Confinement and Chiral Symmetry Breaking from an ensemble of interacting Instanton-dyons(monopoles) in SU(2) QCD

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R. Larsen and E. Shuryak:
arXiv:1511.02237 [PhysRevD] , arXiv:1504.03341 [PhysRevD]

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Overview

- Motivation
  - Understand the mechanism behind **Confinement** and **Chiral Symmetry breaking** in SU(2) using instanton-dyons.
- Tool
  - Minimize free energy density in a gas of interacting dyons
  - Find **Polyakov loop** (Confinement) and **chiral condensate** (Chiral symmetry breaking)
- Introduction
  - Properties of dyons and 2-point interactions
- Free energy density f
  - Results for pure gauge
- Add Fermions
  - Obtain chiral condensate from eigenvalue distribution
  - Results for $N_f = 2$

Instanton-Dyons

- Dyons appear for non-zero expectation value of A_4 field in a color direction

$$< A_4^3 > \equiv 2\pi T\nu, \qquad holonomy \;\; \nu \tag{1}$$
$$Polyakov \;\; loop \;\; P = \cos(\pi\nu) \tag{2}$$

- Lee, Lu[hep-th/9802108] and Kraan, van Baal [arXiv:hep-th/9806034]
- Dyons are topological **solutions to Equations of Motion**
- N_c **dyons** make up one Caloron (finite temperature Instanton)
- $N_c = 2$: Two dyons called M and L dyons

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- $N_c = 2$: Two dyons called M and L dyons
- Confined phase is $P = 0$ and $\nu = 0.5$
- Deconfined phase is $P = 1$ and $\nu = 0$
- Topological charge is ν for M and $\bar{\nu} = 1 - \nu$ for L dyons

	M	$\bar{M}$	L	$\bar{L}$
Sg^2	$8\pi^2\nu$	$8\pi^2\nu$	$8\pi^2(1-\nu)$	$8\pi^2(1-\nu)$
e	1	1	-1	-1
m	1	-1	-1	1

Classical interaction

- Coulomb like interaction, with opposite sign on electric charges

$$\begin{aligned} \Delta S &= \frac{8\pi^2\nu}{g^2}\left(-e_1 e_2 \frac{1}{x} + m_1 m_2 \frac{1}{x}\right) \\ x &= 2\pi\nu r T \end{aligned} \tag{3}$$

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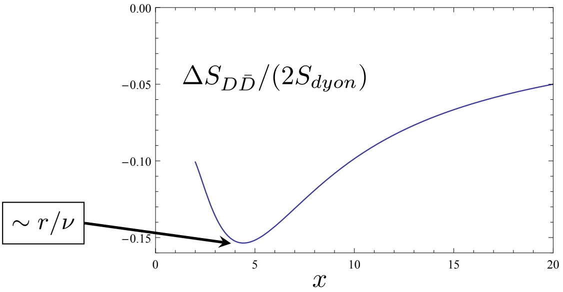
- r is distance between dyons
- Attractive case obtained from gradient flow



- Lack of states within $x < 2$ simulated as a hard core

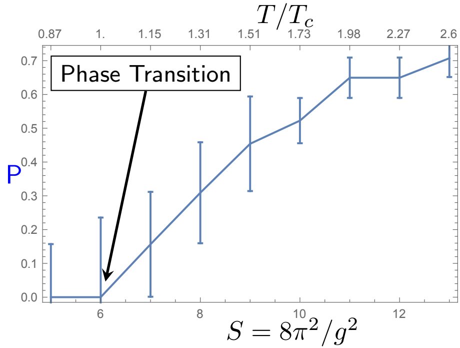
Free Energy Density

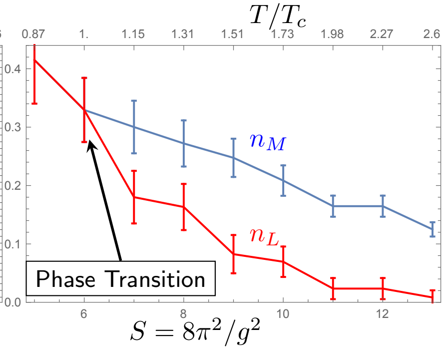
- At volume $V \to \infty$, the dominating configuration is the parameters that minimizes free energy density

$$f = \frac{4\pi^2}{3}\nu^2\bar{\nu}^2 - 2n_M \ln\left[\frac{d_\nu e}{n_M}\right] - 2n_L \ln\left[\frac{d_{\bar{\nu}} e}{n_L}\right] + \Delta f_{Interactions} \tag{4}$$

- Free energy density contains 3 items
  - The **GPY potential** that prefer **trivial** Holonomy
  - The **entropy** due to the dyons moving around
  - ($\Delta f_{Interactions}$) **Corrections** to the energy due to the interactions of the dyons
- GPY potential [Gross, Pisarski, Yaffe, Rev. Mod. Phys. 53, 43]
- d_ν [Diakonov, Gromov, Petrov, Slizovskiy: arXiv:hep-th/0404042]

- The Polyakov loop P (left) and **density** of M and L dyons (right) as a function of action/temperature



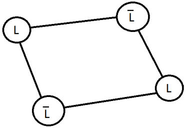


Dyons ensemble gives Confinement-Deconfinement phase transition.

