Flux Tubes Interacting with Superfluids

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Superconductor Recap

Type I: Zero resistance, complete expulsion of magnetic fields: Meissner effect
Destroyed by: critical current, critical temperature T_c,
critical magnetic field H_c.

Type II: Zero resistance, Meissner effect only below first critical field H_{c1}
Magnetic flux enters above H_{c1} in form of flux tubes
Normal conducting state reached via T_c or second critical field H_{c2}

Fluxtube: Above H_{c1} magnetic fields enters SC in tubes, core of flux tube is
normal conducting,
field only enters in ***n*** units of fundamental flux quantum

$$\Phi = B/A = n\Phi_0$$

Ginzburg-Landau parameter κ : Material parameter determining type of SC

Motivation

Protons and neutrons in compact stars form a Cooper pair condensate
→ interacting multi fluid system of a superconductor (SC) and a superfluid (SF)

“Common wisdom”: Protons form type II SC (simple model calculations: $\kappa > \frac{1}{\sqrt{2}}$)

But: some observations support type I (long period precession)

In general: type I → II transition as function of density expected

? What does this transition look like ?

? How is this simple picture modified by the presence of the superfluid ?

? How does the phase diagram of the system look ?

K. Glampedakis, N. Andersson, L. Samuelsson, MNRAS, 410, 802-829, arXiv:1001.4046
K. Buckley, M. Metlitski, A. Zhitnitsky, PRC 69, 055803

Model

Ginzburg-Landau Theory from Microscopic Model

Effective field-theoretical model: two coupled, complex, **gauged** scalar fields with self interaction and interaction between the fields.

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_i + \mathcal{L}_{Int}$$

$$\mathcal{L}_i = \sum_{i=1,2} \left[D_{i,\mu}\varphi_i \left(D_i^\mu \varphi_i\right)^* - m_i^2|\varphi_i|^2 - \lambda_i|\varphi_i|^4 \right]$$

$$\mathcal{L}_{Int} = 2h|\varphi_1|^2|\varphi_2|^2 - \frac{G}{2}\left(\varphi_1\varphi_2\left(D_{1,\mu}\varphi_1\right)^*\left(D_2^\mu\varphi_2\right)^* + perm.\right)$$

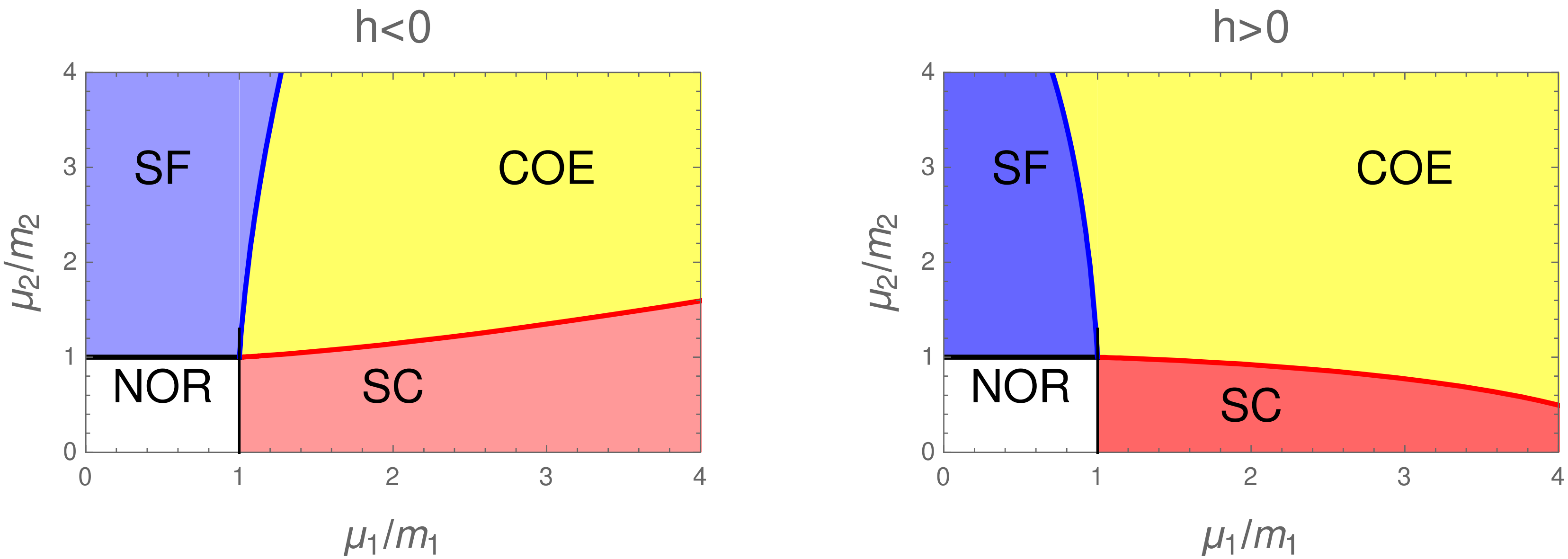
tree-level potential = Ginzburg-Landau free energy + entrainment term

