

*Scattering of Charmed Mesons from Lattice **QCD***

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Hadron Spectrum Collaboration

University of Cambridge

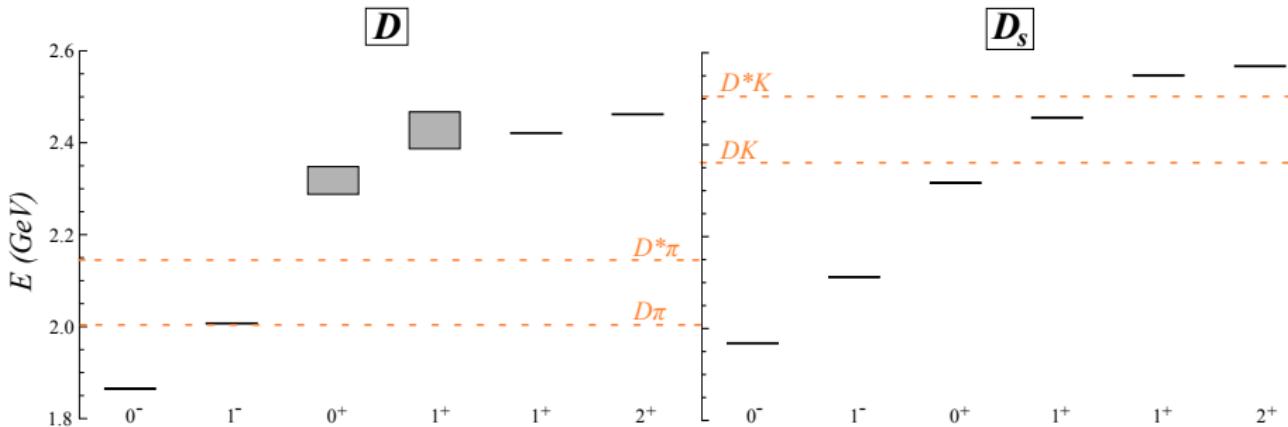


29th August 2016

**XIIth Quark Confinement and the Hadron Spectrum
Thessaloniki, Greece**

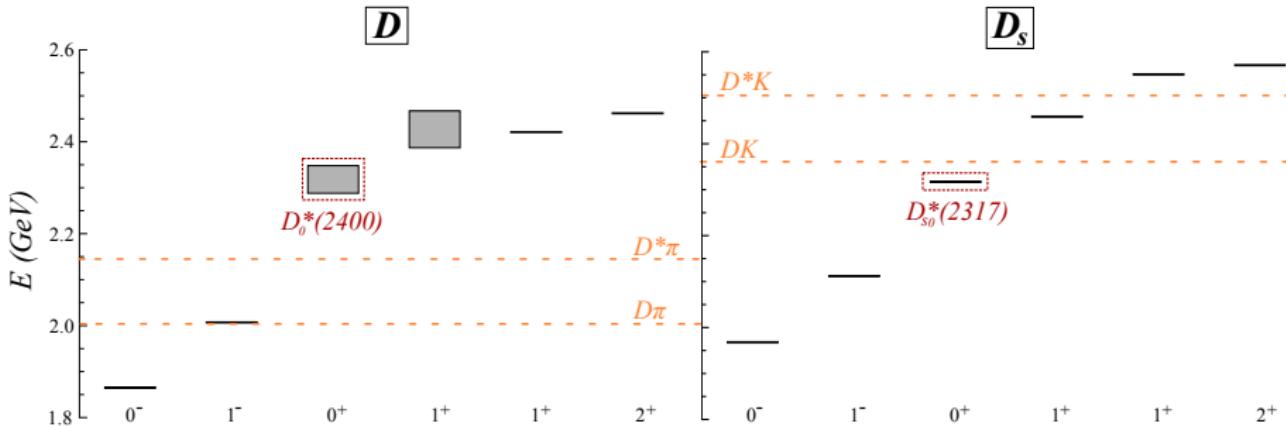
Open-Charm Mesons

(Taken from the PDG)



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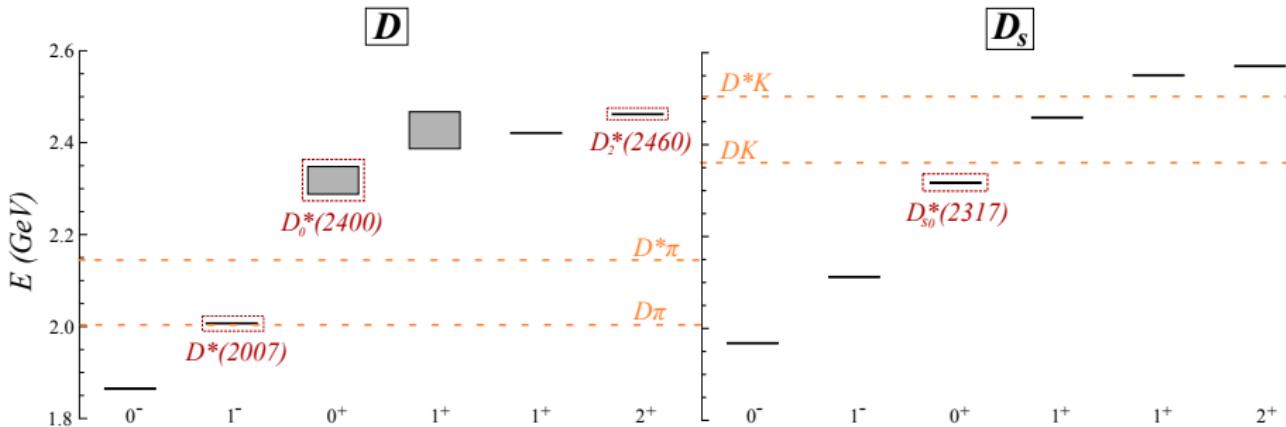


