Molecular partners of the X(3872) from heavy-quark spin symmetry: a fresh look

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Plenty of experimentally observed $XYZ$ states do not fit in quark model predictions $X(3872), Zc(3900), Y(4260), Zb(10610), Zb(10650), ...$  

Enigmatic example: $X(3872)$ seen by Belle, CDF, D0, BABAR, LHCb, BESIII  

because of its mass, width and quantum numbers does not fit to charmonium interpretation  

$X(3872)$ is a $J^{PC} = 1^{++}$ state residing near the $D \bar{D}^*$ threshold:  

$$E_X = m_0 + m_0^* - M_X = 0.12 \pm 0.30 \text{ MeV}$$  

Molecular interpretation: bound state of $D$ and $\bar{D}^*$ in an S wave  

Other interpretations: tetraquark, a mixture of a molecular and a charmonium, ...
Heavy quark spin symmetry (HQSS)

In the limit \( \Lambda_{QCD}/m_Q \to 0 \) strong interactions are independent of HQ spin.

Consequences of HQSS are different for different scenarios. 

Search for spin partner states \( \implies \) insights into the nature of XYZ states
Heavy quark spin symmetry (HQSS)

In the limit \( \Lambda_{QCD}/m_Q \rightarrow 0 \) strong interactions are independent of HQ spin.

Consequences of HQSS are different for different scenarios.\[\text{Cleven et al. (2015)}\]

Search for spin partner states \(\implies\) insights into the nature of XYZ states

Molecule picture

- HQSS molecular partners of isovector states \(Zb^+(10610)\) and \(Zb^+(10650)\)\[\text{Bondar et al. (2011), Voloshin (2011), Mehen and Powell (2011)}\]

- \(J^{PC}=2^{++}\) partner of the \(X(3872)\) as a shallow bound state in the \(D^*D^*\) system\[\text{Nieves and Valderrama (2012), Guo et al. (2013)}\]

The width of the \(2^{++}\) partner: a few — a dozen MeV using an EFT with contact interactions and perturbative pions\[\text{Albaladejo et al. (2015)}\]

This Talk: Revisit HQSS predictions for the \(X(3872)\) in the molecular scenario

for HQSS predictions in the case of hadrocharmonium see\[\text{Cincioglu et al. (2016)}\]
Molecular partners: contact theory

- Basis states \( J^{PC} \) made of \( D \) and \( \bar{D}^* \)

  \( D \bar{D}^*(\pm) = \frac{1}{\sqrt{2}} (D \bar{D}^* \pm D^* \bar{D}) \)

  - C-parity states: \( C = \pm \)
  - \( D \bar{D}^*(\pm) \) includes actions (4)-(7) respectively. We define the C-parity eigenstates as total spin, the angular momentum, and the total momentum of the two-meson system, where the individual partial waves are labelled as ++, −−, +−, and −+. For the quantum numbers \( JPC \) of the two-meson states, the leading-order EFT \( (10) \): the same linear combination

- S-wave derivativeless contact interactions respecting HQSS

  \[
  V_{LO}^{(0++)} = \begin{pmatrix}
  C_{0a} & -\sqrt{3}C_{0b} \\
  -\sqrt{3}C_{0b} & C_{0a} - 2C_{0b}
  \end{pmatrix},
  \]
  \[
  V_{LO}^{(1+-)} = \begin{pmatrix}
  C_{0a} - C_{0b} & 2C_{0b} \\
  2C_{0b} & C_{0a} - C_{0b}
  \end{pmatrix},
  \]
  \[
  V_{LO}^{(1++)} = C_{0a} + C_{0b},
  \]
  \[
  V_{LO}^{(2++)} = C_{0a} + C_{0b}.
  \]

  - Grinstein et al. (1992), AlFiky et al. (2006), Nieves and Valderrama (2012)

    \( V_{LO}^{(1++)} \) and \( V_{LO}^{(2++)} \): the same linear combination

    - two LECS at LO \( C_{0a} \) and \( C_{0b} \)

- Lippmann-Schwinger type integral equations:

  \[
  T^{(JPC)}(p, p') = V^{(JPC)}(p, p') - \int dk \; k^2 \; V^{(JPC)}(p, k) G(k) T^{(JPC)}(k, p')
  \]

