pNRQCD determination of E1 radiative transitions

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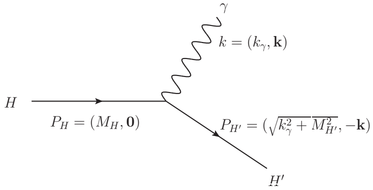
Outline

Motivation

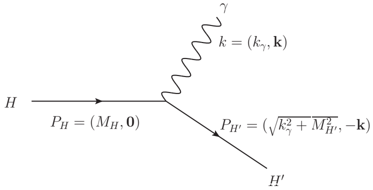


$$\Leftrightarrow \quad 2^3P_{J=0,1,2} \to 1^3S_1 + \gamma$$

Why to study radiative decays?

- They are one of the possible ways to access heavy quarkonium states.
- In particular, they are one of the simplest transitions to be measured for quarkonia below open-flavor threshold.
- They provide a probe to the internal structure of hadrons

Motivation



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$$\Gamma^{(0)}_{E1} = \frac{4}{9}\alpha_{e/m} e_Q^2 k_\gamma^3 \left[\int_0^\infty \mathrm{d}r\, r^2 R_{n'0}(r)\, r\, R_{n1}(r)\right]^2 \qquad \Delta\ell = 1, \Delta s = 0$$

$$\Gamma^{(0)}_{M1} = \frac{4}{3}\alpha_{e/m} e_Q^2 \frac{k_\gamma^3}{m^2} \qquad \Delta\ell = 0, \Delta s = 1$$

Motivation & Introduction

Beyond LO, EFTs are the tools of choice for systematic computations.

pNRQCD

The effective Lagrangian is organized as an expansion in $\frac{1}{m}$, $\alpha_s(m)$ and $\frac{1}{p} \sim r$:

$$\mathcal{L}_{\mathrm{pNRQCD}} = \int \mathrm{d}^3 r \sum_n \sum_k \frac{c_n(\alpha_s(m),\nu)}{m^n} \times V_{n,k}(r,\nu) r^k \times \mathcal{O}_k(\nu, mv^2, \dots),$$

where a multipole expansion of the gluon field has been performed.
The Wilson coefficients of pNRQCD depend on the distance r (and scale ν):

- The $V_{n,0}$ are the potentials in the Schrödinger equation.
- The $V_{n,k\neq 0}$ are the couplings with the low-energy degrees of freedom, which provide corrections to the potential picture.

Motivation & Introduction

The decay width in pNRQCD

$$\Gamma_{n^3P_j \to n'^3S_1} = \Gamma^{(0)}_{E_1}\Bigg[1 + R^{S=1}(J) - \frac{k_\gamma}{6m} - \frac{k_\gamma^2}{60}\frac{I_5^{(0)}(n1\to n'0)}{I_3^{(0)}(n1\to n'0)}$$
$$+\left(\frac{J(J+1)}{2}-2\right)\left(-(1+\kappa_Q^{em})\frac{k_\gamma}{2m} + \frac{1}{m^2}(1+2\kappa_Q^{em})\frac{I_2^{(1)}(n1\to n'0)+2I_1^{(0)}(n1\to n'0)}{I_3^{(0)}(n1\to n'0)}\right)\Bigg]$$

$$I_N^{(k)}(n\ell \to n'\ell') = \int_0^\infty \mathrm{d}r\, r^2 r^{N-2} R_{n'\ell'}\left(\frac{\mathrm{d}^k}{\mathrm{d}r^k}R_{n\ell}\right)$$

Experimental branching ratios - PDG[1]

Mode	Fraction (Γ_i/Γ)
$\chi_{b_0}(1P) \to \Upsilon(1S) + \gamma$	$(1.76 \pm 0.35)\%$
$\chi_{b_1}(1P) \to \Upsilon(1S) + \gamma$	$(33.9 \pm 2.2)\%$
$\chi_{b_2}(1P) \to \Upsilon(1S) + \gamma$	$(19.1 \pm 1.2)\%$

[1] K. A. Olive et al. “Review of Particle Physics”. In: *Chin. Phys.* (1 2014)

Quantum mechanical perturbation theory I

We assume Coulombic bound states (pNRQCD at weak coupling):

The Schrödinger equation

$$\left(\frac{-1}{2m_{\mathrm{r}}}\nabla^2 + V_s^{(0)}(r)\right)\psi_{n\ell m}^{(0)}(\vec{r}) = E_n^{(0)}\psi_{n\ell m}^{(0)}(\vec{r})$$

The static potential $V_s = -C_F\frac{\alpha_s}{r}\left[1 + \sum_{k=1}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^k a_k(r)\right]$ with coefficients

$$a_1(r) = a_1 + 2\beta_0 \ln(\nu e^{\gamma_E} r)$$

$$a_2(r) = a_2 + \frac{\pi^2}{3}\beta_0^2 + (4a_1\beta_0 + 2\beta_1)\ln(\nu e^{\gamma_E} r) + 4\beta_0^2\ln^2(\nu e^{\gamma_E} r)$$

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The solution:

$$\psi_{n\ell m}^{(0)}(\vec{r}) = R_{nl}(r)Y_{\ell m}(\Omega_r) = N_{n\ell}\, e^{-\frac{\rho_n}{2}}\rho_n^{\ell}\, L_{n-\ell-1}^{2\ell+1}(\rho_n)Y_{\ell m}(\Omega_r)$$

$$E_n^{(0)} = -\frac{m_{\mathrm{r}} C_F^2\alpha_s^2}{2n^2}\,, \quad \rho_n = \frac{2r}{na}$$

