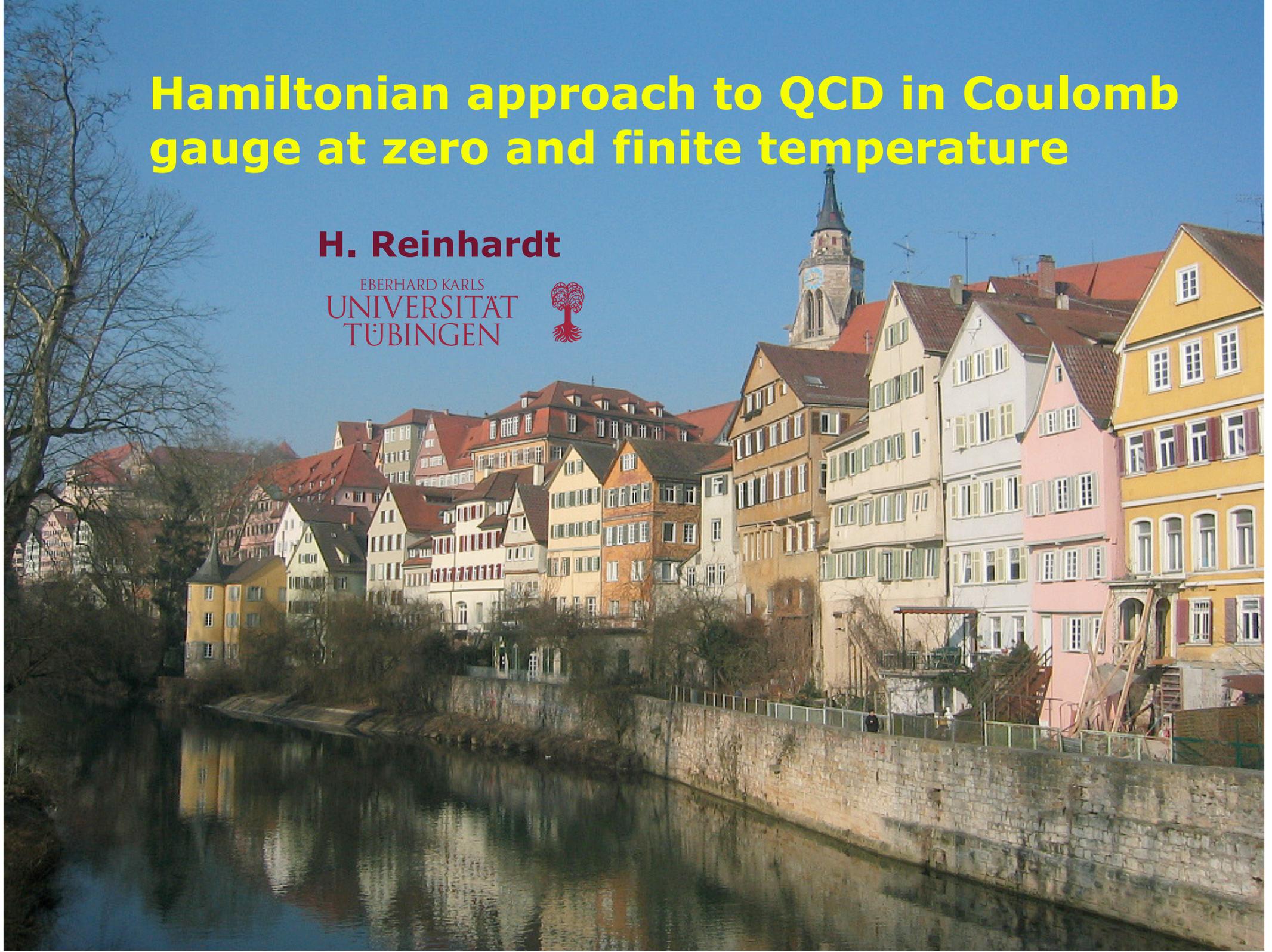


Hamiltonian approach to QCD in Coulomb gauge at zero and finite temperature

H. Reinhardt

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN



collaborators:

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN



G. Burgio, D. Campagnari, J. Heffner, M. Quandt,

P. Vastag, H. Vogt, E. Ebadati

Hamiltonian approach to QCD in Coulomb gauge

- introduction
- Coulomb string tension: center vortices
- quark sector
- finite T by compactification of a spatial dimension
 - chiral & dual quark condensate
- continuum & lattice
- conclusions

Hamiltonian approach to YMT in Coulomb gauge $\partial A = 0$

$$H = \frac{1}{2} \int (J^{-1} \Pi J \Pi + B^2) + H_C \quad \Pi = \delta / i \delta A$$

Christ and Lee

$$J(A^\perp) = \text{Det}(-D\partial) \quad D^{ab} = \delta^{ab} \partial + g f^{abc} A^c$$

$$H_C = \frac{1}{2} \int J^{-1} \rho J (-D\partial)^{-1} (-\partial^2) (-D\partial)^{-1} \rho \quad \text{Coulomb term}$$

color charge density $\rho^a = -f^{abc} A^b \Pi^c + \rho_m^a$

$$\langle \phi | \dots | \psi \rangle = \int DA J(A) \phi^*(A) \dots \psi(A)$$

$$H\psi[A] = E\psi[A]$$

Variational approach to YMT

- Gaussian ansatz,

$$\Psi(A) = \exp\left[-\frac{1}{2} \int dx dy A(x) \omega(x, y) A(y)\right]$$

D. Schütte 1984

.....

A. Szczepaniak & E. Swanson 2002

C. Feuchter & H. R. 2004

- ansatz
- FP determinant
- renormalization

- Greensite, Matevosyan, Olejnik, Quandt, Reinhardt, Szczepaniak, PRD83

Variational approach to YMT

■ trial ansatz

C. Feuchter & H. R. PRD70(2004)

$$\Psi(A) = \frac{1}{\sqrt{\text{Det}(-D\partial)}} \exp\left[-\frac{1}{2} \int dx dy A(x) \omega(x, y) A(y)\right]$$

gluon propagator

$$\langle A(x) A(y) \rangle = (2\omega(x, y))^{-1}$$

variational kernel

$\omega(x, x')$

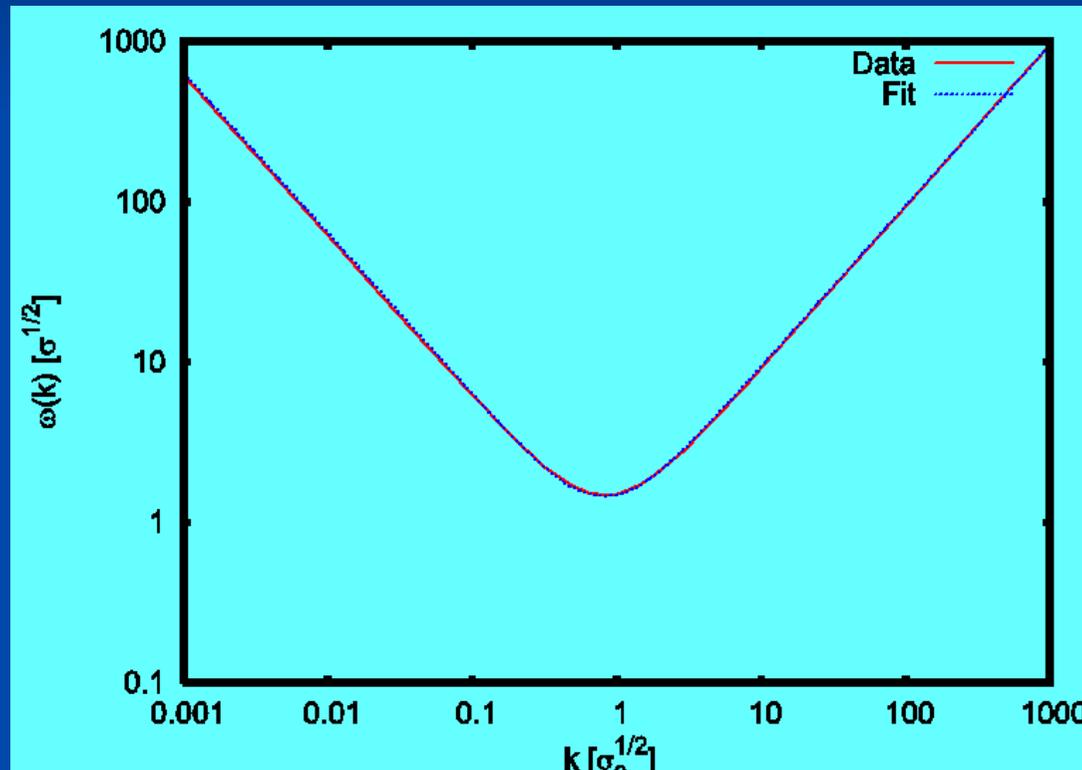
determined from

$$\langle \Psi | H | \Psi \rangle \rightarrow \min$$

Numerical results

gluon energy

D. Epple, H. R., W. Schleifenbaum, PRD
75 (2007)



IR: $\omega(k) \sim 1/k$

UV: $\omega(k) \sim k$

Static gluon propagator in D=3+1

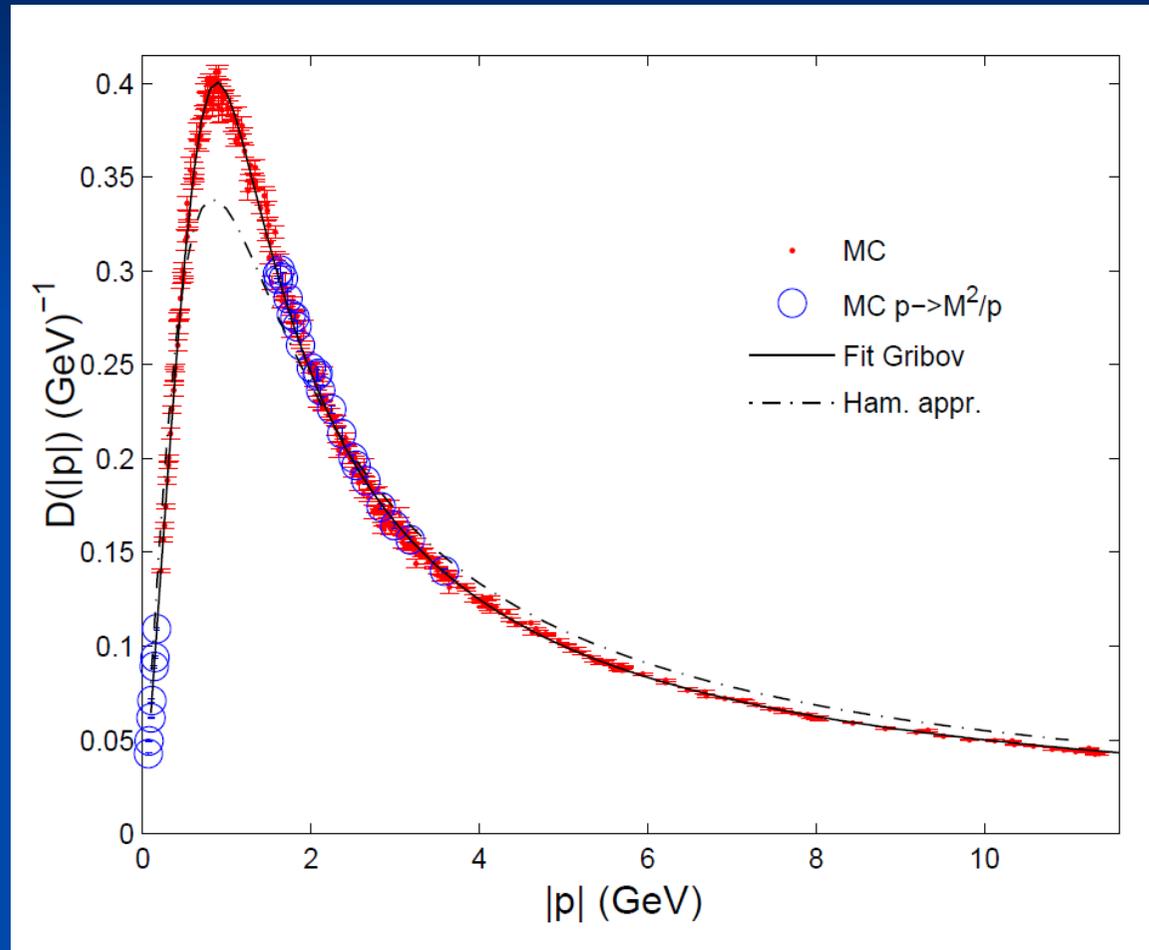
$$D(k) = (2\omega(k))^{-1}$$

Gribov's formula

$$\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}}$$

$$M = 0.88 \text{ GeV}$$

missing strength in
mid momentum regime:
missing gluon loop



G. Burgio, M.Quandt , H.R., **PRL102(2009)**

Variational approach to YMT with non-Gaussian wave functional

D. Campagnari & H.R,
Phys.Rev.D82(2010)
Phys.Rev.D92(2015)

wave functional

$$|\psi[A]|^2 = \exp(-S[A])$$

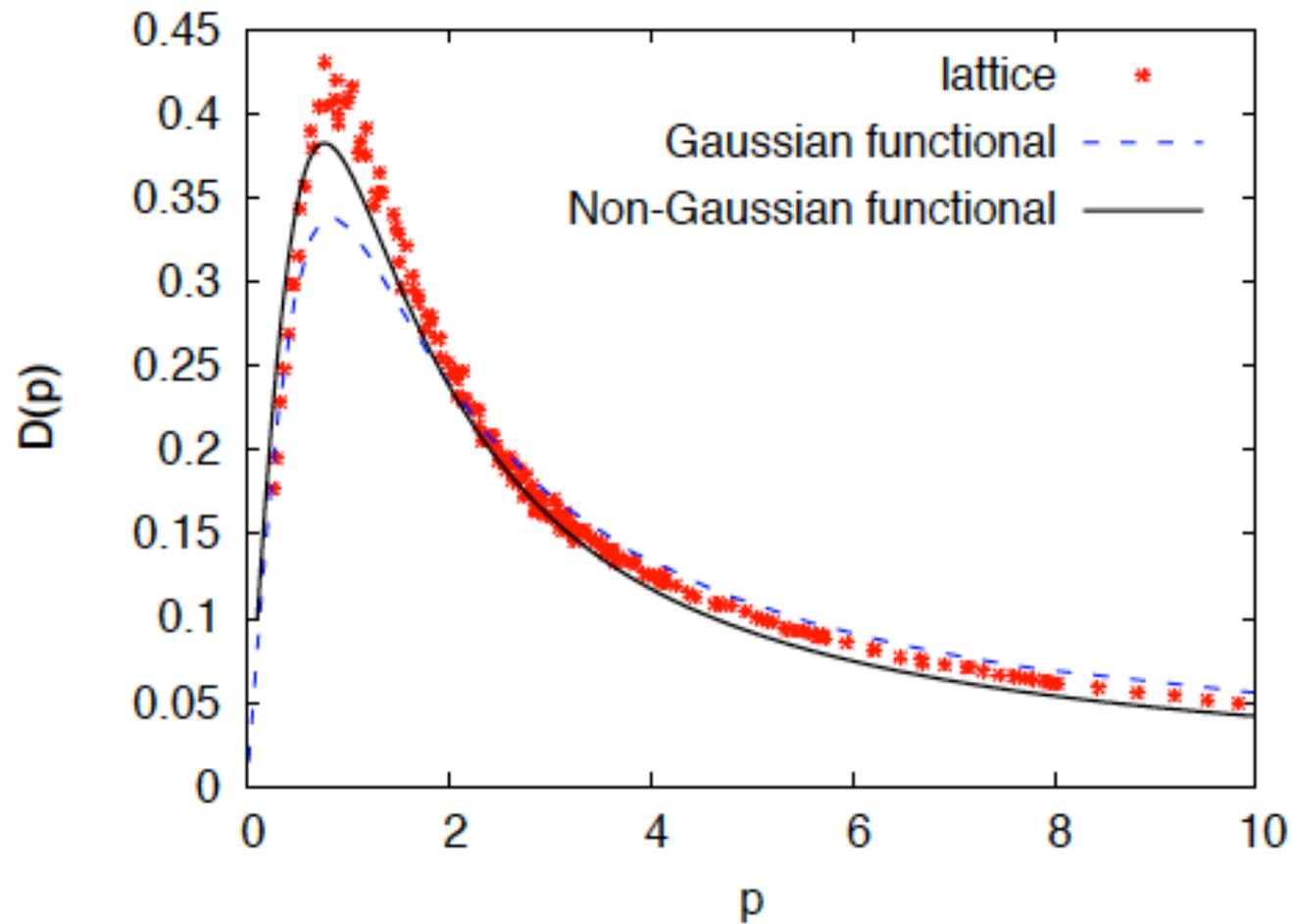
ansatz

$$S[A] = \int \omega A^2 + \frac{1}{3!} \int \gamma^{(3)} A^3 + \frac{1}{4!} \int \gamma^{(4)} A^4$$

exploit DSE

see talk by D. Campagnari

Corrections to the gluon propagator



D. Campagnari & H.R, Phys.Rev.D82(2010)

YM Hamiltonian in $\partial A = 0$

$$H = \frac{1}{2} \int (J^{-1} \Pi J \Pi + B^2) + H_C \quad \Pi = \delta / i \delta A$$

