Superconformal Algebraic Approach to Hadron Structure: The Perturbative-Nonperturbative Interface in QCD

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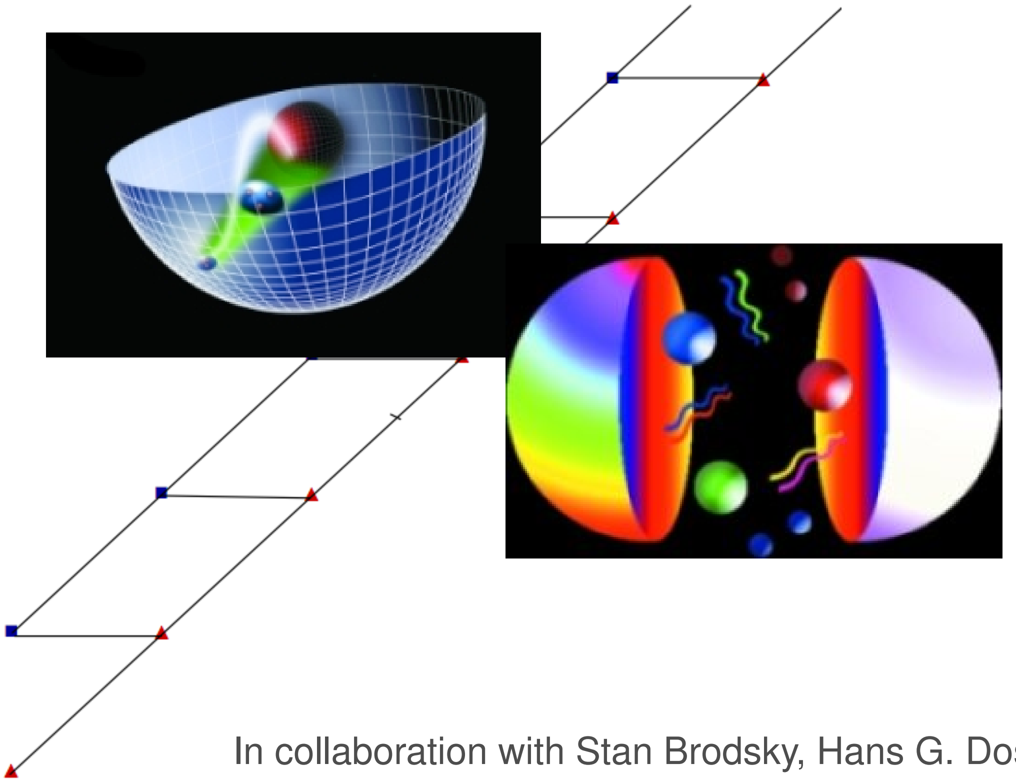
XII$^{\mathrm{th}}$ QUARK CONFINEMENT AND THE HADRONIC SPECTRUM

Thessaloniki, Greece

29 Aug - 3 Sep 2016



In collaboration with Stan Brodsky, Hans G. Dosch, Alexandre Deur and Cédric Lorce

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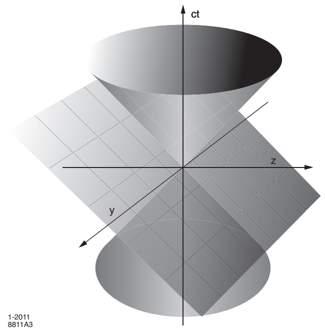
1 Preamble: The complexity of QCD

- The QCD Lagrangian in the limit of massless quarks has no scale: still confinement and a mass scale should emerge from the quantum theory built upon the classical QCD conformal theory
- Increase of the QCD coupling at low energies implies that an infinite number of quark and gluons are strongly coupled
- Description of the dynamics is vastly complex and understanding the mechanism of confinement is an unsolved problem
- We require a semiclassical approximation which captures essential aspects of the nonperturbative confinement dynamics, which are not obvious from the QCD Lagrangian
- Recent progress: nonperturbative QCD dynamics is well captured in a semiclassical effective theory based on superconformal quantum mechanics in the light-front and its holographic embedding on a higher dimensional AdS space (gauge/gravity correspondence)

2 Semiclassical approximation to light front QCD

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]



- Start with $SU(3)_C$ QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \tfrac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Express the hadron 4-momentum generator $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, in terms of dynamical fields $\psi_+ = \Lambda_+ \psi$ and $\mathbf{A}_\perp$ $\left(\Lambda_\pm = \gamma^0\gamma^\pm\right)$ quantized in null plane $x^+ = x^0 + x^3 = 0$

$$\begin{aligned}
P^- &= \tfrac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \, \gamma^+ \frac{\left(i\nabla_\perp\right)^2 + m^2}{i\partial^+}\psi_+ + \text{interactions} \\
P^+ &= \int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i\partial^+ \psi_+ \\
\mathbf{P}_\perp &= \tfrac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i\nabla_\perp \psi_+
\end{aligned}$$

- Construct LF invariant Hamiltonian $P^2 = P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2$ from mass-shell relation

$$P^2 |\psi(P)\rangle = M^2 |\psi(P)\rangle, \qquad |\psi\rangle = \sum_n \psi_n |n\rangle$$

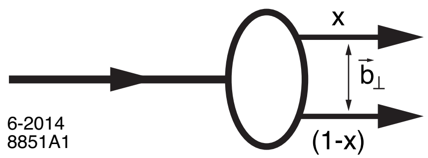
- Simple structure of LF vacuum allows a quantum-mechanical probabilistic interpretation of hadronic states in terms of wave functions: $\psi_n = \langle n|\psi\rangle$

