The EOS of neutron matter, and the effect of Λ hyperons to neutron star structure

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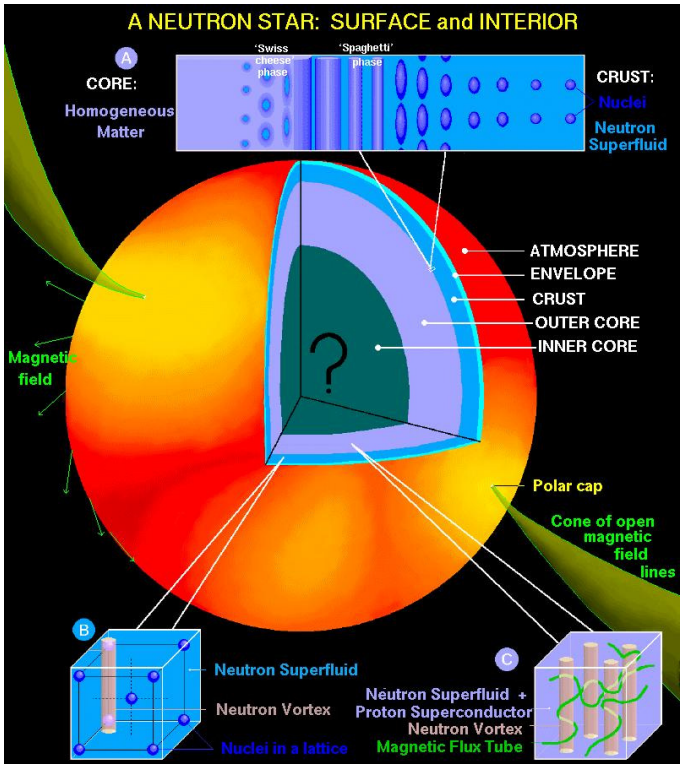
Neutron stars

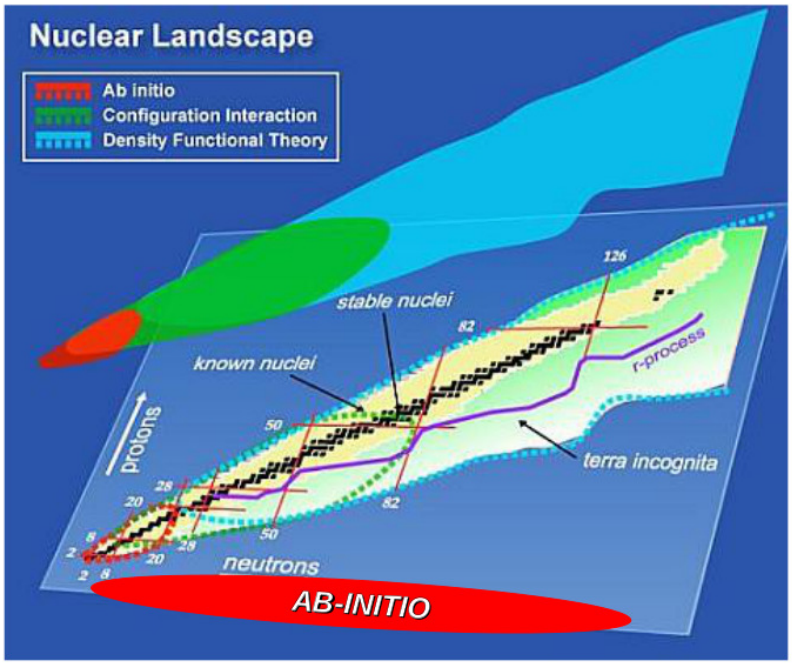
Neutron star is a wonderful natural laboratory

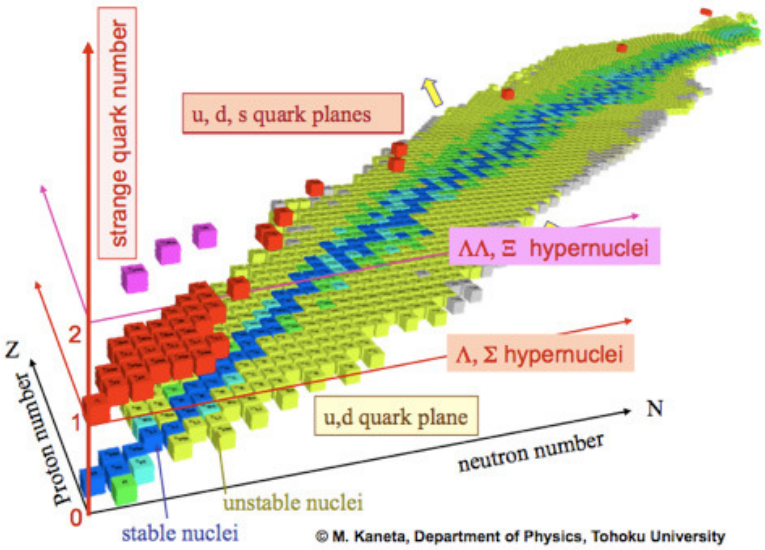


D. Page

- Atmosphere: atomic and plasma physics
- Crust: physics of superfluids (neutrons, vortex), solid state physics (nuclei)
- Inner crust: deformed nuclei, pasta phase
- Outer core: nuclear matter
- Inner core: hyperons? quark matter? π or K condensates?

Nuclei and hypernuclei





Few thousands of binding energies for normal nuclei are known.

Only few tens for hypernuclei.

Outline

- The model and the method
- Equation of state of neutron matter
- Neutron star structure (I) - radius
- Λ-hypernuclei and Λ-neutron matter
- Neutron star structure (II) - maximum mass
- Conclusions

Nuclear Hamiltonian

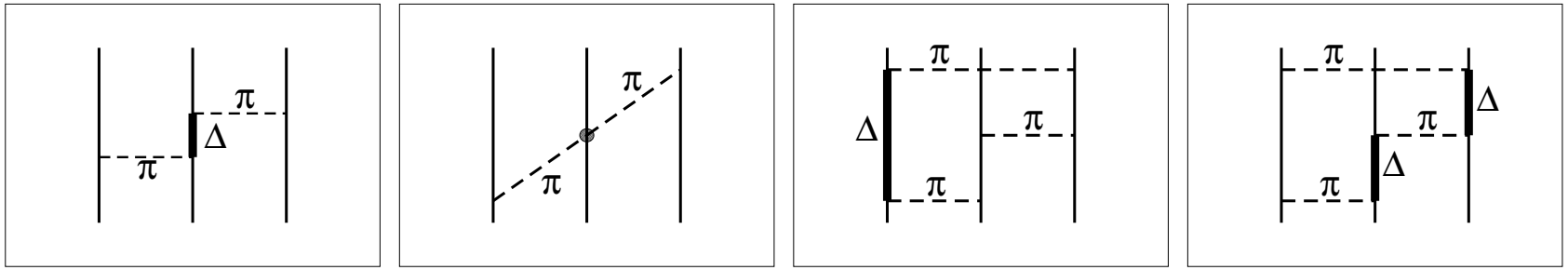
Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

v_{ij} NN (Argonne AV8') fitted on scattering data. Sum of operators:

$$v_{ij} = \sum O_{ij}^{p=1,8} v^p(r_{ij})\,, \quad O_{ij}^p = (1, \vec{\sigma}_i\cdot\vec{\sigma}_j, S_{ij}, \vec{L}_{ij}\cdot\vec{S}_{ij}) \times (1, \vec{\tau}_i\cdot\vec{\tau}_j)$$

Urbana–Illinois V_{ijk} models processes like



$+$ short-range correlations (spin/isospin independent).

Quantum Monte Carlo

$$H\,\psi(\vec{r}_1 \ldots \vec{r}_N) = E\,\psi(\vec{r}_1 \ldots \vec{r}_N) \qquad \psi(t) = e^{-(H-E_T)t}\psi(0)$$

Ground-state extracted in the limit of $t \to \infty$.

Propagation performed by

$$\psi(R,t) = \langle R|\psi(t)\rangle = \int dR'\,G(R,R',t)\psi(R',0)$$

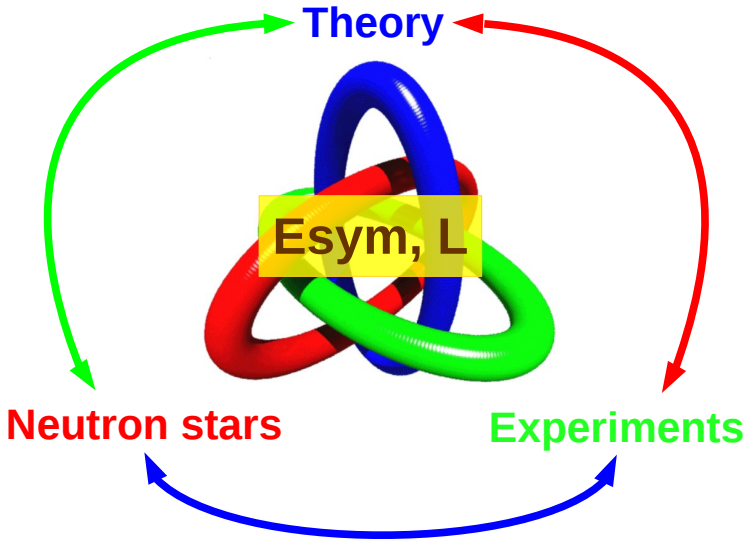
- Importance sampling: $G(R,R',t) \to G(R,R',t)\,\Psi_I(R')/\Psi_I(R)$
- Constrained-path approximation to control the sign problem. Unconstrained calculation possible in several cases (exact).

Ground–state obtained in a **non-perturbative way.** Systematic uncertainties within 1-2 %.

