From QCD's n-point functions to nucleon resonances

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GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, 1606.09602, Prog. Part. Nucl. Phys. (in press)

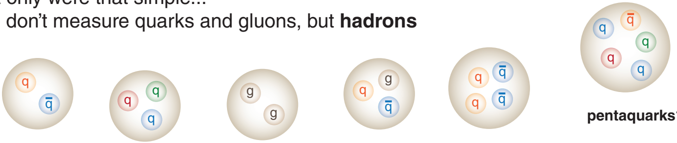
GE, Fischer, Sanchis-Alepuz, 1607.05748

XIIth Quark Confinement and the Hadron Spectrum
Thessaloniki, Greece
August 29, 2016

Introduction

QCD Lagrangian: $\mathcal{L} = \bar{\psi}\,(\partial\!\!\!/ + ig A\!\!\!/ + m)\,\psi + \frac{1}{4}\,F^{a}_{\mu\nu}\,F^{\mu\nu}_{a}$

- if it only were that simple...
  we don't measure quarks and gluons, but **hadrons**



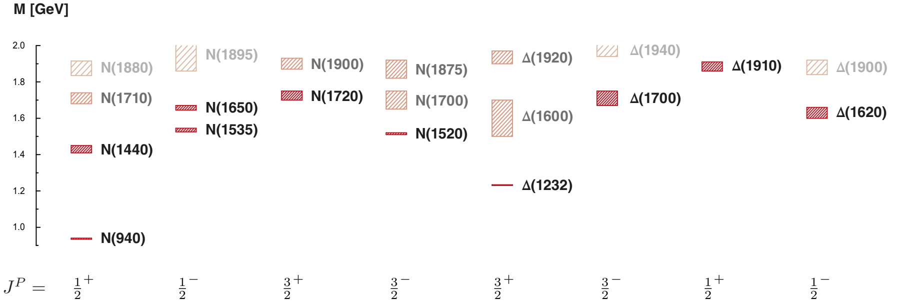
mesons **baryons** **glueballs?** **hybrids?** **tetraquarks?** **pentaquarks??**

- origin of **mass generation** and **confinement?**

	u	d	s	c	b	t
Current mass [GeV]	**0.003**	**0.005**	**0.1**	**1**	**4**	**175**
„Constituent“ mass [GeV]	**0.35**	**0.35**	**0.5**	**1.5**	**4.5**	**175**

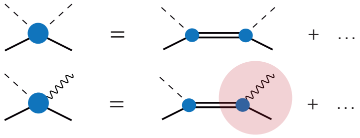
- need to understand **spectrum and interactions!**

Light baryon spectrum



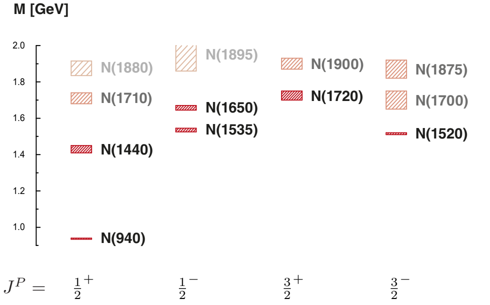
Experimentally extracted from πN scattering, meson photo- and electroproduction

- Nature of **Roper** (level ordering)?
- Three-quark vs. **quark-diquark?**
- "Quark core" vs. meson-baryon **coupled channel effects?**
- **Hybrid baryons?**



extraction of **transition form factors**

Light baryon spectrum



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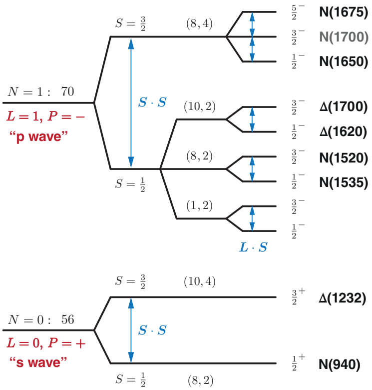
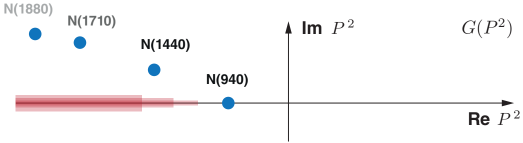
Nonrelativistic quark model:
$P = (-1)^L$

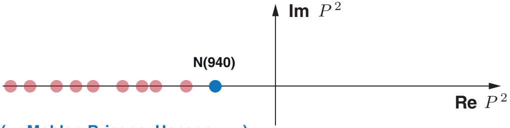


Lattice QCD

Extract baryon poles from (gauge-invariant) two-point correlators:

$$G(x-y) = \langle 0 \,|\, T \, \underbrace{[\Gamma_{\alpha\beta\gamma}\, \psi_\alpha\, \psi_\beta\, \psi_\gamma]\,(x)}_{J(x)} \, \underbrace{[\bar{\Gamma}_{\rho\sigma\tau}\, \bar{\psi}_\rho\, \bar{\psi}_\sigma\, \bar{\psi}_\tau]\,(y)}_{\bar{J}(y)} \,|\, 0 \,\rangle \;=\; \int \mathcal{D}[\psi,\bar{\psi},A]\, e^{-S}\, J(x)\, \bar{J}(y)$$



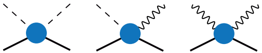


(→ Mohler, Briceno, Hansen, . . .)

- Spectral decomposition:

$$\sum_\lambda |\lambda\rangle\langle\lambda| \quad \rightarrow \quad \sum_\lambda \frac{\cdots}{P^2 + m_i^2}$$

- Same singularity structure in **any** n-point function:



- Pole in momentum space $\Longrightarrow$ exp. decay in Euclidean time

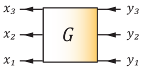
$$G(x-y) \;\rightarrow\; e^{-m\tau}$$

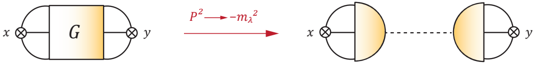
Bethe-Salpeter

Extract baryon poles from (gauge-invariant) two-point correlators:

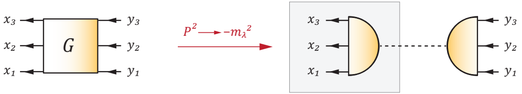
$$G(x-y) = \langle 0\,|\,T\,\underbrace{[\Gamma_{\alpha\beta\gamma}\,\psi_\alpha\,\psi_\beta\,\psi_\gamma]\,(x)}_{J(x)}\;\underbrace{[\bar\Gamma_{\rho\sigma\tau}\,\bar\psi_\rho\,\bar\psi_\sigma\,\bar\psi_\tau]\,(y)}_{\bar J(y)}\,|\,0\,\rangle \;=\; \int \mathcal{D}[\psi,\bar\psi,A]\,e^{-S}\,J(x)\,\bar J(y)$$

$$= \lim_{\substack{x_i \to x \\ y_i \to y}} \Gamma_{\alpha\beta\gamma}\,\bar\Gamma_{\rho\sigma\tau}\,\boxed{\langle 0\,|\,T\,\psi_\alpha(x_1)\,\psi_\beta(x_2)\,\psi_\gamma(x_3)\,\bar\psi_\rho(y_1)\,\bar\psi_\sigma(y_2)\,\bar\psi_\tau(y_3)\,|\,0\,\rangle}$$



