QCD inspired determination of NJL-model parameters

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Motivation

The detailed structure of the **QCD phase diagram** at high chemical potential is still under investigation. **Low-energy models** provide us with some guidance **but**:

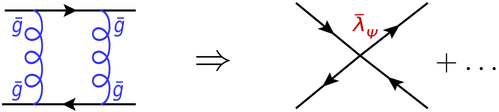
- parameters of the models are fine-tuned (scale Λ_{NJL} and couplings at Λ_{NJL})
- parameters do not depend on T and μ
- typically not all possible interaction channels are included (not Fierz complete)

$\Rightarrow$ **Models require improvement from the point of view of the underlying fundamental theory**

QCD

QCD (chiral limit):

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \bar{g}A\!\!\!/ + \mathrm{i}\gamma_0\mu)\psi + \frac{Z_A}{4}F_z^{\mu\nu}F^z_{\mu\nu}$$



- Quark self-interactions $\bar{\lambda}_\psi$ are induced by quark-gluon interactions
- **Strong** $\bar{\lambda}_\psi$ triggers **chiral symmetry breaking**

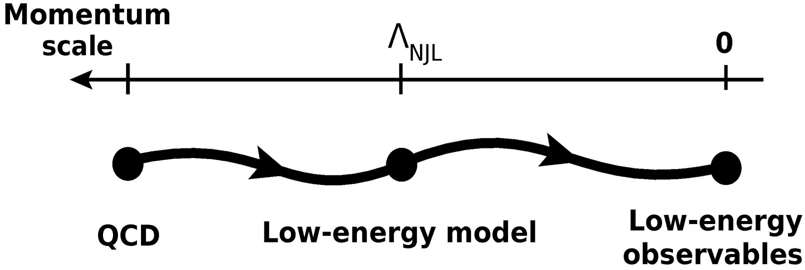
e.g. [H. Gies, J. Jaeckel, 2006], [J. Braun, H. Gies, 2006]

- Consider only 4-fermion interactions

  $\Rightarrow$ Fierz-complete ansatz can be projected onto the channels used in NJL-model

The Basic Idea

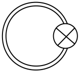
Integrating out quantum fluctuations



Method

Wetterich Flow Equation: IR-regulated action

$$k\partial_k \Gamma_k[\Phi_k] = \frac{1}{2}\,\mathrm{Tr}\, k\partial_k R_k \left(\Gamma_k^{(2)}[\Phi_k] + R_k\right)^{-1} =$$

$$= \frac{1}{2}$$ 

[C. Wetterich, 1993]

Γ_k: scale-dependent effective action

$$\lim_{k\to\Lambda} \Gamma_k[\phi_k] = S[\phi]$$

R_k: IR cut-off function

$$\lim_{k\to 0} \Gamma_k[\phi_k] = \Gamma[\phi]$$

RG flows in QCD

Ansatz:

$$\Gamma_k = \int_x \bar{\psi}(i\partial\!\!\!/ + \bar{g} A\!\!\!/ + i\gamma_0\mu)\psi + \frac{Z_A}{4} F_z^{\mu\nu} F_{\mu\nu}^z + \frac{(\partial_\mu A^\mu)^2}{2\xi}$$
$$+ \frac{1}{2}\Big[\bar{\lambda}_-(\mathrm{V}-\mathrm{A}) + \bar{\lambda}_+(\mathrm{V}+\mathrm{A}) + \bar{\lambda}_\sigma(\mathrm{S}-\mathrm{P})$$
$$+ \bar{\lambda}_{\mathrm{VA}}[2(\mathrm{V}-\mathrm{A})^{\mathrm{adj}} + 1/\mathrm{N_C}(\mathrm{V}-\mathrm{A})]\Big] ,$$

[H. Gies, J. Jaeckel, 2006], [J. Braun, H. Gies, 2006]

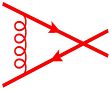
with

- $(\mathrm{V}-\mathrm{A}) = (\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5\psi)^2$
- $(\mathrm{V}+\mathrm{A}) = (\bar{\psi}\gamma_\mu\psi)^2 - (\bar{\psi}\gamma_\mu\gamma_5\psi)^2$
- $(\mathrm{S}-\mathrm{P}) = (\bar{\psi}^a\psi^b)(\bar{\psi}^b\psi^a) - (\bar{\psi}^a\gamma_5\psi^b)(\bar{\psi}^b\gamma_5\psi^a)$,
  with $(a, b, \ldots)$ flavor indices
- $(\mathrm{V}-\mathrm{A})^{\mathrm{adj}} = (\bar{\psi}\gamma_\mu T^z\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5 T^z\psi)^2$,
  $(T^z)_{ij}$ denotes $\mathrm{SU(N_C)}$ generators and $(i, j, \ldots)$ color indices
- Landau gauge
- IR cut-off function: $3d$ linear regulator [Litim, Pawlowski 2006]