G: derivative/entrainment coupling (Andreev-Bashkin effect)
h: non-entrainment coupling

Ungauged: Haber, Schmitt, Stetina, Phys.Rev. D93 (2016) no.2, 025011
Non-relativisitc: Alford, Good, Phys.Rev. B78 (2008) 024510

Phase Structure at B = 0

Assumptions: B=0, T=0, q_2=0 →(SC+SF)



- NOR: normal phase, no superfluidity
- SF : only uncharged field is condensed (superfluid)
- SC : only charged field is condensed (superconductor)
- COE: coexistence phase, both fluids are condensed (SC+SF)

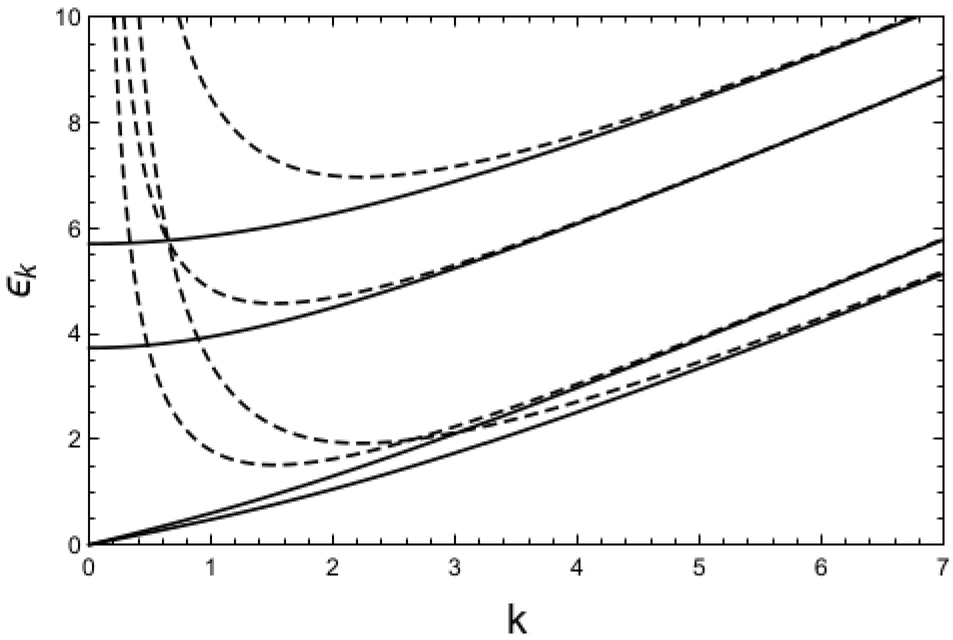
Observation: no dependence on entr. coupling **G**

Finite Temperature I

Large Temperature Expansion

large T = high momentum expansion of dispersion relation

$$\epsilon_k = k + c_1 + \frac{c_2^2}{k} + \dots$$



take all four quasi particle modes into account, compute determinant of propagator:

$$\begin{aligned}\Omega &= U\left(\rho_{P_0}, \rho_{N_0}\right) + T \int \frac{d^3k}{(2\pi)^3} \ln\left(1 - e^{-\varepsilon_i/T}\right) \\ &\approx U\left(\rho_{P_0}, \rho_{N_0}\right) - \frac{\pi^2 T^4}{90} + \frac{c_1 \zeta(3) T^3}{\pi^2} + \frac{\left(c_2^2 - c_1^1\right) T^2}{12}\end{aligned}$$