Quantum mechanical perturbation theory I

We can organize relativistic corrections[2] as follows

$$\delta H = -\frac{\nabla^4}{4m^3} + \frac{V^{(1)}}{m} + \frac{V_{SI}^{(2)}}{m^2} + \frac{V_{SD}^{(2)}}{m^2}$$

$$V_{SI}^{(2)} = V_r^{(2)} + \frac{1}{2}\{V_{p^2}^{(2)}, -\nabla^2\} + V_{L^2}^{(2)}\vec{L}^2$$
$$V_{SD}^{(2)} = V_{LS}^{(2)}\vec{L}\cdot\vec{S} + V_{S^2}^{(2)}\vec{S}^2 + V_{S_{12}}^{(2)}S_{12}\,.$$

$$V^{(1)} = -\frac{C_F C_A \alpha_s^2}{2r^2}\,, \quad V_r^{(2)} = \pi C_F \alpha_s \delta^{(3)}(\vec{r})\,,$$
$$V_{p^2}^{(2)} = -\frac{C_F\alpha_s}{r}\,, \quad V_{L^2}^{(2)} = \frac{C_F\alpha_s}{2r^3}\,, \quad V_{LS}^{(2)} = \frac{3C_F\alpha_s}{2r^3}\,,$$
$$V_{S^2}^{(2)} = \frac{4\pi C_F\alpha_s}{3}\delta^{(3)}(\vec{r})\,, \quad V_{S_{12}}^{(2)} = \frac{C_F\alpha_s}{4r^3}\,.$$

[2] Nora Brambilla, Piotr Pietrulewicz, and Antonio Vairo. "Model-independent Study of Electric Dipole Transitions in Quarkonium". In: *Phys. Rev.* (2012). arXiv: 1203.3020 [hep-ph].

Quantum mechanical perturbation theory II

First order correction to the wave function

$$|n\ell\rangle^{(1)} = \sum_{n'\neq n} \frac{\langle n'\ell'|V|n\ell\rangle}{E_n^{(0)} - E_{n'}^{(0)}} |n'\ell'\rangle = \sum_{n'\neq n} \frac{|n'\ell'\rangle\langle n'\ell'|}{E_n^{(0)} - E_{n'}^{(0)}} V|n\ell\rangle$$

$$\sum_{n'\neq n} \frac{|n'\ell'\rangle\langle n'\ell'|}{E_n^{(0)}-E_{n'}^{(0)}} = \sum_{n'} \frac{|n'\ell'\rangle\langle n'\ell'|}{E_n^{(0)}-E_{n'}^{(0)}} - \sum_{n'=n} \frac{|n'\ell'\rangle\langle n'\ell'|}{E_n^{(0)}-E_{n'}^{(0)}} = \lim_{E\to E_n^{(0)}} \left(\frac{\mathbb{1}}{E-H} - \frac{\mathcal{M}(n)}{E-E_n^{(0)}} \right) \equiv \frac{1}{(E_n-H)'}$$

Quantum mechanical perturbation theory II

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The Coulomb green function

$$G'(\vec{r}_1, \vec{r}_2) \equiv (-1) \times \lim_{E\to E_n} \left(G(\vec{r}_1, \vec{r}_2, E) - \frac{|\psi_{n\ell}|^2}{E - E_n} \right)$$

- $G(\vec{r}_1, \vec{r}_2, E) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} P_\ell(\hat{r}_1 \cdot \hat{r}_2) G_\ell(r_1, r_2)$

  $G_\ell(r_1, r_2) = \sum_{\nu=\ell+1}^{\infty} G_{\nu\ell}(r_1, r_2)$

  $G_{\nu\ell}(r_1, r_2) = m_r a^2 \left(\frac{\nu^4}{\lambda}\right) \frac{R_{\nu\ell}(\rho_{\lambda,1}) R_{\nu\ell}(\rho_{\lambda,2})}{\nu-\lambda}$
- $E = -\frac{m_r C_F^2 \alpha_s^2}{2\lambda^2} = E_n(1-\epsilon)\big|_{\epsilon\to 0} = -\frac{m_r C_F^2 \alpha_s^2}{2n^2}(1-\epsilon)\Big|_{\epsilon\to 0} \quad \Rightarrow \quad \lambda = \frac{n}{\sqrt{1-\epsilon}}\Big|_{\epsilon\to 0}$

