J.G. Körner

Institute of Physics
PRISMA Cluster of Excellence,
University of Mainz

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Identical particle effects in Higgs Decay to 4 Tauons

Lepton mass effects and identical particle effects

- **Lepton mass effects**
  - **Scale of lepton mass effects is set by off-shellness of the Z-boson**

    $$4m_\ell^2 \le q^2 \le (m_H - m_Z)^2$$

    and not by the Higgs mass m_H.
  - τ **mass effects are not negligible. ($m_{e,\mu} = 0$ is a good approximation).**
  - **Numerical example:**

    $$\Gamma(H \to Z + Z^*(\to \tau\tau))/\Gamma(H \to Z + Z^*(\to \mu\mu)) = 0.96 \quad (-4.0\,\%)$$

  - **Angular decay distribution of leptons changes. Can mimic the contributions of new effective operators. Later.**
  - **Test of lepton universality**
  - **Branching ratio $BR(H \to eeee) = 3.27 \times 10^{-5}$ (Higgs handbook (Denner et al.))**
- **Identical particle effects**
  - **In the decay $Z \to (\tau^+\tau^+)(\tau^-\tau^-)$ the two tauons in the two pairs $(\tau^+\tau^+)$ and $(\tau^-\tau^-)$ are undistinguishable.**
  - **Statistical factor of $1/4$**
  - **Quantum interference effects, i.e. more Feynman diagrams**

Two Feynman diagrams

There are two Feynman diagrams that contribute to $H \to \tau^+\tau^-\tau^+\tau^-$

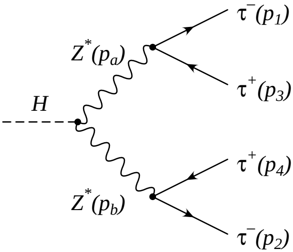

Figure: Feynman diagrams (A) and (B) contributing to $H \to \tau^+\tau^-\tau^+\tau^-$.

They contribute to the rate as follows

$$|M_A + M_B|^2 = |M_A|^2 + 2Re(M_A M_B^*) + |M_B|^2$$

The diagonal contributions

- **Diagrams describing the contributions from $|M_A|^2$ and $|M_B|^2$ are topologically equivalent, i.e. $|M_A|^2 = |M_B|^2$**
- **Diagonal terms contribute as (add statistical factor $1/2!\,2!$)**

$$\frac{1}{4}\left(|M_A|^2 + |M_B|^2\right) = \frac{1}{2}\,|M_A|^2$$

- **If interference contribution $2Re(M_A M_B^*)$ (and lepton mass effects) are neglected one finds**

$$\Gamma(H \to \tau^+\tau^-\tau^+\tau^-) \quad = \quad \frac{1}{2} \quad \Gamma(H \to \tau^+\tau^-\mu^+\mu^-)$$

Sign of interference contribution

The interference contribution is given by the absorptive part of a one-loop contribution compared to the two-loop contributions of the diagonal graphs. Take a minus sign into account.

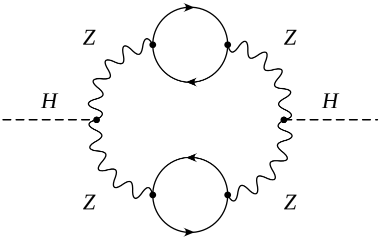

Figure: Squared Feynman diagrams $\sim |M_A|^2$ and $\sim Re(M_A M_B^*)$ contributing to $H \to \tau^+\tau^-\tau^+\tau^-$.

Including the dynamics the interference contribution adds constructively.