Finite Temperature II

Temperature Dependent Mass and Coupling at small G

$$m_{i,T}^2 \equiv m_i^2 + \frac{2\lambda_i - h}{6} T^2 \,, \qquad h_T \equiv h\left(1 + \frac{GT^2}{6}\right)$$

- all T – dependence is absorbed in effective mass and coupling
- gives intuitive expectations for finite T effects
- entrainment dependence of condensates through finite T!

without entrainment, G = 0

$$\rho_{P_0/N_0}^2(T) = \rho_{P_0/N_0}^2(T=0)\left(1 - \frac{T^2}{T_{c_{P/N}}^2}\right)$$

known form from G-L theory

$$T_{c_P}^2 = 3\frac{1 - \frac{h^2}{\lambda_1\lambda_2}}{1 + \frac{h}{2\lambda_1}\left(1 - \frac{h}{\lambda_2}\right)}\rho_{P_0}^2$$

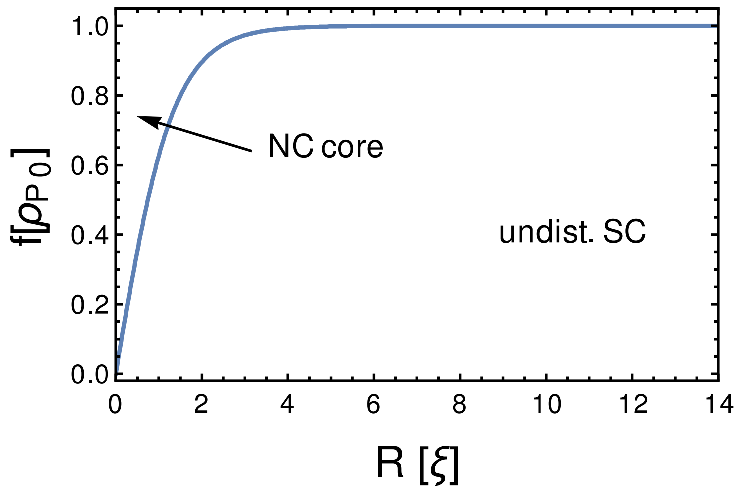
critical temperature modified due to interaction with second field!

(exchange neutron-proton parameters for superfluid T_C)

Flux Tube Profile n=1

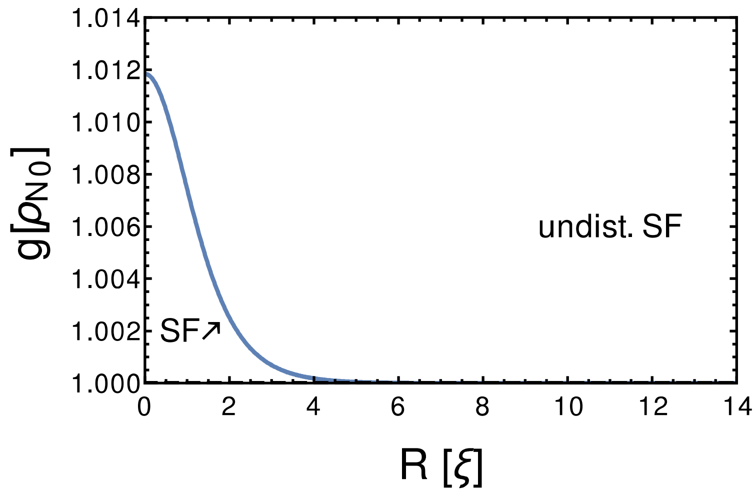
Solve EL-EOM numerically (relaxation method) with radially symmetric ansatz for flux tubes

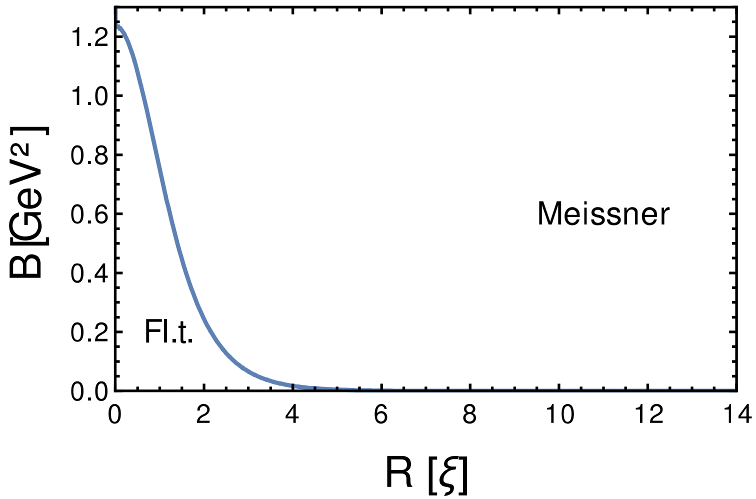
Assumptions: $q_2=0 \rightarrow$ (SC+SF), G=0, h<0, T=0, exterior B-Field



