Properties of QCD with nonzero chiral chemical potential

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1 September 2016

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The results are presented in papers:

- JHEP 1506 (2015) 094
- Phys.Rev. D93 (2016) 034509
- Phys.Rev. D93 (2016) 105025

Outline:

- Introduction
- SU(2) QCD with nonzero chiral chemical potential
- SU(3) QCD with nonzero chiral chemical potential
- Theoretical explanation
- Conclusion

Introduction

Motivation:

- QCD action can be separated into right and left parts: $S_{QCD} = S_R + S_L$
- There is symmetry between right and left parts
- One can introduce asymmetry of the form $\mu_5 Q_5, \quad Q_5 = Q_R - Q_L$
- $S_{QCD}(\mu_5) = S_R(\mu = \mu_5) + S_L(\mu = -\mu_5)$
- $\mu_5 \neq 0$ can be created in
  - Heavy ion collisions
  - Neutron stars and supernovae
  - Early Universe
  - Dirac and Weyl semimetals $(\vec{E}||\vec{H})$
  - Elastic deformation

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How nonzero chiral chemical potential influences the properties of QCD

Studies of the phase diagram of chiral QCD

- "Chiral magnetic effect in the PNJL model“,
  Kenji Fukushima, Marco Ruggieri, Raoul Gatto, Phys.Rev. D81 (2010) 114031
- "Phase diagram of chirally imbalanced QCD matter“,
  M.N. Chernodub, A.S. Nedelin, Phys.Rev. D83 (2011) 105008
- "Hot Quark Matter with an Axial Chemical Potential“,
  Raoul Gatto, Marco Ruggieri, Phys.Rev. D85 (2012) 054013
- "Inverse magnetic catalysis induced by sphalerons“,
  Jingyi Chao, Pengcheng Chu, Mei Huang, Phys.Rev. D88 (2013) 054009
- "Spontaneous generation of local CP violation and inverse magnetic catalysis“,
  Lang Yu, Hao Liu, Mei Huang, Phys.Rev. D90 (2014) 7, 074009
- "The effect of the chiral chemical potential on the chiral phase transition in the NJL model with different regularization schemes“,
  Lang Yu, Hao Liu, Mei Huang, arXiv:1511.03073
- ...

Results:

- Decrease of the critical temperature with chiral chemical potential
- Decrease of the chiral condensate with chiral chemical potential

Studies of the phase diagram of chiral QCD

- "Universality of phase diagrams in QCD and QCD-like theories", M. Hanada, N. Yamamoto, PoS LATTICE2011, 221 (2011), arXiv:1111.3391
- "Chemical potentials and parity breaking: the Nambu-Jona-Lasinio model", Alexander A. Andrianov, Domenec Espriu, Xumeu Planells, Eur.Phys.J. C74 (2014) 2, 2776
- "Effect of the chiral chemical potential on the position of the critical endpoint", Bin Wang, Yong-Long Wang, Zhu-Fang Cui, Hong-Shi Zong, Phys.Rev. D91 (2015) 3, 034017
- "Chiral phase transition with a chiral chemical potential in the framework of Dyson-Schwinger equations", Shu-Sheng Xu, Zhu-Fang Cui, Bin Wang, Yuan-Mei Shi, You-Chang Yang, Hong-Shi Zong, Phys.Rev. D91 (2015) 5, 056003
- "Critical Temperature of Chiral Symmetry Restoration for Quark Matter with a Chiral Chemical Potential", M. Ruggieri, G.X. Peng, arXiv:1602.05250
- "Nonlocal Nambu-Jona-Lasinio model and chiral chemical potential", Marco Frasca, arXiv:1602.04654
- "Critical Temperature of Chiral Symmetry Restoration for Quark Matter with a Chiral Chemical Potential", M. Ruggieri, G.X. Peng, arXiv:1602.03651
- "Quark matter with a chiral imbalance in the Nambu–Jona-Lasinio model" R.L.S. Farias, Dyana C. Duarte, Gastão Krein, Rudnei O. Ramos, arXiv:1604.04518
- ...

