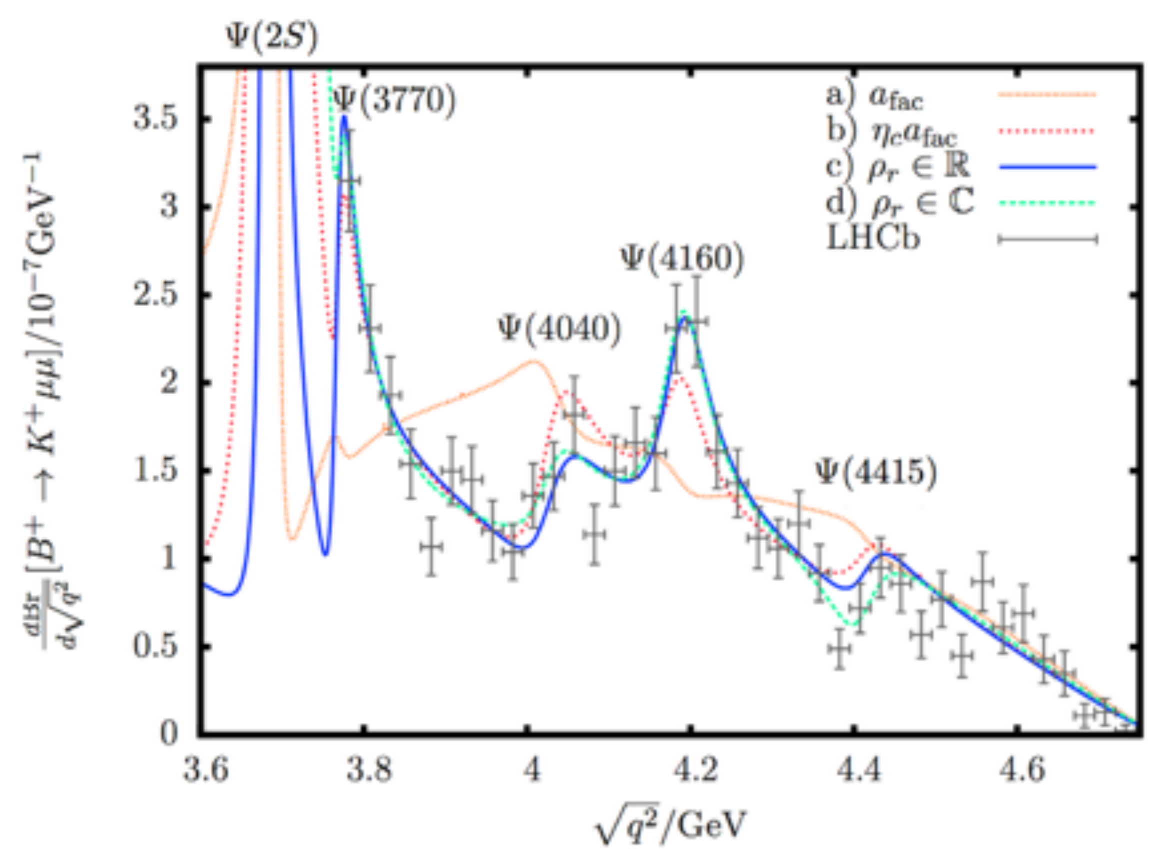
- Using a fit to BES-II data $e^+e^- \to$ hadrons able to check status of “naive” factorisation at high q^2 in $B \to Kll$