- First principles \Rightarrow lattice QCD

| Meson | J^P | Mass (MeV) | Width (MeV) |
|------------------|-------|------------|-------------|
| $D_0^*(2400)$ | 0^+ | 2318(29) | 267(40) |
| $D_{s0}^*(2317)$ | 0^+ | 2317.6(6) | < 3.8 |

Open-Charm Mesons

(Taken from the PDG)



- First principles \Rightarrow lattice QCD

This talk

- Coupled-channel $D\pi$, $D\eta$ and $D_s\bar{K}$ scattering ($I = 1/2$) [arXiv:1607.07093]
- DK scattering ($I = 0$) (Preliminary)

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Scattering on the Lattice

Discrete finite-volume spectrum is given by energies solving

$$\det \left[t_{ij}^{-1}(E) + \mathcal{M}_{ij}(E, L) \right] = 0$$

- i, j label channels
- $t_{ij}(E)$ is the **infinite-volume** scattering t -matrix . . . mixes channels
- $\mathcal{M}_{ij}(E, L)$ are known finite-volume functions . . . mixes partial waves

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Lattice QCD spectrum \Rightarrow infinite volume t -matrix

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Lattice QCD spectrum \Rightarrow infinite volume t -matrix

The bad news

- N coupled-channels \Rightarrow t -matrix has $(N^2 + N)/2$ unknowns (per energy level)
 \Rightarrow very under-constrained

Scattering on the Lattice

A work-around

- Parametrise the t -matrix using a “few” free parameters
- Use many more than a “few” lattice QCD energy levels to constrain $t(E)$

We want to

- Preserve S -matrix **unitarity**
- Analytically continue into the complex $s = E_{cm}^2$ plane
- Examine the **pole content** of the parametrised t -matrix

Make use of many parametrisations

- \mathcal{K} -matrix
- Breit-Wigner
- Effective Range Expansion
- ...

Coupled-channel $D\pi$, $D\eta$ and $D_s\bar{K}$ Scattering ($l = 1/2$)

[arXiv:1607.07093]

Finite-Volume Energy Levels

Reduced rotational symmetry

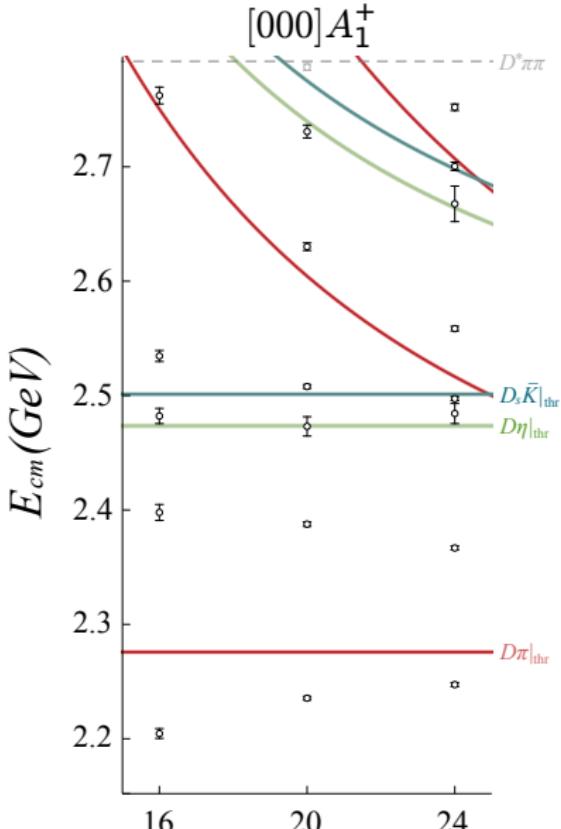
- Partial waves mix in each lattice irrep $[\vec{P}]\Lambda$
- $[000]A_1^+ : \ell = 0, 4, \dots$

Finite-volume energies

- $C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$
- Large basis of “single” and “two-meson” operators for each lattice irrep
- $D\pi(I = 1/2)$ $[000]A_1^+$: $D\pi(4)$, $D\eta(3)$, $D_s\bar{K}(3)$, $D(11)$

Note

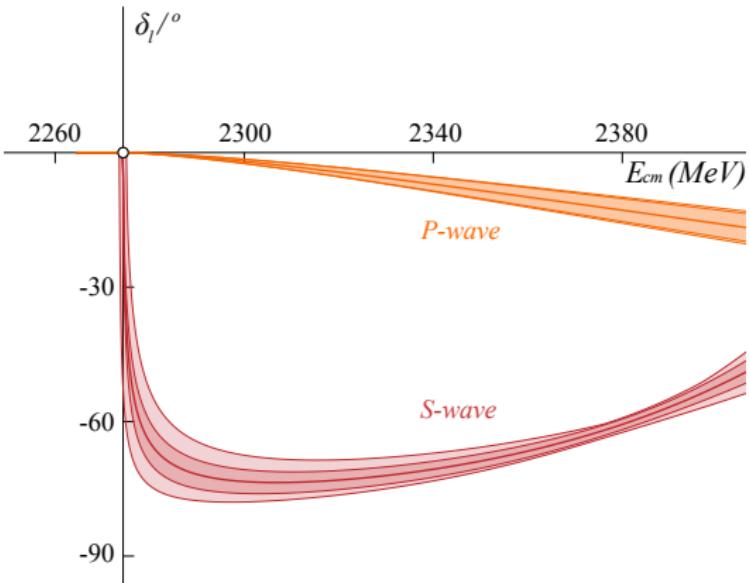
- $M_\pi \approx 391$ MeV
- $M_D \approx 1885$ MeV
- $M_{D_s} \approx 1950$ MeV