- Compute the hadron matrix element $\langle\psi(P')|P_\mu P^\mu|\psi(P)\rangle$ for a two-parton bound state
- Factor out the longitudinal $X(x)$ and orbital kinematical dependence from LFWF ψ

$$\psi(x,\zeta,\varphi) = e^{iL\varphi}X(x)\frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

with invariant transverse impact variable

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2$$



- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple $(L = L^z)$

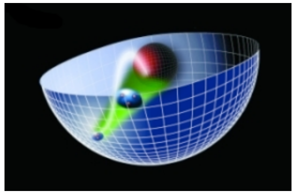
$$M^2 = \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta\, \phi^*(\zeta)\, U(\zeta)\, \phi(\zeta)$$

where effective potential U includes all interactions, including those from higher Fock states

- LF Hamiltonian equation $P_\mu P^\mu|\psi\rangle = M^2|\psi\rangle$ is a LF wave equation for ϕ

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)}$$

- Critical value $L = 0$ corresponds to lowest possible stable solution
- Relativistic and frame-independent LF Schrödinger equation: U is instantaneous in LF time



3 Higher integer-spin wave equations in AdS space

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]

- Why is AdS space important? AdS$_5$ is a 5-dim space of maximal symmetry (conformal), negative curvature and a four-dim boundary: Minkowski space $ds^2 = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)$
- Isomorphism of $SO(4,2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D$ with the group of isometries of AdS$_5$
- Integer spin-J in AdS conveniently described by tensor field $\Phi_{N_1 \cdots N_J}$ with effective action

$$S_{\mathit{eff}} = \int d^d x\, dz\, \sqrt{|g|}\, e^{\varphi(z)}\, g^{N_1 N_1'} \cdots g^{N_J N_J'} \Big(g^{MM'} D_M \Phi^*_{N_1 \dots N_J}\, D_{M'} \Phi_{N_1' \dots N_J'} - \mu^2_{\mathit{eff}}(z)\, \Phi^*_{N_1 \dots N_J}\, \Phi_{N_1' \dots N_J'} \Big)$$

  Covariant derivative D_M includes affine connection and dilaton $\varphi(z)$ effectively breaks conformality

- Effective mass $\mu_{\mathit{eff}}(z)$ is determined by precise mapping to light-front physics
- Non-trivial geometry of pure AdS encodes the kinematics and additional deformations of AdS encode the dynamics, including confinement

- Physical hadron has plane-wave and spin components along physical coordinates $\Phi_P(x,z)_{\mu_1\cdots\mu_J} = e^{iP\cdot x}\Phi(z)_{\mu_1\cdots\mu_J}$ with invariant hadronic mass $P_\mu P^\mu = M^2$

- Variation of the action gives AdS wave equation for spin-J field $\Phi(z)_{\mu_1\cdots\mu_J} = \Phi_J(z)\epsilon_{\mu_1\cdots\mu_J}(P)$

$$\boxed{\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi(z)}}{z^{d-1-2J}}\partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right]\Phi_J = M^2\Phi_J}$$

- Upon substitution $\Phi_J(z) \sim z^{(d-1)/2-J}e^{-\varphi(z)/2}\,\phi_J(z)$ and $z\to\zeta$ we find LFWE (slide 5)

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

for ($d = 4$) where

$$\boxed{U(\zeta) = \tfrac{1}{2}\varphi''(\zeta) + \frac{1}{4}\varphi'(\zeta)^2 + \frac{2J-3}{2z}\varphi'(\zeta)}$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- Unmodified AdS equations correspond to the kinetic energy terms for the partons, and effective LF potential $U(\zeta)$ corresponds to the IR modification of AdS space

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$

4 Higher half-integer-spin wave equations in AdS space

[J. Polchinski and M. J. Strassler, JHEP **0305**, 012 (2003)]

[GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]

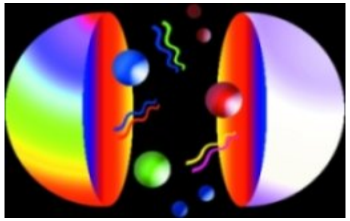
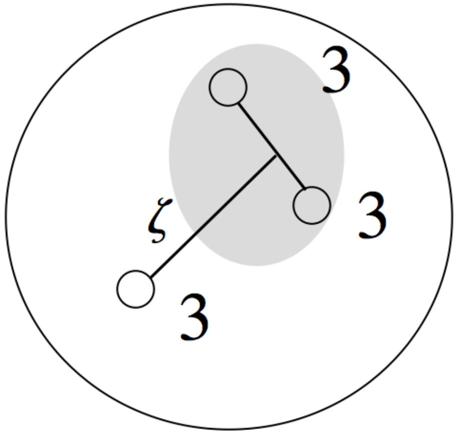


Image credit: N. Evans

- Extension of holographic ideas to spin-$\frac{1}{2}$ (and higher half-integral J) hadrons by considering wave equations for Rarita-Schwinger spinor fields in AdS space and their mapping to light-front physics
- LF clustering decomposition of invariant variable ζ: same multiplicity of states for mesons and baryons
- The invariant variable ζ is the light-front weighted distribution of the spectator diquark cluster relative to the active quark

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{N-1} x_j \mathbf{b}_{\perp j} \right|$$



- LF cluster decomposition follows from mapping of EM form factor in AdS to the light front [S. J. Brodsky and GdT, PRL **96**, 201601 (2006)]

- Half-integer spin $J = T + \frac{1}{2}$ represented by RS spinor $\left[\Psi_{N_1 \cdots N_T}\right]_\alpha$ with effective AdS action

$$S_{\mathit{eff}} = \frac{1}{2} \int d^d x \, dz \, \sqrt{|g|} \, g^{N_1 \, N_1'} \cdots g^{N_T \, N_T'} \\ \left[\overline{\Psi}_{N_1 \cdots N_T} \Big(i \, \Gamma^A \, e_A^M \, D_M - \mu - \rho(z) \Big) \Psi_{N_1' \cdots N_T'} + h.c. \right]$$

where the covariant derivative D_M includes the affine connection and the spin connection

