From QCD’s n-point functions to nucleon resonances

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XIIth Quark Confinement and the Hadron Spectrum
Thessaloniki, Greece
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GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, 1606.09602, Prog. Part. Nucl. Phys. (in press)

GE, Fischer, Sanchis-Alepuz, 1607.05748
Introduction

QCD Lagrangian: \[ \mathcal{L} = \bar{\psi} (\partial + igA + m) \psi + \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a \]

- if it only were that simple...
  we don’t measure quarks and gluons, but **hadrons**

- origin of **mass generation** and **confinement**?

<table>
<thead>
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<th>b</th>
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</thead>
<tbody>
<tr>
<td>Current mass [GeV]</td>
<td>0.003</td>
<td>0.005</td>
<td>0.1</td>
<td>1</td>
<td>4</td>
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</tr>
<tr>
<td>„Constituent“ mass [GeV]</td>
<td>0.35</td>
<td>0.35</td>
<td>0.5</td>
<td>1.5</td>
<td>4.5</td>
<td>175</td>
</tr>
</tbody>
</table>

- need to understand **spectrum and interactions**!
Light baryon spectrum

Experimentally extracted from $\pi N$ scattering, meson photo- and electroproduction

- Nature of Roper (level ordering)?
- Three-quark vs. quark-diquark?
- “Quark core” vs. meson-baryon coupled channel effects?
- Hybrid baryons?
Light baryon spectrum

Nonrelativistic quark model:

\[ P = (-1)^L \]

\[ J^P = \begin{pmatrix} \frac{1}{2}^+ \\ \frac{1}{2}^- \\ \frac{3}{2}^+ \\ \frac{3}{2}^- \end{pmatrix} \]

Experimentally extracted from \( \pi N \) scattering, meson photo- and electroproduction

- Nature of Roper (level ordering)?
- Three-quark vs. quark-diquark?
- “Quark core” vs. meson-baryon coupled channel effects?
- Hybrid baryons?
Lattice QCD

Extract baryon poles from (gauge-invariant) two-point correlators:

\[ G(x - y) = \langle 0 \vert T \left[ \Gamma_{\alpha\beta\gamma} \psi_{\alpha} \psi_{\beta} \psi_{\gamma} \right] (x) \left[ \bar{\Gamma}_{\rho\sigma\tau} \bar{\psi}_{\rho} \bar{\psi}_{\sigma} \bar{\psi}_{\tau} \right] (y) \vert 0 \rangle = \int \mathcal{D}[\psi, \bar{\psi}, A] e^{-S} J(x) \bar{J}(y) \]

- Spectral decomposition:
  \[ \sum_{\lambda} |\lambda\rangle \langle \lambda| \quad \rightarrow \quad \sum_{\lambda} \frac{\cdot \cdot \cdot}{P^2 + m_i^2} \]

- Same singularity structure in any n-point function:

- Pole in momentum space \( \Rightarrow \) exp. decay in Euclidean time
  \[ G(x - y) \rightarrow e^{-m\tau} \]

\( \rightarrow \) Mohler, Briceno, Hansen, \ldots
Extract baryon poles from (gauge-invariant) two-point correlators:

\[
G(x - y) = \langle 0 | T \left[ \Gamma_{\alpha\beta\gamma} \psi_\alpha \psi_\beta \psi_\gamma (x) \right] \left[ \bar{\psi}_\rho \bar{\psi}_\sigma \bar{\psi}_\tau (y) \right] | 0 \rangle = \int \mathcal{D}[\psi, \bar{\psi}, A] e^{-S} J(x) \bar{J}(y)
\]

\[
= \lim_{x_i \to x, y_i \to y} \Gamma_{\alpha\beta\gamma} \bar{\Gamma}_{\rho\sigma\tau} \langle 0 | T \psi_\alpha(x_1) \psi_\beta(x_2) \psi_\gamma(x_3) \bar{\psi}_\rho(y_1) \bar{\psi}_\sigma(y_2) \bar{\psi}_\tau(y_3) | 0 \rangle
\]