Quantum mechanical perturbation theory II

$$
\begin{aligned}
\langle n'\ell'|\mathcal{O}|n\ell\rangle^{(1)} &= \langle n'\ell'|\mathcal{O}\frac{1}{(E_n-H)'}V|n\ell\rangle \\
&= \int \mathrm{d}^3 r_1\, \mathrm{d}^3 r_2\, \psi^*_{n'\ell'}(\vec{r}_2)\,\mathcal{O}(\vec{r}_2)\, G'(\vec{r}_2,\vec{r}_1)\, V(\vec{r}_1)\,\psi_{n\ell}(\vec{r}_1) \\
&= \lim_{\epsilon\to 0,\mathcal{O}(\epsilon^0)} -N_{n'\ell'}N_{n\ell}\frac{4m_\mathrm{r}}{a\lambda}\sum_{\ell''=0}^{\infty}\sum_{s=0}^{\infty}\frac{s!}{(s+\ell''+1-\lambda)(s+2\ell''+1)!} \\
&\times \int_0^\infty \mathrm{d}r_1\, r_1^2 V(r_1)\rho_{\lambda,1}^{\ell''}\rho_{n,1}^{\ell}\mathrm{e}^{-\frac{1}{2}\rho_{\lambda,1}}\mathrm{e}^{-\frac{1}{2}\rho_{n,1}}L_s^{2\ell''+1}(\rho_{\lambda,1})L_{n-\ell-1}^{2\ell+1}(\rho_{n,1}) \\
&\times \int_0^\infty \mathrm{d}r_2\, r_2^2 \mathcal{O}(r_2)\rho_{\lambda,2}^{\ell''}\rho_{n',2}^{\ell'}\mathrm{e}^{-\frac{1}{2}\rho_{\lambda,2}}\mathrm{e}^{-\frac{1}{2}\rho_{n',2}}L_s^{2\ell''+1}(\rho_{\lambda,2})L_{n'-\ell'-1}^{2\ell'+1}(\rho_{n',2}) \\
&\times \sum_{m''=-\ell''}^{\ell''}\int \mathrm{d}\Omega_1\, Y_{\ell''}^{m''*}(\Omega_1)V(\Omega_1)Y_\ell^m(\Omega_1)\int \mathrm{d}\Omega_2\, Y_{\ell'}^{m'*}(\Omega_2)\mathcal{O}(\Omega_2)Y_{\ell''}^{m''}(\Omega_2)
\end{aligned}
$$

- We calculate with $\lambda = n$ the finite contribution, which means performing the sum in s without the pole ($s \neq n-\ell''-1$)
- We compute the divergent term of the sum using $\lambda = \frac{n}{\sqrt{1-\epsilon}}$, expand in ϵ and finally picking up the finite term only ($\mathcal{O}(\epsilon^0)$ when $\epsilon \to 0$).

Quantum mechanical perturbation theory III

Second order correction to the wave function

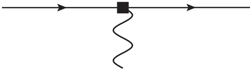
$$|n\ell\rangle^{(2)} = \sum_{k_1\neq n}\left[\sum_{k_2\neq n}\frac{\langle k_1\ell|V|k_2\ell\rangle\langle k_2\ell|V|n\ell\rangle}{(E_n-E_{k_1})(E_n-E_{k_2})} - \frac{\langle k_1\ell|V|n\ell\rangle\langle n\ell|V|n\ell\rangle}{(E_n-E_{k_1})^2}\right]|k_1\ell\rangle$$
$$-\frac{1}{2}\sum_{k_2\neq n}\frac{|\langle k_2\ell|V|n\ell\rangle|^2}{(E_n-E_{k_2})^2}|n\ell\rangle$$

$$\langle n'\ell'|\mathcal{O}|n\ell\rangle^{(2)} = \langle n'\ell'|\mathcal{O}\tfrac{1}{(E_n-H)'}V\tfrac{1}{(E_n-H)'}V|n\ell\rangle - \langle n\ell|V|n\ell\rangle\langle n'\ell'|\mathcal{O}\tfrac{1}{(E_n-H)'}\mathbb{1}\tfrac{1}{(E_n-H)'}V|n\ell\rangle$$
$$-\tfrac{1}{2}\langle n'\ell'|\mathcal{O}|n\ell\rangle\langle n\ell|V\tfrac{1}{(E_n-H)'}\mathbb{1}\tfrac{1}{(E_n-H)'}V|n\ell\rangle$$

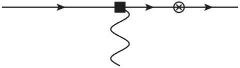
$$\propto \lim_{\substack{\epsilon\to 0,\,\mathcal{O}(\epsilon^0)\\ \epsilon'\to 0,\,\mathcal{O}(\epsilon'^0)}} (-1)^2\sum_{s,s'=0}^{\infty}\int \mathrm{d}\vec{r}_1\mathrm{d}\vec{r}_2\mathrm{d}\vec{r}_3\ldots$$

$\times$	$s'\neq n-\ell-1$	$s'=n-\ell-1$
$s\neq n-\ell-1$	$\lambda=n,\lambda'=n$	$\lambda=n,\lambda'=\frac{n}{\sqrt{1-\epsilon'}}$
$s=n-\ell-1$	$\lambda=\frac{n}{\sqrt{1-\epsilon}},\lambda'=n$	$\lambda=\frac{n}{\sqrt{1-\epsilon}},\lambda'=\frac{n}{\sqrt{1-\epsilon'}}$

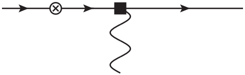
Perturbation theory & decay width



$$= \frac{4}{9}\alpha_{e/m} e_Q^2 k_\gamma^3 \left[I_3^{(0)}(n1 \to n'0) \right]^2 \propto |\,^{(0)}\langle n'0|\mathcal{O}_{E1}|n1\rangle^{(0)}|^2$$



$$\propto |\,^{(1)}\langle n', \ell', s', j'; k_\gamma|\mathcal{O}_{E1}|n, \ell, s, j; 0\rangle^{(0)}|^2$$



