Molecular partners of the X(3872) from heavy-quark spin symmetry: a fresh look

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Introduction

- Plenty of experimentally observed *XYZ* states do not fit in quark model predictions

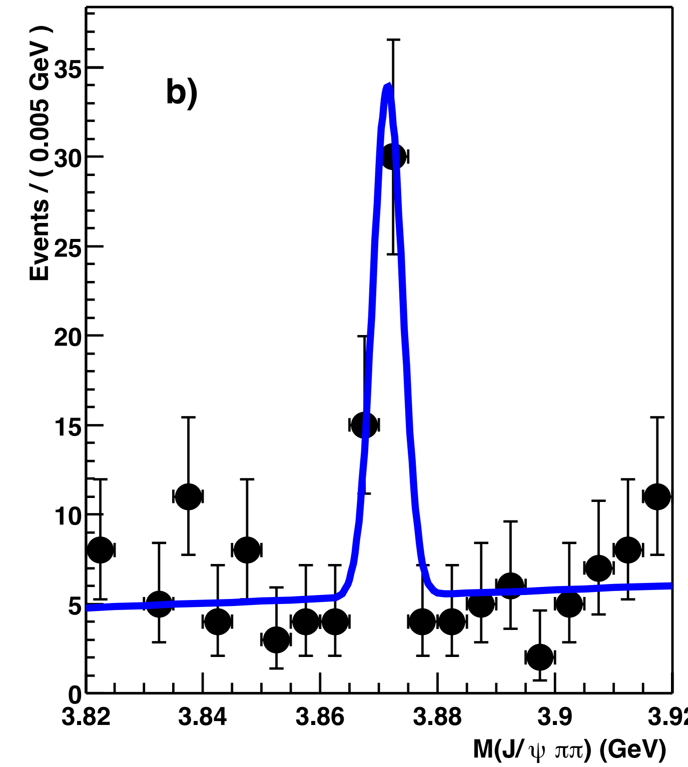
X(3872), Zc(3900), Y(4260), Zb(10610), Zb(10650),… *review article*: Brambilla et al. (2011)

Enigmatic example: X(3872) seen by Belle, CDF, D0, BABAR, LHCb, BESIII

☛ because of its mass, width and quantum numbers does not fit to charmonium interpretation

☛ X(3872) is a $J^{PC} = 1^{++}$ state residing near the $D\bar{D}^*$ threshold:

$$E_X = m_0 + m_0^* - M_X = 0.12 \pm 0.30 \quad \text{MeV}$$



$\Longrightarrow$ Molecular interpretation: bound state of D and $\bar{D}^*$ in an S wave

$\Longrightarrow$ Other interpretations: tetraquark, a mixture of a molecular and a charmonium, …

review article: Brambilla et al. (2011)

Heavy quark spin symmetry (HQSS)

- ☛ In the limit $\Lambda_{\rm QCD}/m_Q \to 0$ strong interactions are independent of HQ spin

- ☛ Consequences of HQSS are different for different scenarios Cleven et al. (2015)

$\Longrightarrow$ Search for spin partner states $\Longrightarrow$ insights into the nature of XYZ states

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Molecule picture

- HQSS molecular partners of isovector states $Z_b^+(10610)$ and $Z_b^+(10650)$

Bondar et al. (2011), Voloshin (2011), Mehen and Powell (2011)

- $J^{PC}= 2^{++}$ partner of the X(3872) as a shallow bound state in the D*D* system

Nieves and Valderrama (2012), Guo et al. (2013)

☛ The width of the 2^{++} partner: a few — a dozen MeV
using an EFT with contact interactions and perturbative pions Albaladejo et al. (2015)

This Talk: Revisit *HQSS* predictions for the X(3872) in the molecular scenario

for HQSS predictions in the case of hadrocharmonium see Cincioglu et al. (2016)

Molecular partners: contact theory

- Basis states $\mathbf{J^{PC}}$ made of D and $\bar{D}^*$

  C-parity states: $C = \pm$

$$D\bar{D}^*(\pm) = \frac{1}{\sqrt{2}}\left(D\bar{D}^* \pm D^*\bar{D}\right)$$

$$\begin{aligned}
0^{++}: &\quad \left\{D\bar{D}(^1S_0), D^*\bar{D}^*(^1S_0)\right\},\\
1^{+-}: &\quad \left\{D\bar{D}^*(^3S_1,-), D^*\bar{D}^*(^3S_1)\right\},\\
1^{++}: &\quad \left\{D\bar{D}^*(^3S_1,+)\right\},\\
2^{++}: &\quad \left\{D^*\bar{D}^*(^5S_2)\right\},
\end{aligned}$$

