Stability of superfluid vortices in dense quark matter

M. G. Alford, S. K. Mallavarapu, T. Vachaspati and A. Windisch

[arXiv:1601.04656] (Phys Rev C)

U.S. DEPARTMENT OF ENERGY | Office of Science

FWF Der Wissenschaftsfonds.

XIIth Quark Confinement and the Hadron Spectrum

from 28 August 2016 to 4 September 2016

Vortices in superfluid CFL quark matter

Q1: Why are CFL superfluid vortices unstable?

A1: Semi-superfluid fluxtube arrangement lowers the energy.

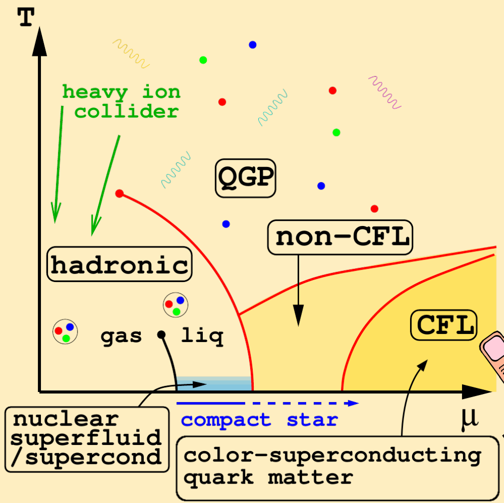
Q2: Are CFL superfluid vortices unstable or metastable?

A2: We numerically mapped out the space of the couplings to identify regions of metastability/instability.

Q3: Can we identify a mode responsible for the decay?

A3: We analytically construct a mode that is sufficient to trigger the decay of a superfluid vortex into semi-superfluid strings.

QCD phase diagram



M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, [arXiv:0709.4635] (Rev Mod Phys)

A. Schmitt, [arXiv:1001.3294] (Lect Notes Phys)

Color superconductivity

QCD:

quarks + attractive interaction $\Rightarrow$ Cooper pairs

Cooper pairs of quarks with pattern P (18^2 c-f-s matrix)

$$\langle \mathbf{q}^{\alpha}_{\mathbf{a}\xi} \mathbf{q}^{\beta}_{\mathbf{b}\zeta} \rangle = \Delta_{\mathbf{P}} \, \mathbf{P}^{\alpha\beta}_{\mathbf{ab}\xi\zeta}, \quad \text{where} \begin{cases} \alpha, \ \beta \in \{r, \ g, \ b\} \\ a, \ b \in \{u, d, s\} \\ \xi, \ \zeta \in \{\uparrow, \downarrow\} \end{cases}$$

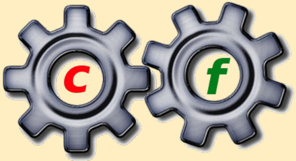
Attractive channel

- **space symmetric** [s-wave pairing]
- **color antisymmetric** [strongest attraction]
- **spin antisymmetric** [isotropic]
- $\Rightarrow$ **flavor antisymmetric**

Color-flavor locked phase

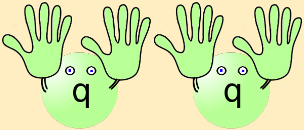
Symmetry breaking

$$\mathrm{SU}(3)_c \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{U}(1)_B \rightarrow \mathrm{SU}(3)_{c+L+R} \times \mathbb{Z}_2$$



CFL condensate Alford, Rajagopal, Wilczek [hep-ph/9804403]

$$\langle q^\alpha_a q^\beta_b \rangle \sim \delta^\alpha_a \delta^\beta_b - \delta^\alpha_b \delta^\beta_a = \epsilon^{\alpha\beta r}\epsilon_{abr}, \quad \text{where} \quad \begin{cases} \alpha,\ \beta \in \{r,\ g,\ b\} \\ a,\ b \in \{u, d, s\} \end{cases}$$



Some properties

- Chiral symmetry broken (not through $\langle \bar{q}q \rangle$)
- Baryon number broken $\Rightarrow$ superfluid

CFL rotational vortices

Angular momentum carried by vortices. The phase of the quark condensate circulates around the core.