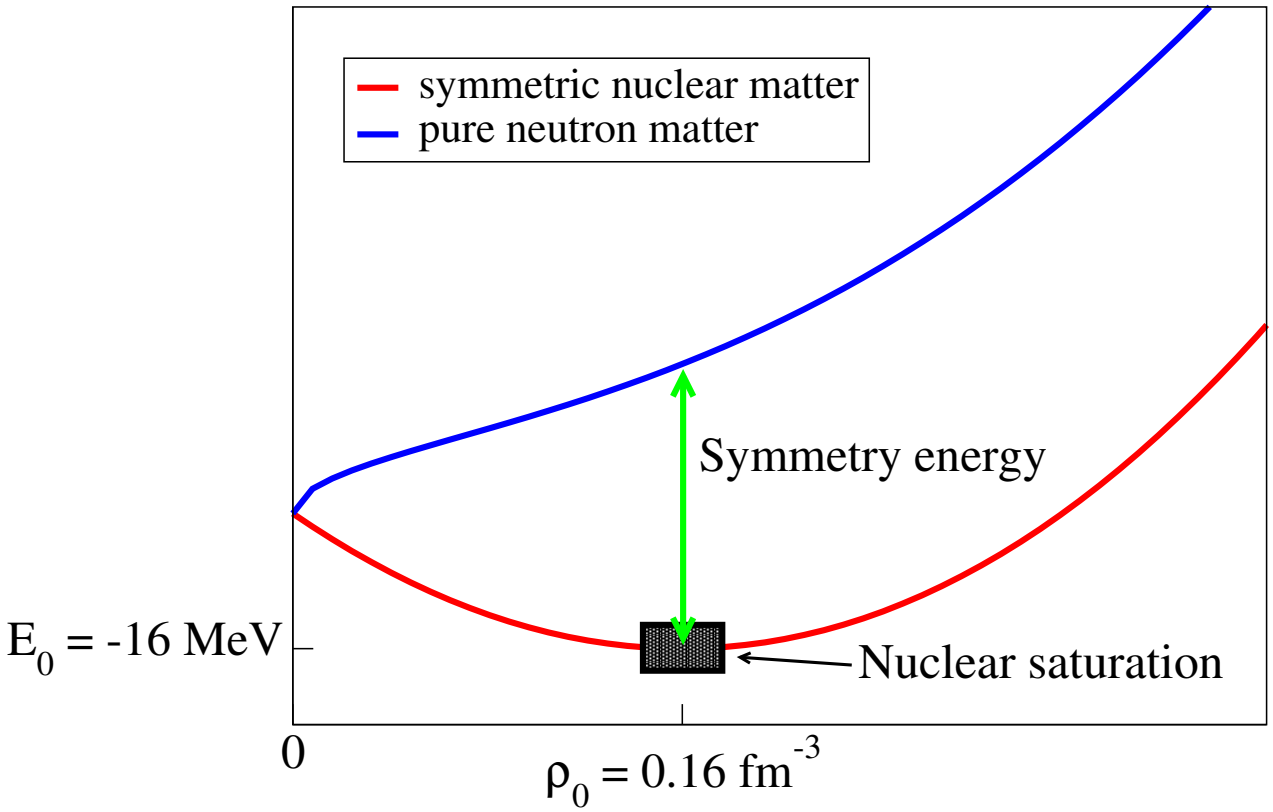
Neutron matter equation of state

Neutron matter is an "exotic" system. Why do we care?

- EOS of neutron matter gives the symmetry energy and its slope.
- The three-neutron force ($T = 3/2$) very weak in light nuclei, while $T = 1/2$ is the dominant part. No direct $T = 3/2$ experiments available.
- Determines properties of neutron stars.



What is the Symmetry energy?



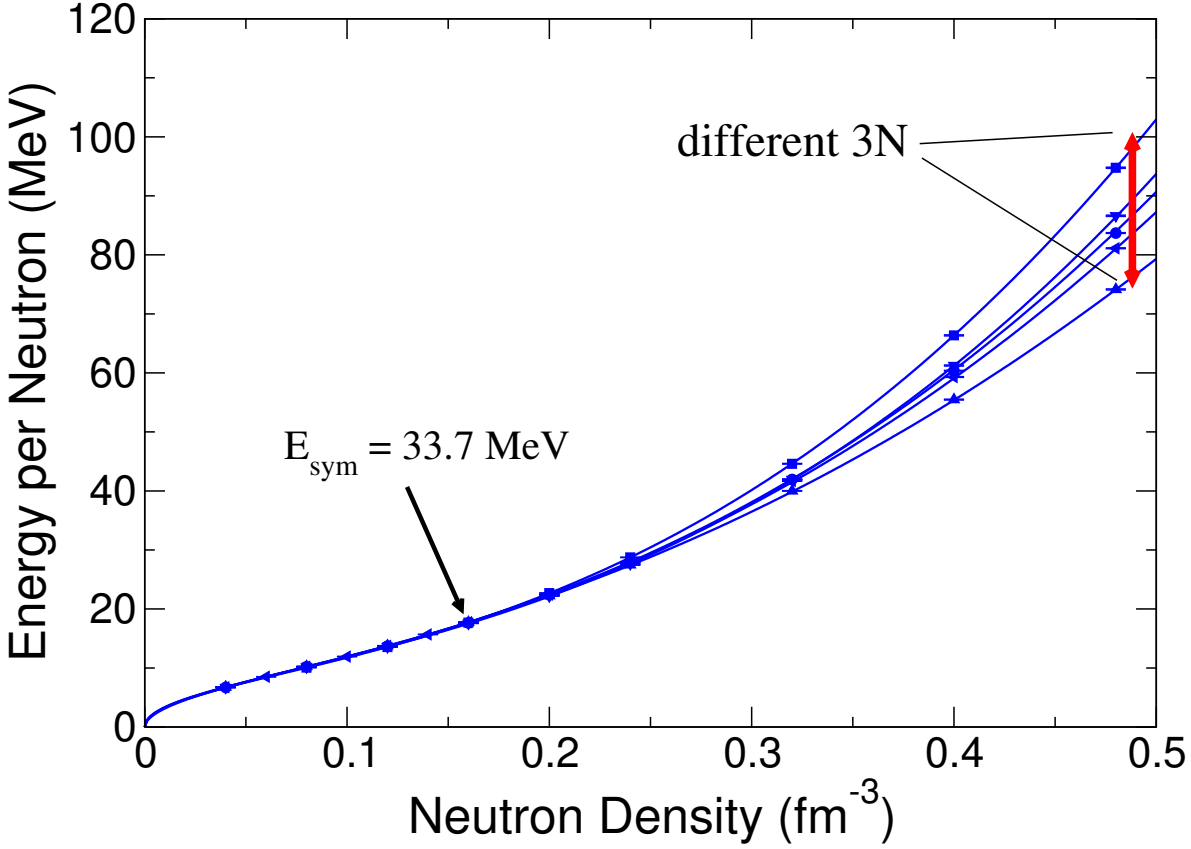
Assumption from experiments:

$$E_{SNM}(\rho_0) = -16 MeV\,, \quad \rho_0 = 0.16 fm^{-3}\,, \quad E_{sym} = E_{PNM}(\rho_0) + 16$$

At ρ_0 we access E_{sym} by studying PNM.

Neutron matter

We consider different forms of three-neutron interaction by only requiring a particular value of E_{sym} at saturation.

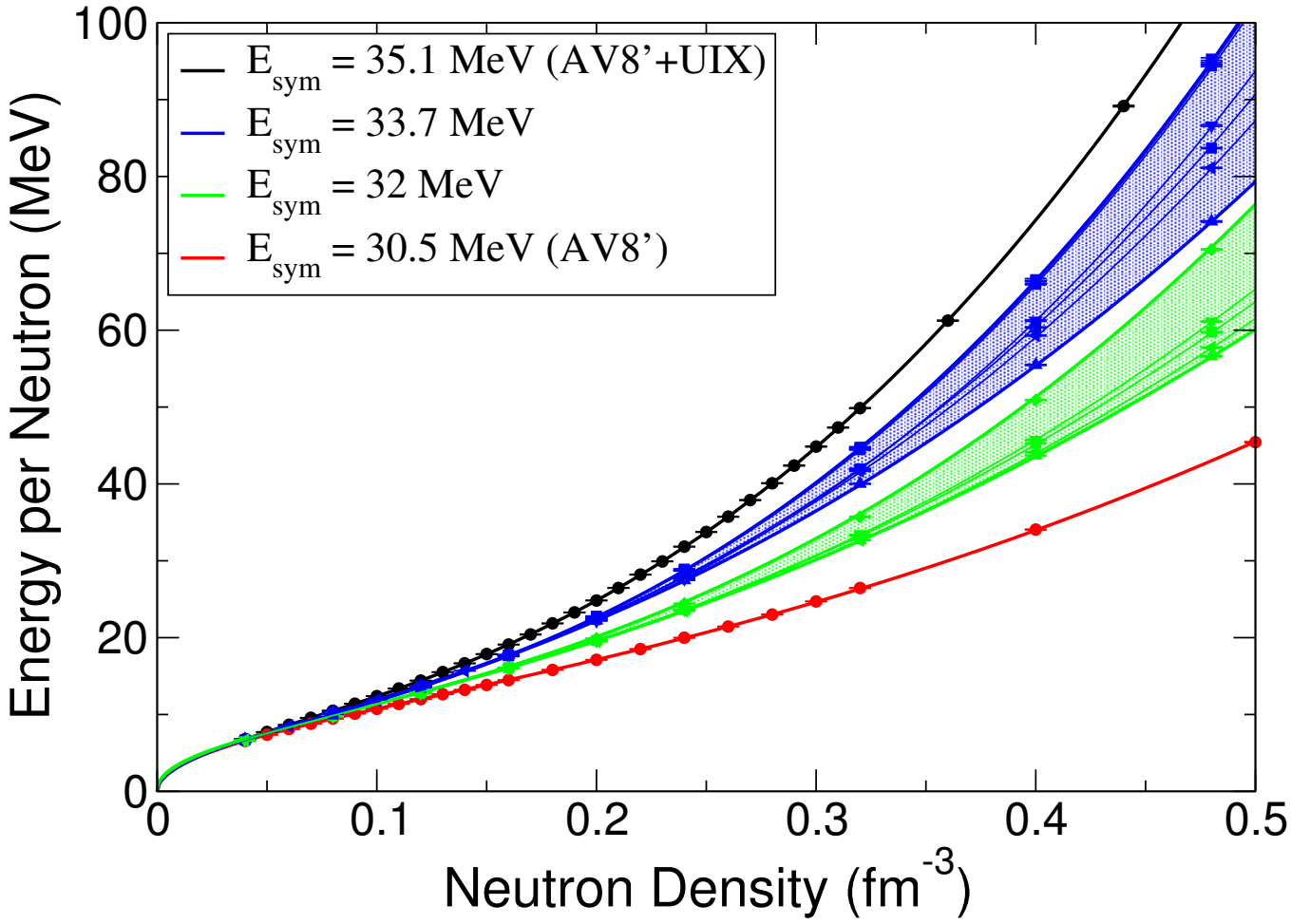


different 3N:

- $V_{2\pi} + \alpha V_R$
- $V_{2\pi} + \alpha V_R^{\mu}$ (several μ)
- $V_{2\pi} + \alpha \tilde{V}_R$
- $V_{3\pi} + \alpha V_R$

Neutron matter

Equation of state of neutron matter using Argonne forces:

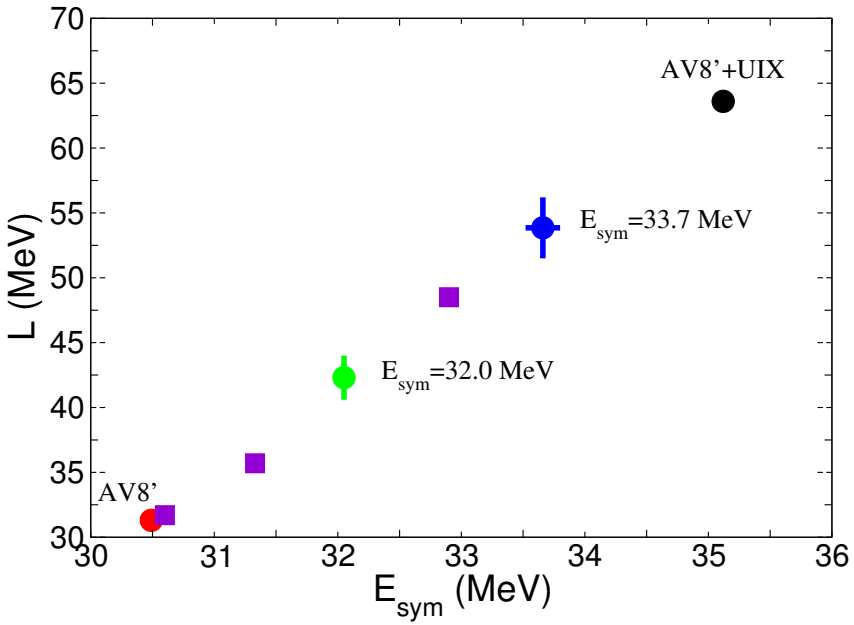


Gandolfi, Carlson, Reddy, PRC (2012)

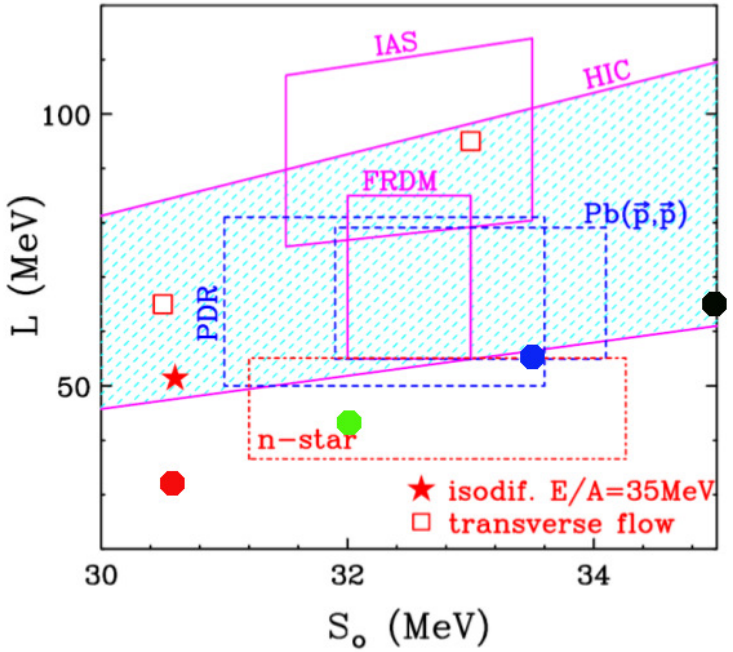
Neutron matter and symmetry energy

From the EOS, we can fit the symmetry energy around ρ_0 using

$$E_{sym}(\rho) = E_{sym} + \frac{L}{3}\frac{\rho - 0.16}{0.16} + \cdots$$



Gandolfi *et al.*, EPJ (2014)



Tsang *et al.*, PRC (2012)

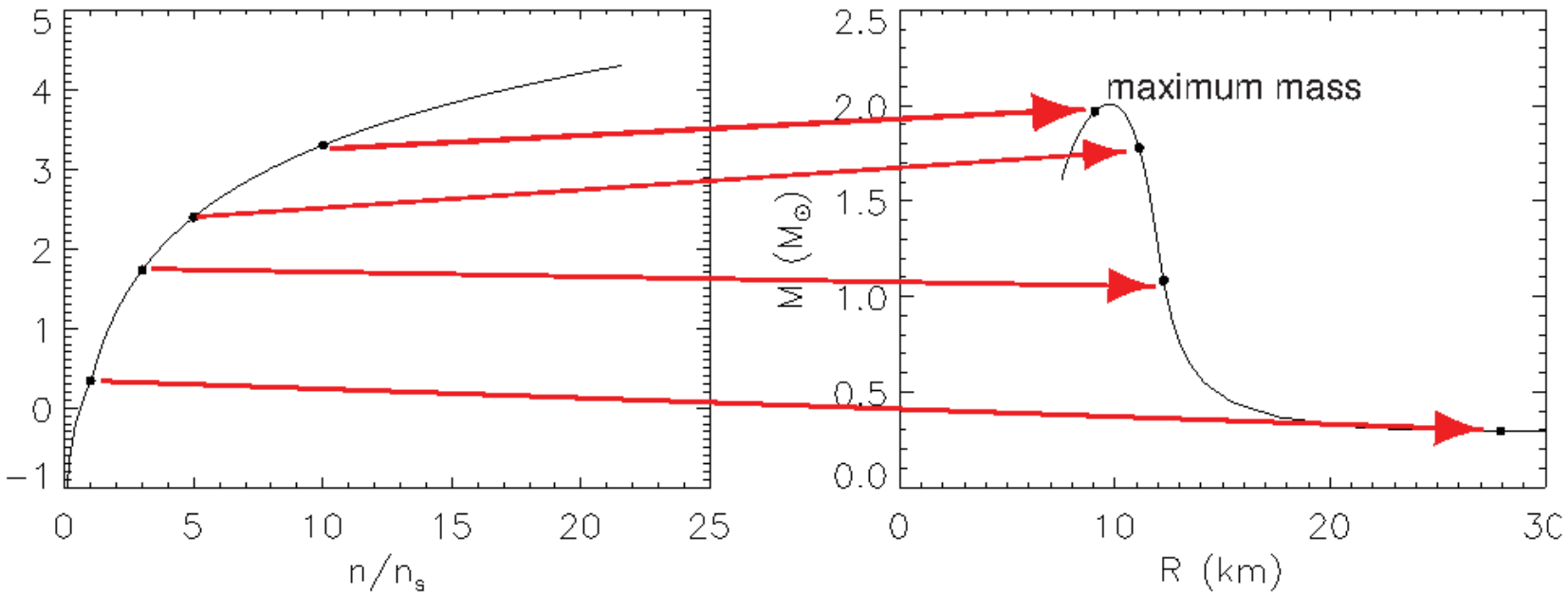
Very weak dependence to the model of 3N force for a given E_{sym}.
Knowing E_{sym} or L useful to constrain 3N! (within this model...)

Neutron matter and neutron star structure

TOV equations:

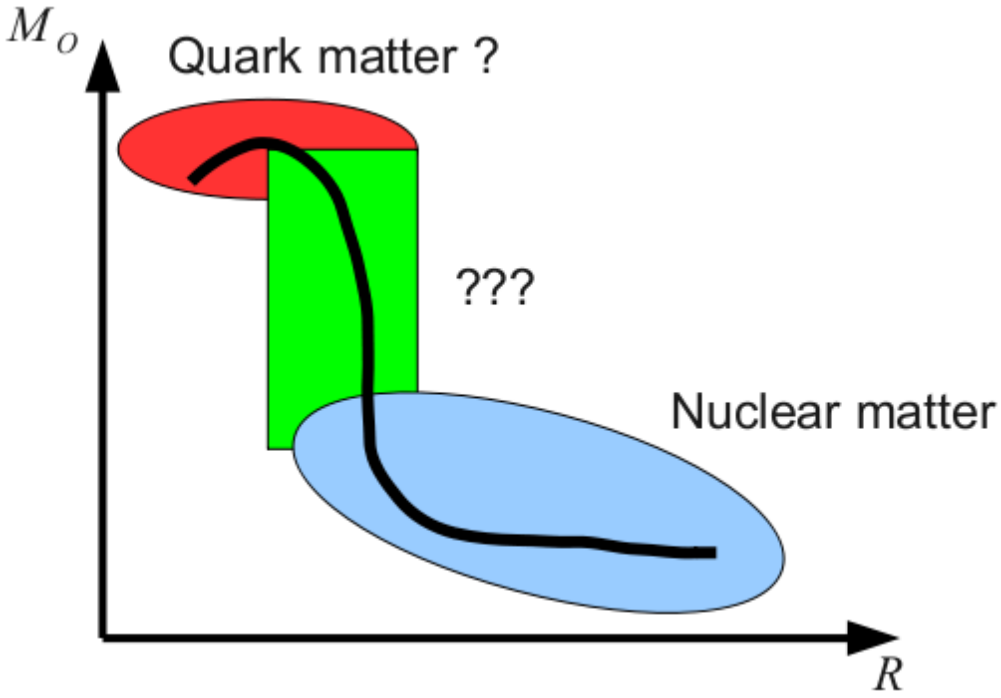
$$\frac{dP}{dr} = -\frac{G[m(r) + 4\pi r^3 P/c^2][\epsilon + P/c^2]}{r[r - 2Gm(r)/c^2]} ,$$

