

This talk is dedicated to the memory of
Martin Schaden

Instantaneous Dynamics of QCD in Coulomb Gauge from Local Action

Patrick Cooper and Daniel Zwanziger
New York University, New York, NY 10003

Conference: XII Quark Confinement and the
Hadron Spectrum 2016,
Thessaloniki, Greece

NO CONFINEMENT WITHOUT COULOMB CONFINEMENT

Gribov's insight into the mechanism of confinement is substantiated by the theorem,

$$V_{\text{coul}}(R) \geq V_{\text{wilson}}(R),$$

for $R \longrightarrow$ infinity. Here the color-Coulomb potential is the temporal gluon propagator in Coulomb gauge,

$$D_{00}(R, T) = V_{\text{coul}}(R) \delta(T) + \text{N.I.}$$

When the Wilson potential is linearly rising, the color-Coulomb potential is linear or super-linear.

MOTIVATION AND METHOD OF PRESENT APPROACH

The motivation for the present approach is to work within the framework of local quantum field theory with a local action that encodes the cut-off at the Gribov horizon, and moreover that is BRST-invariant, thus preserving the geometric property of a gauge theory. This is achieved by the method of Maggiore-Schaden. BRST-symmetry is spontaneously broken. A conjecture is offered for the identification of physical operators and the physical subspace (Martin Schaden and DZ arXiv: 1412.4823). However this important subject will not be discussed today.

METHOD: Dyson-Schwinger equation. Extension of Alkofer, Maas, Z, 0905.4954 from Faddeev-Popov action to include cut-off at Gribov horizon

Hypothesis: Dominance of instantaneous dynamics encoded in instantaneous vertex functions Γ_I

HORIZON FUNCTION AND NON-LOCAL ACTION

$$S = S_{\text{FP}} + \gamma H - \gamma \int d^d x d(N^2 - 1)$$

Euclidean action, where the horizon function is given by

$$H = \int d^d x d^d y D_{\mu}^a(x) D_{\mu}^b(y) (M^{-1})^{ab}(x, y; A).$$

The horizon function cuts off the functional integral at the Gribov horizon as one sees from the eigenfunction expansion

$$(M^{-1})^{ce}(x, y; A) = \sum_n \frac{\psi_n^c(x) \psi_n^e(y)}{\lambda_n(A)}$$

HORIZON CONDITION AND KUGO OJIMA CONFINEMENT CONDITION

The Gribov parameter is fixed by the horizon condition

$$\langle H \rangle = d(N^2 - 1) \int d^d x$$

It is a remarkable fact that the horizon condition and the famous Kugo-Ojima confinement condition are the same statement:

$$-i \int d^d x \langle (D_\mu c)^a(x) (D_\mu \bar{c})^a(0) \rangle = d(N^2 - 1).$$

Thus color confinement is assured in this theory.

HORIZON CONDITION AND DUAL MEISSNER EFFECT

It is also a remarkable fact that, the horizon condition is equivalent to the statement that the QCD vacuum is a perfect color-electric superconductor, which is dual Meissner effect (H. Reinhardt, Phys. Rev. Lett. 101, 061602 (2008), 08030504.)

$$G(\vec{k}) = \frac{d(\vec{k})}{\vec{k}^2} = \frac{1}{\epsilon(\vec{k}) \vec{k}^2}$$

$$d^{-1}(\vec{k} = 0) = 0 \iff \epsilon(\vec{k} = 0)$$

where $G(k)$ is the ghost propagator.

AUXILIARY GHOSTS

Just as the Faddeev-Popov determinant is localized by introducing ghosts

$$\det M = \int dc d\bar{c} \exp \left(- \int d^d x \bar{c} M c \right),$$

Likewise, the horizon function in the action may be localized by introducing “auxiliary” ghosts,

$$\exp(-\gamma H) = \int d\varphi d\bar{\varphi} d\omega d\bar{\omega} \exp \left(- \int d^d x \left[\bar{\varphi} M \varphi - \bar{\omega} M \omega + \gamma^{1/2} D \cdot (\varphi - \bar{\varphi}) \right] \right)$$

The Coulomb gauge is plagued by energy divergences:

$$\int d^3 p d p_0 \frac{1}{\vec{p}^2} \frac{1}{(\vec{p} - \vec{k})^2}.$$

The integrand of the ghost loop in Coulomb gauge is independent of p_0 .

