Confinining properties of QCD in strong magnetic backgrounds

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Based on arXiv:1607.08160, in collaboration with

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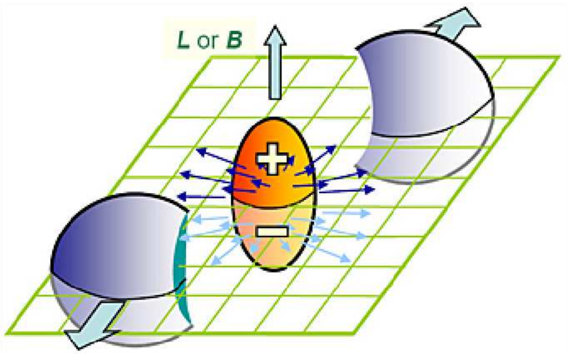
QCD IN EXTERNAL MAGNETIC BACKGROUNDS

Quarks are subject to electroweak interactions, which in general induce small corrections to strong interaction dynamics. Exceptions are expected in the presence of strong e.m. backgrounds, a situation relevant to many contexts:

- Large magnetic fields are expected in a class of neutron stars known as magnetars ($B \sim 10^{10}$ Tesla on the surface) (Duncan-Thompson, 1992).
- Large magnetic fields ($B \sim 10^{16}$ Tesla, $\sqrt{|e|B} \sim 1.5$ GeV), may have been produced at the cosmological electroweak phase transition (Vachaspati, 1991).
- 

in non-central heavy ion collisions, largest magnetic fields ever created in a laboratory (B up to 10^{15} Tesla at LHC) with a possible rich associated phenomenology (e.g., chiral magnetic effect)

Numerical QCD+QED studies go back to the early days of LQCD

- G. Martinelli, G. Parisi, R. Petronzio and F. Rapuano, Phys. Lett. B 116, 434 (1982).
- C. Bernard, T. Draper, K. Olynyk and M. Rushton, Phys. Rev. Lett. 49, 1076 (1982).

An e.m. background field a_μ modifies the covariant derivative as follows:

$$D_\mu = \partial_\mu + i\, g A^a_\mu T^a \quad \rightarrow \quad \partial_\mu + i\, g A^a_\mu T^a + i\, q a_\mu$$

in the lattice formulation:

$$D_\mu \psi \rightarrow \frac{1}{2a}\big(U_\mu(n) u_\mu(n) \psi(n+\hat{\mu}) - U^\dagger_\mu(n-\hat{\mu}) u^*_\mu(n-\hat{\mu})\psi(n-\hat{\mu})\big)$$

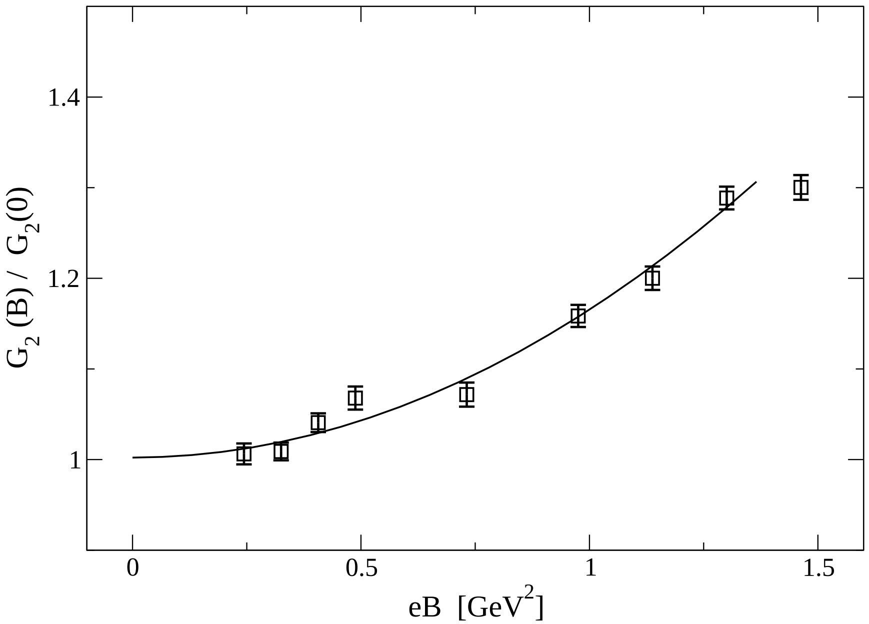
$U_\mu \in SU(3)$ $\qquad$ $\mathbf{u}_\mu \simeq \exp(\mathbf{i\, q\, a}_\mu(\mathbf{n})) \in \mathbf{U(1)}$

- $F^{(em)}_{ij} \neq 0 \quad \Longrightarrow$ **non-zero magnetic fiel (no sign problem)**
- $F^{(em)}_{0i} \neq 0 \quad \Longrightarrow$ **non-zero imaginary electric field (sign problem for real e. f.)**
- **Uniform background field are quantized in the presence of periodic boundary conditions**

