The Dark Side of the Propagators:
exploring their analytic properties by a massive expansion

Fabio Siringo

Department of Physics and Astronomy
University of Catania, Italy

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EUCLIDEAN
SIDE
MINKOWSKI
SIDE

How Far from a Fully Analytical QCD in the IR?

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Optimized Perturbation Theory (OPT) [Stevenson 1985, 2013]
Change the expansion point $\rightarrow$ Massive Expansion
Can we evaluate everything from first principles ?

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Not self-consistent yet

Regularization Scheme dependence $\rightarrow$ Optim. by Lattice
From first principles
We can analytically continue to Minkowski space!

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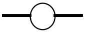
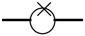
 $+$  $=$ IR finite

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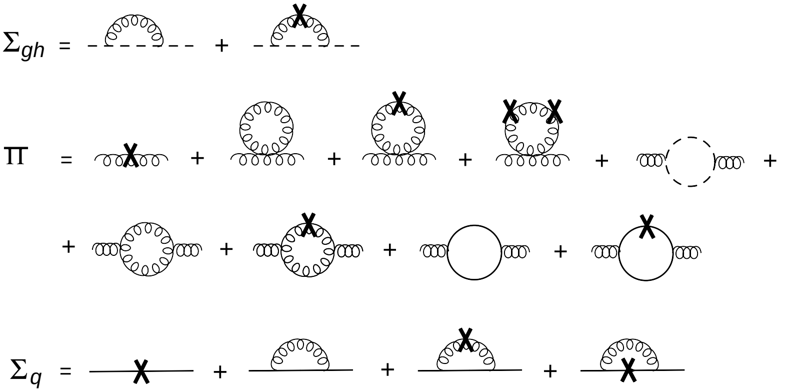
- Standard UV behavior

The original Lagrangian is not modified (first principles)

One-Loop third-order double expansion
(Landau Gauge)

Yang-Mills → F.S., Nucl. Phys. B **907** 572 (2016)
QCD → F.S., arXiv:1607.02040



UNIVERSAL SCALING

Ignoring RG effects, $\alpha \sim N\alpha_s$

$$\Sigma(p) = \alpha\Sigma^{(1)}(p) + \alpha^2\Sigma^{(2)}(p,N) + \cdots$$

$$\Sigma^{(1)} = -p^2\,F(p^2/m^2); \qquad \frac{\Sigma(p)}{\alpha p^2} = -F(p^2/m^2) + \mathcal{O}(\alpha)$$

$$\Delta(p) = \frac{Z}{p^2 - \Sigma(p)} = \frac{J(p)}{p^2}$$

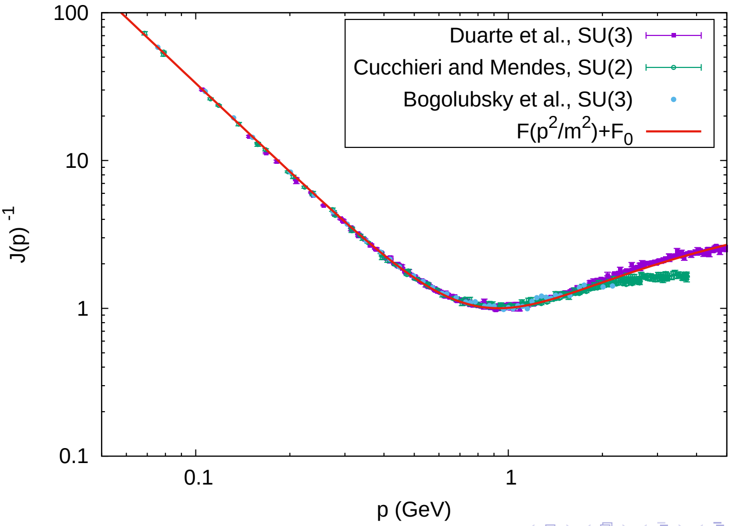
Setting $\quad Z = z\,(1 + \alpha\delta Z) \quad$ (one-loop):

$$z\,J(p)^{-1} = 1 + \alpha\left[F(p^2/m^2) - \delta Z\right] + \mathcal{O}(\alpha^2)$$

$$z\,J(p)^{-1} = 1 + \alpha\left[F(p^2/m^2) - F(\mu^2/m^2)\right] + \mathcal{O}(\alpha^2)$$

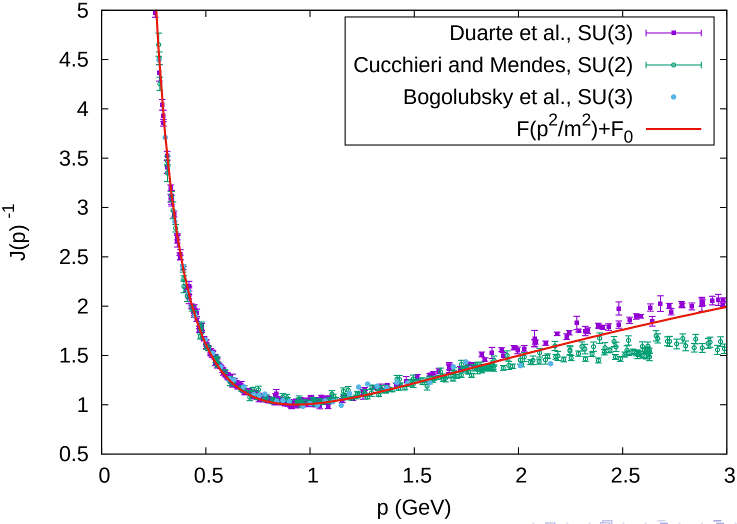
Must exist x, y, z:

$$\boxed{z\,J(p/x)^{-1} + y = F(p^2/m^2) + F_0 + \mathcal{O}(\alpha)}$$

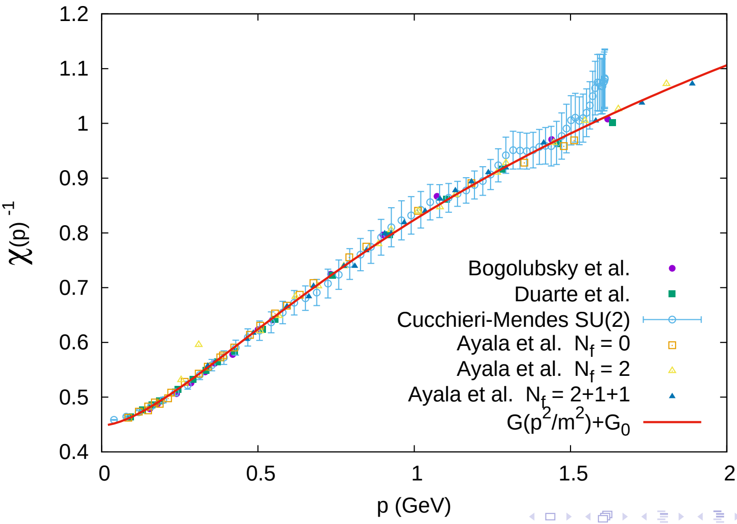


100
10
1
0.1
0.1
1
p (GeV)
J(p) -1
Duarte et al., SU(3)
Cucchieri and Mendes, SU(2)
Bogolubsky et al., SU(3)
F(p2/m2)+F0

UNIVERSAL SCALING
GLUON INVERSE DRESSING FUNCTION

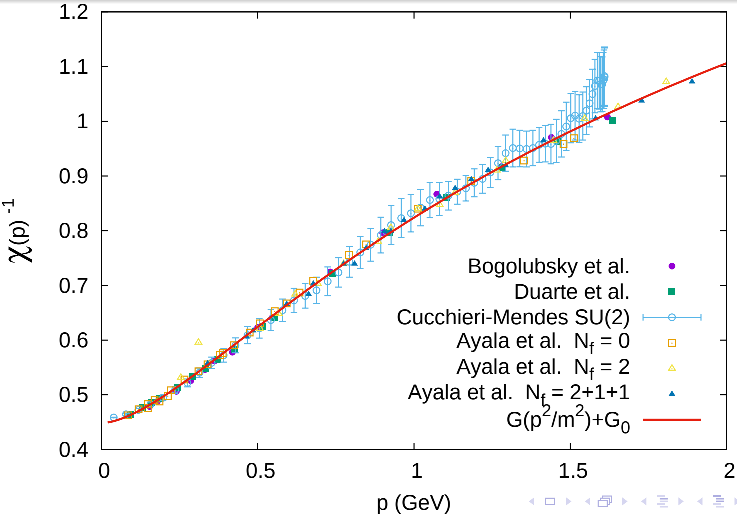


Denoting by $G(s)$ the ghost universal function $(F(s) \to G(s))$



The ghost universal function is just

$$G(s) = \frac{1}{12}\left[2 + \frac{1}{s} - 2s\log s + \frac{1}{s^2}(1+s)^2(2s-1)\log(1+s)\right]$$



