TECHNISCHE UNIVERSITÄT WIEN

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Polyakov line actions from SU(3) lattice gauge theory with dynamical fermions via relative weights

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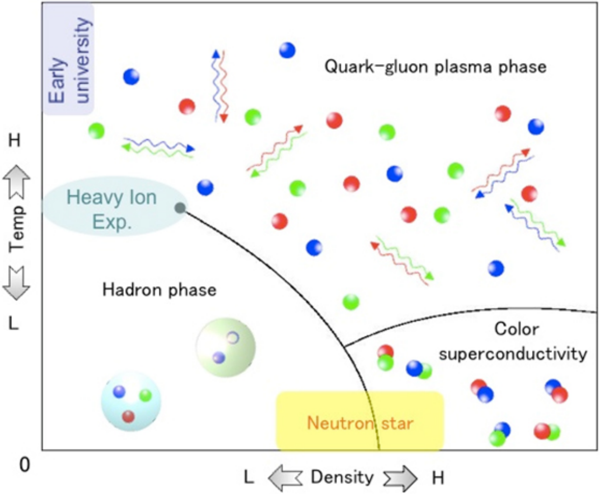
Agenda

Motivation The Phase Diagram of QCD



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determinant is complex and satisfies

$$[\det M(\mu)]^* = \det M(-\mu^*)$$

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- finite chemical potential μ favors propagation of quarks

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- **Worldline formalism and strong coupling limit:** change order of integration, partial integration over loops and hopping parameter expansion

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- hard to compute $\exp[S_P(U_x)]$ directly, but action ratios are easily computed as expectation values $\rightarrow$ use relative weights...

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$$\frac{1}{L^3}\left(\frac{\partial S_P}{\partial a_k}\right)_{a_k=\alpha} = 2K(k)\alpha + \frac{p}{L^3}\sum_x (3he^{ikx} + 3h^2 e^{-ikx} + c.c.)$$

Fitting to lattice data

- lattice momenta $k = 2\sqrt{\sum_{i=1}^{3} \sin^2(k_i/2)}$, $k_i = 2\pi m_i/L$, where $L = 16$ and we use mode numbers $m_i = (000), (100), (110), (200), (210), (300),$ $(311), (400), (322), (430), (333), (433), (443), (444), (554)$

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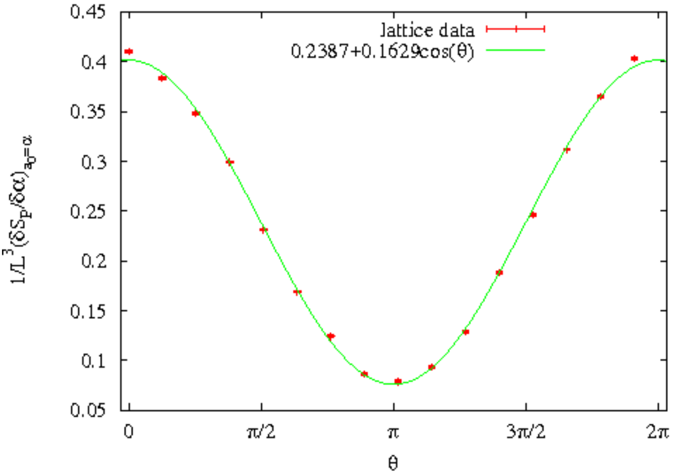

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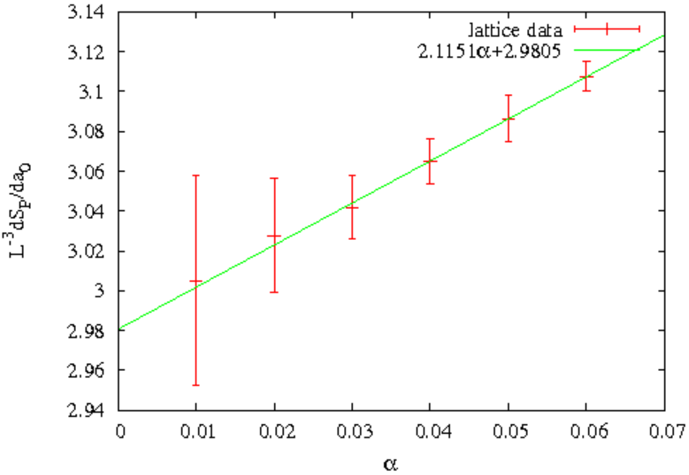