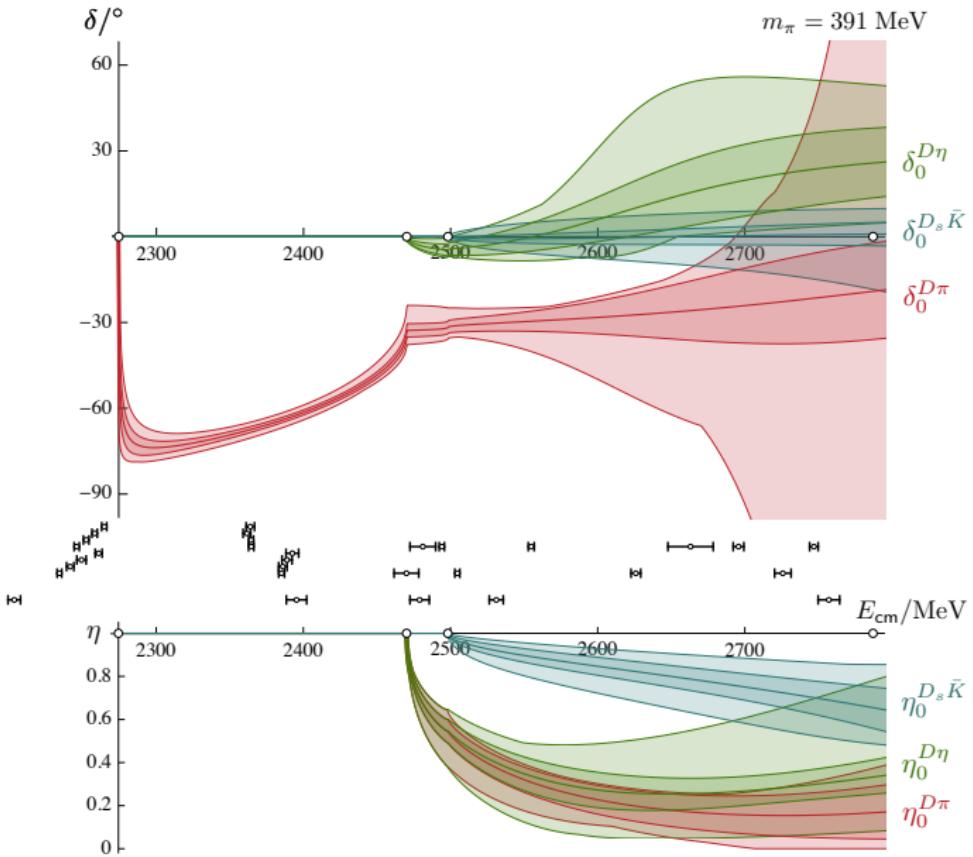
Elastic $D\pi$ Scattering

All irreps with $\ell = 0, 1$

- Minimise χ^2 describing difference between *parametrised* and lattice QCD spectra
- S -wave: rapid phase variation near $D\pi$ threshold
- P -wave: slowly varying phase