Recent years have seen an increasing activity in the lattice study of QCD in magnetic backgrounds (with significant contributions coming from two friends that we miss, Michael Müller-Preussker and Misha Polikarpov) An incomplete summary of results:

Magnetic catalysis (increase of chiral symmetry breaking) of the QCD vacuum has been extensively verified (P. V. Buividovich et al. 2010; MD, F. Negro, 2011; G. S. Bali et al. 2012; E.-M. Ilgenfritz et al., 2012, 2014)

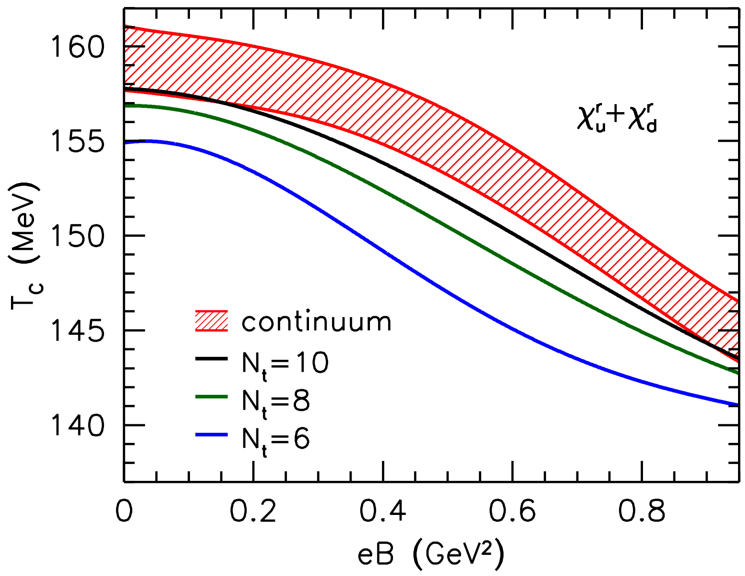
A large effect on gluon fields manifests in anisotropies of gauge observable and in an increase of the gluon condensate as a function of B (gluon magnetic catalysis) (M. Ilgenfritz et al, arXiv:1203.3360; G. Bali et al., arXiv:1303.1328; MD, M. Mesiti, E. Meggiolaro and F. Negro, arXiv:1510.07012)



from arXiv:1510.07012

The magnetic field has strong effects also on QCD thermodynamics and leads to a decrease of the pseudo-critical temperature **(inverse magnetic catalysis)**

G. S. Bali et al., arXiv:1111.4956

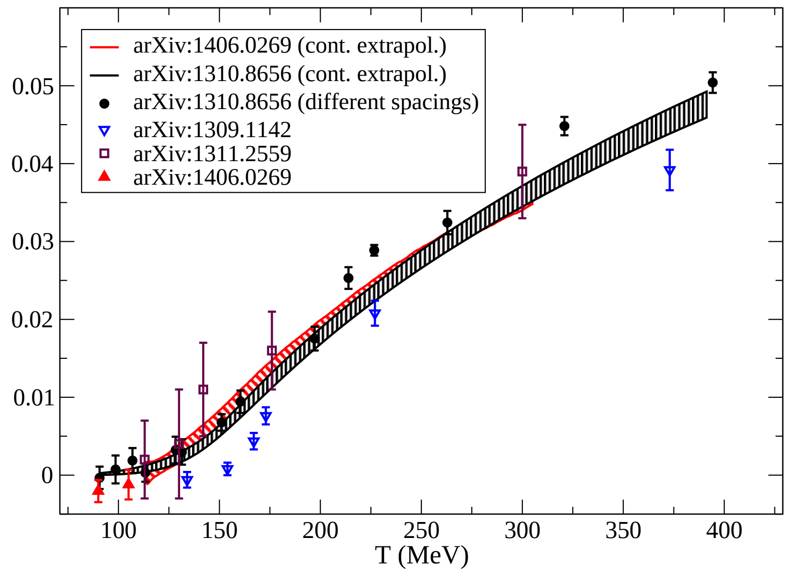


The thermal QCD medium becomes strongly paramagnetic right above T_c

C. Bonati et al., arXiv:1307.8063, arXiv:1310.8656;

L. Levkova and C. DeTar, arXiv:1309.1142;

G. S. Bali et al., arXiv:1406.0269



magnetic susceptibility

Focus of this talk:

Effects of the magnetic field on the static quark potential

- A previous study has shown that the quark-antiquark potential becomes anisotropic, with a string tension smaller (larger) in the direction parallel to $\vec{B}$

  (C. Bonati et al., arXiv:1403.6094)

- The issue is interesting both by itself and for possible phenomenological consequences, e.g. for heavy quark bound states.