- S-wave derivativeless contact interactions respecting HQSS

$$V_{\rm LO}^{(0++)} = \begin{pmatrix} C_{0a} & -\sqrt{3}C_{0b} \\ -\sqrt{3}C_{0b} & C_{0a}-2C_{0b}\end{pmatrix},$$

$$V_{\rm LO}^{(1+-)} = \begin{pmatrix} C_{0a}-C_{0b} & 2C_{0b} \\ 2C_{0b} & C_{0a}-C_{0b}\end{pmatrix},$$

$$V_{\rm LO}^{(1++)} = C_{0a}+C_{0b},$$

$$V_{\rm LO}^{(2++)} = C_{0a}+C_{0b},$$

Grinstein et al. (1992),
AlFiky et al. (2006),
Nieves and Valderrama (2012)

☛ two LECs at LO C_{0a} and C_{0b}

☛ $V_{\rm LO}^{(1++)}$ and $V_{\rm LO}^{(2++)}$: the same linear combination

- Lippmann-Schwinger type integral equations:

$$T^{(JPC)}(p,p') = V^{(JPC)}(p,p') - \int dk\, k^2\, V^{(JPC)}(p,k)G(k)T^{(JPC)}(k,p')$$

- position of poles:

$$\det\left[1 + \int dk\, k^2\, V^{(JPC)}G(k)\right] = 0$$

Molecular partners: contact theory

- Strict HQSS: $m = m_* = \bar{m}$ $\Longrightarrow$ Green functions coincide in all channels

Consequences of HQSS

- Poles of T^{1++} and T^{2++} coincide Nieves, Valderrama (2012)
- By making unitarity transform *U* for 0^{++} and 1^{+-} potentials $U = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$

$$\tilde{V}^{(JPC)} = UV^{(JPC)}U^\dagger = \begin{pmatrix} C_0 & 0 \\ 0 & C_0' \end{pmatrix} \qquad \begin{aligned} C_0 &= C_{0a} + C_{0b} \\ C_0' &= C_{0a} - 3C_{0b} \end{aligned}$$

Coupled-channel LS Eq. decouples into two Eqs. with V= C_0 and V= C_0'

☛ In the strict HQSS limit $T^{(JPC)}$ possess two decoupled solutions:

$$E^{(0)}_{X_{1++}} = E^{(0)}_{X_{2++}} = E^{(0)}_{X_{1+-}} = E^{(0)}_{X_{0++}} \quad \text{and} \quad E^{(0)}_{X'_{0++}} = E^{(0)}_{X'_{1+-}}$$

our finding is in line with Hidalgo-Duque et al. (2013)

☛ While the partner states corresponding to V= C_0 are predictable using the X(3872) as input, an additional experimental input is needed to determine the location of the states controlled by V= C_0'

Contact theory with HQSS breaking

- Introduce D*–D mass splitting and average mass:

$$\delta = m_* - m = 141 \text{ MeV}$$
$$\bar{m} = \frac{1}{4}(3m_* + m) = 1973 \text{ MeV}$$
$$\delta/\bar{m} \simeq 7\%$$

- Leading effect: the states reside near the corresponding thresholds: $D\bar{D}, D\bar{D}^*$ and $D^*\bar{D}^*$

For example: $M_{X_{2++}} = M_{X_{1++}} + \delta$

- HQSS based relations between the binding momenta:

$$\gamma_{X_{2++}} = \left(1 - \frac{\delta}{2\bar{m}}\right)\gamma_{X_{1++}} + \frac{\delta\,\Lambda}{\pi\bar{m}} + O\left(\frac{\delta^2\Lambda}{\bar{m}^2}, \frac{\gamma^2_{X_{1++}}}{\Lambda}\right)$$

$$\gamma_{X'_{1+-}} = \left(1 - \frac{\delta}{2\bar{m}}\right)\gamma_{X_{1+-}} + \frac{\delta\,\Lambda}{\pi\bar{m}} - \frac{(\gamma_{X_{1+-}} - \gamma_{X_{1++}})^2}{\sqrt{\bar{m}\delta}} + i\frac{(\gamma_{X_{1+-}} - \gamma_{X_{1++}})^2}{\sqrt{\bar{m}\delta}} + \dots$$

☛ Correction at *O(δ)* is cutoff dependent ⇒ extra contact term for renormalization
⇒ But small impact on the location of the states

