The EOS of neutron matter, and the effect of $\Lambda$ hyperons to neutron star structure

Stefano Gandolfi

Los Alamos National Laboratory (LANL)

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www.computingnuclei.org
Neutron star is a wonderful natural laboratory

- **Atmosphere**: atomic and plasma physics
- **Crust**: physics of superfluids (neutrons, vortex), solid state physics (nuclei)
- **Inner crust**: deformed nuclei, pasta phase
- **Outer core**: nuclear matter
- **Inner core**: hyperons? quark matter? $\pi$ or $K$ condensates?
Few thousands of binding energies for normal nuclei are known. Only few tens for hypernuclei.
The model and the method
Equation of state of neutron matter
Neutron star structure (I) - radius
Λ-hypernuclei and Λ-neutron matter
Neutron star structure (II) - maximum mass
Conclusions
Nuclear Hamiltonian

Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

\[
H = -\frac{\hbar^2}{2m} \sum_{i=1}^{A} \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}
\]

\(v_{ij}\) NN (Argonne AV8’) fitted on scattering data. Sum of operators:

\[
v_{ij} = \sum O_{ij}^{p=1,8} v^p (r_{ij}), \quad O_{ij}^p = (1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \vec{\tau}_i \cdot \vec{\tau}_j)
\]

Urbana–Illinois \(V_{ijk}\) models processes like

\[\pi \pi \Delta \pi \pi \Delta\]  + short-range correlations (spin/isospin independent).

Stefano Gandolfi (LANL) - stefano@lanl.gov
The EOS of neutron matter, and the effect of \(\Lambda\) hyperons
Quantum Monte Carlo

\[ H \psi(\vec{r}_1 \ldots \vec{r}_N) = E \psi(\vec{r}_1 \ldots \vec{r}_N) \]
\[ \psi(t) = e^{-\left(H-E_T\right)t} \psi(0) \]

Ground-state extracted in the limit of \( t \to \infty \).

Propagation performed by

\[ \psi(R, t) = \langle R | \psi(t) \rangle = \int dR' G(R, R', t) \psi(R', 0) \]

- Importance sampling: \( G(R, R', t) \to G(R, R', t) \frac{\Psi_I(R')}{\Psi_I(R)} \)
- Constrained-path approximation to control the sign problem. Unconstrained calculation possible in several cases (exact).

Ground-state obtained in a **non-perturbative way.** Systematic uncertainties within 1-2 \%.
Neutron matter is an "exotic" system. Why do we care?

- EOS of neutron matter gives the symmetry energy and its slope.
- The three-neutron force ($T = 3/2$) very weak in light nuclei, while $T = 1/2$ is the dominant part. No direct $T = 3/2$ experiments available.
- Determines properties of neutron stars.

![Diagram showing the relationship between theory, experiments, Esym, L, and neutron stars.](image)
What is the Symmetry energy?

Assumption from experiments:

\[ E_{SNM}(\rho_0) = -16\,\text{MeV}, \quad \rho_0 = 0.16\,\text{fm}^{-3}, \quad E_{sym} = E_{PNM}(\rho_0) + 16 \]

At \( \rho_0 \) we access \( E_{sym} \) by studying PNM.
We consider different forms of three-neutron interaction by only requiring a particular value of $E_{\text{sym}}$ at saturation.

different 3N:
- $V_{2\pi} + \alpha V_R$
- $V_{2\pi} + \alpha V_R^\mu$
- ($\text{several } \mu$)
- $V_{2\pi} + \alpha \tilde{V}_R$
- $V_{3\pi} + \alpha V_R$
Neutron matter

Equation of state of neutron matter using Argonne forces:

\[ E_{\text{sym}} = 35.1 \text{ MeV (AV8'+UIX)} \]
\[ E_{\text{sym}} = 33.7 \text{ MeV} \]
\[ E_{\text{sym}} = 32 \text{ MeV} \]
\[ E_{\text{sym}} = 30.5 \text{ MeV (AV8')} \]

Gandolfi, Carlson, Reddy, PRC (2012)
Neutron matter and symmetry energy

From the EOS, we can fit the symmetry energy around $\rho_0$ using

$$E_{\text{sym}}(\rho) = E_{\text{sym}} + \frac{L \rho - 0.16}{0.16} + \cdots$$

Gandolfi et al., EPJ (2014)