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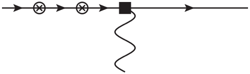
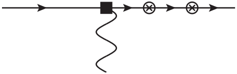
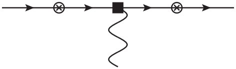


$$\propto |\,^{(0)}\langle n', \ell', s', j'; k_\gamma|\mathcal{O}_{E1}|n, \ell, s, j; 0\rangle^{(2)}|^2$$



$$\propto |\,^{(2)}\langle n', \ell', s', j'; k_\gamma|\mathcal{O}_{E1}|n, \ell, s, j; 0\rangle^{(0)}|^2$$



$$\propto |\,^{(1)}\langle n', \ell', s', j'; k_\gamma|\mathcal{O}_{E1}|n, \ell, s, j; 0\rangle^{(1)}|^2$$

Decay widths of electric dipole transitions

$$\Gamma = \frac{k_\gamma}{(2\pi)} \overline{|\mathcal{M}_{fi}|^2} = \frac{k_\gamma}{(2\pi)} \frac{1}{N_\lambda} \sum_{\lambda,\lambda',\sigma} |\mathcal{M}_{fi}|^2$$

$$\mathcal{O}_{E1} = e e_Q (\vec{r} \cdot \vec{E}) = e e_Q (\hat{e}_r \cdot \vec{E}) \sum_{\mu=-1}^{1} \sqrt{\frac{4\pi}{3}} r Y_1^{\mu *}(\Omega_r)$$

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$$\Phi^{(0)}_{n^3P_0}(\mathbf{r}) = \sqrt{\frac{1}{4\pi}}\, R_{n1}(r)\, \frac{\vec{\sigma}\cdot\vec{r}}{\sqrt{2}} \qquad \Phi^{(0)}_{n^3P_1(\lambda)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}}\, R_{n1}(r)\, \frac{\vec{\sigma}\cdot(\vec{r}\times\vec{e}_{n^3P_1}(\lambda))}{\sqrt{2}}$$

$$\Phi^{(0)}_{n^3P_2(\lambda)}(\mathbf{r}) = \sqrt{\frac{3}{4\pi}}\, R_{n1}(r)\, \frac{\vec{\sigma}^i\, h^{ij}_{n^3P_2}(\lambda)\, \hat{r}^j}{\sqrt{2}} \qquad \Phi^{(0)}_{n^3S_1(\lambda)}(\mathbf{r}) = \frac{1}{\sqrt{4\pi}}\, R_{n0}(r)\, \frac{\vec{\sigma}\cdot\vec{e}_{n^3S_1}(\lambda)}{\sqrt{2}}$$

where the polarization vectors and tensors are normalized as

$$\vec{e}^{\,*}_{n^3S_1}(\lambda)\cdot\vec{e}_{n^3S_1}(\lambda') = \vec{e}^{\,*}_{n^1P_1}(\lambda)\cdot\vec{e}_{n^1P_1}(\lambda') = \vec{e}^{\,*}_{n^3P_1}(\lambda)\cdot\vec{e}_{n^3P_1}(\lambda') = \delta_{\lambda\lambda'}$$

$$h^{ij\,*}_{n^3P_2}(\lambda)\, h^{ji}_{n^3P_2}(\lambda') = \delta_{\lambda\lambda'}$$

The tensor potential: *s*-*d*-wave mixing

- $V_T(\vec{r}) = \frac{1}{m^2}\frac{C_F\alpha_s}{4r^3}S_{12}(\hat{r}) \equiv V_{S_{12}}S_{12}(\hat{r})$ mixes states with $\Delta\ell = 2$, since $S_{12}(\hat{r}) = 3(\hat{r}\cdot\vec{\sigma}_1)(\hat{r}\cdot\vec{\sigma}_2) - \vec{\sigma}_1\cdot\vec{\sigma}_2 = 3\sqrt{5}\left\{\{\hat{r}\otimes\hat{r}\}_2\otimes\{\vec{\sigma}_1\otimes\vec{\sigma}_2\}_2\right\}_0$
- diagonal part is included in the spectrum[3], perturbative treatment of off-diagonal matrix elements is derived in[4]

[3] Clara Peset, Antonio Pineda, and Maximilian Stahlhofen. "Potential NRQCD for unequal masses and the Bc spectrum at NNNLO". . In: (2015). arXiv: 1511.08210 [hep-ph].

[4] Yuichiro Kiyo, Go Mishima, and Yukinari Sumino. "On enhanced corrections from quasi-degenerate states to heavy quarkonium observables". In: (2016). arXiv: 1607.05510 [hep-ph].

