Born–Oppenheimer approximation in EFT and quarkonium hybrids

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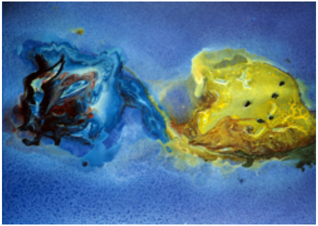

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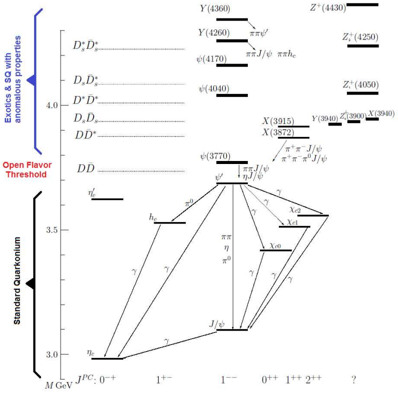
Exotic Quarkonium

- In the last decade many new unexpected states have been found close or above threshold.



Voloshin 2008

- The states that do not fit Quarkonium potential models are called Exotics and labeled Xs, Ys and Zs.
- This states are candidates for **non traditional** hadronic states, including **four constituent quark** or an **excited gluon** constituent.
- Large experimental effort to study normal and Exotic quarkonium: BaBar, Belle2, BESIII, LHCb and Panda (under construction).

Quarkonium Hybrids

What are quarkonium Hybrids?

- A quarkonium hybrid consists of Q, $\bar{Q}$ in a color octet configuration and a gluonic excitation g.

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Key Characteristics

- Heavy quarks are non-relativistic, dynamical time–scale set by the heavy quarks mass.
- Gluons are fast, dynamical time–scale set by Λ_{QCD}.
- The hierarchy between dynamical time–scales can be exploited to describe the system.

Quarkonium hybrids are a similar system to diatomic molecules

- Slow degree–of–freedom: Nuclei $\rightarrow$ Heavy Quark
- Fast degree–of–freedom: Electrons $\rightarrow$ Gluons

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Born-Oppenheimer approximation

1. Solve the Schrödinger equation for the electrons with static nuclei. The electronic energy levels depend on the nuclei positions and are called **static energies**.
2. The molecular energy levels are obtained solving the Schrödinger equation for the nuclei with the **static energies** as background potential.

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Our Aim

- **Systematize** the ideas behind the Born–Oppenheimer approximation for Quarkonium hybrids using **EFT techniques**.

Potential Non–Relativistic QCD

Motivation

- Quarkonium systems are non–relativistic bound states.
- **Multiscale system**: $m \gg mv \gg mv^2$, and Λ_{QCD}. m is the heavy–quark mass, $v \ll 1$ the heavy quark velocity.
- We can exploit the **scale hierarchies** by building an **Effective Field Theory** (EFT).

Matching procedure

- Integrating out the m scale leads to the well known NRQCD. Caswell, Lepage 1986; Bodwin, Braaten and Lepage 1995. Since $m \gg \Lambda_{QCD}$ the matching is perturbative.
- The degrees of freedom in pNRQCD are a **color singlet** (S) and **octet fields** (O^a) and the ultrasoft gluons.
- $\boldsymbol{R}$ is the CoM coordinate and $\boldsymbol{r}$ the relative coordinate of the quark pair.

Born-Oppenheimer EFT for QCD

Let us start from weakly-coupled pNRQCD:

pNRQCD Lagrangian

$$\mathcal{L}_{\mathrm{pNRQCD}} = \int d^3r\, \mathrm{Tr}\left[S^\dagger\left(i\partial_0 + \frac{\boldsymbol{\nabla}_r^2}{M} - V_s(r)\right)S + O^\dagger\left(iD_0 + \frac{\boldsymbol{\nabla}_r^2}{M} - V_o(r)\right)O\right]$$
$$+ gV_A(r)\mathrm{Tr}\left[O^\dagger \boldsymbol{r}\cdot\boldsymbol{E}S + S^\dagger \boldsymbol{r}\cdot\boldsymbol{E}O\right] + \frac{g}{2}V_B(r)\mathrm{Tr}\left[O^\dagger \boldsymbol{r}\cdot\boldsymbol{E}O + O^\dagger O\boldsymbol{r}\cdot\boldsymbol{E}\right]$$
$$-\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\, a}$$

- Hierarchy of scales:
  - Weakly-coupled pNRQCD is valid for short distances: $mv \sim 1/r \gg \Lambda_{QCD}$.
  - Heavy quarks being slower than gluons implies $\Lambda_{QCD} \gg mv^2$.
- *Work plan*:
  - Integrate out the light d.o.f.