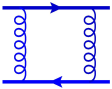
RG flows in QCD

4-fermion couplings $\bar{\lambda}_i$:

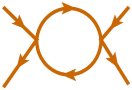
- Four highly coupled differential equations for $\bar{\lambda}_i$ where all of them include contributions proportional to:



,

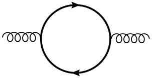


and



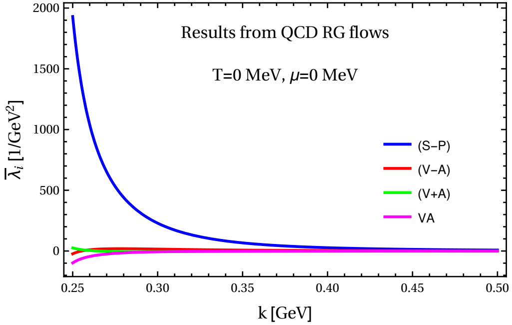
- Boundary condition: $\lim_{k \to \Lambda_{\mathrm{UV,QCD}} \to \infty} \lambda_i = 0$

Strong coupling $g(k)^2$:

- Background-field method: $k\partial_k g(k)^2 = \eta_{A,k} g(k)^2$ [L. Abbott, 1980]
- $\eta_{A,k}(T,\mu) = \eta^{YM}_{A,k}(T) + \Delta\eta_{A,k}(T,\mu)$
  e.g. [J. Braun, 2009], [J. Braun, L. Fister, J. Pawlowski, F. Rennecke, 2014]
- $\eta^{YM}_{A,k}(T)$ is taken from FRG studies [J. Braun, H. Gies, 2006]
- Quark contribution in leading order: $\Delta\eta_{A,k} \sim$ 
- $\alpha = g^2/(4\pi) = 0.163$ at 20 GeV (perturbative limit) e.g. [S. Bethke, 2002]

Defining the low-energy model

- Idea: **Project Fierz-complete ansatz** for the 4-quark interactions from QCD at a particular scale $k = \Lambda_{\mathrm{NJL}}$ **onto** the 4-quark interaction **channels used in conventional NJL/QM-type model** studies
- UV cutoff Λ_{NJL} of NJL model defines IR cutoff for our QCD RG flows



$\Rightarrow$ **Scalar-pseudoscalar channel is dominant**

similar findings in [J. Braun, Diss., 2009], [M. Mitter, J. Pawlowski, N. Strodthoff, 2014]

Defining the low-energy model

- Only **conventional scalar-pseudoscalar channel** in the NJL model:

$$\Gamma_k = \int_x \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \mathrm{i}\gamma_0\mu)\psi + \frac{\bar{\lambda}_{\sigma,\mathrm{NJL}}}{2}[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2] \; .$$

- We work with the partially bosonized version of the NJL model:

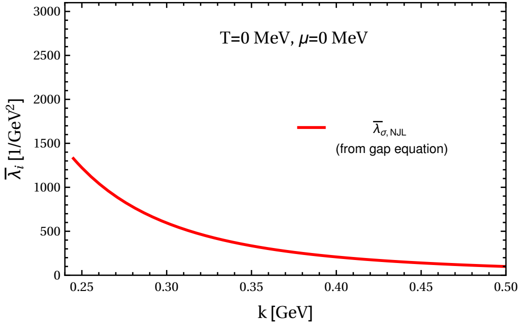
$$\Gamma_k = \int_x \{\bar{\psi}(\mathrm{i}\partial\!\!\!/ + \mathrm{i}\gamma_0\mu)\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2$$
$$+ \mathrm{i}\bar{h}\bar{\psi}(\sigma + \mathrm{i}\vec{\tau}\cdot\vec{\pi}\gamma_5)\psi + \frac{1}{2}\bar{m}^2(\sigma^2 + \vec{\pi}^{\,2})\} \; ,$$

with $\lim_{k\to\Lambda_{\mathrm{NJL}}} \frac{\bar{h}^2}{\bar{m}^2} = \bar{\lambda}_{\sigma,\mathrm{NJL}}(\Lambda_{\mathrm{NJL}})$

- Calculations on the **mean field** level
- In the chiral limit there are two parameters: UV cutoff Λ_{NJL} and $\bar{\lambda}_{\sigma,\mathrm{NJL}}(\Lambda_{\mathrm{NJL}})$

Defining the low-energy model

- Fix Λ_{NJL} such that the low-energy model reproduces the correct results for the constituent quark mass, $M_q = 300$ MeV, and the pion decay constant, $f_\pi = 87$ MeV, in the chiral limit