Alternative: extract \textit{gauge-invariant} baryon poles from \textit{gauge-dependent} quark 6-point function:

\[
G
\]

\[
p^2 \rightarrow -m^2
\]

Bethe-Salpeter wave function: residue at pole, contains all information about baryon
Bethe-Salpeter

- Homogeneous Bethe-Salpeter equation for BS wave function:

\[ \begin{align*}
G & \quad \rightarrow \quad p^2 \rightarrow m^2 \\
\chi & \quad = \quad K \chi
\end{align*} \]

- Depends on QCD’s n-point functions as input, satisfy DSEs = quantum equations of motion

\[ \begin{align*}
G^{-1} & = G^{-1} + \ldots \\
\chi^{-1} & = \chi^{-1} + \ldots
\end{align*} \]

incredibly many coupled equations, in practice truncations:
model / neglect higher n-point functions to obtain closed system
QCD’s n-point functions

- Quark propagator
  \[ A(p^2) \frac{1}{(i\not{p} + M(p^2))} \]

- Gluon propagator
  \[ \frac{D(p^2)}{p^2} \left( \delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \]

- Quark-gluon vertex
  \[ f_1 \gamma^\mu + f_2 i p^\mu + f_3 p^\mu \not{p} + \ldots \]

- Three-gluon vertex
  \[ F_1 \big( \delta^{\mu\nu}(p_1 - p_2)^\rho + \delta^{\nu\rho}(p_2 - p_3)^\mu + \delta^{\rho\mu}(p_3 - p_1)^\nu \big) + \ldots \]

Dynamical chiral symmetry breaking generates ‘constituent-quark masses’

Agreement between lattice, DSE & FRG within reach

(→ Sternbeck, Williams, Huber, Blum, Mitter, Cyrol, Campagnari, . . .)
Bethe-Salpeter

- Homogeneous Bethe-Salpeter equation for BS wave function:

\[ P \chi - m^2 \chi = K \chi \]

- Depends on QCD's n-point functions as input, satisfy DSEs = quantum equations of motion

\[ \begin{align*}
\frac{1}{\lambda} &= \frac{1}{\lambda} + \frac{1}{\lambda} \\
\frac{1}{\lambda} &= \frac{1}{\lambda} + \frac{1}{\lambda} + \frac{1}{\lambda} + \frac{1}{\lambda} + \frac{1}{\lambda} + \ldots
\end{align*} \]

- Kernel can be derived in accordance with chiral symmetry:

\[ = \quad + \quad + \quad + \ldots \quad \rightarrow R. \text{Williams} \]

- Quark propagator

\[ A(p^2) \left(i\gamma + M(p^2)\right) \]

Dynamical chiral symmetry breaking generates ‘constituent-quark masses’

Quark mass function [GeV]:

- Bottom
- Charm
- Strange
- Up/down
- Chiral limit
Bethe-Salpeter

- Homogeneous Bethe-Salpeter equation for BS wave function:

\[ \Gamma - m^2 \]

\[ G \chi \chi K = \]

Depends on QCD’s n-point functions as input, satisfy DSEs = quantum equations of motion

\[ A^{-1} = A^{-1} + \ldots \]

- Kernel can be derived in accordance with chiral symmetry:

Rainbow-ladder: effective gluon exchange

\[ \alpha(k^2) = \alpha_{1R}(k^2, \eta) + \alpha_{UV}(k^2) \]

adjust scale \( \Lambda \) to observable, keep width \( \eta \) as parameter

Maris, Tandy, PRC 60 (1999)

- Quark propagator

Dynamical chiral symmetry breaking generates ‘constituent-quark masses’

Quark mass function [GeV]:

- Bottom
- Charm
- Strange
- Up/down
- Chiral limit
Bethe-Salpeter

- Homogeneous **Bethe-Salpeter equation** for BS wave function:

\[ P\chi - m\chi = G K \chi \]

