Rare B meson decays on the lattice

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Thessaloniki, August 29, 2016

AA, V. Bernard, U.-G. Meißner, A. Rusetsky, Nucl. Phys. B 910 (2016)

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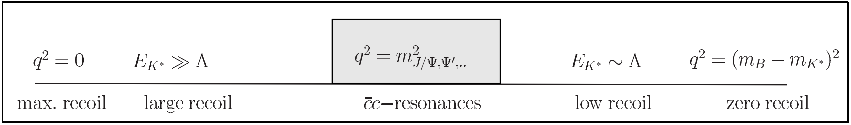
$B \to K^*(892) l^+ l^-$



Source: C. Hambrock et al., Phys. Rev. D 89 (2014), 074014

- Pattern of deviations observed by LHCb and Belle Collaboration

BSM or QCD?

- Hadronic uncertainties: form factors, long-distance effects
- In the low recoil region, lattice QCD simulations are reliable

- For $q^2 \gg \Lambda_{QCD}$, the decay amplitude A is approximately

$$A \approx \sum_{M=1}^{7} c_M(C_7, \ldots) f^M(q^2)$$

$\hookrightarrow$ see, however, J. Lyon and R. Zwicky, arXiv:1406.0566 [hep-ph]

- The seven form factors $f^M(q^2), M = 1, \ldots 7$,

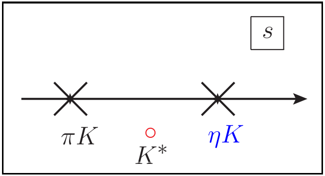
$$\langle K^*(k,\lambda)|\bar{s}\gamma^\mu b|B(p)\rangle = \frac{2iV(q^2)}{m_B + m_V}\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu k_\rho p_\sigma, \quad \text{etc.}$$

- The first unquenched lattice calculation (with a stable K^*) :

R. R. Horgan et al., Phys. Rev. D **89**, 094501 (2014)

$\hookrightarrow$ consistent with LCSR, A. Khodjamirian et al., JHEP **1009**, 089 (2010)

Theoretical issues

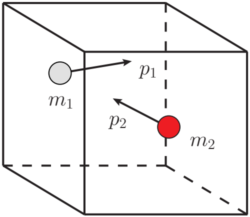


- $K^*(892)$ is a resonance $\rightarrow$ must be determined from lattice data
- Lattice data might be affected by the ηK threshold
- Resonance matrix elements should be properly defined
- Form factors: real axis vs. complex resonance pole

Resonances on the lattice

- scattering phase shift $\leftrightarrow$ finite volume energy spectrum

M. Lüscher, Nucl. Phys. B **354** (1991) 531.



$$\cot\delta(p_n) + \cot\phi(q_n) = 0 \quad \text{(Lüscher equation)}$$

$$p^2 = \lambda(s, m_1^2, m_2^2)/4s, \quad q = \frac{pL}{2\pi}$$

$$\cot\phi(q) = -\frac{1}{2\pi^2 q}\sum_{\mathbf{n}\in\mathbf{Z}^3}\frac{1}{\mathbf{n}^2 - q^2} \quad (l = 0,\ m = 0)$$

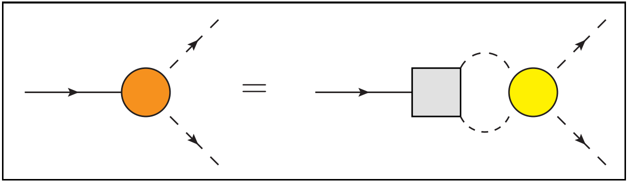
▷ energy levels $\rightarrow$ Lüscher equation $\rightarrow$ scattering phase

▷ effective-range expansion (ERE) $\rightarrow$ resonance pole position E_R

Electroweak decays

- Seminal work on $K \to \pi\pi$ by

  L. Lellouch and M. Lüscher, Commun.Math.Phys. **219**, 31 (2001)



Lellouch-Lüscher formula

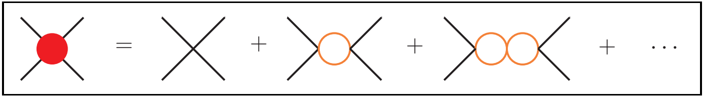
$$|A(K \to \pi\pi)| \propto |\langle \pi\pi | H_{\mathrm{weak}} | K \rangle_L| \times \left(\frac{p^2}{\frac{d\delta(p)}{dp} + \frac{d\phi(q)}{dp}} \right)^{1/2}$$

- The range of applicability:

  large volumes ($m_\pi L \gtrsim 4$), below 3(4)-particle threshold

The framework: non-relativistic EFT

- Ideally suited for the problem we study



Bubble-chain diagrams

$$T \propto 1 + cG + c^2G^2 + \cdots = \frac{1}{1 - cG}$$

J. Gasser, B. Kubis and A. Rusetsky, Nucl. Phys. B **850** (2011) 96

- In a finite volume: $G \to G_L$ (the loop function)
- A bridge: finite volume spectrum $\leftrightarrow$ scattering sector

Form factors

- on the real energy axis $\rightarrow$ model- and process-dependent

$$|\mathrm{Im}\,\mathcal{A}(E_{BW},|\mathbf{q}|)| = \sqrt{\frac{8\pi}{p_{BW}\Gamma}}|F_A(E_{BW},|\mathbf{q}|)|$$

