The Falsificacion of Chiral Nuclear Forces

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Error Analysis and Nuclear Structure

- What is the predictive power of theoretical nuclear physics ?

INPUT from Experiment → CALCULATION → OUTPUT vs Experiment

- Experiment much more precise than theory, but how much ?

$$\Delta M^{\rm exp} < 1{\rm KeV} \ll \Delta M^{\rm th} =?$$

- Theoretical Predictive Power Flow: From light to heavy nuclei

$$H(A) = T + V_{2N} + V_{3N} + V_{4N} + \cdots \to E_2, E_3, E_4, \ldots$$

- Chiral expansion allows to compute $V_{2N}, V_{3N}, V_{4N} \ldots$ systematically so that one has the hierarchy

$$V_{2N} \gg V_{3N} \gg V_{4N} \gg \ldots$$

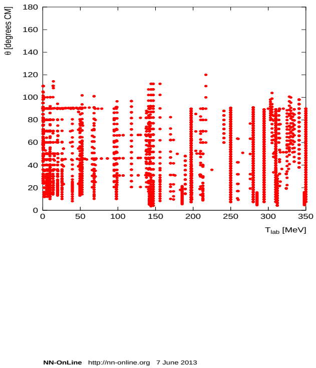
- Forces depend on the renormalization scale allowing to adjust $B_2, B_3, \ldots$

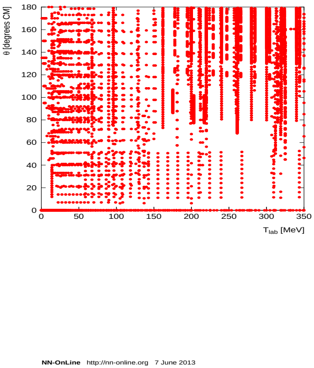
Sources of uncertainties/bias

- Input data from experiment
- Choose the form of the potential
- How to estimate theoretical errors based on INPUT data

$$INPUT = NN, 3N, \cdots \to OUTPUT = 4N, \ldots$$

- First Step: INPUT=NN scattering data
- OUTPUT=NN scattering amplitudes





The issue of predictive power in Chiral Approach

- Chiral forces are UNIVERSAL at long distances

$$V^{\chi}(r) = V^{\pi}(r) + V^{2\pi}(r) + V^{3\pi}(r) + \dots \qquad r \gg r_c$$

- Chiral forces are SINGULAR at short distances

$$V^{\chi}(r) = \frac{a_1}{f_\pi^2 r^3} + \frac{a_2}{f_\pi^4 r^5} + \frac{a_3}{f_\pi^6 r^7} + \dots \qquad r \ll r_c$$

- They trade model independence for regulator dependence
- What is the best theoretical accuracy we can get within "reasonable" cut-offs ?
- What is a reasonable cut-off ?

$$r_c = ?$$

Bottomline

THE PROBLEM

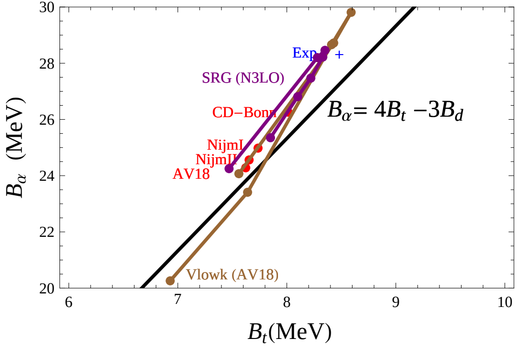
- GOAL: Estimate uncertainties from IGNORANCE of NN,3N,4N interaction
  Reduce computational cost
- Statistical Uncertainties: NN,3N,4N Data
  Data abundance bias
- Systematic Uncertainties: NN,3N,4N potential
  Many forms of potentials possible
- Confidence level of Imperfect theories vs Perfect experiments

OUR APPROACH

- Start with NN
- Fit data WITH ERRORS with a simple interaction
- Compare different interactions
- Estimate uncertainties of Effective Interactions and Matrix elements
- Propagate errors to A=3,4, etc.

Tjon-Line Correlation

- α-triton calculations find a linear correlation between



- SRG argument with on-shell nuclear forces

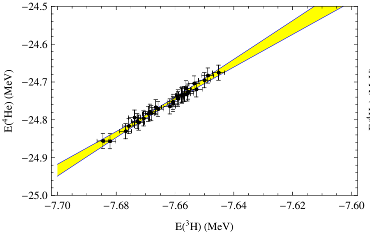
Timoteo, Szpigel, ERA, Few Body 2013

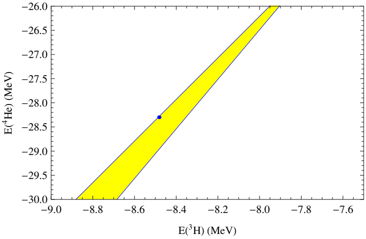
$$B_\alpha = 4B_t - 3B_d \qquad \rightarrow \qquad \frac{\partial B_\alpha}{\partial B_t}\Big|_{B_d} = 4$$

Tjon-Lines: numerical accuracy

(with A. Nogga)

$$\Delta E^{\rm stat}_{\rm triton} = 15 {\rm KeV} \qquad \Delta E^{\rm stat}_{\alpha} = 50 {\rm KeV}$$





- 4-Body forces are masked by numerical noise in the 3 and 4 body calculation if

$$\Delta^{\rm num}_t > 1 {\rm KeV} \qquad \Delta^{\rm num}_t > 20 {\rm KeV}$$

SIMPLE ESTIMATE

Motivation

- Effective Interaction [Skyrme, Moshinsky]
- Useful simplifications in many body calculations [Brink, Vaughterin]
- Power expansion in CM momenta

$$
\begin{aligned}
& V(\mathbf{p}',\mathbf{p}) = \int d^3x e^{-i\mathbf{x}\cdot(\mathbf{p}'-\mathbf{p})}\hat{V}(\mathbf{x}) \\
& = t_0(1+x_0P_\sigma) + \frac{t_1}{2}(1+x_1P_\sigma)(\mathbf{p}'^2+\mathbf{p}^2) \\
& + t_2(1+x_2P_\sigma)\mathbf{p}'\cdot\mathbf{p} + 2it_V\mathbf{S}\cdot(\mathbf{p}'\wedge\mathbf{p}) \\
& + \frac{t_T}{2}\left[\sigma_1\cdot\mathbf{p}\sigma_2\cdot\mathbf{p} + \sigma_1\cdot\mathbf{p}'\sigma_2\cdot\mathbf{p}' - \frac{1}{3}\sigma_1\sigma_2(\mathbf{p}'^2+\mathbf{p}^2)\right] \\
& + \frac{t_U}{2}\left[\sigma_1\cdot\mathbf{p}\sigma_2\cdot\mathbf{p}' + \sigma_1\cdot\mathbf{p}'\sigma_2\cdot\mathbf{p} - \frac{2}{3}\sigma_1\sigma_2\mathbf{p}'\cdot\mathbf{p}\right] \\
& + \mathcal{O}(p^4)
\end{aligned}
$$