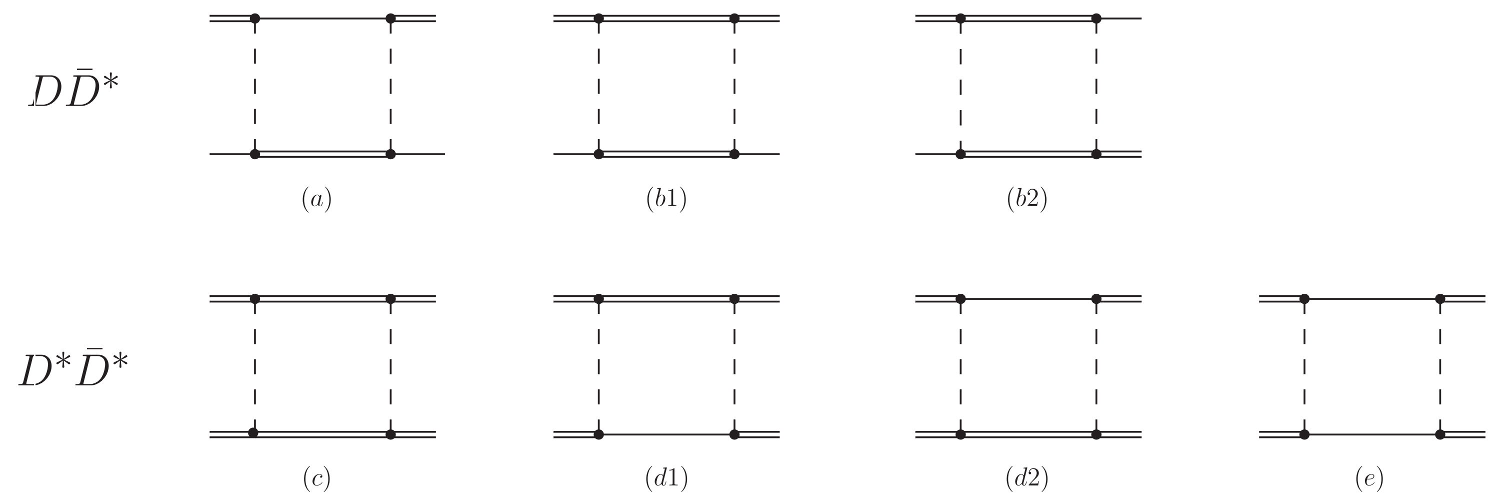
☛ *two exp. inputs needed in the coupled-channel case:* $\gamma_{X_{1++}}$ and $\gamma_{X_{1+-}} \Longrightarrow \gamma_{X'_{1+-}}$

☛ $\gamma_{X'_{1+-}}$ acquires an *Im* part due to coupled-channels $D^*\bar{D}^* \to D\bar{D}^* \to D^*\bar{D}^*$

Strict HQSS limit in the presence of pions

- New transitions due to OPE $\Longrightarrow$ more coupled channels

For example, at one loop:



- Extended basis states:

$$
\begin{aligned}
0^{++}: &\quad \{D\bar{D}(^1S_0), D^*\bar{D}^*(^1S_0), D^*\bar{D}^*(^5D_0)\},\\
1^{+-}: &\quad \{D\bar{D}^*(^3S_1,-), D\bar{D}^*(^3D_1,-), D^*\bar{D}^*(^3S_1), D^*\bar{D}^*(^3D_1)\},\\
1^{++}: &\quad \{D\bar{D}^*(^3S_1,+), D\bar{D}^*(^3D_1,+), D^*\bar{D}^*(^5D_1)\},\\
2^{++}: &\quad \{D\bar{D}(^1D_2), D\bar{D}^*(^3D_2), D^*\bar{D}^*(^5S_2), D^*\bar{D}^*(^1D_2), D^*\bar{D}^*(^5D_2), D^*\bar{D}^*(^5G_2)\}
\end{aligned}
$$

☛ Coupled-channel transitions depend on quantum numbers

- Chiral EFT at LO — contact terms + static OPE — does not depend on the heavy-quark mass $\Longrightarrow$ HQSS multiplets (4+2) found in the contact case should hold!

But how to see this?

Strict HQSS limit in the presence of pions

our work (2016)

- Unitary transform brings the potential to block-diagonal form:

$$\tilde{V}^{(0++)}(3\times 3) = A(2\times 2) \oplus B(1\times 1),$$
$$\tilde{V}^{(1+-)}(4\times 4) = A(2\times 2) \oplus B(1\times 1) \oplus C(1\times 1)$$
$$\tilde{V}^{(1++)}(3\times 3) = A(2\times 2) \oplus D(1\times 1),$$
$$\tilde{V}^{(2++)}(6\times 6) = A(2\times 2) \oplus D(1\times 1) \oplus E(3\times 3)$$

☛ *A(2 x 2) is common for all quantum numbers (related to C0)* $\Longrightarrow$ *4 degenerate states*