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$$\langle n'\ell's'j'm'|S_{12}|n\ell sjm\rangle = 3\delta_{j'j}\delta_{m'm}(-1)^{j+\ell+s'} \begin{Bmatrix} \ell' & \ell & 2 \\ s & s' & j \end{Bmatrix} \langle \ell'||\{\hat{r}\otimes\hat{r}\}_2||\ell\rangle\langle s'||\{\vec{\sigma}_1\otimes\vec{\sigma}_2\}_2||s\rangle$$

using the Wigner-Eckart theorem we find

$$\langle n'\ell's'j'm'|S_{12}|n\ell sjm\rangle = \delta_{j'j}\delta_{m'm}\delta_{s's}\delta_{s1}(-1)^{j+s'}2\sqrt{30}\sqrt{(2\ell+1)(2\ell'+1)} \begin{Bmatrix} \ell' & \ell & 2 \\ s & s' & j \end{Bmatrix} \begin{pmatrix} \ell' & 2 & \ell \\ 0 & 0 & 0 \end{pmatrix},$$

where $\begin{pmatrix} \ell' & 2 & \ell \\ 0 & 0 & 0 \end{pmatrix} \neq 0 \quad \Leftrightarrow \quad \Delta\ell = 2$

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Parameters

$$n_f = 3,\ e_Q = -\frac{1}{3},\ \alpha_{e/m} = \frac{1}{137.0359991}$$

state	$\Upsilon(1S)$	$\chi_{b_0}(1P)$	$\chi_{b_1}(1P)$	$\chi_{b_2}(1P)$
exp. masses[5] $[GeV]$	9.46030	9.85944	9.89278	9.91221

$$k_\gamma = \frac{m_i^2 - m_f^2}{2m_i} = \begin{cases} 0.391061\ GeV\,, & \chi_{b_0}(1P) \to \Upsilon(1S) + \gamma \\ 0.423027\ GeV\,, & \chi_{b_1}(1P) \to \Upsilon(1S) + \gamma \\ 0.441608\ GeV\,, & \chi_{b_2}(1P) \to \Upsilon(1S) + \gamma \end{cases}$$

$$M^{\text{exp.}}(\Upsilon(1S)) = 2m_b(\nu) + E_n^{(0)}(m_b(\nu), \alpha_s(\nu))$$

$$a(\nu) = \frac{1}{C_F m_r(\nu)\alpha_s(\nu)},\ \kappa_Q^{em} \equiv C_F^{e/m} - 1 = C_F \frac{\alpha_s(m_b(\nu))}{2\pi}$$

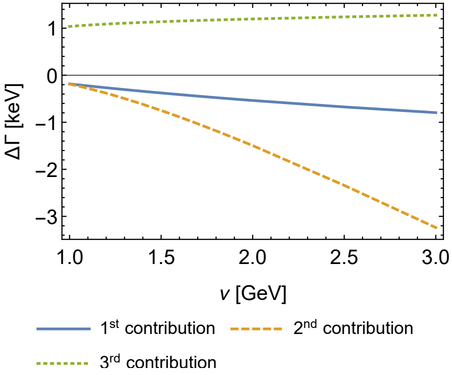
$\nu\,[GeV]$	1.0	1.5	2.0	2.5	3.0
$\alpha_s^{4\text{ loops, } n_f=3}(\nu)$[6]	0.479778	0.345836	0.295478	0.265205	0.245092
$m_b(\nu)\,[GeV]$	4.98515	4.86138	4.82374	4.80525	4.79415
$a(\nu)\,[GeV^{-1}]$	0.627151	0.885286	1.0524	1.17705	1.27659
$\alpha_s^{4\text{ loops, } n_f=3}(m_b(\nu))$	0.203206	0.204915	0.20545	0.205715	0.205876
$C_F^{e/m}(m_b(\nu))$	1.0431217	1.0434843	1.0435978	1.0436541	1.0436882

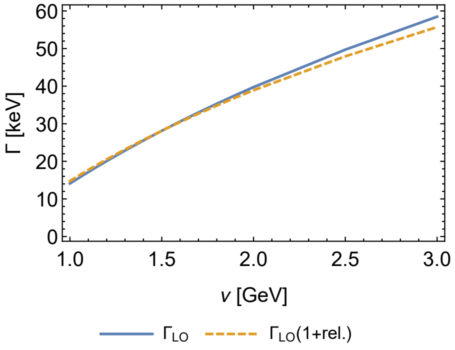
[6] K. A. Olive et al. "Review of Particle Physics". In: *Chin. Phys.* (1 2014)

[6] K. G. Chetyrkin, Johann H. Kuhn, and M. Steinhauser. "RunDec: A Mathematica package for running and decoupling of the strong coupling and quark masses". In: *Comput. Phys. Commun.* (2000). arXiv: hep-ph/0004189 [hep-ph]

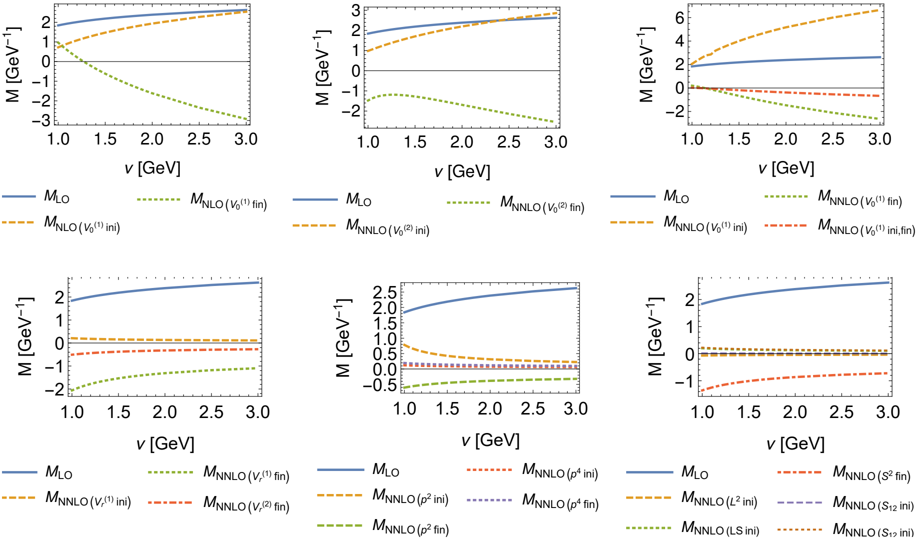
Relativistic corrections to the Lagrangian