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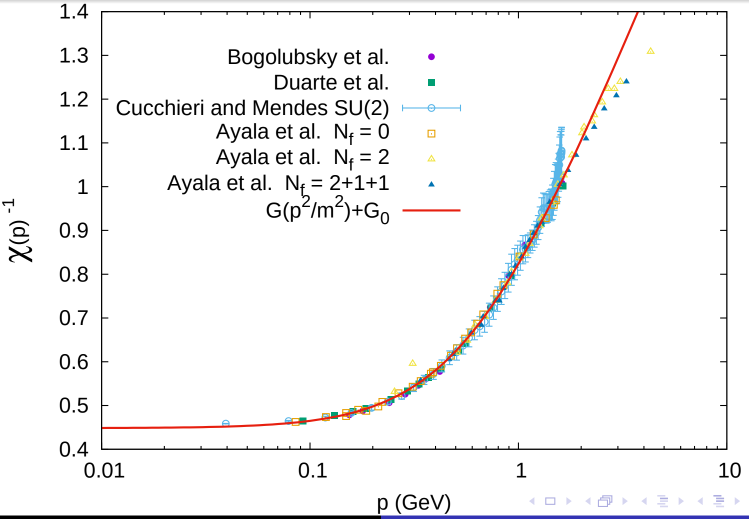
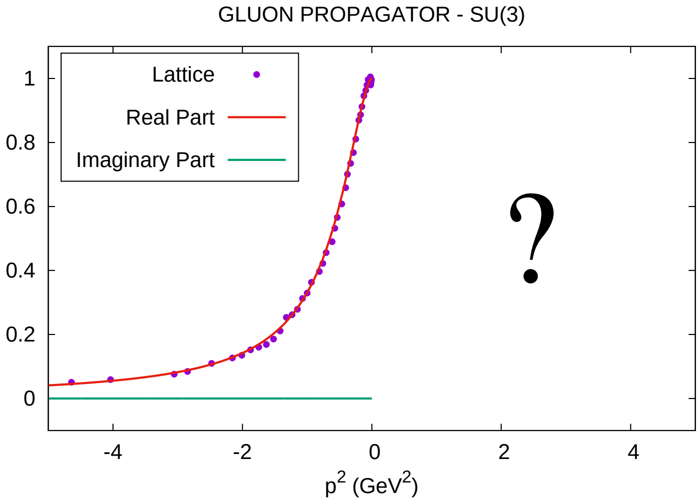
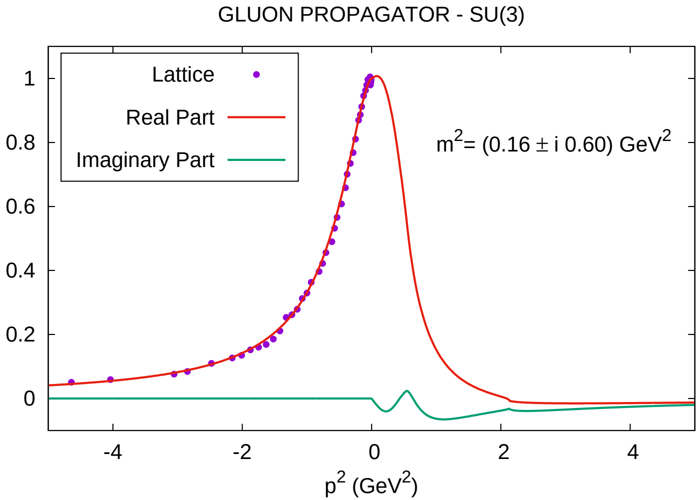




Lattice data are from Bogolubsky et al. (2009)



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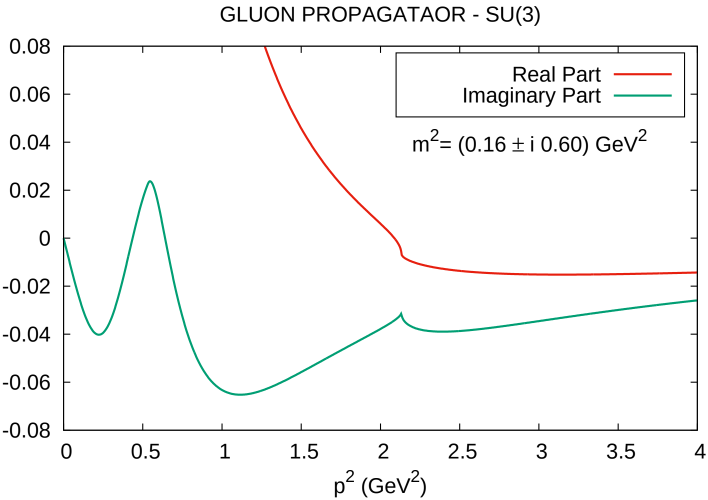


GLUON PROPAGATAOR - SU(3)
Real Part
Imaginary Part
m2= (0.16 ± i 0.60) GeV2
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
0
0.5
1
1.5
2
2.5
3
3.5
4
p2 (GeV2)

If $G(p)$ has complex poles then

$$G(p^2) = G^R(p^2) + \delta G(p^2)$$

where the *rational* function G^R just contains the poles

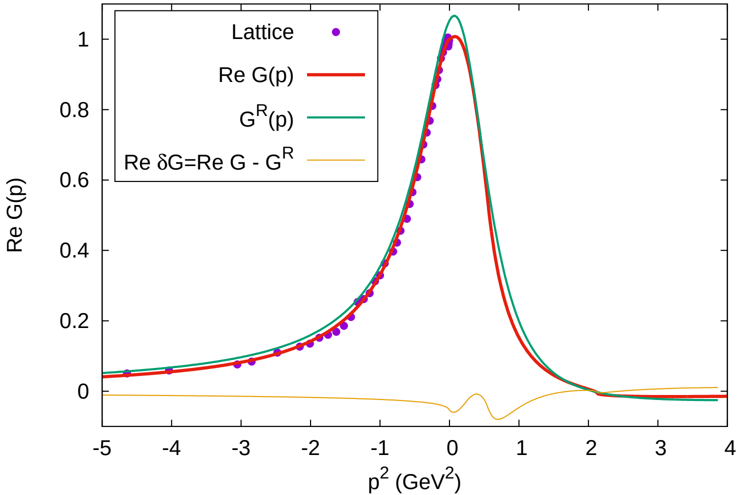
$$G^R(z) = \frac{R}{z-\alpha-i\beta} + \frac{R^\star}{z-\alpha+i\beta}$$

and the finite part δG satisfies usual dispersion relations

$$\mathrm{Re}\,\delta G(p^2) = PV\int_0^{+\infty} \frac{\rho(\omega)}{p^2-\omega}\,\mathrm{d}\omega$$

$$\rho(\omega) = -\frac{1}{\pi}\,\mathrm{Im}\,\delta G(\omega+i\epsilon) = -\frac{1}{\pi}\,\mathrm{Im}\,G(\omega+i\epsilon)$$