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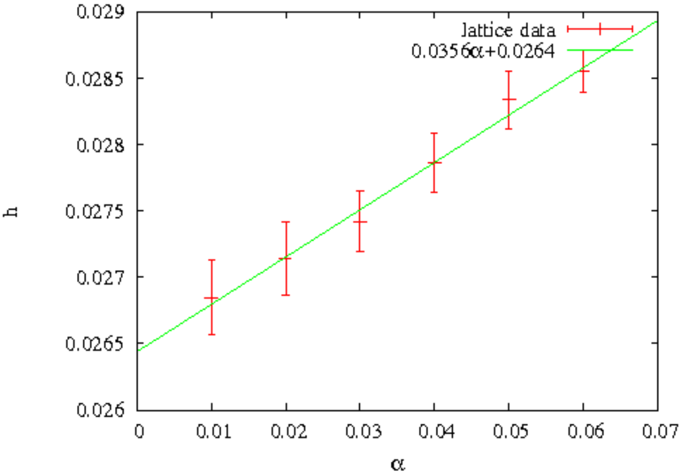

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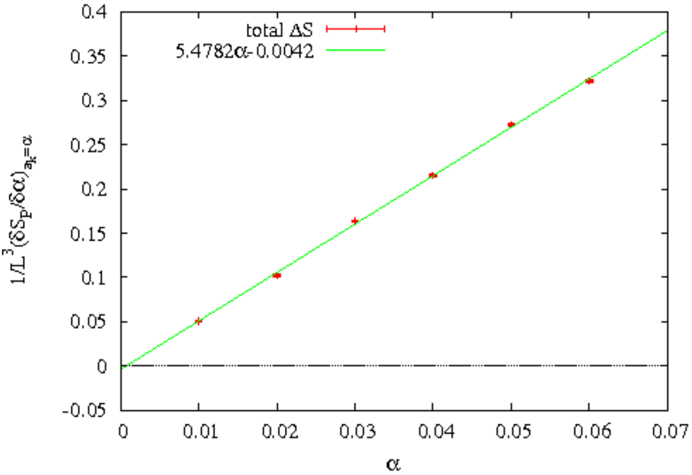

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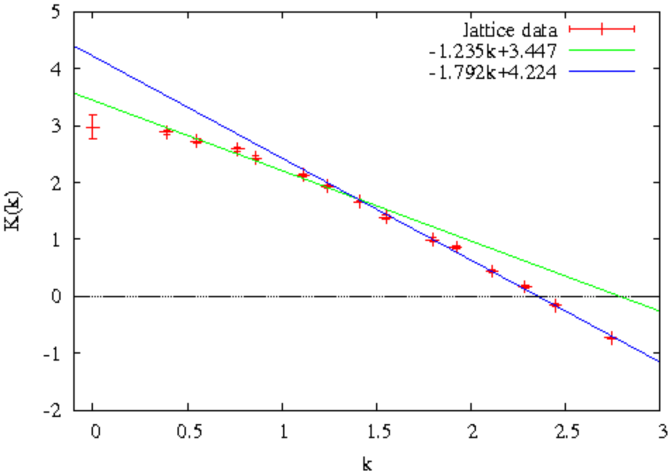

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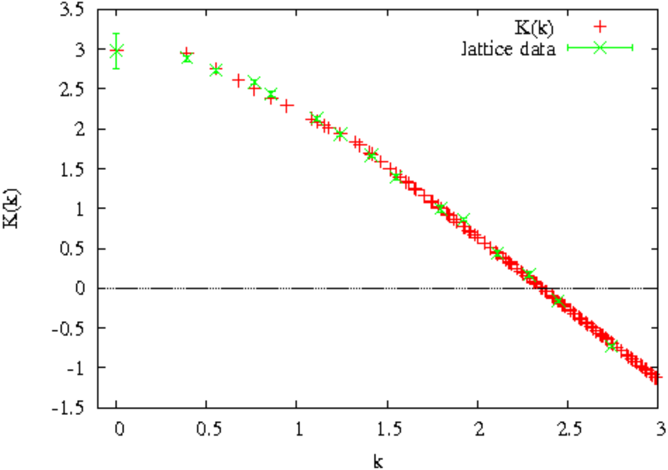
- determine r_{max} by Fourier transforming back and fitting $K(0)$