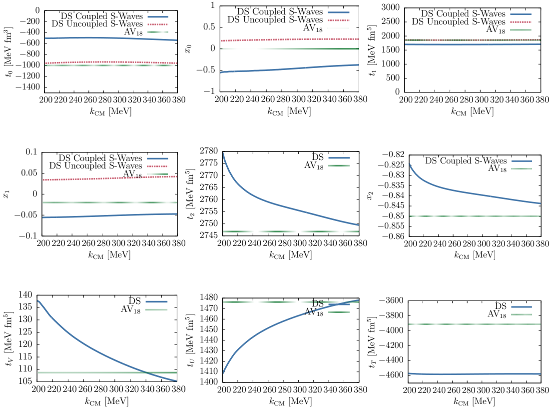
Skyrme parameters as EFT counterterms

- Comparing similar terms

$$
\begin{aligned}
(t_0, x_0 t_0) &= \frac{1}{2}\int d^3x \left[V_{{}^3S_1}(r) \pm V_{{}^1S_0}(r)\right] \\
(t_1, x_1 t_1) &= -\frac{1}{12}\int d^3x r^2 \left[V_{{}^3S_1}(r) \pm V_{{}^1S_0}(r)\right] \\
(t_2, x_2 t_2) &= \frac{1}{54}\int d^3x r^2 \left[V_{{}^3P_0}(r) + 3V_{{}^3P_1}(r) + 5V_{{}^3P_2}(r) \pm 9V_{{}^1P_1}(r)\right] \\
t_V &= \frac{1}{72}\int d^3x r^2 \left[2V_{{}^3P_0}(r) + 3V_{{}^3P_1}(r) - 5V_{{}^3P_2}(r)\right] \\
t_U &= \frac{1}{36}\int d^3x r^2 \left[-2V_{{}^3P_0}(r) + 3V_{{}^3P_1}(r) - V_{{}^3P_2}(r)\right] \\
t_T &= \frac{1}{5\sqrt{2}}\int d^3x r^2 V_{\epsilon_1}(r)
\end{aligned}
$$

Scale dependence

- Skyrme parameters fitting at different energy ranges

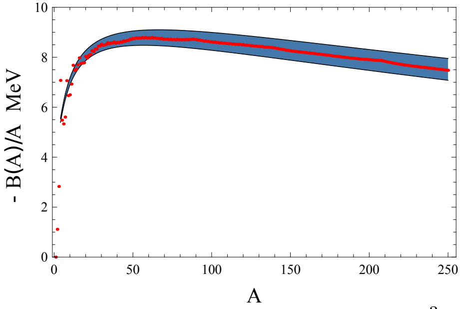


Simple estimate

- Fermi type shape density

$$\rho(x) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$

- Error band for stable nuclei binding energy



$$\frac{\Delta B}{A} = \frac{3}{8A} \Delta t_0 \int d^3x \, \rho(x)^2$$

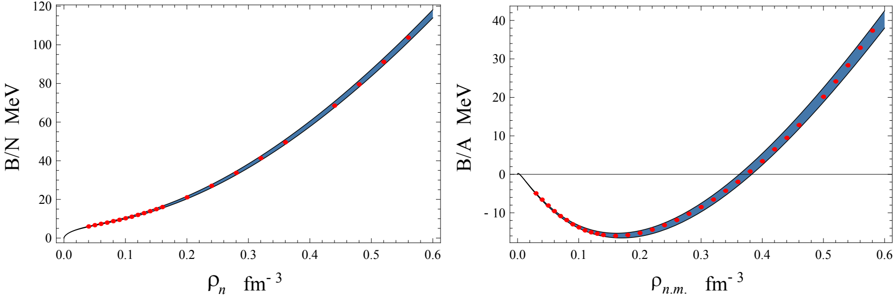
$$\Delta t_0 = 10 \text{MeVfm}^3 \quad \rightarrow \quad 75 \text{MeVfm}^3$$

$$\Delta B/A = 0.6 \text{MeV} \quad \rightarrow \quad 2.5 \text{MeV}$$

Skyrme Parameters

- Nuclear and Neutron matter
  - Error grows linearly with the density

$$\frac{\Delta B_{n.m.}}{A} = \frac{3}{8}\Delta t_0 \rho \sim 3.75\rho$$
$$\frac{\Delta B_n}{A} = \frac{1}{4}\Delta[t_0(1-x_0)]\rho_n \sim 3.5\rho_n$$



The voices of wisdom

- "There is no way to deduce the uncertainty of calculated nuclear ground state energies, obtained in ab initio or any other many-body method, from the variance of a set of phase shift equivalent nucleon-nucleon (NN) interactions.
  The paper does not make any sense to me and is not suitable for publication in any journal. "
  Referee I
- " There is no doubt that even within a (hypothetical) possibility to solve the nuclear many-body problem exactly, errors in knowing the many-body Hamiltonian must carry over to properties of finite nuclei. The idea is novel and very interesting.
  The paper raises a very important question of how the errors related to the estimation of nuclear forces may propagate to the calculated biding energies of nuclei".
  Referee II

Our contribution

- np+pp LARGEST database analysis so far = 6713
- Scrupulous statistical error analysis
- 6 statistically equivalent chiral or non-chiral interactions
- Propagation of errors to light nuclei
- Upgrade of the simple calculation