The energy divergences cancel in the first-order formalism.

In Coulomb gauge, we use first order formalism

$$\exp\left(-\int d^4 x F_{0i}^2/2\right) = \int d^3 \pi \exp\left(\int d^4 x [i\pi_i F_{0i} - \pi_i^2/2]\right)$$

so the energy divergences cancel manifestly. Here $F_{0i} = \partial_0 A_i - D_i A_0$, and the Coulomb gauge condition $\partial_i A_i = 0$ holds identically. The canonical color-electric field

$$\pi_i = \tau_i - \partial_i \lambda$$

decomposes into its transverse τ_i , with $\partial_i \tau_i = 0$, and longitudinal parts λ .

LOCAL ACTION AND PHYSICAL DEGREES OF FREEDOM IN COULOMB GAUGE

$$S = \int d^{d+1}x (\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3)$$

$$\mathcal{L}_1 = i\tau_i D_0 A_i + (1/2)[\tau^2 + (\partial_i \lambda)^2 + B^2]$$

$$\mathcal{L}_2 = i\partial_i \lambda D_i A_0 - \partial_i \bar{c} D_i c + \partial_i \bar{\varphi}_j D_i \varphi_j - \partial_i \bar{\omega}_j D_i \omega_j$$

$$\mathcal{L}_3 = \gamma^{1/2} g f^{abc} A_j (\varphi_j - \bar{\varphi}_j)$$

Only the first term has a time derivative. It acts on the two would-be physical degrees of freedom. All the remaining degrees of freedom impose constraints in the local theory.

The last term mixes bose ghost and gluon. The energy divergences cancel between fermi and bose ghost loops.

$$\begin{pmatrix} D_{\lambda\lambda} & D_{\lambda A_0} \\ D_{A_0\lambda} & D_{A_0 A_0} \end{pmatrix}^{ab} = \delta^{ab} \begin{pmatrix} 0 & -iD_{c\bar{c}}(\vec{k}) \\ -iD_{c\bar{c}}(\vec{k}) & \Gamma_{\lambda\lambda}(\vec{k})D_{c\bar{c}}^2(\vec{k}) \end{pmatrix}$$

Note equality of Faddeev-Popov ghost propagator and of unphysical bose ghost propagator,

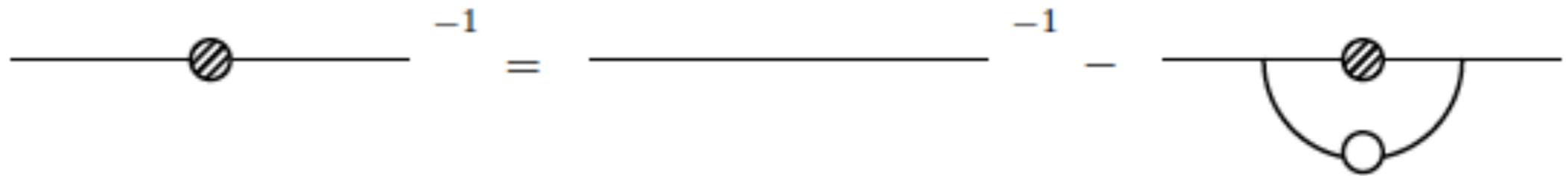
$$D_{\lambda A_0} = -iD_{c\bar{c}}.$$

The corresponding fermi and bose loops yield Faddeev-Popov determinants that cancel exactly:

$$\frac{D(M)}{D(M)} = 1$$

The mixed quantities survive such as the color-Coulomb potential,

$$D_{A_0 A_0} = \Gamma_{\lambda\lambda} D_{c\bar{c}}^2$$



Schematic form of ISDE. The shaded circle represents an instantaneous propagator. The empty circle a non-instantaneous propagator. All 3-vertices and undressed lines correspond to tree-level vertices and propagators respectively

$D_{VPVP}^{(0)} \rightarrow$		$D_{AA}^{(0)} \rightarrow$	
$D_{\tau\tau}^{(0)} \rightarrow$		$D_{A_0A_0}^{(0)} \rightarrow$	
$D_{AV}^{(0)} \rightarrow$		$D_{\lambda A_0}^{(0)} \rightarrow$	

Various propagators. The propagators $D_{\lambda A_0}$ and D_{AV} represent mixing.