- position of poles:

  \[
  \det \left[ 1 + \int dk \; k^2 \; V^{(JPC)} G(k) \right] = 0
  \]
Molecular partners: contact theory

- Strict HQSS: $m = m_\ast = \bar{m} \Rightarrow$ Green functions coincide in all channels

Consequences of HQSS

- Poles of $T^{1++}$ and $T^{2++}$ coincide
  
  Nieves, Valderrama (2012)

- By making unitarity transform $U$ for $0^{++}$ and $1^{+-}$ potentials
  
  $U = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix}$

  
  $\tilde{V}^{(JPC)} = UV^{(JPC)}U^\dagger = \begin{pmatrix} C_0 & 0 \\ 0 & C'_0 \end{pmatrix} \quad C_0 = C_{0a} + C_{0b}$

  $C'_0 = C_{0a} - 3C_{0b}$

  Coupled-channel LS Eq. decouples into two Eqs. with $V = C_0$ and $V = C'_0$

- In the strict HQSS limit $T^{(JPC)}$ possess two decoupled solutions:

  $E^{(0)}_{X_{1^{++}}} = E^{(0)}_{X_{2^{++}}} = E^{(0)}_{X_{1^{+-}}} = E^{(0)}_{X_{0^{++}}} \quad$ and $\quad E^{(0)}_{X'_{0^{++}}} = E^{(0)}_{X'_{1^{+-}}}$

  our finding is in line with Hidalgo-Duque et al. (2013)

- While the partner states corresponding to $V = C_0$ are predictable using the $X(3872)$ as input, an additional experimental input is needed to determine the location of the states controlled by $V = C'_0$
Contact theory with HQSS breaking

- Introduce D*–D mass splitting and average mass:
  \[ \delta = m_* - m = 141 \text{ MeV} \]
  \[ \bar{m} = \frac{1}{4}(3m_* + m) = 1973 \text{ MeV} \]
  \[ \delta/\bar{m} \simeq 7\% \]

- Leading effect: the states reside near the corresponding thresholds: \( D \bar{D}, D \bar{D}^* \) and \( D^* \bar{D}^* \)

  For example:
  \[ M_{X_{2++}} = M_{X_{1++}} + \delta \]

- HQSS based relations between the binding momenta:

  \[ \gamma_{X_{2++}} = \left(1 - \frac{\delta}{2\bar{m}}\right) \gamma_{X_{1++}} + \frac{\delta \Lambda}{\pi \bar{m}} + O \left(\frac{\delta^2 \Lambda}{\bar{m}^2}, \frac{\gamma_{X_{1++}}^2}{\Lambda}\right) \]
  \[ \gamma_{X'_{1+-}} = \left(1 - \frac{\delta}{2\bar{m}}\right) \gamma_{X_{1+-}} + \frac{\delta \Lambda}{\pi \bar{m}} - \frac{(\gamma_{X_{1+-}} - \gamma_{X_{1++}})^2}{\sqrt{\bar{m}\delta}} + \frac{i(\gamma_{X_{1+-}} - \gamma_{X_{1++}})^2}{\sqrt{\bar{m}\delta}} + \ldots \]

  Correction at \( O(\delta) \) is cutoff dependent \( \Rightarrow \) extra contact term for renormalization
  \( \Rightarrow \) But small impact on the location of the states

  two exp. inputs needed in the coupled-channel case: \( \gamma_{X_{1++}} \) and \( \gamma_{X_{1+-}} \) \( \Rightarrow \) \( \gamma_{X'_{1+-}} \)

  \( \gamma_{X'_{1+-}} \) acquires an \( Im \) part due to coupled-channels: \( D^* \bar{D}^* \rightarrow D \bar{D}^* \rightarrow D^* \bar{D}^* \)
Strict HQSS limit in the presence of pions

- New transitions due to OPE \( \rightarrow \) more coupled channels

For example, at one loop:

\[ D\bar{D}^* \]
\[ D^*\bar{D}^* \]