Born-Oppenheimer EFT for QCD

The Hamiltonian density corresponding to the light d.o.f at leading order in the multipole expansion is

$$\hat{h}_0(\boldsymbol{R}) = \frac{1}{2}\left(\boldsymbol{E}^a\boldsymbol{E}^a - \boldsymbol{B}^a\boldsymbol{B}^a\right)$$

Gluelump operators G^a

- G^a are a basis of color-octet eigenstates of $\hat{h}_0(\boldsymbol{R})$ with eigenvalues Λ_κ.

$$\hat{h}_0(\boldsymbol{R})G^a_{i\kappa}(\boldsymbol{R})|0\rangle = \Lambda_\kappa G^a_{i\kappa}(\boldsymbol{R})|0\rangle$$

- Λ_κ is called the gluelump mass and it is a nonperturbative quantity.
  Foster, Michael 1999; Bali, Pineda 2004
- κ labels the $O(3)\times C$ representation (K^{PC} quantum numbers).

The eigenstates of the octet sector Hamiltonian are

$$|\kappa\rangle = O^a\left(\boldsymbol{r},\boldsymbol{R}\right)G^a_{i\kappa}(\boldsymbol{R})|0\rangle\,,$$

We can expand the Lagrangian this basis by projecting into the subspace spanned by

$$\int d^3r\, d^3R \sum_\kappa |\kappa\rangle\, \Psi_{i\kappa}(t,\,\boldsymbol{r},\,\boldsymbol{R})$$

Born-Oppenheimer EFT for QCD

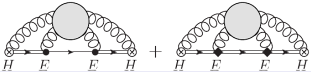
- After projecting and integrating out Λ_{QCD}:

$$\mathcal{L}^{o}_{BO} = \int d^3r \sum_{\kappa} \Psi^{\dagger}_{i\kappa}(t,\, \boldsymbol{r},\, \boldsymbol{R}) \left[\left(i\partial_t + \frac{\boldsymbol{\nabla}^2_r}{M} - V_o(r) - \Lambda_\kappa \right) \delta^{ij} - \sum_{\lambda} P^{i}_{\kappa\lambda} b_{\kappa\lambda} r^2 P^{j}_{\kappa\lambda} + \cdots \right] \Psi_{j\kappa}(t,\, \boldsymbol{r},\, \boldsymbol{R}) + \ldots$$

The $P^{i}_{\kappa\lambda}$ are projectors that select different polarizations of $\Psi_{i\kappa}$.

NLO term: $b_{\kappa\lambda}$

- At finite r the eigenstates must be organized in representations of $D_{\infty\, h}$.
- Proportional to r^2 due to the multipole expansion.



- $b_{\kappa\lambda}$ is a non-perturbative quantity.
- We obtain it from a fit to the lattice data.
- Breaks the $O(3) \times C \;\rightarrow\; D_{\infty\, h}$, $b_{\kappa\lambda} = b_{\kappa-\lambda}$.
- Responsible for the attractive part of the potential.

Born-Oppenheimer EFT for QCD

Defining the projected wavefunction as $\Psi_{\kappa\lambda} = P^{i}_{\kappa\lambda}\Psi_{i\kappa}$ and $\Psi_{i\kappa} = \sum_{\lambda} P_{i\kappa\lambda}\Psi_{\kappa\lambda}$:

$$\mathcal{L}^{o}_{BO} = \int d^3r \sum_{\kappa}\sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t,\, \boldsymbol{r},\, \boldsymbol{R})\Big\{ \Big[i\partial_t - V_o(r) - \Lambda_\kappa - b_{\kappa\lambda} r^2 + \cdots \Big]\delta_{\lambda\lambda'} - P^{i}_{\kappa\lambda}\frac{\boldsymbol{\nabla}^2_r}{M}P_{i\kappa\lambda'} \Big\}\Psi_{\kappa\lambda'}(t,\, \boldsymbol{r},\, \boldsymbol{R})$$