$$\frac{dm(r)}{dr} = 4\pi\epsilon r^2 ,$$



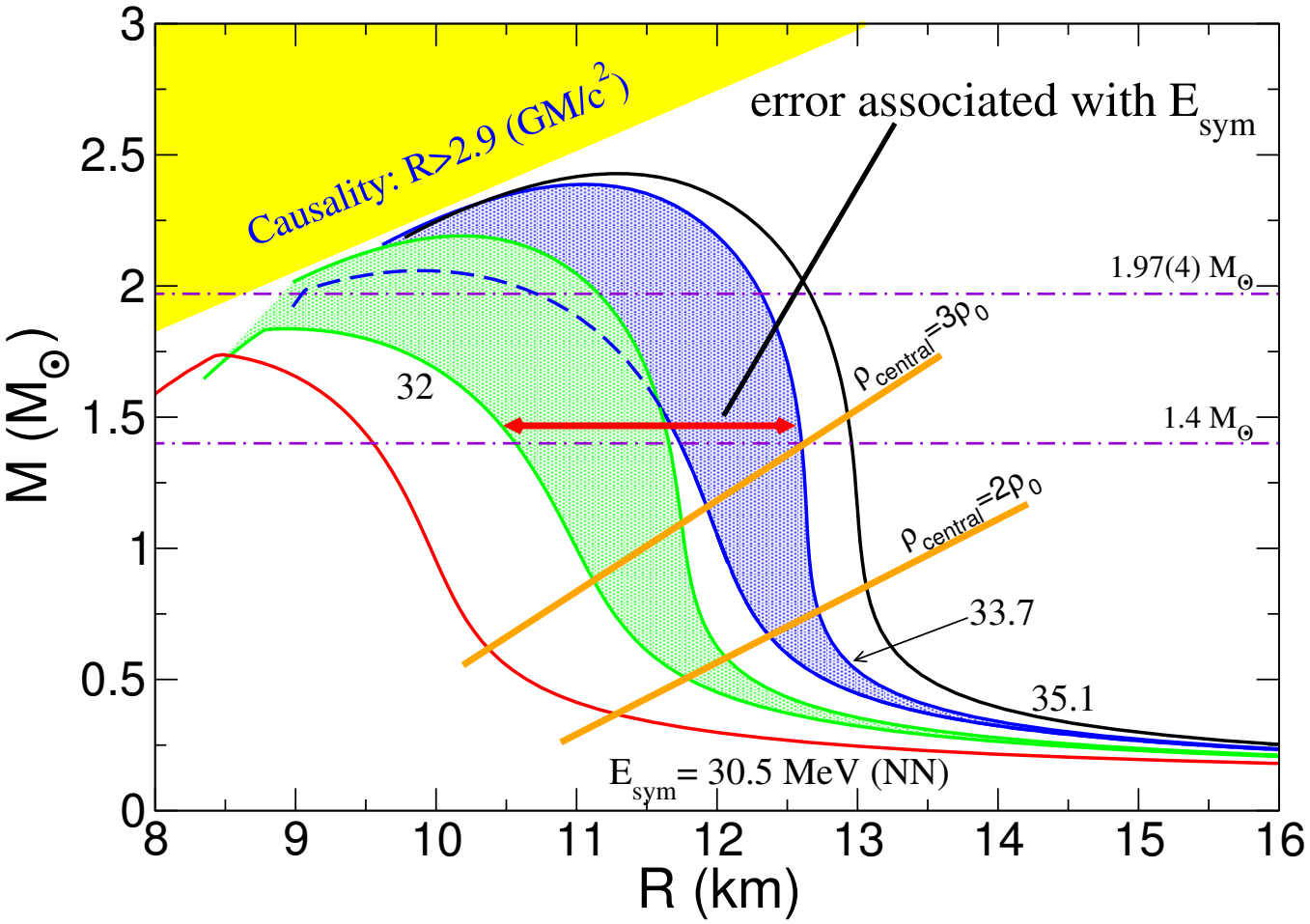
J. Lattimer

Neutron star matter



- Neutron star radii sensitive to EOS around $\rho \sim (1-2)\rho_0$
- Maximum mass depends to higher densities

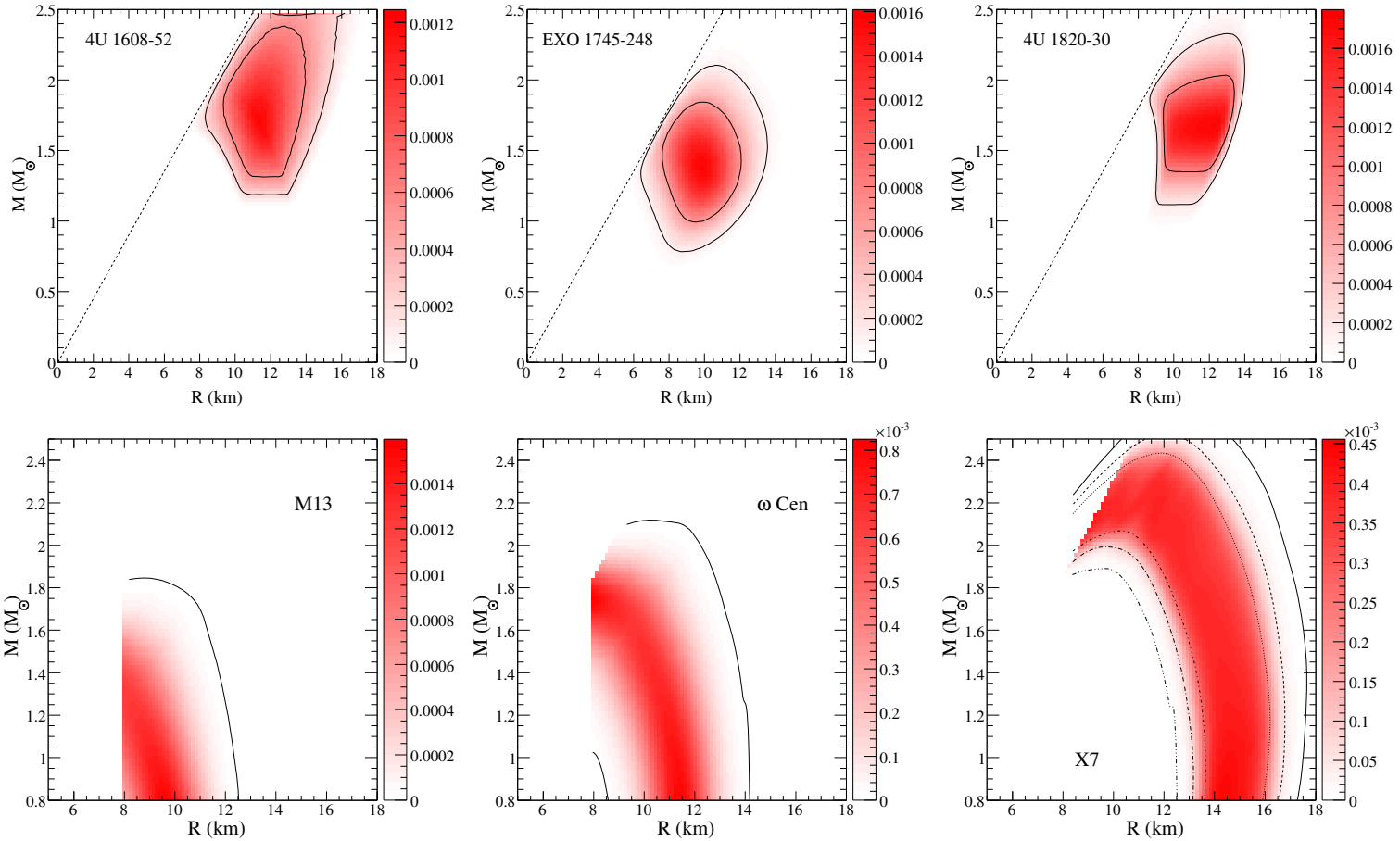
Neutron star structure



Gandolfi, Carlson, Reddy, PRC (2012).

Neutron stars

Observations of the mass-radius relation are becoming available:

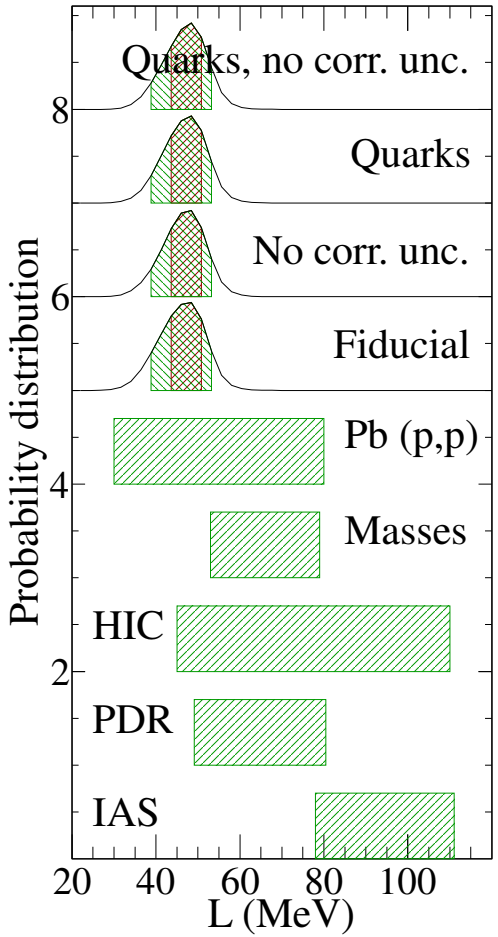


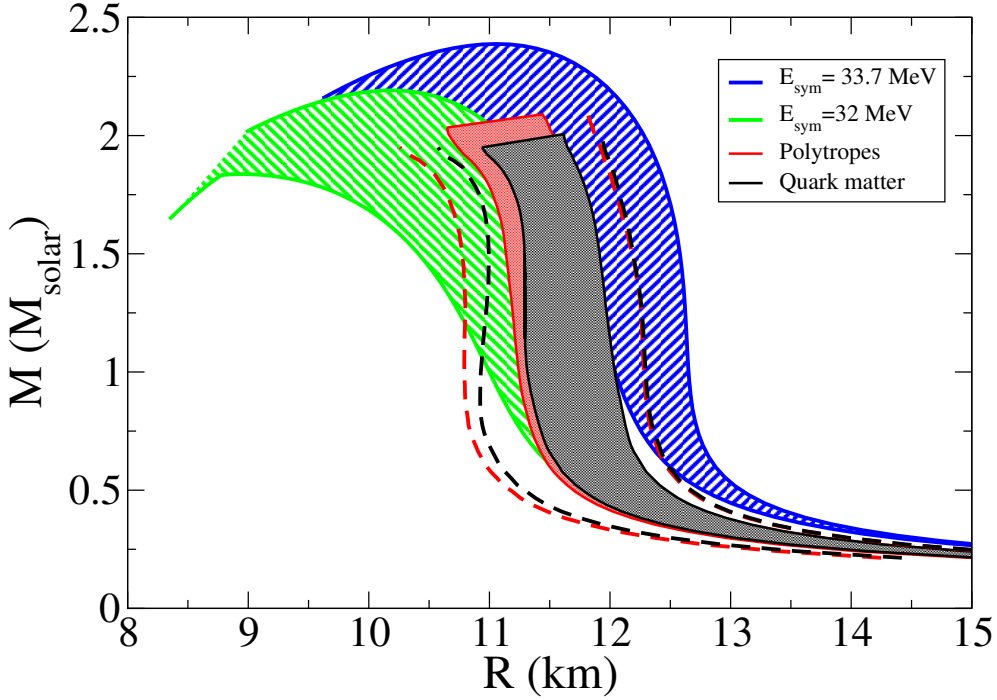
Steiner, Lattimer, Brown, ApJ (2010)

Neutron star observations can be used to constrain the EOS, E_{sym} and L.
(Systematic uncertainties still under debate...)

Neutron star matter really matters!