Christ and Lee

$$J(A^\perp) = \text{Det}(-D\partial) \quad D^{ab} = \delta^{ab} \partial + g f^{abc} A^c$$

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color charge density $\rho^a = -f^{abc} A^b \Pi^c + \rho_m^a$

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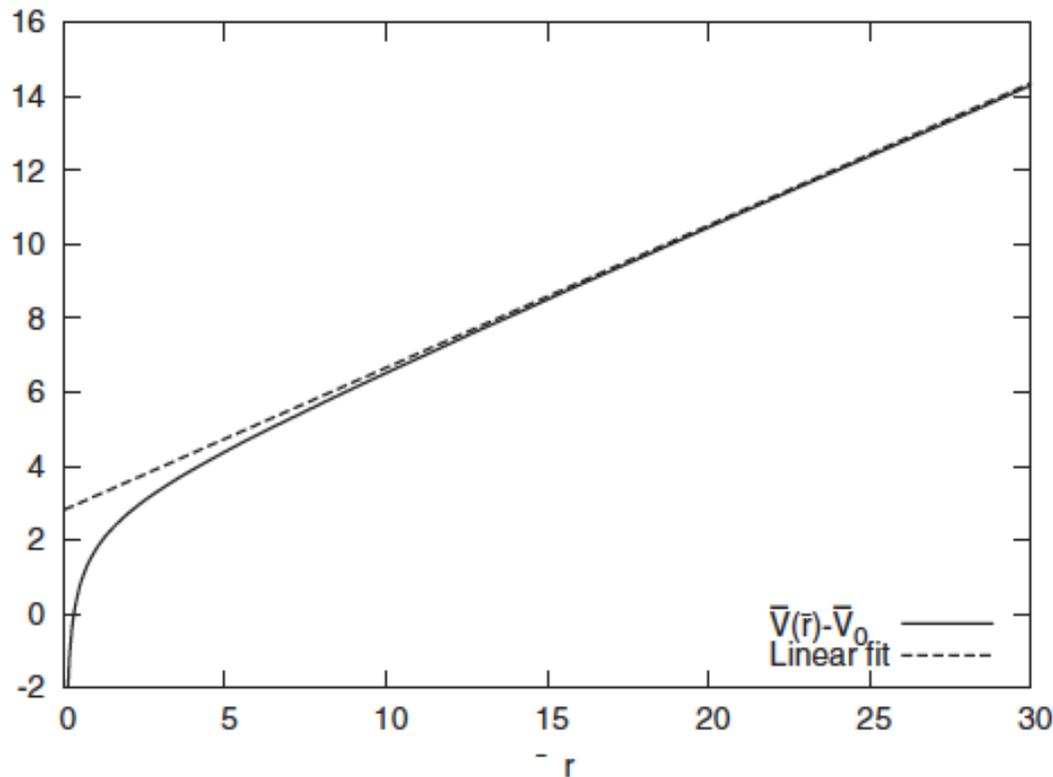
color charge density $\rho^a = -f^{abc} A^b \Pi^c + \rho_m^a$

static quark potential

$$V_C(|\vec{x} - \vec{y}|) = \langle \langle \vec{x} | (-D\partial)^{-1} (-\partial^2) (-D\partial)^{-1} | \vec{y} \rangle \rangle$$

Non-Abelian Coulomb potential

$$V_C(\vec{x}, \vec{y}) = \langle \vec{x} | (-D\partial)^{-1} (-\partial^2) (-D\partial)^{-1} | \vec{y} \rangle$$



D. Epple, H. Reinhardt
W. Schleifenbaum,
PRD 75 (2007)

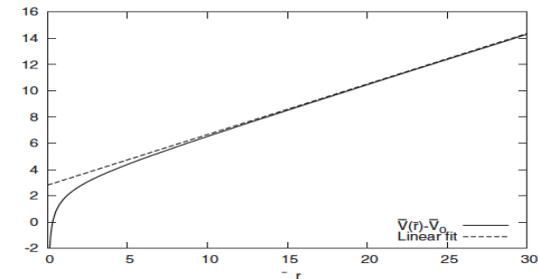
$$V(r) = \xrightarrow{r \rightarrow 0} \sim 1/r$$

$$V(r) = \xrightarrow{r \rightarrow \infty} \sigma_C r,$$

Non-Abelian Coulomb potential

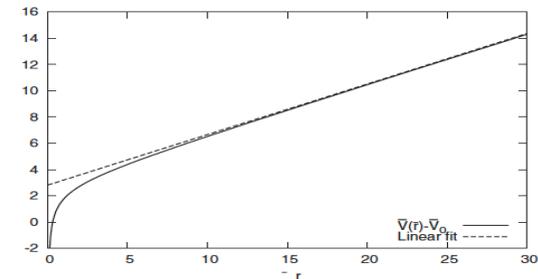
$$V_C(\vec{x}, \vec{y}) = \langle \vec{x} | (-D\partial)^{-1} (-\partial^2) (-D\partial)^{-1} | \vec{y} \rangle$$

$$\xrightarrow{|\vec{x}-\vec{y}| \rightarrow \infty} \sigma_c |\vec{x} - \vec{y}|$$



Non-Abelian Coulomb potential

$$V_C(\vec{x}, \vec{y}) = \langle \vec{x} | (-D\partial)^{-1} (-\partial^2) (-D\partial)^{-1} | \vec{y} \rangle$$
$$\xrightarrow{|\vec{x}-\vec{y}| \rightarrow \infty} \sigma_C |\vec{x} - \vec{y}|$$

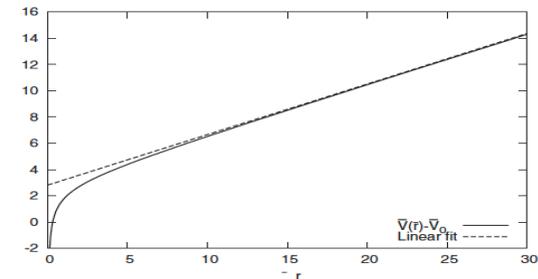


D. Zwanziger $\sigma_C \geq \sigma_W$

Non-Abelian Coulomb potential

$$V_C(\vec{x}, \vec{y}) = \langle \vec{x} | (-D\partial)^{-1} (-\partial^2) (-D\partial)^{-1} | \vec{y} \rangle$$

$\xrightarrow{|\vec{x}-\vec{y}| \rightarrow \infty} \sigma_C |\vec{x} - \vec{y}|$



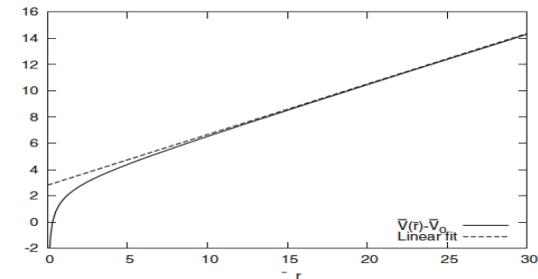
D. Zwanziger $\sigma_C \geq \sigma_W$

lattice: $\sigma_C = 2 \dots 3 \sigma_W$

Non-Abelian Coulomb potential

$$V_C(\vec{x}, \vec{y}) = \langle \vec{x} | (-D\partial)^{-1} (-\partial^2) (-D\partial)^{-1} | \vec{y} \rangle$$

$\xrightarrow{|\vec{x}-\vec{y}| \rightarrow \infty} \sigma_C |\vec{x} - \vec{y}|$



D. Zwanziger $\sigma_C \geq \sigma_W$

lattice: $\sigma_C = 2 \dots 3 \sigma_W$

$$\begin{aligned} aV_C(|\mathbf{x} - \mathbf{y}|) &= -\lim_{t \rightarrow 0} \frac{d}{dt} \log \left\langle \text{tr} \left[P_t(\mathbf{x}) P_t^\dagger(\mathbf{y}) \right] \right\rangle \\ &= -\log \left\langle \text{tr} \left[U_0(\mathbf{x}) U_0^\dagger(\mathbf{y}) \right] \right\rangle, \end{aligned}$$

J. Greensite & S. Olejnik,
Phys.Rev.D67(2003)

center vortices

L. DelDebbio, M.Faber, J.Greensite & S. Olejnik, Phys.Rev.D55(1997)

maximal center gauge $\sum_{i,\mu} |\text{tr} U_\mu(x)|^2 \rightarrow \max$

center projection: $U_\mu(x) \longrightarrow Z_\mu(x) = \pm 1 \in Z(2)$

elimination of center vortices

P. de Forcrand and M.D'Elia, Phys.Rev.Lett. 82, 4582(1999)

$$U_\mu(x) \rightarrow U_\mu(x) Z_\mu(x)$$

removes Wilsonian string tension

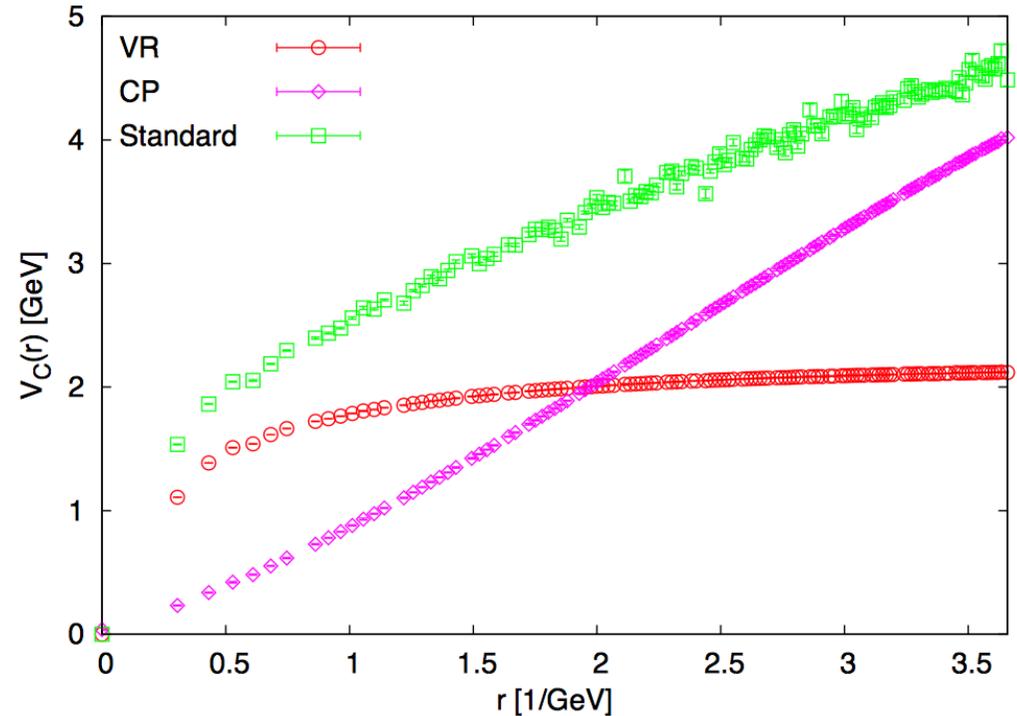
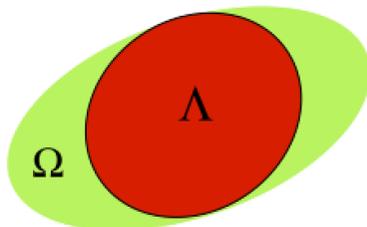
Gribov scenario & center vortex picture

- *Coulomb potential*

*G. Burgio, M. Quandt,
H. R. & H. Vogt,
Phys.Rev.D92(2015)*

- *Coulomb string tension disappears after elimination of center vortices*

- *Center vortices are on the Gribov horizon*



*J.Greensite, S.Olejnik
& D.Zwanziger
JHEP0505(2005)*

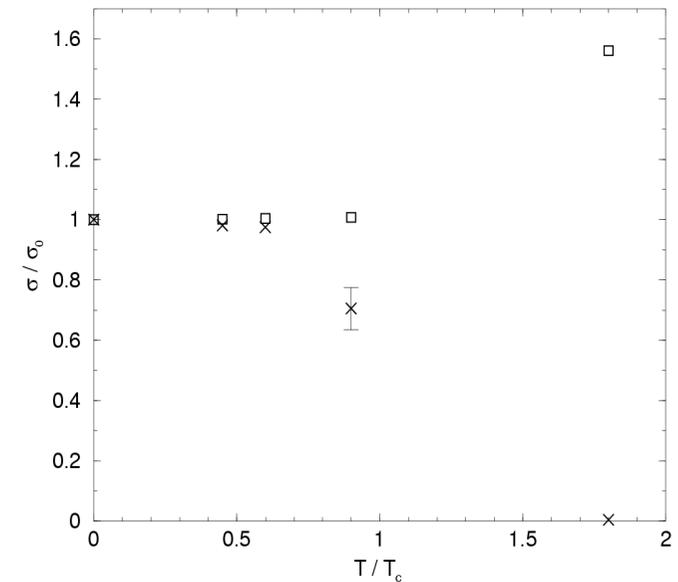
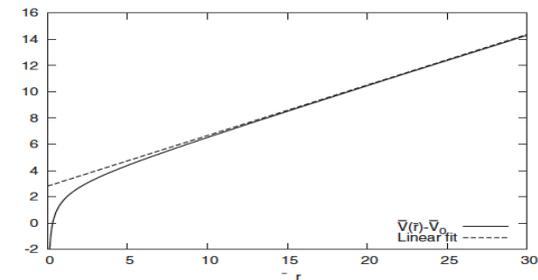
Non-Abelian Coulomb potential

$$V_C(\vec{x}, \vec{y}) = \langle \vec{x} | (-D\partial)^{-1} (-\partial^2) (-D\partial)^{-1} | \vec{y} \rangle$$

$$\xrightarrow{|\vec{x}-\vec{y}| \rightarrow \infty} \sigma_C |\vec{x} - \vec{y}|$$