$=$ 

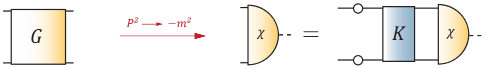
Alternative: extract **gauge-invariant** baryon poles from **gauge-dependent** quark 6-point function:



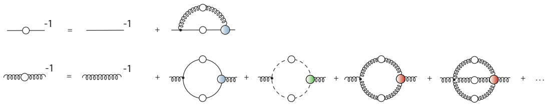
Bethe-Salpeter wave function:
residue at pole, contains all information about baryon

Bethe-Salpeter

- Homogeneous **Bethe-Salpeter equation** for BS wave function:



- Depends on QCD's n-point functions as input,
  satisfy **DSEs = quantum equations of motion**

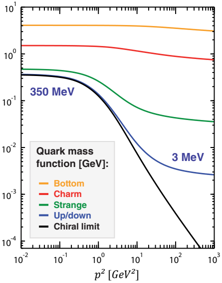


infinitely many coupled equations,
in practice truncations:
model / neglect higher
n-point functions to obtain
closed system

QCD's n-point functions

- **Quark propagator**

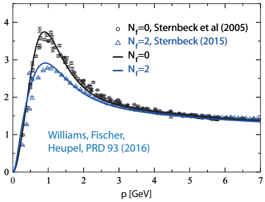
$$A(p^2)\,\big(i\not{p} + M(p^2)\big)^{-1}$$



Dynamical chiral symmetry breaking generates 'constituent-quark masses'

- **Gluon propagator**

$$\frac{D(p^2)}{p^2}\left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)$$



- **Three-gluon vertex**

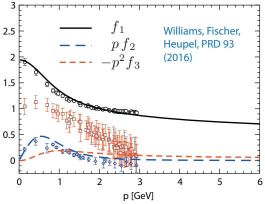
$$F_1\,[\,\delta^{\mu\nu}(p_1 - p_2)^\rho + \delta^{\nu\rho}(p_2 - p_3)^\mu + \delta^{\rho\mu}(p_3 - p_1)^\nu] + \dots$$

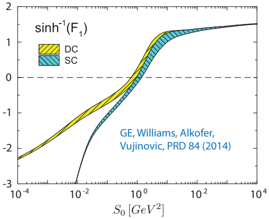
Agreement between lattice, DSE & FRG within reach

($\rightarrow$ **Sternbeck, Williams, Huber, Blum, Mitter, Cyrol, Campagnari, . . .**)

- **Quark-gluon vertex**

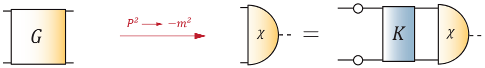
$$f_1\,\gamma^\mu + f_2\, i p^\mu + f_3\, p^\mu \not{p} + \dots$$



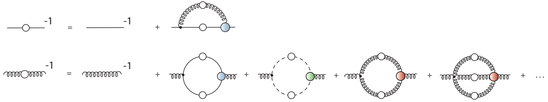


Bethe-Salpeter

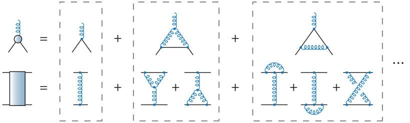
- Homogeneous **Bethe-Salpeter equation** for BS wave function:



- Depends on QCD's n-point functions as input, satisfy **DSEs = quantum equations of motion**

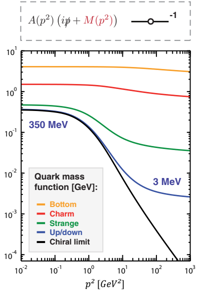


- Kernel can be derived in accordance with **chiral symmetry:**



→ **R. Williams**

- **Quark propagator**



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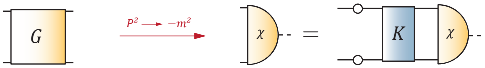

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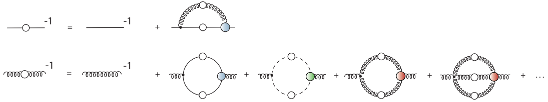

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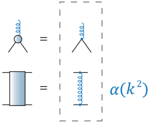
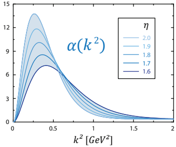




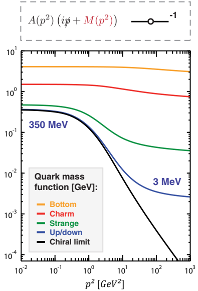
Rainbow-ladder:
effective gluon exchange

$$\alpha(k^2) = \alpha_{\mathrm{IR}}\left(\frac{k^2}{\Lambda^2}, \eta\right) + \alpha_{\mathrm{UV}}(k^2)$$

adjust scale Λ to observable,
keep width η as parameter

Maris, Tandy, PRC 60 (1999)

- **Quark propagator**



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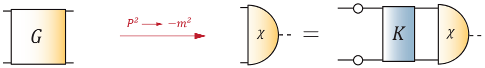

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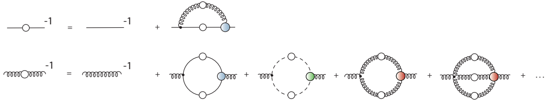

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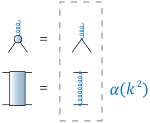

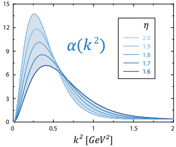

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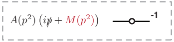
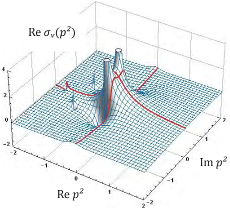
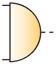




Calculated in **complex plane:** singularities pose restrictions (no physical threshold!)