Here an 'astrophysical measurement'





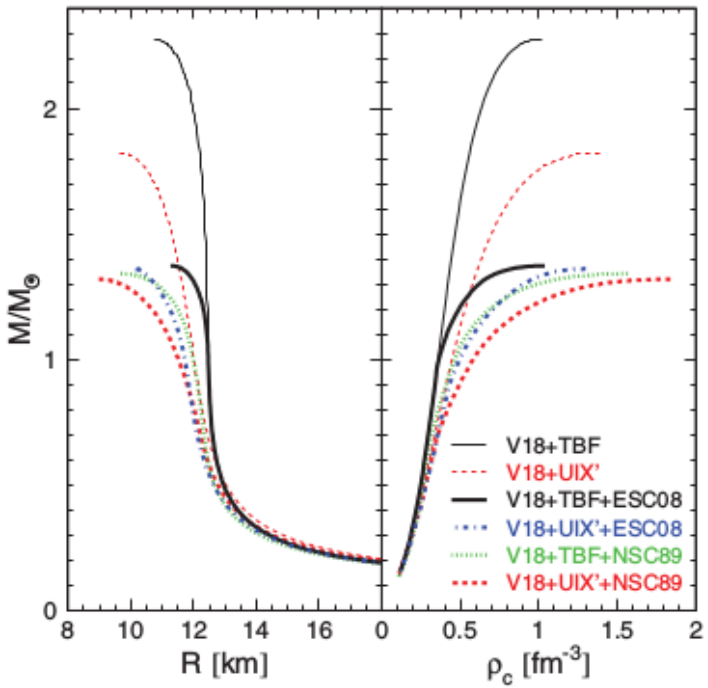
$32 < E_{sym} < 34\ MeV,\ 43 < L < 52\ MeV$

Steiner, Gandolfi, PRL (2012).

High density neutron matter

If chemical potential large enough ($\rho \sim 2 - 3\rho_0$), nucleons produce Λ, Σ, ...

Non-relativistic BHF calculations suggest that available hyperon-nucleon Hamiltonians do not support an EOS with $M > 2M_\odot$:



Schulze and Rijken PRC (2011).

Vidana, Logoteta, Providencia, Polls, Bombaci EPL (2011).

Note: (Some) other relativistic model support $2M_\odot$ neutron stars.

$\longrightarrow$ *Hyperon puzzle*

Λ-systems, outline

Λ-hypernuclei and hypermatter

$$H = H_N + \frac{\hbar^2}{2m_\Lambda} \sum_{i=1}^{A} \nabla_i^2 + \sum_{i<j} v_{ij}^{\Lambda N} + \sum_{i<j<k} V_{ijk}^{\Lambda NN}$$

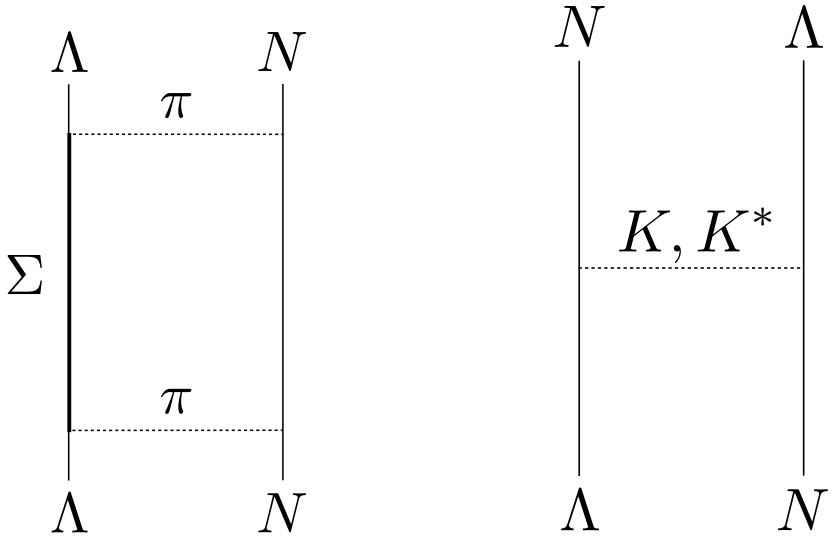
Λ-binding energy calculated as the difference between the system with and without Λ.

Λ-nucleon interaction

The Λ-nucleon interaction is constructed similarly to the Argonne potentials (Usmani).

Argonne NN: $v_{ij} = \sum_p v_p(r_{ij}) O^p_{ij}$, $O_{ij} = (1, \sigma_i \cdot \sigma_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \tau_i \cdot \tau_j)$

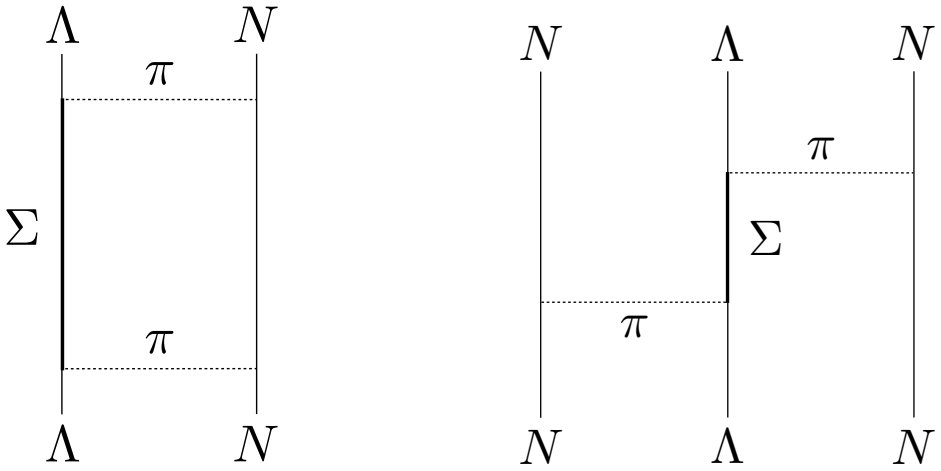
Usmani ΛN: $v_{ij} = \sum_p v_p(r_{ij}) O^p_{ij}$, $O_{\lambda j} = (1, \sigma_\lambda \cdot \sigma_j) \times (1, \tau^z_j)$



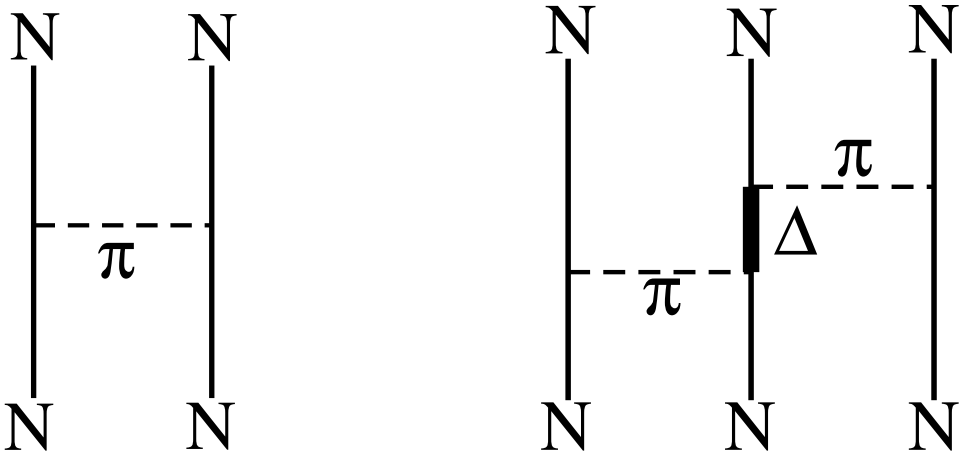
Unfortunately... ∼4500 NN data, ∼30 of ΛN data.

ΛN and ΛNN interactions

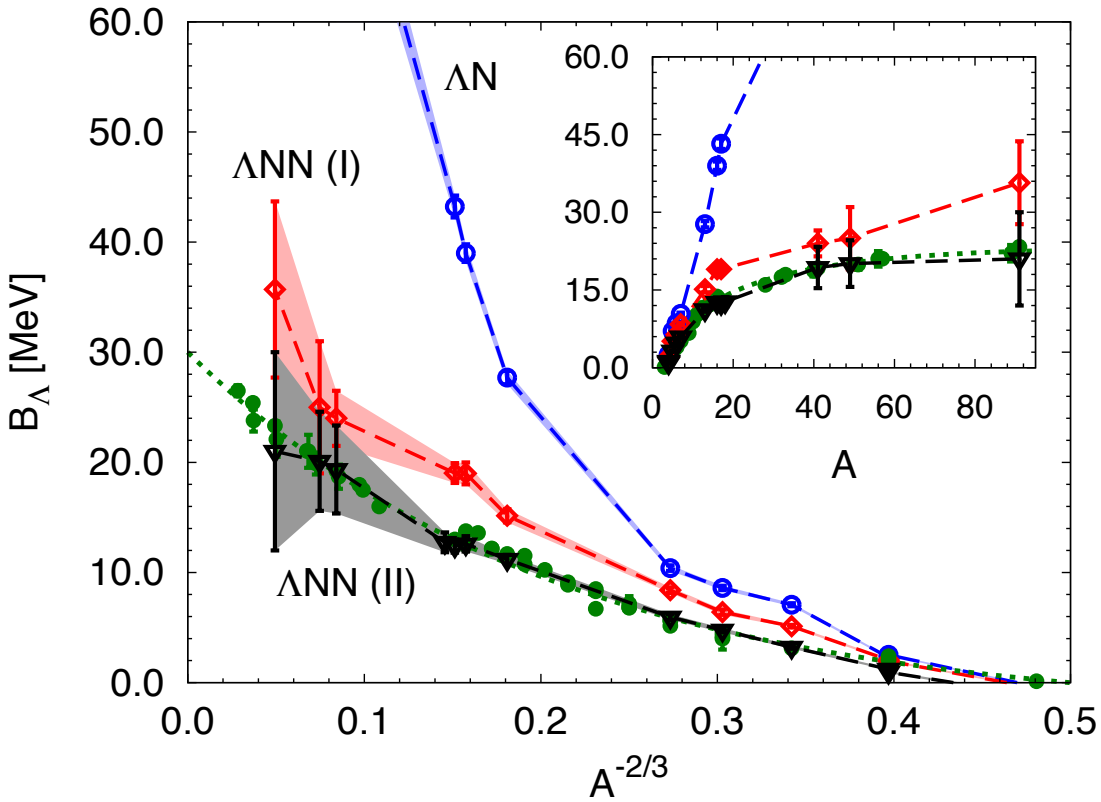
ΛNN has the same range of ΛN



Differently from NN and NNN interactions:



Λ hypernuclei



Lonardoni, Gandolfi, Pederiva, PRC (2013) and PRC (2014).

$V^{\Lambda NN}$ (II) is a new form where the parameters have been readjusted.

ΛNN crucial for saturation.