Width dependence of interference contribution

- **As the width of the Z becomes smaller and smaller the momentum mismatch of the leptons in the interference contribution will become bigger and bigger. One expects**

$$\lim_{\Gamma_Z \to 0} \quad \frac{\Gamma_{\text{interference}}}{\Gamma_{\text{diagonal}}} \quad \Longrightarrow \quad 0$$

- **Question: Does the relative suppression go like (Γ_Z/m_Z) or like $(\Gamma_Z/m_Z)^2$?**

Some numerical results

Numerically one has (m_Z fixed)

Γ_Z [GeV]	$\Gamma_{nondiag}/\Gamma_{diag}$	$\Gamma_{nondiag}/\Gamma_{diag}$ $[\Gamma_Z/m_Z]$
2.4952	10.31 %	3.77
1.0	4.73 %	4.32
0.5	2.50 %	4.56
0.2	1.03 %	4.71
0.1	0.53 %	4.79
0.05	0.27 %	4.89

Table: Dependence of the rate ratios of nondiagonal and diagonal contributions on the Z–width for the decay $H \to Z^*(\to \tau^+\tau^-) + Z^*(\to \tau^+\tau^-)$.

Power of the width suppression

Use the δ–function representation

$$\lim_{\Gamma_Z \to 0} \frac{1}{(q^2 - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} = \frac{\pi}{m_Z \Gamma_Z} \, \delta(q^2 - m_Z^2)$$

to analyze the diagonal contribution in the vicinity of $q^2 = m_Z^2$ (keep M_Z fixed)

$$\begin{aligned}\lim_{\Gamma_Z \to 0} \int dq^2 \frac{F(q^2)}{(q^2 - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} &= \frac{\pi}{m_Z \Gamma_Z} \int dq^2 \, \delta(q^2 - m_Z^2) \, F(q^2) \\ &= \frac{\pi}{m_Z \Gamma_Z} \, F(m_Z^2)\end{aligned}$$

where the function $F(q^2)$ is regular at $q^2 = m_Z^2$. A similar analysis leads to $\lim_{\Gamma_Z \to 0} \Gamma_{\rm interference} = const.$ **One finds**

$$\begin{aligned}\lim_{\Gamma_Z \to 0} \frac{\Gamma_{\rm interference}}{\Gamma_{\rm diagonal}} &= const. \cdot m_Z \, \Gamma_Z \\ &= const. \cdot m_Z^2 \, [\Gamma_Z / m_Z]\end{aligned}$$

The diagonal contribution $\sim |A|^2$

- **The width formula for $H \to Z^*(p^2) + Z^*(q^2)$ (Grau, Pancheri, Phillips 1990)**

$$\Gamma(H \to \mathrm{all}\, Z\, Z) = \int_0^{m_H^2} \frac{dp^2 m_Z \Gamma_Z}{\pi[(p^2 - m_Z^2)^2 + (m_Z \Gamma_Z)^2]} \times \int_0^{(m_H - p)^2} \frac{dq^2 m_Z \Gamma_Z}{\pi[(q^2 - m_Z^2)^2 + (m_Z \Gamma_Z)^2]} \Gamma(H \to Z^* Z^*)$$

where

$$\Gamma(H \to Z^* Z^*) = \frac{1}{2} \frac{g_W^2}{8\pi \cos^2\theta_W} \frac{|\vec{p}|}{m_H^2 m_Z^2} \left(p^2 q^2\right) \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right)$$

Choice of gauge

- unitary gauge

$$\begin{aligned}
\Gamma(H \to Z^* Z^*) &\sim \sum_m \varepsilon^{\alpha}(m, p^2)\varepsilon^{*\,\beta}(m, p^2) \sum_n \varepsilon^{\beta}(n, q^2)\varepsilon^{*\,\alpha}(n, q^2) \\
&= (-g^{\mu\nu} + \frac{p^\mu p^\nu}{m_Z^2})(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_Z^2}) \\
&= \left(4 - \frac{p^2}{m_Z^2} - \frac{q^2}{m_Z^2} + \frac{pq\,pq}{m_Z^4}\right)
\end{aligned}$$

- Spin 1 (Lorenz, Landau) gauge

$$\begin{aligned}
\Gamma(H \to Z^* Z^*) &\sim (-g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2})(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) \\
&= \left(2 + \frac{pq\,pq}{p^2 q^2}\right)
\end{aligned}$$

- Feynman gauge

$$\begin{aligned}
\Gamma(H \to Z^* Z^*) &\sim (-g^{\mu\nu})(-g_{\mu\nu}) \\
&= 4
\end{aligned}$$

Correct result

The result is obviously gauge variant.