- e_M^A is the vielbein and Γ^A tangent space Dirac matrices $\left\{\Gamma^A, \Gamma^B\right\} = \eta^{AB}$

- Variation of the action gives linear equations

$$\begin{aligned} -\frac{d}{dz}\psi_- - \frac{L + \frac{1}{2}}{z}\psi_- - V(z)\psi_- &= M\psi_+ \\ \frac{d}{dz}\psi_+ - \frac{L + \frac{1}{2}}{z}\psi_+ - V(z)\psi_+ &= M\psi_- \end{aligned}$$

where $L = \mu R - \frac{1}{2}$, $\psi \equiv \Psi_T$ and

$$V(z) = \frac{R}{z}\rho(z)$$

a J-independent potential – No spin-orbit coupling along a given trajectory !

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

- Mapping to the light front, $z \to \zeta$ system of linear eqs. is equivalent to the second order equations:

$$\begin{aligned}\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U^+(\zeta)\right)\psi_+ &= M^2\psi_+ \\ \left(-\frac{d^2}{d\zeta^2} - \frac{1-4(L+1)^2}{4\zeta^2} + U^-(\zeta)\right)\psi_- &= M^2\psi_-\end{aligned}$$

where

$$U^\pm(\zeta) = V^2(\zeta) \pm V'(\zeta) + \frac{1+2L}{\zeta}V(\zeta)$$

and

$$V(\zeta) = \frac{R}{\zeta}\rho(\zeta)$$

- The plus and minus component equations correspond to LF orbital angular momentum L and $L+1$

5 Superconformal quantum mechanics and nucleons

[S. Fubini and E. Rabinovici, NPB **245**, 17 (1984)]

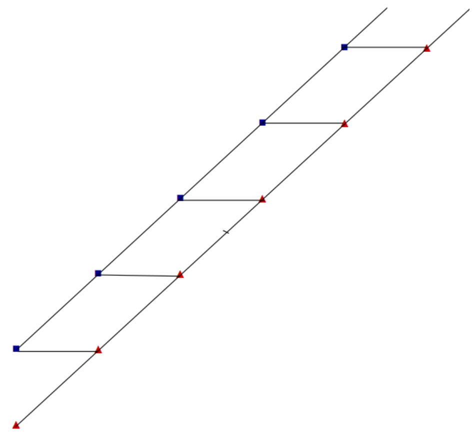
[GdT, H.G. Dosch and S. J. Brodsky, PRD **91**, 045040 (2015)]

- SUSY QM contains two fermionic generators Q and $Q^\dagger$, and a bosonic generator, the Hamiltonian H

  [E. Witten, NPB **188**, 513 (1981)]

- Closure under the graded algebra $sl(1/1)$:

$$\frac{1}{2}\{Q, Q^\dagger\} = H$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$
$$[Q, H] = [Q^\dagger, H] = 0$$



- Since $[Q^\dagger, H] = 0$, the states $|E\rangle$ and $Q^\dagger|E\rangle$ have identical eigenvalues E, but for a zero eigenvalue we can have the trivial solution $|E = 0\rangle = 0$

- For a conformal theory

$$Q = \psi^\dagger\left(-\frac{d}{dx} + \frac{f}{x}\right), \qquad Q^\dagger = \psi\left(\frac{d}{dx} + \frac{f}{x}\right)$$

where ψ and $\psi^\dagger$ are spinor operators with $\{\psi, \psi^\dagger\} = 1$ and f is dimensionless

- Conformal graded-Lie algebra has in addition to the Hamiltonian H and supercharges Q and $Q^\dagger$, a new operator S related to the generator of conformal transformations K

$$S = \chi\, x, \qquad S^\dagger = \chi^\dagger x$$

- Find enlarged algebra (Superconformal algebra of Haag, Lopuszanski and Sohnius (1974))

$$\begin{aligned} \tfrac{1}{2}\{Q, Q^\dagger\} &= H, & \tfrac{1}{2}\{S, S^\dagger\} &= K, \\ \{Q, S^\dagger\} &= f - B + 2iD, & \{Q^\dagger, S\} &= f - B - 2iD \end{aligned}$$

where $B = \frac{1}{2}[\psi^\dagger, \psi]$, and the generators of translation, dilatation and the special conformal transformation H, D and K

$$\begin{aligned} H &= \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{f^2 + 2Bf}{x^2}\right) \\ D &= \frac{i}{4}\left(\frac{d}{dx}x + x\frac{d}{dx}\right) \\ K &= \tfrac{1}{2}x^2 \end{aligned}$$

satisfy the conformal algebra

$$[H, D] = iH, \qquad [H, K] = 2iD, \qquad [K, D] = -iK$$

- Following F&R define a fermionic generator R, a linear combination of the generators Q and S

$$R_\lambda = Q + \lambda\, S$$

which generates a new Hamiltonian

$$G_\lambda = \{R_\lambda, R_\lambda^\dagger\}$$

where by construction

$$\{R_\lambda, R_\lambda\} = \{R_\lambda^\dagger, R_\lambda^\dagger\} = 0, \qquad [R_\lambda, G_\lambda] = [R_\lambda^\dagger, G_\lambda] = 0$$

which also closes under the graded algebra $sl(1/1)$

- In a Pauli matrix representation G_λ is given by

$$G_\lambda = 2H + 2\lambda^2 K + 2\lambda\,(f - \sigma_3)$$

and leads to the eigenvalue equations

$$\left(-\frac{d^2}{dx^2} + \lambda^2\, x^2 + 2\lambda\, f - \lambda + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2}\right)\phi_1 = E\,\phi_1$$

$$\left(-\frac{d^2}{dx^2} + \lambda^2\, x^2 + 2\lambda\, f + \lambda + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2}\right)\phi_2 = E\,\phi_2$$