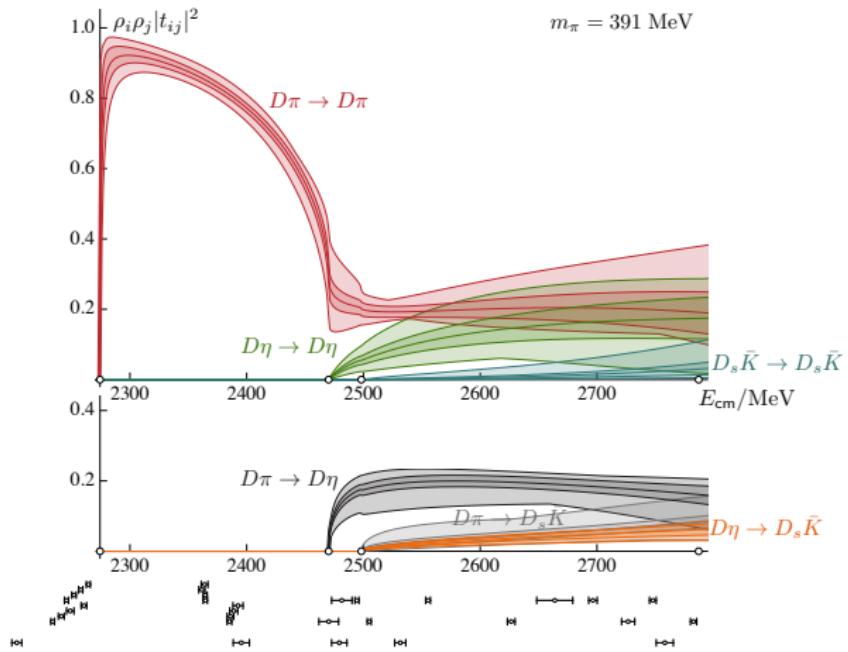


Coupled-Channel $D\pi$, $D\eta$ and $D_s\bar{K}$ Scattering



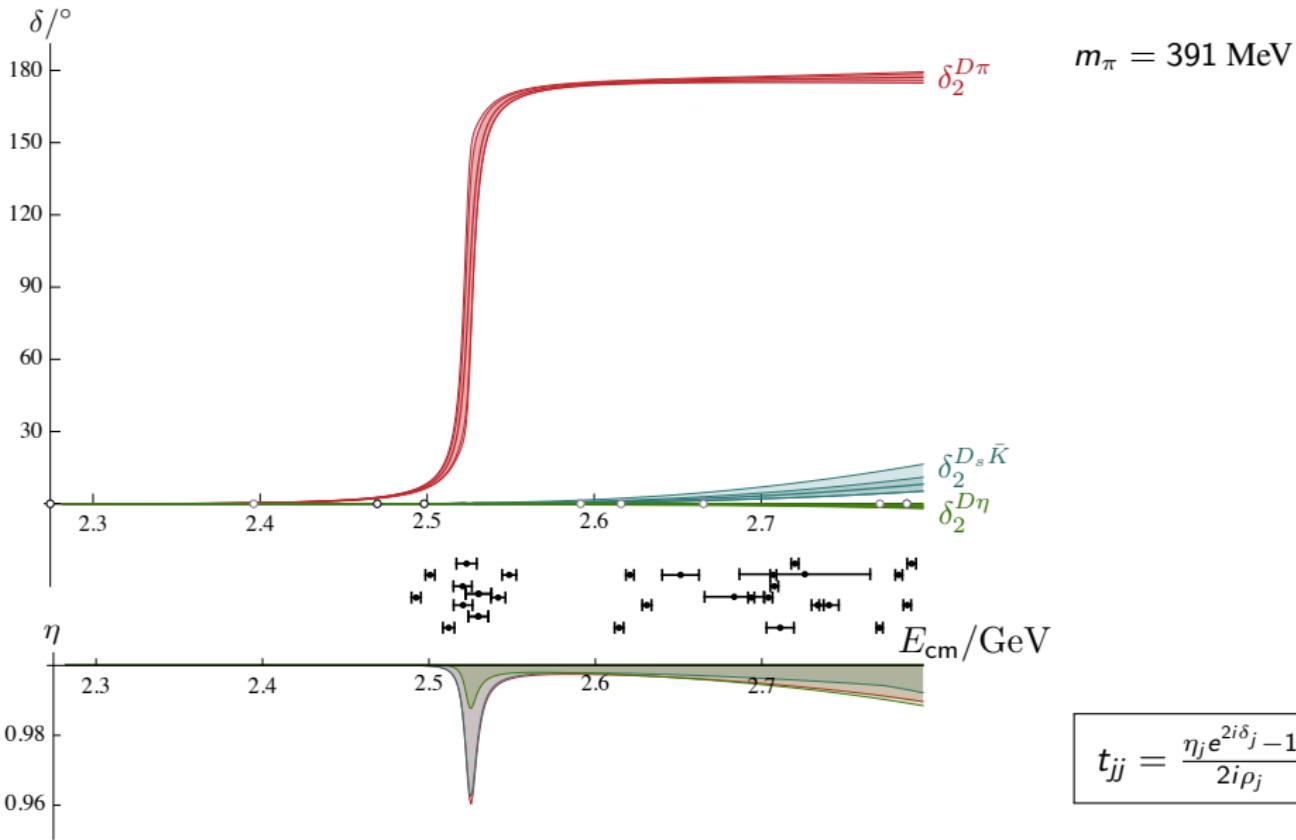
$$t_{jj} = \frac{\eta_j e^{2i\delta_j} - 1}{2i\rho_j}$$