In this talk I discuss results reported in arXiv:1607.08160 (C. Bonati et al.), which try to achieve the following goals:

- A complete determination of the angular dependence of the potential
- An extrapolation to the continuum limit
- An extension to finite temperature

LATTICE SETUP

$$Z(B) = \int \mathcal{D}U \, e^{-\mathcal{S}_{YM}} \prod_{f=u,\,d,\,s} \det \left(D^{f}_{\textrm{st}}[B]\right)^{1/4} .$$

- pure gauge: Symanzik tree level improved gauge action

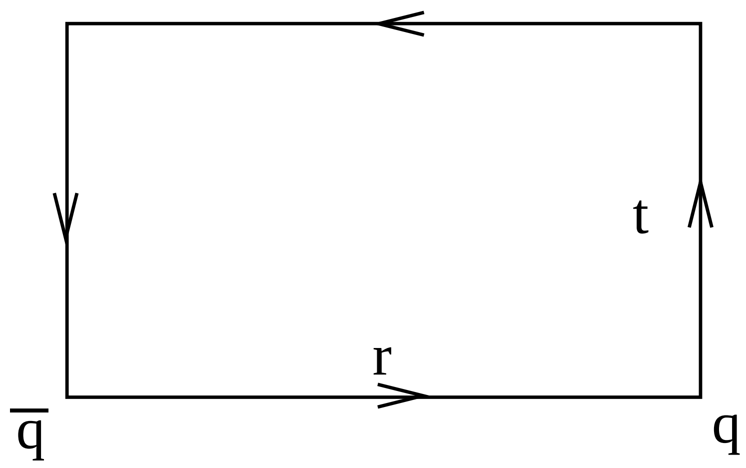
- fermion sector: 2-level stout improved rooted staggered fermions

- physical quark masses

- explored lattice spacings and sizes:
$a = 0.2173,\ 0.1535,\ 0.1249,\ 0.0989$ fm $\quad L_s a \sim 5$ fm in all cases

- numerical simulations on FERMI (BG/Q at CINECA) thanks to PRACE allocation

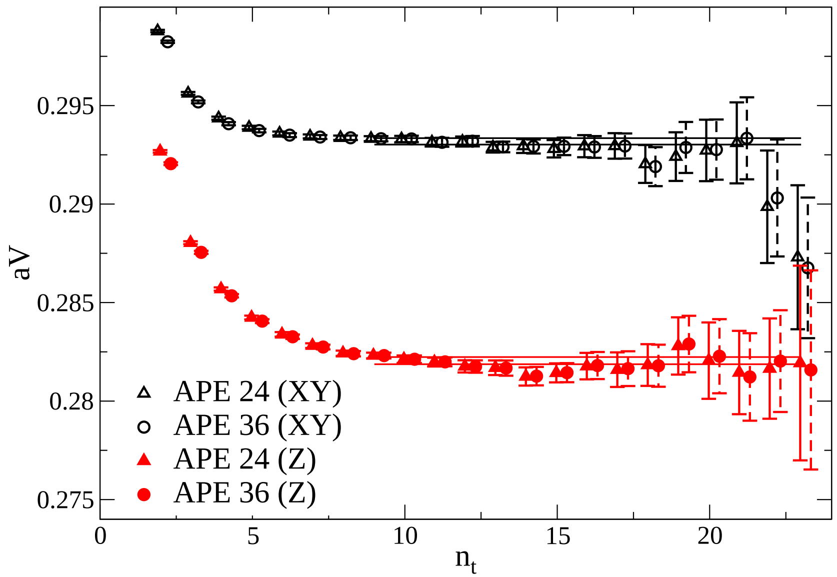
At $T = 0$, the potential is determined through Wilson loop expectation values



1 HYP smearing for temporal links and various APE smearings for spatial links to reduce UV fluctuations

As usual

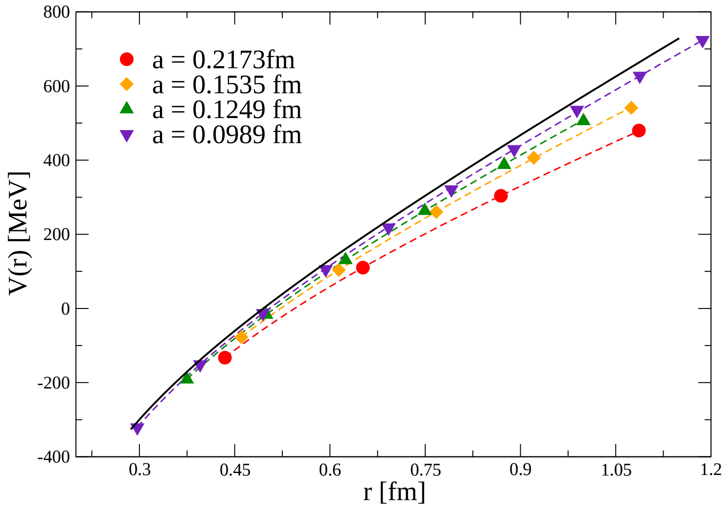
$$aV(a\vec{n}) = \lim_{n_t \to \infty} \log \left(\frac{\langle W(\vec{n}, n_t) \rangle}{\langle W(\vec{n}, n_t + 1) \rangle} \right)$$