Mesons

- The **pion** plays special role in hadron physics:
  quark-antiquark **bound state** ⇔ Goldstone boson of **spontaneous chiral symmetry breaking**



$$= \gamma_5 \left(f_1 + f_2 \not{P} + f_3 \not{q} + f_4 [\not{q}, \not{P}] \right) \otimes \text{Color} \otimes \text{Flavor}$$

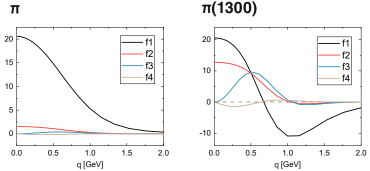
most general Dirac-Lorentz structure,
Lorentz-invariant dressing functions:

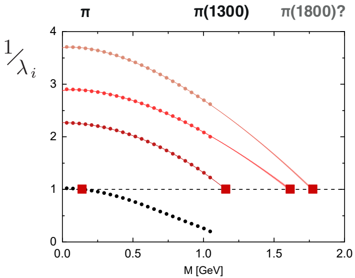
$$f_i = f_i(q^2, q \cdot P, P^2 = -m^2)$$

$\Longrightarrow$ pion is made of **s waves** and **p waves!**
(relative momentum ~ orbital angular momentum)

- Eigenvalue spectrum of BS kernel:
  Holl, Krassnigg, Roberts, PRC 70 (2004)

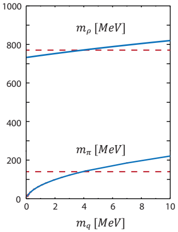
$$K\,\psi_i = \lambda_i(P^2)\,\psi_i\,, \qquad \lambda_i \xrightarrow{P^2 \longrightarrow -m_i^2} 1$$



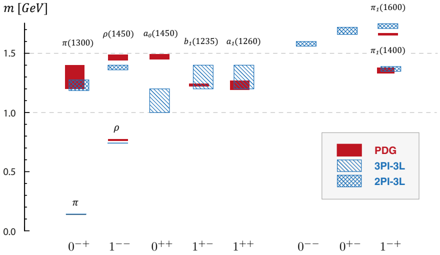


Mesons

- Pion is **Goldstone boson:** $m_\pi{}^2 \sim m_q$



- **Light meson spectrum** beyond rainbow-ladder



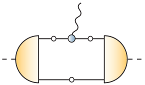
Williams, Fischer, Heupel, PRD 93 (2016)

GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, 1606.09602

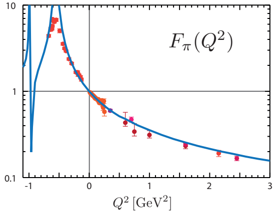
→ **C. Fischer**
R. Williams
T. Hilger

- **Pion electromagnetic form factor:**

Maris & Tandy, PRC 61 (2000), Chang, Cloet, Roberts, Schmidt, Tandy, PRL 111 (2013)



Timelike vector meson poles automatically generated in quark-photon vertex!

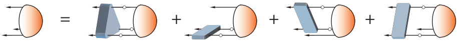


A. Krassnigg, Schladming 2010

Baryons

- Covariant Faddeev equation for **baryons:**
  keep 2-body interactions & rainbow-ladder,
  but no further approximations: $M_N = 0.94\,\mathrm{GeV}$

  GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010), GE, PRD 84 (2011)



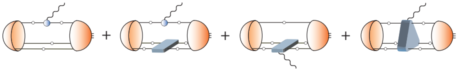
  Relativistic bound states:
  64 / 128 tensor structures for nucleon / Δ

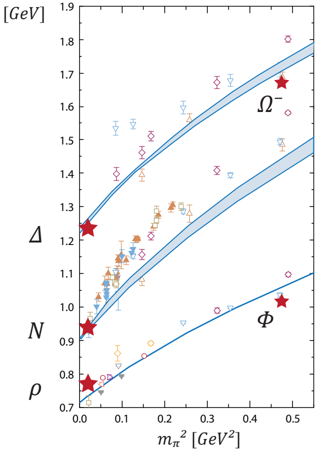
- Octet & decuplet baryons, pion cloud effects,
  first steps beyond rainbow-ladder

  Sanchis-Alepuz, Fischer, PRD 90 (2014), Sanchis-Alepuz, Fischer, Kubrak, PLB 733 (2014), Sanchis-Alepuz, Williams PLB 749 (2015)

- **Baryon form factors:**
  nucleon and Δ FFs, $N\to\Delta\gamma$ transition

  GE, PRD 84 (2011), Sanchis-Alepuz, Williams, Alkofer, PRD 87 (2013), Alkofer, GE, Sanchis-Alepuz, Williams, Hyp. Int. 234 (2015)

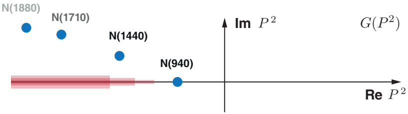




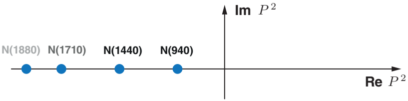
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, 1606.09602

Resonances?

Branch cuts & widths generated by
meson-baryon interactions: Roper $\rightarrow N\pi$, etc.



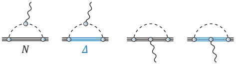
Without them: **bound states without widths**



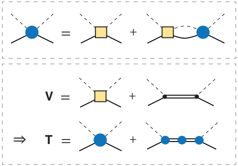
Difficult to implement at **quark-gluon level:**
complicated topologies beyond rainbow-ladder

Different phenomenological pictures
how this could happen:

- **'pion-cloud effects'** affect masses and form factors in light-quark region



- **dynamical generation of resonances:** start with 'bare' seed, hadronic interactions produce new poles



e.g.
Suzuki et al.,
PRL 104 (2010)

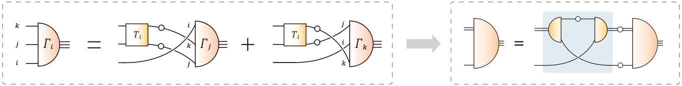
- Three-quark vs. five-quark / molecular components

Diquarks?

- Suggested to resolve **'missing resonances'** in quark model:
  fewer degrees of freedom ⇒ fewer excitations

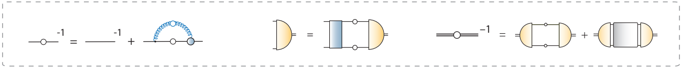
Anselmino et al., Rev. Mod. Phys. 65 (1993),
Klempt, Richard, Rev. Mod. Phys. 82 (2010)

- QCD version: assume qq scattering matrix as sum of diquark correlations
  ⇒ three-body equation simplifies to **quark-diquark BSE**



Oettel, Hellstern, Alkofer, Reinhardt, PRC 58 (1998),
Cloet et al., FBS 46 (2009),
Segovia et al., FBS 55 (2014)

Quark exchange between quark & diquark binds nucleon.
Gluons absorbed in building blocks, to be calculated in advance:



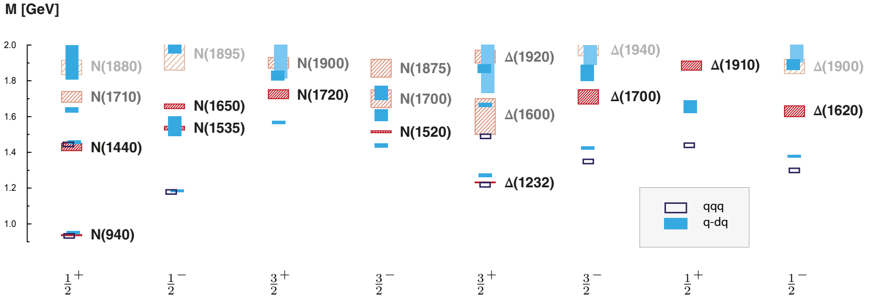
GE, Krassnigg, Schwinzerl, Alkofer, Ann. Phys. 323 (2008),
GE, Cloet, Alkofer, Krassnigg, Roberts, PRC 79 (2009),
Nicmorus, GE, Alkofer, PRD 82 (2010)

Rainbow-ladder: **scalar diquark** ~ **800 MeV, axialvector diquark** ~ **1 GeV**

- N and Δ masses & form factors very similar in quark-diquark and three-quark approach:
  quark-diquark approximation is good. → What about other channels?

Baryon spectrum I

Three-quark vs. quark-diquark in rainbow-ladder: GE, Fischer, Sanchis-Alepuz, 1607.05748



- **Three-body and quark-diquark results agree** (where available): N, Δ, Roper, N(1535)
- Number of levels compatible with experiment: **no states missing**
- N, Δ and their 1st excitations (including **Roper**) agree with experiment
- But remaining states too low ⇒ level ordering between Roper and N(1535) is wrong

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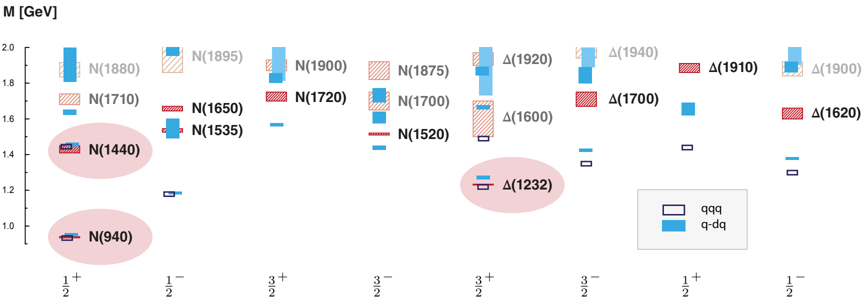

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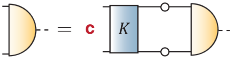
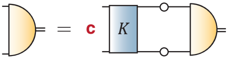
The role of diquarks

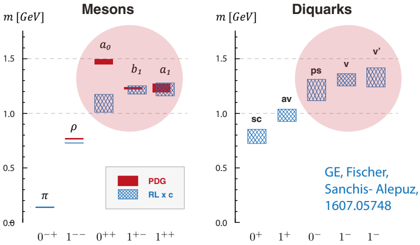
Mesons and ‘diquark’ properties closely related: after taking Dirac, color & flavor traces, only factor 1/2 remains ⇒ **diquarks ‘less bound’ than mesons**





Pseudoscalar & vector mesons already good in rainbow-ladder ⇔ **Scalar & axialvector diquarks** sufficient for nucleon and Δ

Scalar & axialvector mesons too light, repulsion beyond RL ⇔ **Pseudoscalar & vector diquarks** important for remaining channels

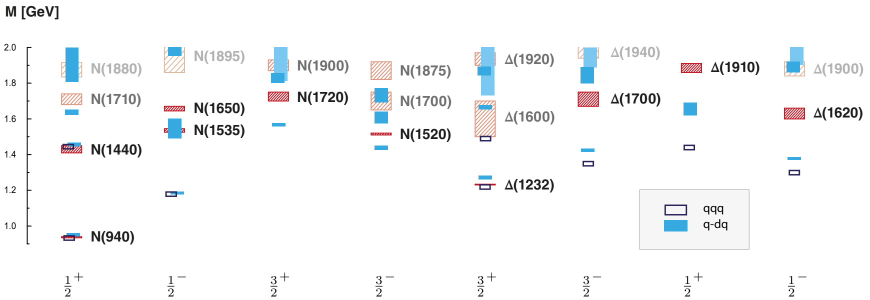


Simple strategy to emulate **beyond-RL effects:**
Roberts, Chang, Cloet, Roberts, FBS 51 (2011), Chen et al., FBS 53 (2012)

- Insert factor **$0 < c < 1$** in ‘bad’ meson and diquark channels ⇒ increases masses
- Fixed in the meson sector ($\rho - a_1$ splitting): c = 0.35

Baryon spectrum I

Three-quark vs. quark-diquark in rainbow-ladder: GE, Fischer, Sanchis-Alepuz, 1607.05748



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- N, Δ and their 1st excitations (including **Roper**) agree with experiment
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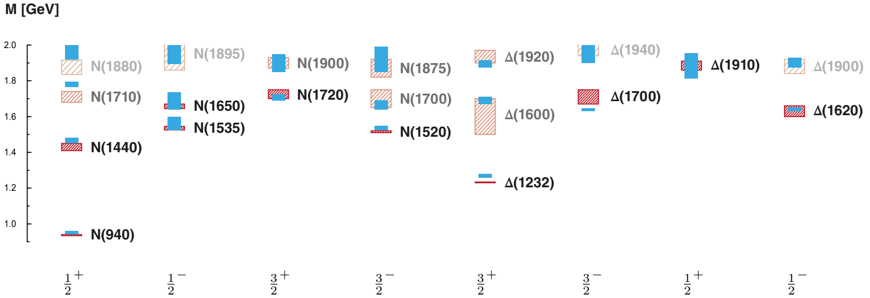
Baryon spectrum II

Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, 1607.05748

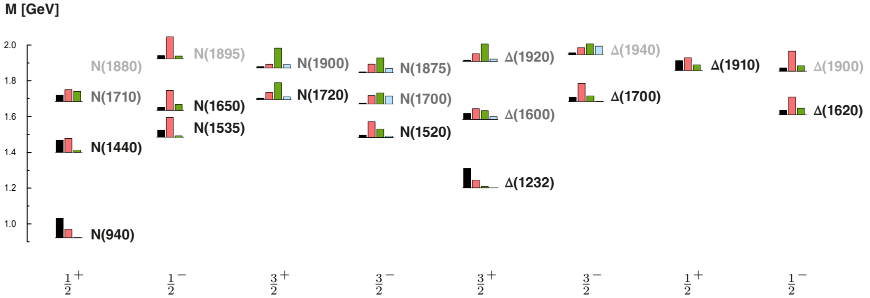


- **Quantitative agreement with experiment**
- N($\frac{1}{2}^+$) and Δ($\frac{3}{2}^+$) channels not affected, but remaining ones were polluted by ps + v diquarks
- Correct level ordering between **Roper and N(1535)**