- Depends on QCD's n-point functions as input, satisfy **DSEs = quantum equations of motion**

\[ -1 = -1 + \]

- Kernel can be derived in accordance with **chiral symmetry**:

**Rainbow-ladder:**

Effective gluon exchange

\[ \alpha(k^2) = \alpha_{1R}(k^2, \eta) + \alpha_{UV}(k^2) \]

Adjust scale \( \Lambda \) to observable, keep width \( \eta \) as parameter

Maris, Tandy, PRC 60 (1999)

- **Quark propagator**

Calculated in **complex plane**: singularities pose restrictions (no physical threshold!)

\[ A(p^2) \{ i\phi + M(p^2) \} \]
Mesons

- The pion plays special role in hadron physics: quark-antiquark bound state $\Leftrightarrow$ Goldstone boson of spontaneous chiral symmetry breaking

\[ \gamma_5 \left( f_1 + f_2 \not{p} + f_3 \not{q} + f_4 [\not{q}, \not{p}] \right) \otimes \text{Color} \otimes \text{Flavor} \]

most general Dirac-Lorentz structure, Lorentz-invariant dressing functions:
\[ f_i = f_i(q^2, q \cdot P, P^2 = -m^2) \]

$p$-wave is made of $s$ waves and $p$ waves!

(relative momentum $\sim$ orbital angular momentum)

- Eigenvalue spectrum of BS kernel:

\[ K \psi_i = \lambda_i(P^2) \psi_i, \quad \lambda_i \xrightarrow{p^2 \rightarrow m_i^2} 1 \]

\[ \frac{1}{\lambda_i} \]

\[ \begin{array}{c|c|c|c}
\pi & \pi(1300) & \pi(1800) \\
\hline
f_1 & f_2 & f_3 & f_4 \\
\end{array} \]
Mesons

- **Pion is Goldstone boson:** \( m_\pi^2 \sim m_q \)

- **Light meson spectrum** beyond rainbow-ladder

- **Pion electromagnetic form factor:**
  

- **Timelike vector meson poles** automatically generated in quark-photon vertex!
Baryons

- Covariant Faddeev equation for baryons:
  keep 2-body interactions & rainbow-ladder, but no further approximations: \( M_N = 0.94 \text{ GeV} \)

GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010), GE, PRD 84 (2011)

\[ \begin{array}{c}
\text{Octet & decuplet baryons, pion cloud effects, first steps beyond rainbow-ladder} \\
\end{array} \]

- Baryon form factors:
  nucleon and \( \Delta \) FFs, \( N \rightarrow \Delta \gamma \) transition

Resonances?

Branch cuts & widths generated by **meson-baryon interactions**: Roper → $N\pi$, etc.

Without them: **bound states without widths**

Difficult to implement at **quark-gluon level**: complicated topologies beyond rainbow-ladder

Different phenomenological pictures how this could happen:

- **‘pion-cloud effects’** affect masses and form factors in light-quark region

- **dynamical generation of resonances**: start with ‘bare’ seed, hadronic interactions produce new poles

- Three-quark vs. five-quark / molecular components

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August 29, 2016 11 / 20
Diquarks?

- Suggested to resolve ‘missing resonances’ in quark model: fewer degrees of freedom ⇒ fewer excitations

- QCD version: assume $qq$ scattering matrix as sum of diquark correlations ⇒ three-body equation simplifies to quark-diquark BSE

![Diagram of quark-diquark BSE]

**Quark exchange** between quark & diquark binds nucleon. Gluons absorbed in building blocks, to be calculated in advance:

Rainbow-ladder: **scalar diquark** $\sim 800$ MeV, **axialvector diquark** $\sim 1$ GeV

- N and $\Delta$ masses & form factors very similar in quark-diquark and three-quark approach: **quark-diquark approximation is good.** → What about other channels?
Three-quark vs. quark-diquark in rainbow-ladder: 