Light-front mapping and nucleon spectrum

- Upon the substitutions $x \mapsto \zeta$, $E \mapsto M^2$ and $f \mapsto L + \frac{1}{2}$ in the AdS wave equations (slide 10) we find $U^+ = \lambda^2\zeta^2 + 2\lambda(L+1)$ and $U^- = \lambda^2\zeta^2 + 2\lambda L$, thus the effective potential $V = \lambda\, z$ and the bound state equations:

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(L+1)\right)\psi_+ = M^2\psi_+$$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4(L+1)^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda L\right)\psi_- = M^2\psi_-,$$

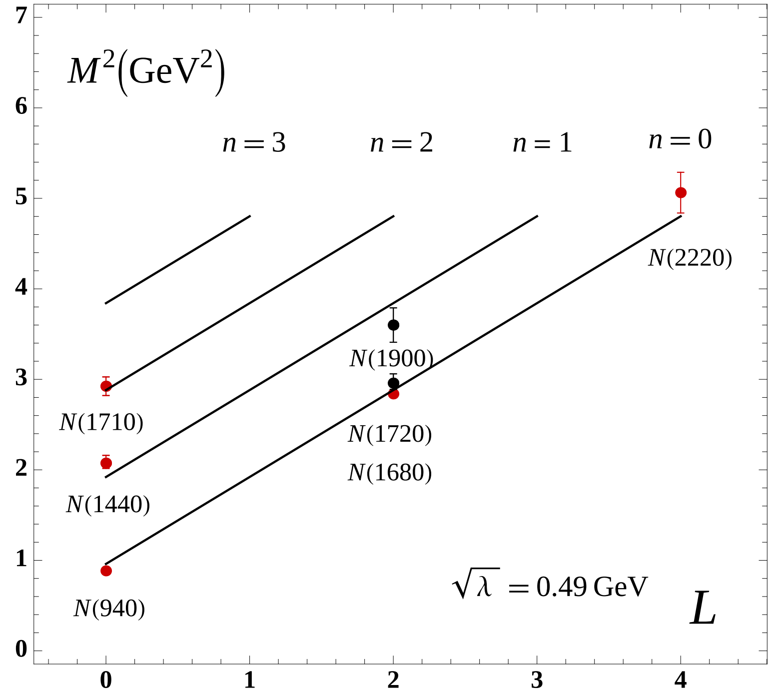
- Eigenfunctions

$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+L} e^{-\lambda\zeta^2/2} L_n^L(\lambda\zeta^2)$$
$$\psi_-(\zeta) \sim \zeta^{\frac{3}{2}+L} e^{-\lambda\zeta^2/2} L_n^{L+1}(\lambda\zeta^2)$$

- Eigenvalues

$$M^2 = 4\lambda(n + L + 1)$$

- L is the relative LF angular momentum between the active quark and spectator cluster



6 Superconformal meson-baryon symmetry

[H.G. Dosch, GdT, and S. J. Brodsky, PRD **91**, 085016 (2015)]

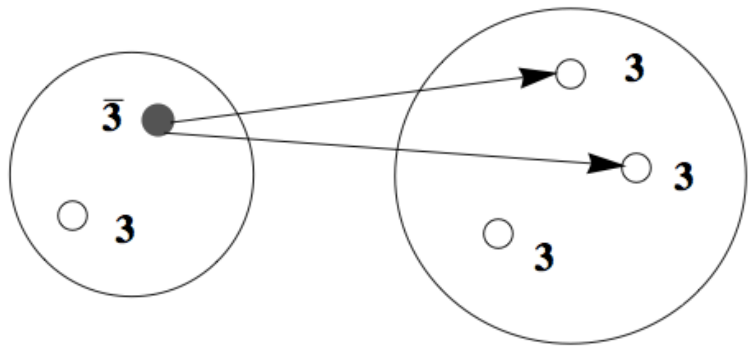
$$|\phi\rangle = \begin{pmatrix} \phi_{\mathrm{Meson}} \\ \phi_{\mathrm{Nucleon}} \end{pmatrix}$$

- Extend superconformal QM to relate bound-state equations for mesons and baryons
- Upon the substitutions (slide 13) $x \mapsto \zeta$, $E \to M^2$, $\lambda \mapsto \lambda_B = \lambda_M$ and $f \mapsto L_M - \frac{1}{2} = L_B + \frac{1}{2}$ find the bound-state equations

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + \lambda_M^2\,\zeta^2 + 2\lambda_M(L_M - 1)\right)\phi_{Meson} = M^2\,\phi_{Meson}$$

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_N^2 - 1}{4\zeta^2} + \lambda_N^2\,\zeta^2 + 2\lambda_N(L_N + 1)\right)\phi_{Nucleon} = M^2\,\phi_{Nucleon}$$

- Superconformal QM imposes the condition $\lambda = \lambda_M = \lambda_B$ (equality of Regge slopes) and the remarkable relation $\Rightarrow$ $\boxed{L_M = L_B + 1}$
- Also $R^\dagger|M, L\rangle = |B, L-1\rangle, \quad R^\dagger|M, L=0\rangle = 0$
  Special role of the pion as a unique state of zero energy
- Emerging dynamical SUSY from SU(3) color
  (Hadronic SUSY introduced by H. Miyazawa (1966))