Fitting to lattice data



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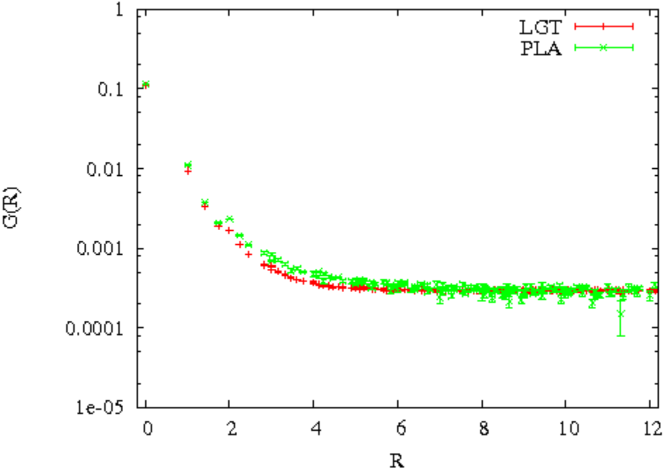

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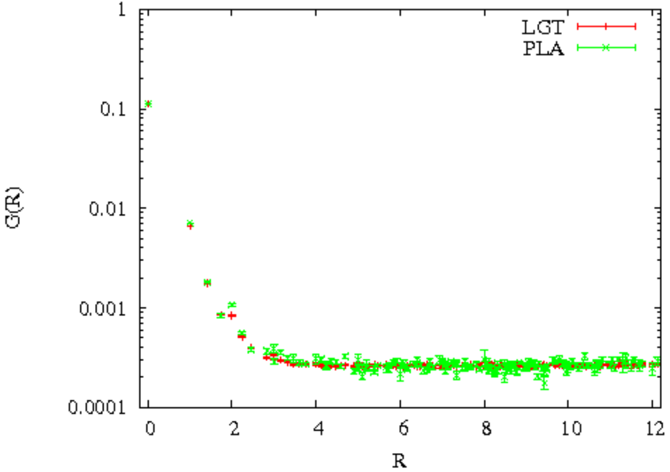

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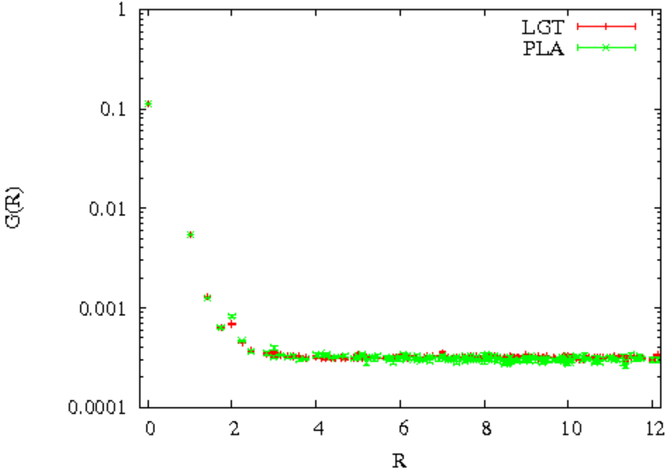

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$$\mathrm{Tr} U_x = (\mathrm{Tr} U_x - u) + u \ , \ \mathrm{Tr} U_x^\dagger = (\mathrm{Tr} U_x^\dagger - v) + v$$

$$
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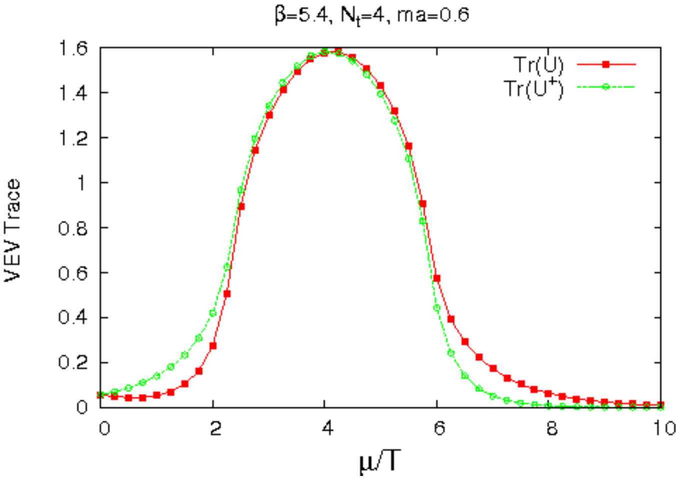
- equivalent to the stationarity of the mean field free energy with respect to variations in u and v $\rightarrow$ solve numerically