D. Zwanziger $\sigma_C \geq \sigma_W$

lattice: $\sigma_C = 2 \dots 3 \sigma_W$



M. Engelhardt & H.R
Nucl.Phys.B585(2000)

spatial center vortices

maximal center gauge $\sum_{x,\mu} |\text{tr} U_\mu(x)|^2 \rightarrow \max$

spatial center projection:

$$U_i(x) \longrightarrow Z_i(x) = \pm 1 \in Z(2)$$

elimination of spatial center vortices

$$U_i(x) \longrightarrow U_i(x) Z_i(x)$$

>removes spatial string tension

>does not change temporal links

Polyakov loops & temporal string tension
are not affected

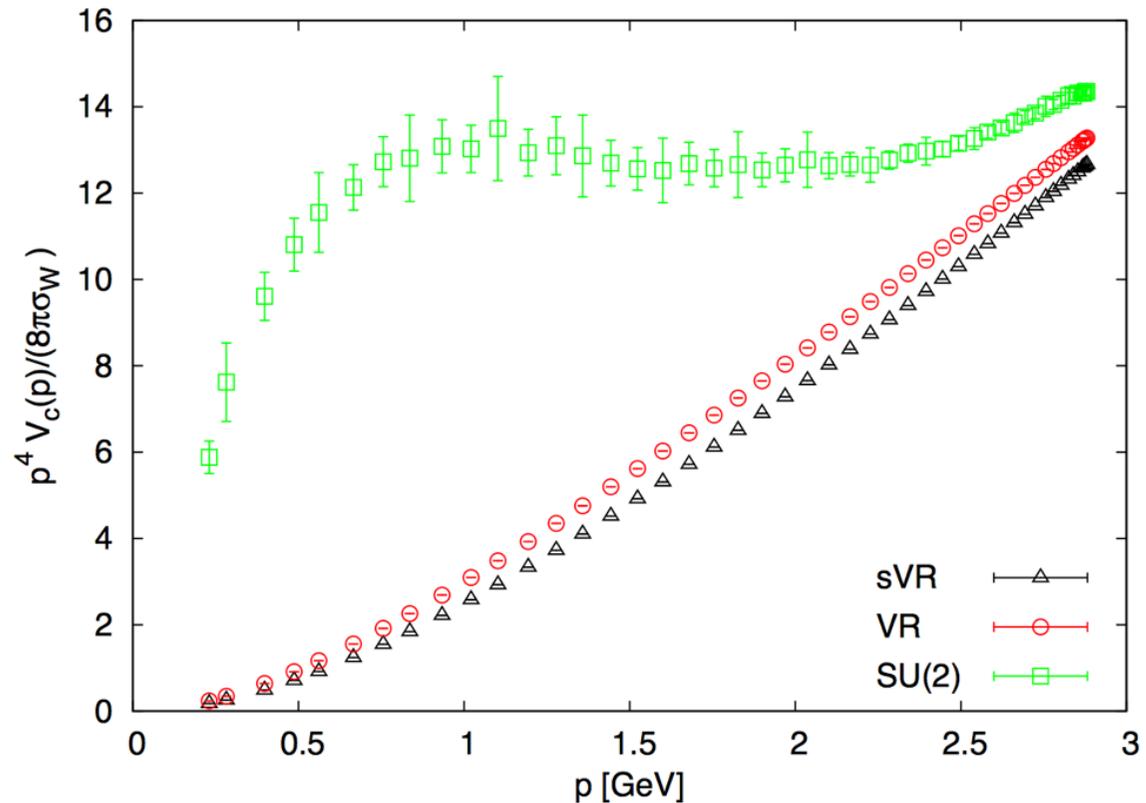
Gribov scenario & center vortex picture

- Coulomb potential

$$V_C(\mathbf{p}) = g^2 \text{tr} \left\langle \left(-\hat{\mathbf{D}} \cdot \nabla \right)^{-1} \left(-\nabla^2 \right) \left(-\hat{\mathbf{D}} \cdot \nabla \right)^{-1} \right\rangle$$

$$p^4 V_C(p) / 8\pi\sigma_w$$

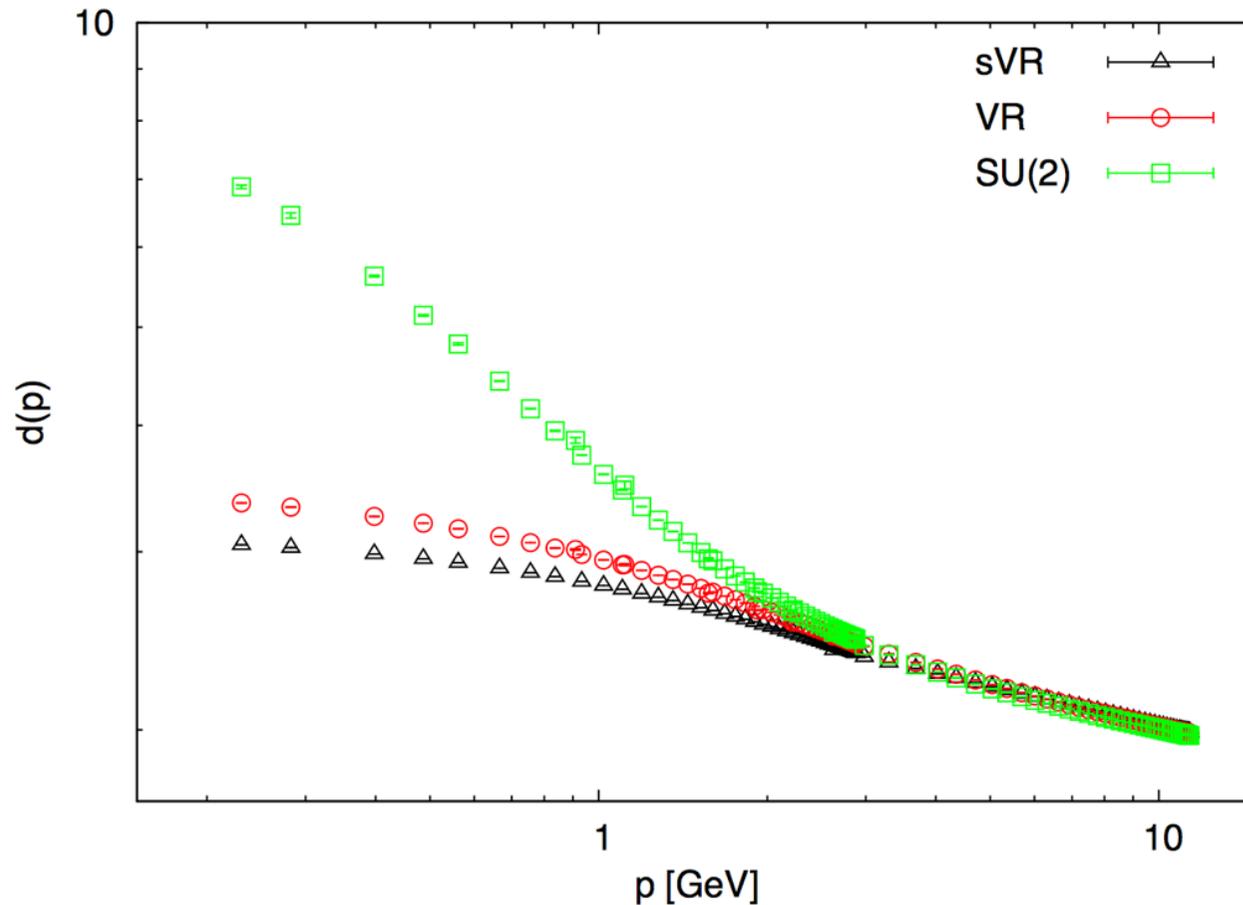
$$\lim_{p \rightarrow 0} p^4 V_C(p) = 8\pi\sigma_c$$



- Coulomb string tension disappears after (spatial)center vortex removal

Gribov scenario & center vortex picture

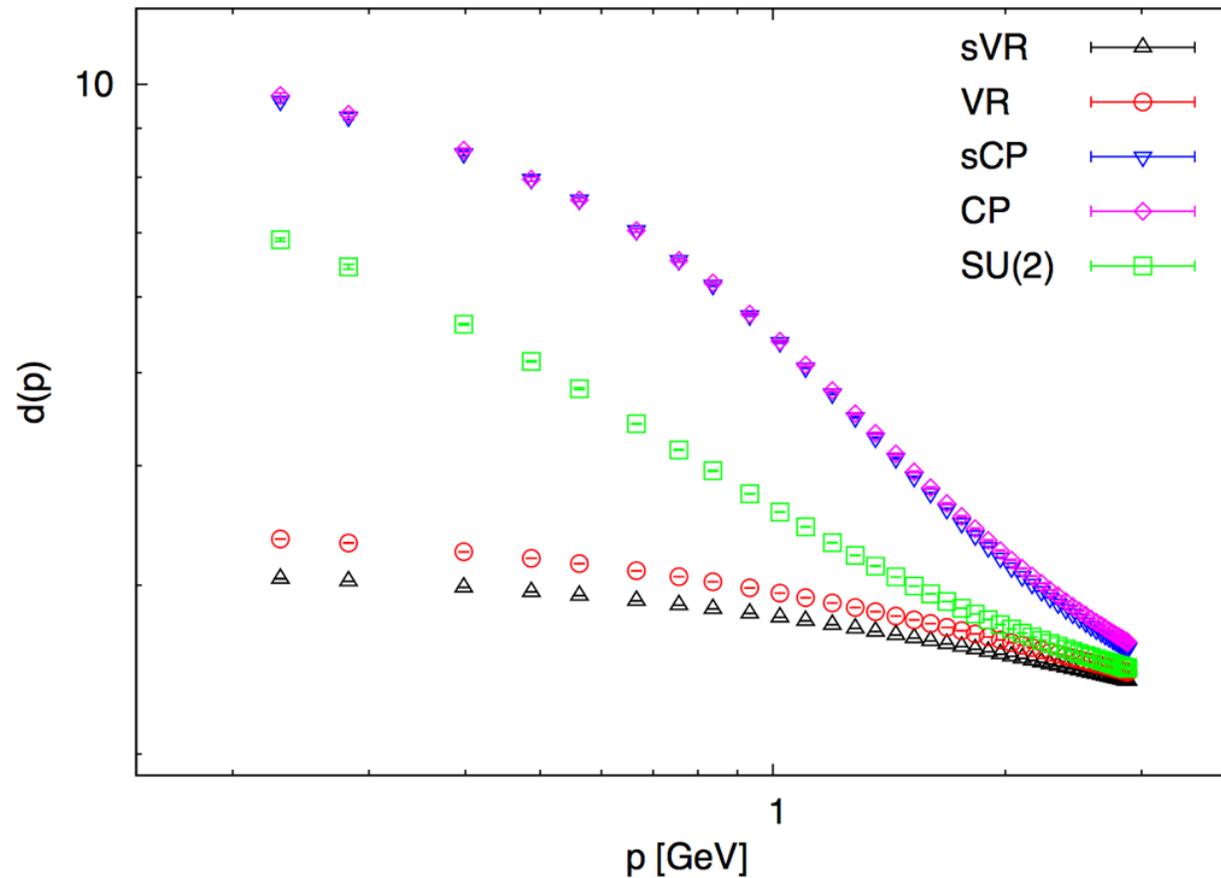
- ghost form factor



- IR enhancement disappears after (spatial) center vortex removal

Gribov scenario & center vortex picture

- ghost form factor

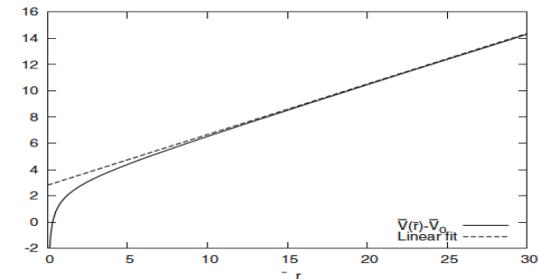


- IR enhancement disappears after (spatial) center vortex removal
- (spatial)center vortex projection increases $d(p)$

Non-Abelian Coulomb potential

$$V_C(\vec{x}, \vec{y}) = \langle \vec{x} | (-D\partial)^{-1} (-\partial^2) (-D\partial)^{-1} | \vec{y} \rangle$$

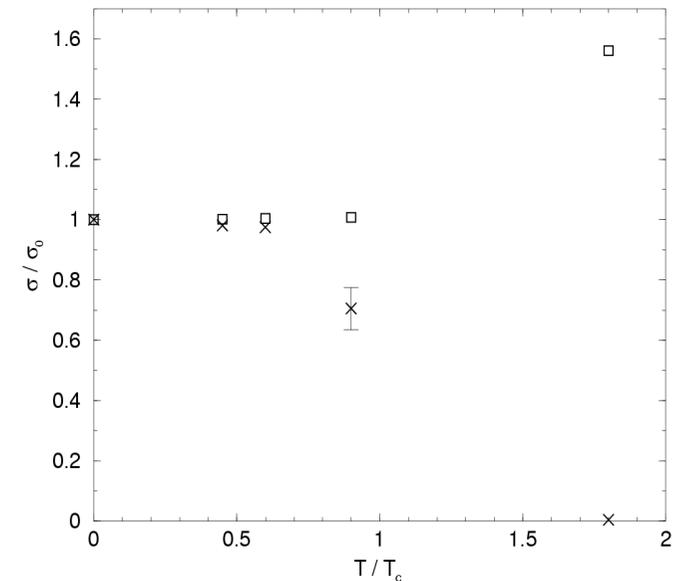
$$\xrightarrow{|\vec{x}-\vec{y}| \rightarrow \infty} \sigma_C |\vec{x} - \vec{y}|$$