$$\Gamma_{n^3P_J\to n'^3S_1} = \Gamma^{(0)}_{E_1}\left[1 + R^{S=1}(J) - \frac{k_\gamma}{6m} - \frac{k_\gamma^2}{60}\frac{I_5^{(0)}(n1\to n'0)}{I_3^{(0)}(n1\to n'0)}\right.$$
$$\left. + \left(\frac{J(J+1)}{2} - 2\right)\left(-(1+\kappa_Q^{em})\frac{k_\gamma}{2m} + \frac{1}{m^2}(1+2\kappa_Q^{em})\frac{I_2^{(1)}(n1\to n'0)+2I_1^{(0)}(n1\to n'0)}{I_3^{(0)}(n1\to n'0)}\right)\right]$$

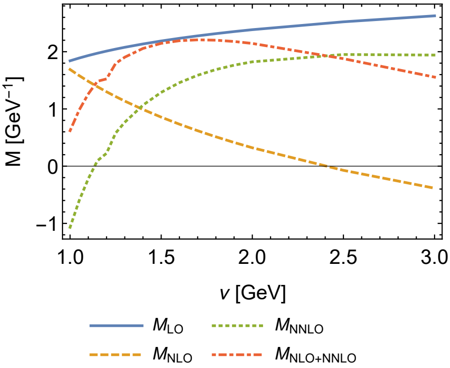


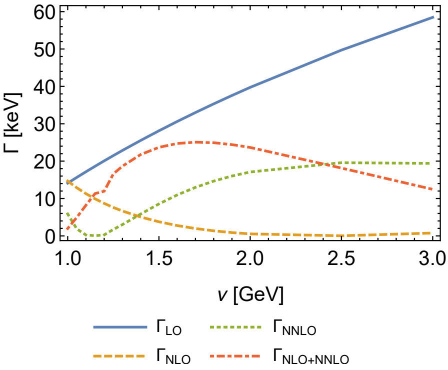


Wave function corrections - partial matrix elements

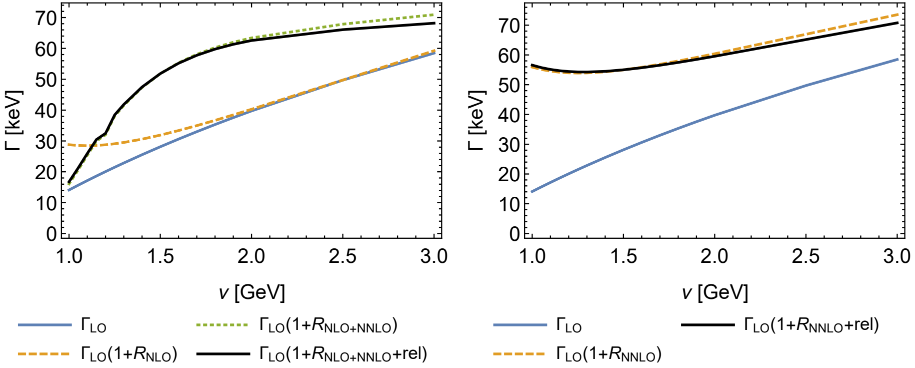


Wave function corrections - matrix elements & decay widths





Wave function corrections - decay widths



Summary & Outlook

Summary

- We presented numerical results for the $E1$-transition $2^3P_{J=0,1,2} \to 1^3S_1 + \gamma$ up to NNLO ($\mathcal{O}(m\alpha_s^4)$) in pNRQCD at weak coupling.
- The results are reasonable but a non-negligible dependence on the scale ν is observable via:
  - Direct ν dependence induced by the corrections to the static potential.
  - Indirect ν dependence via $\alpha_s(\nu)$ and $a(\alpha_s(\nu))$.

Outlook

- Incorporate radiative corrections to the static potential in the Schrödinger equation should diminish the ν dependence. This has been done succesfully for the $M1$ transition[7].
- Non-perturbative effects might be of the same size as the perturbative corrections and should be incorporated.

[7] Nora Brambilla, Yu Jia, and Antonio Vairo. "Model-independent study of magnetic dipole transitions in quarkonium". In: *Phys. Rev.* (2006). arXiv: hep-ph/0512369 [hep-ph];
Antonio Pineda and Jorge Segovia. "Improved determination of heavy quarkonium magnetic dipole transitions in potential nonrelativistic QCD". . In: *Phys. Rev.* 7 (2013). arXiv: 1302.3528 [hep-ph]