Fermions

- L dyons have fermionic **zero modes** ($\not{D}\psi = 0$) for anti-periodic fermions
- The determinant of the Dirac operator = closed loops of hopping over L's [Shuryak, Verbaarschot:Nucl.Phys. B341 (1990) 1-26]

$$Det \begin{vmatrix} 0 & T_{ij} \\ T_{ji} & 0 \end{vmatrix} = \sum_{\text{All combinations}}$$



- Shape of T_{ij} is taken from overlap of fermionic zero-modes

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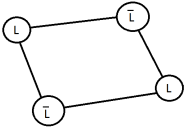

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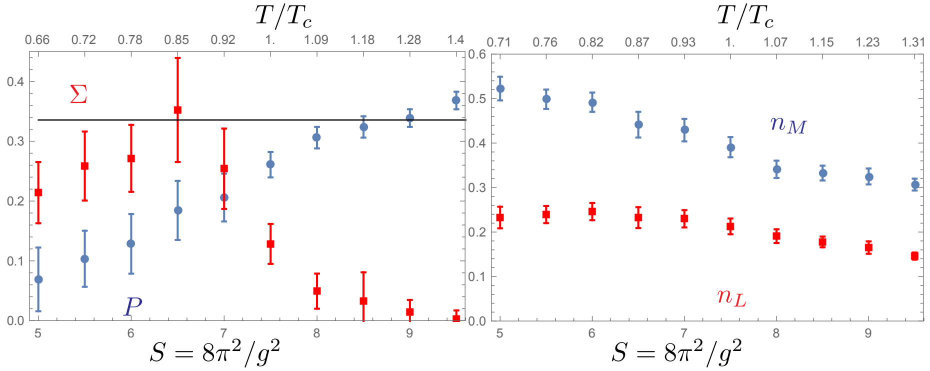
- The Banks-Casher relation for the chiral condensate tells us that

$$| < \bar{\psi}\psi > | \quad = \quad \pi\rho(\lambda)_{\lambda\to 0, m\to 0, V\to\infty} \tag{6}$$

- For finite volume we need to look at eigenvalue distribution around 0
- Eigenvalue distribution fitted to Random matrix theory[arXiv:0910.4134]

Polyakov Loop and Chiral Condensate for $N_f = 2$

- Polyakov loop P and Chiral condensate Σ (left) and densities for M and L dyons (right)
- The drop in the Polyakov loop increases the **effective density** of L dyons, creating a chiral condensate



- Dyons can therefore give **Confinement** and a non-zero **Chiral condensate**.

Summary

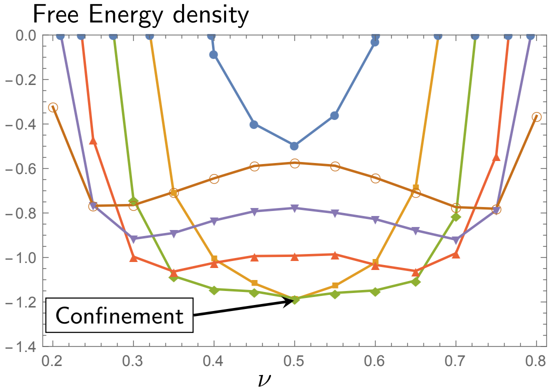
- Pure gauge: Phase transition to zero Polyakov loop
- Add 2 fermions: Non-zero Chiral condensate appear as the Polyakov loop approaches zero
- Transitions driven by density of instanton-dyons
- Chiral condensate dependent on Polyakov loop

- Outlook
  - Expand the ideas to $SU(3)$ QCD
  - Correlation functions for mesons

- New paper on Z_2 symmetric fermions [arXiv:1605.07474]

Finding the dominating Configuration

- We minimize free energy in the following parameters:
  - Density of M dyons n_M and L dyons n_L
  - Holonomy ν
  - Screening mass describing the fall off of the fields



- $\bullet$ $n = 0.53$, $\blacksquare$ $n = 0.37$, $\blacklozenge$ $n = 0.27$, $\blacktriangle$ $n = 0.20$, $\blacktriangledown$ $n = 0.15$, $\circ$ $n = 0.12$