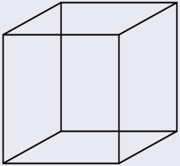
Results:

- Increase of the critical temperature with chiral chemical potential
- Increase of the chiral condensate with chiral chemical potential

SU(2) QCD
with
nonzero chiral chemical potential

Details of the calculation:

- Dynamical staggered fermions ($N_f = 4$) + Wilson action (SU(2))
- Link modification:
  $U \to U e^{\mu_5 \gamma_5}, \quad U^+ \to U^+ e^{-\mu_5 \gamma_5} \Rightarrow$ nonlocal action
- Chiral chemical potential:
  $\delta S_{\mu_5} = \frac{1}{2} \mu_5 a \sum_x (-1)^{x_2} (\bar{\psi}_{x+\delta} \bar{U}_{x+\delta,x} \psi_x - \bar{\psi}_x \bar{U}^{\dagger}_{x+\delta,x} \psi_{x+\delta})$



- Correct continuum limit: $\delta S_{\mu_5}|_{a \to 0} \to \mu_5 \int d^4x \; \bar{Q} (\gamma_4 \gamma_5 \times 1) Q$
- $6 \times 20^3 (m_\pi \sim 300\text{MeV}), \; 10 \times 28^3 (m_\pi \sim 500\text{MeV})$

Observables:

- The Polyakov loop (confinement/deconfinement transition)

  $L = \frac{1}{N_\sigma^3} \sum_{n_1,n_2,n_3} \langle Tr \prod_{n_4=1}^{N_\tau} U_4(n_1,n_2,n_3,n_4) \rangle$

- The chiral condensate (chiral symmetry breaking/restoration transition)

  $a^3 \langle \bar\psi \psi \rangle = -\frac{1}{N_\tau N_\sigma^3} \frac{1}{4} \frac{\partial}{\partial(ma)} \log Z = \frac{1}{N_\tau N_\sigma^3} \frac{1}{4} \langle Tr \frac{1}{D+ma} \rangle$

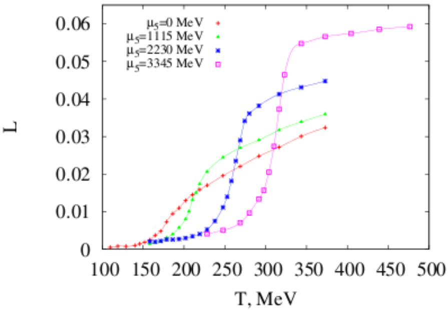
- The Polyakov loop susceptibility (position of the transition)

  $\chi_L = N_\sigma^3 \left(\langle L^2 \rangle - \langle L \rangle^2 \right)$

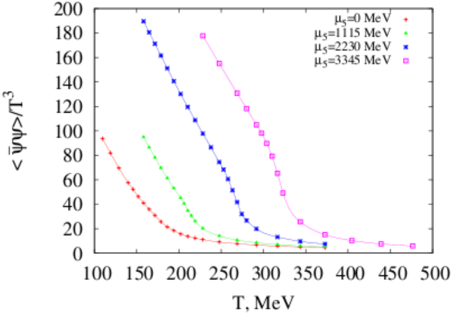
- The disconnected part of the chiral susceptibility (position of the transition)

  $\chi_{disc} = \frac{1}{N_\tau N_\sigma^3} \frac{1}{16} (\langle (Tr \frac{1}{D+ma})^2 \rangle - \langle Tr \frac{1}{D+ma} \rangle^2)$