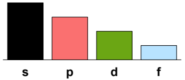
- Scale Λ set by f_π
- Current-quark mass set by m_π
- c adjusted to ρ–a_1 splitting
- η doesn't change much

Baryon spectrum II

Quark-diquark with reduced pseudoscalar + vector diquarks:



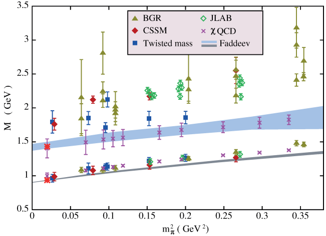
Partial-wave content:



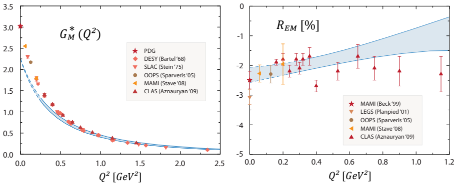
- N and Δ ground states dominated by **s waves,** negative-parity states typically by **p waves** (as expected)
- But 'quark-model forbidden' contributions are always present, e.g. **Roper: dominated by p waves** ⇒ **relativity is important!**

Structure properties

- **Current-mass evolution** of Roper similar to nucleon. Lattice?
  GE, Fischer, Sanchis-Alepuz, 1607.05748



- $\gamma N \to \Delta$ **transition form factors:**
  GE, Nicmorus, PRD 85 (2012)



Discrepancies mainly in **magnetic dipole (G_M^*):**
"Core + 25% pion cloud"

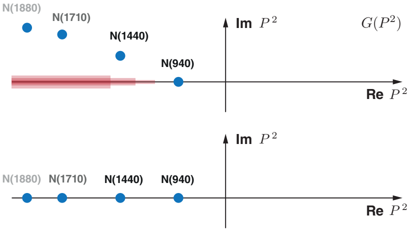
Electric quadrupole ratio
small & negative, encodes deformation.
No pion cloud necessary: **OAM** from **p waves!**

First three-body results similar
Alkofer, GE, Sanchis-Alepuz, Williams, Hyp. Int. 234 (2015)

- All signatures of 1st **radial excitation:** partial-wave content, zero crossing

- **Roper transition form factors** in qualitative agreement with experiment
  Segovia et al., PRL 115 (2015)

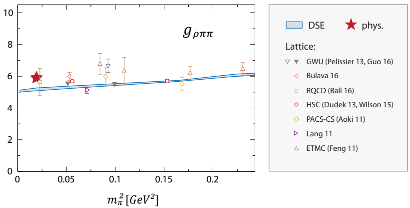
So what does it mean?

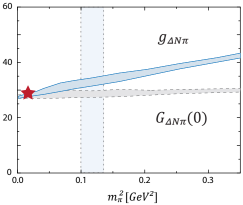


Results favor ‘mild’ scenario:

- spectrum generated by quark-gluon interactions
- meson-baryon effects would merely shift poles into complex plane
- Effects on masses? Scale set by f_π , but pion-cloud affects f_π too so only ‘non-trivial effects’ visible
- Will be interesting to study **transition form factors**

Note: **‘bound states without widths’** doesn’t mean that $\rho \to \pi\pi,\ \Delta \to N\pi, \ldots$ decays are zero!!



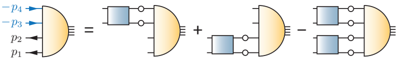


Mader, GE, Blank, Krassnigg, PRD 84 (2011),

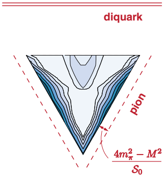
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, 1606.09602

Tetraquarks *are* resonances

- **Light scalar mesons** σ, κ, a_0, f_0 as **tetraquarks:**
  solution of four-body equation reproduces mass pattern
  GE, Fischer, Heupel, PLB 753 (2016)

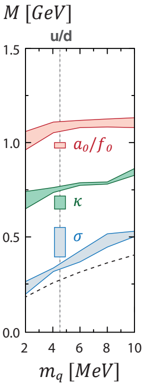


BSE dynamically generates **meson poles** in wave function,
drive σ mass from 1.5 GeV to ~350 MeV



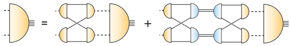
Four quarks rearrange
to **"meson molecule"**

Tetraquarks are "dynamically
generated **resonances**"
(but from the quark level!)



→ **C. Fischer**

- Similar in **meson-meson / diquark-antidiquark** approximation
  (analogue of quark-diquark for baryons) Heupel, GE, Fischer, PLB 718 (2012)



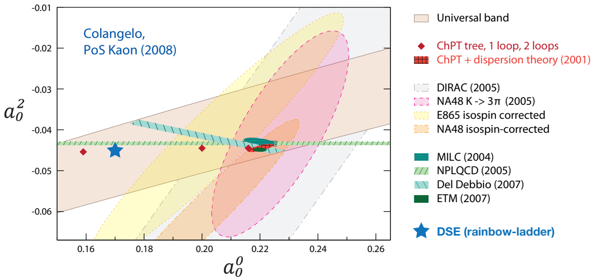
. . . and more

Scattering amplitudes from quark level:

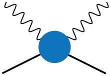
- **$\pi\pi$ scattering**

  Bicudo et al., PRD 65 (2002),

  Cotanch, Maris, PRD 66 (2002)



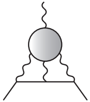
- **Nucleon Compton scattering**



  GE, Fischer, PRD 85 (2012) & PRD 87 (2013), GE, FBS 57 (2016)

- **Hadronic light-by-light scattering**

  Goecke, Fischer, Williams, PLB 704 (2011), GE, Fischer, Heupel, PRD 92 (2015)



Summary

Progress with **Dyson-Schwinger, Bethe-Salpeter** and **Faddeev equations:**

- **Baryon spectrum** quantitatively reproduced

- **Quark-diquark** and **three-quark** spectrum very similar:

  Quark-diquark with **sc, av, ps, v** ~ three-quark **in RL**

  Quark diquark with **sc, av, ps, v** ~ three-quark **beyond RL?**

- Still **"bound states without widths"**,
  because meson-baryon interactions difficult to implement at quark-gluon level.
  But:
    - would mainly shift poles into complex plane (?)
    - decay properties are calculable
    - tetraquarks are genuine resonances (even in RL!)

- For a recent review see:
  GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, arXiv:1606.09602, Prog. Part. Nucl. Phys. (in press)

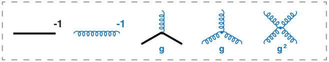
Thank you!