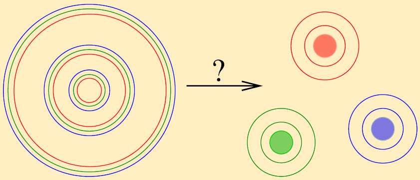
Large r: $\langle qq \rangle \sim e^{i\theta}$

Q1 (stability of vortices)

- Vortices are not stable
- Three separated *semisuperfluid flux tubes* have lower energy

Balachandran, Digal, Matsuura [hep-ph/0509276]

Superfluid to Semi-superfluid



CFL rotational vortices

Angular momentum carried by vortices. The phase of the quark condensate circulates around the core.

Large r: $\langle qq \rangle \sim e^{i\theta}$

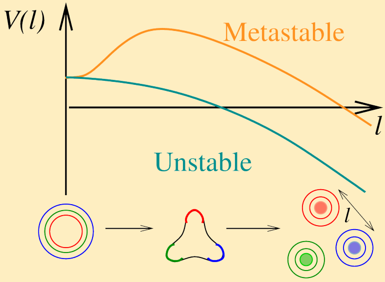
Q2 (unstable or metastable?)

Two cases:

- **Metastable:** Energy barrier prevents immediate decay of SF vortex
- **Unstable:** SF vortex undergoes spontaneous decay

$\Rightarrow$ explore parameter space

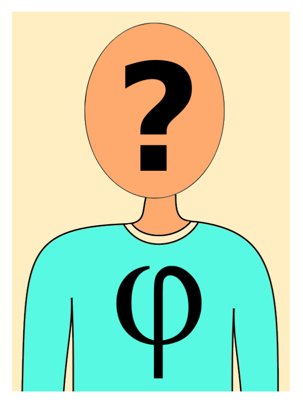
Superfluid to Semi-superfluid



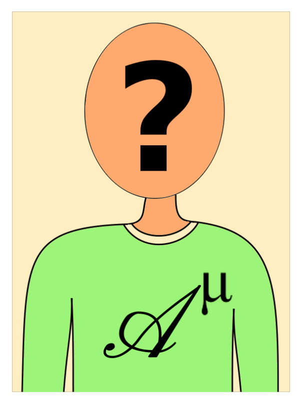
Effective theory (we assume $m_u = m_d = m_s = 0$)

$$\mathcal{L} = \mathrm{Tr}\left\{-\frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} + (\mathcal{D}_\mu\phi)(\mathcal{D}^\mu\phi)^* + \mathbf{m^2}(\phi^\dagger\phi) - \lambda_2(\phi^\dagger\phi)^2\right\} - \lambda_1\left(\mathrm{Tr}\left\{\phi^\dagger\phi\right\}\right)^2$$

Player 1 (scalar field)



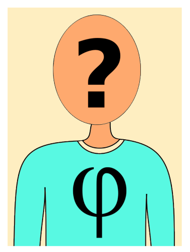
Player 2 (gluons, no EM)



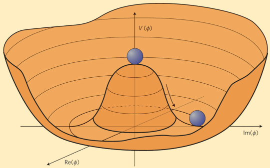
Effective theory (we assume $m_u = m_d = m_s = 0$)

$$\mathcal{L} = \mathrm{Tr}\left\{-\frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} + (\mathcal{D}_\mu\phi)(\mathcal{D}^\mu\phi)^* + \mathbf{m^2}(\phi^\dagger\phi) - \lambda_2(\phi^\dagger\phi)^2\right\} - \lambda_1\left(\mathrm{Tr}\left\{\phi^\dagger\phi\right\}\right)^2$$

Player 1



$\mathbf{m^2} < 0$



Scalar field ϕ $(3_c \times 3_f)$

$$\phi^{\mathbf{a}}_{\alpha} = \epsilon^{\alpha\beta\gamma}\epsilon_{\mathbf{abc}}\langle \mathbf{q}^{\beta}_{\mathbf{b}}\mathbf{q}^{\gamma}_{\mathbf{c}}\rangle$$

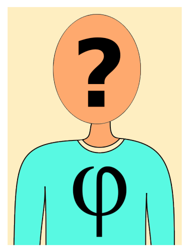
(vev) breaks baryon number $\Rightarrow$ superfluid

$$\langle\phi\rangle = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\bar{\phi}$$