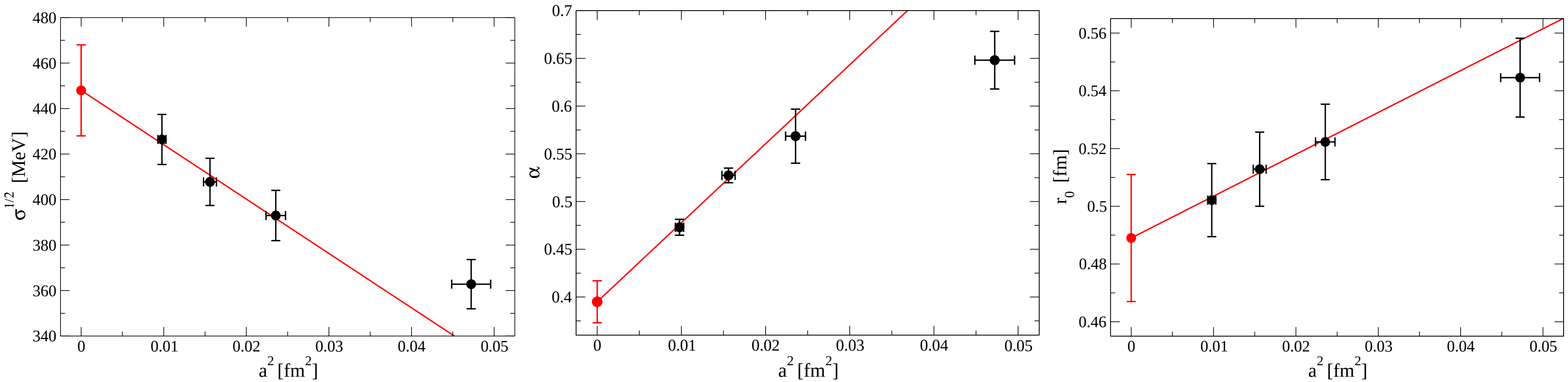


results in the figure refer to two different orientations with respect to $\vec{B} = B\hat{z}$, and for simulations performed at $a \simeq 0.0989$ fm with $|e|B \simeq 1$ GeV^2.

we have first studied the potential at $B = 0$, adopting the Cornell potential as an ansatz for all lattice spacings

$$V(r) = -\frac{\alpha}{r} + \sigma r + V_0 \; ,$$





In this way we obtain continuum extrapolated results for σ, α and for the Sommer parameter r_0

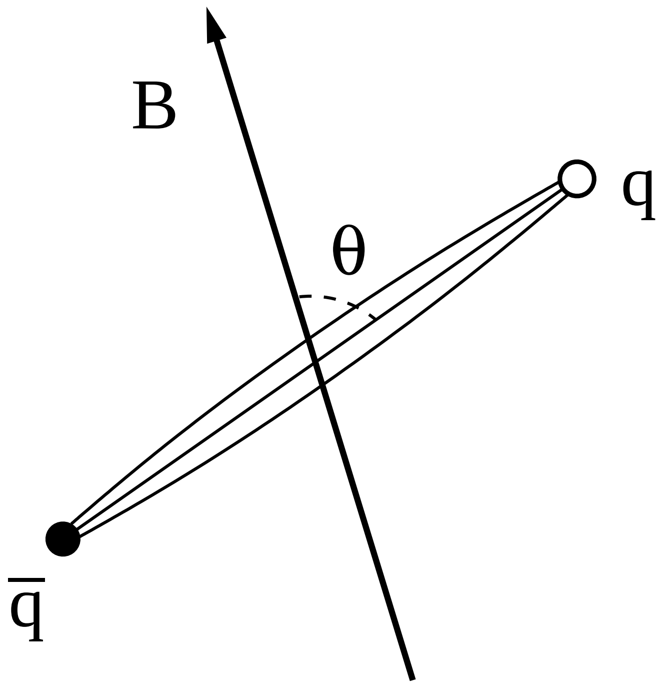
$$r_0^2 \left. \frac{dV}{dr} \right|_{r_0} = 1.65$$

α	**0.395(22)**
$\sqrt{\sigma}$	**448(20) MeV**
r_0	**0.489(20) fm**

For non-zero background field $\vec{B}$, we want to study the potential not just for parallel or orthogonal directions, but for generic orientations.

In principle, one can either rotate the spatial side of the Wilson loop, or rotate $\vec{B}$ and perform new simulations.