Tsang et al., PRC (2012)

Very weak dependence to the model of 3N force for a given $E_{\text{sym}}$. Knowing $E_{\text{sym}}$ or $L$ useful to constrain 3N! (within this model...)
TOV equations:

\[
\frac{dP}{dr} = - \frac{G[m(r) + 4\pi r^3 P/c^2][\epsilon + P/c^2]}{r[r - 2Gm(r)/c^2]},
\]

\[
\frac{dm(r)}{dr} = 4\pi \epsilon r^2,
\]
Neutron star matter

- Neutron star radii sensitive to EOS around $\rho \sim (1 - 2)\rho_0$
- Maximum mass depends to higher densities
Neutron star structure

Causality: $R > 2.9 \ (GM/c^2)$

$\rho_{\text{central}} = 2 \rho_0$

$\rho_{\text{central}} = 3 \rho_0$

Error associated with $E_{\text{sym}}$

$E_{\text{sym}} = 30.5 \ \text{MeV (NN)}$

$1.4 \ M_\odot$

$1.97(4) \ M_\odot$

Observations of the mass-radius relation are becoming available:


Neutron star observations can be used to constrain the EOS, $E_{\text{sym}}$ and $L$.

(Systematic uncertainties still under debate...)
Here an 'astrophysical measurement'

\[ 32 < E_{\text{sym}} < 34 \text{ MeV}, \ 43 < L < 52 \text{ MeV} \]

Steiner, Gandolfi, PRL (2012).
If chemical potential large enough ($\rho \sim 2 - 3\rho_0$), nucleons produce $\Lambda$, $\Sigma$, ...

Non-relativistic BHF calculations suggest that available hyperon-nucleon Hamiltonians do not support an EOS with $M > 2M_\odot$:

Schulze and Rijken PRC (2011).

Note: (Some) other relativistic model support $2M_\odot$ neutron stars.

→ Hyperon puzzle
Λ-hypernuclei and hypermatter

\[
H = H_N + \frac{\hbar^2}{2m_\Lambda} \sum_{i=1}^{A} \nabla_i^2 + \sum_{i<j} v_{ij}^{\Lambda N} + \sum_{i<j<k} V_{ijk}^{\Lambda NN}
\]

Λ-binding energy calculated as the difference between the system with and without Λ.
The $\Lambda$-nucleon interaction is constructed similarly to the Argonne potentials (Usmani).

**Argonne NN:**

$$v_{ij} = \sum_p v_p(r_{ij}) O_{ij}^p, \quad O_{ij} = (1, \sigma_i \cdot \sigma_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \tau_i \cdot \tau_j)$$

**Usmani $\Lambda N$:**

$$v_{ij} = \sum_p v_p(r_{ij}) O_{ij}^\lambda, \quad O_{\lambda j} = (1, \sigma_{\lambda} \cdot \sigma_j) \times (1, \tau_j^z)$$

Unfortunately... $\sim 4500$ NN data, $\sim 30$ of $\Lambda N$ data.
ΛN and ΛNN interactions

ΛNN has the same range of ΛN

\[
\begin{array}{c}
\Lambda \\
\Sigma \\
\Lambda \\
\end{array}
\quad \pi \\
\begin{array}{c}
N \\
\Sigma \\
N \\
\end{array}
\]

\[
\begin{array}{c}
N \\
\pi \\
N \\
\end{array}
\quad \begin{array}{c}
\Lambda \\
\Sigma \\
\Lambda \\
\end{array}
\quad \pi \\
\begin{array}{c}
N \\
\Sigma \\
N \\
\end{array}
\]

Differently from NN and NNN interactions:

\[
\begin{array}{c}
N \\
\pi \\
N \\
\end{array}
\quad \Delta \\
\begin{array}{c}
N \\
\pi \\
N \\
\end{array}
\]

\[
\begin{array}{c}
N \\
\pi \\
N \\
\end{array}
\quad \Delta \\
\begin{array}{c}
N \\
\pi \\
N \\
\end{array}
\]
The EOS of neutron matter, and the effect of Λ hyperons

Lonardoni, Gandolfi, Pederiva, PRC (2013) and PRC (2014).