D. Zwanziger $\sigma_C \geq \sigma_W$

lattice: $\sigma_C = 2...3\sigma_W$

*G. Burgio, M. Quandt,
H. R. & H. Vogt
Phys.Rev.D92(2015)*

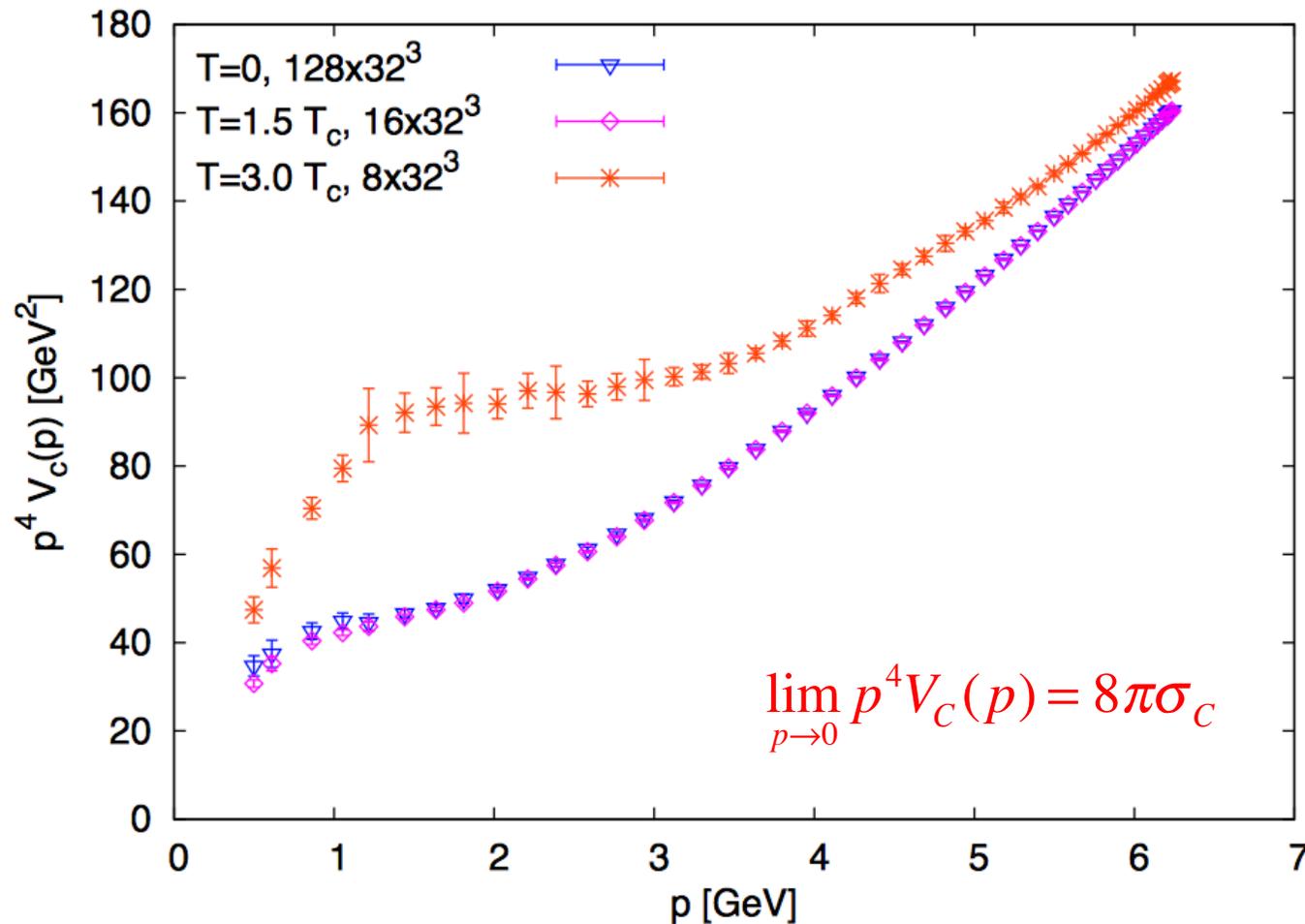


*explains finite temp
behaviour of σ_C*

$$\sigma_C \neq \sigma_T \quad \sigma_C \sim \sigma_S$$

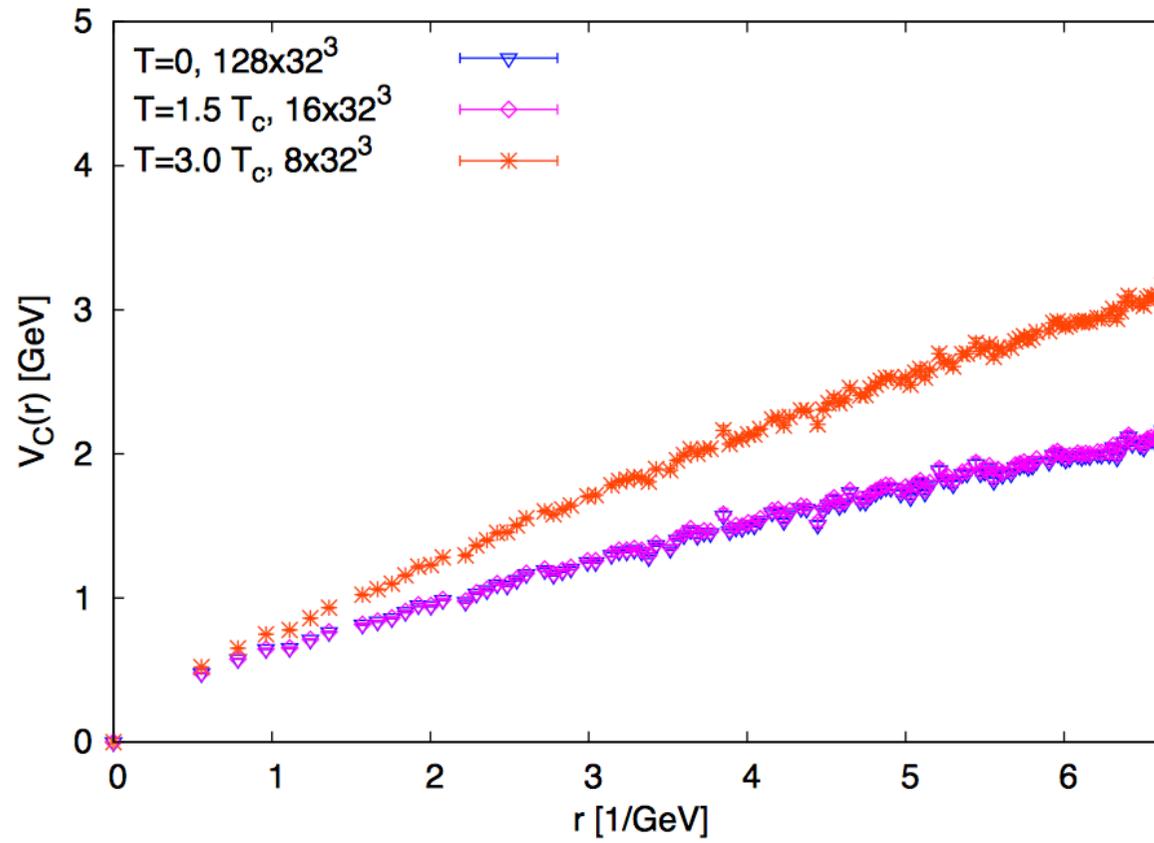
non-Abelian Coulomb potential at finite T

Y. Nakagawa, A. Nakamura, T. Saito, H. Toki, and D. Zwanziger,
Phys. Rev., D73:094504, 2006.



G. Burgio, M. Quandt, H. R. & H. Vogt,
Phys.Rev.D92(2015)

Coulomb potential



conclusion:

-the Coulomb string tension is tied to the spatial string tension, which is known to decouple from the temporal string tension in the deconfinement phase transition

-explains the temperature behaviour of the Coulomb string tension

Hamiltonian approach to QCD in Coulomb gauge

*P. Vastag, H. R. & D. Campagnari
Phys.Rev.D93(2016)*

*D. Campagnari , E. Ebadati, H.R. and P: Vastag,
arXiv:1608.06820*

The QCD Hamiltonian in Coulomb gauge

$$H_{QCD} = H_{YM} + H_C + H_q$$

gluon part

$$H_{YM} = \frac{1}{2} \int (J^{-1} \Pi J \Pi + B^2) \quad \Pi = -i\delta / \delta A \quad J(A^\pm) = \text{Det}(-D\partial)$$

quark part

$$H_q = \int \Psi^\dagger(\mathbf{x}) [\vec{\alpha}(\vec{p} + g\vec{A}) + \beta m_0] \Psi(\mathbf{x}) \quad \vec{\alpha}, \beta - \text{Dirac matrices}$$

Coulomb term

$$H_C = \frac{1}{2} \int J^{-1} \rho (-D\partial)^{-1} (-\partial^2) (-D\partial)^{-1} J \rho$$

color charge density

$$\rho^a = -f^{abc} A^b \Pi^c + \Psi^\dagger(\mathbf{x}) t^a \Psi(\mathbf{x})$$

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A | \Phi \rangle_q = \exp \left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha} \cdot \vec{A} + w\beta\vec{\alpha} \cdot \vec{A}) \Psi_- \right] | 0 \rangle$$

s, v, w – variational kernels $\vec{\alpha}, \beta$ – Dirac matrices

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

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s, v, w – variational kernels $\vec{\alpha}, \beta$ – Dirac matrices

$v=w=0$: BCS – wave function

Finger & Mandula
Adler & Davis, Alkofer

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A | \Phi \rangle_q = \exp \left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha} \cdot \vec{A} + w\beta\vec{\alpha} \cdot \vec{A}) \Psi_- \right] | 0 \rangle$$

s, v, w – variational kernels $\vec{\alpha}, \beta$ – Dirac matrices

$v=w=0$: BCS – wave function

Finger & Mandula
Adler & Davis, Alkofer

$v \neq 0, w=0$: quark - gluon - coupling

Pak & Reinhardt,

quark wave functional

$$\langle A | \Phi \rangle_q = \exp \left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha} \cdot \vec{A} + w\beta\vec{\alpha} \cdot \vec{A}) \Psi_- \right] | 0 \rangle$$

s, v, w – variational kernels $\vec{\alpha}, \beta$ – Dirac matrices

> calculate $\langle H_{\text{QCD}} \rangle$ up to 2 loops

> variation w.r.t. $\mathbf{S}, \mathbf{V}, \mathbf{W}$

$$v(p, q) = f_v[s, \omega] \quad w(p, q) = f_w[s, \omega]$$

$$s(p) = f_s[s, v, w; p] \quad \text{gap equation}$$

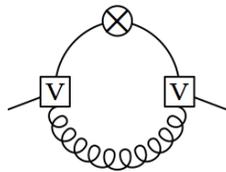
cancelation of all \mathcal{UV} -divergencies

cancellation of UV-divergencies

$$\langle A | \Phi \rangle_q = \exp \left[\int \Psi_+^\dagger (\mathbf{s}\beta + \mathbf{v}\vec{\alpha} \cdot \vec{A} + \mathbf{w}\beta\vec{\alpha} \cdot \vec{A}) \Psi_- \right] | 0 \rangle$$

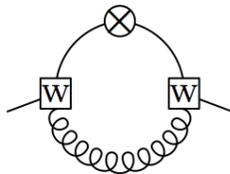
divergent loop contributions to the gap equation

> kernel **V**



$$\frac{C_F}{16\pi^2} g^2 S(k) \left[-2\Lambda + k \ln \frac{\Lambda}{\mu} \left(-\frac{2}{3} + 4P(k) \right) \right]$$

> kernel **W**



$$\frac{C_F}{16\pi^2} g^2 S(k) \left[2\Lambda + k \ln \frac{\Lambda}{\mu} \left(\frac{10}{3} - 4P(k) \right) \right]$$

> Coulomb term **V_C**



$$-\frac{C_F}{6\pi^2} g^2 k S(k) \ln \frac{\Lambda}{\mu}$$

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A | \Phi \rangle_q = \exp \left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha} \cdot \vec{A} + w\beta\vec{\alpha} \cdot \vec{A}) \Psi_- \right] | 0 \rangle$$

s, v, w – variational kernels $\vec{\alpha}, \beta$ – Dirac matrices

numerical calculation

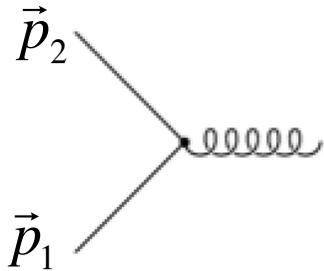
D. Campagnari, E. Ebadati, H.R. and P: Vastag,
arXiv:1608.06820

input: *lattice:* $\sigma_c = 2.5\sigma$

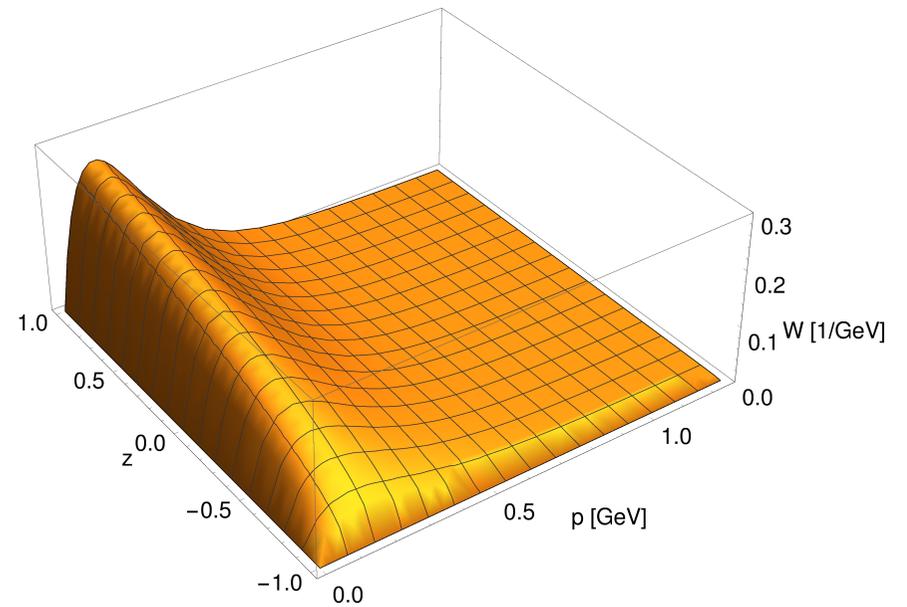
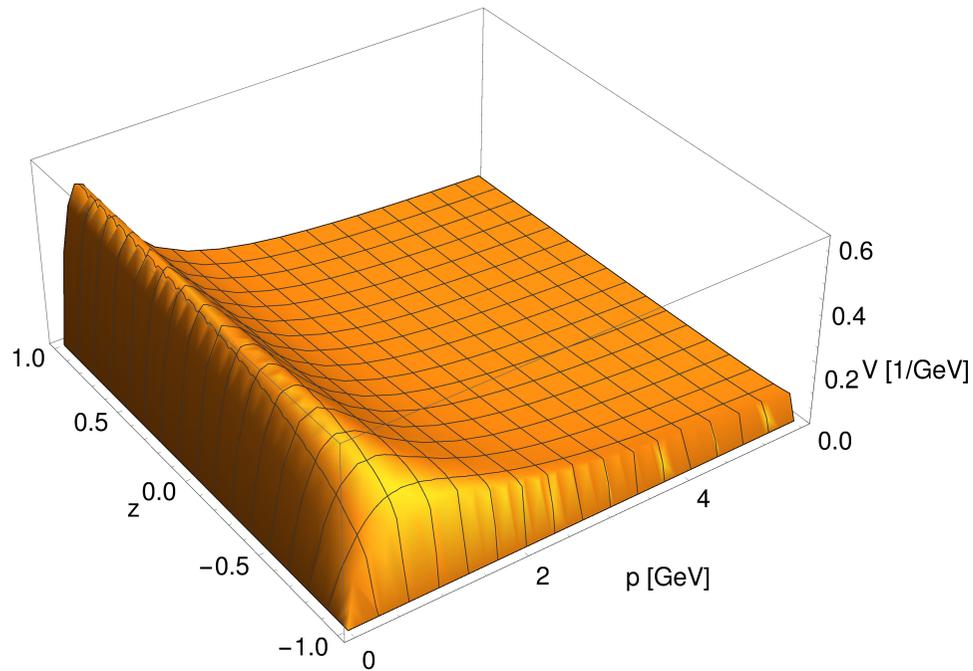
G. Burgio, M.Quandt, H.R.,
PRL102(2009)

choose g to reproduce $\langle \bar{q}q \rangle = (-235 \text{ MeV})^3 \Rightarrow g \approx 2.1$