Polyakov loop and chiral condensate

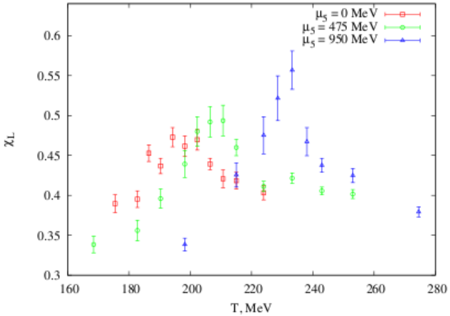


Polyakov loop

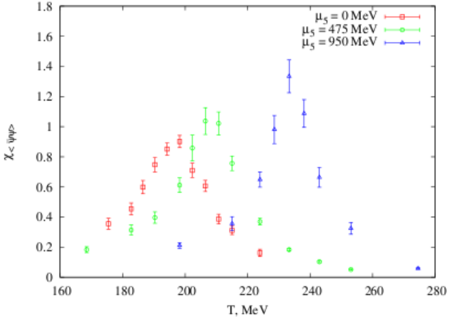


chiral condensate

Susceptibilities

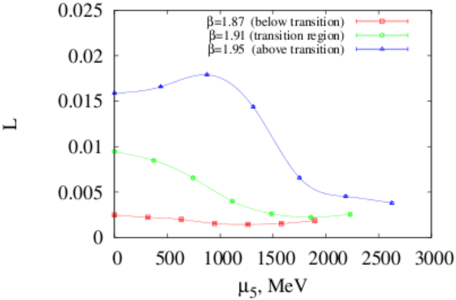


Polyakov loop susceptibility

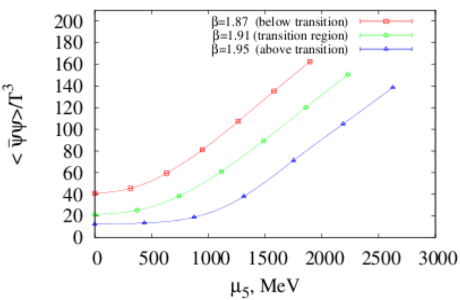


chiral susceptibility

Fixed temperature scan



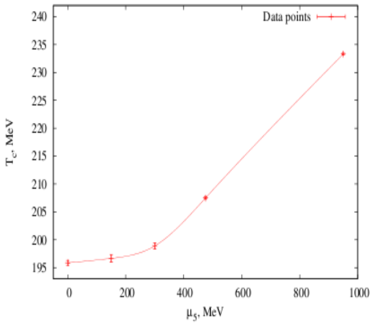
Polyakov loop



chiral condensate

Results of the calculation:

- The critical temperatures increase
- The critical temperatures of the confinement/deconfinement phase transition and of the chiral symmetry breaking/restoration coincide



SU(3) QCD
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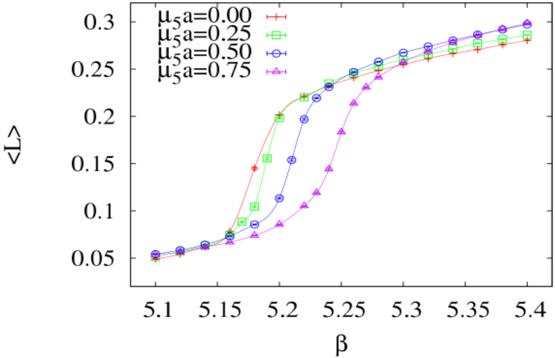
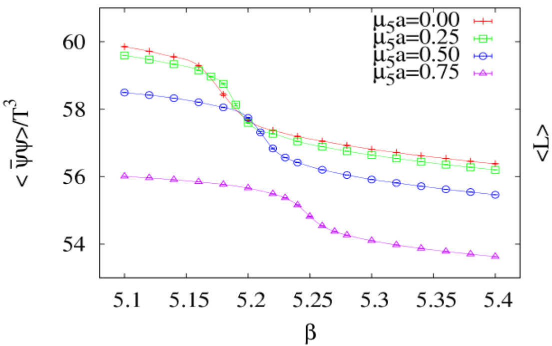
Details of the calculation:

- Dynamical Wilson fermions ($N_f = 2$) + Wilson action (SU(3))
- Link modification:
  $U \to U e^{\mu_5 \gamma_5}, \quad U^+ \to U^+ e^{-\mu_5 \gamma_5}$
- Continuum limit: $\delta S_{\mu_5}|_{a \to 0} \to \mu_5 \int d^4x \; \bar{Q} \big(\gamma_4 \gamma_5 \times 1 \big) Q$
- $4 \times 16^3 (m_\pi \sim 400 \text{MeV})$