Backup slides

... to Dyson-Schwinger equations

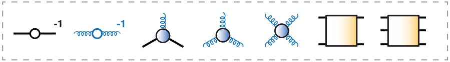
QCD's classical action:

$$S = \int d^4x \left[\bar\psi \left(\partial\!\!\!/ + igA\!\!\!/ + m \right) \psi + \tfrac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \right]$$

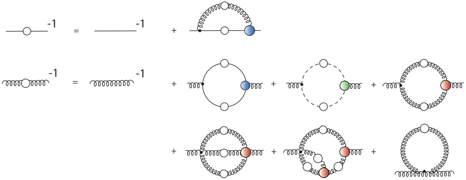
$=$ 

Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar\psi, A]\, e^{-S} = e^{-\Gamma}$$



DSEs = quantum equations of motion:
instead of calculating n-point functions directly, derive eqs. of motion for them from path integral



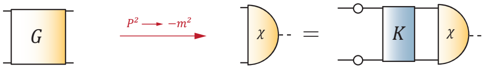
infinitely many coupled eqs., in practice truncations: model / neglect higher n-point functions to obtain **closed system**

For reviews see:

Roberts, Williams, Prog. Part. Nucl. Phys. 33 (1994),
Alkofer, von Smekal, Phys. Rept. 353 (2001)
Fischer, J. Phys. G32 (2006)

Mesons

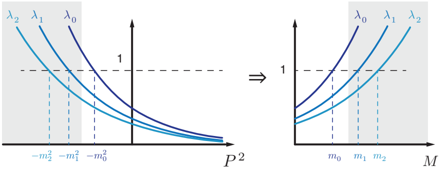
- Homogeneous **Bethe-Salpeter equation** for BS wave function:

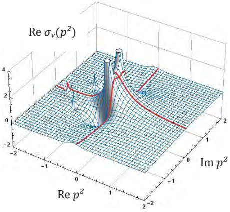


- BS wave function only makes sense **onshell**, but homogeneous BSE = **eigenvalue equation**, can be solved for offshell momenta:

$$K\,\psi_i = \lambda_i(P^2)\,\psi_i\,, \qquad \lambda_i \xrightarrow{P^2 \,\longrightarrow\, -m_i^2} 1$$

Largest eigenvalue ⇔ ground state,
smaller ones ⇔ excitations



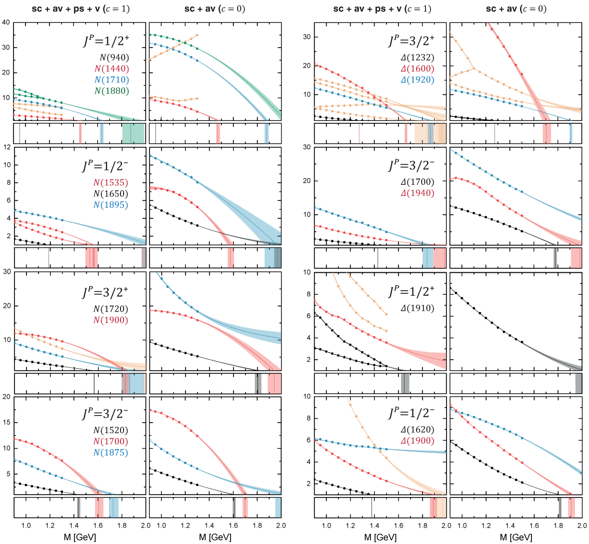


Restricted by singularities in **quark propagator** (no **physical threshold!**):

mesons: $M < 2m_p$
baryons: $M < 3m_p$

$m_p \sim 500 MeV$

Eigenvalue spectra



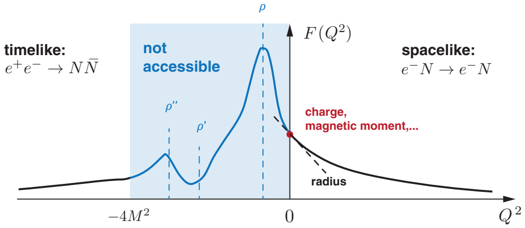
GE, Fischer, Sanchis-Alepuz, 1607.05748

- N($\frac{1}{2}^+$) and Δ($\frac{3}{2}^+$) channels hardly affected by ps, v diquarks

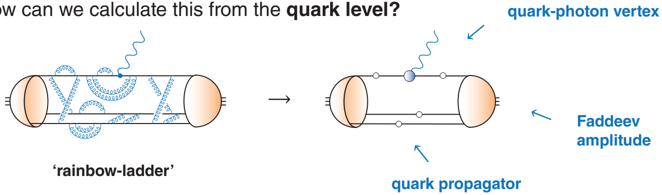
- all other channels:
  sc, av → masses too high
  sc, av, ps, v → masses too low
- not all eigenvalues extrapolate to masses below 2 GeV
- some are complex conjugate (but imaginary parts small), some split into 2 real branches: numerical or truncation artifact?

Form factors

Sketch of a generic electromagnetic form factor:



How can we calculate this from the **quark level?**



Form factors

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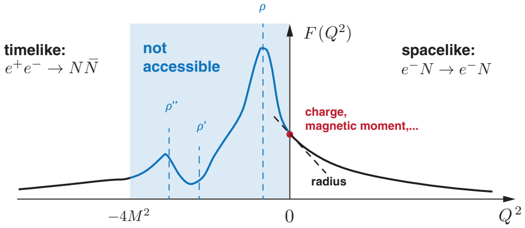
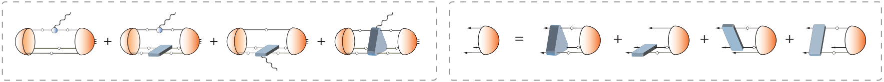
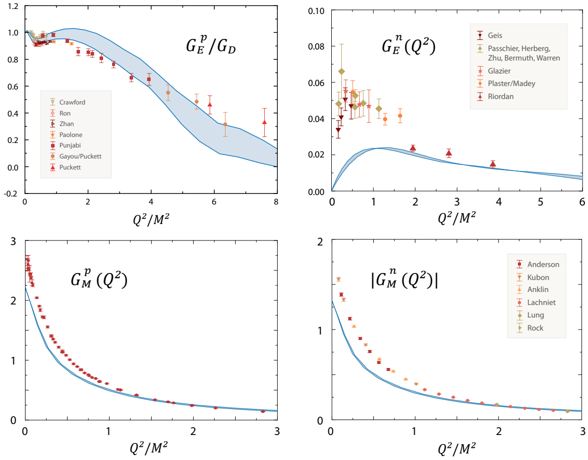


Microscopic decomposition of current matrix element:
satisfies **electromagnetic gauge invariance,** consistent with baryon's Faddeev equation



Nucleon em. form factors



Three-body results:
all ingredients calculated,
model dependence shown
by bands GE, PRD 84 (2011)