$G^R(p^2)$ cannot be reconstructed from $\mathrm{Im}\,G$



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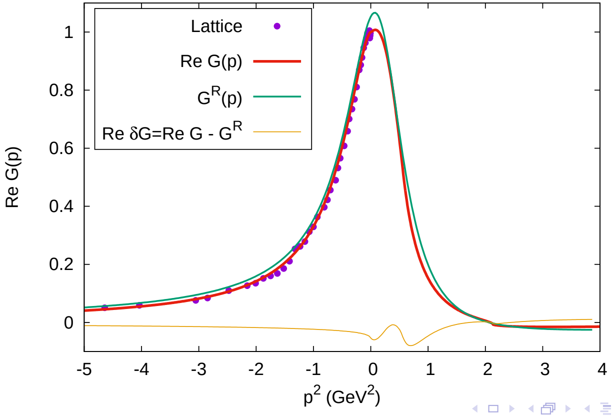
$$G^R(z) = \frac{R}{z-\alpha-i\beta} + \frac{R^\star}{z-\alpha+i\beta} \Longrightarrow \frac{p_E^2 + (\alpha + t\beta)}{p_E^4 + 2\alpha p_E^2 + (\alpha^2+\beta^2)}$$

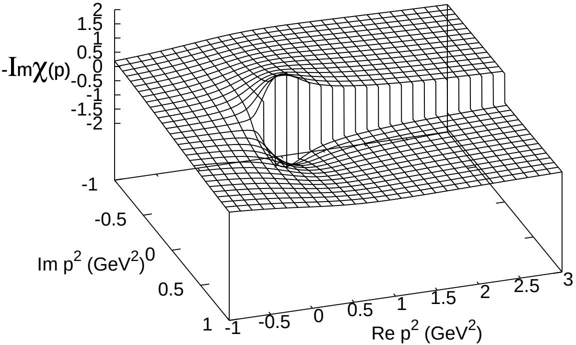
where $t = (\mathrm{Im}\,R)/(\mathrm{Re}\,R) = \tan[\arg(R)]$ RGZ model!



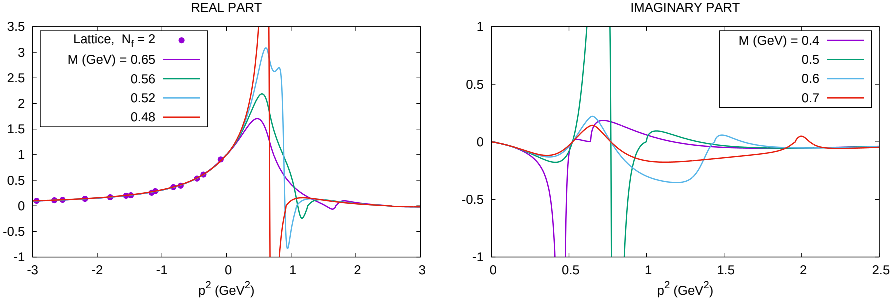
ANALYTIC CONTINUATION

Ghost dressing function: $\mathcal{G}(p^2) = \frac{\chi(p^2)}{p^2}$

$$\rho(p^2) = -\frac{1}{\pi}\,\mathrm{Im}\,\mathcal{G}(p^2 + i\varepsilon) = \chi(0)\,\delta(p^2) - \frac{1}{\pi}\frac{\mathrm{Im}\,\chi(p^2)}{p^2}$$



Optimized by the Lattice $N_f = 2$, $m = 0.8$ GeV $\quad M$ = ?



Lattice data are for two light quarks, from Ayala et al. (2012)

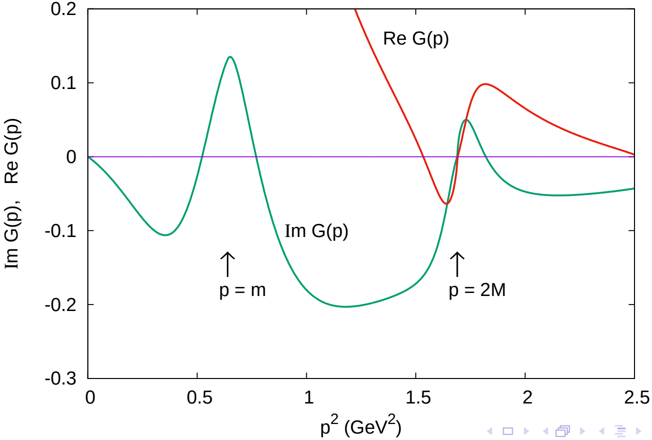
What about poles ?