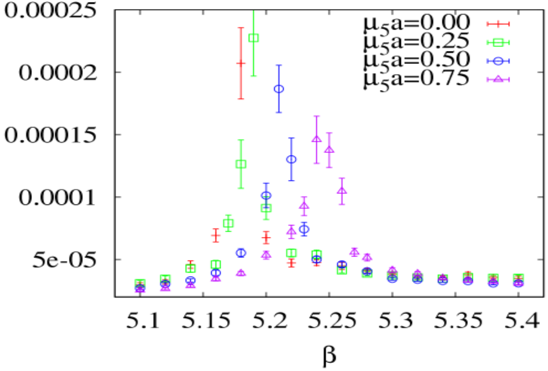
Polyakov loop



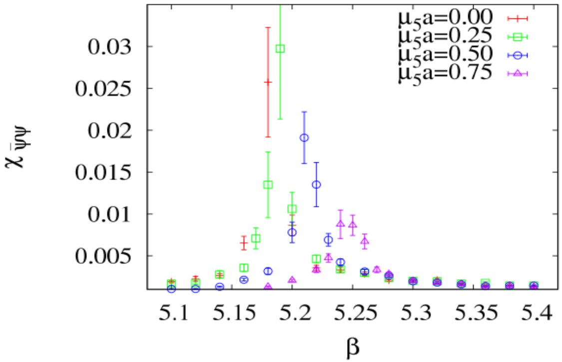
Chiral condensate



Polyakov loop susceptibility



Chiral condensate susceptibility



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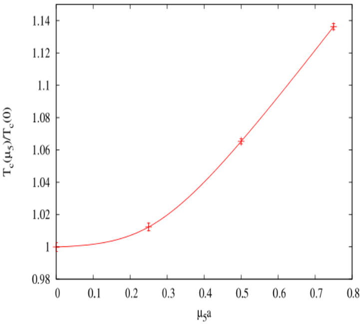

Summary:

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Summary:

- QCD-like theories:
  - SU(2) $N_f = 4$
  - SU(3) $N_f = 2$
- Different ways of introduction of chiral chemical potential
- Similar results (enhancement of chiral symmetry breaking)

Theoretical explanation

NJL model ($U_L(1) \times U_R(1)$, N_c colors)

- $S_E = \int d^4x \left(\bar\psi(\hat\partial + m - \mu_5 \gamma_4 \gamma_5)\psi - G\left[(\bar\psi\psi)^2 + (\bar\psi i\gamma_5\psi)^2\right] \right)$

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- $S_E = \int d^4x \left(\bar\psi (\hat\partial + m - \mu_5 \gamma_4 \gamma_5 + \sigma + i \gamma_5 \pi) \psi + \frac{1}{4G} \left[\sigma^2 + \pi^2 \right] \right)$

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- $S_{eff} = \int d^4x \Big(\frac{1}{4G}(\sigma^2 + \pi^2) - \mathrm{Tr}\log(\hat\partial + m - \mu_5\gamma_4\gamma_5 + \sigma + i\gamma_5\pi) \Big)$

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Gap equation ($N_c \to \infty$)

- $\frac{\delta S_{eff}}{\delta\sigma} = \frac{\sigma}{2G} - N_c \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\frac{1}{i\hat k + m - \mu_5\gamma_4\gamma_5 + \sigma + i\gamma_5\pi} \right] = 0$

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- $\frac{\pi^2}{GN_c} = \int_0^\Lambda k^2 dk \left[\frac{1}{\sqrt{(|\vec k| - \mu_5)^2 + M^2}} + \frac{1}{\sqrt{(|\vec k| + \mu_5)^2 + M^2}}\right]$

Gap equation

$$\frac{1}{\alpha_{NJL}} - 1 = \left(y^2 - \frac{x^2}{2} \right) \log \frac{1}{x^2}$$

$$\alpha_{NJL} = \frac{G N_c \Lambda^2}{\pi^2}, \quad x = \frac{M}{\Lambda}, \quad y = \frac{\mu_5}{\Lambda}$$