Nonadiabatic coupling

We have splitted the kinetic operator acting and the nonadiavatic coupling

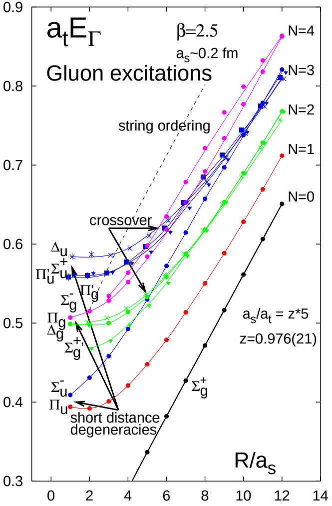
$$P^{i}_{\kappa\lambda}\frac{\boldsymbol{\nabla}^2_r}{M}P_{i\kappa\lambda'} = \frac{\boldsymbol{\nabla}^2_r}{M} + C_{\kappa\lambda\lambda'}$$

with

$$C_{\kappa\lambda\lambda'} = P^{i}_{\kappa\lambda}\left[\frac{\boldsymbol{\nabla}^2_r}{M}, P_{i\kappa\lambda'}\right]$$

- The nonadiabatic coupling mixes states which are different projections of the same gluelump.
- States which are different projections of the same gluelump are degenerate in the limit $r \to 0$.

Lattice data on hybrid static energies



Quenched lattice NRQCD: Juge, Kuti, Morningstar 2002

- The gluonic static energies are are the eigenvalues of the NRQCD Hamiltonian in the static limit.
- The most recent data by Juge, Kuti, Morningstar, 2002 and Bali and Pineda 2003.
- Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are Π_u and Σ_u^-, they are nearly degenerate at short distances.

Lowest energy multiplet Σ_u^-–Π_u

- The lowest mass gluelump has quantum numbers 1^{+-} and $\Lambda^{RS}_{1^{+-}} = 0.87 \pm 0.15$ GeV. Bali, Pineda 2004
- It generates the two lowest laying hybrid static energies Π_u and Σ_u^- which are degenerate at short distances.
- The nonadiavatic coupling mixes these two static energies.

Coupled radial equations for Σ_u^-–Π_u

$$\left[-\frac{\partial_r^2}{m} + \frac{1}{mr^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_\Sigma^{\text{light}} & 0 \\ 0 & E_\Pi^{\text{light}} \end{pmatrix}\right]\begin{pmatrix} \Psi^N_{\epsilon,\,\Sigma} \\ \Psi^N_{\epsilon,\,\Pi} \end{pmatrix} = \mathcal{E}_N \begin{pmatrix} \Psi^N_{\epsilon,\,\Sigma} \\ \Psi^N_{\epsilon,\,\Pi} \end{pmatrix}$$

$$\left[-\frac{\partial_r^2}{m} + \frac{l(l+1)}{mr^2} + E_\Pi^{\text{light}}\right]\psi^N_{-\epsilon,\,\Pi} = \mathcal{E}_N \psi^N_{-\epsilon,\,\Pi}\,.$$

- The coupled Schrödinger equations can be solved numerically.

Hybrid state masses from $V^{(0.25)}$

M. Berwein, N. Brambilla, J.T., A. Vairo. arXiv:1510.04299 Phys.Rev. D92 (2015) 11, 114019

Solving the coupled Schrödinger equations we obtain

GeV	$c\bar{c}$				$b\bar{c}$				$b\bar{b}$			
	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π
H_1	4.15	0.42	0.16	0.82	7.48	0.46	0.13	0.83	10.79	0.53	0.09	0.86
H_1'	4.51	0.34	0.34	0.87	7.76	0.38	0.27	0.87	10.98	0.47	0.19	0.87
H_2	4.28	0.28	0.24	1.00	7.58	0.31	0.19	1.00	10.84	0.37	0.13	1.00
H_2'	4.67	0.25	0.42	1.00	7.89	0.28	0.34	1.00	11.06	0.34	0.23	1.00
H_3	4.59	0.32	0.32	0.00	7.85	0.37	0.27	0.00	11.06	0.46	0.19	0.00
H_4	4.37	0.28	0.27	0.83	7.65	0.31	0.22	0.84	10.90	0.37	0.15	0.87
H_5	4.48	0.23	0.33	1.00	7.73	0.25	0.27	1.00	10.95	0.30	0.18	1.00
H_6	4.57	0.22	0.37	0.85	7.82	0.25	0.30	0.87	11.01	0.30	0.20	0.89
H_7	4.67	0.19	0.43	1.00	7.89	0.22	0.35	1.00	11.05	0.26	0.24	1.00

Consistency test:

1. The multipole expansion requires $\langle 1/r \rangle \gtrsim E_{kin}$.

Conclusion:

- As expected the our approach works better in bottomonium than charmonium

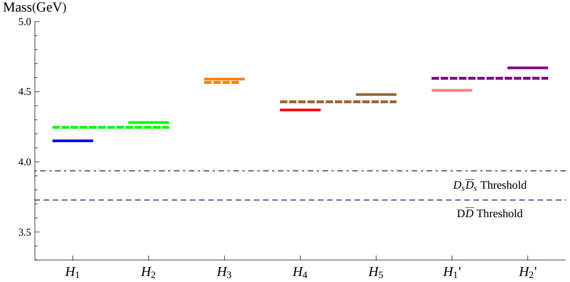
- Spin symmetry multiplets

H_1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^- , Π_u
H_2	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^- , Π_u
H_5	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u
H_6	$\{3^{--}, (2,3,4)^{-+}\}$	Σ_u^- , Π_u
H_7	$\{3^{++}, (2,3,4)^{+-}\}$	Π_u

Λ–doubling effect

- In Braaten *et al* 2014 a similar procedure was followed to obtain the hybrid masses.
- No Λ–doubling effect mixing terms where included, and phenomenological potentials fitting the lattice data.
- We can compare the results to estimate the size of the Λ–doubling effect.

Charmonium sector



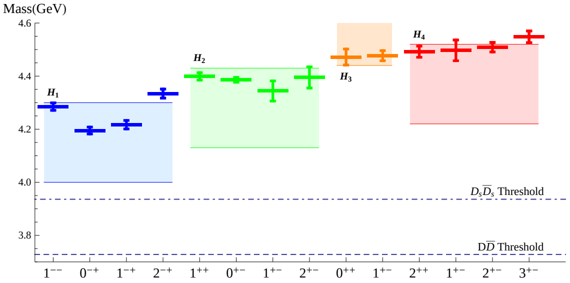
Braaten *et al* 2014 results plotted in dashed lines.

- The mixing lower the mass of the $H_1(H_4)$ multiplet with respect to $H_2(H_4)$.

Comparison with direct lattice computations

Charmonium sector

- Calculations done by the Hadron Spectrum Collaboration using unquenched lattice QCD with a pion mass of 400 MeV. Liu *et all* 2012
- They worked in the constituent gluon picture, which consider the multiplets H_2, H_3, H_4 as part of the same multiplet.
- Their results are given with the η_c mass subtracted.



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

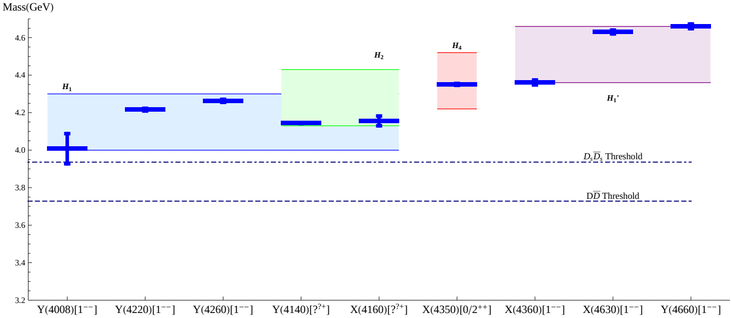
Split (GeV)	Liu	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.10	0.13
$\delta m_{H_4-H_1}$	0.24	0.22
$\delta m_{H_4-H_2}$	0.13	0.09
$\delta m_{H_3-H_1}$	0.20	0.44
$\delta m_{H_3-H_2}$	0.09	0.31

- Our masses are $0.1 - 0.14$ GeV lower except the for the H_3 multiplet, which is the only one dominated by Σ_u^-.
- Good agreement with the mass gaps between multiplets, in particular the Λ-doubling effect ($\delta m_{H_2-H_1}$).

Identification with experimental states

Most of the candidates have 1^{--} or $0^{++}/2^{++}$ since the main observation channels are production by e^+e^- or $\gamma\gamma$ annihilation respectively.

- Charmonium states (Belle, CDF, BESIII, Babar):



- Bottomonium states: $Y_b(10890)[1^{--}]$, $m = 10.8884 \pm 3.0$ (Belle). Possible H_1 candidate, $m_{H_1} = 10.79 \pm 0.15$.

However, some of the candidates decay modes violate Heavy Quark Spin Symmetry.