2 pairs of compex conjugated poles

Optimized by the Lattice:

$m = 0.8$ GeV, $M = 0.65$ GeV

$m_1^2 = (0.54 \pm 0.52i)$ GeV^2, $m_2^2 = (1.69 \pm 0.1i)$ GeV^2



Quark propagator:

$$S(p) = S_p(p^2)\not{p} + S_M(p^2)$$

NO COMPLEX POLES $\Longrightarrow$ Standard Dispersion Relations

$$\rho_M(p^2) = -\frac{1}{\pi}\,\mathrm{Im}\,S_M(p^2)$$
$$\rho_p(p^2) = -\frac{1}{\pi}\,\mathrm{Im}\,S_p(p^2)$$

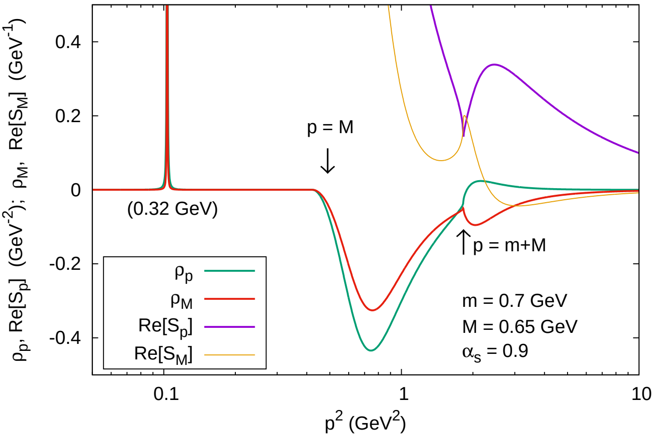
$$S(p) = \int_0^\infty \mathrm{d}q^2\,\frac{\rho_p(q^2)\not{p} + \rho_M(q^2)}{p^2 - q^2 + i\varepsilon}.$$

Positivity Conditions:

$\rho_p(p^2) \geq 0,$ $\qquad p\,\rho_p(p^2) - \rho_M(p^2) \geq 0$

CHIRAL QCD

Quark sector: $N_f = 2$, $M = 0.65$ GeV, $m = 0.7$ GeV



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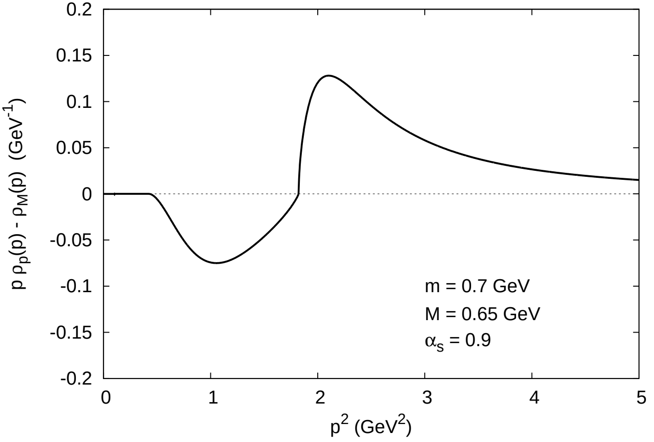

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- Are BRST and chiral invariance satisfied at any finite order?
- Can we *Optimize* the expansion (without Lattice Data) ?
  May be by real observables, like glueball masses.
- Is the theory renormalizable ?
  Special case of Curci and Ferrari (1976).
- What about improving the expansion by RG ?
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- More questions and remarks are welcome!

THANK YOU

BACKUP SLIDES

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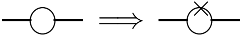

$$\frac{1}{-p^2+m^2} \Longrightarrow \frac{1}{-p^2+m^2}\, m^2\, \frac{1}{-p^2+m^2}$$

The integral is less divergent at each insertion.
A finite number of insertions makes any loop integral convergent:

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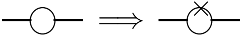

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The integral is less divergent at each insertion.
A finite number of insertions makes any loop integral convergent: divergences must cancel at a finite order

TABLE of OPTIMIZED RENORMALIZATION CONSTANTS: $z\,J(p/x)^{-1} + y = F(p^2/m^2) + F_0$

arXiv:1607.02040

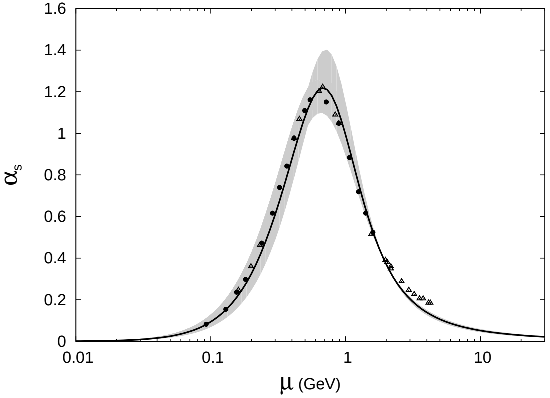
Data set	N	N_f	x	y	z	y'	z'
Bogolubsky et al.	3	0	1	0	3.33	0	1.57
Duarte et al.	3	0	1.1	-0.146	2.65	0.097	1.08
Cucchieri-Mendes	2	0	0.858	-0.254	1.69	0.196	1.09
Ayala et al.	3	0	0.933	-	-	0.045	1.17
Ayala et al.	3	2	1.04	-	-	0.045	1.28
Ayala et al.	3	4	1.04	-	-	0.045	1.28