Mean Field Results at $\beta = 5.4, ma = 0.6$



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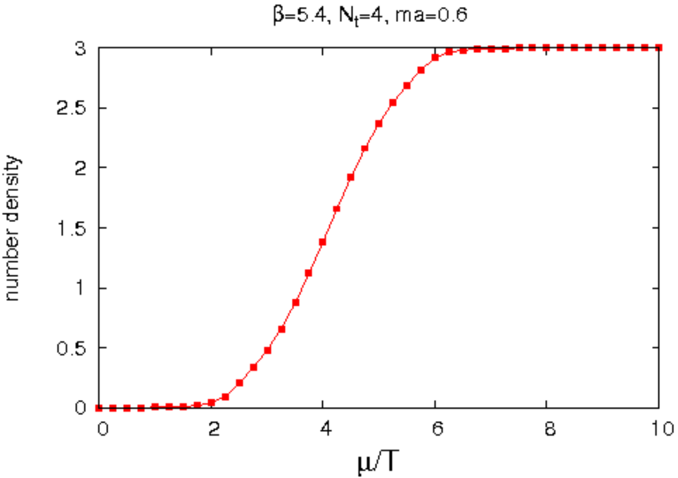

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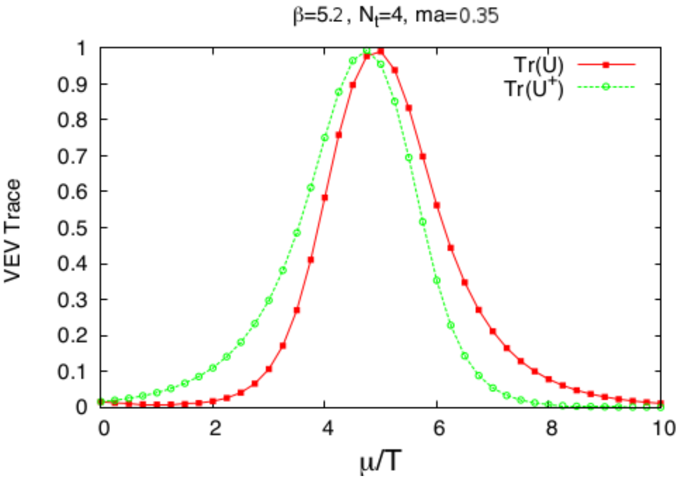

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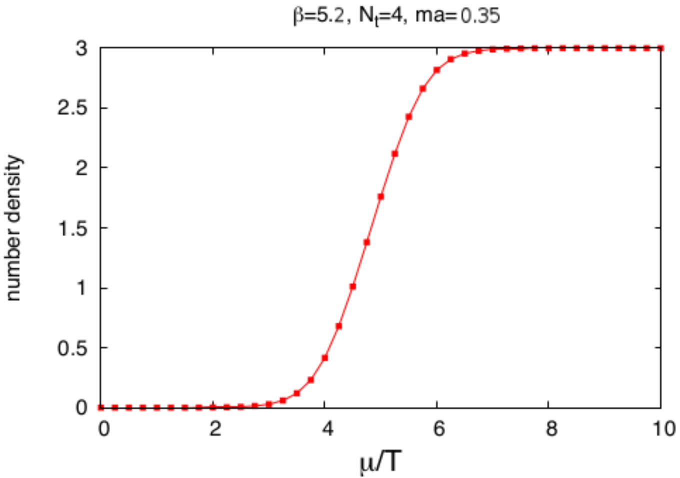

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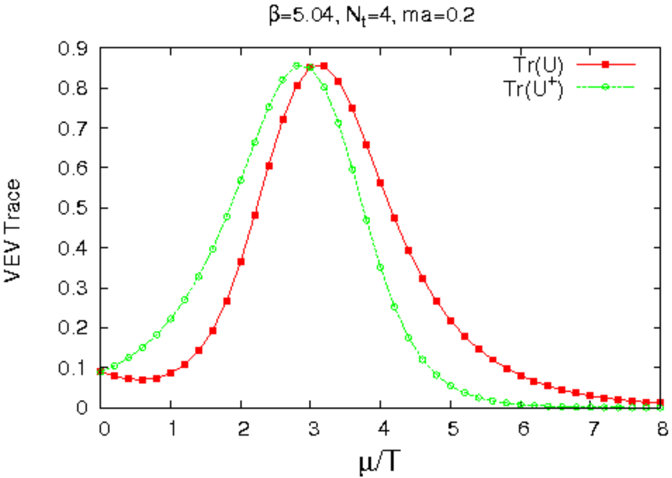