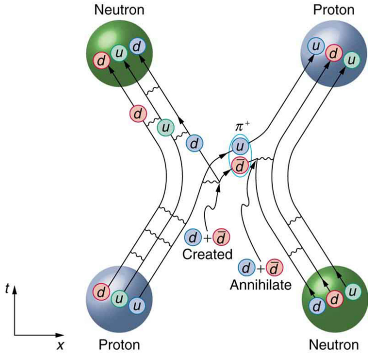
$$\Delta B/A = 2 - 2.4 \mathrm{MeV}$$

The sophisticated calculation

- **Uncertainty analysis and order-by-order optimization of chiral nuclear interactions**
  B.D. Carlsson, A. Ekström, C. Forssén, D.Fahlin Strömberg, G.R. Jansen, O. Lilja, M. Lindby, B.A. Mattsson, K.A. Wendt
  Phys.Rev. X6 (2016) no.1, 011019
- Take into account 2N and 3N chiral forces order by order, different regularizations and estimates of higher orders
$$\frac{\Delta B_{^{16}O}}{16} = 4\mathrm{MeV}$$
- Much worse than the semiempirical mass formula
- Most dependence on the cut-off Λ or the regularization
- Is this the predictive power of theoretical nuclear physics ?
- Conservative vs realistic uncertainty estimates

ANATOMY OF NUCLEAR FORCES

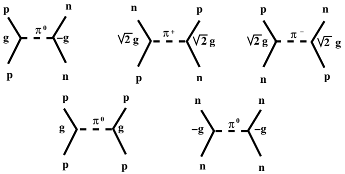
Fundamental approach: QCD



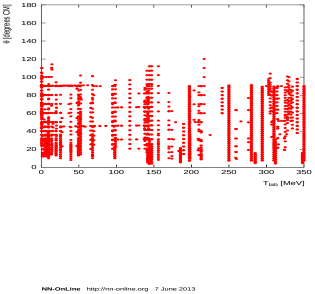
- Lattice form factor $g_{\pi NN} \sim 10 - 12$
- Lattice NN potential $g^2_{\pi NN}/(4\pi) = 12.1 \pm 2.7$
- QCD sum rules $g_{\pi NN} \sim 13(1)$

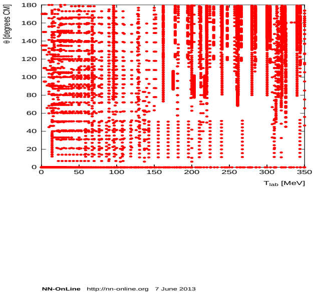
Long distances

- Nucleons exchange JUST one pion



- Low energies (about pion production) 8000 pp + np scattering data (polarizations etc.)





Nucleon-Nucleon Scattering

- Scattering amplitude

$$
\begin{aligned}
M &= a + m(\sigma_1 \cdot \mathbf{n})(\sigma_2 \cdot \mathbf{n}) + (g-h)(\sigma_1 \cdot \mathbf{m})(\sigma_2 \cdot \mathbf{m}) \\
&+ (g+h)(\sigma_1 \cdot \mathbf{l})(\sigma_2 \cdot \mathbf{l}) + c(\sigma_1 + \sigma_2) \cdot \mathbf{n} \\
\mathbf{l} &= \frac{\mathbf{k}_f + \mathbf{k}_i}{|\mathbf{k}_f + \mathbf{k}_i|} \qquad \mathbf{m} = \frac{\mathbf{k}_f - \mathbf{k}_i}{|\mathbf{k}_f - \mathbf{k}_i|} \qquad \mathbf{n} = \frac{\mathbf{k}_f \wedge \mathbf{k}_i}{|\mathbf{k}_f \wedge \mathbf{k}_i|}
\end{aligned}
$$

- 5 complex amplitudes $\rightarrow$ 24 measurable cross-sections and polarization asymmetries
- Partial Wave Expansion

$$
\begin{aligned}
M^{s}_{m'_s, m_s}(\theta) &= \frac{1}{2ik} \sum_{J,l',l} \sqrt{4\pi(2l+1)} Y^{l'}_{m'_s - m_s}(\theta, 0) \\
&\times \; C^{l',s,J}_{m_s - m'_s, m'_s, m_s} i^{l-l'} (S^{J,s}_{l,l'} - \delta_{l',l}) C^{l,s,J}_{0,m_s,m_s}, \qquad (1)
\end{aligned}
$$

- S-matrix

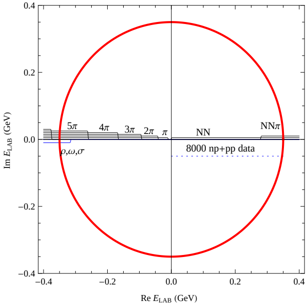
$$
S^J = \begin{pmatrix} e^{2i\delta^{J,1}_{J-1}} \cos 2\epsilon_J & i e^{i(\delta^{J,1}_{J-1} + \delta^{J,1}_{J+1})} \sin 2\epsilon_J \\ i e^{i(\delta^{J,1}_{J-1} + \delta^{J,1}_{J+1})} \sin 2\epsilon_J & e^{2i\delta^{J,1}_{J+1}} \cos 2\epsilon_J \end{pmatrix}, \qquad (2)
$$

Analytical Structure

- $s = 4(M_N^2 + p^2) \to E_{\rm LAB} = 2p^2/M_N$
- Partial Wave Scattering Amplitude analytical for $|p| \le m_\pi/2$

$$T^J_{ll'}(p) \equiv S^J_{ll'}(p) - \delta_{l,l'} = p^{l+l'} \sum_n C_{n,l,l'} p^{2n}$$

- Nucleons behave as elementary (AT WHAT SCALE ?)



- Nucleons are heavy → Local Potentials

$$V_{n\pi}(r) \sim \frac{g^{2n}}{r} e^{-n m_\pi r}$$

Charge dependent One Pion Exchange

$$
\begin{aligned}
V_{\mathrm{OPE},pp}(r) &= f_{pp}^2 V_{m_{\pi^0},\mathrm{OPE}}(r), \\
V_{\mathrm{OPE},np}(r) &= -f_{nn} f_{pp} V_{m_{\pi^0},\mathrm{OPE}}(r) + (-)^{(T+1)} 2 f_c^2 V_{m_{\pi^\pm},\mathrm{OPE}}(r),
\end{aligned}
$$

where $V_{m,\mathrm{OPE}}$ is given by

$$
\begin{aligned}
V_{m,\mathrm{OPE}}(r) &= \left(\frac{m}{m_{\pi^\pm}}\right)^2 \frac{1}{3} m \left[Y_m(r) \sigma_1 \cdot \sigma_2 + T_m(r) S_{1,2} \right], \\
S_{1,2} &= 3 \sigma_1 \cdot \hat{r} \sigma_2 \cdot \hat{r} - \sigma_1 \cdot \sigma_2 \\
Y_m(r) &= \frac{e^{-mr}}{mr} \\
T_m(r) &= \frac{e^{-mr}}{mr} \left[1 + \frac{3}{mr} + \frac{3}{(mr)^2} \right]
\end{aligned}
$$