$\bar{\mathbf{3}} \to \mathbf{3} \times \mathbf{3}$

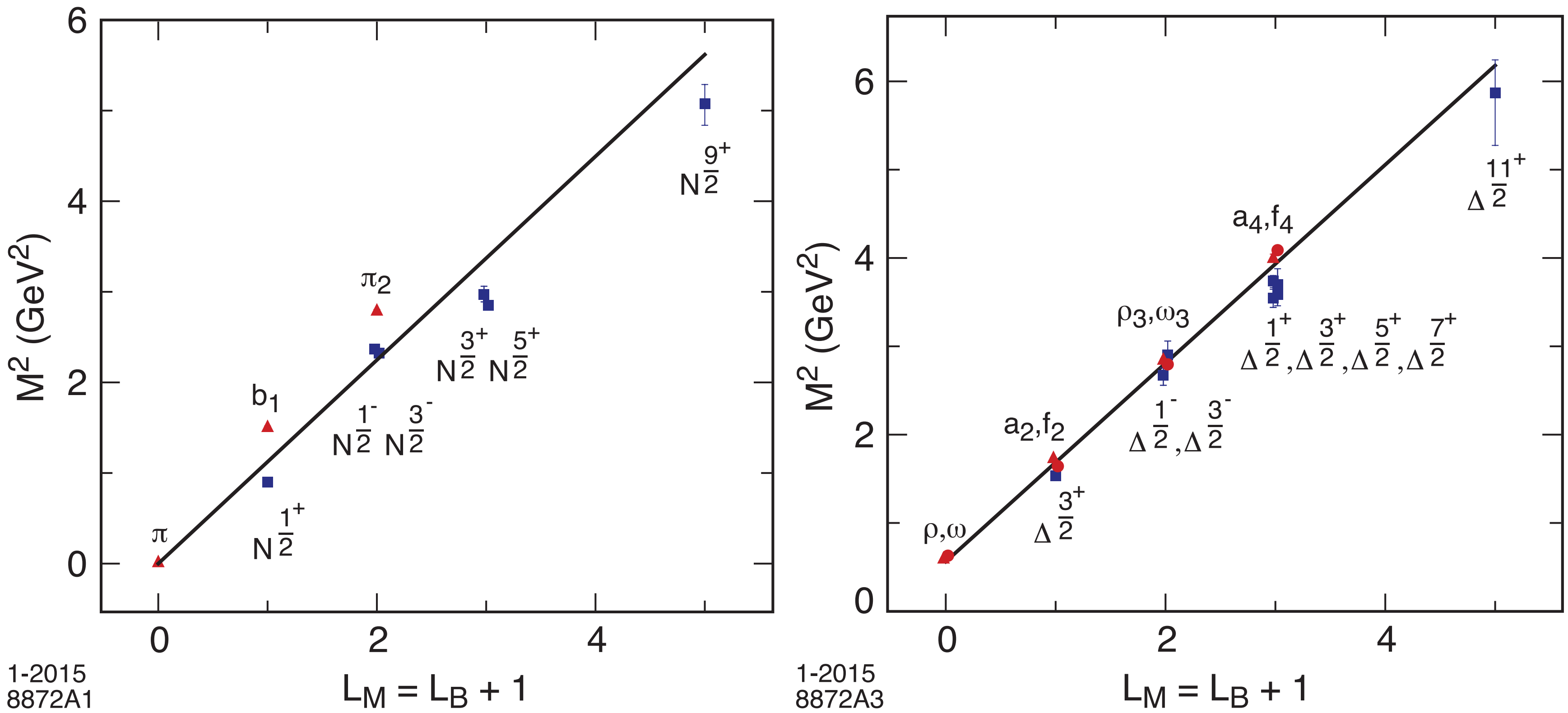
$L \to L - 1$

- Superconformal spin-dependent Hamiltonian to describe mesons and baryons (chiral limit)

  [S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé, PLB **759**, 171 (2016)]