Coupled-Channel $D\pi$, $D\eta$ and $D_s\bar{K}$ Scattering

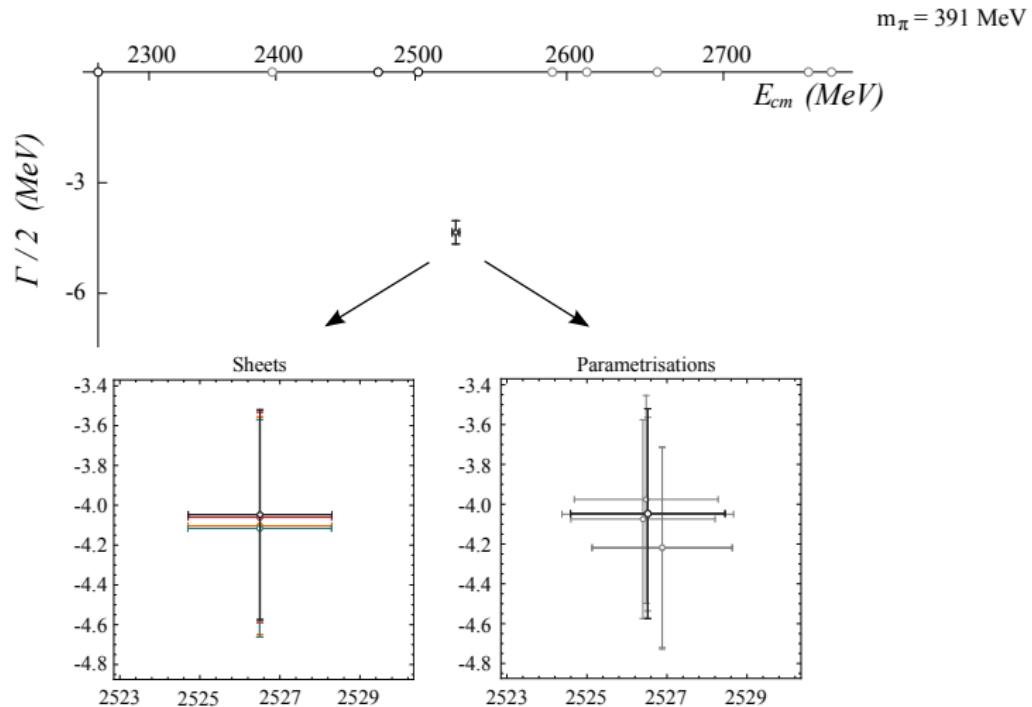


- Near-threshold S -wave pole on the **real-axis** $t_{ij} \sim c_i c_j / (s_{pole} - s)$
- Couplings: $c_{D\pi} = 549(158)$ MeV ; $c_{D\eta} = 436(130)$ MeV ; $c_{D_s\bar{K}} = 221(85)$ MeV.

Coupled-Channel $D\pi$, $D\eta$ and $D_s\bar{K}$ Scattering

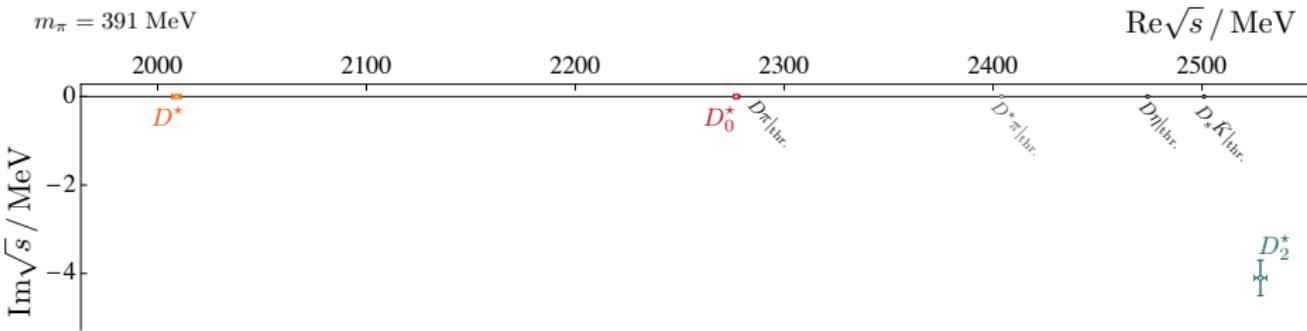


Coupled-Channel $D\pi$, $D\eta$ and $D_s\bar{K}$ Scattering



- Resonance pole on all sheets with $\text{Im}[k_{D\pi}] < 0$

Coupled-Channel $D\pi$, $D\eta$ and $D_s\bar{K}$ Scattering



S-wave

- **Bound-state** pole: 2275.9 ± 0.9 MeV c.f. $D_0^*(2400)$: 2318 ± 29 MeV

P-wave

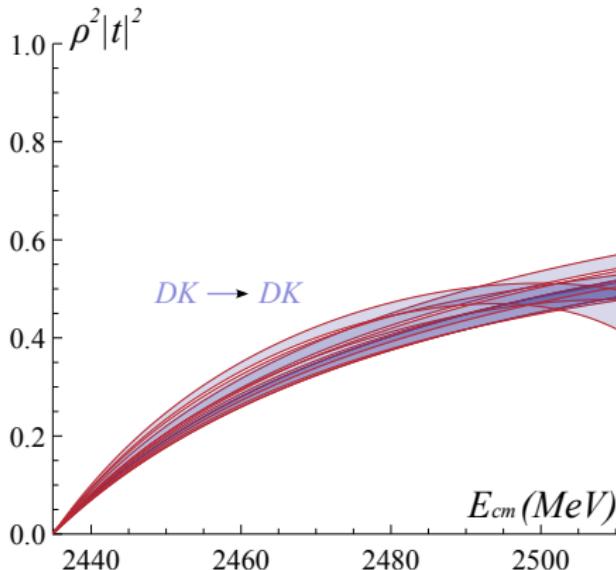
- **Bound-state** pole: 2009 ± 2 MeV c.f. $D^*(2007)$: 2006.9 ± 0.08 MeV

D-wave

- **Resonance** pole: $m = 2527 \pm 3$ MeV ; $\Gamma = 8.2 \pm 0.7$ MeV
c.f. $D_2^*(2460)$: $m = 2462.6 \pm 0.6$ MeV ; $\Gamma = 49.0 \pm 1.3$ MeV

DK Scattering ($I = 0$) (Preliminary)