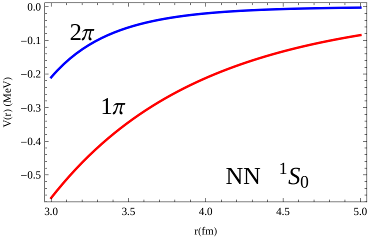
Small short range

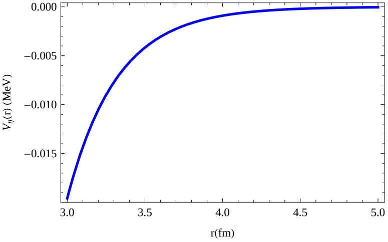
- OPE exchange

$$V_{1\pi}(r) = -f^2_{\pi NN} \frac{e^{-m_\eta r}}{r}$$

- TPE exchange
- η-exchange

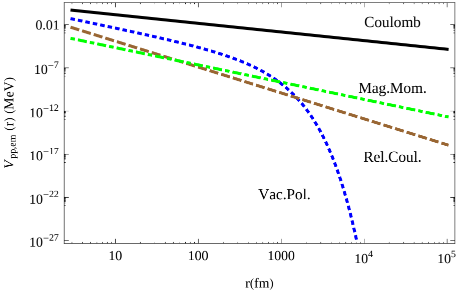
$$V_\eta(r) = -f^2_{\eta NN} \frac{e^{-m_\eta r}}{r}$$





Small but crucial long range

- Coulomb interaction (pp) e/r
- Magnetic moments $\sim \mu_p \mu_n / r^3, \mu_p \mu_p / r^3, \mu_n \mu_n / r^3$
  Lowered $\chi^2/\nu \sim 2 \to \chi^2/\nu \sim 2 \to 1$
  Summing 1000-2000 partial waves
- Vacuum polarization (Uehling potential,Lamb-shift)
- Relativistic corrections $1/r^2$



Effective Elementary

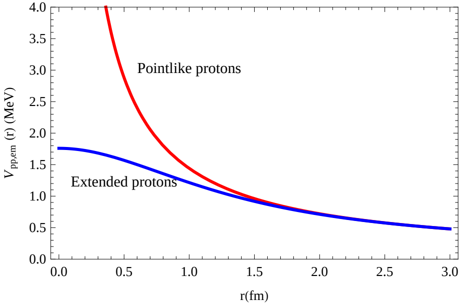
When are two protons interacting as point-like particles ?

- Electromagnetic Form factor

$$F_i(q) = \int d^3 r e^{iq\cdot r} \rho_i(r)$$

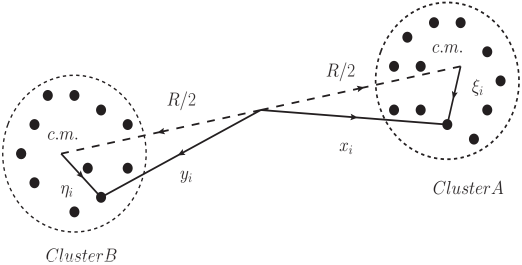
- Electrostatic interaction

$$V_{pp}^{\rm el}(r) = e^2 \int d^3 r_1 d^3 r_2 \frac{\rho_p(r_1)\rho_p(r_2)}{|\vec{r}_1 - \vec{r}_2 - \vec{r}|} \to \frac{e^2}{r} \qquad r > r_e \sim 2\text{fm}$$



Quark Cluster Dynamics (qcd)

- Atomic analogue. Neutral atoms
- Non-overlapping atoms exchange TWO photons (Van der Waals force)
- Overlapping atoms are not locally neutral; ONE photon exchange is possible (Chemical bonding)

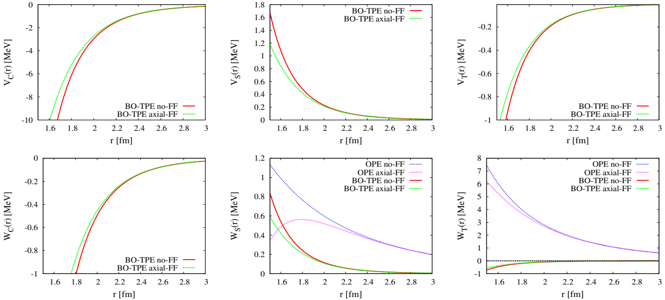


Finite size effects

- NN potential in the Born-Oppenheimer approximation Calle Cordon, RA, '12

$$\bar{V}^{1\pi+2\pi+\cdots}_{NN,NN}(\boldsymbol{r}) = V^{1\pi}_{NN,NN}(\boldsymbol{r}) + 2\,\frac{|V^{1\pi}_{NN,N\Delta}(\boldsymbol{r})|^2}{M_N - M_\Delta} + \frac{1}{2}\,\frac{|V^{1\pi}_{NN,\Delta\Delta}(\boldsymbol{r})|^2}{M_N - M_\Delta} + \mathcal{O}(V^3)\,,$$

- Bulk of TWO-Pion Exchange Chiral forces reproduced
- Finite size effects set in at $2\mathrm{fm}$ $\rightarrow$ exchange quark effects become explicit
- High quality potentials confirm these trends.



COARSE GRAINING

The number of parameters (for $E_{\rm LAB} \le 350$ MeV)

- At what distance look nucleons point-like ?

$$r > 2\text{fm}$$

- When is OPE the ONLY contribution ?

$$r_c > 3\text{fm}$$

- What is the minimal resolution where interaction is elastic ?

$$p_{\rm max} \sim \sqrt{M_N m_\pi} \to \Delta r = 1/p_{\rm max} = 0.6\text{fm}$$

- How many partial waves must be fitted ?