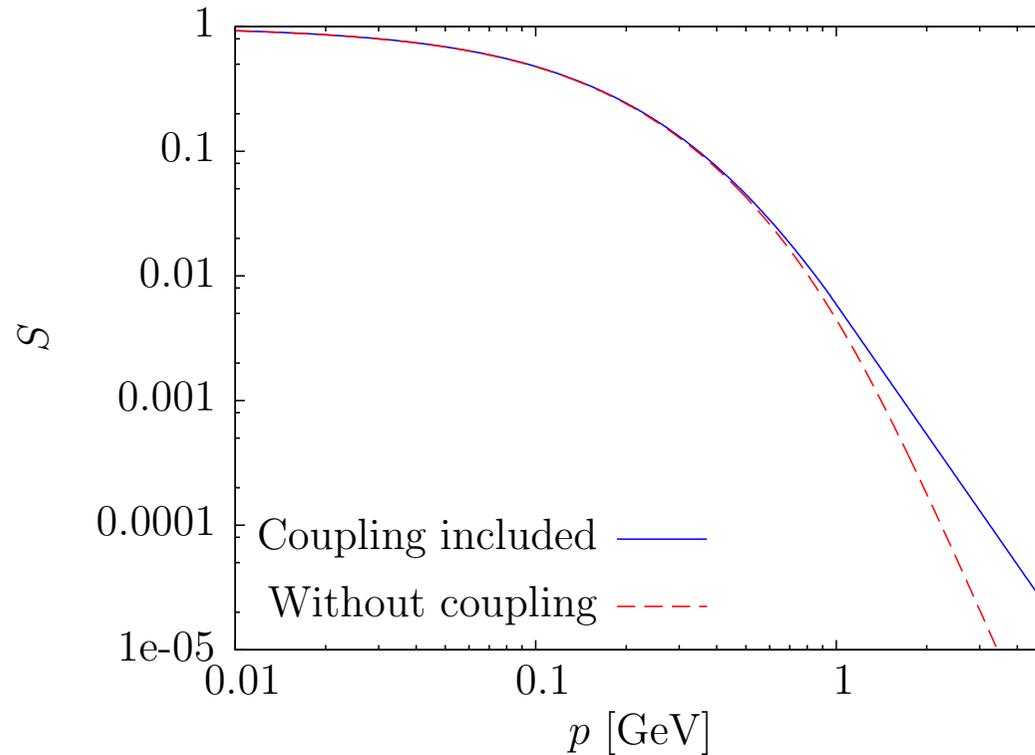
vector form factors v, w



$$v, w(\vec{p}_1, \vec{p}_2): \quad p := |\vec{p}_1| = |\vec{p}_2|, \quad z = \cos \angle(\vec{p}_1, \vec{p}_2)$$



scalar form factor



-quark-gluon coupling modifies only the mid- and high-momentum regime

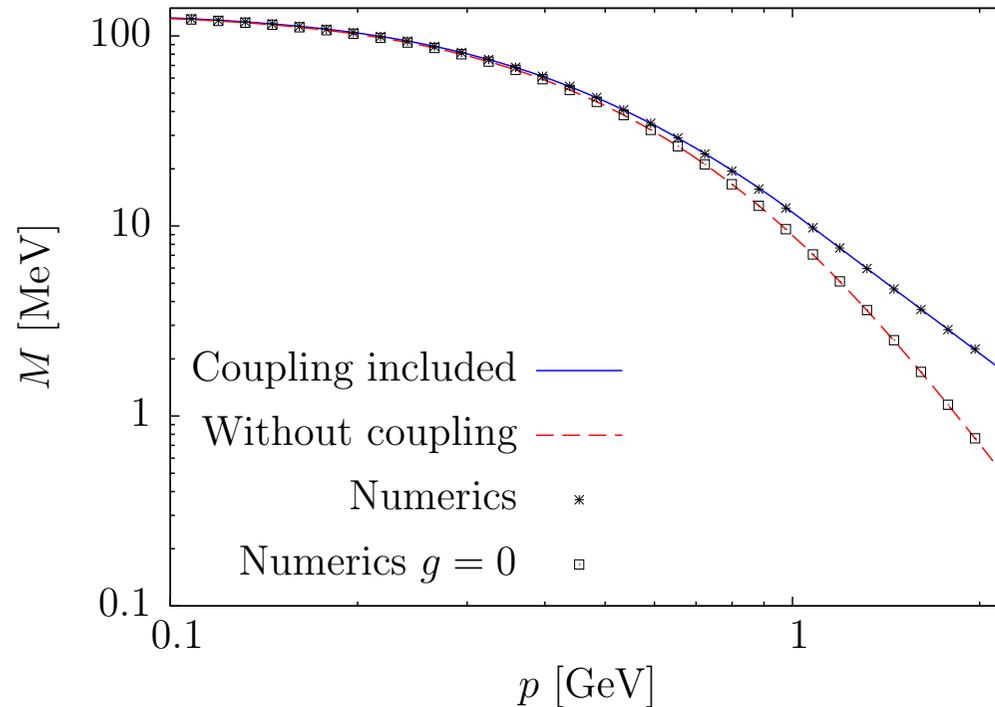
-low-momentum regime is dominated by Coulomb term

effective quark mass

$$M(p) = \frac{2ps(p)}{1-s^2(p)}$$

$$\langle \bar{q}q \rangle_{phen} = (-235 \text{ MeV})^3$$

$$\langle \bar{q}q \rangle_{AD} = (-185 \text{ MeV})^3$$

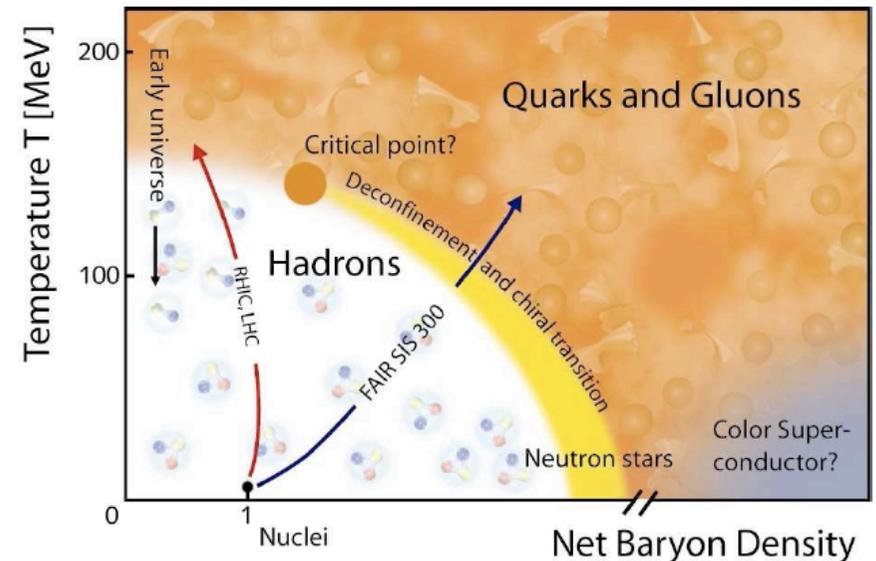


> coupling to transversal gluons substantially increases chiral symmetry breaking

> no chiral symmetry breaking without the Coulomb term

QCD at finite temperature: grand canonical ensemble

- quasi-particle ansatz for the density operator
- minimization of the thermodynamic potential



YM sector:

H.Reinhardt, D.Campagnari & A. Szczepaniak, Phys.Rev.D84(2011)

J.Heffner, H.Reinhardt & D.Campagnari, Phys.Rev.D85(2012)

Alternative Hamiltonian approach to finite temperature QFT

*H. R. arXiv:1604.06273
Phys.Rev.D94(2016)045016*

- no ansatz for the density matrix required
- motivation: Polyakov loop

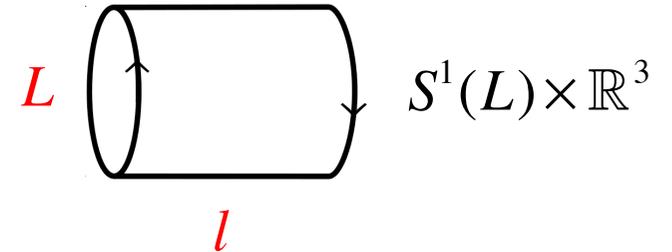
$$P[A_0](\vec{x}) = \frac{1}{d_r} \text{tr} P \exp \left[i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

- $\langle P[A_0] \rangle$ order parameter of confinement
- Hamiltonian approach
 - Weyl gauge $A_0=0$
- How to calculate the Polyakov loop in the Hamiltonian approach?

Finite temperature QFT

- compactification of (Euclidean) time

- bc: $A(x^0 = L/2) = A(x^0 = -L/2)$ Bose fields
 $\psi(x^0 = L/2) = -\psi(x^0 = -L/2)$ Fermi fields



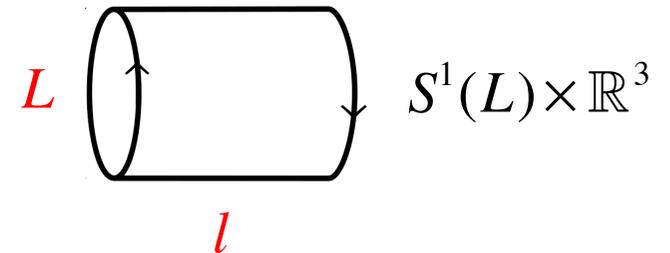
- temperature $T = L^{-1}$ $l \rightarrow \infty$



Finite temperature QFT

- compactification of (Euclidean) time

- bc: $A(x^0 = L/2) = A(x^0 = -L/2)$ Bose fields
 $\psi(x^0 = L/2) = -\psi(x^0 = -L/2)$ Fermi fields



- temperature $T = L^{-1}$ $l \rightarrow \infty$

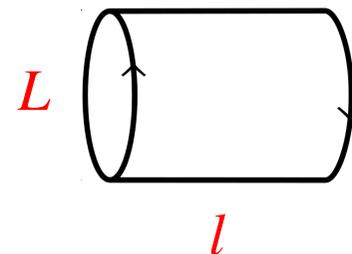
- exploit the $O(4)$ -invariance of the Euclidean Lagrangian

- $O(4)$ -rotation $x^0 \rightarrow x^3$ $A^0 \rightarrow A^3$ $\gamma^0 \rightarrow \gamma^3$
 $x^1 \rightarrow x^0$ $A^1 \rightarrow A^0$ $\gamma^1 \rightarrow \gamma^0$

- one compactified spatial dimension

- bc: $A(x^3 = L/2) = A(x^3 = -L/2)$ Bose fields
 $\psi(x^3 = L/2) = -\psi(x^3 = -L/2)$ Fermi fields

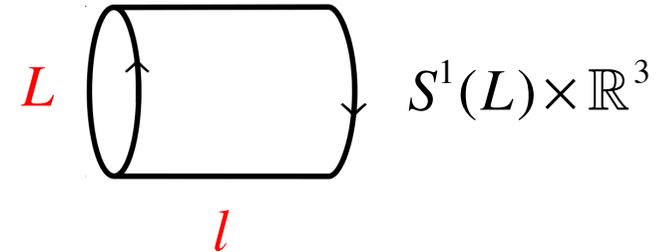
- spatial manifold: $\mathbb{R}^2 \times S^1(L)$



Finite temperature QFT

- compactification of (Euclidean) time

- bc: $A(x^0 = L/2) = A(x^0 = -L/2)$ Bose fields
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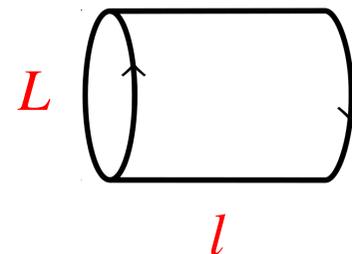
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 $\psi(x^3 = L/2) = -\psi(x^3 = -L/2)$ Fermi fields

- spatial manifold: $\mathbb{R}^2 \times S^1(L)$



Hamiltonian approach

- *temperature is now encoded in one „spatial“ dimension while „time“ has infinite extension independent of the temperature*

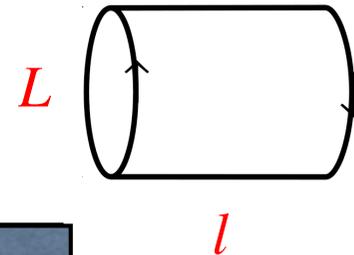
Finite temperature QFT

- partition function

$$Z(L) = \lim_{l \rightarrow \infty} \text{Tr} \exp(-lH(L)) = \lim_{l \rightarrow \infty} \sum_n \exp(-lE_n(L)) = \lim_{l \rightarrow \infty} \exp(-lE_0(L))$$

- ground state energy $E_0(L) = l^2 Le(L)$

- on the spatial manifold: $\mathbb{R}^2 \times S^1(L)$



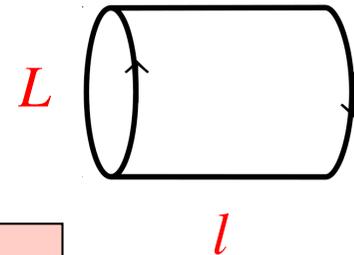
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- ground state energy $E_0(L) = l^2 Le(L)$

- on the spatial manifold: $\mathbb{R}^2 \times S^1(L)$



- pressure:

$$p = -e(L)$$

- energy density:

$$\varepsilon = \partial[Le(L)] / \partial L - \mu \partial e / \partial \mu$$

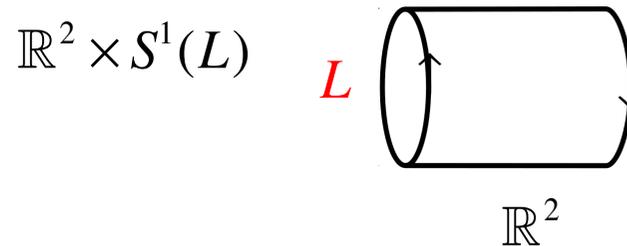
- Dirac fermions with finite chemical potential

$$h = \vec{\alpha} \cdot \vec{p} + \beta m \rightarrow h + i\mu\alpha^3$$

QCD at finite T

- Hamiltonian approach in Coulomb gauge on the partially compactified spatial manifold

$$\mathbb{R}^2 \times S^1(L)$$



- variational solution of the Schrödinger equation for the vacuum
- finite temperature QCD is fully encoded in its vacuum

YMT at finite T

J. Heffner & H. R.
Phys.Rev.D91(2015)

Polyakov loop

H. R. & J. Heffner,
Phys.Rev.D88(2013)

dual quark condensate

H. R. & P. Vastag
arXiv:1605.03740

center symmetry

DECONFINEMENT PHASE TRANSITION:

confined phase: center symmetry
deconfined phase: center symmetry broken

any observable transforming non-trivially under the center may serve as order parameter for confinement

prototype: Polyakov loop

$$P[A_0](\vec{x}) = \frac{1}{d_r} \text{tr} P \exp \left[i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

dual quark condensate -dressed Polyakov loop

Gattringer, PRL. 97(2006)

Synatschke, Wipf, Wozar, Phys. Rev. D75(2007)

....

lattice: Bilgici, Bruckmann, Gattringer, Hagen, PR D77(2008)...

FRG: Braun, Haas, Marhauser, Pawłowski, PRL 106(2011)

DSE: Fischer, Maas, Müller, Eur. Phys. J. 68(2010) ...