The problem is that the concept of an external off-shell gauge boson is not a gauge invariant concept. One must attach fermion pairs to the off-shell gauge boson to get a gauge invariant result. In addition one must use the unitary gauge to get a gauge invariant result.

The unitary gauge

Consider the gauge boson propagator in the general R_ξ gauge and rewrite it into a convenient form.

$$
\begin{aligned}
D^{\mu\nu} &= \frac{i}{q^2 - m_Z^2}\left(-g^{\mu\nu} + \frac{q^\mu q^\nu (1-\xi_Z)}{q^2 - \xi_Z m_Z^2}\right) \\
&= \frac{i}{q^2 - m_Z^2}\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{m_Z^2}\right) - i\,\frac{q^\mu q^\nu}{m_Z^2}\,\frac{1}{q^2 - \xi_Z m_Z^2}.
\end{aligned}
$$

The first term is the unitary propagator. The second gauge-dependent term is cancelled by the contribution of the neutral Goldstone ϕ^0.

Again one needs to attach fermion pairs to the gauge boson and to the neutral Goldstone boson to see the cancellation. Explicit examples of this cancellation can be found in the book of Peskin-Schroeder and in Körner [1402.2787].

Attaching (massless) fermion pairs to the off-shell gauge bosons

Split the unitary gauge propagator into a spin 1 and a spin 0 piece:

$$-g^{\nu\beta}+\frac{q^\nu q^\beta}{m_Z^2}=\underbrace{\left(-g^{\nu\beta}+\frac{q^\nu q^\beta}{q^2}\right)}_{\text{spin }1}-\underbrace{\frac{q^\nu q^\beta}{q^2}\left(1-\frac{q^2}{m_Z^2}\right)}_{\text{spin }0},$$

For massless external fermions the spin 0 piece gives zero contribution. When zero mass fermions are attached to off-shell gauge bosons one can use the spin 1 gauge.

The correct result is

$$\Gamma(H\to Z^*Z^*)\quad\sim\quad\left(\frac{p^2q^2}{m_Z^4}\right)\Gamma(\text{spin 1 gauge})$$

Where do the factors p^2 and q^2 come from? They come from attaching a fermion pair to the off-shell gauge bosons. To keep things simple take $m_\ell=0$.

$$\int d\Omega_p L^{\mu\nu}(p)=\frac{4\pi}{3}\,p^2\,P_1^{\mu\nu}(p))\qquad\int d\Omega_q L^{\mu\nu}(q)=\frac{4\pi}{3}\,q^2\,P_1^{\mu\nu}(q)$$

Attaching (massless) fermion pairs to the off-shell gauge bosons

$$\frac{d\Gamma_{ij}}{dp^2 dq^2}(p^2,q^2) = 2\cdot B_i\,B_j \frac{1}{\pi}\frac{m_Z\Gamma}{(p^2-m_Z^2)^2+m_Z^2\Gamma_Z^2}\frac{1}{\pi}\frac{m_Z\Gamma}{(q^2-m_Z^2)^2+m_Z^2\Gamma_Z^2}$$

$$\cdot\left(\frac{p^2 q^2}{m_Z^4}\right)\Gamma(H\to Z^*+Z^*)_{\text{spin 1 gauge}}$$

Sum over the channels

$$\sum_{i,j} B_i\,B_j \approx \frac{1}{2}\Big(\sum_i B_i\Big)\Big(\sum_j B_j\Big) = \frac{1}{2}$$

$$\begin{array}{lll} i) & i\neq j & B_i\,B_j = B_j\,B_i \\ ii) & i=j & B_i\,B_i \approx \frac{1}{2}\,B_i\,B_j \end{array}$$

The factor $1/2$ is crucial to obtain the idetical particle factor of $1/2$ appropiate for the decay $H\to Z^*\,Z^*$.