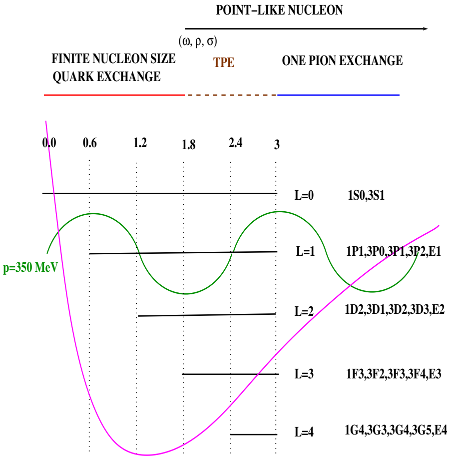
$$l_{\rm max} = p_{\rm max} r_c = r_c / \Delta r = 5$$

- Minimal distance where centrifugal barrier dominates

$$\frac{l(l+1)}{r_{\rm min}^2} \le p^2$$

- How many parameters ?
$({}^1S_0, {}^3S_1)$, $({}^1P_1, {}^3P_0, {}^3P_1, {}^3P_2)$, $({}^1D_2, {}^3D_1, {}^3D_2, {}^3D_3)$, $({}^1F_3, {}^3F_2, {}^3F_3, {}^3F_4)$

$$2 \times 5 + 4 \times 4 + 4 \times 3 + 4 \times 2 + 4 \times 1 = 50$$


POINT–LIKE NUCLEON
(ω, ρ, σ)
FINITE NUCLEON SIZE
QUARK EXCHANGE
TPE
ONE PION EXCHANGE
0.0
0.6
1.2
1.8
2.4
3
L=0
1S0,3S1
L=1
1P1,3P0,3P1,3P2,E1
p=350 MeV
L=2
1D2,3D1,3D2,3D3,E2
L=3
1F3,3F2,3F3,3F4,E3
L=4
1G4,3G3,3G4,3G5,E4

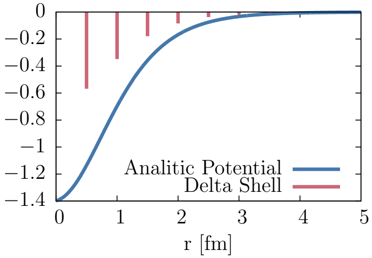
Delta Shell Potential

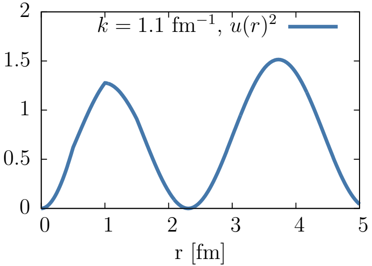
- A sum of delta functions

$$V(r) = \sum_i \frac{\lambda_i}{2\mu} \delta(r - r_i)$$

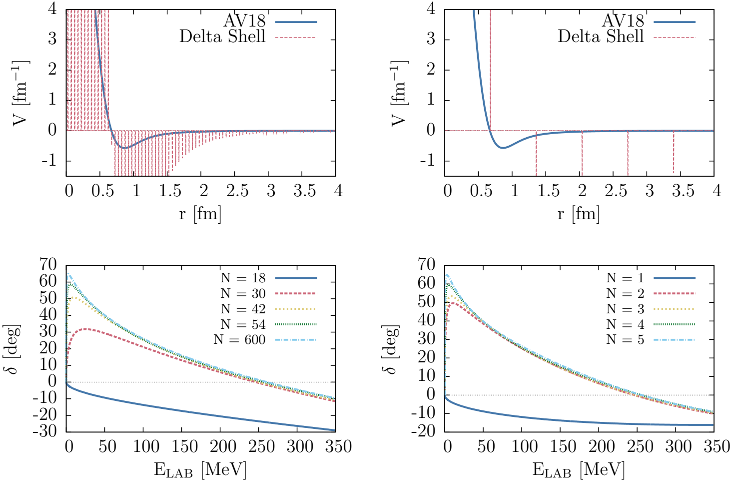
[Aviles, Phys.Rev. C6 (1972) 1467]

- Optimal and minimal sampling of the nuclear interaction
- Pion production threshold $\Delta k \sim 2$ fm^{-1}
- Optimal sampling, $\Delta r \sim 0.5$fm





Coarse Graining the AV18 potential



Delta Shell Potential

- 3 well defined regions
- Innermost region $r \leq 0.5$ fm
  - Short range interaction
  - No delta shell (No repulsive core)
- Intermediate region $0.5 \leq r \leq 3.0$ fm
  - Unknown interaction
  - λ_i parameters fitted to scattering data
- Outermost region $r \geq 3.0$ fm
  - Long range interaction
  - Described by OPE and EM effects
    - Coulomb interaction V_{C1} and relativistic correction V_{C2} (pp)
    - Vacuum polarization V_{VP} (pp)
    - Magnetic moment V_{MM} (pp and np)

STATISTICS

Self-consistent fits

- We test the assumption

$$O_i^{\mathrm{exp}} = O_i^{\mathrm{th}} + \xi_i \Delta O_i \qquad i = 1, \dots, N_{\mathrm{Data}} \qquad \xi_i \in N[0,1]$$

- Least squares minimization $\mathbf{p} = (p_1, \dots,)$

$$\chi^2(\mathbf{p}) = \sum_{i=1}^{N} \left(\frac{O_i^{\mathrm{exp}} - F_i(\mathbf{p})}{\Delta O_i^{\mathrm{exp}}} \right)^2 \to \min_{\lambda_i} \chi^2(\mathbf{p}\chi^2(\mathbf{p}_0) \tag{3}$$

- Are residuals Gaussian ?

$$R_i = \frac{O_i^{\mathrm{exp}} - O_i^{\mathrm{th}}}{\Delta O_i} \qquad O_i^{\mathrm{th}} = F_i(\mathbf{p}_0) \qquad i = 1, \dots, N \tag{4}$$

  If $R_i \in N[0,1]$ self-consistent fit.

- Normality test for a finite sample with N elements → Probability (Confidence level) p-value

$$\chi^2_{\mathrm{min}} = 1 \pm \sigma \sqrt{\frac{2}{\nu}} \qquad \nu = N_{\mathrm{Dat}} - N_{\mathrm{Par}} \qquad p = 1 - \int_\sigma^\sigma dt \frac{e^{-t^2}}{\sqrt{2\pi}}$$

  Histograms, Moments, Kolmogorov-Smirnov, **Tail Sentitive** QQ-plots

Normality tests

- Does the sequence

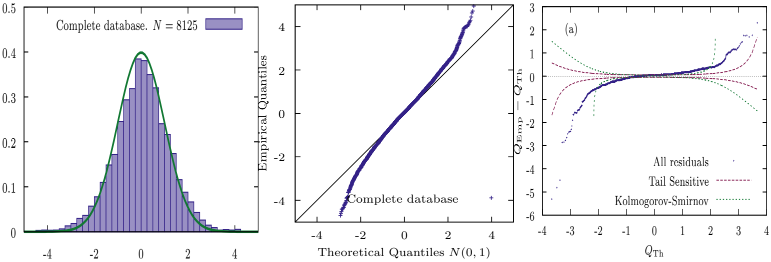
$$x_1^{\rm exp} \le x_2^{\rm exp} \le \cdots \le x_N^{\rm exp} \in N[0,1]$$