Rotating the loop on the lattice introduces new cusps and renormalization effects, so we chose the second solution



Each component of the field gets quantized in the presence of spatial periodic b.c.

$$eB_x = 6\pi b_x/(a^2 N_z N_y)\,; \quad b_x \in \mathbb{Z}$$

$$eB_y = 6\pi b_y/(a^2 N_x N_z)\,; \quad b_y \in \mathbb{Z}$$

$$eB_z = 6\pi b_z/(a^2 N_x N_y)\,; \quad b_z \in \mathbb{Z}$$

we performed different simulations at fixed $B_x^2 + B_y^2 + B_z^2$ and different $\vec{B}$ orientations

Expected symmetries and ansatz for $V(r,\theta,\phi)$

- **by residual rotational symmetry around $\vec{B}$:** $V(r,\theta,\phi) = V(r,\theta)$
- **by symmetry under $\vec{B} \to -\vec{B}$:** $V(r,\pi-\theta) = V(r,\theta)$
- **We make the assumption the potential is Cornell like along each direction**

$$V(r,\theta) = -\frac{\alpha(\theta,B)}{r} + \sigma(\theta,B)r + V_0(\theta,B)$$

and write a Fourier expansion in θ for each term:

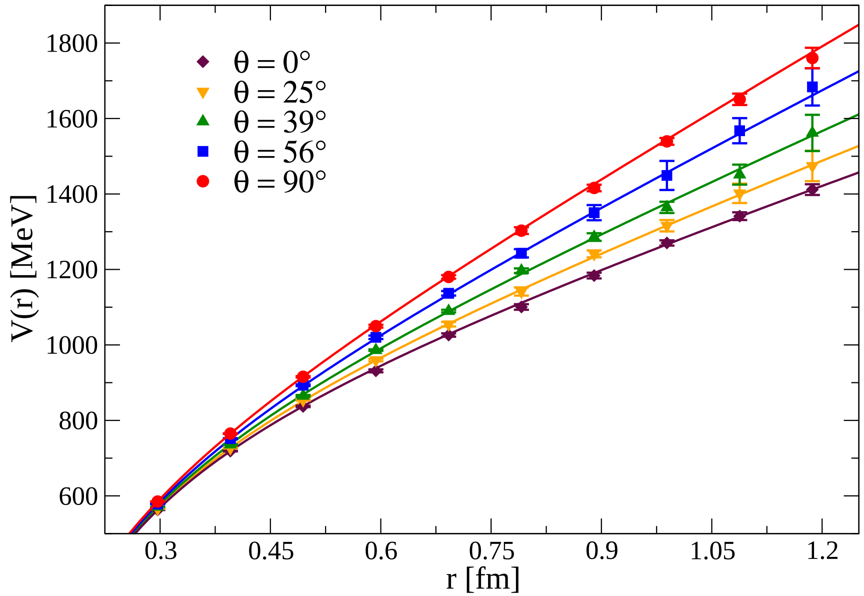
$$\begin{aligned} V(r,\theta) = & -\frac{\bar{\alpha}(B)}{r}\Big(1 - \sum_{n=1} c_{2n}^{\alpha}(B)\cos(2n\theta)\Big) \\ & + \bar{\sigma}(B)r\Big(1 - \sum_{n=1} c_{2n}^{\sigma}(B)\cos(2n\theta)\Big) \\ & + \bar{V}_0(B)\Big(1 - \sum_{n=1} c_{2n}^{V_0}(B)\cos(2n\theta)\Big) . \end{aligned}$$

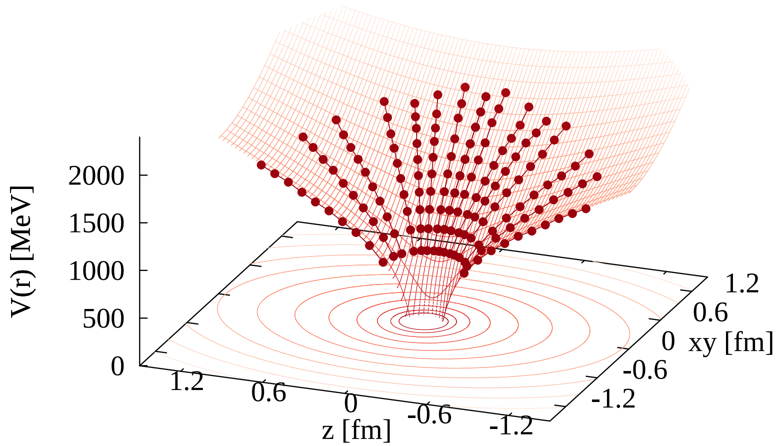
RESULTS

- **full angular dependence studied at just two lattice spacings, $a \simeq 0.1, 0.15$ fm and for $eB \sim 1$ GeV^2. Results shown for $a \sim 0.1$ fm**

- **At fixed r, the potential is an increasing function of the angle and reaches a maximum for orthogonal directions**

- **Our ansatz works well ($\chi^2/d.o.f. \sim 1$) with only the first term in the expansion $c_2 \neq 0$ (quadrupole-like deformation)**





Strategy followed for other B and a: The simplified angular dependence (only $c_2 \neq 0$), permits to reconstruct V from data at $\theta = 0, \pi$ only.