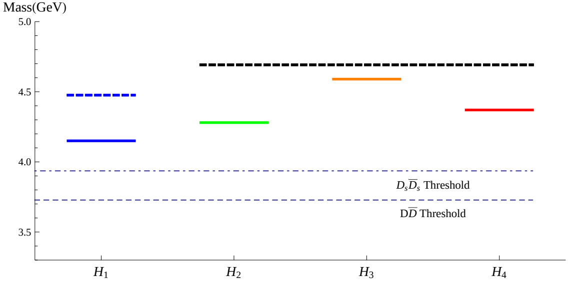
Conclusions

- We have obtained an EFT formulation of the Born-Oppenheimer approximation for quarkonium hybrids.
- The potential is obtained by matching to the static energies computed in Lattice NRQCD.
- The nonadiavatic coupling mixing terms are important due to the short range degeneracy of the static energies.
- There are several experimental candidates for charmonium hybrids and one candidate for bottomonium hybrids.

Thank you for your attention

Comparison with Guo, Szczepaniak, Galata, Vassallo, Santopino (2008)

Charmonium sector

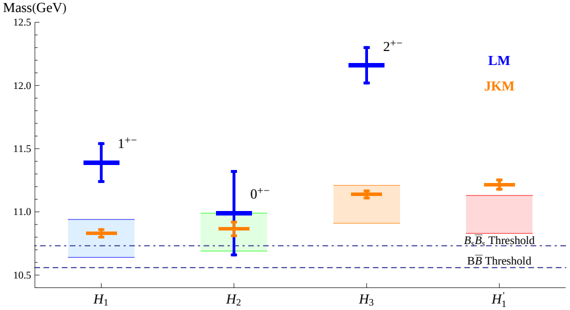


- For the H_1 multiplet they are about 300 MeV higher, and for the H_2, H_3 and H_4, (degenerate in the constituent gluon picture) they are $100 - 400$ MeV higher depending on the multiplet.
- Different multiplet structure (constituent gluon vs Born-Oppenheimer pictures).
- Overall mass shift due to the different origin of the potential.

Comparison with direct lattice computations

Bottomonium sector

- Calculations done by Juge, Kuti, Morningstar 1999 and Liao, Manke 2002 using quenched lattice QCD.
- Juge, Kuti, Morningstar 1999 included no spin or relativistic effects.
- Liao, Manke 2002 calculations are fully relativistic.



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

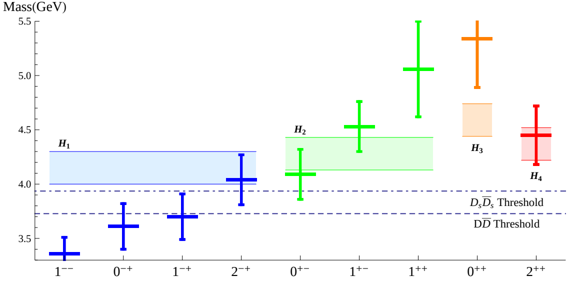
Split (GeV)	JKM	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.04	0.05
$\delta m_{H_3-H_1}$	0.33	0.27
$\delta m_{H_3-H_2}$	0.30	0.22
$\delta m_{H_1'-H_1}$	0.42	0.19

- Our masses are $0.15-0.25$ GeV lower except the for the H_1' multiplet, which is larger by 0.36 GeV.
- Good agreement with the mass gaps between multiplets, in particular the Λ-doubling effect ($\delta m_{H_2-H_1}$).

Comparison with QCD sum rules

- A recent analysis of QCD sum rules for hybrid operators has been performed by Chen *et al* 2013, 2014 for $b\bar{b}$ and $c\bar{c}$ hybrids, and $b\bar{c}$ hybrids respectively.
- Correlation functions and spectral functions were computed up to dimension six condensates which stabilized the mass predictions compared to previous calculations which only included up to dimension 4 condensates.

Charmonium sector Chen *et al* 2013

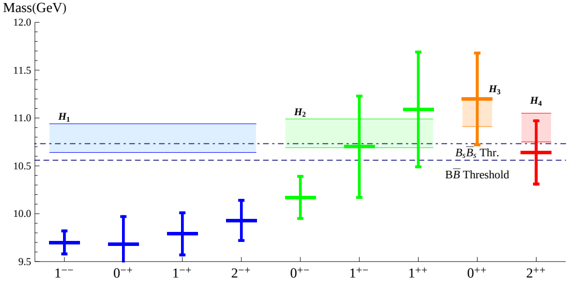


Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

- The spin average of the H_1 multiplet is 0.4 GeV lower than our mass.
- H_2, H_3 and H_4 multiplets are incomplete.
- Large uncertainties compared to direct lattice calculations.

Comparison with QCD sum rules

Bottomonium sector Chen *et al* 2013



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

- The spin average of the H_1 multiplet is 0.98 GeV lower than our mass.
- H_2, H_3 and H_4 multiplets are incomplete.
- Large uncertainties compared to direct lattice calculations.