- We compute the theoretical points

$$\frac{n}{N+1} = \int_{-\infty}^{x_n^{\rm th}} dt \frac{e^{-t^2/2}}{\sqrt{2\pi}}$$

- The Q-Q plot is $x_n^{\rm th}$ vs $x_n^{\rm exp}$
- For large N

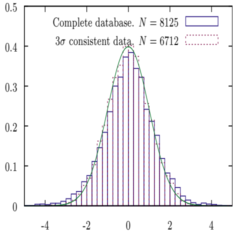
$$x_n^{\rm th} - x_n^{\rm exp} = \mathcal{O}(1/\sqrt{N})$$

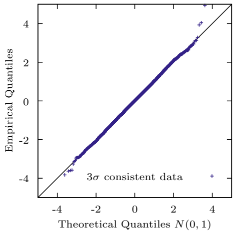


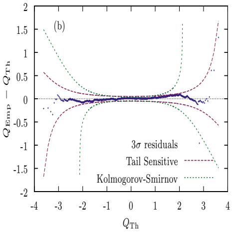
Granada-2013 np+pp database

Selection criterium

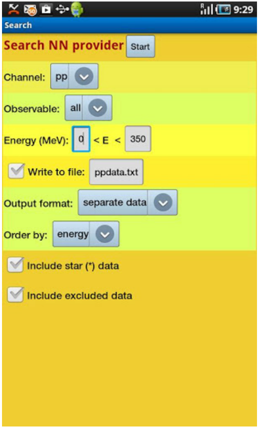
- Mutually incompatible data. Which experiment is correct? Is any of the two correct?
- Maximization of experimental consensus
- Exclude data sets inconsistent with the rest of the database
  1. Fit to all data ($\chi^2/\nu > 1$)
  2. Remove data sets with improbably high or low χ^2 (3σ criterion)
  3. Refit parameters
  4. Re-apply 3σ criterion to all data
  5. Repeat until no more data is excluded or recovered







Fitting NN observables



- Database of NN scattering data obtained till 2013
  - http://nn-online.org/
  - http://gwdac.phys.gwu.edu/
  - NN provider for Android
    - Google Play Store

[J.E. Amaro, R. Navarro-Perez, and E. Ruiz-Arriola]

- 2868 pp data and 4991 np data
- 3σ criterion by Nijmegen to remove possible outliers

Fitting NN observables

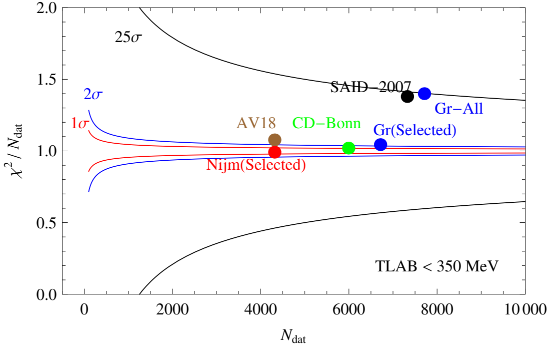
- Delta shell potential in every partial wave

$$V_{l,l'}^{JS}(r) = \frac{1}{2\mu_{\alpha\beta}} \sum_{n=1}^{N} (\lambda_n)_{l,l'}^{JS} \delta(r - r_n) \qquad r \leq r_c = 3.0\text{fm}$$

- Strength coefficients λ_n as fit parameters
- Fixed and equidistant concentration radii $\Delta r = 0.6$ fm
- EM interaction is crucial for pp scattering amplitude

$$\begin{aligned} V_{C1}(r) &= \frac{\alpha'}{r} , \\ V_{C2}(r) &\approx -\frac{\alpha\alpha'}{M_p r^2} , \\ V_{VP}(r) &= \frac{2\alpha\alpha'}{3\pi r} \int_1^\infty dx \, e^{-2m_e r x} \left[1 + \frac{1}{2x^2}\right] \frac{(x^2-1)^{1/2}}{x^2} , \\ V_{MM}(r) &= -\frac{\alpha}{4M_p^2 r^3} \left[\mu_p^2 S_{ij} + 2(4\mu_p - 1)\mathbf{L}\cdot\mathbf{S}\right] \end{aligned}$$

To believe or not to believe



$$\chi^2_{\rm min}/\nu = 1 \pm \sqrt{2/\nu}$$

A=2 STATISTICAL UNCERTAINTIES

To fit or not to fit

- N- Experimental Data $O_i^{\rm exp} \pm \Delta O_i$ with $i = 1, \dots, N$
- Theory with P-parameters $O_i^{\rm th} = O_i(p_1, \dots, p_P)$
- Distance between theory and data

$$||O^{\rm th}(p) - O^{\rm exp}||^2 = \sum_i \left[\frac{O_i^{\rm th} - O_i^{\rm exp}}{\Delta O_i} \right]^2$$

- Closest theory to data (Least Squares Method, Lagrange)

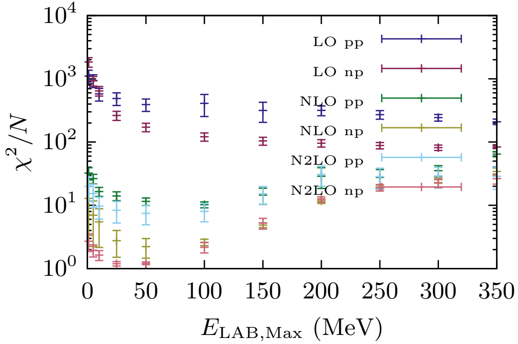
$$D^2_{\rm min} = {\rm min}_p ||O^{\rm th}(p) - O^{\rm exp}||^2$$

- Statistical interpretation: Maximum likelihood
- Propose fits with natural parameters
- Reject fits with bound states other than the deuteron
- Check fit residuals *a posteriori*

$$\chi^2_{\rm min}/\nu = 1 \pm \sqrt{2/\nu}$$

Fit Phases or fit data ?