Let $\mathcal{O}$ be α, σ, V and define

$$\delta^{\mathcal{O}}(|e|B) = \frac{\mathcal{O}_{XY}(|e|B) - \mathcal{O}_Z(|e|B)}{\mathcal{O}_{XY}(|e|B) + \mathcal{O}_Z(|e|B)} \quad ; \qquad R^{\mathcal{O}}(|e|B) = \frac{\mathcal{O}_{XY}(|e|B) + \mathcal{O}_Z(|e|B)}{2\mathcal{O}(|e|B = 0)}$$

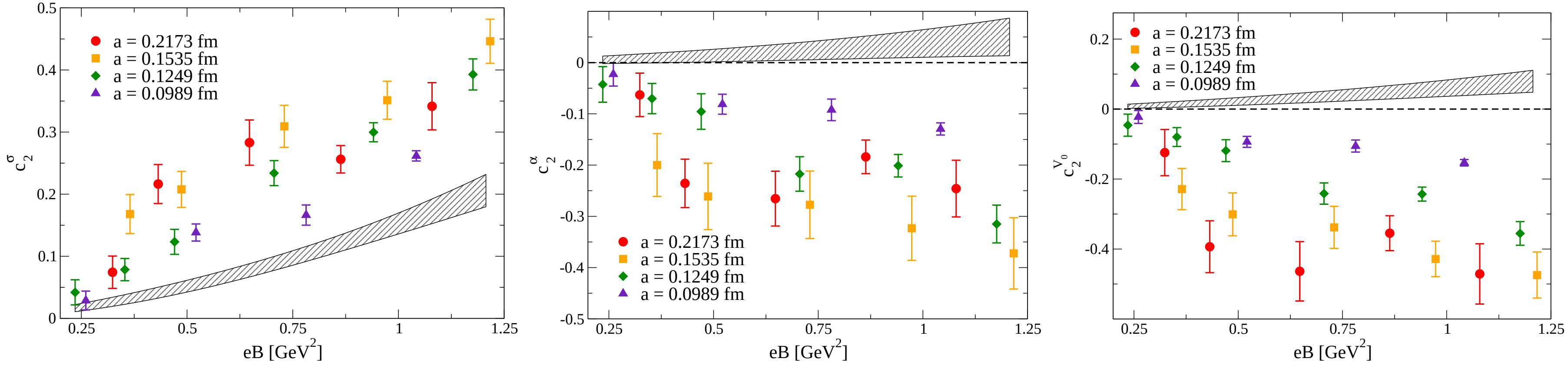
then

$$\delta^{\mathcal{O}} = c_2^{\mathcal{O}} + c_4^{\mathcal{O}} + \cdots = \sum_n c_{2n}^{\mathcal{O}} \simeq c_2^{\mathcal{O}}$$

$$R^{\mathcal{O}}(|e|B) = \frac{\bar{\mathcal{O}}(|e|B)}{\mathcal{O}(|e|B = 0)} \left(1 - \sum_{n \text{ even}} c_{2n}^{\mathcal{O}} \right) \simeq \frac{\bar{\mathcal{O}}(|e|B)}{\mathcal{O}(|e|B = 0)}$$

i.e. such quantities are enough to fix all the coefficients giving a non-trivial contribution to $V(r, \theta, B)$.

We perform the continuum extrapolation from data along longitudinal and orthogonal directions only

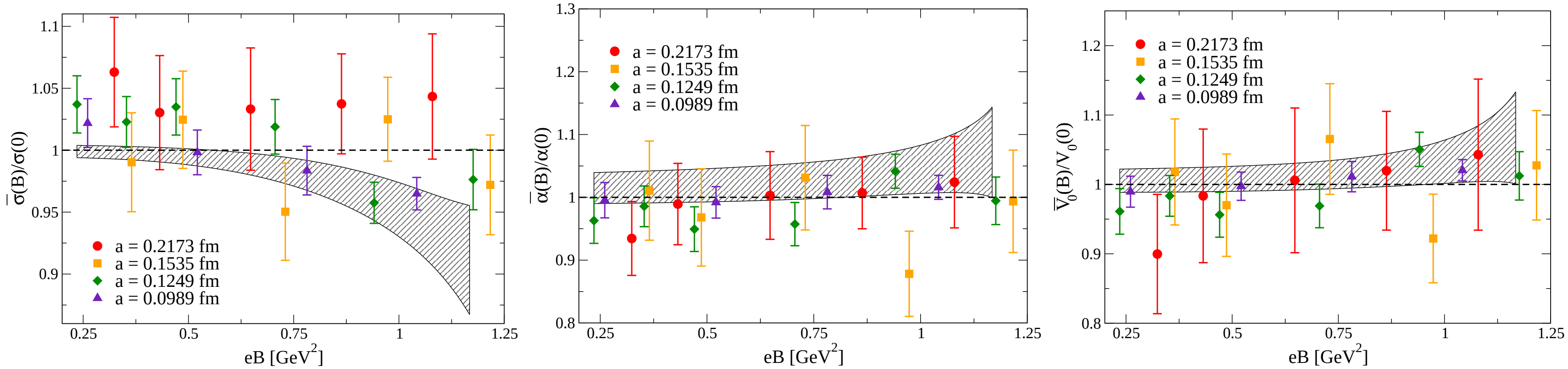


Continuum limit extrapolation of our data according to the following power-law ansatz