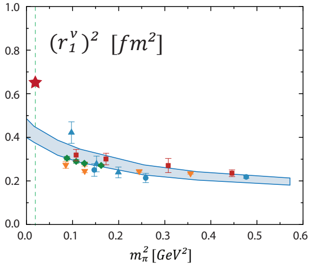
- **electric proton form factor:** consistent with data, possible zero crossing
- **magnetic form factors:** missing pion effects at low Q^2
- Similar for axial & ps. FFs, Δ elastic and $N \to \Delta\gamma$ transition
  GE, Fischer, EPJ A 48 (2012),
  Sanchis-Alepuz et al., PRD 87 (2013),
  Alkofer et al., Hyp. Int. 234 (2015)

$\Rightarrow$ **“quark core without pion-cloud effects”**

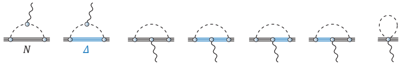
Nucleon em. form factors

Nucleon charge radii:

isovector (p-n) Dirac (F1) radius

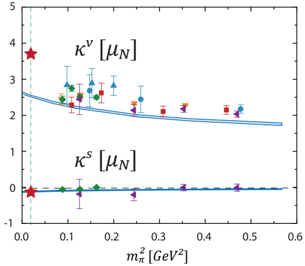


- **Pion-cloud effects** missing (⇒ divergence!), agreement with lattice at larger quark masses.



Nucleon magnetic moments:

isovector (p-n), isoscalar (p+n)



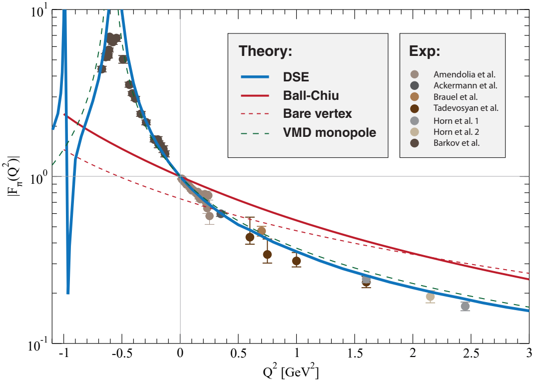
- **But:** pion-cloud **cancels** in κ^s ⇔ **quark core**

Exp: $\kappa^s = -0.12$

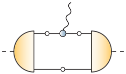
Calc: $\kappa^s = -0.12(1)$ **!!**

GE, PRD 84 (2011)

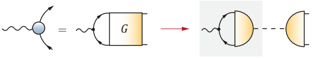
Pion form factor



A. Krassnigg (Schladming 2010),
Maris & Tandy, Nucl. Phys. Proc. Suppl. 161 (2006)

- Form factor from 

- **Timelike vector meson poles** automatically generated by quark-photon vertex BSE!



$\Rightarrow \Gamma^\mu =$ **Ball-Chiu** (em. gauge invariance)

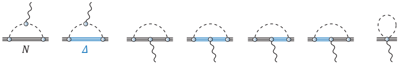
$+$ **Transverse part** (vm. poles & dominance)

- Form factor at large Q^2
  Chang, Cloet, Roberts, Schmidt, Tandy, PRL 111 (2013)

- Include **pion cloud** effects:
  GE, Fischer, Kubrak, Williams, in preparation

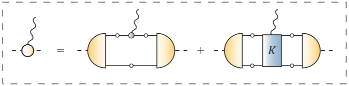
Pion cloud effects

- **Hadron level:**
  $N\pi$ contributions to nucleon self-energy;
  charge radii diverge in chiral limit,
  $\Delta \to N\pi$ decay cusps, etc.

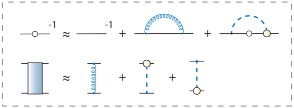


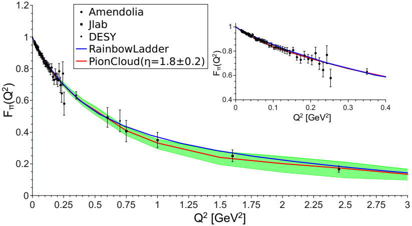
- **Baryons:** pion effects reduce N, Δ masses but also f_π (sets the scale) by similar amount: net effect small Sanchis-Alepuz, Fischer, Kubrak, PLB 733 (2014)

- **Pion form factor:** photon also couples to pion (necessary for gauge invariance), π exchange in quark-photon vertex



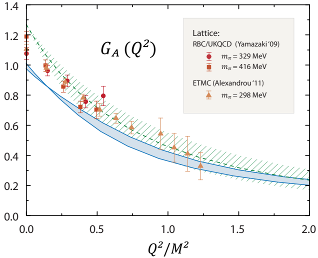
- **Quark level:**
  π contributions to quark self-energy,
  effective π exchange between quarks;
  pion not elementary field!
  Fischer, Nickel, Wambach, PRD 76 (2007)





GE, Fischer, Kubrak, Williams, in preparation

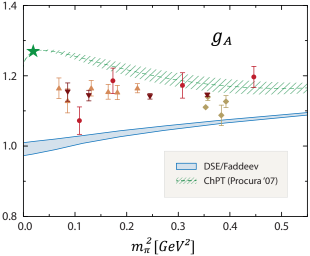
Axial form factors

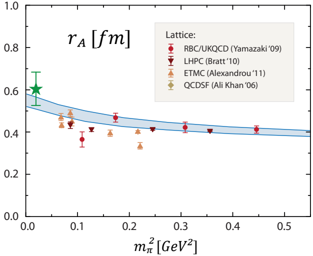


- looks like magnetic form factors:
  missing structure at low Q^2 $\Rightarrow$ g_A **too small**
- **Timelike meson poles:**
  a_1 in G_A, π & $\pi(1300)$ in G_P , $G_{\pi NN}$
- **Goldberger-Treiman relation**
  reproduced for **all** quark masses:

$$G_A(0) = \frac{f_\pi}{M_N}\,G_{\pi NN}(0)$$

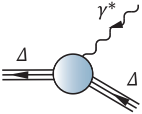
GE & Fischer, EPJ A 48 (2012)





Δ electromagnetic FFs

Almost no experimental information since Δ unstable: $\Delta \to N\pi$

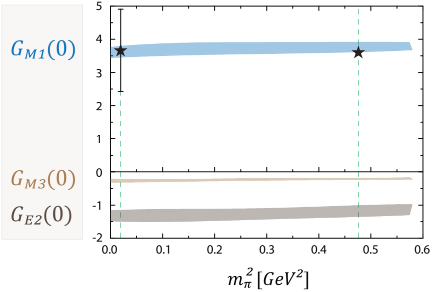


Magnetic moment $\mu_\Delta \sim 3.5$ with large errors (Δ^+).
But Ω^- (spin 3/2, *sss*) is stable w.r.t strong interaction,
magnetic moment $|\mu_\Omega| = 3.6(1)$. Accidental?

