Phenomenology of near-threshold states: a practical parametrisation for the line shapes

A.V. Nefediev

(ITEP & MEPhI, Moscow, Russia)

in collaboration with

F.-K. Guo, C. Hanhart, Yu. S. Kalashnikova, P. Matuschek, R. V. Mizuk, Q. Wang, J.-L. Wynen

Confinement XII

Outline

- Motivation of the research
- Coupled-channel approach to near-threshold states
- Practical parametrisation for the line shapes
- A paradigmatic example—combined data analysis for

$$\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi B^{(*)}\bar{B}^*$$
$$\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi\pi\Upsilon(nS) \quad n = 1,2,3$$
$$\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi\pi h_b(mP) \quad m = 1,2$$

$$Z_b = Z_b(10610) \qquad Z_b' = Z_b(10650)$$

- Nature of $Z_b(10610)$ and $Z_b(10650)$ from data
- Conclusions

Relevant referenes:
C. Hanhart et al. Phys. Rev. Lett. 115 (2015), 202001 [arXiv:1507.00382 [hep-ph]]
F.-K. Guo et al., Phys. Rev. D 93 (2016), 074031 [arXiv:1602.00940 [hep-ph]]

Motivation

Experimental background:

- Many exotic hadrons are discovered which do not fit into simple Quark Model picture [$Z_b(10610)$ & $Z_b(10650)$]
- Most of them reside near strong thresholds [$B\bar{B}^*$ & $B^*\bar{B}^*$]
- Overlapping structures [$Z_b(10610)$ & $Z_b(10650)$]

Goal:

- Simple but phenomenologically adequate tool to analyse data on exotic states
- Preserve unitarity and analyticity
- Combined analysis of all relevant channels $\Longrightarrow$ use full information contained in the data [$B\bar{B}^*$, $B^*\bar{B}^*$, $\pi\Upsilon(1S)$, $\pi\Upsilon(2S)$, $\pi\Upsilon(3S)$, $\pi h_b(1P)$, $\pi h_b(2P)$]
- Override gap between theory and experiment

Why not just to use Breit-Wigners?

Note:

BW implies substitution loop operator $\to$ constant width

Then:

- No threshold phenomena in BW
  (Im part does not change across threshold)
- Notions "mass" and "width" are misleading near threshold(s)
  (e.g. for cusp $M_{\rm peak} = M_{\rm threshold}$ and $\Gamma_{\rm visible} < \sum \Gamma_{\rm partial}$)
- BW has problems with analyticity
  (Only one pole of two symmetric poles is picked up. This works fine near the resonance but both poles are important near threshold)
- Naive sum of BW's violates unitarity
  ($\text{Im}(BW) \propto |BW|^2$ but $\text{Im}(BW_1 + BW_2) \not\propto |BW_1 + BW_2|^2$)

Conclusion:

BW's should never be used for near-threshold states

Alternative approach: Coupled channels

- The most general formulation of the problem:
  - $N_{\rm p} = \#$ of bare poles (elementary states) $[N_{\rm p} = 0]$
  - $N_{\rm e} = \#$ of elastic (open-flavour) channels $[N_{\rm e} = 2]$
  - $N_{\rm in} = \#$ of inelastic (hidden-flavour) channels $[N_{\rm in} = 5]$
- Lippmann-Schwinger equations (LSE) used guarantee that
  - Unitarity is preserved (all channels iterated to all orders)
  - Threshold effects are captured (width $\to$ loop operators)
  - Analyticity is preserved (both Re(loop) and Im(loop) kept)
- Parameters (couplings *etc*) have clear physical interpretation
- Additional input (symmetries, lattice measurements, theoretical predictions, etc) is straightforward to implement

Coupled channels: Problems and solutions

Problems:

- Typically, $N_{\rm p}=0..2$, $N_{\rm e}=1..2$ however $N_{\rm in}\gg 1$
- Extra inelastic channels entail reformulation of entire problem
- LSE cannot be solved analytically in general terms