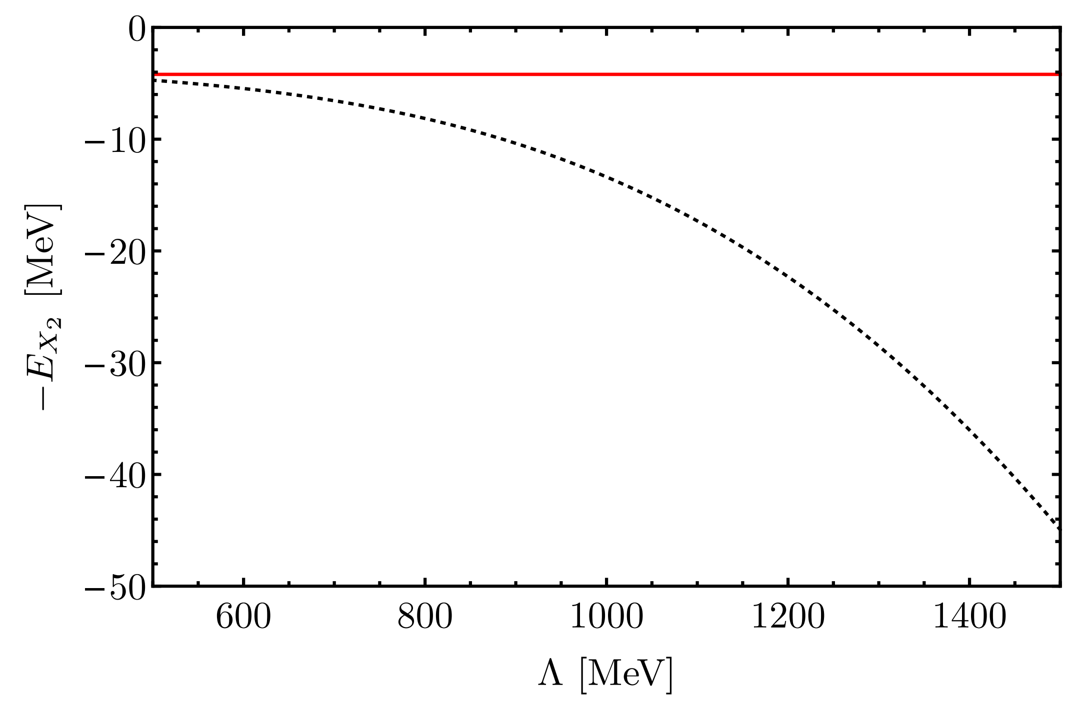
☛ *B(1 x 1) is related to C_0'* $\Longrightarrow$ *bring about 2 more degenerate states* $0^{++}, 1^{+-}$

- But these HQSS predictions hold only if all particle coupled channels are included!

Example: <u>2⁺⁺ partner state</u>

Full coupled-channel dynamics: $E_{X_{2++}} = E_{X_{1++}}$

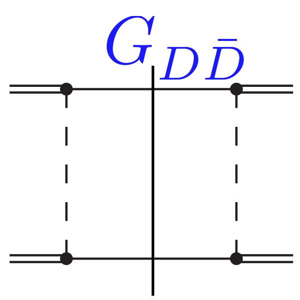
Neglecting $D^*\bar{D}^* \to D\bar{D} \to D^*\bar{D}^*$, $D^*\bar{D}^* \to D\bar{D}^* \to D^*\bar{D}^*$ transitions, as done by Nieves, Valderrama (2012), leads to severe violation of HQSS predictions

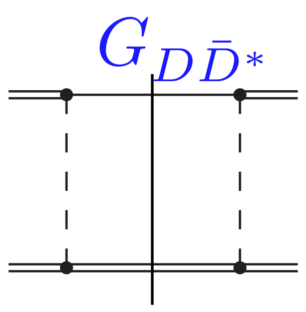


Contact + OPE interactions: beyond strict HQSS limit

- Switch on D*–D mass splitting $\Longrightarrow$ 2^{++} $D^*\bar{D}^*$ state acquires the finite width

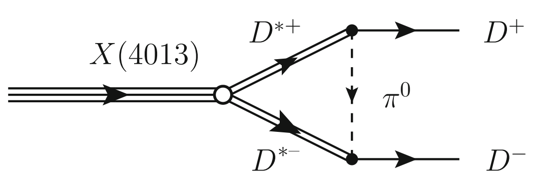
Example of transitions which cause the Imaginary part of the amplitude:





- perturbative pions*: the width* $\Gamma_{X_{2++}}$ *~ 1—20 MeV*

For example:



	Without pion-exchange FF		With pion-exchange FF	
	$\Lambda = 0.5$ GeV	$\Lambda = 1$ GeV	$\Lambda = 0.5$ GeV	$\Lambda = 1$ GeV
$\Gamma(X_2 \to D^+D^-)$ [MeV]	$3.3^{+3.4}_{-1.4}$	$7.3^{+7.9}_{-2.1}$	$0.5^{+0.5}_{-0.2}$	$0.8^{+0.7}_{-0.2}$
$\Gamma(X_2 \to D^0\bar{D}^0)$ [MeV]	$2.7^{+3.1}_{-1.2}$	$5.7^{+7.8}_{-1.8}$	$0.4^{+0.5}_{-0.2}$	$0.6^{+0.7}_{-0.2}$
$\Gamma(X_2 \to D^+D^{*-})$ [MeV]	$2.4^{+2.1}_{-1.0}$	$4.4^{+3.1}_{-1.2}$	$0.7^{+0.6}_{-0.3}$	$1.0^{+0.5}_{-0.2}$
$\Gamma(X_2 \to D^0\bar{D}^{*0})$ [MeV]	$2.0^{+2.1}_{-0.9}$	$3.5^{+3.5}_{-1.0}$	$0.5^{+0.6}_{-0.2}$	$0.7^{+0.5}_{-0.2}$

Albaladejo, Guo, Hidaldo-Duque, Nieves, Valderrama (2015)

- But the *relevant momentum scales* stem from coupled-channels induced by OPE

$$q_1 = \sqrt{2\delta\bar{m}} \approx 700 \text{ MeV} \qquad \text{from} \qquad G_{D\bar{D}} = \frac{1}{k^2/2\mu - 2\delta - E - i0}$$

$$q_2 = \sqrt{\delta\bar{m}} \approx 500 \text{ MeV} \qquad \text{from} \qquad G_{D\bar{D}^*} = \frac{1}{k^2/2\mu_* - \delta - E - i0}$$

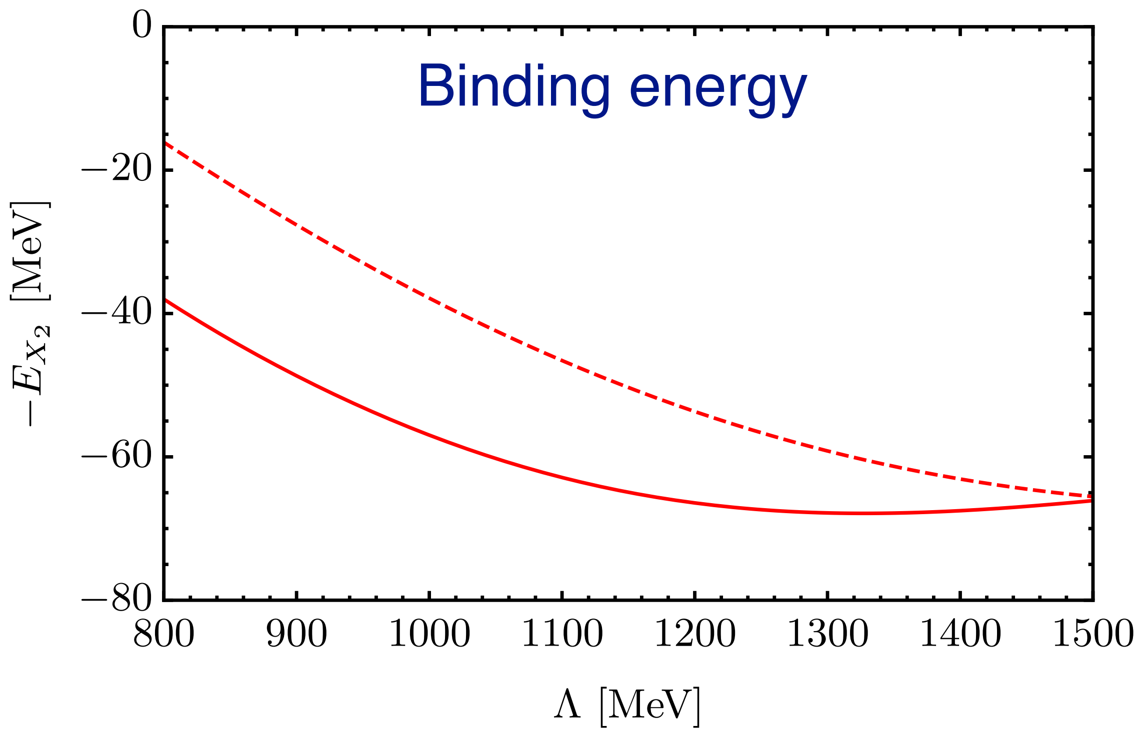
$\Longrightarrow$ Coupled-channel transitions in D-waves are not suppressed relative to S-wave interactions