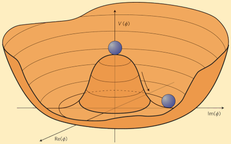
Effective theory (we assume $m_u = m_d = m_s = 0$)

$$\mathcal{L} = \mathrm{Tr}\left\{ -\frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} + (\mathcal{D}_\mu\phi)(\mathcal{D}^\mu\phi)^* + \mathbf{m^2}\left(\phi^\dagger\phi\right) - \lambda_2\left(\phi^\dagger\phi\right)^2 \right\} - \lambda_1\left(\mathrm{Tr}\left\{\phi^\dagger\phi\right\}\right)^2$$

Player 1



$\mathbf{m}^2 < 0$



Superfluid vortex ($A_i = 0$)

$$\phi^{(\mathbf{sf})\mathbf{a}}_{\alpha} = \delta^{\mathbf{a}}_{\alpha}\mathbf{e}^{i\theta}\beta(\mathbf{r})\bar{\phi}$$

Matrix structure ($\sim$ 3 global vortices)

$$\phi^{(sf)} = \begin{pmatrix} e^{i\theta} & 0 & 0 \\ 0 & e^{i\theta} & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \bar{\phi}\beta(r)$$

Our simulation: Superfluid-vortex decays into three semi-superfluid flux tubes

See also:

Balachandran, Digal, Matsuura

[hep-ph/0509276]

See also:

Eto, Nitta

[arXiv:0907.1278]

$U(\phi) \overset{r\to\infty}{\longrightarrow} 0$

Energetics: superfluid vs. semi-superfluid

Superfluid vortex

- $\phi = \bar{\phi} e^{in\theta}$
- $A = 0$
- $\varepsilon \propto |\vec{\nabla}\phi|^2 = n^2 \bar{\phi}^2 / r^2$
- $E_{vortex} \sim E_{core} + n^2 \bar{\phi}^2 \ln\left(\frac{R_{vol}}{R_{core}}\right)$
- **Large r: repulsion**

Semisuperfluid flux tube

- $\phi = \bar{\phi} e^{in\theta}$
- $A_\theta = -\frac{n}{gr}$
- $\varepsilon \propto |\vec{D}\phi|^2 = |\vec{\nabla}\phi - ig\vec{A}\phi|^2 = 0$
- $E_{flux\ tube} \sim E_{core}$
- **Large r: ?**

$$U(\phi) \overset{r\to\infty}{\longrightarrow} 0$$

Energetics: superfluid vs. semi-superfluid

Superfluid vortex

- $\phi^{(sf)} \approx \begin{pmatrix} e^{i\theta} & 0 & 0 \\ 0 & e^{i\theta} & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \bar{\phi}$
- $A_\theta^{(sf)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Semisuperfluid flux tube

- $\phi_G^{(ssf)} = \begin{pmatrix} e^{i\frac{\theta}{3}} & 0 & 0 \\ 0 & e^{i\frac{\theta}{3}} & 0 \\ 0 & 0 & e^{i\frac{\theta}{3}} \end{pmatrix} \bar{\phi}$
- $A_\theta^{(ssf)} = \begin{pmatrix} e^{-i\frac{\theta}{3}} & 0 & 0 \\ 0 & e^{-i\frac{\theta}{3}} & 0 \\ 0 & 0 & e^{i\frac{2\theta}{3}} \end{pmatrix}$

Energy density (n=1)

$$\varepsilon \sim 3 \times 1^2 \times \bar{\phi}^2/r^2 = 3\frac{\bar{\phi}^2}{r^2}$$

Energy density (n=1/3)

$$\varepsilon \sim 3 \times \left(\frac{1}{3}\right)^2 \times \bar{\phi}^2/r^2 = \frac{1}{3}\frac{\bar{\phi}^2}{r^2}$$