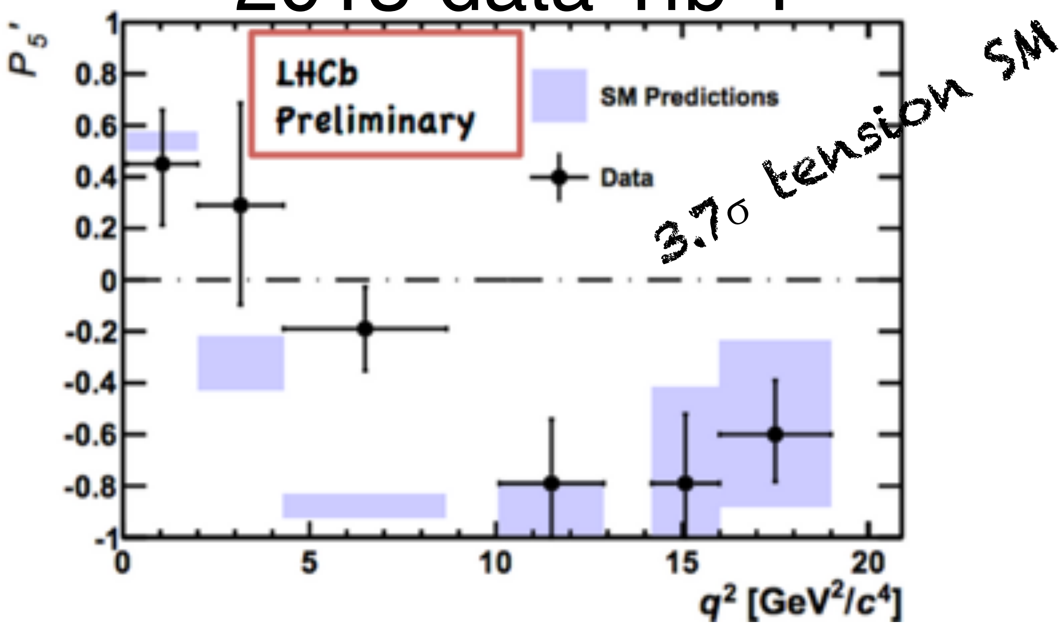


hight of resonances in naive fac. by factor ~(-2.5) fits the data well

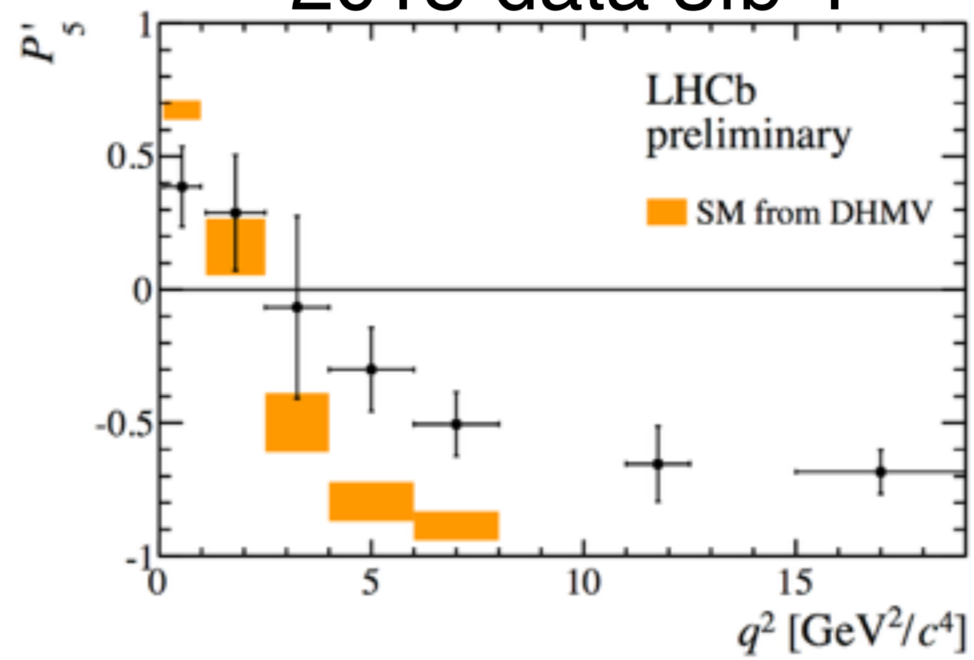


- Led us to speculate P_5'-anomaly in $B \to K^{(*)} ll$ might be related to charm (since charm pronounced)