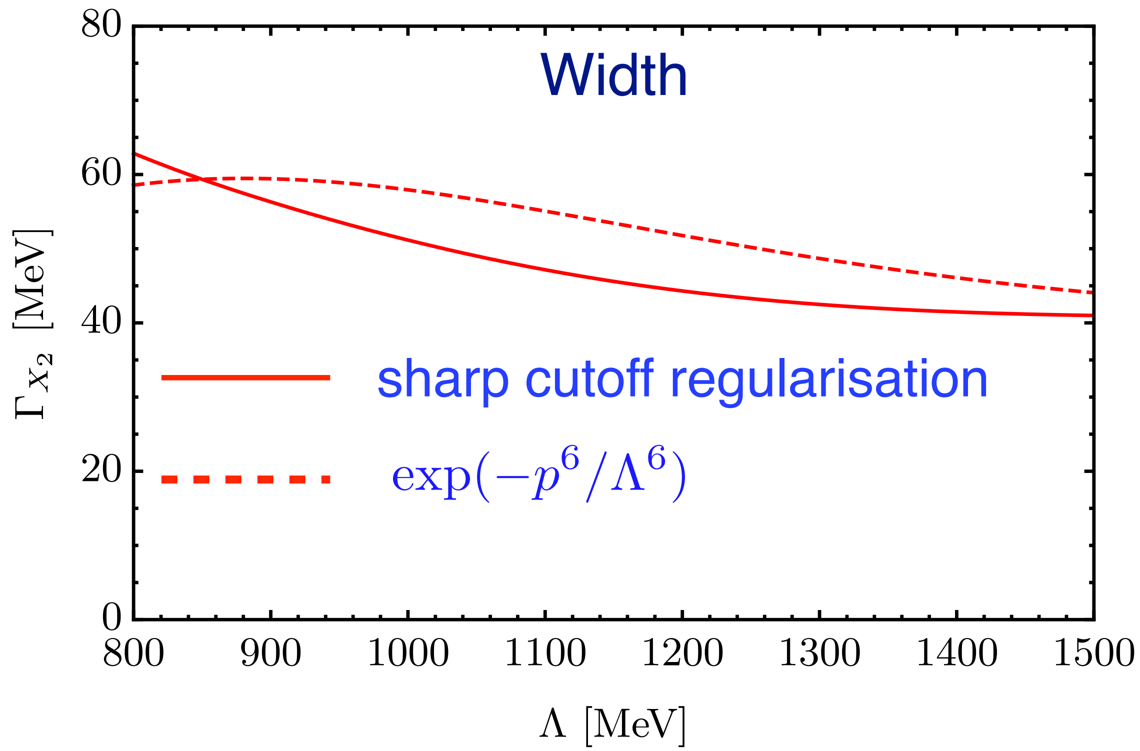
$\Longrightarrow$ Iterations of OPE to all orders are expected to be important

Contact + OPE interactions at LO: Non-perturbative Results

- 2^{++} partner from the nonperturb. solution of the LS equation with $V = C_0 + V_{\mathrm{OPE}}$

$$V_{\mathrm{OPE}}(\vec{p},\vec{p}\,') \sim -\frac{g_c^2}{(4\pi f_\pi)^2}\frac{(\vec{A}_1^* \cdot \vec{q})(\vec{A}_2 \cdot \vec{q})}{\vec{q}^{\,2} + M_\pi^2}$$





Relatively Λ independent due to unitarity

☛ Significant shift of $E_{X_{2++}}$ and large width $\Gamma_{X_{2++}} \simeq 50 \pm 10$ MeV

much larger than in the perturbative study
Albaladejo et al. (2015)

☛ Cutoff variation $\Longrightarrow$ estimate of a higher-order contact term at *O(δ)*

Open Questions and Theory To-Do List

- Relatively small separation of scales may call the convergence of the EFT into question

  ☛ include explicitly the members of SU(3) pseudoscalar octet as well as vector mesons

- Investigate the role of three-body effects in the OPE potential

  For the role of three-body dynamics for the X(3872) see Fleming et al. (2007), our works (2010-2015), Jansen et al. (2015), Guo et al. (2014)

  ☛ Since the main contribution to the width of the 2^{++} D*D* state stems from coupled channels, three-body effects are not expected to change the picture qualitatively