Angular decay distribution 1

Covariant expression:

$$W(\theta_p, \theta_q, \chi) = g_{\alpha\alpha'}\, P_1^{\alpha\mu}(p)\, P_{0\oplus 1}^{\alpha'\mu'}(q)\, L_{\mu\nu}^{(p)}(p)\, L_{\mu'\nu'}^{(q)}(q)\, P_1^{\nu\beta}(p)\, P_{0\oplus 1}^{\nu'\beta'}(q)\, g^*_{\beta\beta'}$$

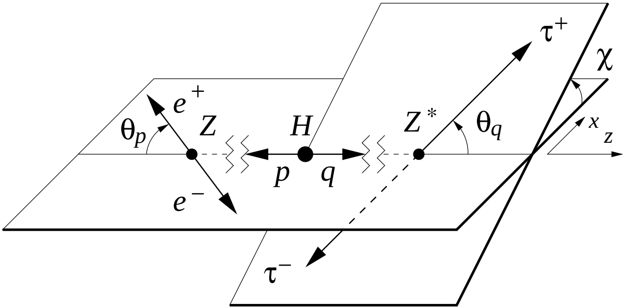

Figure: Definition of the momenta p and q, the polar angles θ_p and θ_q, and the azimuthal angle χ in the cascade decay $H \to Z(\to e^+e^-) + Z^*(\to \tau^+\tau^-)$

Angular decay distribution 2

Two routes to proceed:

- **Define momenta in three frames. Boost momenta to Higgs rest frame. Do the contractions. (Cabibbo, Maksymowicz 1965, Buchalla et al. 2014)**
- **Helicity method (Jacob, Wick 1959)**

Transform covariant distribution to a helicity distribution with the help of the completeness relations for polarization vectors. To make life a bit simpler we treat the on-shell (p)– off-shell (q) case.

- **Off-shell spin 1+spin 0 propagator (unitary gauge)**

$$P_{0\oplus 1}^{\mu'\alpha'}(q) = -g^{\mu'\alpha'} + \frac{q^{\mu'}q^{\alpha'}}{m_V^2} = -\sum_{\lambda_{V^*}=t,\pm1,0} \varepsilon^{\mu'}(\lambda_{V^*})\varepsilon^{*\,\alpha'}(\lambda_{V^*})\,\hat{g}_{\lambda_{V^*}\lambda_{V^*}}\,.$$

- **On-shell spin 1 propagator ($p^2 = m_Z^2$)**

$$P_1^{\alpha\mu}(p) = -g^{\alpha\mu} + \frac{p^\alpha p^\mu}{p^2} = \sum_{\lambda_V=\pm1,0} \bar{\varepsilon}^{\alpha}(\lambda_V)\bar{\varepsilon}^{*\,\mu}(\lambda_V)$$

Angular decay distribution 3

Helicity representation of angular decay distribution

$$W(\theta_p, \theta_q, \chi) = \sum_{\substack{\lambda_V, \lambda'_V \\ J, J' \lambda_{V*}, \lambda'_{V*}}} (-F_S)^{2-J-J'} L^{(p)}_{\lambda_V\, \lambda'_V}(\cos\theta_p) H_{\lambda_V, \lambda_{V*}} H^*_{\lambda'_V, \lambda'_{V*}} L^{(q)}_{\lambda_{V*}\, \lambda'_{V*}}(\cos\theta_q, \chi)$$

with $J, J' = 0, 1 \quad \lambda_{V^*}, \lambda'_{V^*} = t, \pm 1, 0, \lambda_V, \lambda'_V = \pm 1, 0$

Helicity amplitudes for $H \to Z\, Z^*$ **:**

$$H_{\lambda_V, \lambda_{V*}} = g_{\alpha\beta} \bar{\varepsilon}^{*\alpha}(\lambda_V) \varepsilon^{*\beta}(\lambda_{V^*})$$