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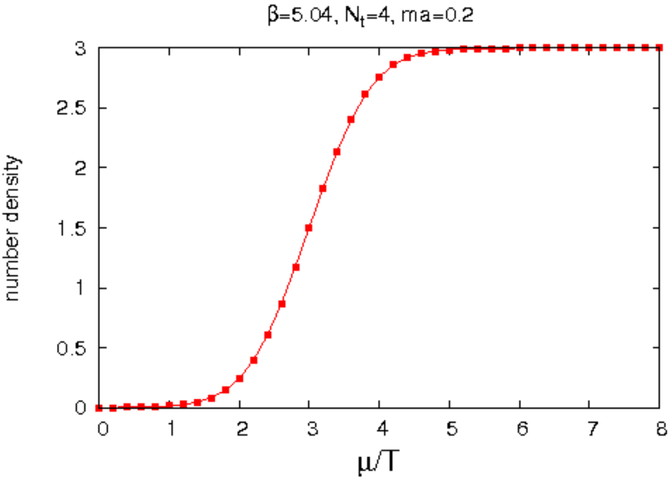

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- go on to smaller quark masses...

Questions?

Thank You &

Tareq Alhalholy, Derar Altarawneh, Michael Engelhardt, Manfried Faber, Martin Gal, Jeff Greensite, Urs M. Heller, James Hettrick, Andrei Ivanov, Thomas Layer, Štefan Olejnik, Luis Oxman, Mario Pitschmann, Jesus Saenz, Thomas Schweigler, Wolfgang Söldner, David Vercauteren, Markus Wellenzohn

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- unfortunate ambiguity for highly non-local S_P, not a finite density issue!

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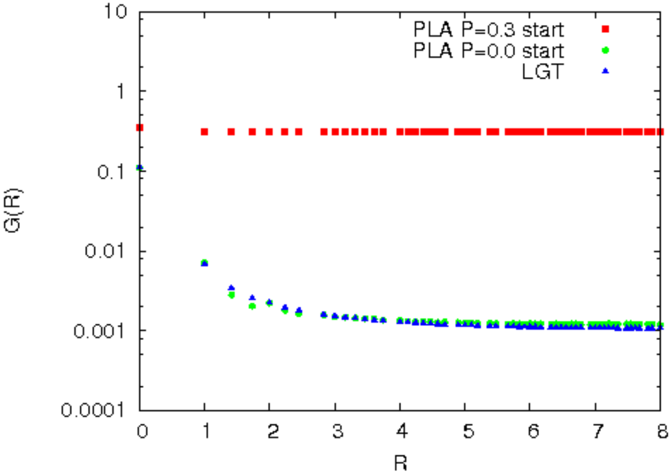

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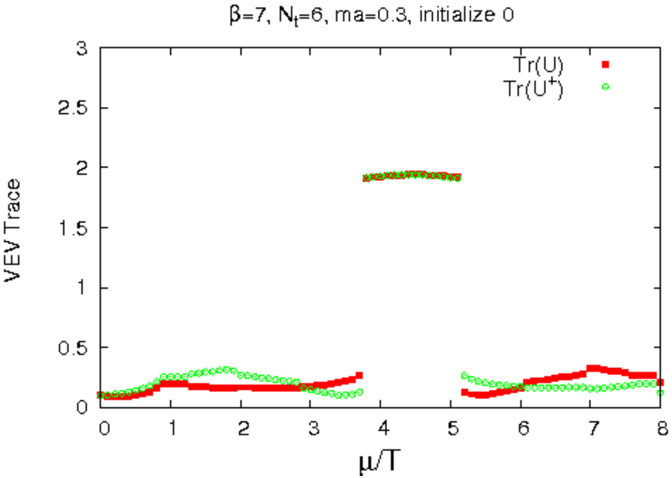