- Extended basis states:
  
  \[
  \begin{align*}
  0^{++} : & \quad \{ D\bar{D}(1S_0), D^*\bar{D}^*(1S_0), D^*\bar{D}^*(5D_0) \}, \\
  1^{+-} : & \quad \{ D\bar{D}^*(3S_1, -), D\bar{D}^*(3D_1, -), D^*\bar{D}^*(3S_1), D^*\bar{D}^*(3D_1) \}, \\
  1^{++} : & \quad \{ D\bar{D}^*(3S_1, +), D\bar{D}^*(3D_1, +), D^*\bar{D}^*(5D_1) \}, \\
  2^{++} : & \quad \{ D\bar{D}(1D_2), D\bar{D}^*(3D_2), D^*\bar{D}^*(5S_2), D^*\bar{D}^*(1D_2), D^*\bar{D}^*(5D_2), D^*\bar{D}^*(5G_2) \}
  \end{align*}
\]

\[ \text{Coupled-channel transitions depend on quantum numbers} \]

- Chiral EFT at LO \( \rightarrow \) contact terms + static OPE \( \rightarrow \) does not depend on the heavy-quark mass \( \rightarrow \) HQSS multiplets (4+2) found in the contact case should hold!

But how to see this?
Strict HQSS limit in the presence of pions

- Unitary transform brings the potential to block-diagonal form:
  \[ \tilde{V}^{(0++)}(3 \times 3) = A(2 \times 2) \oplus B(1 \times 1), \]
  \[ \tilde{V}^{(1+-)}(4 \times 4) = A(2 \times 2) \oplus B(1 \times 1) \oplus C(1 \times 1) \]
  \[ \tilde{V}^{(1++)}(3 \times 3) = A(2 \times 2) \oplus D(1 \times 1), \]
  \[ \tilde{V}^{(2++)}(6 \times 6) = A(2 \times 2) \oplus D(1 \times 1) \oplus E(3 \times 3) \]

- \(A(2 \times 2)\) is common for all quantum numbers (related to \(C_0\)) \(\implies\) 4 degenerate states

- \(B(1 \times 1)\) is related to \(C_0'\) \(\implies\) bring about 2 more degenerate states \(0^{++}, 1^{+-}\)

- But these HQSS predictions hold only if all particle coupled channels are included!

**Example: \(2^{++}\) partner state**

- Full coupled-channel dynamics: \(E_{X_{2^{++}}} = E_{X_{1^{++}}}\)

- Neglecting \(D^*D^* \to D\bar{D} \to D^*\bar{D}^*, \quad D^*D^* \to DD^* \to D^*D^*\) transitions, as done by Nieves, Valderrama (2012), leads to severe violation of HQSS predictions

![Graph showing binding energy of the 2\(^{++}\) state]
Contact + OPE interactions: beyond strict HQSS limit

- Switch on $D^*-D$ mass splitting $\implies$ 2$^{++}$ $D^*\bar{D}^*$ state acquires the finite width

Example of transitions which cause the imaginary part of the amplitude:

- perturbative pions: the width $\Gamma_{X^{2++}} \sim 1-20$ MeV

<table>
<thead>
<tr>
<th></th>
<th>Without pion-exchange FF</th>
<th>With pion-exchange FF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Lambda = 0.5$ GeV</td>
<td>$\Lambda = 1$ GeV</td>
</tr>
<tr>
<td></td>
<td>$\Lambda = 0.5$ GeV</td>
<td>$\Lambda = 1$ GeV</td>
</tr>
<tr>
<td>$\Gamma(X_1 \rightarrow D^+D^-)$ [MeV]</td>
<td>$3.3^{+3.4}_{-1.4}$</td>
<td>$7.3^{+7.9}_{-2.1}$</td>
</tr>
<tr>
<td>$\Gamma(X_2 \rightarrow D^0\bar{D}^0)$ [MeV]</td>
<td>$2.7^{+3.1}_{-1.2}$</td>
<td>$5.7^{+7.8}_{-1.8}$</td>
</tr>
<tr>
<td>$\Gamma(X_2 \rightarrow D^+D^{*-})$ [MeV]</td>
<td>$2.4^{+2.1}_{-1.0}$</td>
<td>$4.4^{+3.1}_{-1.2}$</td>
</tr>
<tr>
<td>$\Gamma(X_2 \rightarrow D^0\bar{D}^{*0})$ [MeV]</td>
<td>$2.0^{+2.1}_{-0.9}$</td>
<td>$3.5^{+3.5}_{-1.0}$</td>
</tr>
</tbody>
</table>