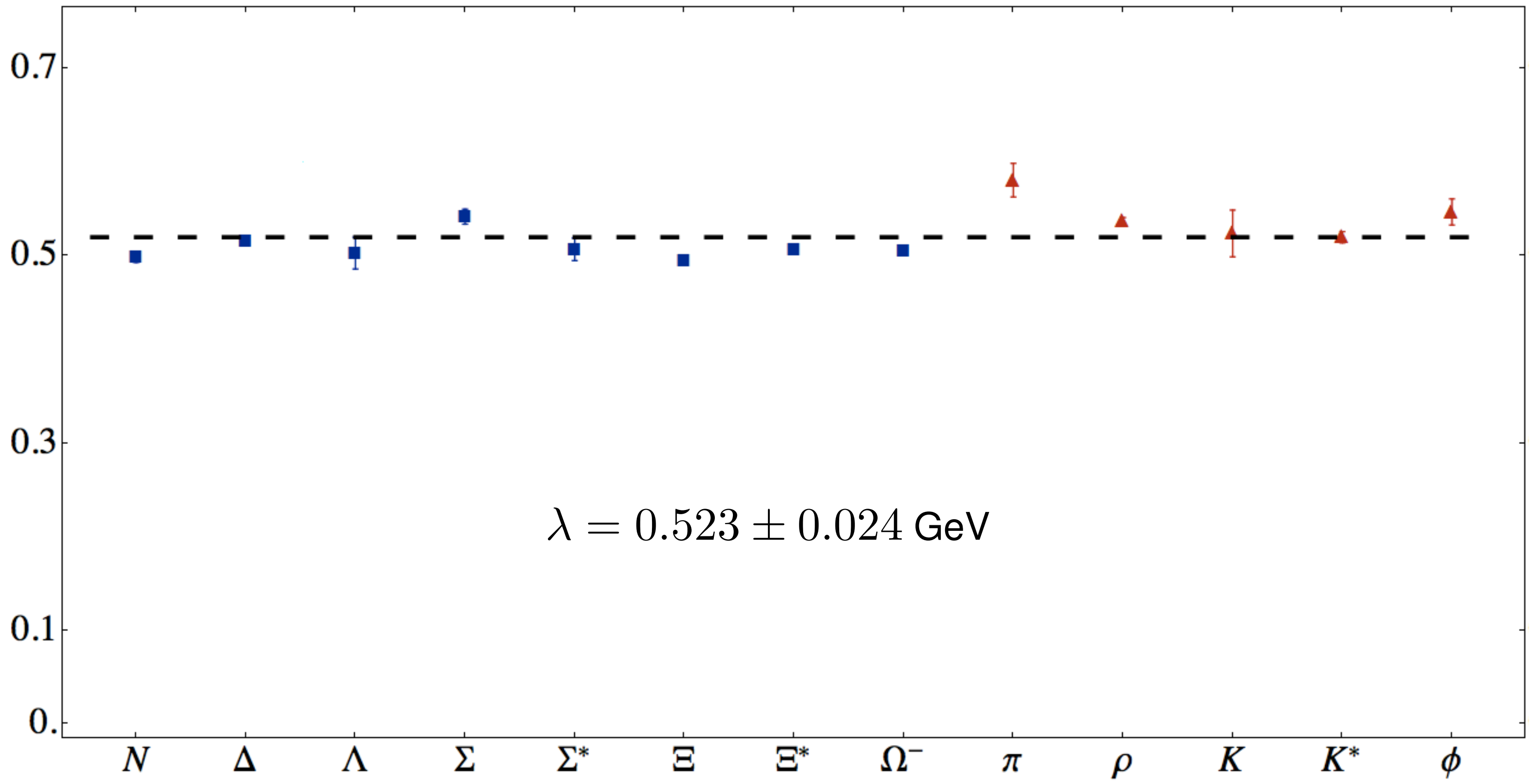
$$\boxed{G = \{R_\lambda^\dagger, R_\lambda\} + 2\lambda\, S} \qquad\qquad S = 0, 1$$

$$\text{Mesons} : M^2 = 4\lambda\,(n + L_M) + 2\lambda S, \quad \text{Baryons} : M^2 = 4\lambda\,(n + L_B + 1) + 2\lambda S$$



Superconformal meson-nucleon partners: solid line corresponds to $\sqrt{\lambda} = 0.53$ GeV

- How universal is the semiclassical approximation based on superconformal QM and its LF holographic embedding? [S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé, PLB **759**, 171 (2016)]]



Best fit for hadronic scale $\sqrt{\lambda}$ from the different sectors including radial and orbital excitations

7 Supersymmetry across the heavy-light hadronic spectrum

[H.G. Dosch, GdT, and S. J. Brodsky, Phys. Rev. D **92**, 074010 (2015)]

[H.G. Dosch, GdT, and S. J. Brodsky **IN PREPARATION**]

- Introduction of quark masses breaks conformal symmetry without violating supersymmetry
- Embedding of SUSY bound-state equations in AdS to determine the confinement potential
- SUSY potential for a meson can be read-off from slide 10 $(V_{\rm light} = \lambda\zeta)$

$$U_{susy} = V^2 - V' + \frac{2L+1}{\zeta} V$$

where V is an arbitrary superpotential and $L = L_M - 1$

- Embedding meson bound-state equation in AdS the LF potential is $(\varphi_{\rm light} = \lambda\zeta^2)$

$$U_{AdS} = \frac{1}{4}(\varphi')^2 + \frac{1}{2}\varphi'' + \frac{2L-1}{2\zeta}\varphi'$$

where φ is an arbitrary dilaton (slide 7) and $L = L_M - 1$ for $J = L_M$

- Equating the SUSY and the dilaton potentials $U_{susy} = U_{AdS}$ we find:

$$\begin{aligned} \varphi'(\zeta) &= \int d\zeta \left(\lambda\zeta\sigma(\zeta) - \frac{\lambda^2\zeta^2\sigma'(\zeta)}{\lambda^2\zeta^2\sigma(\zeta) + 2(L_M - 1)\lambda} \right) \\ V(\zeta) &= \frac{1}{2}\left(\lambda\zeta\sigma(\zeta) + \frac{\lambda^2\zeta^2\sigma'(\zeta)}{\lambda^2\zeta^2\sigma(\zeta) + 2(L_M - 1)\lambda} \right) \end{aligned}$$