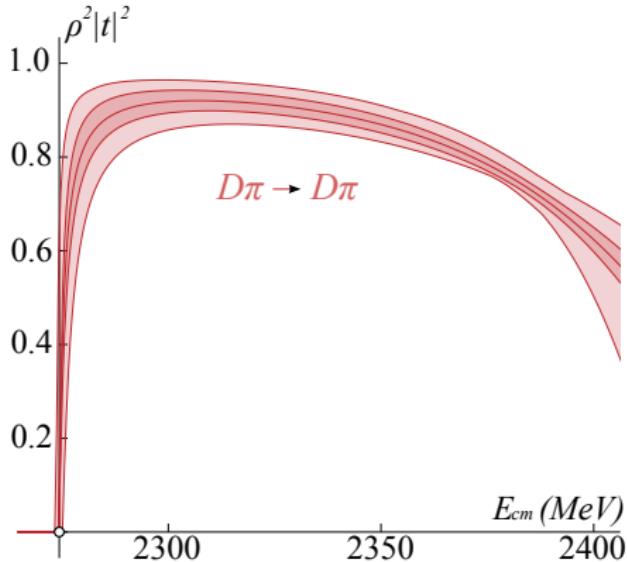
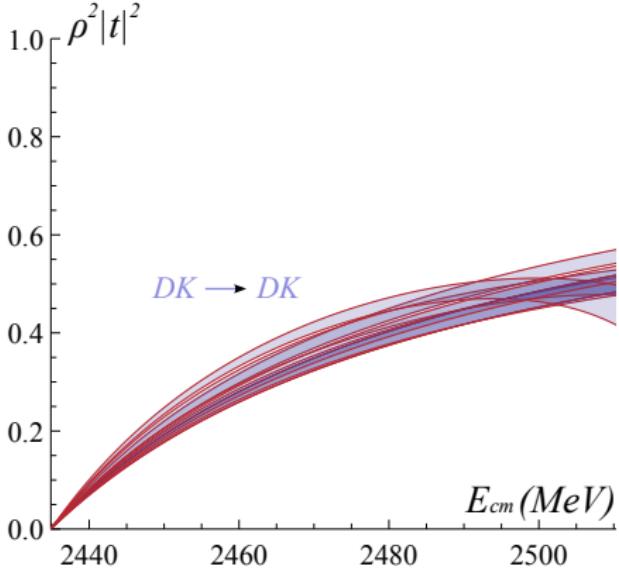
DK Scattering (Preliminary)



S-wave

- **Bound-state** pole ≈ 2380 MeV ; ≈ 55 MeV below DK threshold (at $m_\pi = 391$)
- Expt. $D_{s0}^*(2317)$: 2317.7 ± 0.6 MeV ; ≈ 45 MeV below DK threshold

DK Scattering (Preliminary)



S-wave

- **Bound-state** pole ≈ 2380 MeV ; ≈ 55 MeV below DK threshold (at $m_\pi = 391$)
- Expt. $D_{s0}^*(2317)$: 2317.7 ± 0.6 MeV ; ≈ 45 MeV below DK threshold
- c.f. S-wave pole in $D\pi$ channel: ≈ 1 MeV below threshold

Summary and Outlook

Summary:

First lattice QCD calculation of coupled-channel scattering including heavy quarks

$D\pi$ ($I = 1/2$) channel (at $m_\pi = 391$ MeV)

- 0^+ : near-threshold bound-state pole c.f. $D_0^*(2400)$
- 1^- : deeply-bound pole c.f. $D^*(2007)$
- 2^+ : narrow resonance c.f. $D_2^*(2460)$

DK ($I = 0$) channel (at $m_\pi = 391$ MeV)

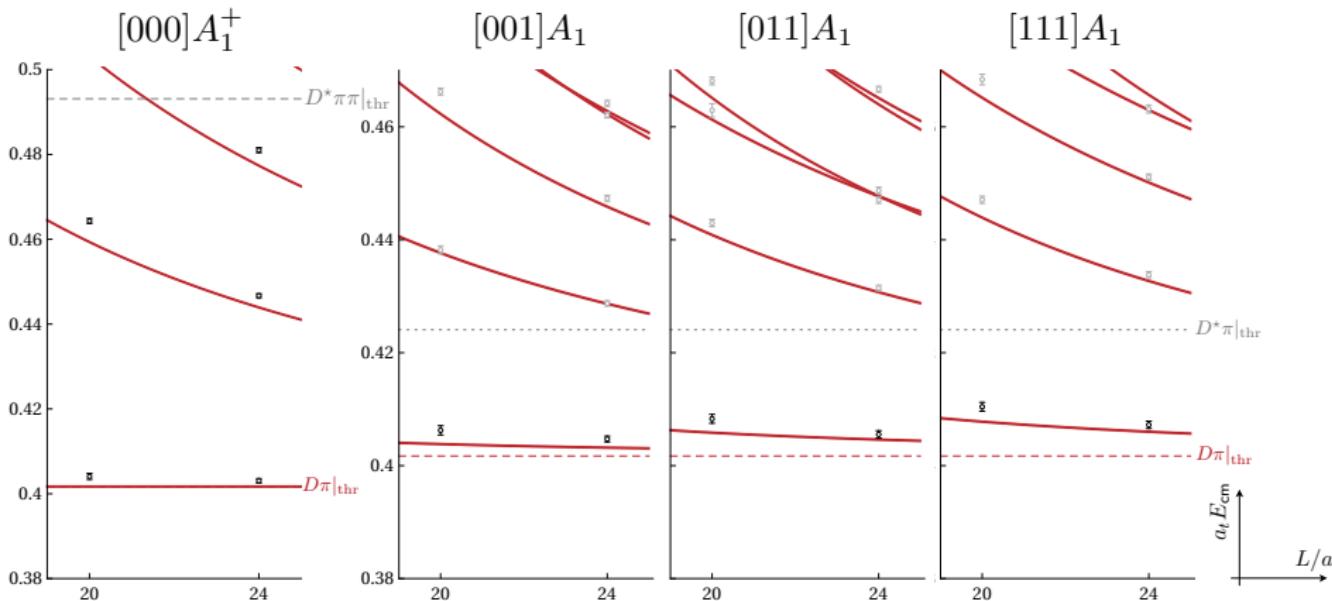
- 0^+ : bound-state pole c.f. $D_{s0}^*(2317)$