Table: Scaling constants x, y, z (gluon) and y', z' (ghost). The constant shifts $F_0 = -1.05$, $G_0 = 0.24$ and the mass $m = 0.73$ GeV are optimized by requiring that $x = 1$ and $y = y' = 0$ for the lattice data of Bogolubsky et al. (2009)

RG invariant product (Landau Gauge – MOM-Taylor scheme):

$$\alpha_s(\mu) = \alpha_s(\mu_0)\frac{J(\mu)\chi(\mu)^2}{J(\mu_0)\chi(\mu_0)^2}$$

What if $\delta F_0 = \delta G_0 = \pm 25\%$?



$\mu_0 = 2$ GeV, $\alpha_s = 0.37$, data of Bogolubsky et al.(2009).

RG invariant product (Landau Gauge – MOM-Taylor scheme):

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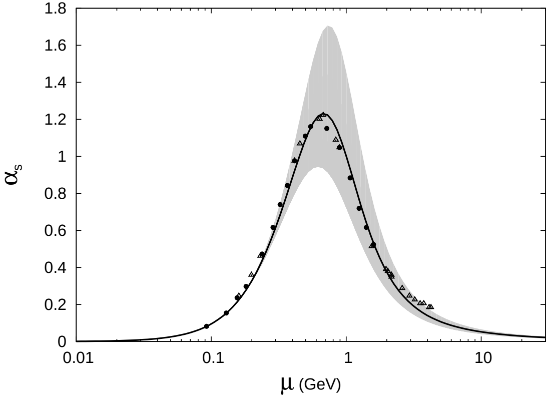

$\mu_0 = 0.15$ GeV, $\alpha_s = 0.2$, data of Bogolubsky et al.(2009).

RG invariant product (Landau Gauge – MOM-Taylor scheme):

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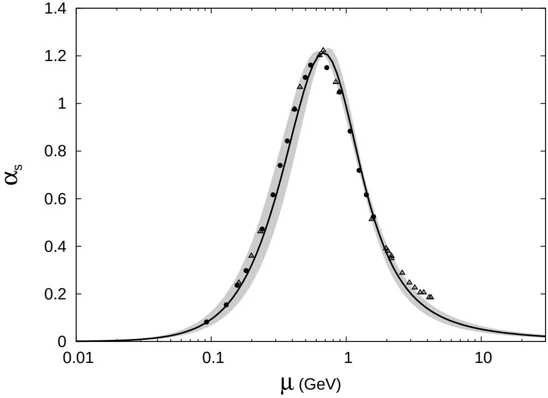

$\mu_0 = 0.67$ GeV, $\alpha_s = 1.21$, data of Bogolubsky et al.(2009).

ANALYTIC CONTINUATION

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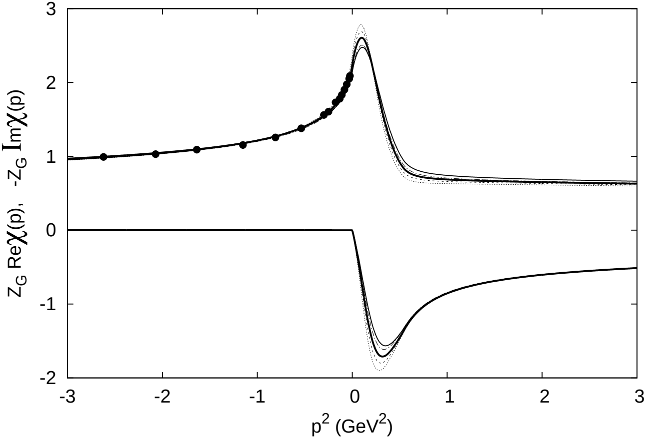

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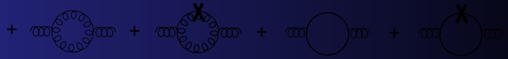
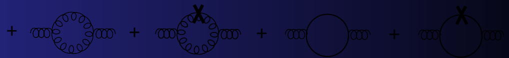
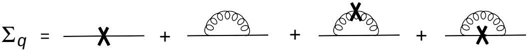