- Three-body and quark-diquark results agree (where available): N, Δ, Roper, N(1535)

- Number of levels compatible with experiment: no states missing

- N, Δ and their 1st excitations (including Roper) agree with experiment

- But remaining states too low ⇒ level ordering between Roper and N(1535) is wrong
Three-quark vs. quark-diquark in rainbow-ladder: GE, Fischer, Sanchis-Alepuz, 1607.05748

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The role of diquarks

Mesons and ‘diquark’ properties closely related: after taking Dirac, color & flavor traces, only factor 1/2 remains ⇒ diquarks ‘less bound’ than mesons

\[ \text{Pseudoscalar & vector mesons already good in rainbow-ladder} \quad \Leftrightarrow \quad \text{Scalar & axialvector diquarks sufficient for nucleon and } \Delta \]

\[ \text{Scalar & axialvector mesons too light, repulsion beyond RL} \quad \Leftrightarrow \quad \text{Pseudoscalar & vector diquarks important for remaining channels} \]

Simple strategy to emulate beyond-RL effects:

- Insert factor \( 0 < c < 1 \) in ‘bad’ meson and diquark channels ⇒ increases masses
- Fixed in the meson sector (\( \rho-a_1 \) splitting): \( c = 0.35 \)
Three-body and quark-diquark results agree (where available): N, Δ, Roper, N(1535)

Number of levels compatible with experiment: no states missing

N, Δ and their 1st excitations (including Roper) agree with experiment

But remaining states too low ⇒ level ordering between Roper and N(1535) is wrong
Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, 1607.05748

- Quantitative agreement with experiment
- $N(\frac{1}{2}^+)$ and $\Delta(\frac{3}{2}^+)$ channels not affected, but remaining ones were polluted by $ps + v$ diquarks
- Correct level ordering between Roper and $N(1535)$
- Scale $\Lambda$ set by $f_\pi$
- Current-quark mass set by $m_\pi$
- $c$ adjusted to $\rho - a_1$ splitting
- $\eta$ doesn’t change much
Quark-diquark with reduced pseudoscalar + vector diquarks:

Partial-wave content:

- N and Δ ground states dominated by s waves, negative-parity states typically by p waves (as expected)
- But ‘quark-model forbidden’ contributions are always present, e.g. Roper: dominated by p waves ⇒ relativity is important!
Structure properties

- **Current-mass evolution** of Roper similar to nucleon. Lattice?
  
  GE, Fischer, Sanchis-Alepuz, 1607.05748

- **γN→Δ transition form factors:**
  
  GE, Nicmorus, PRD 85 (2012)

- Discrepancies mainly in magnetic dipole ($G^*_M$): “Core + 25% pion cloud”

- **Electric quadrupole ratio**
  small & negative, encodes deformation.
  No pion cloud necessary: OAM from p waves!

- **Roper transition form factors** in qualitative agreement with experiment
  
  Segovia et al., PRL 115 (2015)

- All signatures of 1st radial excitation:
  partial-wave content, zero crossing

- First three-body results similar
  
So what does it mean?

Results favor ‘mild’ scenario:

- spectrum generated by quark-gluon interactions
- meson-baryon effects would merely shift poles into complex plane
- Effects on masses? Scale set by $f_\pi$, but pion-cloud affects $f_\pi$ too so only ‘non-trivial effects’ visible
- Will be interesting to study transition form factors

Note: ‘bound states without widths’ doesn’t mean that $\rho \to \pi\pi$, $\Delta \to N\pi$, ... decays are zero!!

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Mader, GE, Blank, Krassnigg, PRD 84 (2011),
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, 1606.09602

---

\[ g_{\rho\pi\pi} \]

\[ G(\rho^2) \]

\[ G(\Delta N\pi)(0) \]

\[ G_{\Delta N\pi}(0) \]

---

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Tetraquarks are resonances

- Light scalar mesons $\sigma$, $\kappa$, $a_0$, $f_0$ as tetraquarks: solution of four-body equation reproduces mass pattern

GE, Fischer, Heupel, PLB 753 (2016)

BSE dynamically generates meson poles in wave function, drive $\sigma$ mass from 1.5 GeV to $\sim$350 MeV

Four quarks rearrange to "meson molecule"

Tetraquarks are "dynamically generated resonances" (but from the quark level!)