$$c_2^{\mathcal{O}} = A^{\mathcal{O}}(1 + C^{\mathcal{O}}a^2)(|e|B)^{D^{\mathcal{O}}(1+E^{\mathcal{O}}a^2)}$$

$$R^{\mathcal{O}} = 1 + \bar{A}^{\mathcal{O}}(1 + \bar{C}^{\mathcal{O}}a^2)(|e|B)^{\bar{D}^{\mathcal{O}}}$$

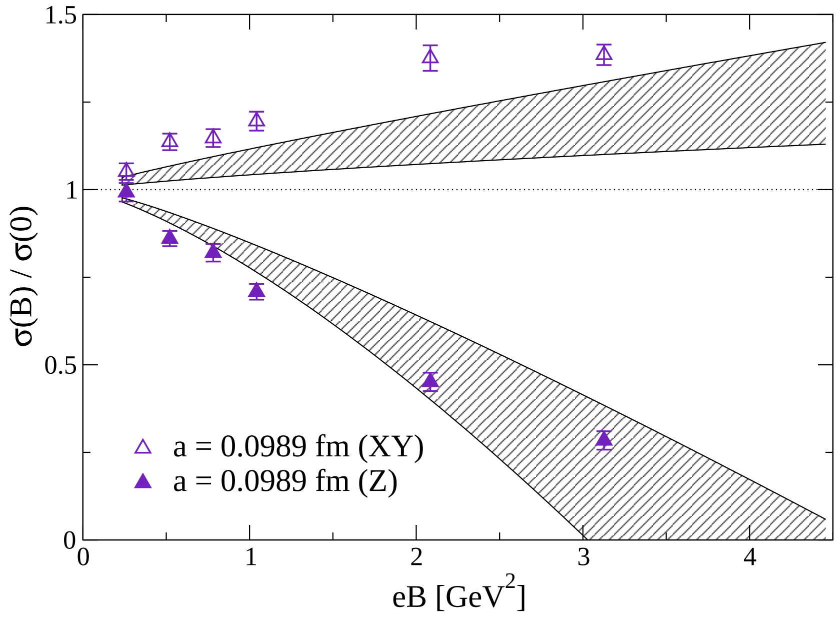
→ The only quantities with non-trivial B dependence in the continuum are c_2^σ and $\bar{\sigma}$



The continuum extrapolated results for σ predict a vanishing longitudinal string tension for $eB \sim 4$ GeV^2

This is outside the range explored for the continuum extrapolation, $eB \lesssim 1$ GeV^2.

Can we trust the prediction?



Cut-off effects are large for $eB \gtrsim 1/a^2$. We could extend to larger B just on the finest lattice spacing.

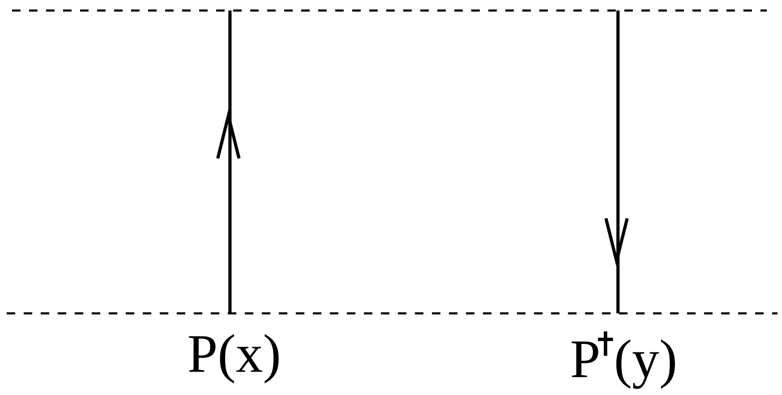
The decrease of $\sigma_{\|}$ is steady, even if it somewhat undershoots the continuum band extrapolated to large B.

Simulations at finer lattice spacings should clarify the issue in the future.