Results:

- No superfluid vortices induced
- Superfluid density enhanced/diminished in fluxtube depending on sgn(h)
- Multi flux quantum configurations can be computed



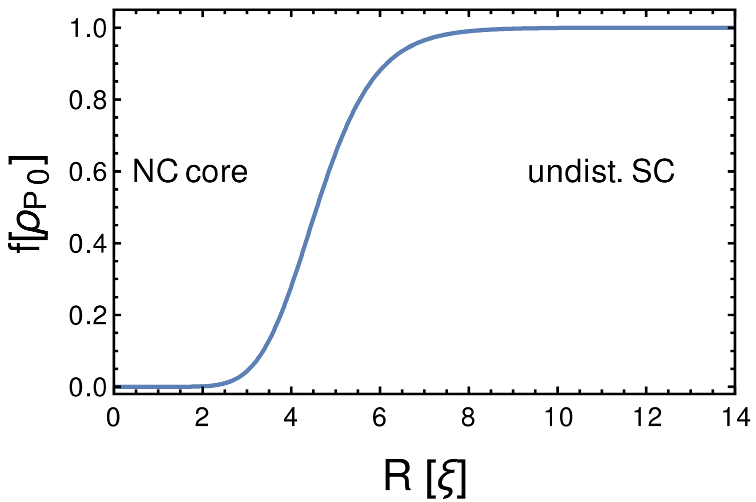


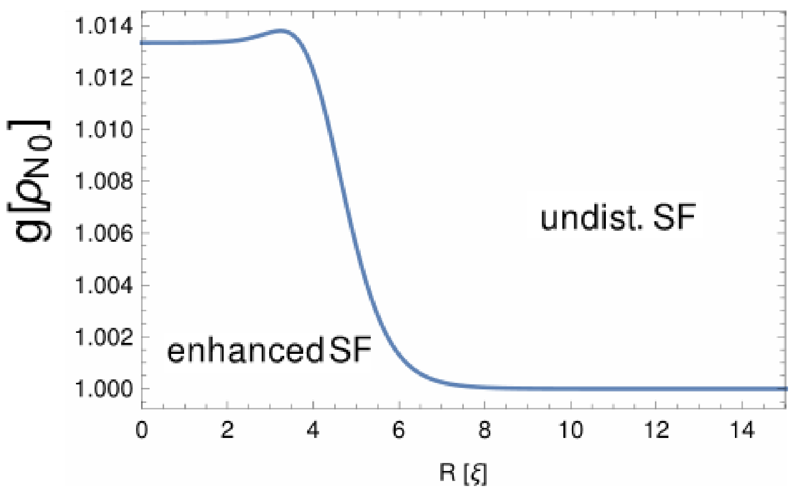
Flux Tube Profile n=11

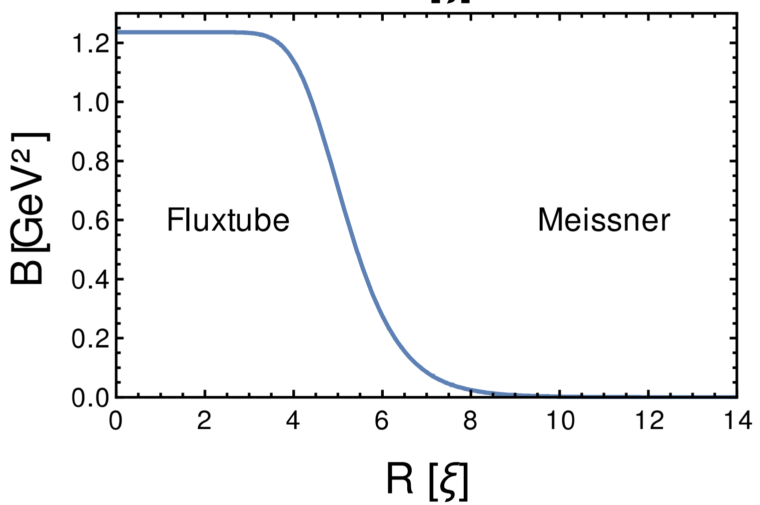
Assumptions: q_2=0 → (SC+SF),
G<0, h<0, T=0, exterior B-Field