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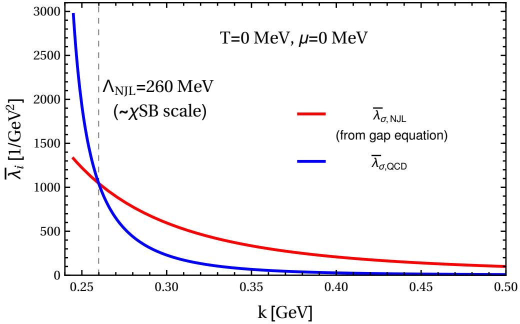
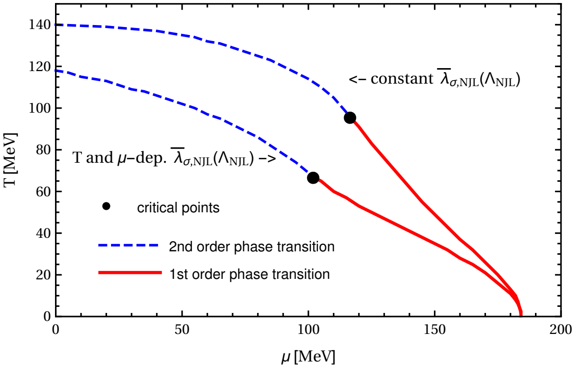


- Keep $\boxed{\Lambda_{\text{NJL}} = 260 \text{ MeV}}$ fixed for finite T- and μ-studies

$\Rightarrow \bar{\lambda}_{\sigma,\text{NJL}}(\Lambda_{\text{NJL}})$ is now a **function of T and μ**
(as predicted from QCD RG flows)

Results



- T-dependent $\bar{\lambda}_{\sigma,\mathrm{NJL}}(\Lambda_{\mathrm{NJL}})$ lowers T_c
- T- and μ-dependent $\bar{\lambda}_{\sigma,\mathrm{NJL}}(\Lambda_{\mathrm{NJL}})$ shifts the position of the critical point (T_c, μ_c) to smaller values
- Too small critical temperatures because of too small Λ_{NJL}

Conclusion and Outlook

Conclusion:

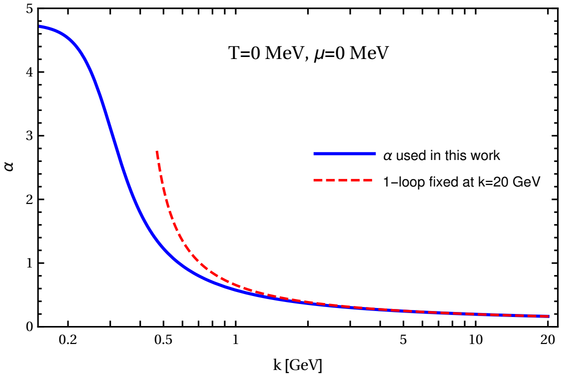
- Improvement of low-energy QCD model from QCD RG-flows
- Current UV cutoff Λ_{NJL} is smaller than in conventional studies
- NJL parameters as function of T and μ
- T- and μ-dependent model parameters impact the phase diagram (lower T_c and shift the position of the critical point to smaller values)

Outlook:

- Include vector channel in low-energy model
- Beyond mean-field calculations for the low-energy model
- Improved RG flows for α, crosscheck against lattice QCD and DSE

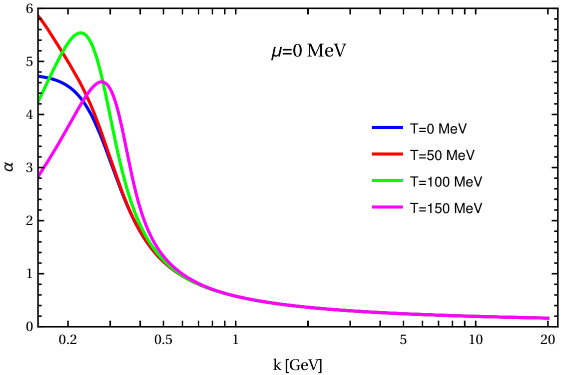
Thank you for your attention!

Backup: Strong coupling



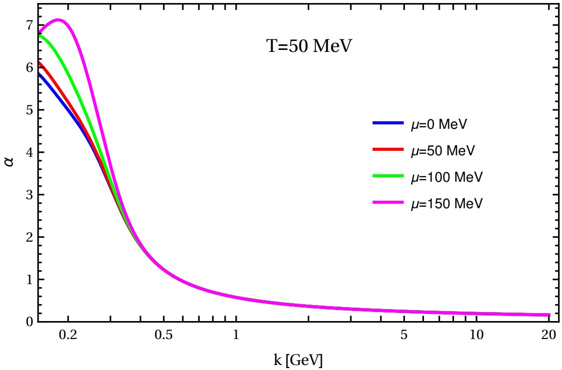
- Perfect agreement with perturbative result in UV regime
- In IR regime: α grows and approaches a fixed point