Angular decay distribution 4; a sample result

Normalized angular decay distribution ($\int_{\mathrm{angles}} \widetilde{W}(\theta_p, \theta_q, \chi) = 1$)

$$\widetilde{W}(\theta_p, \theta_q, \chi) = \frac{1}{8\pi}\left(1 + \sum_{i=1}^{7} \widetilde{\mathcal{F}}_i h_i(\theta_p, \theta_q, \chi)\right)$$

i	$\widetilde{\mathcal{F}}_i^Z$ $(m_\ell = 0)$	$\widetilde{\mathcal{F}}_i^Z$ $(m_\ell = m_\tau)$	$h_i(\theta_p, \theta_q, \chi)$
1	-0.9115	-0.6257	$P_2(\cos\theta_q)$
2	-0.9115	-0.9391	$P_2(\cos\theta_p)$
3	$+0.9557$	$+0.6561$	$P_2(\cos\theta_p)P_2(\cos\theta_q)$
4	$+0.0030$	$+0.0023$	$\cos\theta_p\cos\theta_q$
5	$+0.0167$	$+0.0132$	$\sin\theta_p\sin\theta_q\cos\chi$
6	$+0.1875$	$+0.1287$	$\sin 2\theta_p\sin 2\theta_q\cos\chi$
7	$+0.0332$	$+0.0228$	$\sin^2\theta_p\sin^2\theta_q\cos 2\chi$

Table: Numerical results for the normalized coefficient functions $\widetilde{\mathcal{F}}_i(q^2)$ at $q^2 = 50\,\mathrm{GeV}^2$. Legendre polynomial $P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$.

Helicity composition of the gauge bosons

On-shell – off-shell case

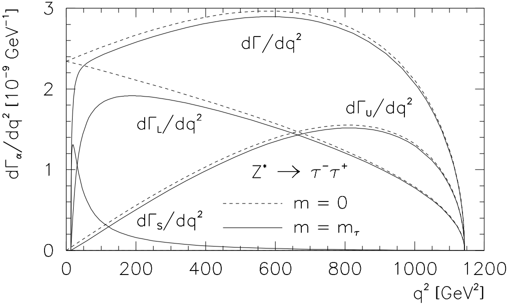

Figure: Differential rates $d\Gamma^Z_\alpha/dq^2$ (indices $\alpha = U, L, S$ for the decay $H \to Z(\to e^+e^-) + Z^*(\to \ell^+\ell^-)$ with $m_\ell = 0$ and $m_\ell = m_\tau$.

- **L refers to** (Z^*Z^*) **double density matrix element** ρ_{LL}
- **U** – // – ρ_{TT}
- **S** – // – ρ_{LS}

On-shell – off-shell vs. Off-shell – off-shell decays

	Γ^Z [GeV]	Γ^Z_U/Γ^Z	Γ^Z_L/Γ^Z	Γ^Z_S/Γ^Z
$H \to Z(\to e^+e^-) + Z^*(\to \ell^+\ell^-)$				
$(m_\ell = m_\mu)$	1.01×10^{-7} GeV	0.41	0.59	0
$(m_\ell = m_\tau)$	0.97×10^{-7} GeV	0.41	0.55	0.04
$H \to Z^*(\to e^+e^-) + Z^*(\to \ell^+\ell^-)$				
$(m_\ell = m_\mu)$	1.22×10^{-7} GeV	0.39	0.61	0
$(m_\ell = m_\tau)$	1.20×10^{-7} GeV	0.39	0.59	0.02

Table: Total and normalized partial decay rates for the four-body decays $H \to Z(\to e^+e^-) + Z^*(\to \ell^+\ell^-)$ and $H \to Z^*(\to e^+e^-) + Z^*(\to \ell^+\ell^-)$.

Some final remarks

- **There is a great deal of interesting physics in the process $H \to \tau^+\tau^-\tau^+\tau^-$. Gauge invariance, identical particle effects, lepton mass effects, topology of Feynman diagrams, momentum mismatch, no external off-shell gauge bosons, ...**
- **Experimentalists are getting better and better in the identification of tauons. Taus from the process $H \to \tau^+\tau^-\tau^+\tau^-$ are used to train tau finding algorithms for the gold plated process $H \to \tau^+\tau^-$.**
- **My office mate and friend Jian Wang calculated many of the rates in minutes using MadGraph. We took two years. There is essential agreement but some differences in detail. My congratulations for the MadGraph team for having done a tantalizingly good job.**
- **Many thanks to my collaborators Stefan Groote and Lauri Kaldamäe for collaboration and Matthias Neubert for support.**