Hyper-neutron matter

Neutrons and Λ particles:

$$\rho = \rho_n + \rho_\Lambda \,, \qquad\qquad x = \frac{\rho_\Lambda}{\rho}$$

$$E_{\mathrm{HNM}}(\rho, x) = \Big[E_{\mathrm{PNM}}((1-x)\rho) + m_n\Big](1-x) + \Big[E_{\mathrm{P\Lambda M}}(x\rho) + m_\Lambda\Big]x + f(\rho, x)$$

where $E_{P\Lambda M}$ is the non-interacting energy (no $v_{\Lambda\Lambda}$ interaction),

$$E_{PNM}(\rho) = a\left(\frac{\rho}{\rho_0}\right)^{\alpha} + b\left(\frac{\rho}{\rho_0}\right)^{\beta}$$

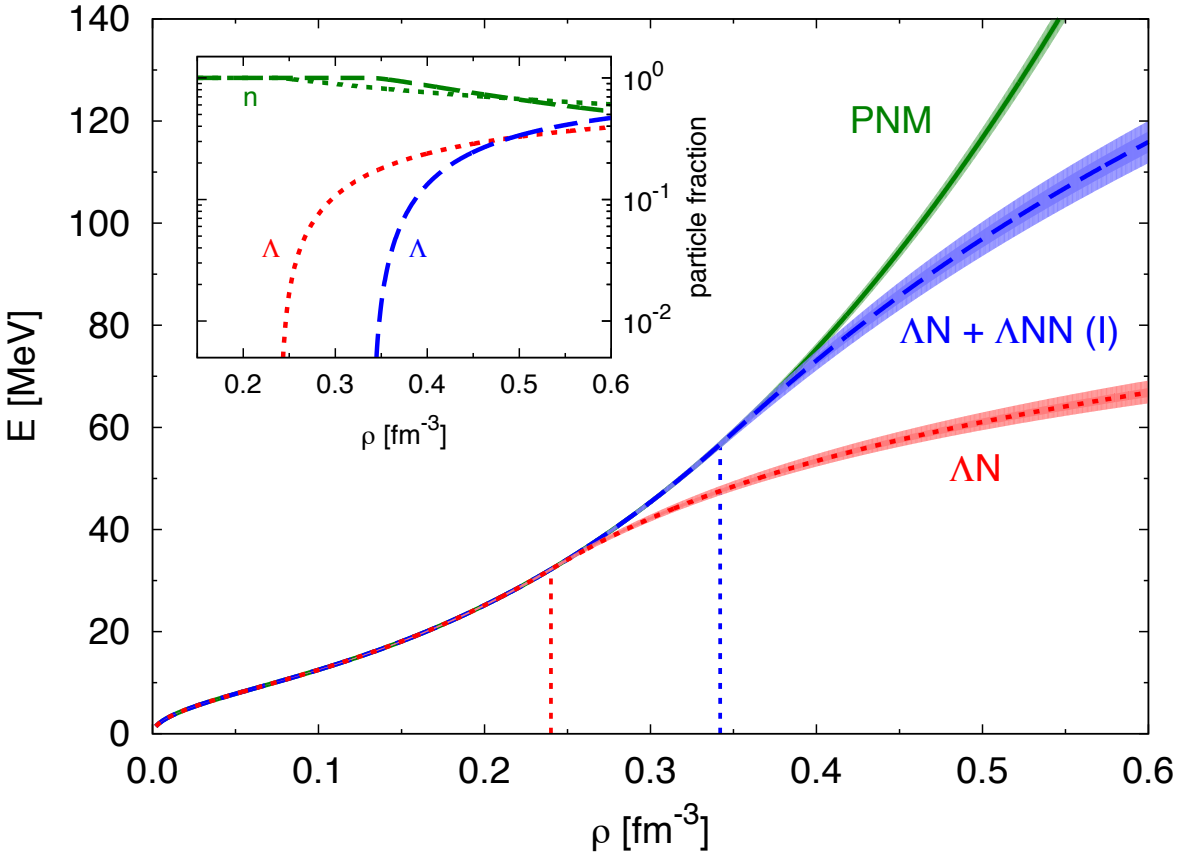
and

$$f(\rho, x) = c_1 \frac{x(1-x)\rho}{\rho_0} + c_2 \frac{x(1-x)^2\rho^2}{\rho_0^2}$$

All the parameters are fit to Quantum Monte Carlo results.

Λ-neutron matter

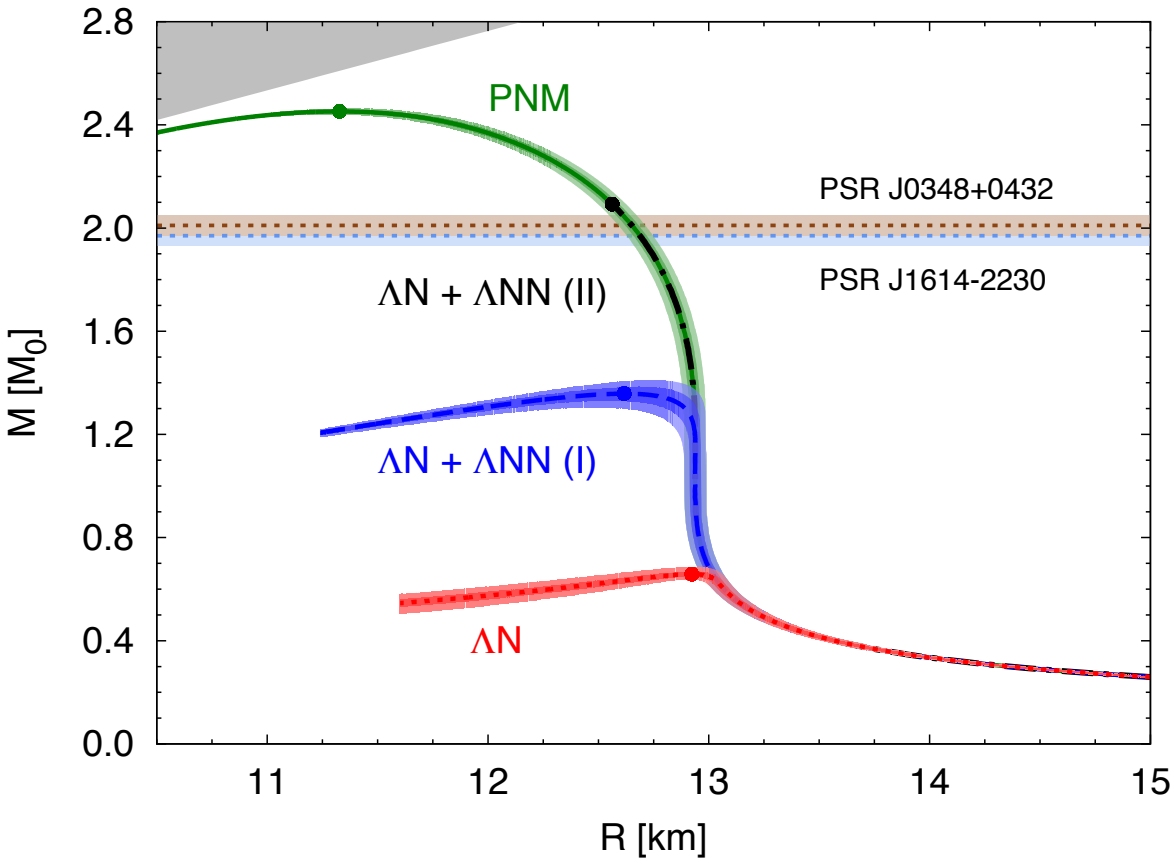
EOS obtained by solving for $\mu_\Lambda(\rho, x) = \mu_n(\rho, x)$



Lonardoni, Lovato, Gandolfi, Pederiva, PRL (2015)

No hyperons up to $\rho = 0.5$ fm^{-3} using ΛNN (II)!!!

Λ-neutron matter



Lonardoni, Lovato, Gandolfi, Pederiva, PRL (2015)

Drastic role played by ΛNN. Calculations can be compatible with neutron star observations.

Note: no $v_{\Lambda\Lambda}$, no protons, and no other hyperons included yet...

Summary

- EOS of pure neutron matter qualitatively well understood.
- Λ-nucleon data very limited, but ΛNN is very important.
- Role of Λ in neutron stars far to be understood. We cannot conclude **anything** for neutron stars with present models...

Future needs:

- Accurate and precise measurement of E_{sym} and L.
- More ΛN experimental data needed. Input from Lattice QCD? Femtoscopy @HADES (talk by Piotr Salabura)?
- Light and medium Λ-nuclei measurements needed, especially $N \neq Z$ (JLAB exp. approved)