Backup: Strong coupling



- $T > 0$: quark degrees of freedom have only hard Matsubara modes $\Rightarrow$ suppression of quark contributions $\Rightarrow$ larger α
- For $p^2 < T^2$, wavelength of fluctuations is larger than extent of compactified Euclidean time direction $\Rightarrow$ dimensional reduction of the theory: $\alpha_{4d} = (k/T)\alpha_{3d}$ [J. Braun, H. Gies, 2006]

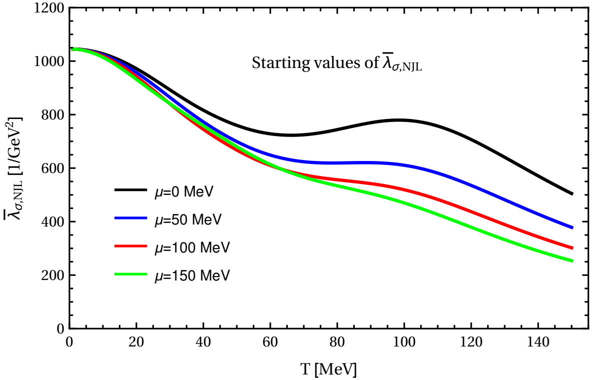
Backup: Strong coupling



- $\mu > 0$: further suppression of quark contributions

Backup: Starting value $\bar{\lambda}_{\sigma,\mathrm{NJL}}(\Lambda_{\mathrm{NJL}})$

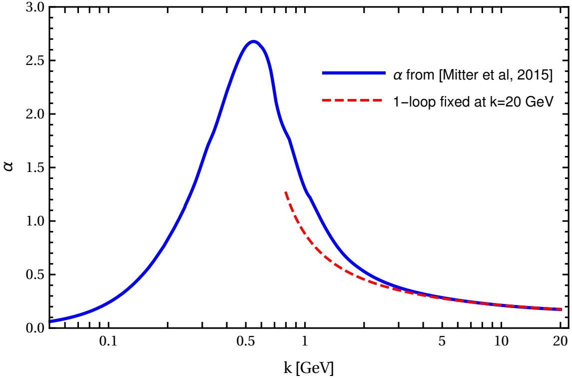
- $\bar{\lambda}_{\sigma,\mathrm{NJL}}(\Lambda_{\mathrm{NJL}})$ is now a function of T and μ



- $\bar{\lambda}_{\sigma,\mathrm{NJL}}$ becomes generally smaller with increasing T: it reflects the restoration of chiral symmetry in QCD
- Non-monotonous behavior for small μ: thermal decoupling of quarks

Backup: An alternative input for the strong coupling

- α from FRG in Landau gauge [Mitter et al, 2015]
- Modified Ward-Takahashi identities insure the gauge invariance.



Disadvantages:

- α only for $T = 0$, $\mu = 0$
- No data for $\eta_{A,k}^{\text{gluons}}$ which enters our threshold functions

Nonetheless, consider two approximations:

- $\eta_{A,k}^{\text{gluons}} \approx 0$
- $\eta_{A,k}^{\text{gluons}} \approx (\partial_t \alpha)/\alpha$

Backup: An alternative input for the strong coupling

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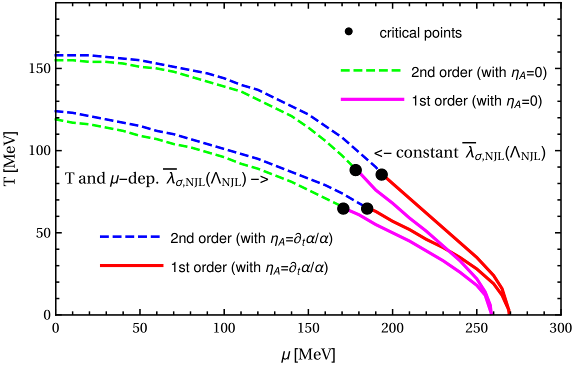
- α only for $T = 0$, $\mu = 0$
- No data for $\eta_{A,k}^{\text{gluons}}$ which enters our threshold functions

Nonetheless, consider two approximations:

- $\eta_{A,k}^{\text{gluons}} \approx 0 \quad \rightarrow \quad \Lambda_{\text{NJL}} = 502$ MeV!
- $\eta_{A,k}^{\text{gluons}} \approx (\partial_t \alpha)/\alpha \quad \rightarrow \quad \Lambda_{\text{NJL}} = 557$ MeV!

Backup: Results



- Results for constant $\bar{\lambda}_{\sigma,\mathrm{NJL}}(\Lambda_{\mathrm{NJL}})$ are in quantitative agreement with typical model results.
- Also here, T-dependent $\bar{\lambda}_{\sigma,\mathrm{NJL}}(\Lambda_{\mathrm{NJL}})$ lowers T_c