For example:

- But the relevant momentum scales stem from coupled-channels induced by OPE

\[
q_1 = \sqrt{2\delta \tilde{m}} \approx 700 \text{ MeV}
\]

\[
q_2 = \sqrt{\delta \tilde{m}} \approx 500 \text{ MeV}
\]

$\implies$ Coupled-channel transitions in D-waves are not suppressed relative to S-wave interactions

$\implies$ Iterations of OPE to all orders are expected to be important

Contact + OPE interactions at LO: Non-perturbative Results

- $2^{++}$ partner from the nonperturb. solution of the LS equation with $V = C_0 + V_{\text{OPE}}$

\[
V_{\text{OPE}}(\vec{p}, \vec{p}') \sim -\frac{g_c^2}{(4\pi f)^2} \frac{\vec{A}_1 \cdot \vec{q}}{\vec{q}^2 + M_\pi^2} \cdot \vec{A}_2 \cdot \vec{q}
\]

- Significant shift of $E_{X^{2++}}$ and large width $\Gamma_{X^{2++}} \approx 50 \pm 10 \text{ MeV}$

- Relatively $\Lambda$ independent due to unitarity

- Cutoff variation $\Rightarrow$ estimate of a higher-order contact term at $O(\delta)$

- Cutoff variation $\Rightarrow$ estimate of a higher-order contact term at $O(\delta)$

Figure 3: The energy and the width of the $X^{2++}$ state from the nonperturbative solution of the LS equation. The sharp cutoff regularisation and the exponential regularisation are shown as solid and dashed lines, respectively. The binding energy $E_{X^{2++}}$ and the width $\Gamma_{X^{2++}}$ are shown as functions of the cutoff parameter $\Lambda$. The results are compared to the mass of the $X(3872)$ and the production rate $\Gamma_{X(3872)}$. The conclusions are drawn from the shape of the production rate, Eq. (42), as functions of the cutoff parameter $\Lambda$. The conclusion is related to unitarity and therefore is a reliable prediction of the spin-symmetry-violating terms in the full model. On the contrary, neglecting these quantum numbers and found that the coupled-channel effects (in the Lippmann-Schwinger equations) turn out to be independent due to unitarity. This is related to unitarity and therefore is a reliable prediction of the $X(3872)$ binding energy $\Gamma_{X(3872)}$.
Open Questions and Theory To-Do List

- Relatively small separation of scales may call the convergence of the EFT into question. Include explicitly the members of SU(3) pseudoscalar octet as well as vector mesons.

- Investigate the role of three-body effects in the OPE potential. For the role of three-body dynamics for the $X(3872)$ see Fleming et al. (2007), our works (2010-2015), Jansen et al. (2015), Guo et al. (2014).

  Since the main contribution to the width of the $2^{++}$ $D^*D^*$ state stems from coupled channels, three-body effects are not expected to change the picture qualitatively.

- Bring additional Imaginary parts from the right-hand cut.

- Bring additional HQSS corrections due to $D, D^*$ energies.

- Estimate HQSS violating contact terms more reliably.
Summary

- We confirm that in the *strict HQSS* limit there are two degenerate multiplets of isoscalar molecular partner states with

\[ E^{(0)}_{X_{1++}} = E^{(0)}_{X_{2++}} = E^{(0)}_{X_{1+-}} = E^{(0)}_{X_{0++}} \quad \text{and} \quad E^{(0)}_{X'_{0++}} = E^{(0)}_{X'_{1+-}} \]

- This conclusion holds in the presence of OPE interactions if and only if all particle coupled-channel transitions are included.

- Leading HQSS breaking correction stems from D*-D mass splitting.
  - Molecular states reside in the vicinity of their thresholds.
  - Predictions for the 2^{++} partner of the the X(3872) are possible.
  - One additional experimental input is needed to predict 0^{++} and 1^{+-} partners.