- Similar in meson-meson / diquark-antidiquark approximation (analogue of quark-diquark for baryons) Heupel, GE, Fischer, PLB 718 (2012)
Scattering amplitudes from quark level:

- **$\pi\pi$ scattering**
  
  Bicudo et al., PRD 65 (2002),

- **Nucleon Compton scattering**
  

- **Hadronic light-by-light scattering**
  
  Goecke, Fischer, Williams, PLB 704 (2011),
  GE, Fischer, Heupel, PRD 92 (2015)
Progress with Dyson-Schwinger, Bethe-Salpeter and Faddeev equations:

- **Baryon spectrum** quantitatively reproduced

- **Quark-diquark** and **three-quark** spectrum very similar:
  - Quark-diquark with *sc, av, ps, v* ~ three-quark in RL
  - Quark diquark with *sc, av, ps, v* ~ three-quark beyond RL?

- Still “**bound states without widths**”,
  because meson-baryon interactions difficult to implement at quark-gluon level.
  But:
    - would mainly shift poles into complex plane (?)
    - decay properties are calculable
    - tetraquarks are genuine resonances (even in RL!)

- For a recent review see:

Thank you!
Backup slides
... to Dyson-Schwinger equations

QCD’s classical action:

\[
S = \int d^4x \left[ \bar{\psi} \left( \frac{\partial}{\partial x} + igA + m \right) \psi + \frac{1}{4} F_{\mu\nu}^a F_{a\mu\nu}^a \right]
\]

Quantum “effective action”:

\[
\int D[\psi, \bar{\psi}, A] e^{-S} = e^{-\Gamma}
\]

DSEs = quantum equations of motion:
instead of calculating n-point functions directly, derive eqs. of motion for them from path integral

\[
\frac{1}{1} - \frac{1}{1} = \frac{1}{1} + \frac{1}{1}
\]

\[
\frac{1}{1} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1}
\]

infinite many coupled eqs., in practice truncations: model / neglect higher n-point functions to obtain closed system

For reviews see:
Mesons

- Homogeneous **Bethe-Salpeter equation** for BS wave function:

\[
G \rightarrow_{p^2} -m^2 \quad \chi \quad = \quad K \chi
\]

- BS wave function only makes sense **onshell**, but homogeneous BSE = **eigenvalue equation**, can be solved for offshell momenta:

\[
K \psi_i = \lambda_i (P^2) \psi_i,
\]

\[
\lambda_i \rightarrow_{p^2} -m_i^2 \rightarrow 1
\]

Largest eigenvalue ⇔ ground state, smaller ones ⇔ excitations

Restricted by singularities in **quark propagator** (no physical threshold!):

- mesons: \( M < 2m_p \)
- baryons: \( M < 3m_p \)

\( m_p \sim 500 MeV \)
N($\frac{1}{2}^+$) and $\Delta(\frac{3}{2}^+$) channels hardly affected by ps, v diquarks

- all other channels:
  sc, av $\rightarrow$ masses too high
  sc, av, ps, v $\rightarrow$ masses too low

- not all eigenvalues extrapolate to masses below 2 GeV

- some are complex conjugate (but imaginary parts small), some split into 2 real branches: numerical or truncation artifact?
Form factors

Sketch of a generic electromagnetic form factor:

How can we calculate this from the quark level?

