Three-point functions in YM Theory and QCD in Landau gauge

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Der Wissenschaftsfonds.

NAWI Graz
Natural Sciences

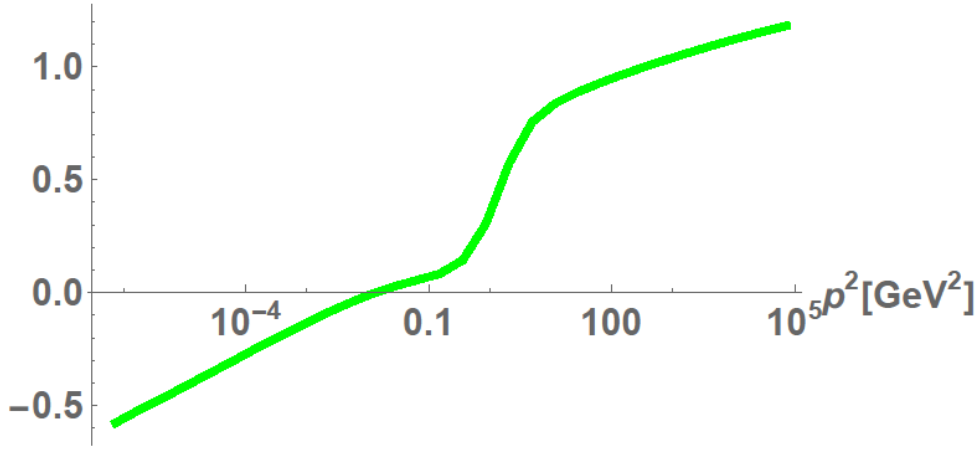
Doktoratskolleg "Hadrons in Vacuum, Nuclei and Stars"

Outline

Three-gluon Vertex:

(YM)

- appearance due to non-Abelian nature of QCD
- linked to confinement
- special feature: zero crossing



Symmetric point: $p^2 = q^2$, cos(α)=-0.5

Outline

Three-gluon Vertex:

(YM)

- appearance due to non-Abelian nature of QCD
- linked to confinement
- special feature: zero crossing

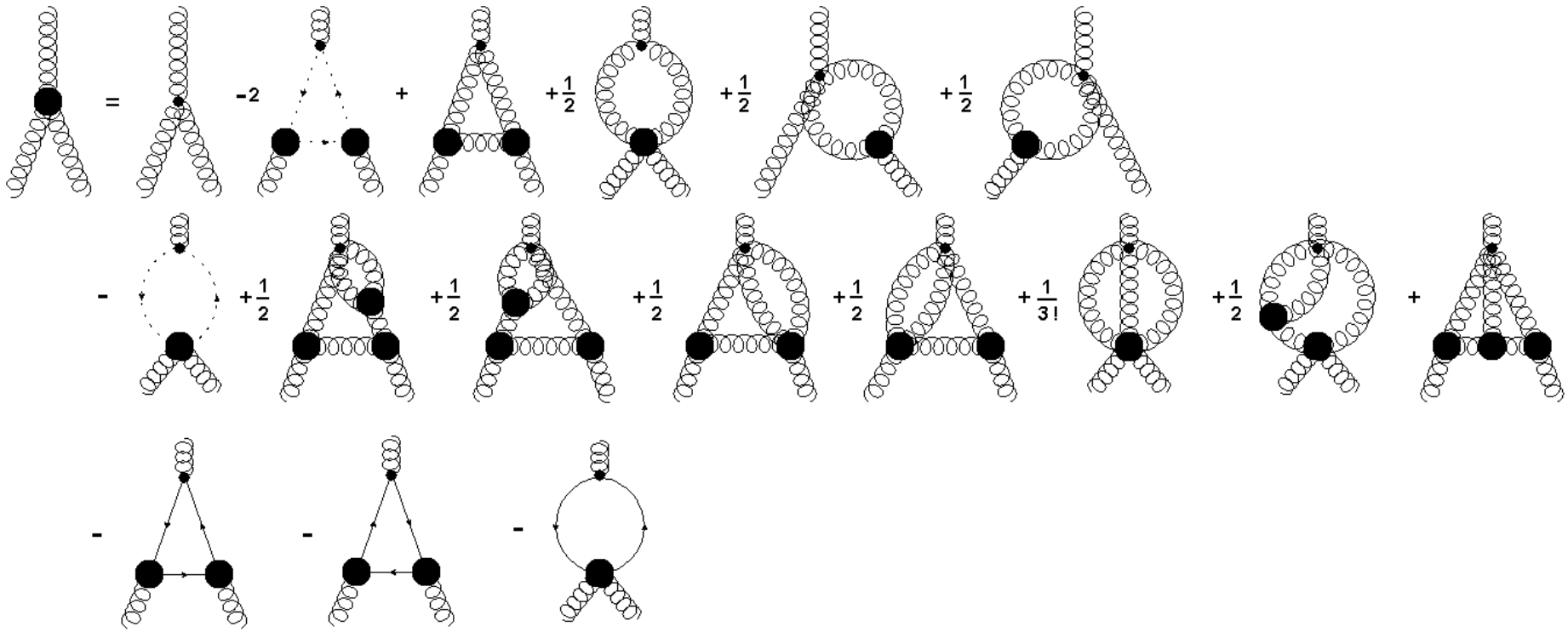
Quark-gluon Vertex:

- connects gauge and matter sector
- crucial for chiral symmetry-breaking and generation of mass

Three-gluon + Quark-gluon Vertex:

- physical quantities depend on unquenched system
- How much is position of zero-crossing altered by unquenching effects ?

The three-gluon vertex DSE



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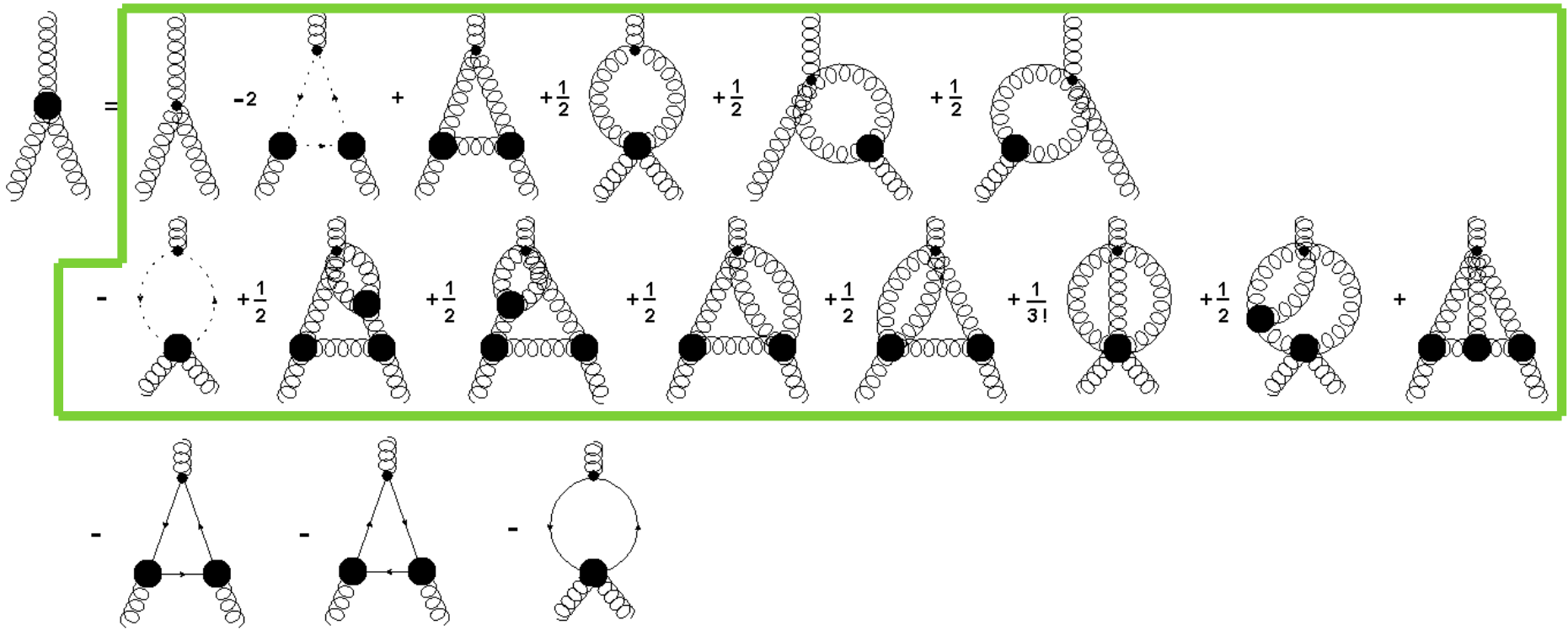