- Phase shifts are NOT experimental observables UNLESS we have a complete set of 10 measurements (cross sections and polarization asymmetries) for a given energy.
- Data is difficult since magnetic moments are $1/r^3$ difficult 1000-2000 partial waves in momentum space approaches
- Fitting phases is easier but

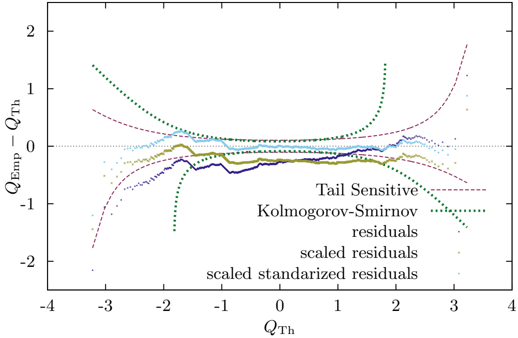


Local chiral effective field theory interactions and quantum Monte Carlo applications A. Gezerlis, I. Tews, E. Epelbaum, M. Freunek, S. Gandolfi, K. Hebeler, A. Nogga and A. Schwenk, Phys. Rev. C **90** (2014) no.5, 054323

Are residuals irrelevant ?

- Even if you have a “good” fit , it can be inconsistent
- Fit N2LO for $T_{\rm LAB} 125 {\rm MeV}$

  Optimized Chiral Nucleon-Nucleon Interaction at Next-to-Next-to-Leading Order; A. Ekström, G. Baardsen, C. Forssén, G. Hagen, M. Hjorth-Jensen, G.R. Jansen, R. Machleidt, W. Nazarewicz, T. Papenbrock, J. Sarich; Phys.Rev.Lett. 110 (2013) no.19, 192502



- Gross Systematic errors !!

Frequentist or Bayes ?

- Frequentist: What is the probability that given a theory data are correct ?
- Bayesian: What is the probability that given the data the theory is correct ? (priors, random parameters)

$$\chi^2_{\mathrm{augmented}} \to \chi^2_{\mathrm{data}} + \chi^2_{\mathrm{parameters}}$$

- Both approaches agree for $N_{\mathrm{Data}} \gg N_{\mathrm{Parameters}}$. We have $N_{\mathrm{Dat}} \sim 7000$ and $N_{\mathrm{Par}} = 40$

Maximal energy vs shortest distance

- The full potential is separated into two pieces

$$V(r) = V_{\rm short}(r)\theta(r_c - r) + V^{\chi}_{\rm long}(r)\theta(r - r_c)$$

- Data are fitted up to a maximal $T_{\rm LAB}$

$$T_{\rm LAB} \le \max T_{\rm LAB} \leftrightarrow p_{\rm CM} \le \Lambda$$

Max $T_{\rm LAB}$ MeV	r_c fm	c_1 GeV^{-1}	c_3 GeV^{-1}	c_4 GeV^{-1}	Highest counterterm	χ^2/ν
350	1.8	-0.4(11)	-4.7(6)	4.3(2)	F	1.08
350	1.2	-9.8(2)	0.3(1)	2.84(5)	F	1.26
125	1.8	-0.3(29)	-5.8(16)	4.2(7)	D	1.03
125	1.2	-0.92	-3.89	4.31	P	1.70
125	1.2	-14.9(6)	2.7(2)	3.51(9)	P	1.05

- D-waves, forbidden by Weinberg counting, are indispensable !
- Data and χ-N2LO do not support $r_c < 1.8{\rm fm}$!
  (Several χ-potentials take $r_c = 0.9 - 1.1{\rm fm}$)

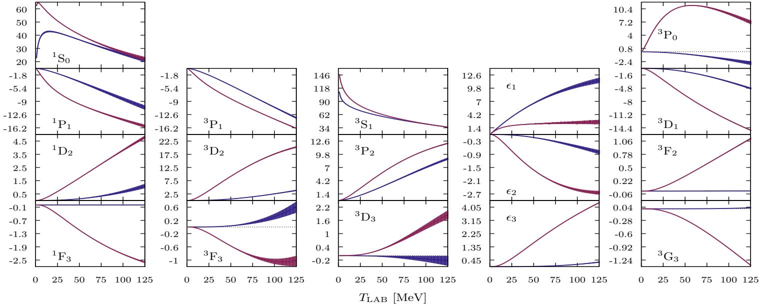
Deconstructing chiral forces

- The full potential is separated into two pieces

$$V(r) = V_{\rm short}(r)\theta(r_c - r) + V^{\chi}_{\rm long}(r)\theta(r - r_c)$$

- Under what conditions are the short distance phases compatible with zero

$$|\delta_{\rm short}| \le \Delta\delta \qquad r_c = 1.8 {\rm fm}$$



- This is equivalent to set counterterms in given partial waves directly to cero

$$\delta_{\rm short} = 0 \leftrightarrow C_{\rm short} = 0$$

To count or not to count

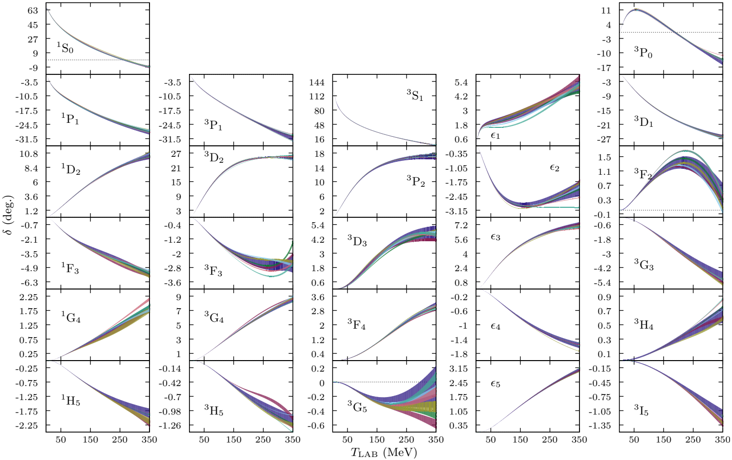
- We can fit CHIRAL forces to ANY energy and look if counterterms are compatible with zero within errors
- We find that if $E_{\rm LAB} \le 125{\rm MeV}$ Weinberg counting is INCOMPATIBLE with data.
- You have to promote D-wave counterterms.
  N2LO-Chiral TPE + N3LO-Counterterms → Residuals are normal
  Piarulli,Girlanda,Schiavilla,Navarro Pérez,Amaro,RA, PRC
- We find that if $E_{\rm LAB} \le 40{\rm MeV}$ TPE is INVISIBLE
- We find that peripheral waves predicted by 5th-order chiral perturbation theory ARE NOT consistent with data within uncertainties

$$|\delta^{\rm Ch,N4LO} - \delta^{\rm PWA}| > \Delta\delta^{\rm PWA,stat}$$

Dominant contributions to the nucleon-nucleon interaction at sixth order of chiral perturbation theory D.R. Entem, N. Kaiser, R. Machleidt, Y. Nosyk Phys.Rev. C92 (2015) no.6, 064001