'trainbow-ladder'  \[ \rightarrow \]

quark-photon vertex

Faddeev amplitude

quark propagator
Form factors

Sketch of a generic electromagnetic form factor:

Microscopic decomposition of current matrix element: satisfies electromagnetic gauge invariance, consistent with baryon's Faddeev equation
Nucleon em. form factors

Three-body results:
all ingredients calculated,
model dependence shown by bands

- electric proton form factor: consistent with data, possible zero crossing
- magnetic form factors: missing pion effects at low $Q^2$
- Similar for axial & ps. FFs, $\Delta$ elastic and $N\rightarrow\Delta\gamma$ transition

GE, Fischer, EPJ A 48 (2012),
Sanchis-Alepuz et al., PRD 87 (2013),
Alkofer et al., Hyp. Int. 234 (2015)

⇒ “quark core without pion-cloud effects”
Nucleon em. form factors

Nucleon magnetic moments:
*isovector (p−n), isoscalar (p+n)*

\[ \mu^v \]
\[ \mu^s \]

But: pion-cloud cancels \( \Rightarrow \) quark core

Exp: \( \kappa^s = -0.12 \)
Calc: \( \kappa^s = -0.12(1) \) \( \boxed{\text{GE, PRD 84 (2011)}} \)
Pion form factor

![Graph showing pion form factor]

**Theory:**
- DSE
- Ball-Chiu
- Bare vertex
- VMD monopole

**Exp:**
- Amendolia et al.
- Ackermann et al.
- Brauel et al.
- Tadevosyan et al.
- Horn et al. 1
- Horn et al. 2
- Barkov et al.

- **Form factor from**
- **Timelike vector meson poles** automatically generated by quark-photon vertex BSE!

\[ \Gamma^\mu = \text{Ball-Chiu} \]  
(em. gauge invariance)

- **Transverse part** (vm. poles & dominance)

- **Form factor at large** \( Q^2 \)
  Chang, Cloet, Roberts, Schmidt, Tandy, PRL 111 (2013)

- **Include pion cloud effects:**
  GE, Fischer, Kubrak, Williams, in preparation

Pion cloud effects

- **Hadron level:**
  \( N\pi \) contributions to nucleon self-energy; charge radii diverge in chiral limit, \( \Delta \rightarrow N\pi \) decay cusps, etc.

- **Quark level:**
  \( \pi \) contributions to quark self-energy, effective \( \pi \) exchange between quarks; pion not elementary field!

- **Baryons:** pion effects reduce \( N, \Delta \) masses but also \( f_\pi \) (sets the scale) by similar amount: net effect small

- **Pion form factor:** photon also couples to pion (necessary for gauge invariance), \( \pi \) exchange in quark-photon vertex

Sanchis-Alepuz, Fischer, Kubrak, PLB 733 (2014)

Fischer, Nickel, Wambach, PRD 76 (2007)

GE, Fischer, Kubrak, Williams, in preparation
Axial form factors

• looks like magnetic form factors: missing structure at low \( Q^2 \Rightarrow g_A \) too small

• Timelike meson poles: 
  \( a_1 \) in \( G_A, \pi & \pi(1300) \) in \( G_P, G_{\pi NN} \)

• Goldberger-Treiman relation reproduced for all quark masses:

\[
G_A(0) = \frac{f_\pi}{M_N} G_{\pi NN}(0)
\]

GE & Fischer, EPJ A 48 (2012)
**Δ electromagnetic FFs**

Almost no experimental information since Δ unstable: \( \Delta \to N\pi \)

**Magnetic moment** \( \mu_\Delta \sim 3.5 \) with large errors (\( \Delta^+ \)).
But \( \Omega^- \) (spin 3/2, sss) is stable w.r.t strong interaction, magnetic moment \( |\mu_\Omega| = 3.6(1) \). Accidental?

\[
J^{\mu,\rho\sigma}(P,Q) = i \mathbb{P}^{\rho\alpha}(P_f) \left[ \left( F_1^* \gamma^\mu - F_2^* \frac{\sigma^{\mu\nu} Q^\nu}{2M_\Delta} \right) \delta^{\alpha\beta} - \left( F_3^* \gamma^\mu - F_4^* \frac{\sigma^{\mu\nu} Q^\nu}{2M_\Delta} \right) \frac{Q^\alpha Q^\beta}{4M_\Delta^2} \right] \mathbb{P}^{\beta\sigma}(P_i)
\]