Σ_q =

- The counterterm $\delta\Gamma = -M$ cancels the mass at tree-level
- A massive propagator from *loops* $\rightarrow S(p) = \frac{Z(p)}{\not{p} - M(p)}$
- A new parameter $x = M/m$

but





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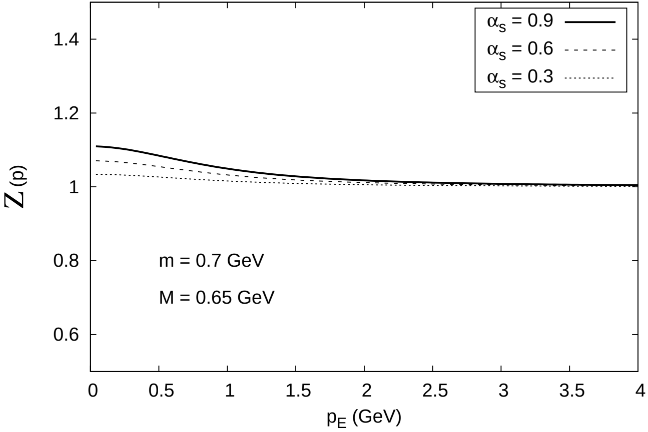
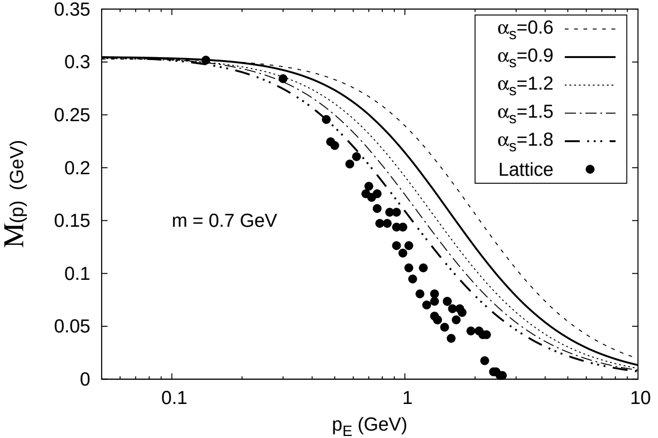
- A massive propagator from *loops* $\rightarrow S(p) = \frac{Z(p)}{\not{p} - M(p)}$
- A new parameter $x = M/m$

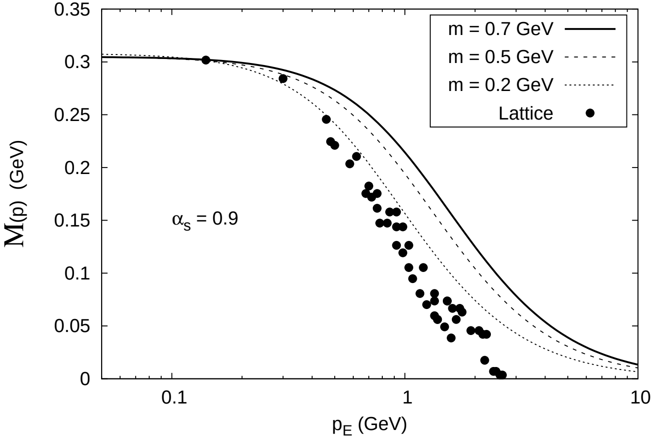
but

- Agreement not as good as for pure YM theory ($Z(p)$ is decreasing)
- $M(p)$ depends on α_s
- Optimization is not easy without RG corrections!


α_s = 0.9
α_s = 0.6
α_s = 0.3
1.4
1.2
1
0.8
0.6
Z(p)
m = 0.7 GeV
M = 0.65 GeV
0
0.5
1
1.5
2
2.5
3
3.5
4
p_E (GeV)



Lattice data are: *unquenched*, $N_f = 2$, in the CHIRAL limit
Bowman et al. (2005)