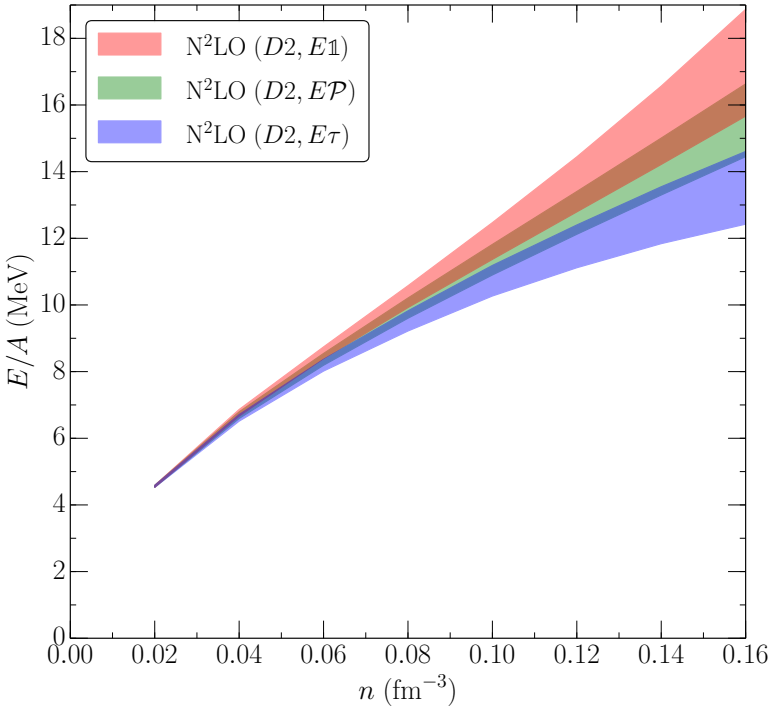
Acknowledgments

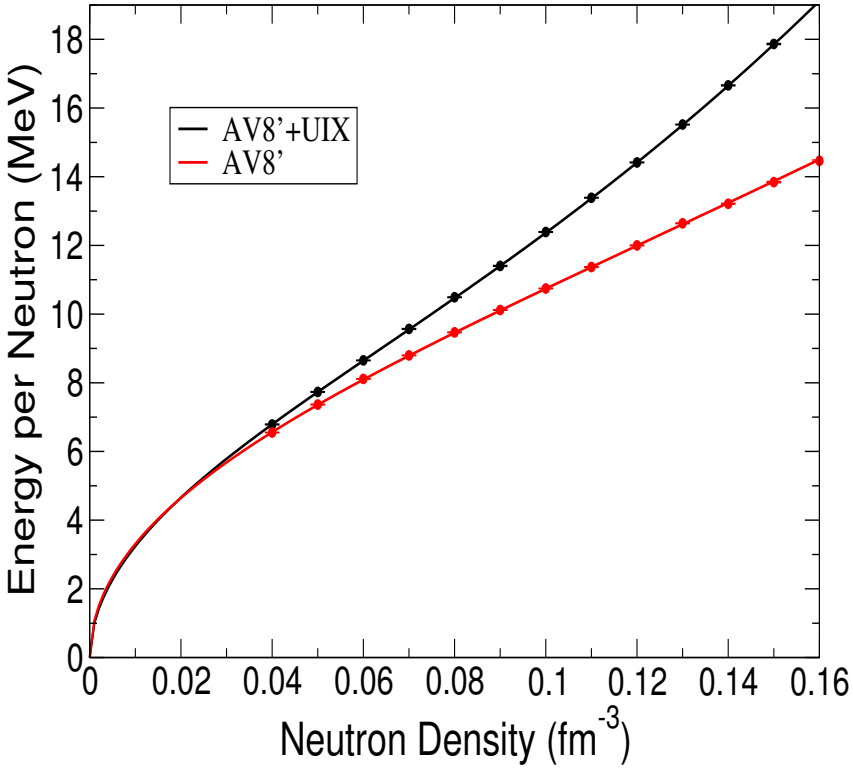
- J. Carlson, D. Lonardoni (LANL)
- A. Lovato (ANL)
- F. Pederiva (Trento)
- S. Reddy (INT)
- A. Steiner (UT/ORNL)

Extra slides

Neutron matter at N2LO

EOS of pure neutron matter at N2LO, R_0=1.0 fm.
Error quantification estimated as previously.





Lynn, Tews, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk (2015).

Nuclear Hamiltonian

Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

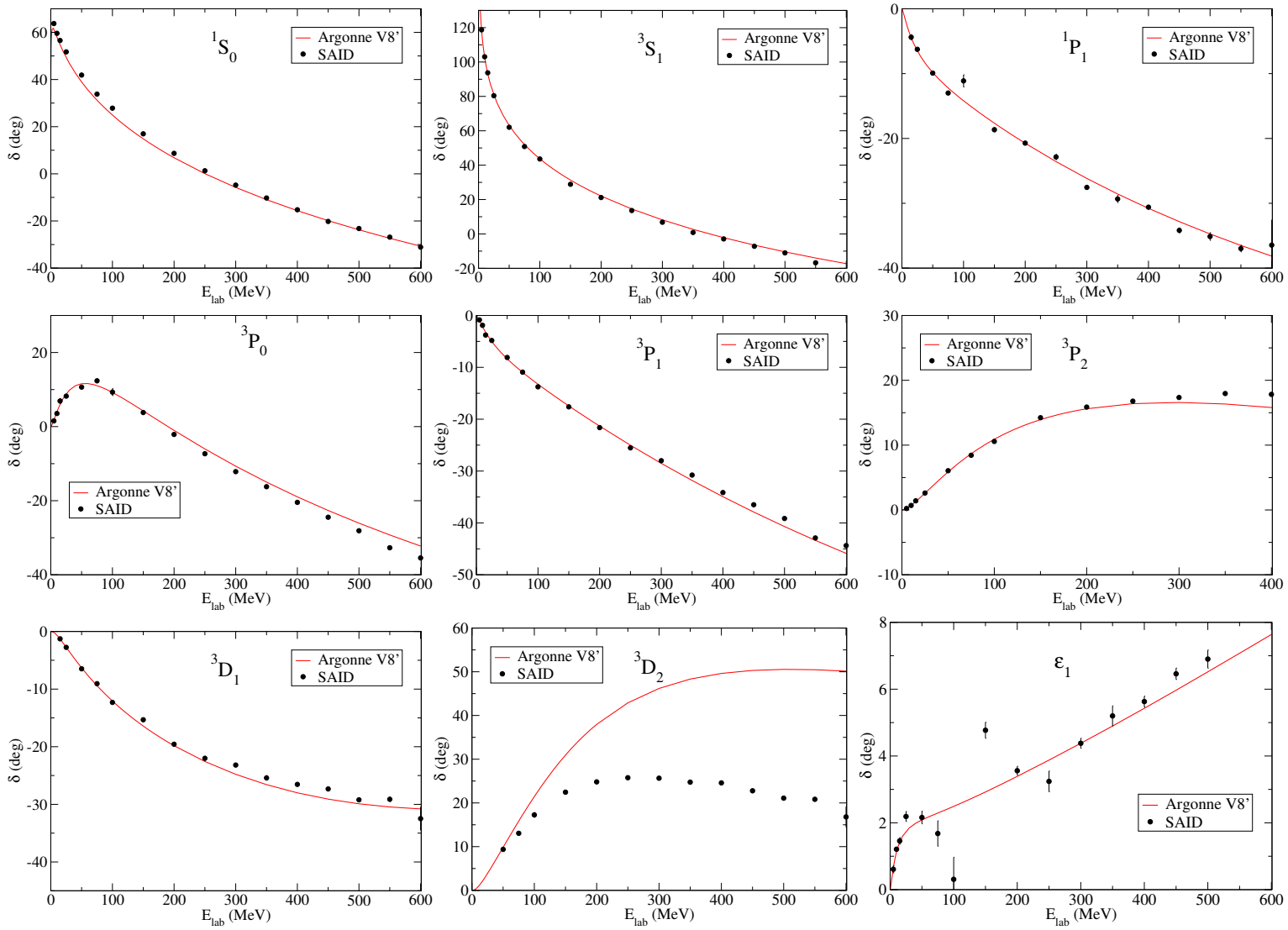
$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

v_{ij} NN fitted on scattering data. Sum of operators:

$$v_{ij} = \sum O_{ij}^{p=1,8} v^p(r_{ij})\,, \quad O_{ij}^p = (1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \vec{\tau}_i \cdot \vec{\tau}_j)$$

NN interaction - Argonne AV8' and AV6'.

Phase shifts, AV8'



Difference AV8′-AV18 less than 0.2 MeV per nucleon up to A=12.

Scattering data and neutron matter

Two neutrons have

$$k \approx \sqrt{E_{lab}\, m/2}\,, \qquad \rightarrow k_F$$

that correspond to

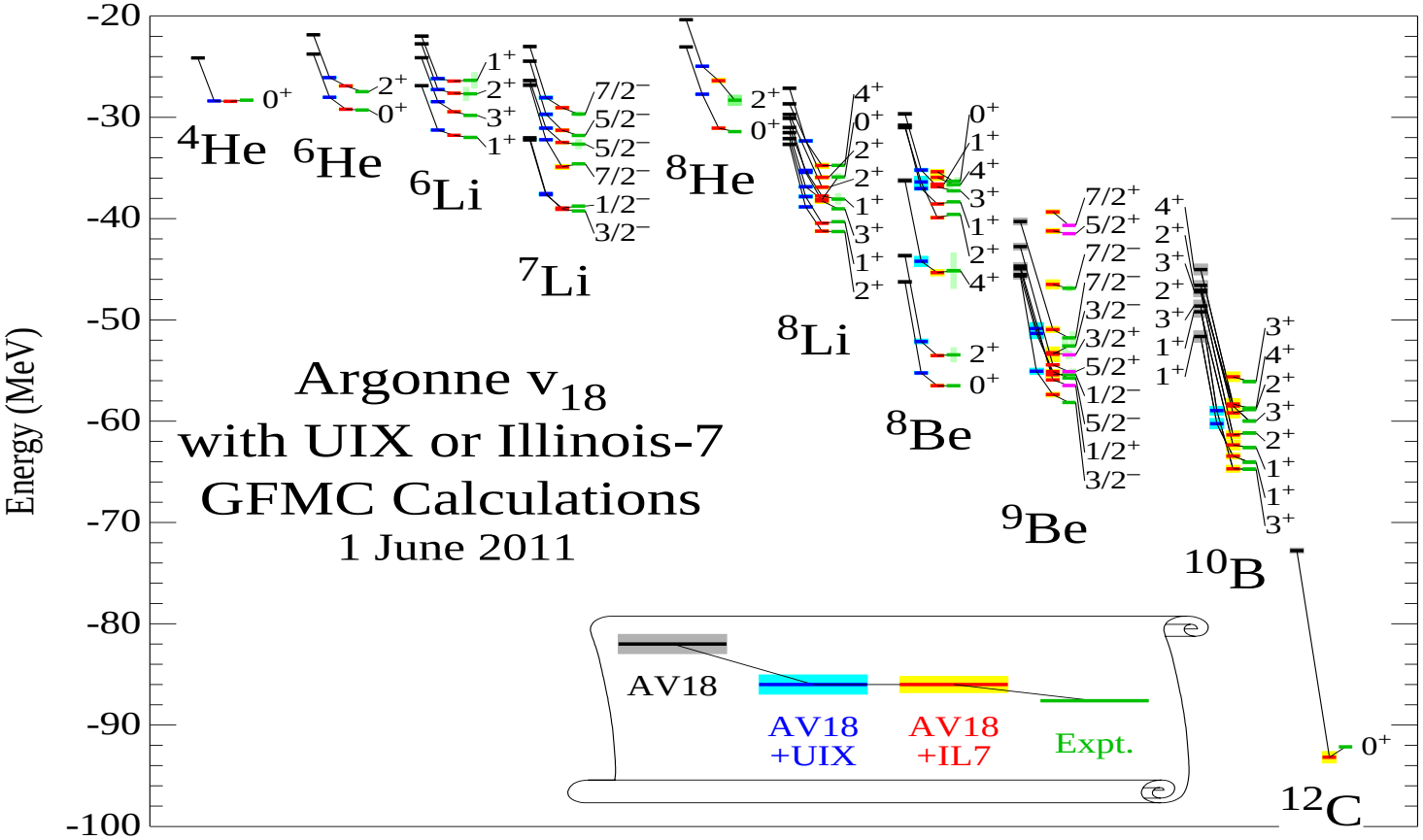
$$k_F \rightarrow \rho \approx (E_{lab}\, m/2)^{3/2}/2\pi^2\,.$$

E_{lab}=150 MeV corresponds to about 0.12 fm^{-3}.

E_{lab}=350 MeV to 0.44 fm^{-3}.