Simplifications:

- Neglect direct interaction between inelastic channels (for example, $\rho(\bar{Q}Q)\leftrightarrow\omega(\bar{Q}Q)$ or $\pi(\bar{Q}Q)\leftrightarrow\pi(\bar{Q}Q)$)
- Assume elastic-to-inelastic form factors in a separable form

Outcome:

- All channels involved are completely disentangled
- LSE are solved analytically; solution $\to$ parametrisation
- Inelastic channels enter additively (e.g. $\sum_{i=1}^{N_{\rm in}}$ )
- The problem reduces to matrices $N_{\rm e}\times N_{\rm e}$ and $N_{\rm p}\times N_{\rm p}$

Practical parametrisation

- Direct interaction elastic t matrix [2 parameters—see below]
- Couplings

Vertex	Transition
$v_{\alpha a}$	elastic S-wave channels $\Leftrightarrow$ bare poles
$v_{ai}(\boldsymbol{k}) = \lambda_{ai}\lvert\boldsymbol{k}\rvert^{l_i}$	inelastic l_i-wave channels $\Leftrightarrow$ bare poles
$v_{i\alpha}(\boldsymbol{k}) = g_{i\alpha}\lvert\boldsymbol{k}\rvert^{l_i}$	S-wave elastic $\Leftrightarrow$ l_i-wave inelastic channels

$$\left[\begin{array}{lll} g_{[\pi\Upsilon(nS)][B^{(*)}\bar{B}^*]} & (n=1,2,3) & \text{6 parameters} \\ g_{[\pi h_b(mP)][B^{(*)}\bar{B}^*]} & (m=1,2) & \text{4 parameters} \end{array}\right]$$

- Ratios of production sources ξ_α $\left[\xi = \dfrac{g_{[\pi\Upsilon(5S)][B^*\bar{B}^*]}}{g_{[\pi\Upsilon(5S)][B\bar{B}^*]}}\right]$
- Norm in each distribution [7 channels = 7 norms]

Note!

- All parameters are real, imaginary parts come from loops
- If additional inelasticity is needed then data set is incomplete

Constraints from Heavy Quark Spin Symmetry

$m_b \gg \Lambda_{\rm QCD} \quad \Longrightarrow \quad$ Heavy-quark spin decouples

- Spin w.f.'s of $B^{(*)}\bar{B}^*$ pairs with quantum numbers 1^{+-} read

$$\begin{array}{l} |B\bar{B}^*\rangle = 0^-_{\bar{b}b} \otimes 1^-_{\bar{q}q} + 1^-_{\bar{b}b} \otimes 0^-_{\bar{q}q} \\ |B^*\bar{B}^*\rangle = 0^-_{\bar{b}b} \otimes 1^-_{\bar{q}q} - 1^-_{\bar{b}b} \otimes 0^-_{\bar{q}q} \end{array} \Longrightarrow \frac{g_{[\pi h_b(mP)][B^*\bar{B}^*]}}{g_{[\pi h_b(mP)][B\bar{B}^*]}} = -\frac{g_{[\pi \Upsilon(nS)][B^*\bar{B}^*]}}{g_{[\pi \Upsilon(nS)][B\bar{B}^*]}} = 1$$

A.E. Bondar et al. PRD 84 (2011) 054010

- Direct interaction elastic potential

$$V(1^{+-}) = \begin{pmatrix} V_{B\bar{B}^*\to B\bar{B}^*} & V_{B\bar{B}^*\to B^*\bar{B}^*} \\ V_{B^*\bar{B}^*\to B\bar{B}^*} & V_{B^*\bar{B}^*\to B^*\bar{B}^*} \end{pmatrix} \propto \begin{pmatrix} \gamma_s^{-1} + \gamma_t^{-1} & \gamma_s^{-1} - \gamma_t^{-1} \\ \gamma_s^{-1} - \gamma_t^{-1} & \gamma_s^{-1} + \gamma_t^{-1} \end{pmatrix}$$