2013-data 1fb-1

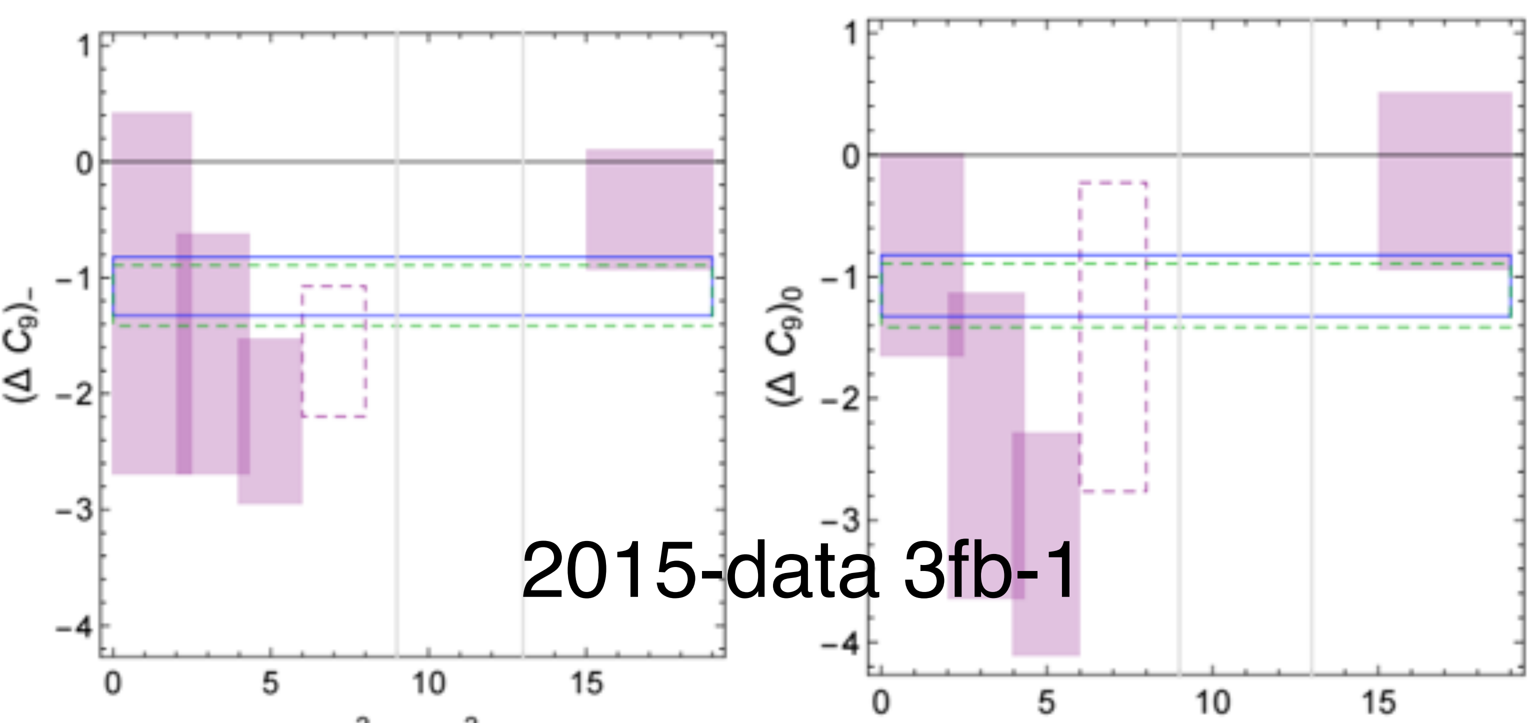


2015-data 3fb-1



- 1) pronounced to J/Ψ 2) accommodated by photon penguin C_{10} not nec.

Straub's talk Moriond'15 (proceedings & Wolfgang's talk)



- effect same sign as in naive fac. in "-" versus "0" helicity
- <u>my comment</u>: that's what B→ J/Ψ K* experimental angular analysis predicts for J/Ψ,Ψ(2S)-contributions