Argonne potentials useful to study dense matter above ρ_0=0.16 fm^{-3}

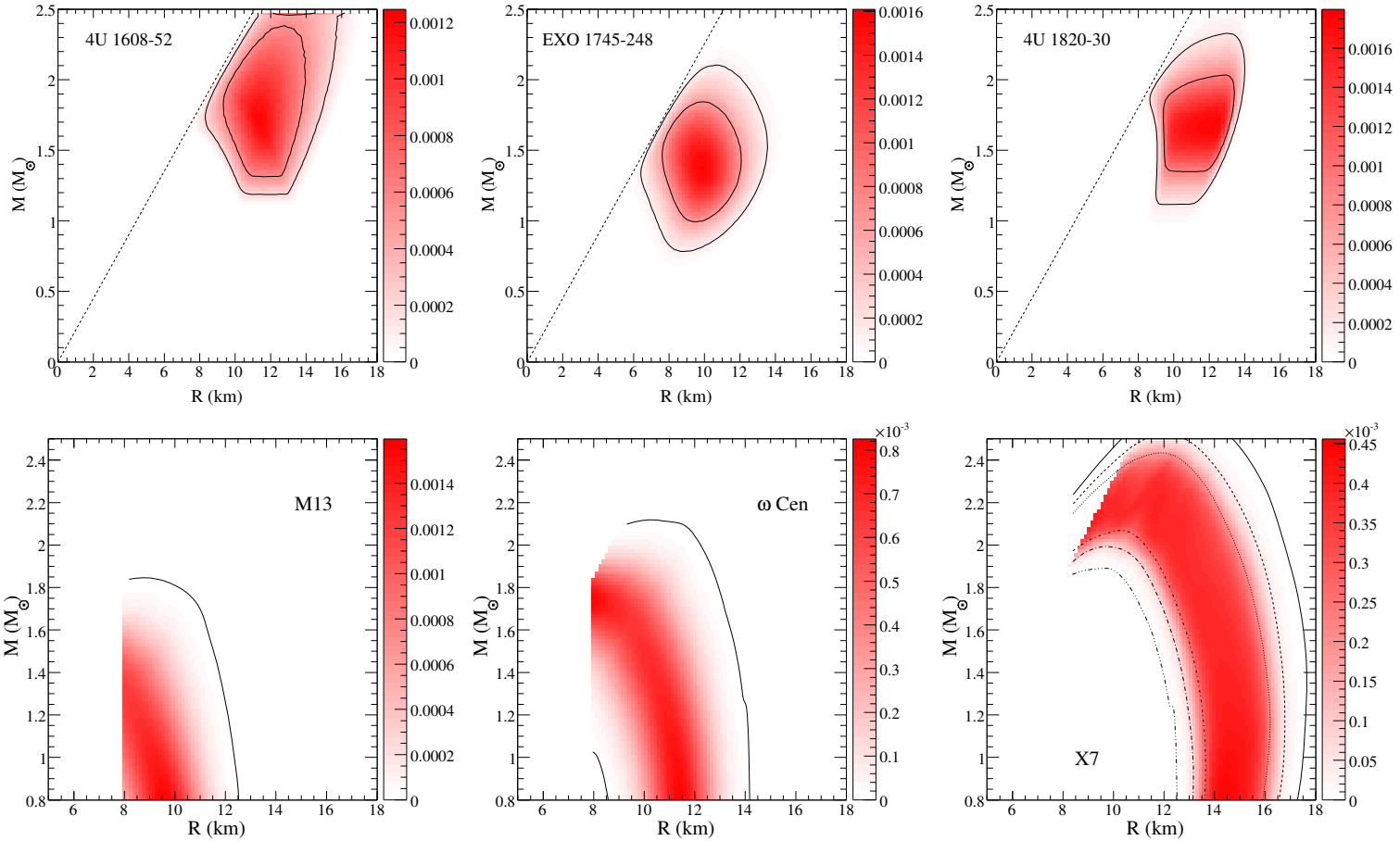
Light nuclei spectrum computed with GFMC



Carlson, *et al.*, arXiv:1412.3081, RMP (2015)

Neutron stars

Observations of the mass-radius relation are becoming available:



Steiner, Lattimer, Brown, ApJ (2010)

Neutron star observations can be used to 'measure' the EOS and constrain E_{sym} and L. (Systematic uncertainties still under debate...)

Neutron star matter

Neutron star matter model:

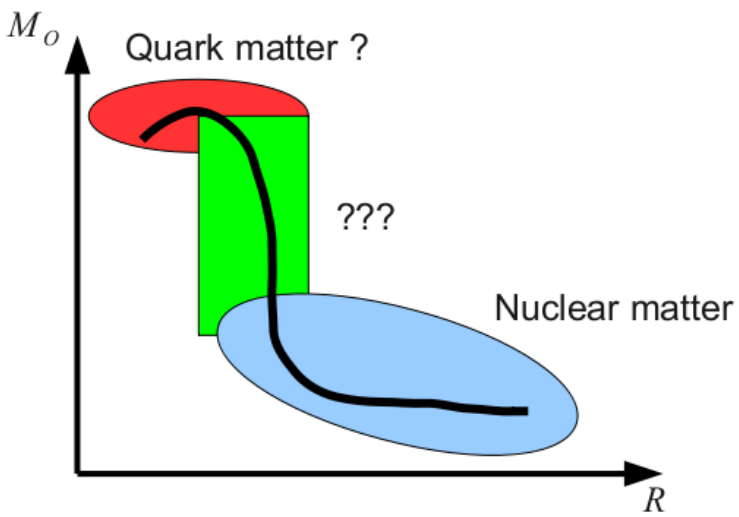
$$E_{NSM} = a\left(\frac{\rho}{\rho_0}\right)^{\alpha} + b\left(\frac{\rho}{\rho_0}\right)^{\beta}, \quad \rho < \rho_t$$

(form suggested by QMC simulations),

and a high density model for $\rho > \rho_t$

i) two polytropes

ii) polytrope+quark matter model

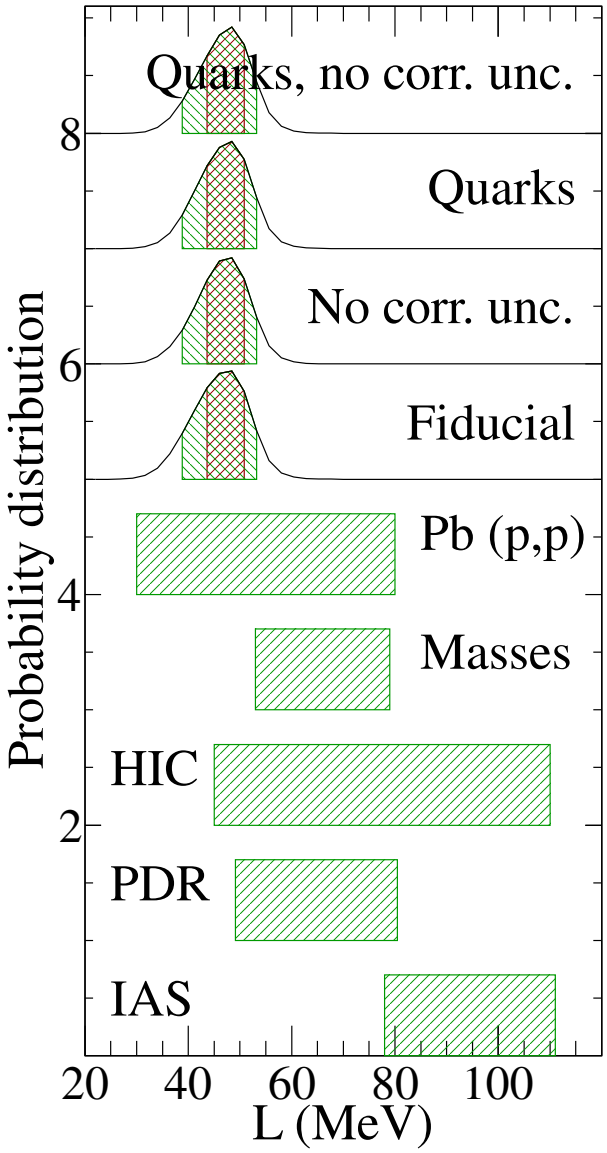


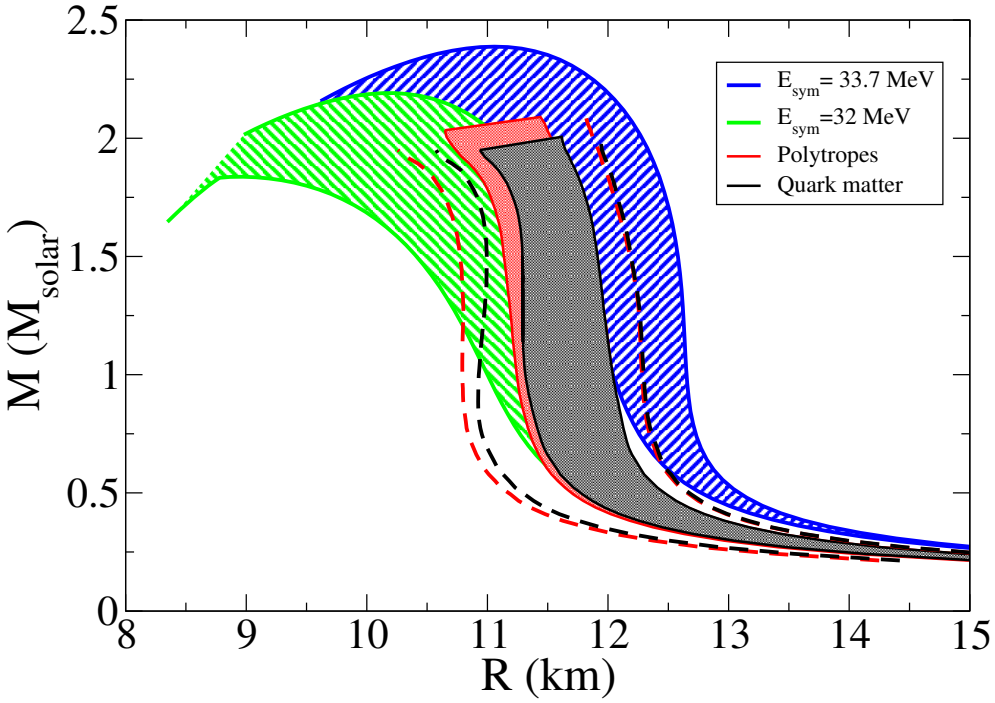
Neutron star radius sensitive to the EOS at nuclear densities!

Direct way to extract E_{sym} and L from neutron stars observations:

$$E_{sym} = a + b + 16\,, \quad L = 3(a\alpha + b\beta)$$

Neutron star matter really matters!





$32 < E_{sym} < 34\ MeV$

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Steiner, Gandolfi, PRL (2012).