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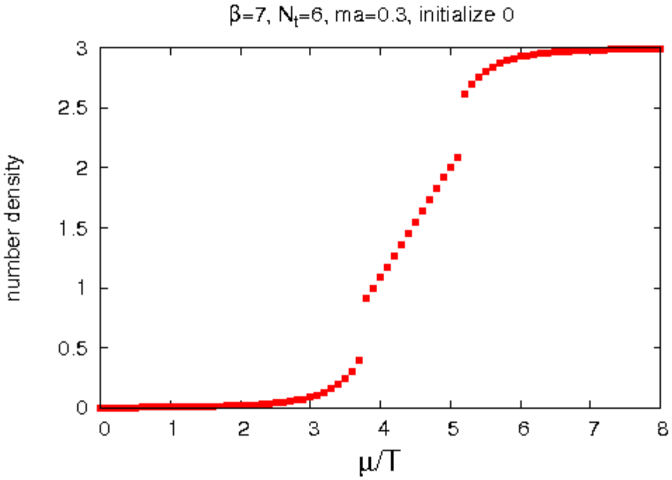

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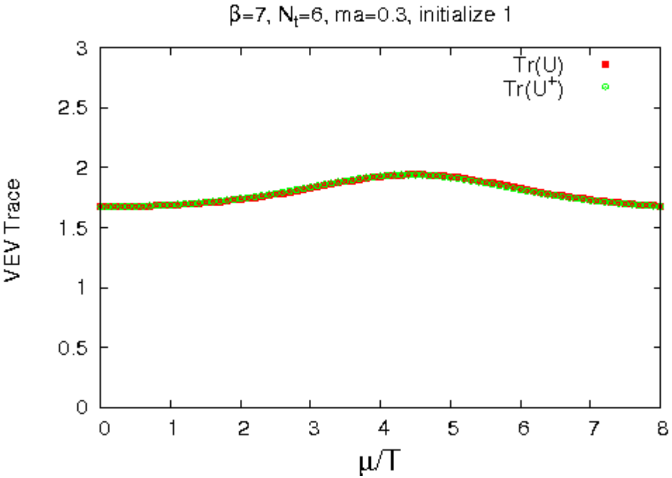

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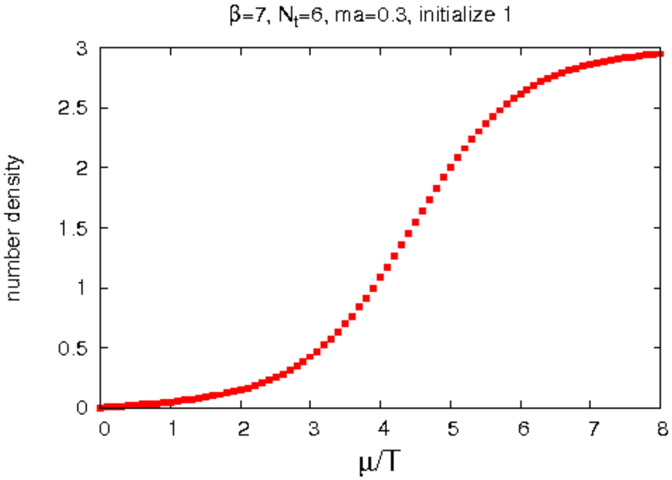

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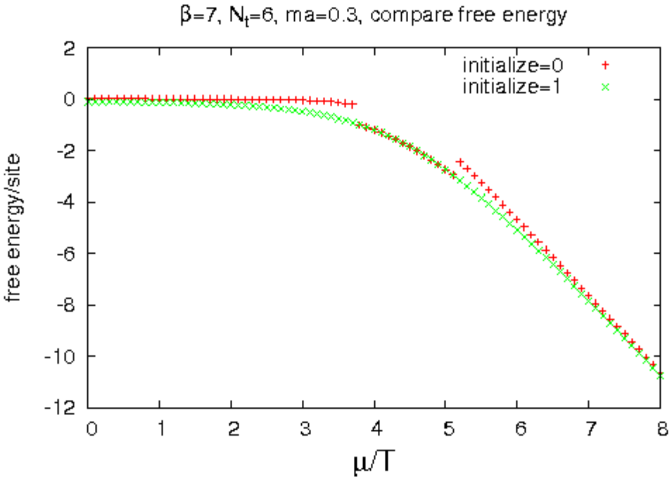

Polyakov line actions from SU(3) lattice gauge theory with dynamical fermions via relative weights

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