Outlook:

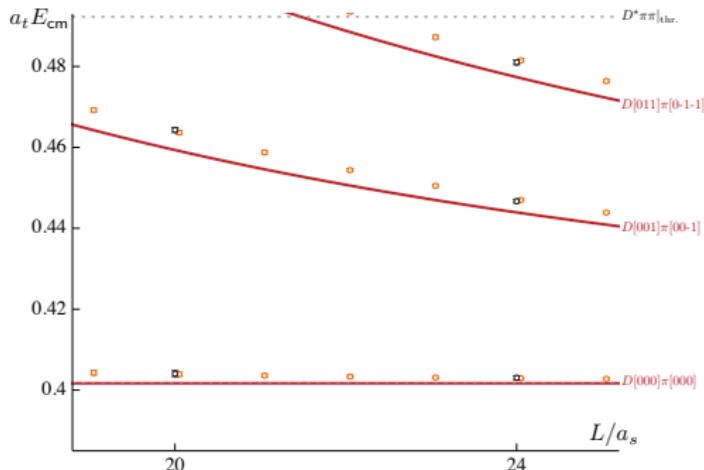
- Pole dependence on m_π under-way
- Address X,Y,Z's ?

Extra Slides

Isospin-3/2: Finite-Volume Energy Levels

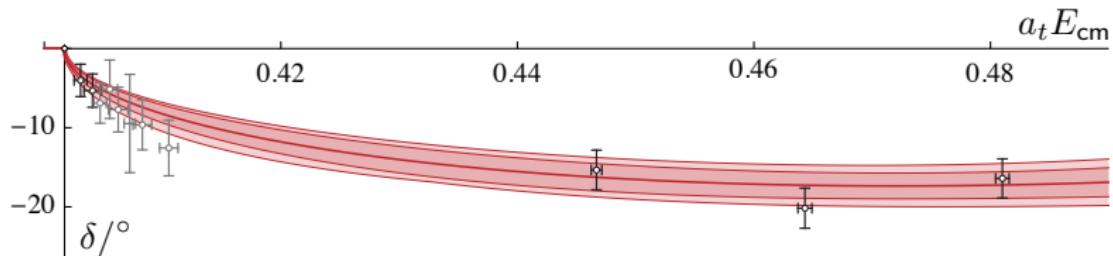


Isospin-3/2: $D\pi$ Phase-Shift



- $\chi^2/N_{\text{dof}} = 1.29 :$

- $a_0 = -0.19(5) \text{ fm}$
- $r_0 = -0.9(4) \text{ fm.}$



\mathcal{K} -Matrix Parametrisation

A *convenient choice* is given by a **\mathcal{K} -matrix** description

$$t_{ij}^{-1}(s) = \mathcal{K}_{ij}^{-1}(s) + \mathcal{I}_{ij}(s)$$

Preservation of unitarity

- Above kinematic threshold: $\text{Im}[\mathcal{I}_{ij}(s)] = -\delta_{ij}\rho_i(s)$
- Below kinematic threshold: $\text{Im}[\mathcal{I}_{ij}(s)] = 0$

Flexibility in $\text{Re}[\mathcal{I}_{ij}(s)]$

- Zero above kinematic threshold
- Chew-Mandelstam prescription

Most freedom comes from parametrising $\mathcal{K}_{ij}(s)$, for example,

$$\mathcal{K}_{ij}(s) = \sum_p \frac{g_i^{(\rho)} g_j^{(\rho)}}{m_p^s - s} + \sum_n \gamma_{ij}^{(n)} s^n$$

Extracting Finite-Volume Energy Levels

Solve a GEVP for a matrix of 2-pt functions constructed using

① Distillation

- Improves overlap with low-lying states
- **Efficient** re-use of perambulators

② Large bases of interpolating operators for each irrep $[\vec{P}]\Lambda$

- “Single-meson” $\sim \bar{\psi}\Gamma D \dots \psi$ with up to 3 derivatives for $\vec{P} = 0$ (2 for $\vec{P} \neq 0$)
- “Two-meson” $\sim \Omega_{\vec{p}_1}^{(1)\dagger} \Omega_{\vec{p}_2}^{(2)\dagger}$ for **variationally optimised** “single-mesons” $\Omega_{\vec{p}}^{(i)\dagger}$