The Coulomb gauge is a unitary gauge. Consequently in the first-order formalism, the $c - \bar{c}$ ghost loop cancels the $\lambda - A_0$ loop, etc.

$$\int d^{(d+1)}p [D_{A_0\lambda}(\vec{p})D_{A_0\lambda}(\vec{k} + \vec{p}) - D_{c\bar{c}}(\vec{p})D_{c\bar{c}}(\vec{k} + \vec{p})] = 0.$$

This gets rid of the horrible energy divergences,

$$dp_0 \ 1 = \infty.$$

INSTANTANEOUS DYNAMICS

In Coulomb gauge the propagators in general decompose into an instantaneous part and a non-instantaneous part,

$$D(x) = D_I(\vec{x})\delta(x_0) + D_N(\vec{x}, x_0)$$

$$D(p) = D_I(\vec{p}) + D_N(\vec{p}, p_0)$$

where $\lim_{p_0 \rightarrow \infty} D_N(\vec{p}, p_0) = 0$.

Loops with two instantaneous propagators cancel between bosons and fermions.

$$\int d^{d+1}p [D_{Iboson}(\vec{p})D_{Iboson}(\vec{p} + \vec{k}) - (boson \rightarrow fermion)] = 0$$

Keep loops with one instantaneous propagator and one non-instantaneous propagator

$$\int d^{d+1}p D_N(\vec{p}, \mathbf{p}_0)D_I(\vec{p} + \vec{k})$$

Neglect loops with two non-instantaneous propagators

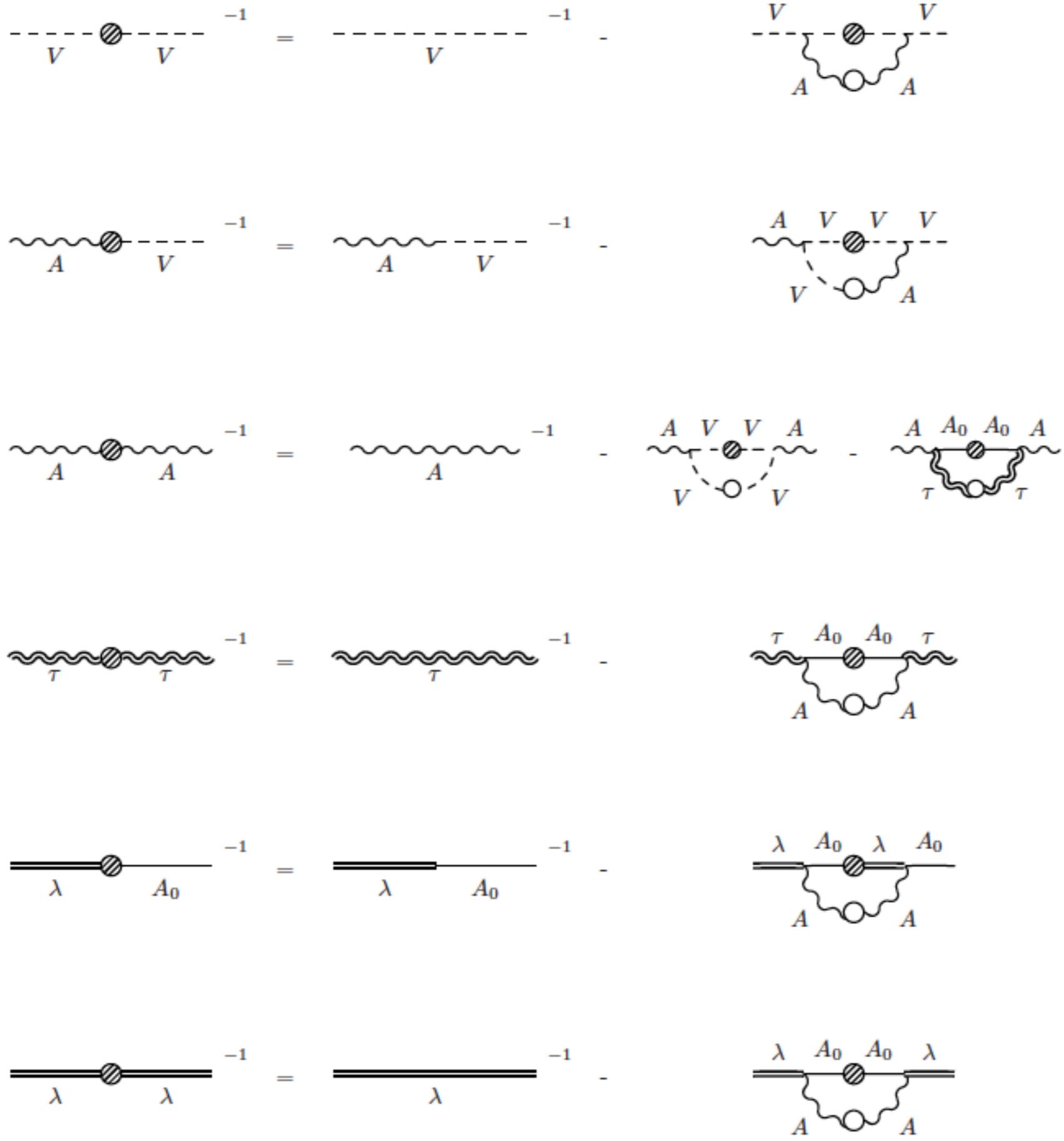
$$\int d^{d+1}p D_N(p)D_N(p + k) \rightarrow 0$$