Hamiltonian approach:

H. R. & P. Vastag, [arXiv:1605.03740](https://arxiv.org/abs/1605.03740)

dual quark condensate -dressed Polyakov loop

$$\Sigma_n = \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-in\varphi} \langle (\bar{q}q)_\varphi \rangle \quad q(\beta) = e^{i\varphi} q(0)$$

Σ_n loops winding n -times around the compact time axis

Σ_1 dressed Polyakov loop

Gattringer
PRL97(2006)

imaginary chemical potential : $\mu = i \frac{\pi - \varphi}{\beta}$

compactified 3-axis potential : $p_3 = \Omega_n + i\mu = \frac{2\pi n + \varphi}{\beta}$

Dual quark condensate in the Hamiltonian approach in $\partial A=0$

H. R. & P. Vastag, arXiv:1605.03740

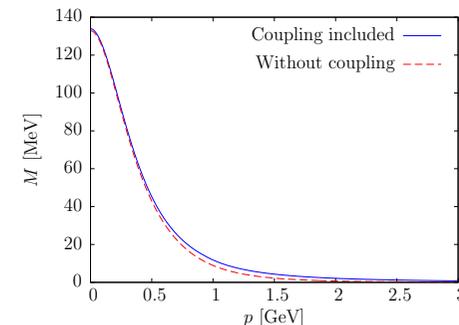
$$\Sigma_n = \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-in\varphi} \langle (\bar{q}q)_\varphi \rangle \quad q(\beta) = e^{i\varphi} q(0)$$

after Poisson resummation

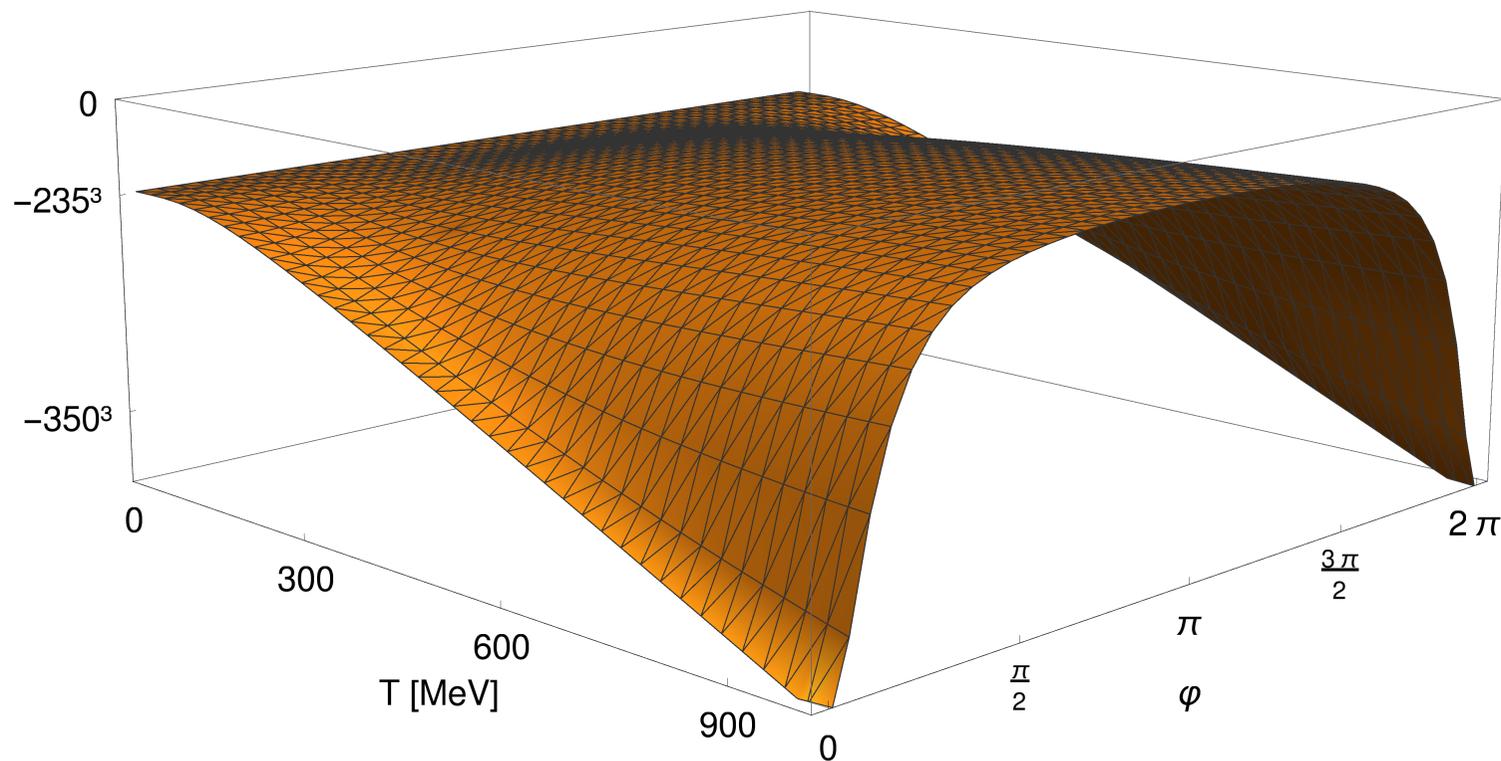
$$\Sigma_n = (-)^{n+1} \frac{2N_c}{2\pi^2} \int_0^\infty dp p \frac{\sin(nLp)}{nL} \frac{M(p)}{\sqrt{p^2 + M^2(p)}}$$

effective quark mass $M(p)$

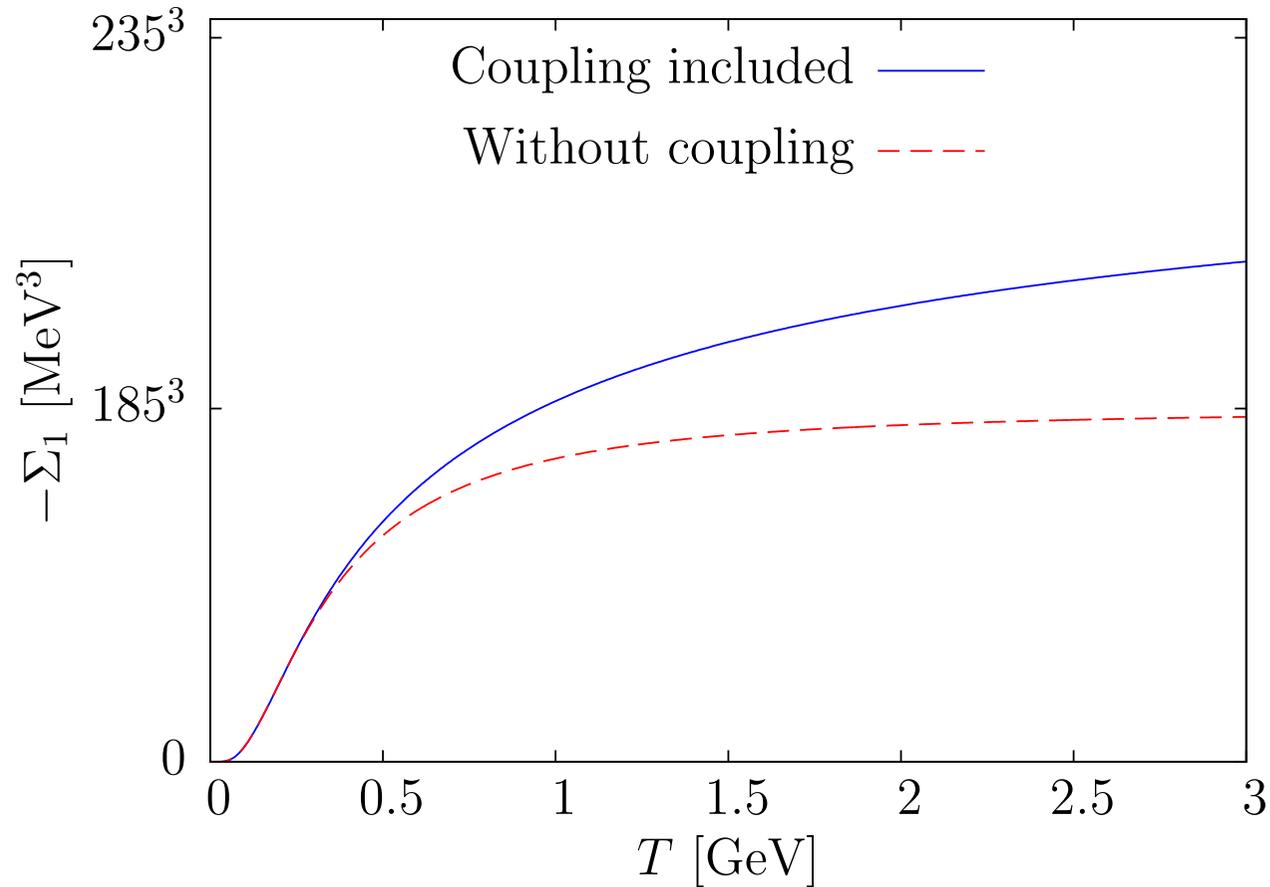
D. Campagnari, E. Ebadati,
H.R. and P. Vastag,
arXiv:1608.06820



quark condensate $\langle (\bar{q}q)_\varphi \rangle$

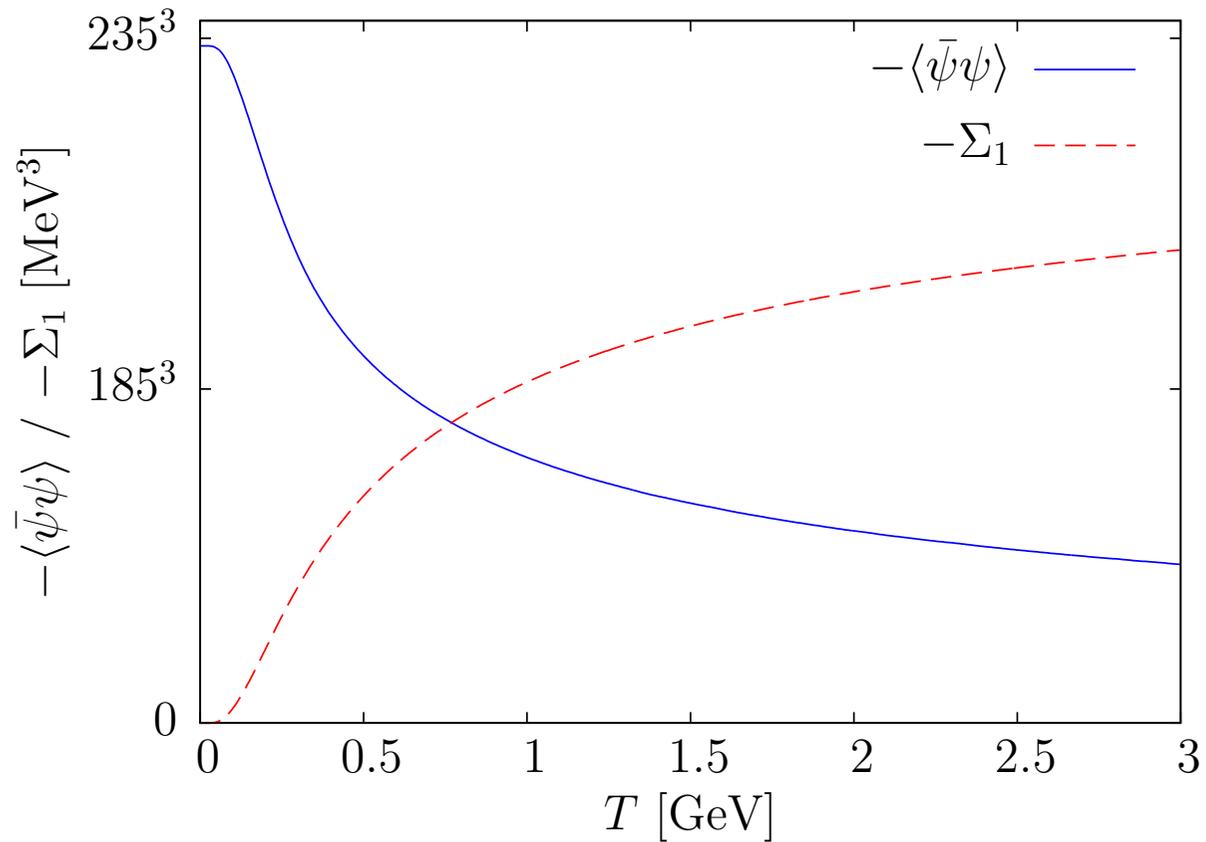


dual quark condensate



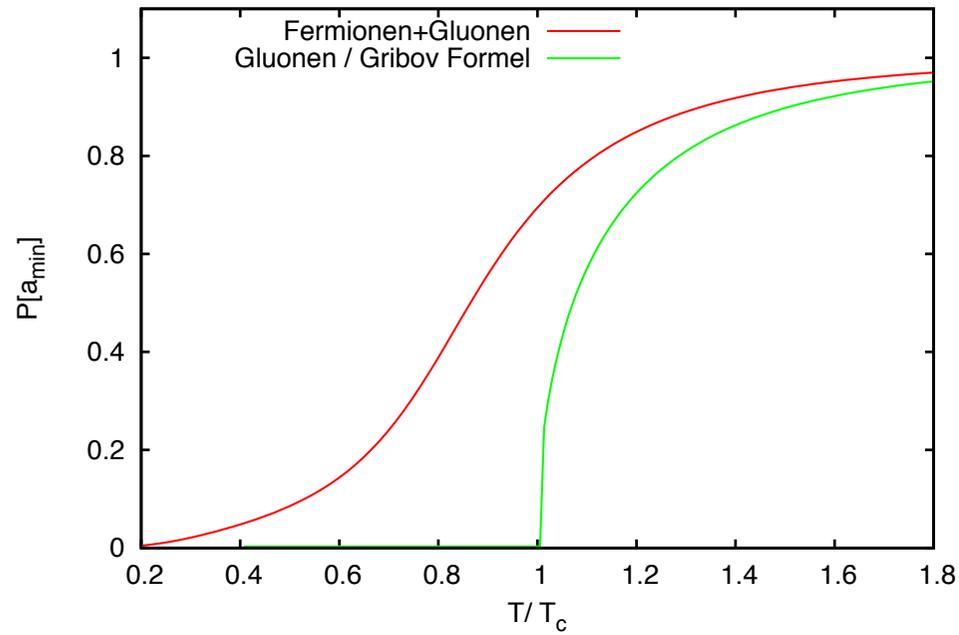
$$\sigma_C = 2.5\sigma \quad (T_{PC})_C \simeq 196 \text{ MeV}$$

chiral & dual condensate

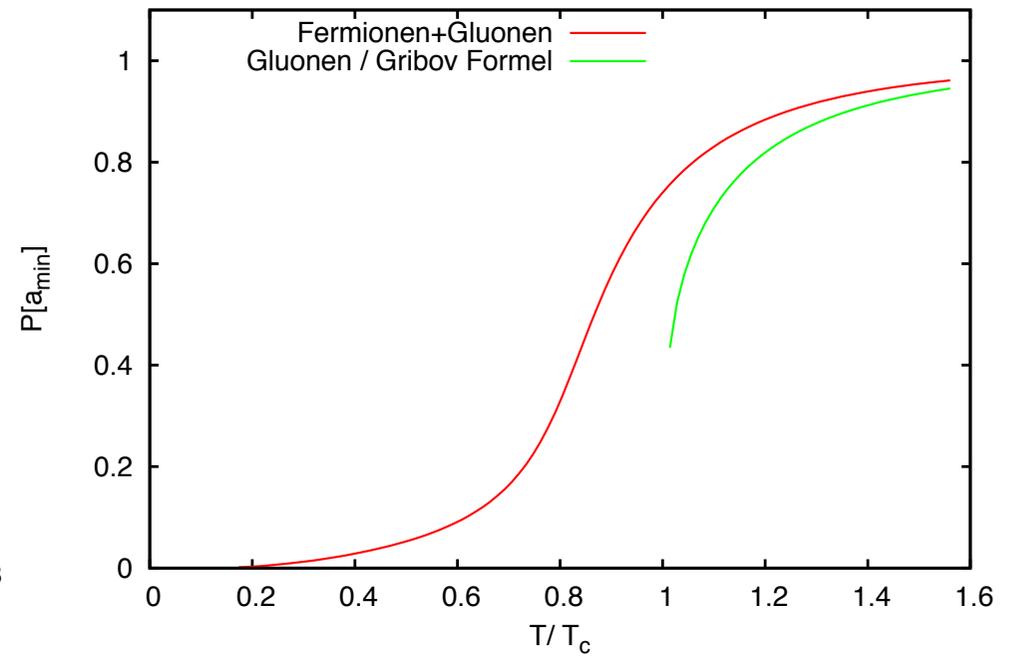


$$\sigma_C = 2.5\sigma \quad (T_{PC})_\chi \simeq 168 \text{ MeV} \quad (T_{PC})_C \simeq 196 \text{ MeV}$$