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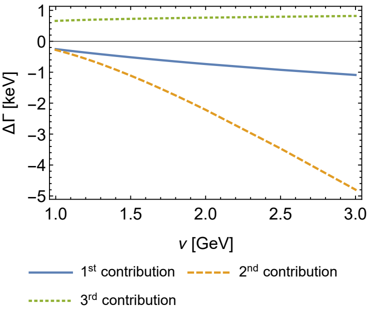
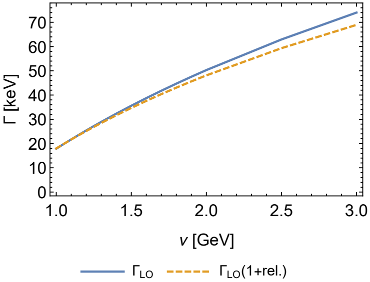
BACKUP SLIDES


ΔΓ [keV]
v [GeV]
1.0
1.5
2.0
2.5
3.0
1
0
−1
−2
−3
−4
−5
1st contribution
2nd contribution
3rd contribution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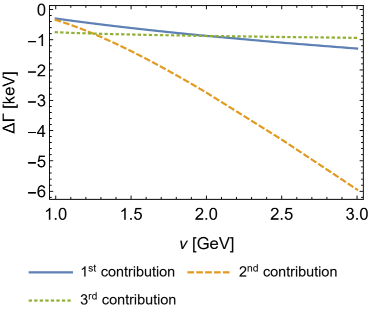
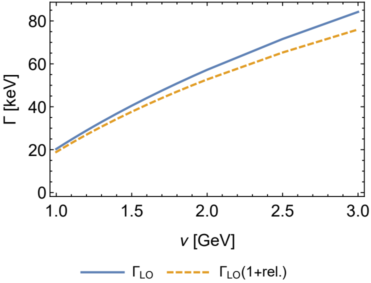
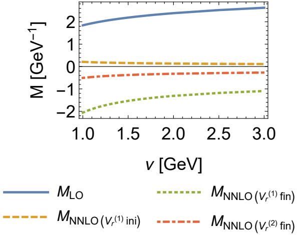
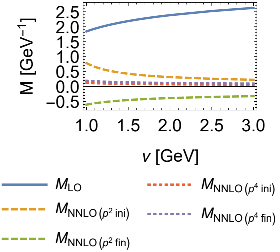
v [GeV]
0
10
20
30
40
50
60
70
1.0
1.5
2.0
2.5
3.0
Γ_LO
Γ_LO(1+rel.)


ΔΓ [keV]
v [GeV]
0
−1
−2
−3
−4
−5
−6
1.0
1.5
2.0
2.5
3.0
1st contribution
2nd contribution
3rd contribution

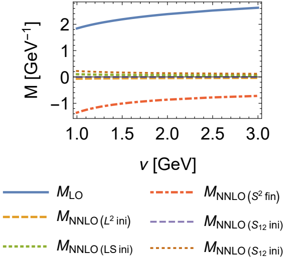

Γ [keV]
v [GeV]
0
20
40
60
80
1.0
1.5
2.0
2.5
3.0
Γ_LO
Γ_LO(1+rel.)



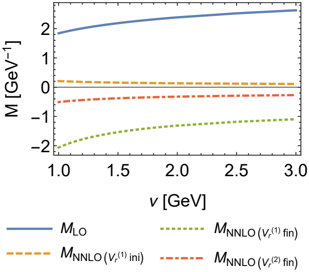
M [GeV^{-1}]
2
1
0
−1
−2
1.0 1.5 2.0 2.5 3.0
v [GeV]
M_{LO}
$M_{\text{NNLO}\,(V_r^{(1)}\,\text{fin})}$
$M_{\text{NNLO}\,(V_r^{(1)}\,\text{ini})}$
$M_{\text{NNLO}\,(V_r^{(2)}\,\text{fin})}$



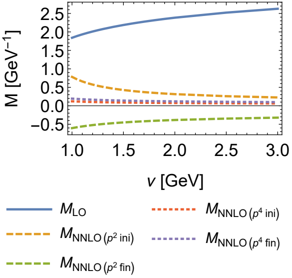
M [GeV^{-1}]
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
1.0 1.5 2.0 2.5 3.0
v [GeV]
M_{LO}
$M_{\text{NNLO}\,(p^4\,\text{ini})}$
$M_{\text{NNLO}\,(p^2\,\text{ini})}$
$M_{\text{NNLO}\,(p^4\,\text{fin})}$
$M_{\text{NNLO}\,(p^2\,\text{fin})}$



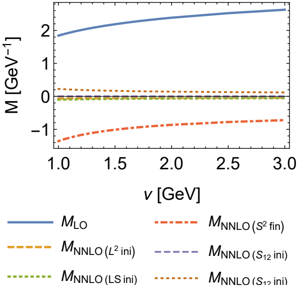
M [GeV^{-1}]
2
1
0
−1
1.0 1.5 2.0 2.5 3.0
v [GeV]
M_{LO}
$M_{\text{NNLO}\,(S^2\,\text{fin})}$
$M_{\text{NNLO}\,(L^2\,\text{ini})}$
$M_{\text{NNLO}\,(S_{12}\,\text{ini})}$
$M_{\text{NNLO}\,(LS\,\text{ini})}$
$M_{\text{NNLO}\,(S_{12}\,\text{ini})}$