Only equal-time parts of non-instantaneous propagator contribute to the graphs we consider,

$$\int d^{d+1}p D_N(\vec{p}, \mathbf{p}_0) D_I(\vec{p} + \vec{k}) = \int d^d p D^{\text{ET}}(\vec{p}) D_I(\vec{p} + \vec{k})$$

$$\int dp_0 D_N(\vec{p}, \mathbf{p}_0) \equiv D^{\text{ET}}(\vec{p})$$

so in all graphs that we consider, the non-instantaneous part may be replaced by the equal-time part,



$$\begin{pmatrix} D_{\tau\tau} & D_{\tau A} & D_{\tau VP} \\ D_{A\tau} & D_{AA} & D_{AVP} \\ D_{VP\tau} & D_{VPA} & D_{VPVP} \end{pmatrix}^{ET}(\vec{p}) \equiv \int \frac{dp_0}{2\pi} \begin{pmatrix} D_{\tau\tau} & D_{\tau A} & D_{\tau VP} \\ D_{A\tau} & D_{AA} & D_{AVP} \\ D_{VP\tau} & D_{VPA} & D_{VPVP} \end{pmatrix}_N(p_0, \vec{p}).$$

$$\begin{pmatrix} D_{\tau\tau} & D_{\tau A} & D_{\tau VP} \\ D_{A\tau} & D_{AA} & D_{AVP} \\ D_{VP\tau} & D_{VPA} & D_{VPVP} \end{pmatrix}^{ET}(\vec{p}) = \delta^{ab} T_{ij} \begin{pmatrix} (4D_{AA}^{ET})^{-1} & 0 & 0 \\ 0 & D_{AA}^{ET} & -i\tilde{\Gamma}_{AV} D_{c\bar{c}} D_{AA}^{ET} \\ 0 & -i\tilde{\Gamma}_{AV} D_{c\bar{c}} D_{AA}^{ET} & -\tilde{\Gamma}_{AV}^2 D_{c\bar{c}}^2 D_{AA}^{ET} \end{pmatrix}(\vec{p}).$$

The vanishing matrix elements come from integrating an odd function of p_0 .

$D_{c\bar{c}}$	$\sim ak^{-\alpha}$
$g^{-1}\tilde{\Gamma}_{AV}$	$\sim bk^{\beta}$
D_{AA}	$\sim ck^{-\gamma}$
$g^2 D_{IA_0A_0}$	$\sim dk^{-\delta}$
$g^2\Gamma_{\lambda\lambda}$	$\sim ek^{\epsilon}$

Ansatz for infrared limit: Infrared critical exponents

Limits on the critical exponents are obtained by requiring that the loop integrals converge in the infrared and the ultraviolet.