$$U(\phi) \stackrel{r\to\infty}{\rightarrow} 0$$

Energetics: superfluid vs. semi-superfluid

Superfluid vortex

scalar field	wdg
	$+1$
	$+1$
	$+1$
Total	$+3$

Semisuperfluid flux tube

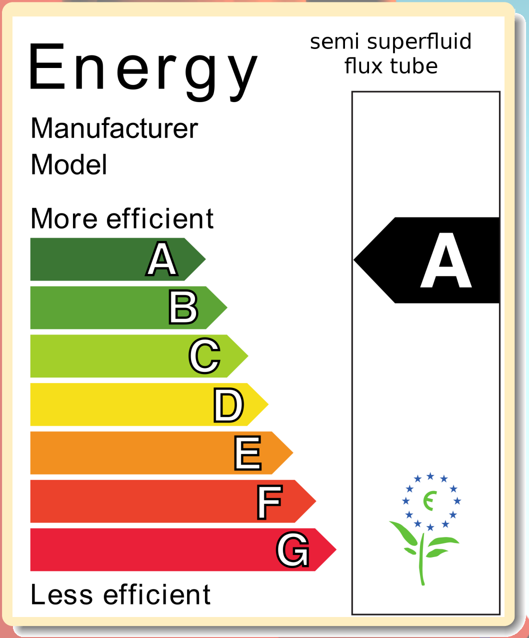
scalar + gauge field	wdg
	$+\frac{1}{3}$
	$+\frac{1}{3}$
	$+\frac{1}{3}$
Total	$+1$

Energy density (n=+1)

$$|\vec{\nabla}\phi|^2 \sim 3 \times (+1)^2 = 3$$

Energy density (n=+1/3)

$$|\vec{D}\phi|^2 \sim 3 \times (+1/3)^2 = 1/3$$

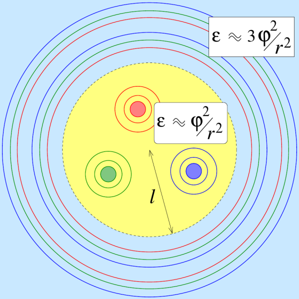

Energy
semi superfluid
flux tube
Manufacturer
Model
More efficient
A
B
C
D
E
F
G
Less efficient
A
€

Vortices in superfluid CFL quark matter

Q1: Why are CFL superfluid vortices unstable?

A1: Semi-superfluid fluxtube arrangement lowers the energy by canceling gradient energy of the winding partially.

Long range repulsion



$r \lesssim l$: low energy density

$$\varepsilon \sim 3 \times \tfrac{1}{3}\bar{\phi}^2/r^2 = \bar{\phi}^2/r^2$$

$r \gtrsim l$: high energy density

$$\varepsilon \sim 3\bar{\phi}^2/r^2$$

Increasing l: lower E-density

$$V(l) \sim \bar{\phi}^2 \int_0^l r dr \frac{1-3}{r^2}$$
$$\sim const - \bar{\phi}^2 \ln(l)$$

Q2: Metastable vs. unstable

Numerical framework (2d lattice)

Initialization

Set up $n = 1$ superfluid vortex

Choose couplings

gauge coupling g, self-couplings $\lambda_1,\ \lambda_2$
($\lambda \equiv 3\lambda_1 + \lambda_2$)

Add small perturbation

use analytic or random mode

Q2: Metastable vs. unstable

Numerical framework (2d lattice)

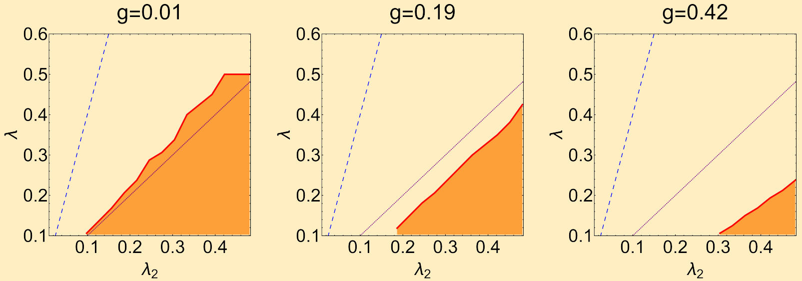
Evolve system

Look for signature of decay in evolution

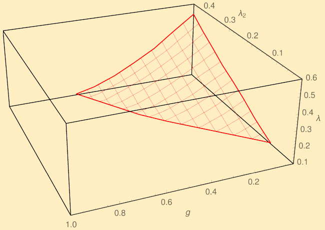
Create map of parameter space

Vary the couplings, look for exponential growth

Results from numerical analysis ($\lambda \equiv 3\lambda_1 + \lambda_2$)