  where $\sigma(\zeta)$ is an arbitrary function

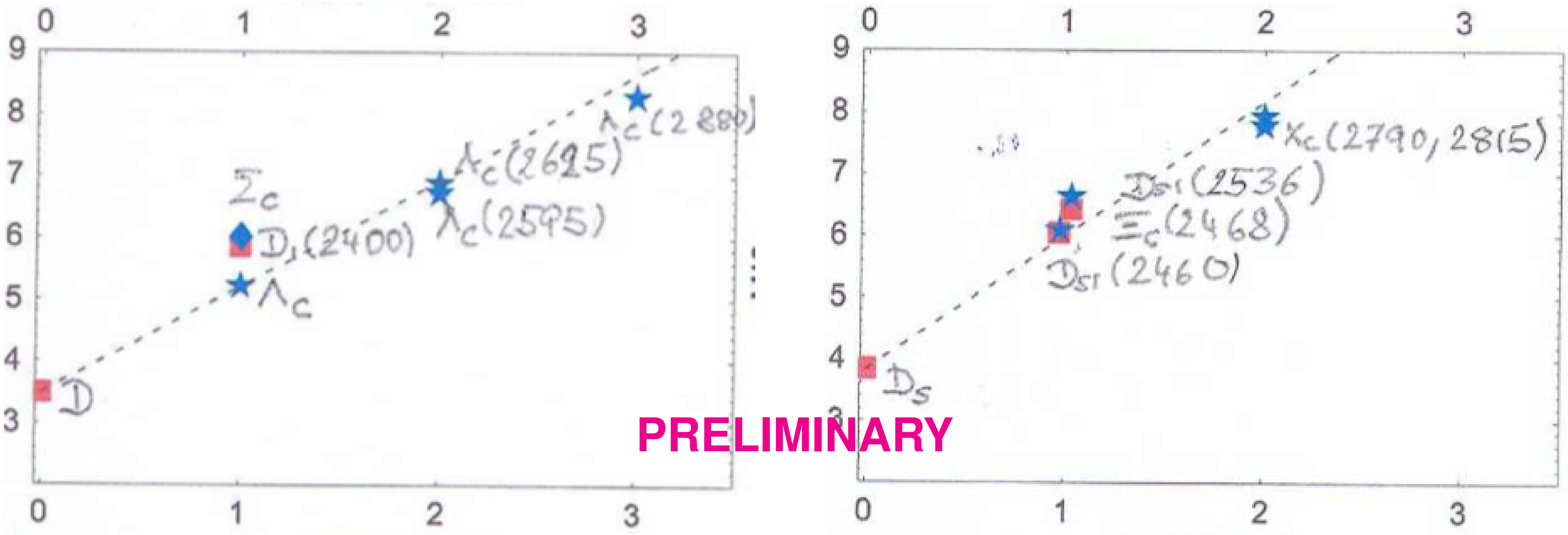
- If dilaton profile is independent of angular momentum one has $\sigma'(\zeta) = 0$ and thus $\sigma = \mathrm{const} \equiv 2A$

- Solution:

$$\varphi = \lambda A\zeta^2 + B \quad \text{and} \quad V(\zeta) = \lambda A\zeta$$

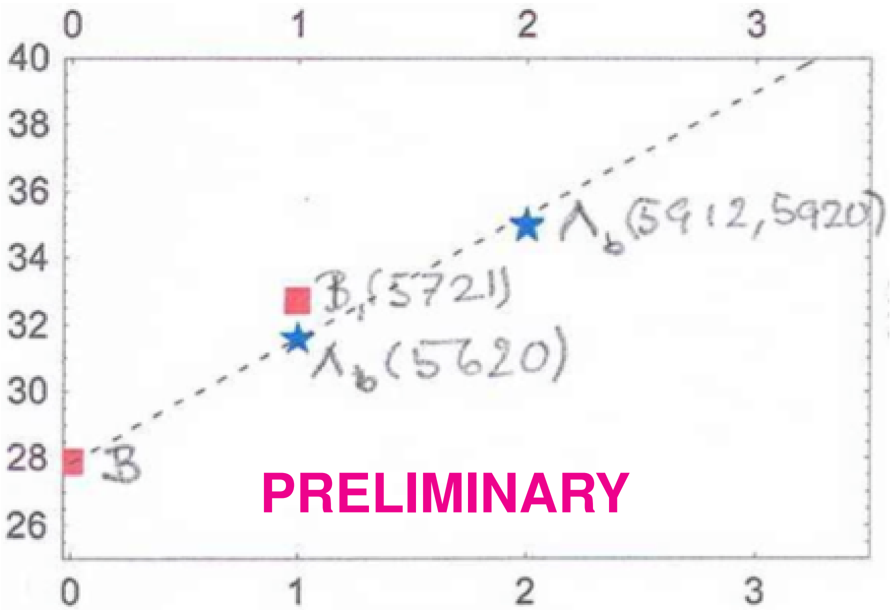
- For strongly broken conformal invariance the dilaton is still quadratic, but since A is arbitrary the strength of the potential is not determined

M^2 vs $L_M = L_B + 1$



D/Λ_c trayectory (left) and D_s/Ξ_c trajectory (right)

M^2 vs $L_M = L_B + 1$



B/Λ_b trayectory

$$\sqrt{\lambda_{\rm light}} \simeq 0.52 \,{\rm GeV}, \quad \sqrt{\lambda_{\rm charm}} \simeq 0.7 \,{\rm GeV}, \quad \sqrt{\lambda_{\rm bottom}} \simeq 1 \,{\rm GeV},$$

8 Infrared behavior of the strong coupling in light-front holographic QCD

[S. J. Brodsky, GdT and A. Deur, PRD **81** (2010) 096010]

[A. Deur, S. J. Brodsky and GdT, PLB **750**, 528 (2015); PLB **757**, 275 (2016), arXiv:1608.04933 [hep-ph]]

- Effective coupling $\alpha_{g_1} = g_1^2/4\pi$ defined from and observable: g_1 scheme from Bjorken sum rule

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A} \int_0^1 dx\, g_1^{p-n}(x, Q^2)$$

- Infrared behavior of strong coupling in LF holographic QCD from Fourier transform of the LF transverse coupling:

$$\alpha_{g_1}^{AdS}(Q) = \pi \exp\left(-Q^2/4\lambda\right)$$

- Large Q-dependence of α_s is computed from the pQCD β series:

$$Q^2 d\alpha_s/dQ^2 = \beta(Q) = -(\beta_0 \alpha_s^2 + \beta_1 \alpha_s^3 + \beta_2 \alpha_s^4 + \cdots)$$

where coefficients β_i are known up to β_4 in $\overline{MS}$ scheme (five-loops):