V^{ΛNN} (II) is a new form where the parameters have been readjusted. ΛNN crucial for saturation.
Hyper-neutron matter

Neutrons and $\Lambda$ particles:

$$\rho = \rho_n + \rho_\Lambda, \quad x = \frac{\rho_\Lambda}{\rho}$$

$$E_{\text{HNM}}(\rho, x) = \left[ E_{\text{PNM}}((1-x)\rho) + m_n \right](1-x) + \left[ E_{\text{PAM}}(x\rho) + m_\Lambda \right]x + f(\rho, x)$$

where $E_{\text{PAM}}$ is the non-interacting energy (no $\nu_{\Lambda\Lambda}$ interaction),

$$E_{\text{PNM}}(\rho) = a \left( \frac{\rho}{\rho_0} \right)^\alpha + b \left( \frac{\rho}{\rho_0} \right)^\beta$$

and

$$f(\rho, x) = c_1 \frac{x(1-x)\rho}{\rho_0} + c_2 \frac{x(1-x)^2\rho^2}{\rho_0^2}$$

All the parameters are fit to Quantum Monte Carlo results.
Λ-neutron matter

EOS obtained by solving for \( \mu_\Lambda(\rho, x) = \mu_n(\rho, x) \)

Lonardoni, Lovato, Gandolfi, Pederiva, PRL (2015)

No hyperons up to \( \rho = 0.5 \text{ fm}^{-3} \) using \( \Lambda NN \) (II)!!!
Drastic role played by $\Lambda NN$. Calculations can be compatible with neutron star observations.

Note: no $\nu_{\Lambda\Lambda}$, no protons, and no other hyperons included yet...
Summary

- EOS of pure neutron matter qualitatively well understood.
- Λ-nucleon data very limited, but ΛNN is very important.
- Role of Λ in neutron stars far to be understood. We cannot conclude anything for neutron stars with present models...

Future needs:

- Accurate and precise measurement of $E_{\text{sym}}$ and $L$.
- More ΛN experimental data needed. Input from Lattice QCD? Femtoscopy @HADES (talk by Piotr Salabura)?
- Light and medium Λ-nuclei measurements needed, especially $N \neq Z$ (JLAB exp. approved)

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- S. Reddy (INT)
- A. Steiner (UT/ORNL)
Extra slides
Neutron matter at N2LO

EOS of pure neutron matter at N2LO, $R_0=1.0$ fm. Error quantification estimated as previously.

Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

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\]

NN interaction - Argonne AV8’ and AV6’.
Phase shifts, AV8′

Difference AV8′-AV18 less than 0.2 MeV per nucleon up to A=12.

Stefano Gandolfi (LANL) - stefano@lanl.gov
Two neutrons have

\[ k \approx \sqrt{E_{lab} m/2} \rightarrow k_F \]

that correspond to

\[ k_F \rightarrow \rho \approx (E_{lab} m/2)^{3/2}/2\pi^2. \]

\( E_{lab} = 150 \) MeV corresponds to about 0.12 fm\(^{-3}\).

\( E_{lab} = 350 \) MeV to 0.44 fm\(^{-3}\).

Argonne potentials useful to study dense matter above \( \rho_0 = 0.16 \) fm\(^{-3}\).
Light nuclei spectrum computed with GFMC

Argonne $v_{18}$ with UIX or Illinois-7 GFMC Calculations
1 June 2011

Observations of the mass-radius relation are becoming available:


Neutron star observations can be used to 'measure' the EOS and constrain $E_{\text{sym}}$ and $L$. (Systematic uncertainties still under debate...)
Neutron star matter

Neutron star matter model:

\[ E_{NSM} = a \left( \frac{\rho}{\rho_0} \right)^\alpha + b \left( \frac{\rho}{\rho_0} \right)^\beta, \quad \rho < \rho_t \]

(form suggested by QMC simulations),

and a high density model for \( \rho > \rho_t \)

i) two polytropes

ii) polytrope+quark matter model

Neutron star radius sensitive to the EOS at nuclear densities!

Direct way to extract \( E_{sym} \) and \( L \) from neutron stars observations:

\[ E_{sym} = a + b + 16, \quad L = 3(a\alpha + b\beta) \]
Neutron star matter really matters!

32 < \( E_{sym} < 34 \text{ MeV} \)
43 < \( L < 52 \text{ MeV} \)

Steiner, Gandolfi, PRL (2012).