We obtain

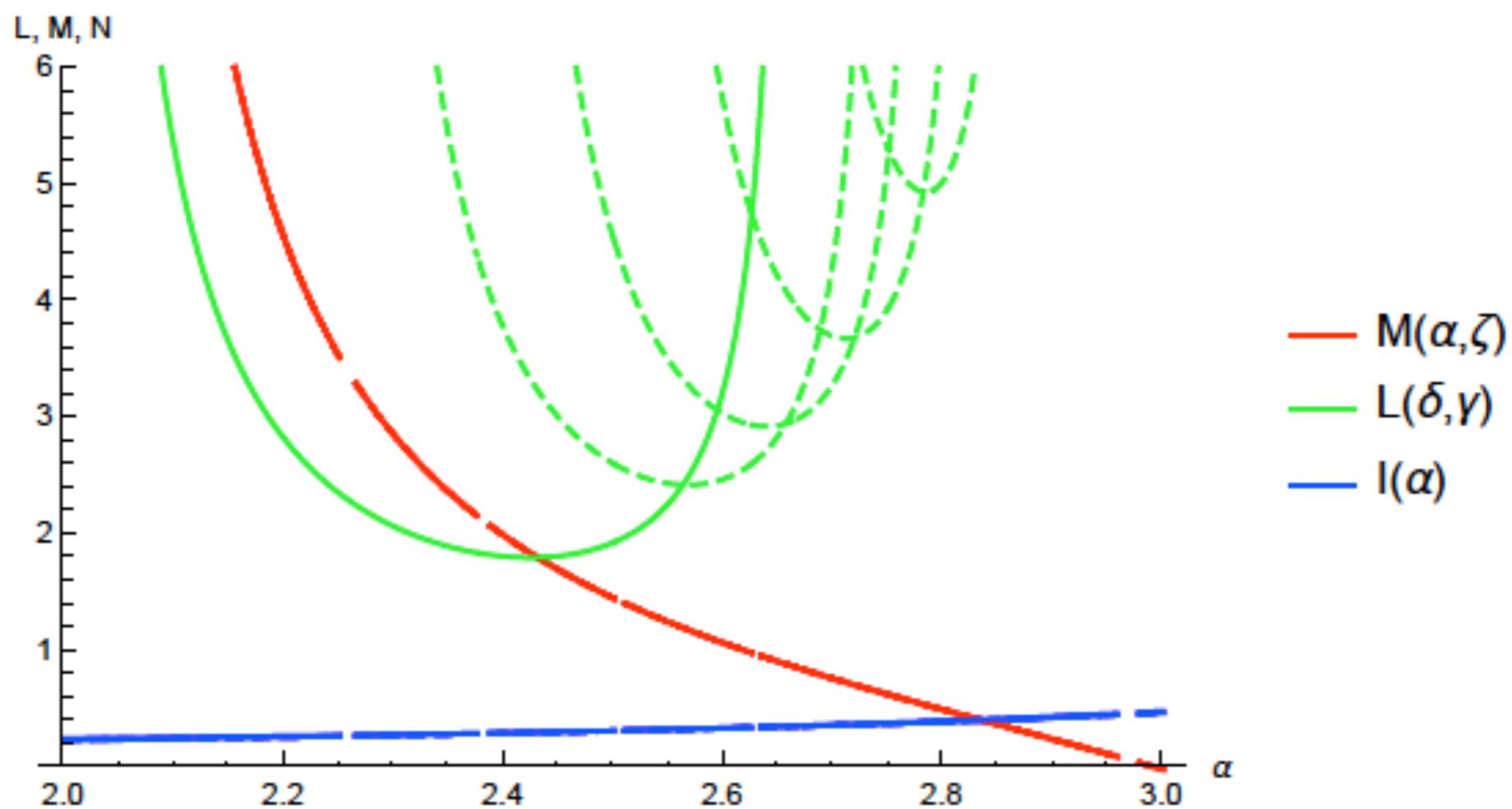
$\alpha > 2$, that is, enhancement of the ghost propagator,

$\gamma < 0$, so the would-be physical gluon propagator vanishes at $k = 0$, and

$14/3 < \delta$, so the color-Coulomb potential is long range and in fact super-linear.