- Non-perturbative pion dynamics plays the crucial role for the determination of the parameters of the 2^{++} partner state.
  - The predicted width — \( \Gamma_{X_{2++}} \approx 50 \pm 10 \text{ MeV} \) — is much larger than in a theory with perturbative pions.
Spires
**X(3872) as a molecular state**

- Prediction of the $D\bar{D}^*$ molecular state similar to the deuteron
  
  ![Diagram](image)

- Experience from nuclear physics: One pion exchange (OPE) potential governs the lowest energy left-hand singularity of the NN force $\implies$ crucial influence on the NN parameters
  
  ![Diagram](image)

- OPE between $D\bar{D}^*$ has even richer applications—many three-body thresholds around

  ![Graph](image)

  $X(3872)\rightarrow D\bar{D}\pi$ and for chiral extrapolations of the $X$-pole

  ![Graph](image)

  Investigate the role of OPE for the molecular partner states
Contact theory with HQSS breaking: coupled channels

- Green functions of $D\bar{D}$, $D\bar{D}^*$ and $D^*\bar{D}^*$ states are not the same anymore

$$G^{(1++)}_{\text{LO}}(k) = \begin{pmatrix} (k^2/2\mu_* - E - i0)^{-1} & 0 \\ 0 & (k^2/2\mu_{**} - E - i0)^{-1} \end{pmatrix}, \quad G^{(0++)}_{\text{LO}}(k) = \begin{pmatrix} (k^2/2\mu - E - i0)^{-1} & 0 \\ 0 & (k^2/2\mu_{**} - E - i0)^{-1} \end{pmatrix},$$

$$2\mu = \bar{m} - \frac{3}{4}\delta, \quad 2\mu_* = \bar{m} - \frac{\delta}{4}, \quad 2\mu_{**} = \bar{m} + \frac{1}{4}\delta,$$

$\Rightarrow$ VG in the Lippmann-Schwinger Eq. can not be diagonalized

- Poles are determined by both $C_0$ and $C_0'$ simultaneously — two exp. inputs needed

- $C_0'$ can be fixed assuming the X(3915) to be a $0^{++} D^*\bar{D}^*$ partner of the X(3872)

But $X(3915)$ is about 100 MeV below $D^*\bar{D}^*$

$\Rightarrow$ Zhou et al. (PRL 2015) argue that the $X(3915)$ and $X(3930)$ is the same $2^{++}$ state
Strict HQSS limit in the presence of pions

- New transitions due to OPE \[\implies\] more coupled channels

  For example, at one loop:

  \[
  D\bar{D}^* \quad (a) \quad (b) \quad (b2) \]

  \[
  D^*\bar{D}^* \quad (c) \quad (d1) \quad (d2) \quad (e) \]

- Extended basis states:

  \[
  0^{++} : \{ D\bar{D}(1S_0), D^*\bar{D}^*(1S_0), D^*\bar{D}^*(5D_0) \}, \\
  1^{++} : \{ D\bar{D}^*(3S_1, -), D\bar{D}^*(3D_1, -), D^*\bar{D}^*(3S_1), D^*\bar{D}^*(3D_1) \}, \\
  1^{+-} : \{ D\bar{D}^*(3S_1, +), D\bar{D}^*(3D_1, +), D^*\bar{D}^*(5D_1) \}, \\
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  \]

- Can one absorb the divergencies of the loop diagrams with different J^{PC} to a single contact term C_0? **Yes but only if all coupled-channel transitions are included!**

For each J^{PC} the coefficient in front of the leading divergence is the same once all coupled-channels are included

\[
\begin{array}{|c|c|c|c|c|}
\hline
& D\bar{D} & D\bar{D}^* & D^*\bar{D}^* & \text{Sum} \\
\hline
1^{++} & 2^{S+1}L_J & - & 3S_1 & 3D_1 & 5D_1 \\
& \text{Coeff.} & - & 1/9 & 2/9 & 2/3 \\
\hline
2^{++} & 2^{S+1}L_J & 1D_2 & 3D_2 & 5S_2 & 1D_2 & 5D_2 & 5G_2 \\
& \text{Coeff.} & 2/15 & 2/5 & 1/9 & 2/45 & 14/45 & 0 & 1 \\
\hline
\end{array}
\]

\[\implies\text{successful renormalization program}\]