$$J^{\mu,\rho\sigma}(P,Q) = i\,\mathbb{P}^{\rho\alpha}(P_f)\left[\left(F_1^\star\,\gamma^\mu - F_2^\star\,\frac{\sigma^{\mu\nu}Q^\nu}{2M_\Delta}\right)\delta^{\alpha\beta} - \left(F_3^\star\,\gamma^\mu - F_4^\star\,\frac{\sigma^{\mu\nu}Q^\nu}{2M_\Delta}\right)\frac{Q^\alpha Q^\beta}{4M_\Delta^2}\right]\mathbb{P}^{\beta\sigma}(P_i)$$

Form factors at $Q^2=0$:

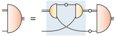
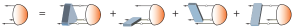
$G_{E_0}(0) = e_\Delta$	charge
$G_{E_2}(0) = \mathcal{Q}$	electric quadrupole moment
$G_{M_1}(0) = \mu_\Delta$	magnetic dipole moment
$G_{M_3}(0) = \mathcal{O}$	magnetic octupole moment



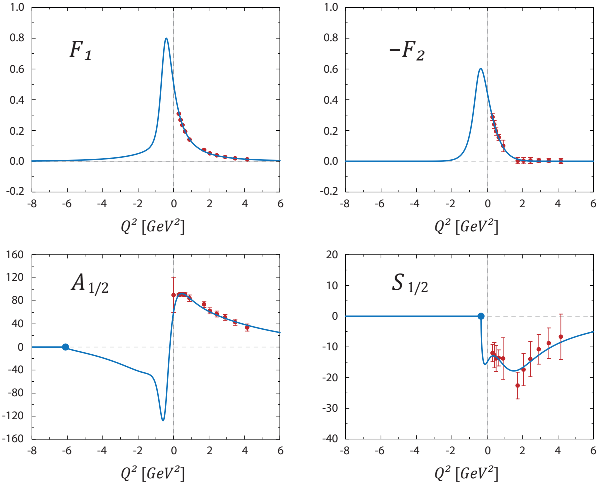
almost quark-mass independent,
match Ω^- magnetic moment
Nicmorus, GE, Alkofer, PRD 82 (2010)

Three-body results similar (except G_{M_3})
Sanchis-Alepuz, Alkofer, Williams, PRD 87 (2013)

DSE / Faddeev landscape $N \to N^*\gamma$

	Quark-diquark 			Three-quark 		
	Contact interaction	**QCD-based model**	**DSE (RL)**	**RL**	**bRL**	**bRL + 3q**
N, Δ masses	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\dots$
N, Δ em. FFs	$\surd$	$\surd$	$\surd$	$\surd$		
$N \to \Delta\gamma$	$\surd$	$\surd$	$\surd$	$\dots$		
Roper	$\surd$	$\surd$		$\dots$		
$N \to N^*\gamma$	$\surd$	$\surd$		$\dots$		
$N^*(1535), \dots$	$\dots$	$\dots$		$\dots$	$\dots$	
$N \to N^*\gamma$	$\dots$	$\dots$				

N*(1535)?



Form factors:
no kinematic constraints

CLAS data &
toy parametrization
with "ρ bump"

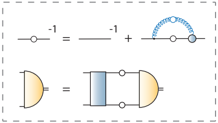
...vs. **helicity amplitudes**
in $[10^{-3} GeV^{-1/2}]$

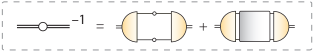
kinematic zeros at
$Q^2 = -(m_R \pm m)^2$

see also
Ramalho & Tsushima, PRD 84 (2011)

N*(1535): the recipe

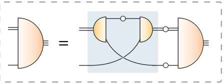
- Calculate quark DSE and **(pseudoscalar, vector)** diquark BSEs & propagators in complex plane



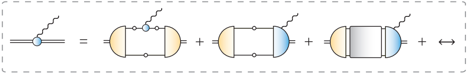


pseudoscalar diquark ~ 1 GeV
vector diquark ~ 1.1 GeV

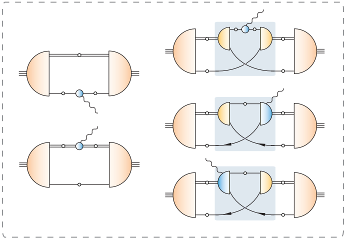
- Solve Faddeev equation, obtain N*(1535) mass and wave function



- Calculate quark-photon and **(pseudoscalar, vector scalar, axialvector)** diquark-photon vertices

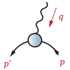


- Insert everything here and calculate **transition form factor:**



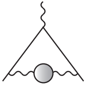
Muon g-2

- **Muon anomalous magnetic moment:** total SM prediction deviates from exp. by ~3σ

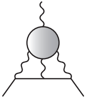


$$= ie\,\bar{u}(p')\left[F_1(q^2)\,\gamma^\mu - F_2(q^2)\,\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

- Theory uncertainty dominated by **QCD:** Is QCD contribution under control?



Hadronic vacuum polarization

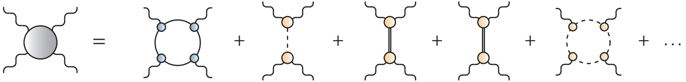


Hadronic light-by-light scattering

Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

$a_\mu\ [10^{-10}]$			
Exp:	11 659 208.9	(6.3)	
QED:	11 658 471.9	(0.0)	
EW:	15.3	(0.2)	
Hadronic:			
• VP (LO+HO)	685.1	(4.3)	
• **LBL**	**10.5**	**(2.6)**	**?**
SM:	11 659 182.8	(4.9)	
Diff:	26.1	(8.0)	

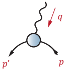
- **LbL amplitude:** ENJL & MD model results
Bijnens 1995, Hakayawa 1995, Knecht 2002, Melnikov 2004, Prades 2009, Jegerlehner 2009, Pauk 2014



Quark loop	pseudoscalar exchange	scalar exchange	axialvector exchange	π, K loop	
2	8 ... 11	-1	2	-2	$(\times 10^{-10})$

Muon g-2

- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by ~3σ



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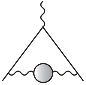

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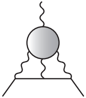

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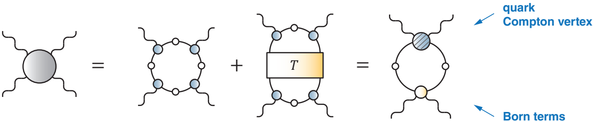
- **LbL amplitude** at quark level, derived from **gauge invariance:**
  GE, Fischer, PRD 85 (2012), Goecke, Fischer, Williams, PRD 87 (2013)



- **no double-counting, gauge invariant!**
- need to understand **structure of amplitude**
  GE, Fischer, Heupel, PRD 92 (2015)