Results:

- Superfluid density enhanced/diminished in fluxtube depending on sgn(h)
- "Bump" of the neutron condensate at the beginning of the descent due to gradient term







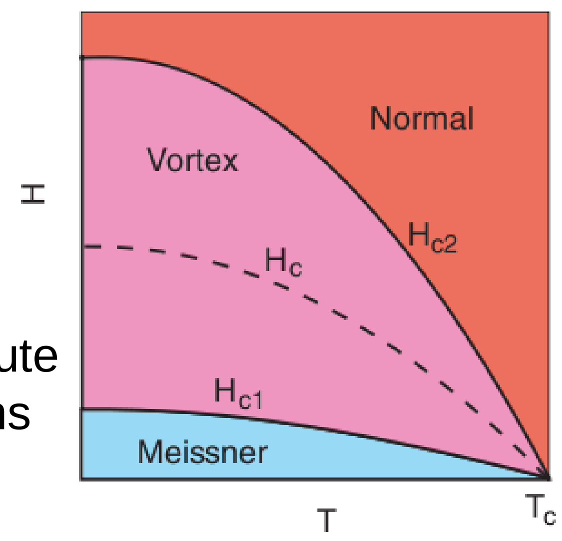
Critical Magnetic Fields I

Goal: calculate critical magnetic fields

H_c : compare free energy of Meissner phase with free energy of normal phase

$$H_c = \sqrt{2\pi\lambda_1}\sqrt{1 - \frac{h_T^2}{\lambda_1\lambda_2}\rho_{P_0}^2}\,(T)$$

finite T sketch



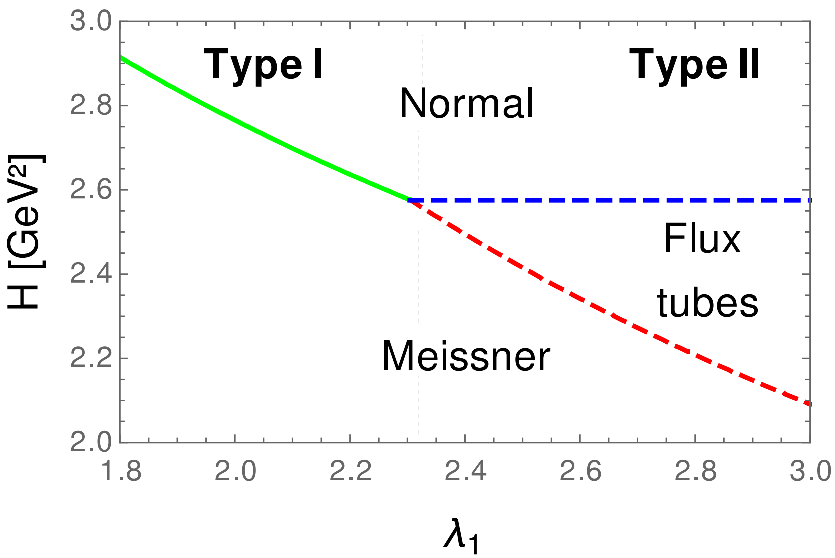
H_{c2} : linearize equations of motion (= assume 2nd order phase transition), compute maximal magnetic field which allows solutions

$$H_{c_2} = \sqrt{2}\kappa\sqrt{1 - \frac{h_T^2}{\lambda_1\lambda_2}}H_c\,(T)$$

H_{c1} : compare Gibbs free energy of a **single** flux tube with **winding number n** with the Meissner phase demands full numerical calculation of the flux tube profiles