The Polyakov loop



SU(2)



SU(3)

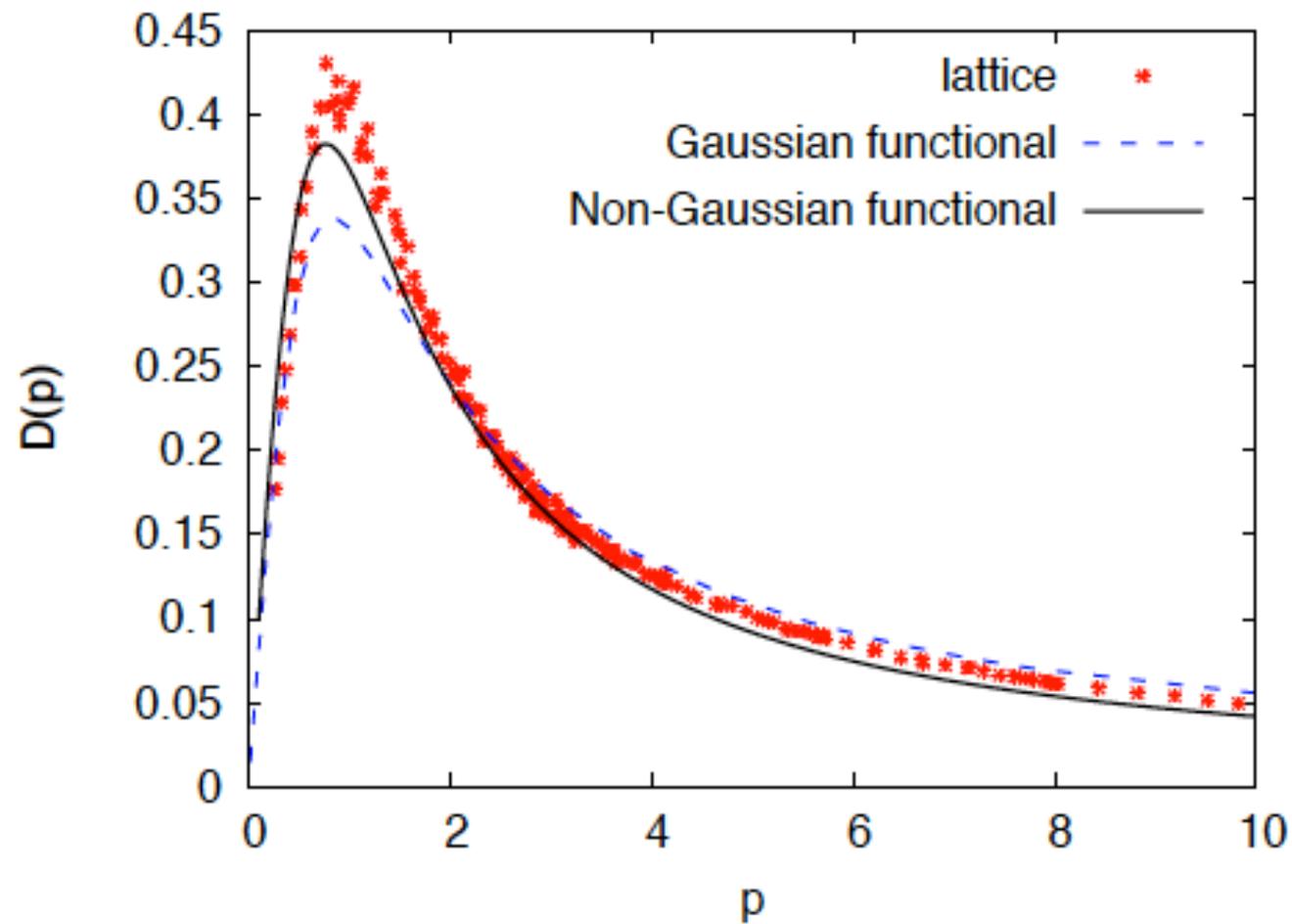
J. Heffner, H. Reinhardt & P. Vastag, to be published

Conclusions

- *Hamiltonian approach to QCD in Coulomb gauge at $T=0$*
 - *Coulomb string tension - spatial string tension*
 - *no chiral symmetry breaking without Coulomb confinement*
- *QCD at finite temperature*
 - *compactification of a spatial dimension $R^2 \times S^1$*
 - *chiral & dual quark condensate*
 - *effective potential of the Polyakov loop*
 - *deconfinement phase transition in YMT*
 - *SU(2): 2.order*
 - *SU(3): 1.order*
 - *inclusion of quarks:*
 - *deconfinement phase transition is turned into a crossover*
 - *pressure & energy density*

*Confronting the continuum
approach with the lattice*

Corrections to the gluon propagator



D. Campagnari & H.R, Phys.Rev.D82(2010)

Infrared analysis

Lerche & von Smekal, PRD65
Schleifenbaum, Leder, H.R.
PRD73

gluon energy

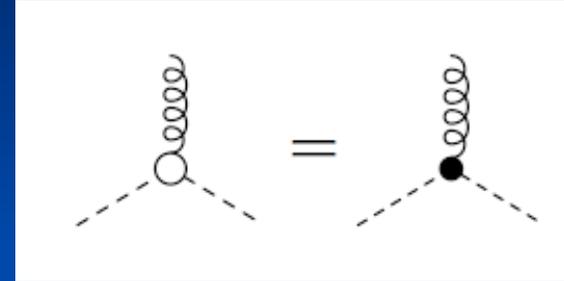
$$\omega(p) = A / p^\alpha$$

ghost form factor

$$d(p) = B / p^\beta$$

assumptions:

bare ghost gluon vertex



horizon condition $d^{-1}(0) = 0$

supported by the lattice

sum rule

$$\alpha = 2\beta + 2 - d$$

$$d = 3$$

$$\alpha = 1 \text{ (0.6)}$$

$$\beta = 1 \text{ (0.8)}$$

lattice:

$$\alpha \approx 1$$

$$\beta \approx 0.5$$

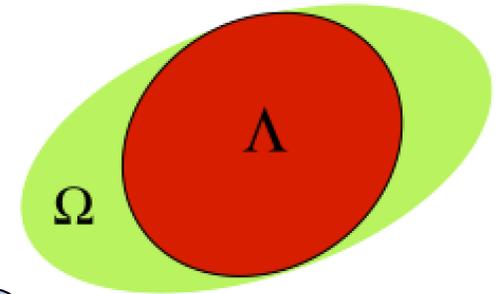
lattice propagators violate the sum rule

Coulomb gauge fixing on the lattice & Gribov copies

$$F_t^U [g] = \sum_{\vec{x}, i} \text{Re tr} \left[U_i^g(t, \vec{x}) \right] \rightarrow \max$$

local maximum: first Gribov region

global maximum: fundamental modular region



bc: „best“ Gribov copy: largest local maximum
= „best representative“ of the global maximum!?

counter example: U(1)-LGT on S^2 de Forcrand & Hetrick

Sternbeck & Müller-Preußker, Maas, (Landau-gauge)

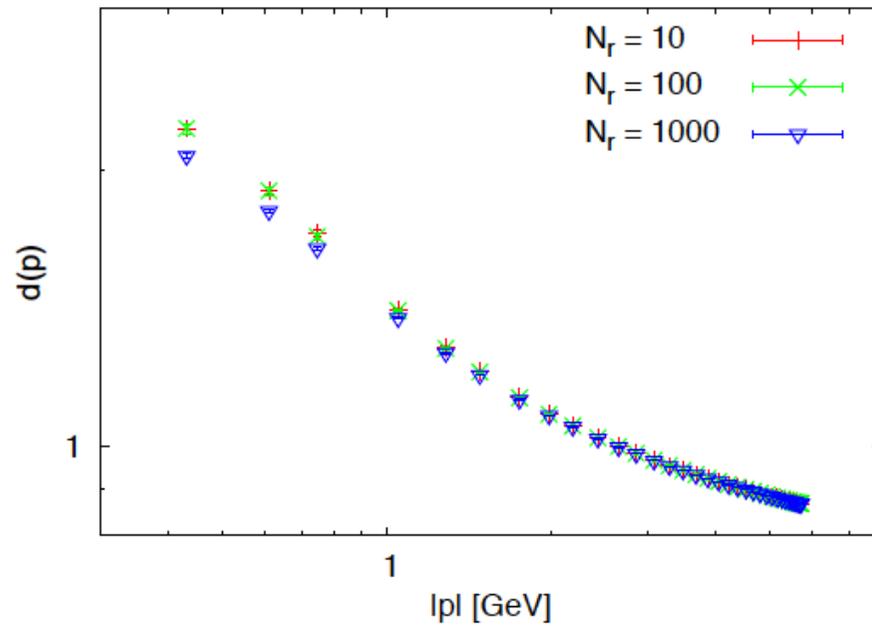
lc: „lowest“ Gribov copy: lowest (n.t.) eigenvalue of the FP operator

Cooper & Zwanziger: results closer to the continuum

G. Burgio, M. Quandt, H. R. & H. Vogt, arXiv:1608.05795

Coulomb gauge fixing on the lattice & Gribov copies: ghost form factor

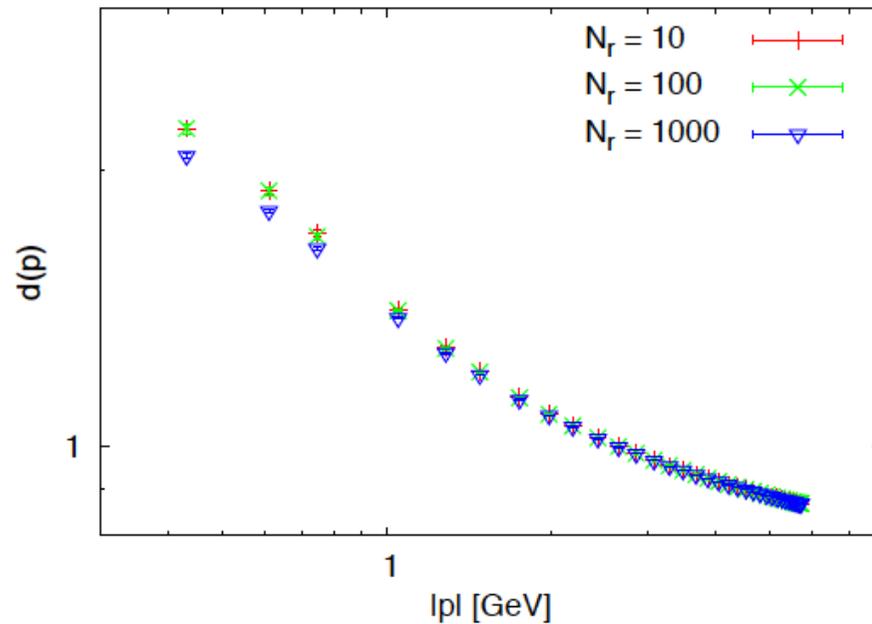
best copy



*IR-exponent
fixed at 0.5*

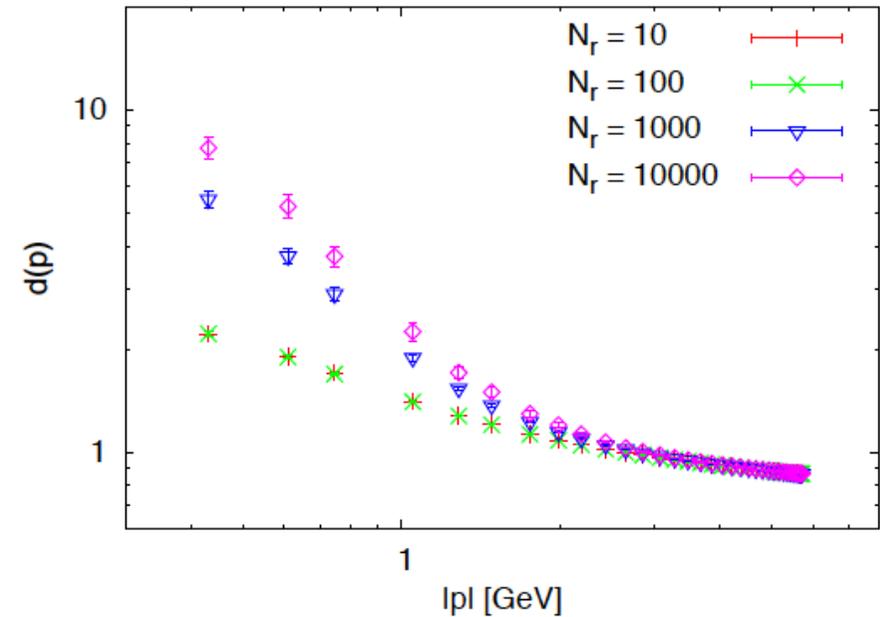
Coulomb gauge fixing on the lattice & Gribov copies: ghost form factor