| $[000]A_1^+$ | $[001]A_1$ | $[011]A_1$ | $[111]A_1$ |
|------------------------------------|------------------------------------|------------------------------------|------------------------------------|
| $D_{000}\pi_{000}$ | $D_{001}\pi_{000}$ | $D_{011}\pi_{000}$ | $D_{111}\pi_{000}$ |
| $D_{001}\pi_{00-1}$ | $D_{000}\pi_{00-1}$ | $D_{000}\pi_{-0-1-1}$ | $D_{000}\pi_{-1-1-1}$ |
| $D_{011}\pi_{0-1-1}$ | $D_{011}\pi_{00-1}$ | $D_{001}\pi_{-0-10}$ | $D_{011}\pi_{-100}$ |
| * $D_{111}\pi_{-1-1-1}$ | $D_{001}\pi_{-0-1-1}$ | $D_{011}\pi_{-1-10}$ | $D_{001}\pi_{-1-10}$ |
| $D_{000}\eta_{000}$ | $D_{111}\pi_{0-1-1}$ | $D_{111}\pi_{-00-1}$ | $D_{002}\pi_{-1-1-1}$ |
| $D_{001}\eta_{00-1}$ | $D_{011}\pi_{-1-1-1}$ | $D_{001}\pi_{-1-1-1}$ | $D_{111}\pi_{00-2}$ |
| * $D_{011}\eta_{0-1-1}$ | $D_{002}\pi_{00-1}$ | $D_{002}\pi_{-0-1-1}$ | $D_{111}\eta_{000}$ |
| $D_s 000 K_{000}$ | $D_{001}\eta_{000}$ | $D_{011}\eta_{000}$ | $D_{000}\eta_{-1-1-1}$ |
| $D_s 001 K_{00-1}$ | $D_{000}\eta_{00-1}$ | $D_{000}\eta_{0-1-1}$ | $D_{011}\eta_{-100}$ |
| * $D_s 011 K_{0-1-1}$ | $D_{011}\eta_{00-1}$ | $D_{001}\eta_{0-10}$ | $D_{001}\eta_{-1-10}$ |
| | $D_{001}\eta_{0-1-1}$ | $D_{111}\eta_{00-1}$ | $D_s 111 K_{000}$ |
| | $D_{002}\eta_{00-1}$ | $D_s 011 K_{000}$ | $D_s 000 K_{-1-1-1}$ |
| | $D_s 001 K_{000}$ | $D_s 000 K_{0-1-1}$ | $D_s 011 K_{-100}$ |
| | $D_s 000 K_{00-1}$ | $D_s 001 K_{0-10}$ | $D_s 001 K_{-1-10}$ |
| | $D_s 011 K_{00-1}$ | $D_s 111 K_{00-1}$ | |
| | $D_s 001 K_{0-1-1}$ | | |
| $(\bar{\psi}\Gamma\psi) \times 11$ | $(\bar{\psi}\Gamma\psi) \times 32$ | $(\bar{\psi}\Gamma\psi) \times 52$ | $(\bar{\psi}\Gamma\psi) \times 37$ |

Ensemble Details

Calculations are performed on anisotropic $N_f = 2 + 1$ ensembles generated by the **Hadron Spectrum Collaboration**

Anisotropic actions

- Gauge: Symanzik-improved (tree-level tadpole improvement)
- Fermionic: Sheikholeslami-Wohlert (tree-level clover coefficients)

Anisotropy: $\xi = a_s/a_t \approx 3.5$, where $a_s \approx 0.12$ fm

Pion mass: $M_\pi \approx 391$ MeV

Volumes:

- $L = 16^3 \times 128 \Rightarrow M_\pi L \approx 3.8$ (479 cfgs)
- $L = 20^3 \times 128 \Rightarrow M_\pi L \approx 4.8$ (603 cfgs)
- $L = 24^3 \times 128 \Rightarrow M_\pi L \approx 5.7$ (553 cfgs)

Elastic $D\pi$ Scattering

Parametrisation

χ^2/N_{dof}

K-matrix with Chew-Mandelstam $I(s)$ & $K_1 = \frac{g_1^2}{m_{1-s}^2} + \gamma_1$

| | |
|--|------|
| $K = \frac{g^2}{m_{1-s}^2} + \gamma^{(0)}$ | 1.64 |
| $K = \frac{g^2}{m_{1-s}^2} + \gamma^{(1)}s$ | 1.63 |
| $K = \frac{(g^{(1)})^2 s}{m_{1-s}^2} + \gamma^{(0)}$ | 1.64 |
| $K = \frac{(g+g^{(1)})^2 s}{m_{1-s}^2}$ | 1.66 |

K-matrix with Chew-Mandelstam $I(s)$ & $K_1 = \frac{g_1^2}{m_{1-s}^2}$

| | |
|---|------|
| $K = \frac{g^2}{m_{1-s}^2} + \gamma^{(0)}$ | 1.82 |
| $K = \frac{g^2}{m_{1-s}^2} + \gamma^{(1)}s$ | 1.82 |

K-matrix with $I(s) = -i\rho(s)$ & $K_1 = \frac{g_1^2}{m_{1-s}^2} + \gamma_1$

| | |
|---|------|
| $K = \frac{g^2}{m_{1-s}^2} + \gamma^{(0)}$ | 1.61 |
| $K = \frac{g^2}{m_{1-s}^2} + \gamma^{(1)}s$ | 1.64 |

K-matrix with $I(s) = -i\rho(s)$ & $K_1 = \frac{g_1^2}{m_{1-s}^2}$

| | |
|---|------|
| $K = \frac{g^2}{m_{1-s}^2} + \gamma^{(0)}$ | 1.81 |
| $K = \frac{g^2}{m_{1-s}^2} + \gamma^{(1)}s$ | 1.80 |

Effective range expansion in $\ell = 0$ & $K_1 = \frac{g_1^2}{m_{1-s}^2} + \gamma_1$

$$k_{D\pi} \cot \delta_0^{D\pi} = \frac{1}{a} + \frac{1}{2} r^2 k_{D\pi}^2 + P_2 k_{D\pi}^4 \quad 1.91$$