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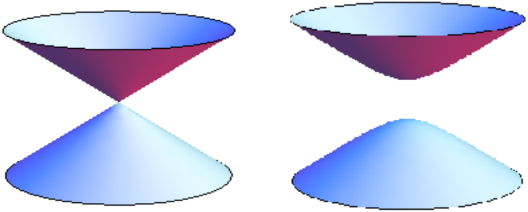
Properties ($\mu_5 = 0$):

- $\alpha_{NJL} < 1$ no solutions
- $\alpha_{NJL} > 1$ there is solution $M \neq 0$



Weakly coupled chiral plasma ($\alpha_{NJL} \ll 1$)

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μ_5 creates dynamical chiral symmetry breaking

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$\mu_5 \sim M$: $$M^2 \simeq 2\mu_5^2 \left(1 - \frac{1-\alpha_{NJL}}{\alpha_{NJL}} \frac{1}{2y^2 \log\left(\frac{1}{2y^2}\right)}\right).$$

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Strongly coupled chiral plasma ($\alpha_{NJL} > 1$)

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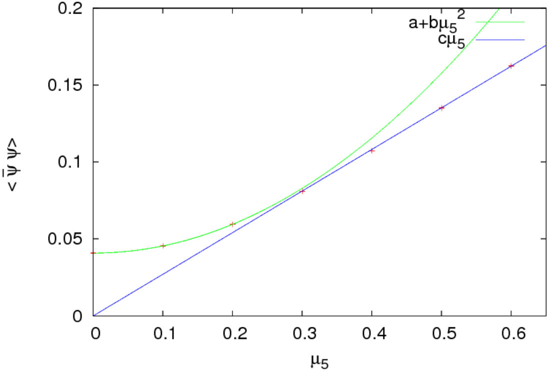
μ_5 enhances dynamical chiral symmetry breaking

Prediction:

In strong coupling region NJL model predicts that dynamical fermion mass is quadratically rising function at small μ_5 which switches to linear rising behaviour at large μ_5


0.2
0.15
0.1
0.05
0
0
0.1
0.2
0.3
0.4
0.5
0.6
$<\bar{\psi}\psi>$
μ_5
$a+b\mu_5^2$
$c\mu_5$

Chiral symmetry breaking as condensation of Cooper pairs (BCS theory)

- Vacuum: $|vac\rangle = \hat{G}_1 \hat{G}_2 \hat{G}_3 |p_F\rangle$,
  $\hat{G}_1 = \prod_{\boldsymbol{p}} \left(\cos(\theta_L) - \sin(\theta_L) \hat{a}^+_{L,\boldsymbol{p}} \hat{b}^+_{L,-\boldsymbol{p}} \right)$
  $\hat{G}_2 = \prod_{\boldsymbol{p} > \mu_5} \left(\cos(\theta_R) + \sin(\theta_R) \hat{a}^+_{R,\boldsymbol{p}} \hat{b}^+_{R,-\boldsymbol{p}} \right)$
  $\hat{G}_3 = \prod_{\boldsymbol{p} < \mu_5} \left(\cos(\tilde{\theta}_R) + \sin(\tilde{\theta}_R) \hat{b}_{R,-\boldsymbol{p}} \hat{a}_{R,\boldsymbol{p}} \right)$

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  $\hat{G}_3 = \prod_{p<\mu_5} \Big(\cos(\tilde{\theta}_R) + \sin(\tilde{\theta}_R) \hat{b}_{R,-p} \hat{a}_{R,p} \Big)$
- Energy:

$$E_{vac} = 2N_c \Big[\int_{p<\mu_5} \frac{d^3p}{(2\pi)^3} (p-\mu) \cos^2 \tilde{\theta}_R + \int_{p>\mu_5} \frac{d^3p}{(2\pi)^3} (p-\mu) \sin^2 \theta_R + \int \frac{d^3p}{(2\pi)^3} (p+\mu) \sin^2 \theta_L \Big]$$

$$-GN_c^2 \Big(\int_{p<\mu_5} \frac{d^3p}{(2\pi)^3} \sin 2\tilde{\theta}_R + \int_{p>\mu_5} \frac{d^3p}{(2\pi)^3} \sin 2\theta_R + \int \frac{d^3p}{(2\pi)^3} \sin 2\theta_L \Big)^2$$