▷ $F_A(E_{BW},|\mathbf{q}|) \rightarrow$ current matrix elements, Γ – resonance width

D. Drechsel et al., Nucl. Phys. A 645 (1999) 145

- at the resonance pole $\rightarrow$ process-independent $\Rightarrow$ favourable!

$$\langle P,\,\mathrm{resonance}|J(0)|Q,\,\mathrm{stable}\rangle = \lim_{P^2\to s_R,\;Q^2\to M^2} Z_R^{-1/2} Z^{-1/2}(s_R - P^2)(M^2 - Q^2)F(P,Q)$$

▷ $F(P,Q)$ – 3-point Green's function

▷ Z, Z_R – wave-function renormalization constants

generalization of: S. Mandelstam, Proc. Roy. Soc. Lond. A **233** (1955) 248

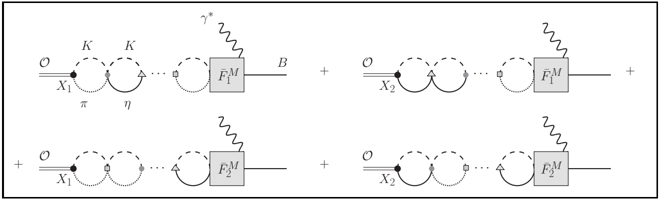
Kinematics

Little group	Irrep	Form factor
C_{4v}	$\mathbb{E}$	V, A_1, T_1, T_2
	$\mathbb{A}_1$	A_0, A_{12}, T_{23}

Table: The irreps without $S - P$ partial-wave mixing

- K^* is at rest $\mathbf{k} = 0$ and the B momentum is $\mathbf{p} = \mathbf{q} = \frac{2\pi}{L}(0, 0, n)$
- broken rotational symmetry $\rightarrow$ cubic group: choose some irrep(s)

Lellouch-Lüscher formula for $B \to K^*(892) l^+ l^-$



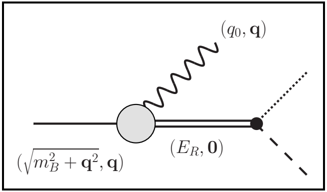
$$|F^M(E_n, |\mathbf{q}|)| = \frac{\mathcal{V}^{-1}}{8\pi E} \left| v_1 \bar{F}_1^M + v_2 \bar{F}_2^M \right| \Big|_{E=E_n}$$

- ▷ $v_1 = v_1(E, L)$, $v_2 = v_2(E, L)$ – known functions (gener. of LL factor)
- ▷ $\mathcal{V} = \eta L^3$, $\eta \in (0,1)$ (asymmetric volumes)
- ▷ $F^M(E_n, |\mathbf{q}|) \to$ current matrix elements, measured on the lattice

- $\bar{F}_1^M(E_n, |\mathbf{q}|)$, $\bar{F}_2^M(E_n, |\mathbf{q}|) \to$ decay amplitudes $\mathcal{A}^M(B \to \pi K l^+ l^-)$

↪ Watson's theorem, agrees with
S. Sharpe and M. Hansen, Phys. Rev. D **86** (2012) 016007

Form factors at the K^* pole



- Suppose, the K^* pole is on the Riemann sheet *II*
- Analytic continuation: vary E with $|\mathbf{q}|$ fixed (analog of the ERE)

$$F_R^M(E_R, |\mathbf{q}|) = -\frac{i}{8\pi E}\left(w_1 \bar{F}_1^M - w_2 \bar{F}_2^M\right)\Big|_{E=E_R}$$

▷ $w_1 = w_1(E)$, $w_2 = w_2(E)$ – volume-independent quantities

- Form factors at the pole → complex

Infinitely narrow width

- Results are simplified in the limit $\Gamma \to 0$ (K^* is above ηK)
- Assume the Breit-Wigner form in the vicinity of $E = E_{BW}$
- Real axis:

$$\left|F^M(E_n, |\mathbf{q}|)\right| = \frac{\mathcal{V}^{-1}}{\sqrt{2E_n}} \left|F_A^M(E_n, |\mathbf{q}|)\right| + O(\Gamma^{1/2}), \quad E_n = E_{BW} + O(\Gamma)$$

- Complex plane ($E_R \to E_{BW}$):

$$F_R^M(E_R, |\mathbf{q}|)\Big|_{\Gamma \to 0} = F_A^M(E_{BW}, |\mathbf{q}|) + O(\Gamma^{1/2})$$

$\hookrightarrow$ coincides with the one-channel case of the $\Delta N \gamma^*$ transition:
AA, V. Bernard, U.-G. Meißner and A. Rusetsky, Nucl. Phys. B **886** (2014) 1199

Conclusions

- Extraction of the $B \to K^*$ form factors on the lattice is studied
- Possible admixture of the ηK to πK channels is taken into account
- Equation for the $B \to K^* l^+ l^-$ amplitude at low recoil is derived
- Form factors at the K^* pole are determined
- Infinitely-narrow width approximation of the results is considered

- An open issue: long-distance effects $\Leftrightarrow$ non-local matrix elements

Progress is needed!