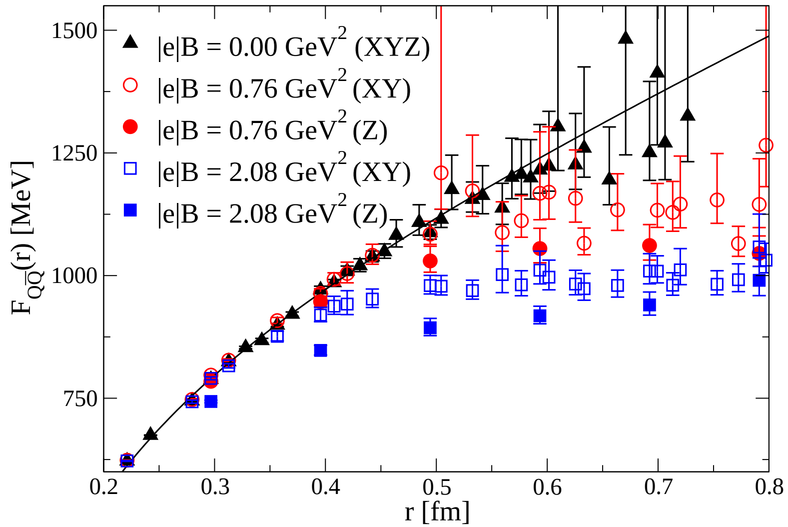
Finite T results

At finite T, the quark-antiquark potential is measured from Polyakov loop correlators

$$\langle \mathrm{Tr} P(\vec{x})\, \mathrm{Tr} P^\dagger(\vec{y}) \rangle \sim \exp\left(-\frac{F_{\bar{q}q}(r,T)}{T} \right)$$



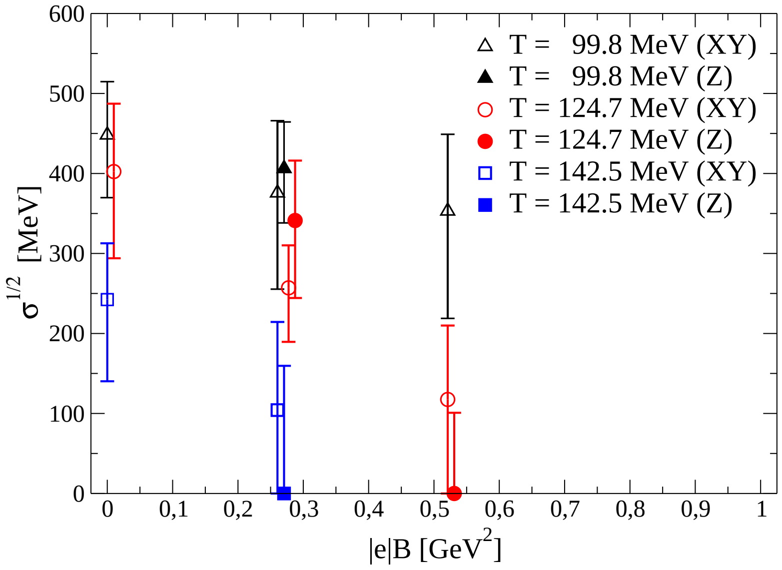
Results at $T \sim 100$ MeV on a $N_t = 20$ lattice



Although a small anisotropy is still visible, the main effect of B seems to suppress the potential in all directions

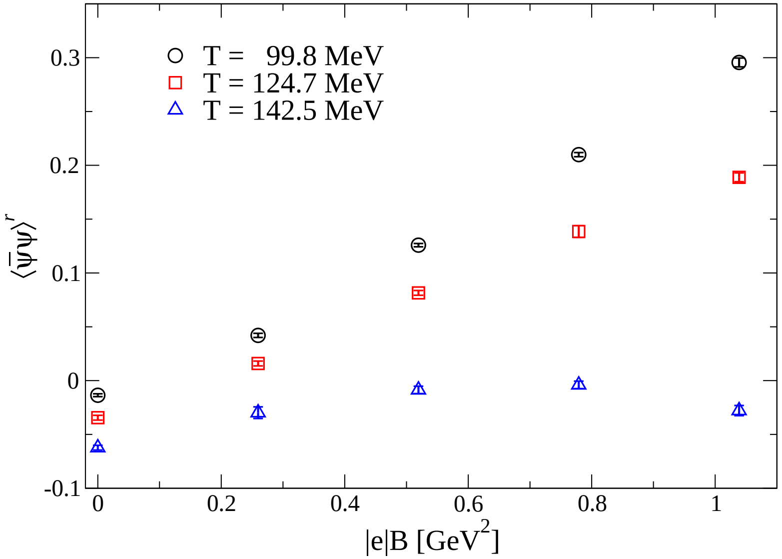
The string tension tends to disappear

A fit to the Cornell potential works in a limited range of distances and permits to obtain a determination of σ, which shows a steady decrease in all directions.



We can call this effect deconfinement catalysis

It is interesting to notice that this happens before (in temperature) inverse magnetic catalysis is visible in the chiral condensate



Is the decrease of T_c as a function of B related to a change in the confining properties?

CONCLUSIONS

- **The magnetic field leads to a quadrupole-like deformation of the static quark-antiquark potential**

- **Most of the effect seems related to a modification of the string tension**

- **We have hints that $\sigma_{||}$ could vanish in the vacuum for eB of the order of 10 GeV. Future simulations on finer lattice spacings could confirm this possibility.**

- **At finite T, the main effect is a general suppression of the potential leading to a precocious loss of confining properties: deconfinement catalysis.**
  That could be important for heavy ion physics in the thermal medium, think for instance of J/ψ suppression and related issues.