- $\alpha_{g_1}^{pQCD}(Q)$ expressed as a perturbative expansion in $\alpha_{\overline{MS}}(Q)$:

$$\alpha_{g_1}^{pQCD}(Q) = \pi \left[\alpha_{\overline{MS}}/\pi + a_1 \left(\alpha_{\overline{MS}}/\pi\right)^2 + a_2 \left(\alpha_{\overline{MS}}/\pi\right)^3 + \cdots\right]$$

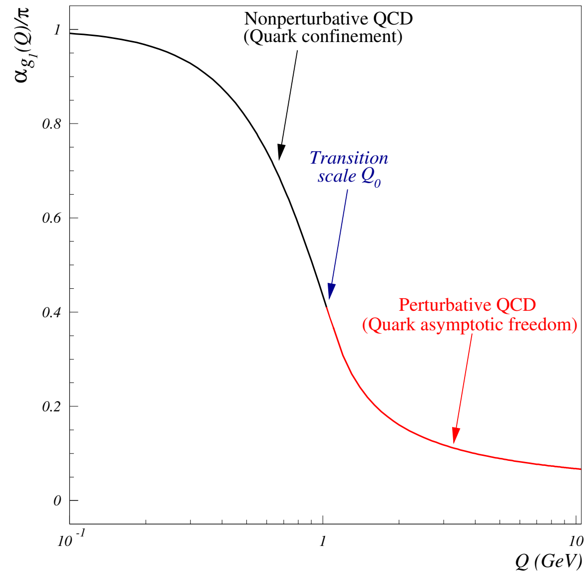
The coefficients a_i are known up to order a_4

- Λ_{QCD} and transition scale Q_0 from matching perturbative and nonperturbative regimes:

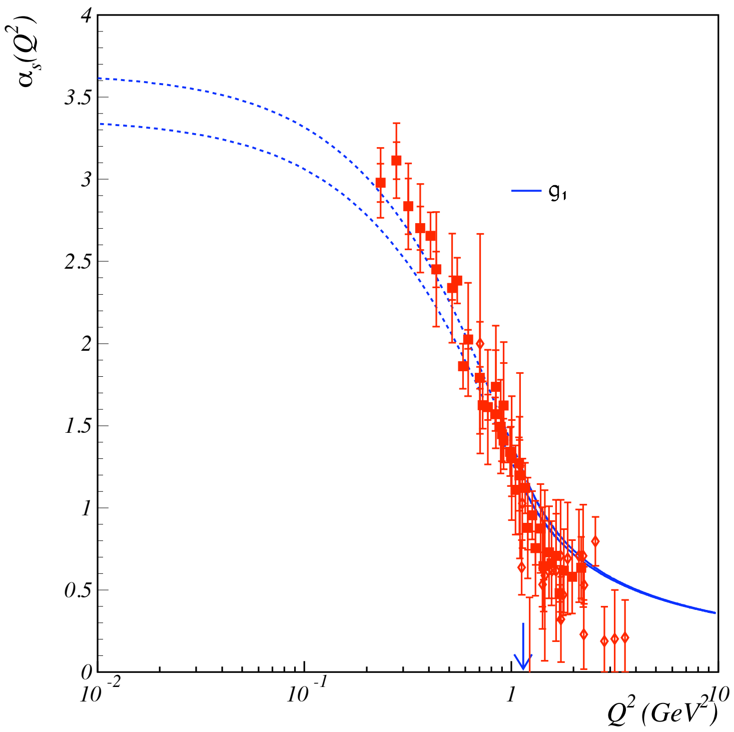
  $\Lambda_{\overline{MS}} = 0.339 \pm 0.019$ GeV

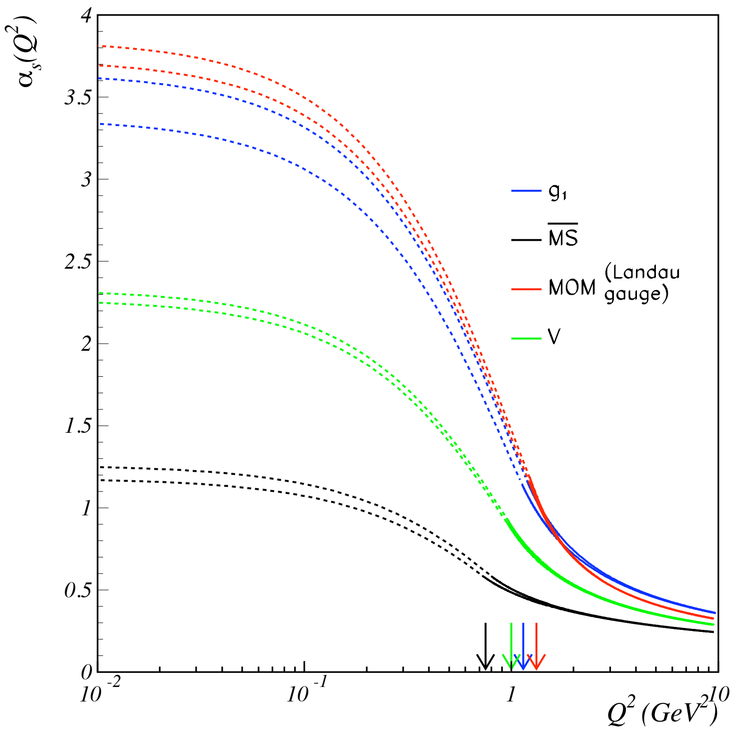
  (World Average: $\Lambda_{\overline{MS}} = 0.332 \pm 0.019$ GeV)

  Transition scale: $Q_0^2 \simeq 1$ GeV^2



- Renormalization scheme dependence





Thanks !

Review of LFHQCD, see S. J. Brodsky, GdT, H. G. Dosch and J. Erlich, Phys. Rept. **584**, 1 (2015)