Statistical vs Systematics

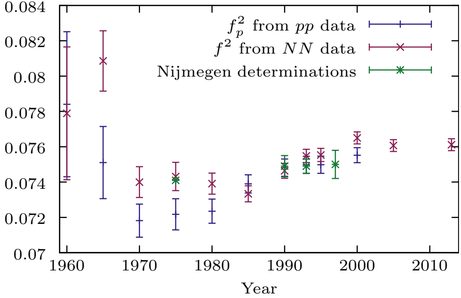
Statistically equivalent interactions with $\chi^2_{\min}/\nu = 1 \pm \sqrt{2/\nu}$ DO NOT overlapp



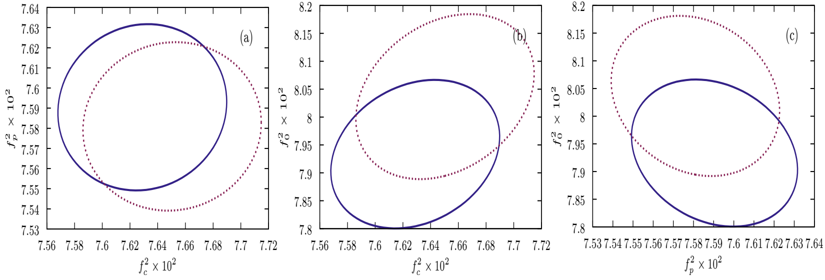
COUPLING CONSTANTS

Arqueological Flashback

Chronological recreation of pion-nucleon coupling constants



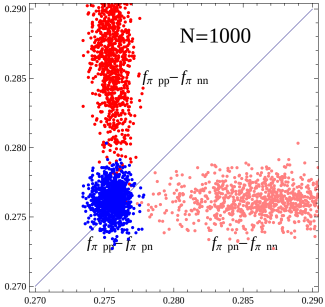
The pion-nucleon coupling constants f_p^2, f_0^2 and f_c^2

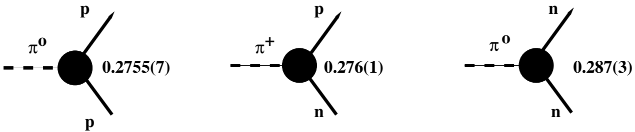


Fits to the Granada-2013 database.

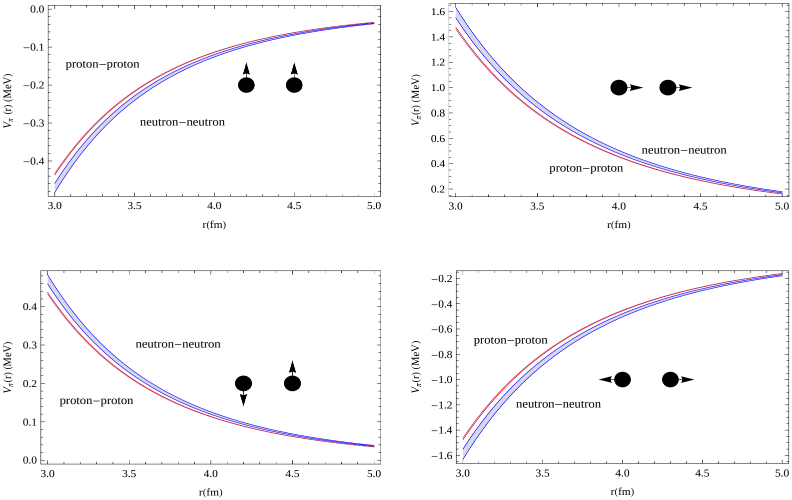
f^2	f_0^2	f_c^2	CD-waves	χ^2_{pp}	χ^2_{np}	$N_{\rm Dat}$	$N_{\rm Par}$	χ^2/ν
0.075	idem	idem	$^1S_0$	3051	3951	6713	46	1.051
0.0761(3)	idem	idem	$^1S_0$	3051	3951	6713	46+1	1.051
-	-	-	$^1S_0, P$	2999	3951.40	6713	46+3	1.043
0.0759(4)	0.079(1)	0.0763(6)	$^1S_0, P$	3045	3870	6713	46+3+9	1.039

The $\pi N N$ vertices



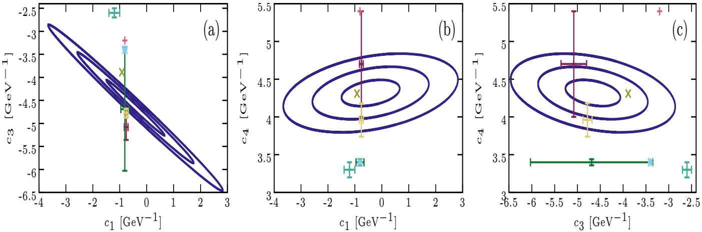


Neutrons interact more strongly than protons

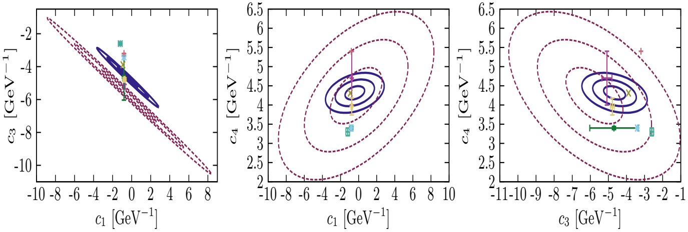


Chiral Two Pion Exchange

Fit to from Granada-2013 np+pp database $T_{\mathrm{LAB}} < 350\mathrm{MeV}$



Fit to from Granada-2013 np+pp database $T_{\mathrm{LAB}} < 125\mathrm{MeV}$



CONCLUSIONS

Conclusions

- Chiral nuclear forces have MANY advantages
- Weinberg's paper has over 1000 citations
- Many calculations have been undertaken using 2,3,4 body forces
  BUT:
- Many forms of Chiral nuclear forces can be falsified
- Only validated forces against data (and not phases) allow to make serious error analysis
- One should be able to make REALISTIC error estimates not just conservative ones
- Cut-off dependence is the largest source of error

We must not count our chickens before they are hatched !