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  $\hat{G}_1 = \prod_p \Big(\cos(\theta_L) - \sin(\theta_L) \hat{a}^+_{L,p} \hat{b}^+_{L,-p} \Big)$

  $\hat{G}_2 = \prod_{p>\mu_5} \Big(\cos(\theta_R) + \sin(\theta_R) \hat{a}^+_{R,p} \hat{b}^+_{R,-p} \Big)$

  $\hat{G}_3 = \prod_{p<\mu_5} \Big(\cos(\tilde{\theta}_R) + \sin(\tilde{\theta}_R) \hat{b}_{R,-p} \hat{a}_{R,p} \Big)$

- Energy:

$$E_{vac} = 2N_c \Big[\int_{p<\mu_5} \frac{d^3p}{(2\pi)^3} (p-\mu) \cos^2 \tilde{\theta}_R + \int_{p>\mu_5} \frac{d^3p}{(2\pi)^3} (p-\mu) \sin^2 \theta_R + \int \frac{d^3p}{(2\pi)^3} (p+\mu) \sin^2 \theta_L \Big]$$

$$-GN_c^2 \Big(\int_{p<\mu_5} \frac{d^3p}{(2\pi)^3} \sin 2\tilde{\theta}_R + \int_{p>\mu_5} \frac{d^3p}{(2\pi)^3} \sin 2\theta_R + \int \frac{d^3p}{(2\pi)^3} \sin 2\theta_L \Big)^2$$

- Minimum energy is realized at the gap equation

Chiral symmetry breaking as condensation of Cooper pairs (BCS theory)

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- $\mu_5 > 0$ creates Fermi spheres of right particles and right antiparticles
- Due the Fermi spheres additional fermion states participate in chiral symmetry breaking
- μ_5 plays role of catalyst of chiral symmetry breaking due to additional fermion states (model independent result applicable not only in QCD)

Chiral symmetry breaking as condensation of Cooper pairs (BCS theory)

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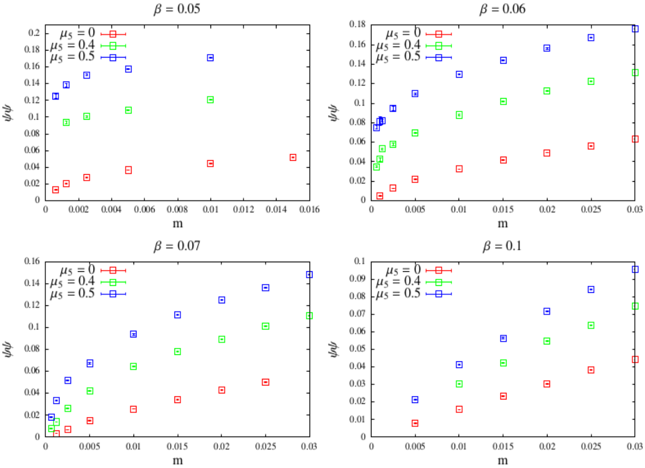
We called this phenomena: CHIRAL CATALYSIS

Numerical simulation of Dirac semimetals

- Staggered fermions with rooting
- Instantaneous Coulomb interaction with α_{eff}
- Asymmetry in Fermi velocity
- The first study of the phase diagram in $(\alpha_{eff}, v_\perp/v_\parallel)$ plane
  V.V. Braguta, M.I. Katsnelson, A. Yu. Kotov, A.A. Nikolaev, arXiv:1608.07162



Conclusion

Conclusion:

- We carried out numerical study of SU(2) and SU(3) QCD with nonzero chiral chemical potential
- Critical temperatures of confinement/deconfinement and breaking/restoration of chiral symmetry transitions rise with chiral chemical potential
- CHIRAL CATALYSIS: Chiral chemical potential creates/enhances chiral symmetry breaking
- Chiral plasma is unstable with respect to chiral symmetry breaking and condensation of Cooper pairs for $T < T_c$