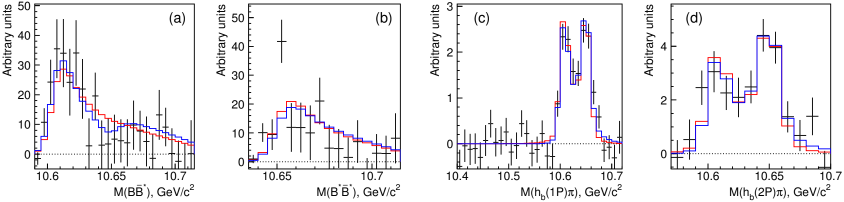
Note! $\gamma_s = \gamma_t$ implies no $B\bar{B}^* \leftrightarrow B^*\bar{B}^*$ direct transitions
Light-quark spin symmetry???

M.B. Voloshin, PRD 93 (2016) 074011

Fits for the data

Fit	γ_s,MeV	γ_t,MeV	ξ	$\frac{g_{[\pi h_b(1P)][B^*\bar{B}^*]}}{g_{[\pi h_b(1P)][B\bar{B}^*]}}$	$\frac{g_{[\pi h_b(2P)][B^*\bar{B}^*]}}{g_{[\pi h_b(2P)][B\bar{B}^*]}}$	C.L.
A	35^{+38}_{-56}	-228^{+68}_{-61}	$-0.83^{+0.08}_{-0.07}$	$1.73^{+0.68}_{-0.42}$	$1.72^{+0.70}_{-0.43}$	55%
B	-86^{+32}_{-36}	-93^{+35}_{-39}	-1^*	1^*	1^*	47%

* Constrained from HQSS (7 norms + 7 parameters for shapes)



Data:

A. Garmash et al. [Belle Collab.], PRL 116 (2016) 212001 [arXiv:1512.07419]
A. Bondar et al. [Belle Collab.], PRL 108 (2012) 122001 [arXiv:1110.2251]

Conclusions from the fits

Fit	γ_s,MeV	γ_t,MeV	ξ	$\frac{g_{[\pi h_b(1P)][B^*\bar{B}^*]}}{g_{[\pi h_b(1P)][B\bar{B}^*]}}$	$\frac{g_{[\pi h_b(2P)][B^*\bar{B}^*]}}{g_{[\pi h_b(2P)][B\bar{B}^*]}}$	C.L.
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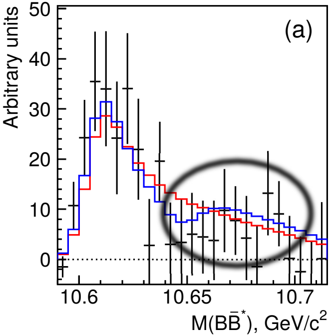
B	-86^{+32}_{-36}	-93^{+35}_{-39}	-1	1	1	47%



- Fits A and B have similar (high) quality
- Both fits give similar S-matrix poles
- Both Z_b's are virtual states with $\varepsilon_B \sim 1$ MeV
- $Z_b^{(\prime)}$ w.f.'s have (only?) two-meson components
- More precise data needed to fix parameters
- HQSS may be better met in updated data
- Fit B: HQSS implies LQSS ???

Conclusions

- Coupled channels + natural assumptions = simple but phenomenologically adequate parametrisation
- Parametrisation is well suited for combined data analysis
- Parameters have clear physical interpretation $\Longrightarrow$ way to override the gap between theory and experiment
- Unitariry is preserved $\Longrightarrow$ if the fit requires additional inelasticity then the data set is incomplete
- Easy to generalise, namely
  - (i) to extend the basis of coupled channels
  - (ii) to implement symmetry constraints
  - (iii) to use info from complementary approaches (e.g. lattice)
- Further developments to include:
  - (i) final-state interaction
  - (ii) one-pion exchange
  - (iii) additional production mechanisms