Form factors at \( Q^2=0 \):

\[
G_{E_0}(0) = e_\Delta \quad \text{charge}
\]
\[
G_{E_2}(0) = Q \quad \text{electric quadrupole moment}
\]
\[
G_{M_1}(0) = \mu_\Delta \quad \text{magnetic dipole moment}
\]
\[
G_{M_3}(0) = 0 \quad \text{magnetic octupole moment}
\]

almost quark-mass independent, match \( \Omega^- \) magnetic moment

Nicmorus, GE, Alkofer, PRD 82 (2010)

Three-body results similar (except \( G_{M_3} \))

Sanchis-Alepuz, Alkofer, Williams, PRD 87 (2013)
### DSE / Faddeev landscape \( N \rightarrow N^*\gamma \)

<table>
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<tr>
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<th>Contact interaction</th>
<th>QCD-based model</th>
<th>DSE (RL)</th>
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<th>bRL</th>
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*Contact interaction, QCD-based model, DSE (RL), RL, bRL, bRL + 3q.*

(see backup slides for references)
**$N^*(1535)?$**

**Form factors:**
- no kinematic constraints
- CLAS data & toy parametrization with "$\rho$ bump"

...vs. helicity amplitudes in $[10^{-3} GeV^{-1/2}]$

kinematic zeros at $Q^2 = -(m_R \pm m)^2$

see also
Ramalho & Tsushima, PRD 84 (2011)
N*(1535): the recipe

- Calculate quark DSE and (pseudoscalar, vector) diquark BSEs & propagators in complex plane

\[ \begin{align*}
\text{pseudoscalar diquark} & \sim 1 \text{ GeV} \\
\text{vector diquark} & \sim 1.1 \text{ GeV}
\end{align*} \]

- Solve Faddeev equation, obtain N*(1535) mass and wave function

- Insert everything here and calculate transition form factor:

\[ \begin{align*}
\text{pseudoscalar diquark} & \sim 1 \text{ GeV} \\
\text{vector diquark} & \sim 1.1 \text{ GeV}
\end{align*} \]
Muons anomalous magnetic moment:

- **Total SM prediction deviates from exp. by \( \sim 3\sigma \)**

\[
\frac{g-2}{2} = \frac{\alpha}{\pi} \left[ F_1(q^2) - F_2(q^2) \frac{\sigma^{\mu\nu} q_{\mu\nu}}{2m^2} \right] u(p)
\]

- **Theory uncertainty dominated by QCD:**
  Is QCD contribution under control?

- **LBL amplitude:** ENJL & MD model results

\[ a_\mu \begin{bmatrix} 10^{-10} \end{bmatrix} \]

<table>
<thead>
<tr>
<th>Exp: ( 11 ) 659 208.9 ( (6.3) )</th>
<th>QED: ( 11 ) 658 471.9 ( (0.0) )</th>
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Gernot Eichmann (Uni Giessen)
**Muon g-2**

- **Muon anomalous magnetic moment:**
  - total SM prediction deviates from exp. by $\sim 3\sigma$
  
  \[
  \gamma(q) = i e \bar{u}(p') \left[ F_1(q^2) \gamma^\mu - F_2(q^2) \frac{\sigma^{\mu\nu} q_\nu}{2m} \right] u(p) \]

- Theory uncertainty dominated by QCD:
  - Is QCD contribution under control?

  ![Hadronic vacuum polarization](image1) ![Hadronic light-by-light scattering](image2)

- **LBL amplitude** at quark level, derived from **gauge invariance**:
  
  ![Quark Compton vertex](image3) ![Born terms](image4)

  - no double-counting, gauge invariant!
  - need to understand **structure of amplitude**

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