Critical Magnetic Fields II

Uncoupled Superconductor



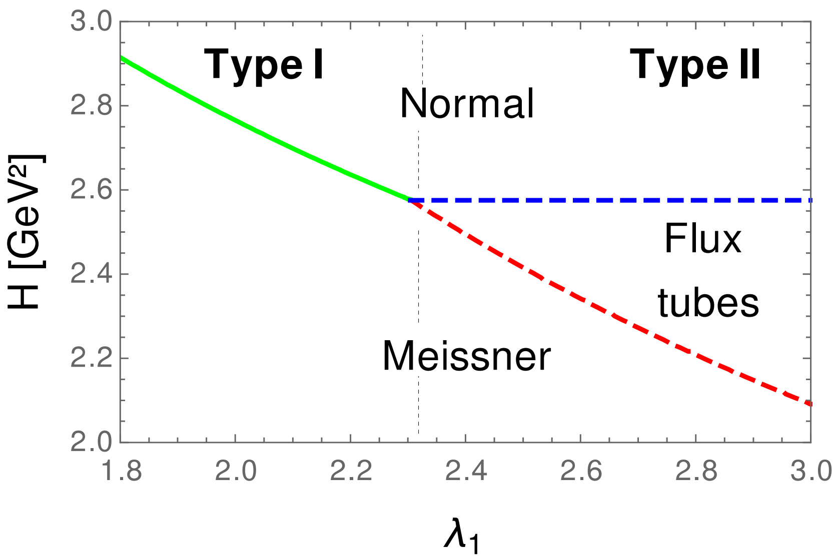
$$\kappa = \sqrt{\frac{\lambda_1}{4\pi q^2}}$$

Normal SC without interaction: consistent phase structure

Analytical result: SC phase is preferred directly below H_{c2} if phase transition is 2^{nd} order

Critical Magnetic Fields II

Uncoupled Superconductor



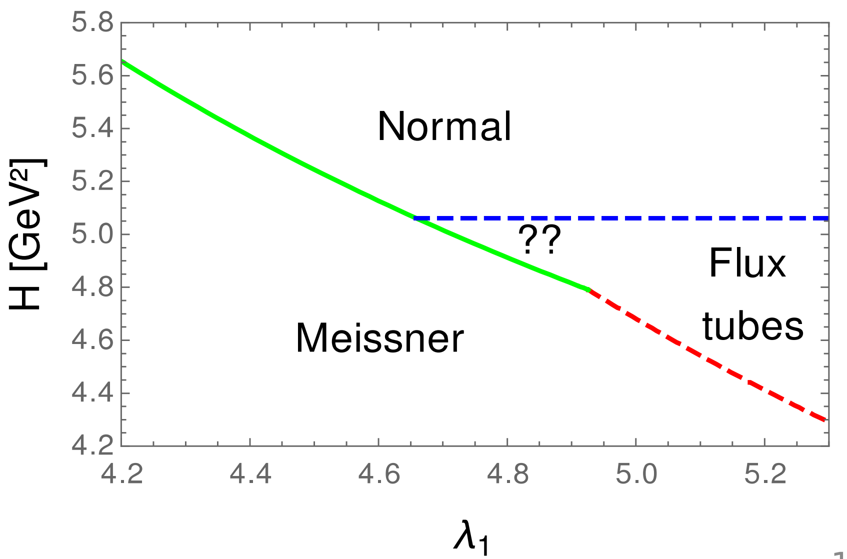
Normal SC without interaction: consistent phase structure

Analytical result: SC phase is preferred directly below H_{c2} if phase transition is 2nd order

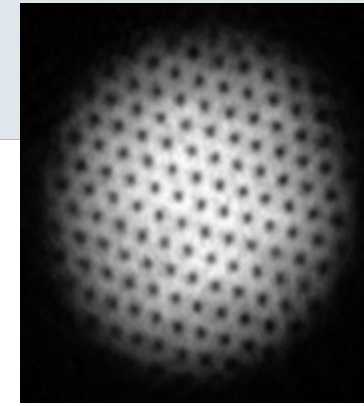
Coupled System

SC coupled to SF: phase structure more complicated

Exact phase structure depends on interaction between flux tubes



Flux Tube Interaction I



BEC vortex lattice by MIT

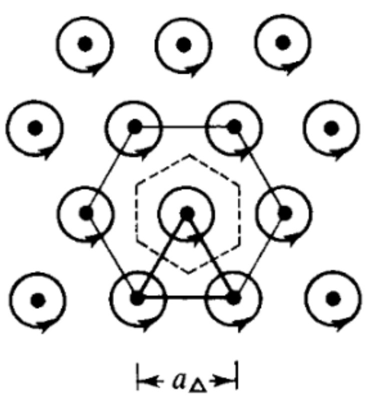
Two main contributions:

1) Repulsive part due to Lorentz force: magnetic field of one vortex reaches its neighbor $\vec{F}_L = q\vec{v}_2 \times \vec{h}_1 (r_0)$

2) Attractive part: flux tubes want to overlap to gain condensation energy

$$\frac{F_{int}(r_0)}{L} = 2\rho_{P0}^2 r_0 \int_{r_0/2}^{\infty} \frac{dR}{\sqrt{R^2 - \left(\frac{r_0}{2}\right)^2}} \left[\frac{\kappa^2 n^2 a' (1-a)}{R^2} - (1-f) f' - x^2 (1-g) g' \right]$$

K. Buckley, M. Metlitski, A. Zhitnitsky, PRC 69, 055803



Tinkham, Introduction to SC, Dover Press

Assumptions:
- Nearest neighbor approximation
- Large distance between tubes → linearization of EOM
- Hexagonal lattice

Flux Tube Interaction II

Integral can be solved analytically using asymptotic solutions in form of modified Bessel functions

$$F_{Gibbs} = U_{COE} + \frac{n\nu}{2q}\left[H_{c_1}(n) - H\right] + \frac{\#_{NN}\nu}{2}\frac{F_{int}(r_0)}{L}$$

with the flux tube area density $\nu = \frac{2}{\sqrt{3}r_0^2}$ and $\#_{NN} = 6$

→ dynamically compute lattice spacing by minimization of free energy

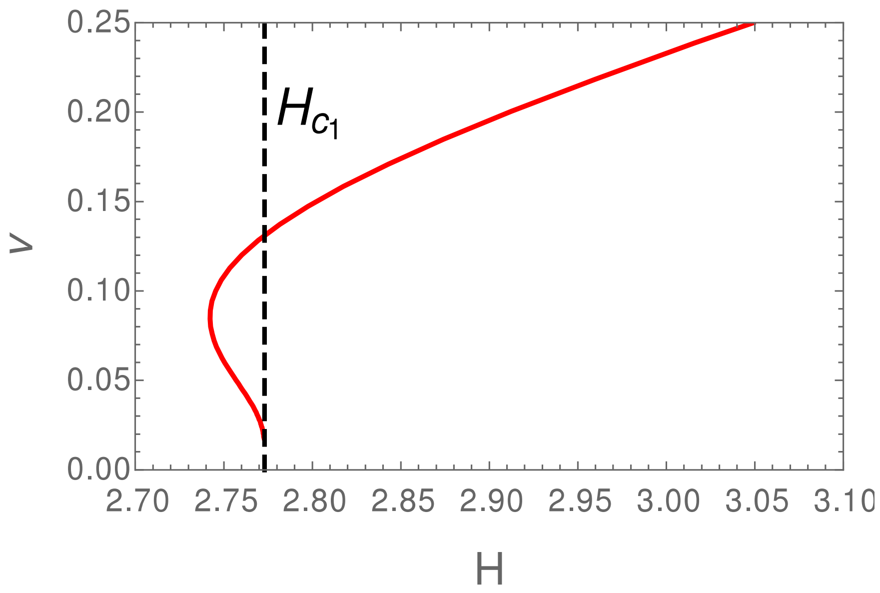
Possible effect: attractive term allows for flux tubes **below** H_{c1}

Analog: Baryon onset Energy of single flux tube ↔ baryon mass
Attractive flux tube interaction ↔ binding energy

Lattice Spacing / Density

First Order Phase Transition

$$\nu = \frac{2}{\sqrt{3} r_0^2}$$



without flux tube interaction:

- no solutions below H_{c1} can be found
- for very sparse lattice: single flux tube result is restored → density asymptotically approaches H_{c1}

Caveat: so far only tiny effect in energetically not realized phases → systematic investigation of parameter space and comparison of free energies is necessary

Summary & Outlook

Summary

- Behavior of a superconductor is altered by interaction with superfluid
- Effective parameter kappa due to interaction with SF
- Topology of phase diagrams complicated, flux tube interactions crucial
- Attractive interaction might lead to earlier 1st order onset of flux tube phase

Outlook

- Further investigation of phase structure with flux tube interaction
- Relate proton Cooper pair self-coupling to actual density profile of compact stars
- Parameter fit to observational values, e.g. critical temperature
- Summarize everything and write a paper