  ☛ Bring additional Imaginary parts from the right-hand cut

  ☛ Bring additional HQSS corrections due to D, D* energies

- Estimate HQSS violating contact terms more reliably

Summary

- We confirm that in the *strict HQSS* limit there are two degenerate multiplets of isoscalar molecular partner states with

$$E^{(0)}_{X_{1++}} = E^{(0)}_{X_{2++}} = E^{(0)}_{X_{1+-}} = E^{(0)}_{X_{0++}} \quad \text{and} \quad E^{(0)}_{X'_{0++}} = E^{(0)}_{X'_{1+-}}$$

- This conclusion holds in the presence of OPE interactions if and only if all particle coupled-channel transitions are included

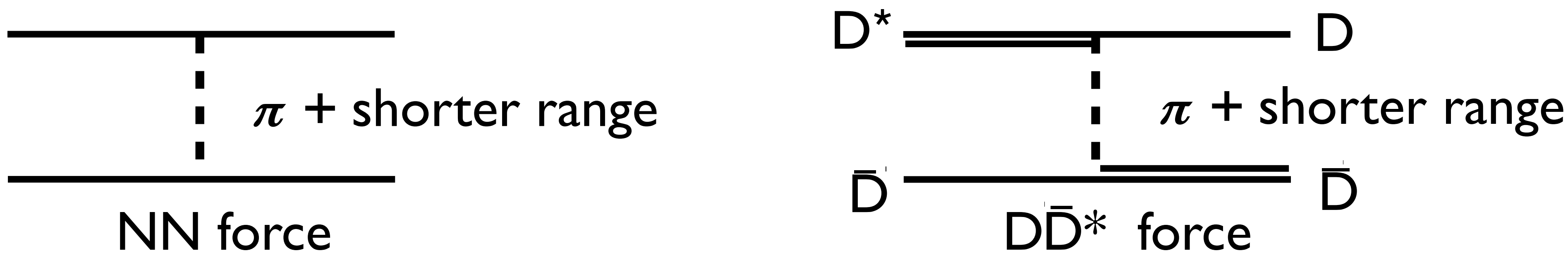
- Leading HQSS breaking correction stems from D*-D mass splitting
  - ☛ Molecular states reside in the vicinity of their thresholds
  - ☛ Predictions for the 2^{++} partner of the the X(3872) are possible
  - ☛ One additional experimental input is needed to predict 0^{++} and 1^{+-} partners

- Non-perturbative pion dynamics plays the crucial role for the determination of the parameters of the 2^{++} partner state
  - ☛ The predicted width — $\Gamma_{X_{2++}} \simeq 50 \pm 10\ \mathrm{MeV}$ — is much larger than in a theory with perturbative pions

Spires

X(3872) as a molecular state

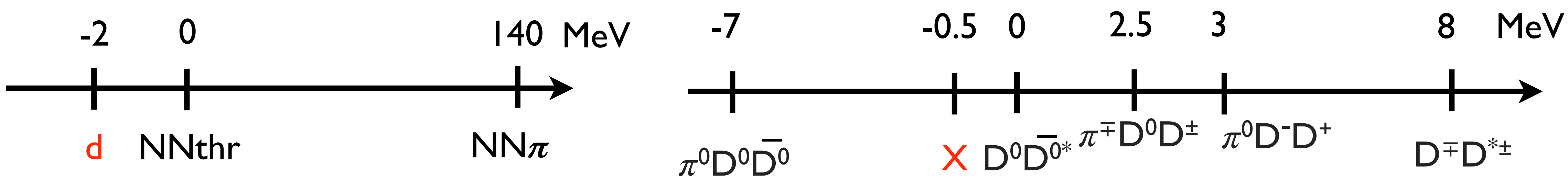
- Prediction of the $D\bar{D}^*$ molecular state similar to the deuteron — Okun, Voloshin (1976), Törnqvist (1991)



- Experience from nuclear physics: One pion exchange (OPE) potential governs the lowest energy left-hand singularity of the NN force $\Longrightarrow$ crucial influence on the NN parameters
  Epelbaum, Gegelia (2012), V.B., Filin, Epelbaum,Gegelia (2015)

- OPE between $D\bar{D}^*$ has even richer applications—many three-body thresholds around

Thresholds



☛ OPE is important for the decay rate $X \to D\bar{D}\pi$ and for chiral extrapolations of the X-pole
Fleming et al. (2007), our works (2010-2015), Jansen et al. (2015), Guo et al. (2014)