Boundary in parameter space

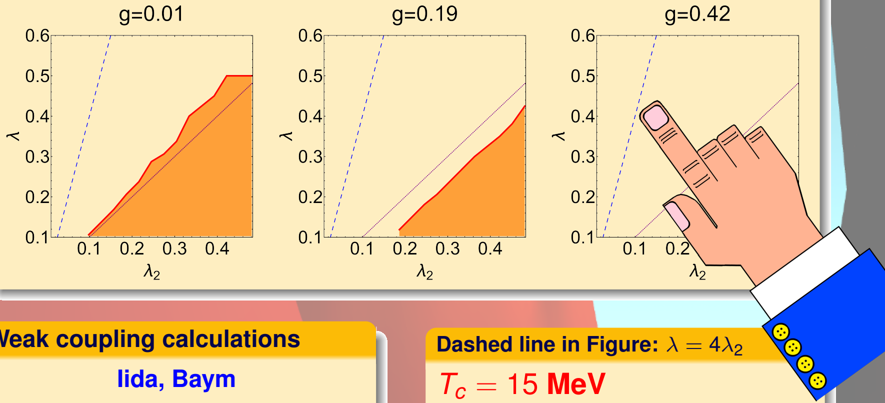


Findings

- **Metastable:**
  $\lambda_1 \lesssim -0.16g$
- **Unstable:**
  Increasing $g,\ \lambda_1$

Fixed $g,\ \lambda_1$ **: changing** λ_2 **has little/no effect**

Results from numerical analysis ($\lambda \equiv 3\lambda_1 + \lambda_2$)



Weak coupling calculations

Iida, Baym

[hep-ph/0011229]

$$\lambda_1 = \lambda_2 \approx 420(T_c/\mu_q)^2$$

Giannakis, Ren

[hep-ph/0108256]

Dashed line in Figure: $\lambda = 4\lambda_2$

$T_c = 15$ **MeV**

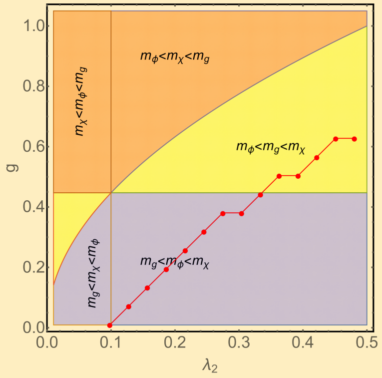
$\mu_q = 400$ **MeV**

$\Rightarrow \lambda_2 \approx 0.6$

Extrapolation to neutron star densities

CFL vortices appear to be *always* unstable.

Stability boundary and mass hierarchy
($\lambda = 0.1,\ m^2 = 0.25$)



Gluon mass

$$m_g = \sqrt{2} g \bar{\phi}$$

Scalar mass ϕ

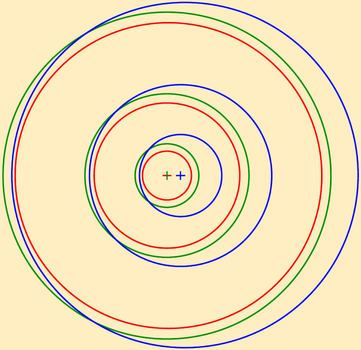
$$m_\phi = \sqrt{2} m$$

Scalar mass χ

$$m_\chi = 2\sqrt{\lambda_2}\bar{\phi}$$

Structure of unstable mode

Sufficient to trigger decay ($g = 0$)



Mode lowers energy

$$\delta E = -\varepsilon^2 \lambda_1 \frac{3\pi m^4}{(3\lambda_1+\lambda_2)^2} \int_0^\infty \left(\frac{d\beta}{dr}\right)^2 \beta^2 r dr$$

Superfluid vortex, $\varphi(\vec{r}) = \bar{\phi} e^{i\theta} \beta(r)$

$$\phi^{(sf)} = \begin{pmatrix} \varphi(\vec{r}) & 0 & 0 \\ 0 & \varphi(\vec{r}) & 0 \\ 0 & 0 & \varphi(\vec{r}) \end{pmatrix}$$

Superfluid vortex with perturbation

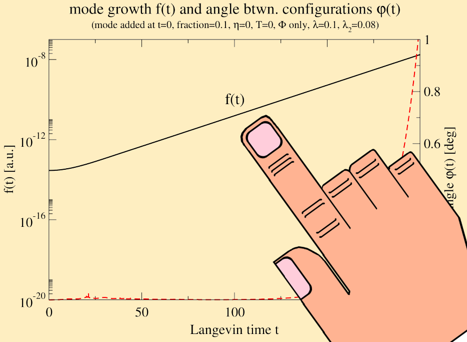
$$\phi^{(sf)}_{pert} = \begin{pmatrix} \varphi(\vec{r}+\varepsilon\hat{x}) & 0 & 0 \\ 0 & \varphi(\vec{r}+\varepsilon\hat{x}) & 0 \\ 0 & 0 & \varphi(\vec{r}-2\varepsilon\hat{x}) \end{pmatrix}$$