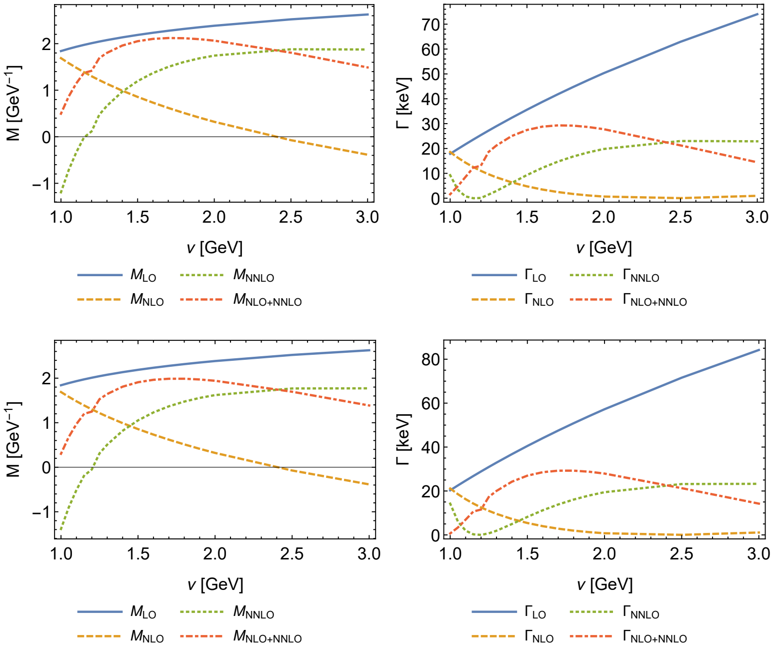
M [GeV^{-1}]
2
1
0
−1
−2
1.0 1.5 2.0 2.5 3.0
v [GeV]
M_{LO}
$M_{\text{NNLO}\,(V_r^{(1)}\,\text{fin})}$
$M_{\text{NNLO}\,(V_r^{(1)}\,\text{ini})}$
$M_{\text{NNLO}\,(V_r^{(2)}\,\text{fin})}$

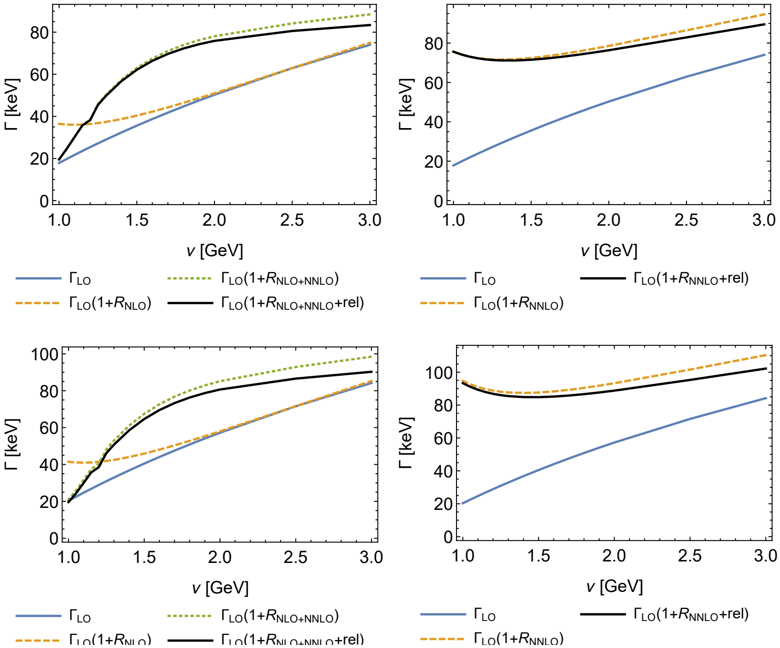


M [GeV^{-1}]
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
1.0 1.5 2.0 2.5 3.0
v [GeV]
M_{LO}
$M_{\text{NNLO}\,(p^4\,\text{ini})}$
$M_{\text{NNLO}\,(p^2\,\text{ini})}$
$M_{\text{NNLO}\,(p^4\,\text{fin})}$
$M_{\text{NNLO}\,(p^2\,\text{fin})}$



M [GeV^{-1}]
2
1
0
−1
1.0 1.5 2.0 2.5 3.0
v [GeV]
M_{LO}
$M_{\text{NNLO}\,(S^2\,\text{fin})}$
$M_{\text{NNLO}\,(L^2\,\text{ini})}$
$M_{\text{NNLO}\,(S_{12}\,\text{ini})}$
$M_{\text{NNLO}\,(LS\,\text{ini})}$
$M_{\text{NNLO}\,(S_{12}\,\text{ini})}$


M [GeV⁻¹]
v [GeV]
M_LO
M_NNLO
M_NLO
M_NLO+NNLO
Γ [keV]
v [GeV]
Γ_LO
Γ_NNLO
Γ_NLO
Γ_NLO+NNLO
M [GeV⁻¹]
v [GeV]
M_LO
M_NNLO
M_NLO
M_NLO+NNLO
Γ [keV]
v [GeV]
Γ_LO
Γ_NNLO
Γ_NLO
Γ_NLO+NNLO


Γ [keV]
v [GeV]
1.0 1.5 2.0 2.5 3.0
0 20 40 60 80 100
Γ_LO
Γ_LO(1+R_NLO)
Γ_LO(1+R_NLO+NNLO)
Γ_LO(1+R_NLO+NNLO+rel)
Γ_LO(1+R_NNLO+rel)
Γ_LO(1+R_NNLO)