Indeed the theorem “No confinement without Coulomb confinement,” which gives $4 < \delta$, is satisfied. Thus

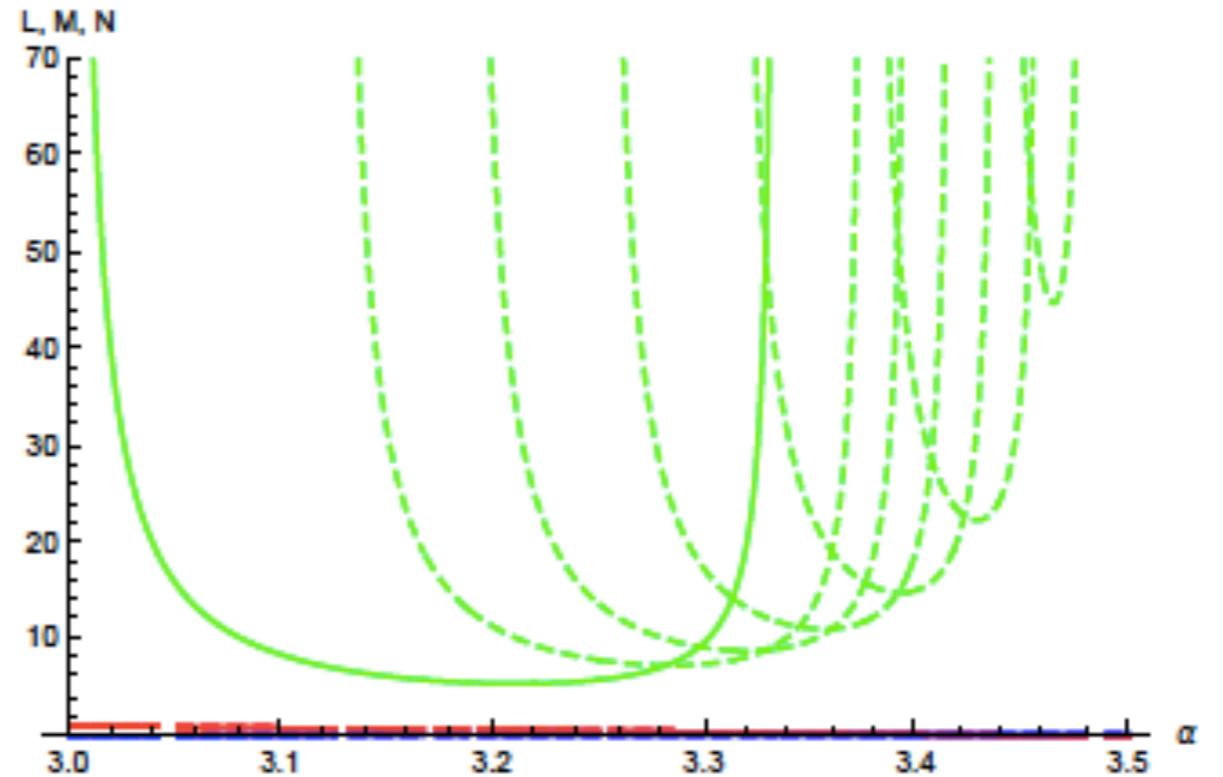
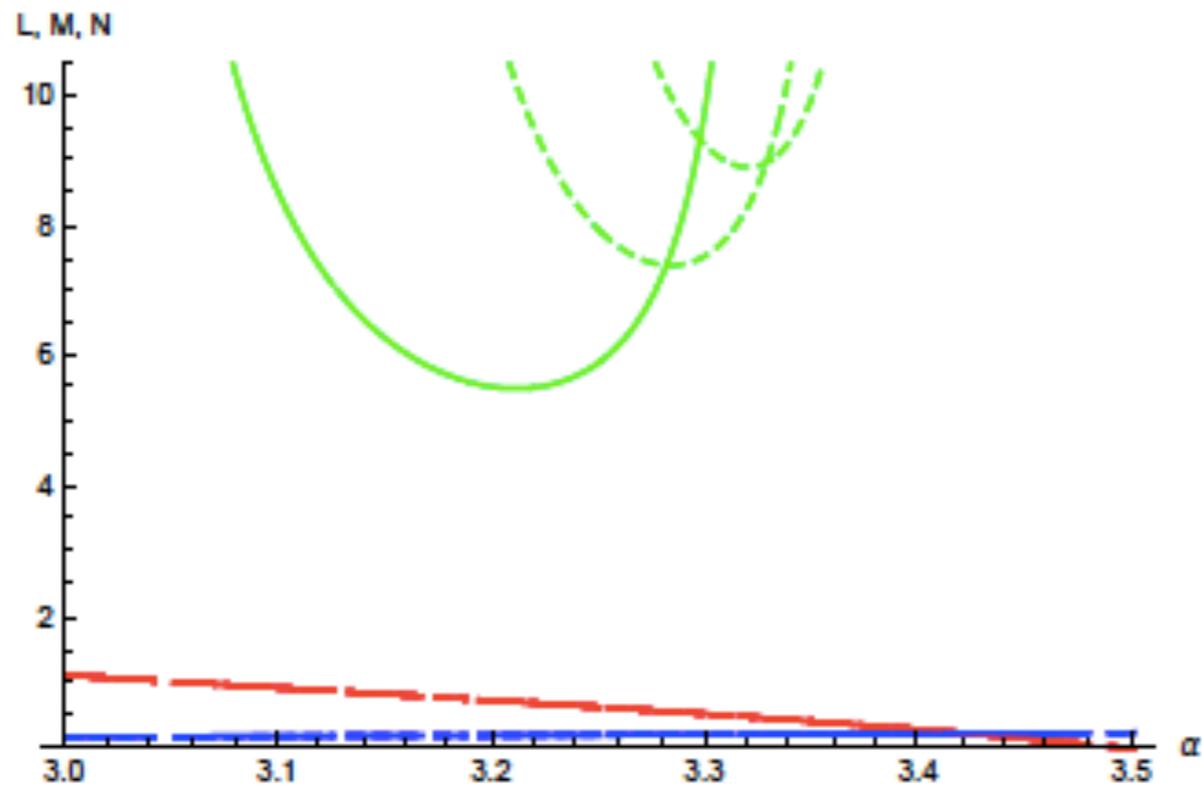
all the elements of the Gribov scenario are realized.