$\Longrightarrow$ Investigate the role of OPE for the molecular partner states

Contact theory with HQSS breaking: coupled channels

- Green functions of $D\bar{D},\ D\bar{D}^*$ and $D^*\bar{D}^*$ states are not the same anymore

$$G^{(1+-)}_{\rm LO}(k) = \begin{pmatrix} (k^2/2\mu_* - E - i0)^{-1} & 0 \\ 0 & (k^2/2\mu_{**} - E - i0)^{-1} \end{pmatrix}, \qquad G^{(0++)}_{\rm LO}(k) = \begin{pmatrix} (k^2/2\mu - E - i0)^{-1} & 0 \\ 0 & (k^2/2\mu_{**} - E - i0)^{-1} \end{pmatrix},$$

$$2\mu = \bar{m} - \frac{3}{4}\delta, \quad 2\mu_* = \bar{m} - \frac{\delta}{4}, \quad 2\mu_{**} = \bar{m} + \frac{1}{4}\delta,$$

⇒ *VG in the Lippmann-Schwinger Eq.* can not be *diagonalized*

☛ *poles are determined by both C_0 and C_0' simultaneously — two exp. inputs needed*

- C_0' can be fixed assuming the X(3915) to be a 0^{++} $D^*\bar{D}^*$ partner of the X(3872)

Nieves and Valderrama (2012)

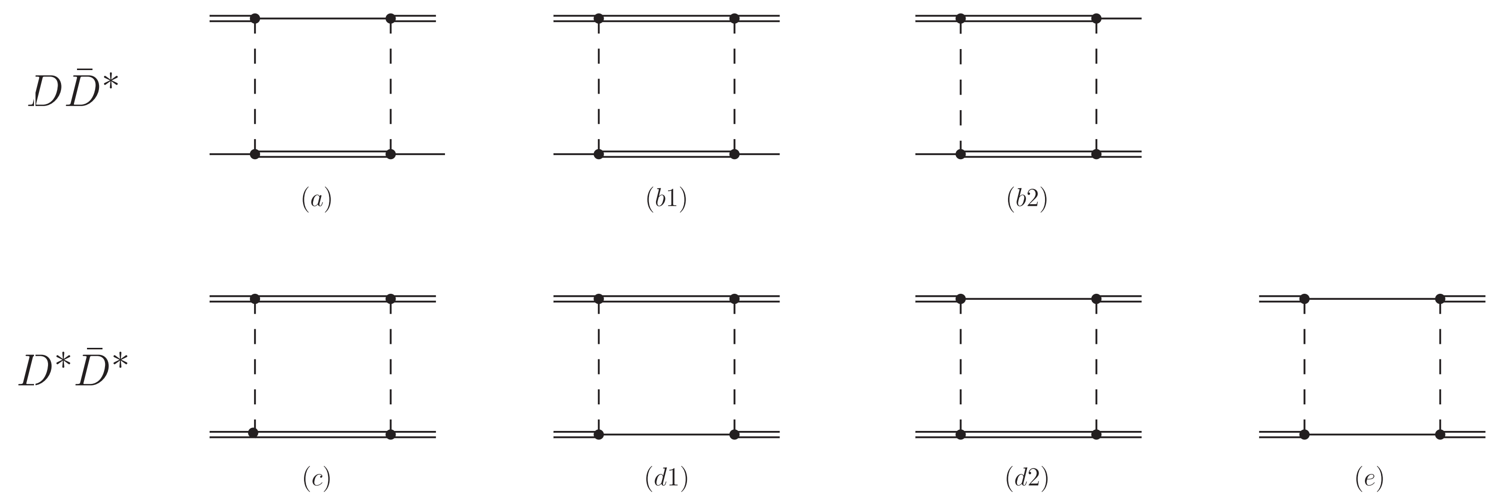
But ☛ *X(3915) is about 100 MeV below* $D^*\bar{D}^*$

☛ *Zhou et al. (PRL 2015) argue that the X(3915) and X(3930) is the same* 2^{++}*state*

Strict HQSS limit in the presence of pions

- New transitions due to OPE $\Longrightarrow$ more coupled channels

For example, at one loop:



- Extended basis states:

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\end{aligned}
$$

- Can one absorb the divergencies of the loop diagrams with different J^{PC} to a single contact term C_0? *Yes but only if all coupled-channel transitions are included!*

For each J^{PC} the coefficient in front of the leading divergence is the same once *all coupled-channels* are included

		$D\bar{D}$	$D\bar{D}^*$	$D^*\bar{D}^*$	Sum
1^{++}	$^{2S+1}L_J$	—	$^3S_1$ $^3D_1$	$^5D_1$	
	Coeff.	—	$1/9$ $2/9$	$2/3$	1
2^{++}	$^{2S+1}L_J$	$^1D_2$	$^3D_2$	$^5S_2$ $^1D_2$ $^5D_2$ $^5G_2$	
	Coeff.	$2/15$	$2/5$	$1/9$ $2/45$ $14/45$ 0	1

$\Longrightarrow$ successful renormalization program