best copy



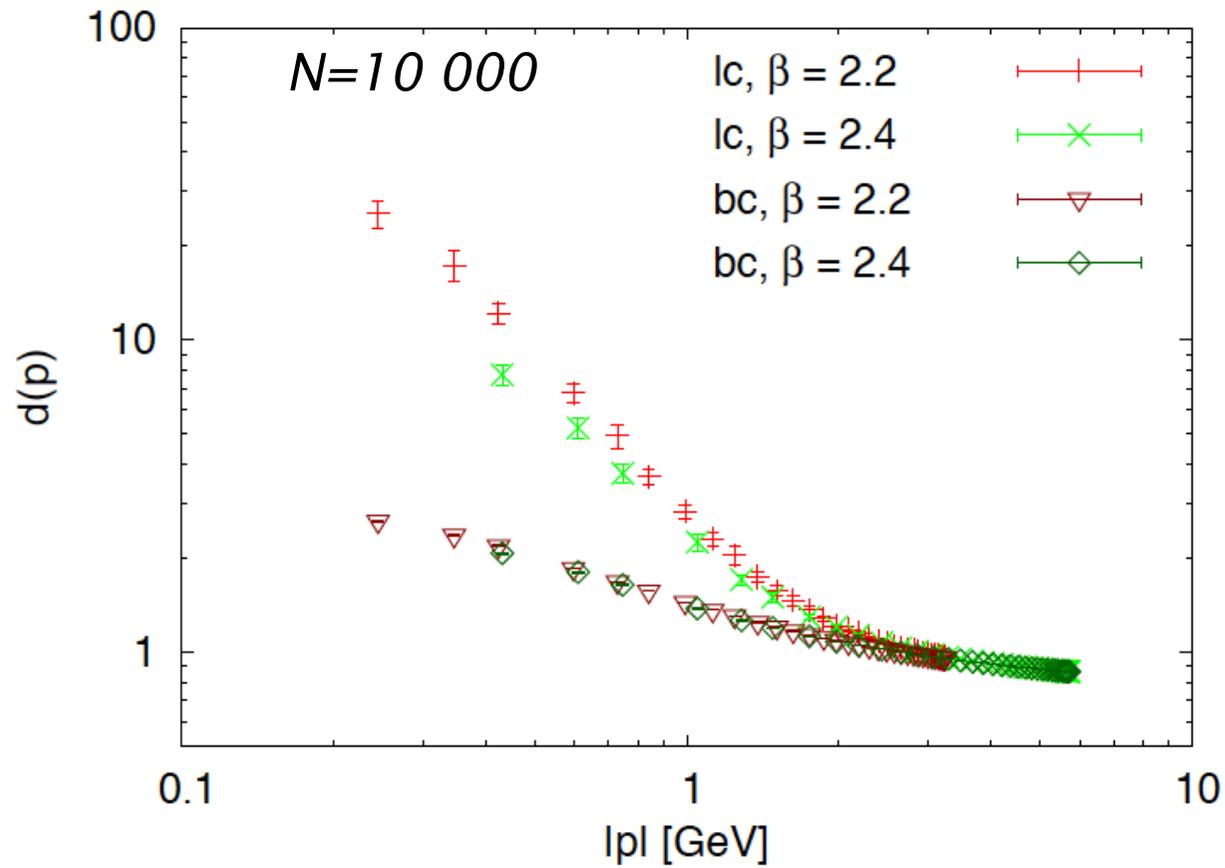
*IR-exponent
fixed at 0.5*

lowest copy



*IR-exponent
increases and
„saturates“ near 1*

Coulomb gauge fixing on the lattice & Gribov copies: ghost form factor



Coulomb gauge fixing on the lattice & Gribov copies: gluon propagator

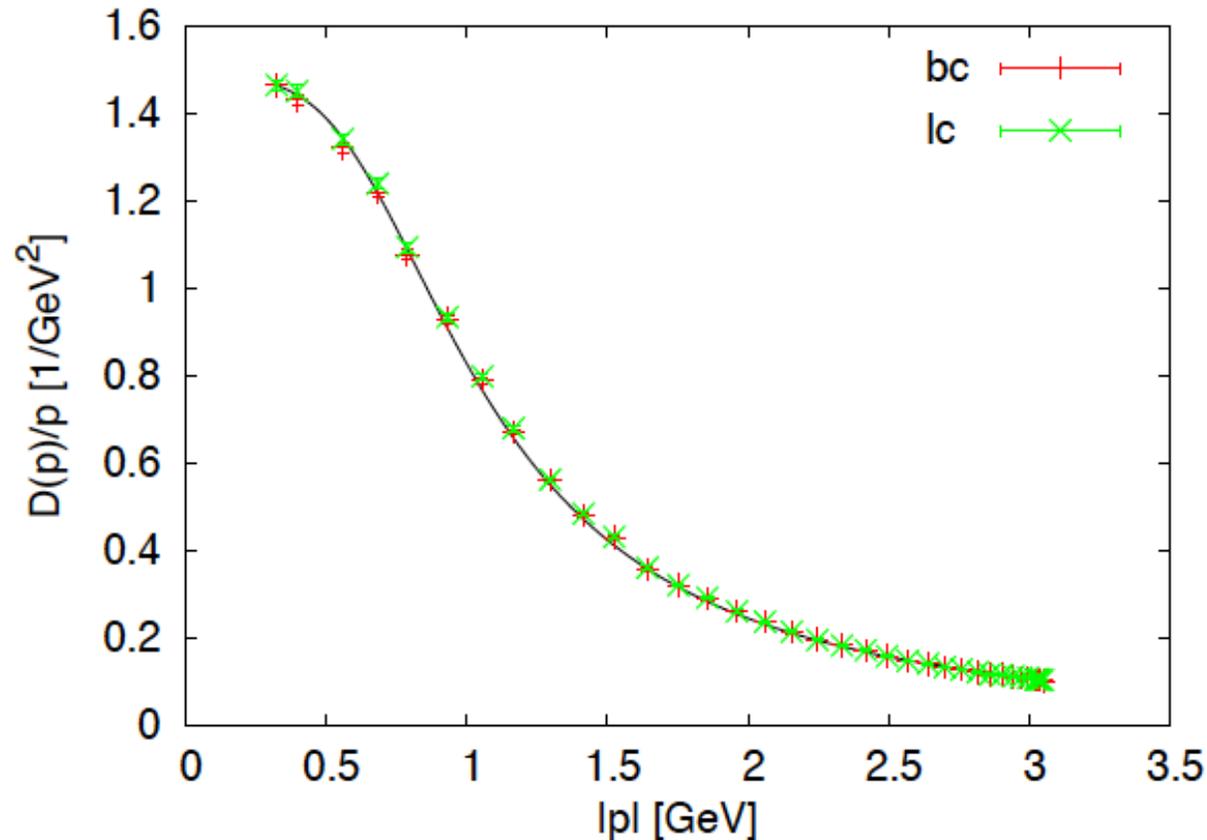


FIG. 6. The gluon propagator for the B1 lattice with the bc and the lc-approach from 1000 trials. The solid line is a fit to the Gribov formula [14, 15]. The choice of Gribov copy apparently makes no visible difference.

Coulomb gauge fixing on the lattice & Gribov copies: Coulomb potential(bc)

$$p^4 V_C(p) / 8\pi\sigma_W$$

$$\lim_{p \rightarrow 0} p^4 V_C(p) = 8\pi\sigma_C$$

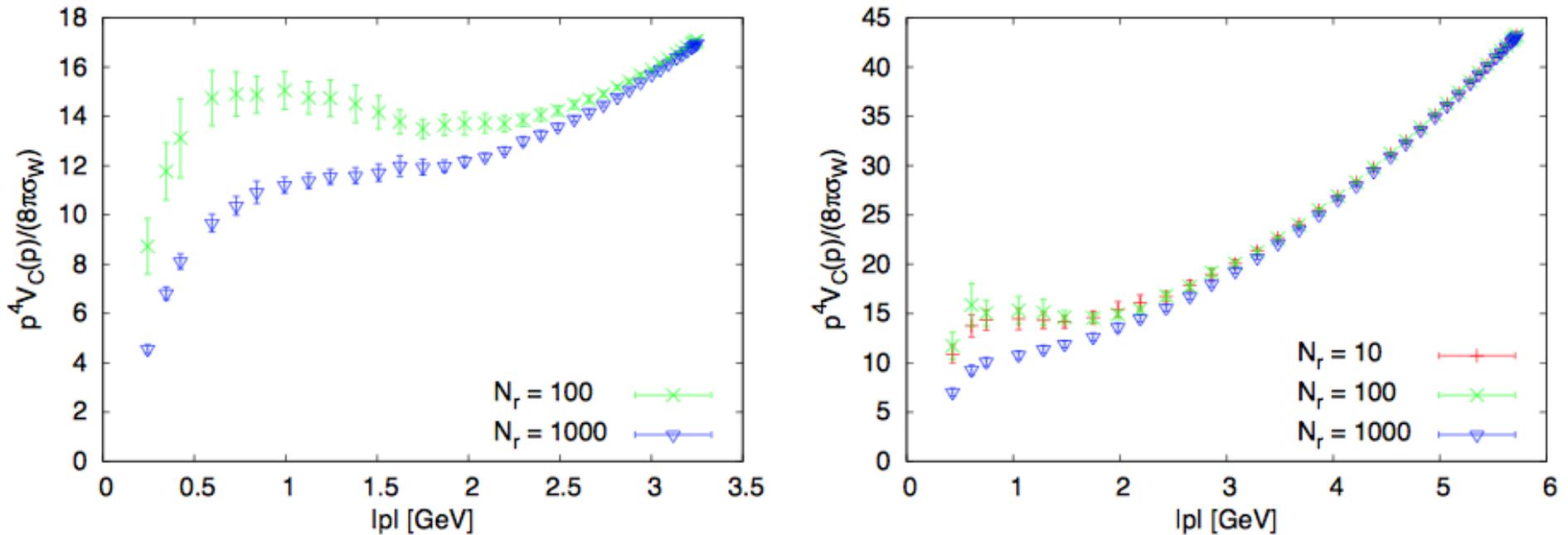


FIG. 11. Coulomb potential in the bc-approach as a function of N_r . The data with $N_r = 10,000$ are omitted, since they show no difference as compared to $N_r = 1,000$, see Fig. 4.

Coulomb gauge fixing on the lattice & Gribov copies: Coulomb potential-bc & lc

$$V_C(\vec{x}, \vec{y}) = \langle \vec{x} | (-D\partial)^{-1} (-\partial^2) (-D\partial)^{-1} | \vec{y} \rangle$$

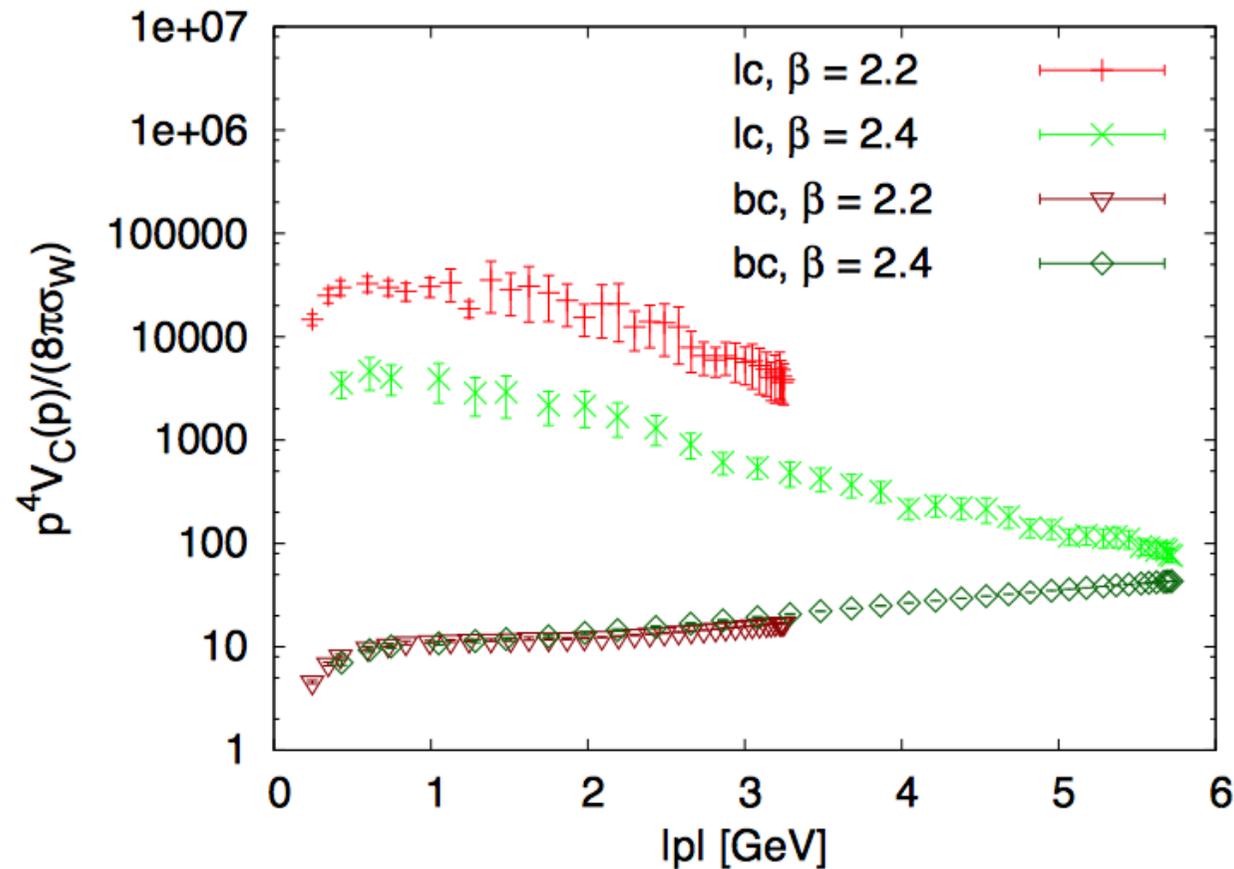


FIG. 12. The Coulomb potential from the lattices B1 and B3 after 10,000 copies.

Coulomb gauge fixing on the lattice & Gribov copies: Coulomb potential

$$\begin{aligned}
 aV_C(|\mathbf{x} - \mathbf{y}|) &= -\lim_{t \rightarrow 0} \frac{d}{dt} \log \left\langle \text{tr} \left[P_t(\mathbf{x}) P_t^\dagger(\mathbf{y}) \right] \right\rangle \\
 &= -\log \left\langle \text{tr} \left[U_0(\mathbf{x}) U_0^\dagger(\mathbf{y}) \right] \right\rangle,
 \end{aligned}$$

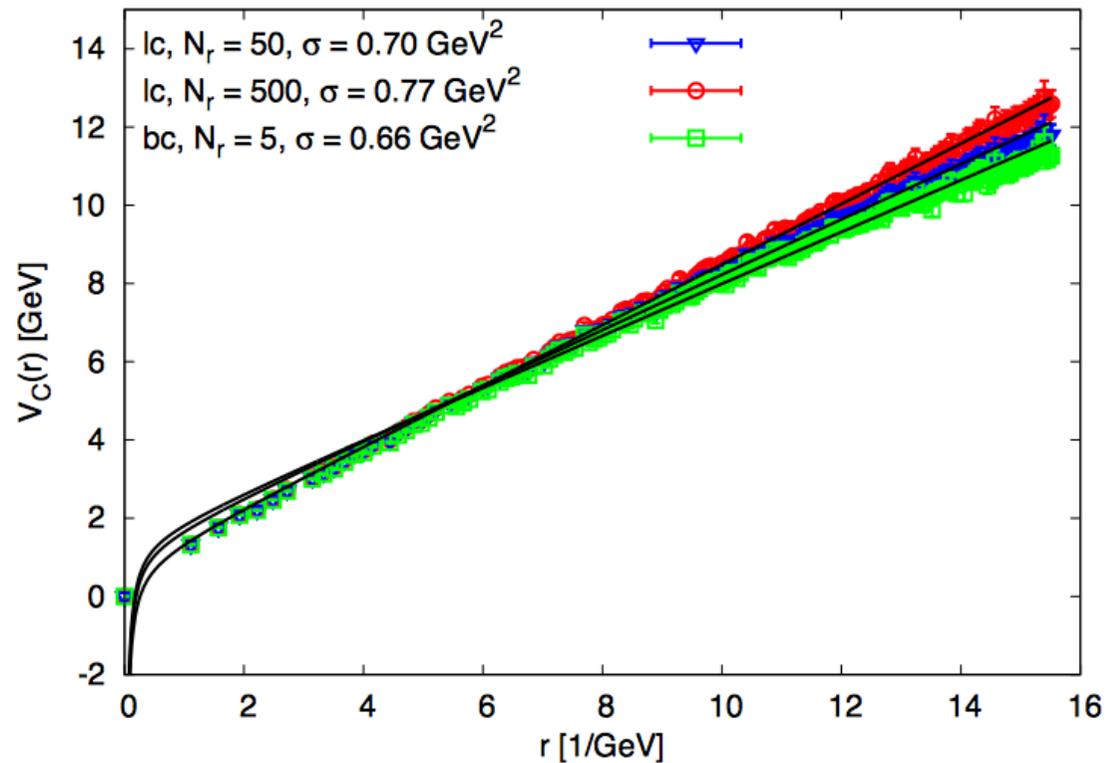


FIG. 13. Coulomb potential in position space from the $\langle U_0 U_0^\dagger \rangle$ correlator in eq. (11) (D1 lattice). The bc configurations are preconditioned with simulated annealing.