The critical exponents α and β are determined by

$$I(\alpha) = M(\alpha, \beta) = L(\alpha, \beta).$$

Space dimension $D = 2$. Plot of $I(\alpha)$, $M(\alpha, \beta)$ and $L(\alpha, \beta)$ as a function of critical exponent α for seven different values of β , equally spaced in the region of convergence. We see that M and I intersect, but as β sweeps through the allowed region, L remains far away from this intersection. We see that M depends weakly on β , whereas L is strongly β -dependent. I is β -independent.



The critical exponents α and β are determined by

$$I(\alpha) = M(\alpha, \beta) = L(\alpha, \beta).$$

Space dimension $D = 3$. Above are two identical plots, with two different vertical scales of the three functions $I(\alpha)$, $M(\alpha, \beta)$ and $L(\alpha, \beta)$ as a function of critical exponent α for seven different values of β , equally spaced in the region of convergence. Again we see that M and I intersect, but as β sweeps through the allowed region, L remains far away from this intersection. We again notice that M depends weakly on β , whereas L is strongly β -dependent. I is β -independent.

CONCLUSION

We have a local, renormalizable quantum field theory with the following interesting properties:

- It provides a cut-off at the Gribov horizon.

- The Kugo-Ojima color confinement condition is satisfied.

The vacuum is a perfect dielectric

- There is an alternate derivation by the Maggiore-Schaden method that provides a BRST-invariant Lagrangian.
- BRST-symmetry is spontaneously broken, but perhaps only in the unphysical sector. Further work is required here.
- In Coulomb gauge the color-Coulomb potential is linear or super-linear when the Wilson potential is linearly rising, but dynamical calculations remain to be done.