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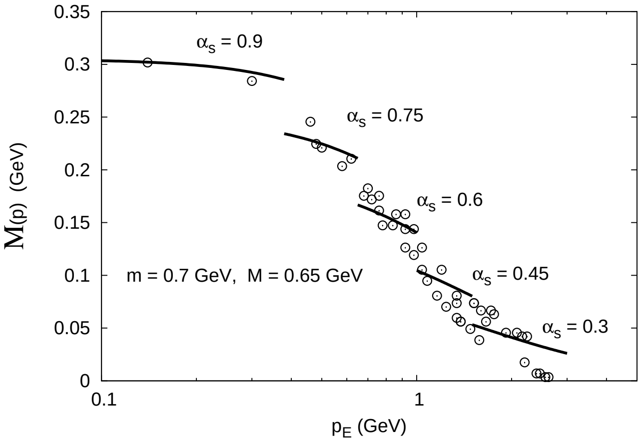
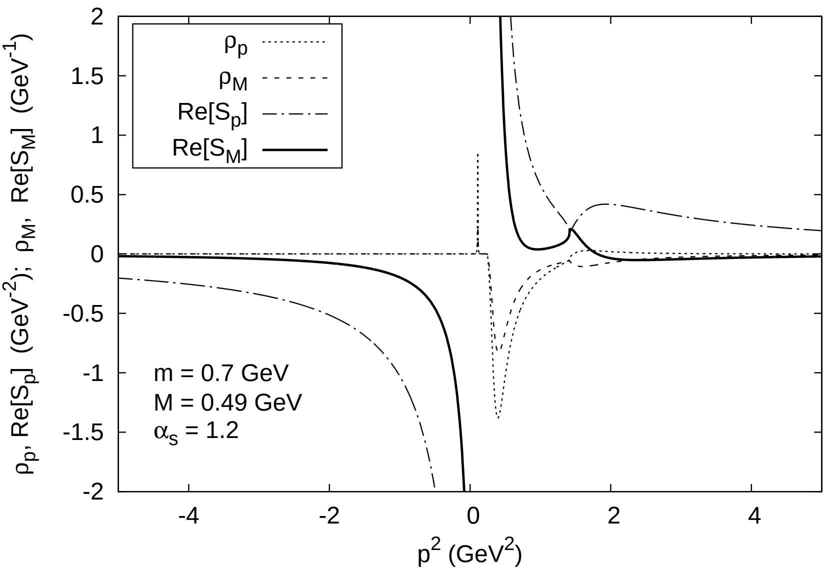
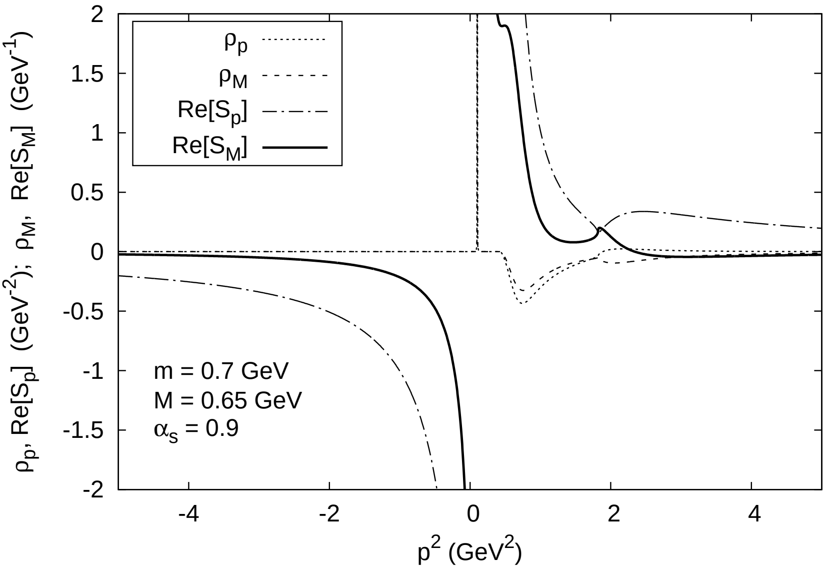


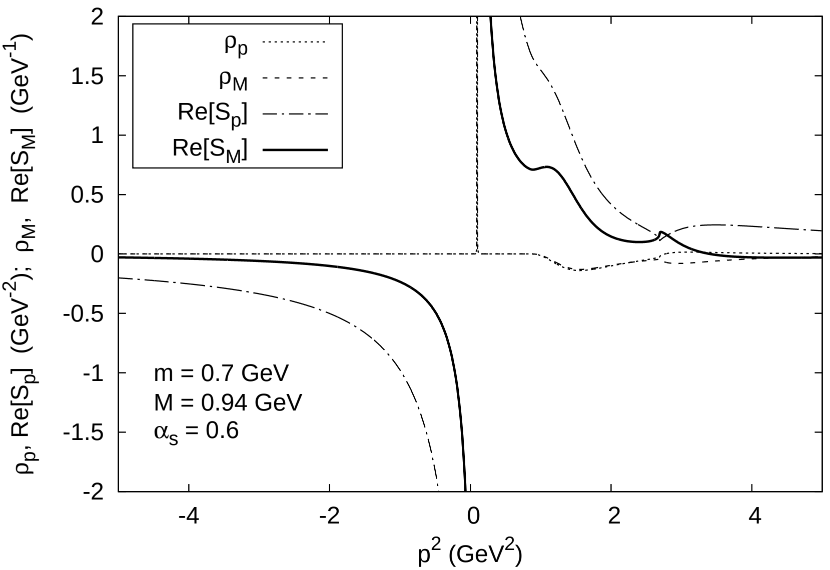
Lattice data are: *unquenched*, $N_f = 2$, in the CHIRAL limit
Bowman et al. (2005)



ρ_p
ρ_M
$Re[S_p]$
$Re[S_M]$
m = 0.7 GeV
M = 0.49 GeV
α_s = 1.2
ρ_p, $Re[S_p]$ (GeV^{-2}); ρ_M, $Re[S_M]$ (GeV^{-1})
p^2 (GeV^2)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-4
-2
0
2
4



ρ_p
ρ_M
$Re[S_p]$
$Re[S_M]$
m = 0.7 GeV
M = 0.65 GeV
α_s = 0.9
ρ_p, $Re[S_p]$ (GeV^{-2}); ρ_M, $Re[S_M]$ (GeV^{-1})
p^2 (GeV^2)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-4
-2
0
2
4



ρp
ρM
Re[Sp]
Re[SM]
m = 0.7 GeV
M = 0.94 GeV
αs = 0.6
ρp, Re[Sp] (GeV-2); ρM, Re[SM] (GeV-1)
p2 (GeV2)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-4
-2
0
2
4