Unstable mode

$$\delta\phi^a_\alpha = \varepsilon\hat{x}\cdot\vec{\nabla}\varphi(\vec{r})\, T^a_{8\ \alpha}$$

Dynamics of unstable mode

Mode growth and angle config/perturbation



Scalar product

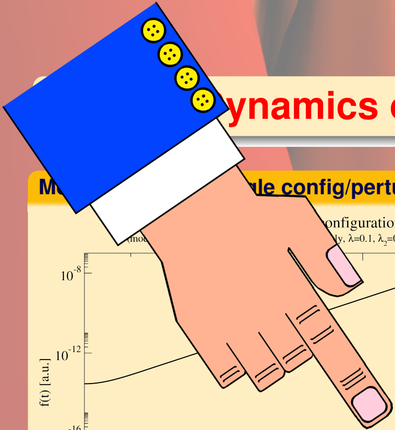
$$\langle A, B\rangle \equiv \int d^2x \mathrm{Tr}\{B^\dagger A\}$$

Mode growth

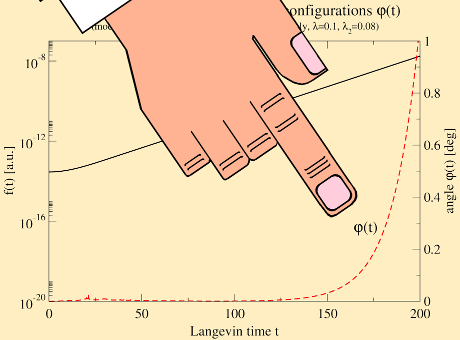
$$f(t) \equiv \langle \delta\Phi^{(p)} | \left(|\Phi\rangle_t - |\Phi^{(sf)}\rangle \right) \propto e^{\gamma t}$$

$$t_{min} < t < t_{max}$$

ynamics of unstable mode



M le config/perturbation



Scalar product

$$\langle A, B\rangle \equiv \int d^2x \mathrm{Tr}\{B^\dagger A\}$$

Angle config/perturbation

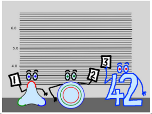
$$\varphi(t) \equiv \arccos \frac{|\langle A,B\rangle|}{||A||\ ||B||}$$

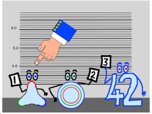
$$A \equiv \left(|\Phi\rangle_t - |\Phi^{(sf)}\rangle\right)$$

$$B \equiv |\delta\Phi^{(p)}\rangle$$

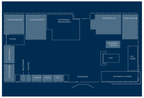
Summary

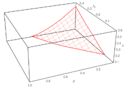
Q1: Identify configuration with least energy



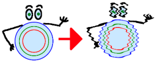


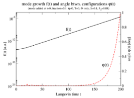
Q2: Map out parameter space





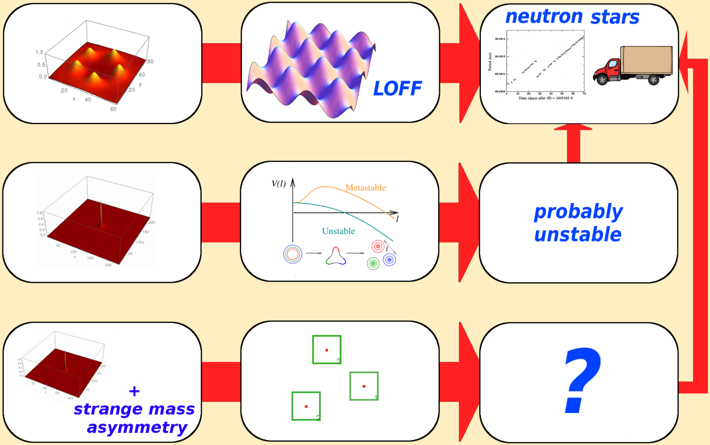
Q3: Study mode responsible for decay





Moving on: Introducing mass asymmetry

Implications



$\theta\alpha\nu\kappa\sigma$!