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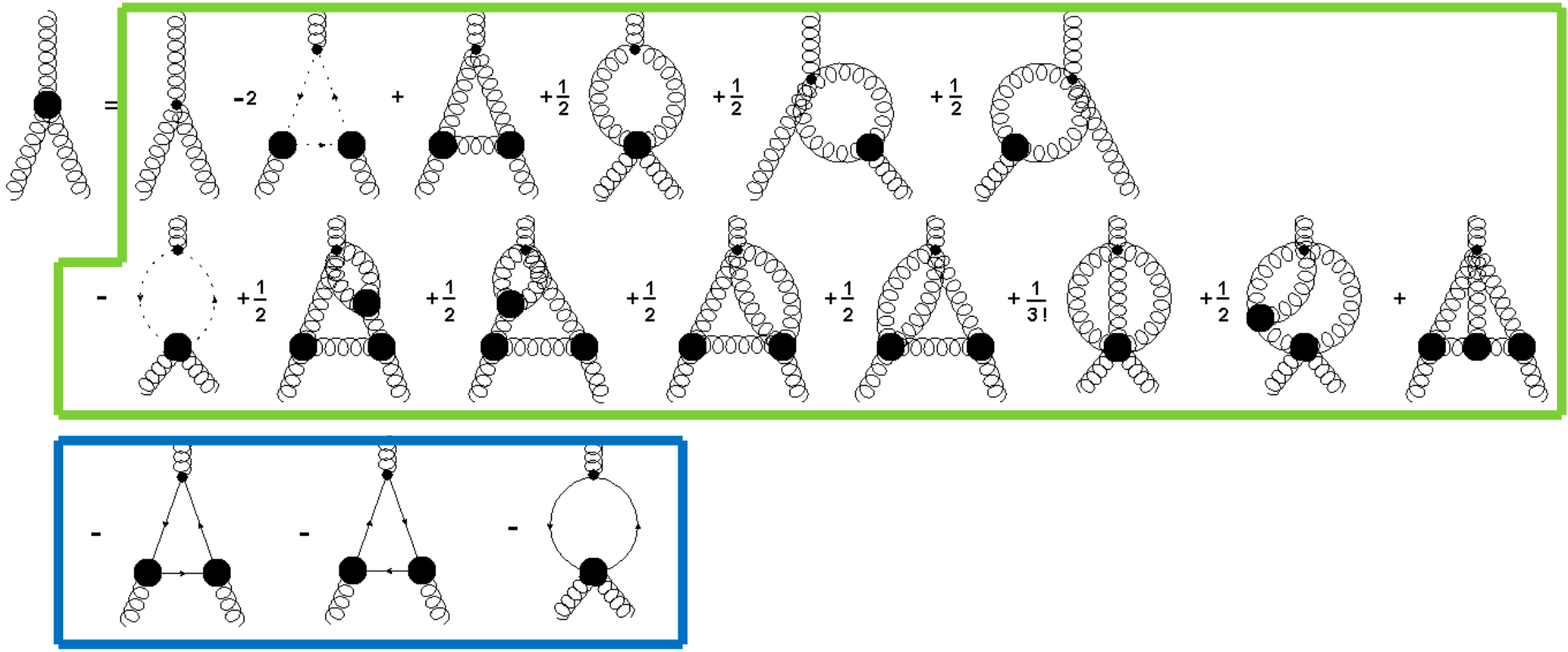
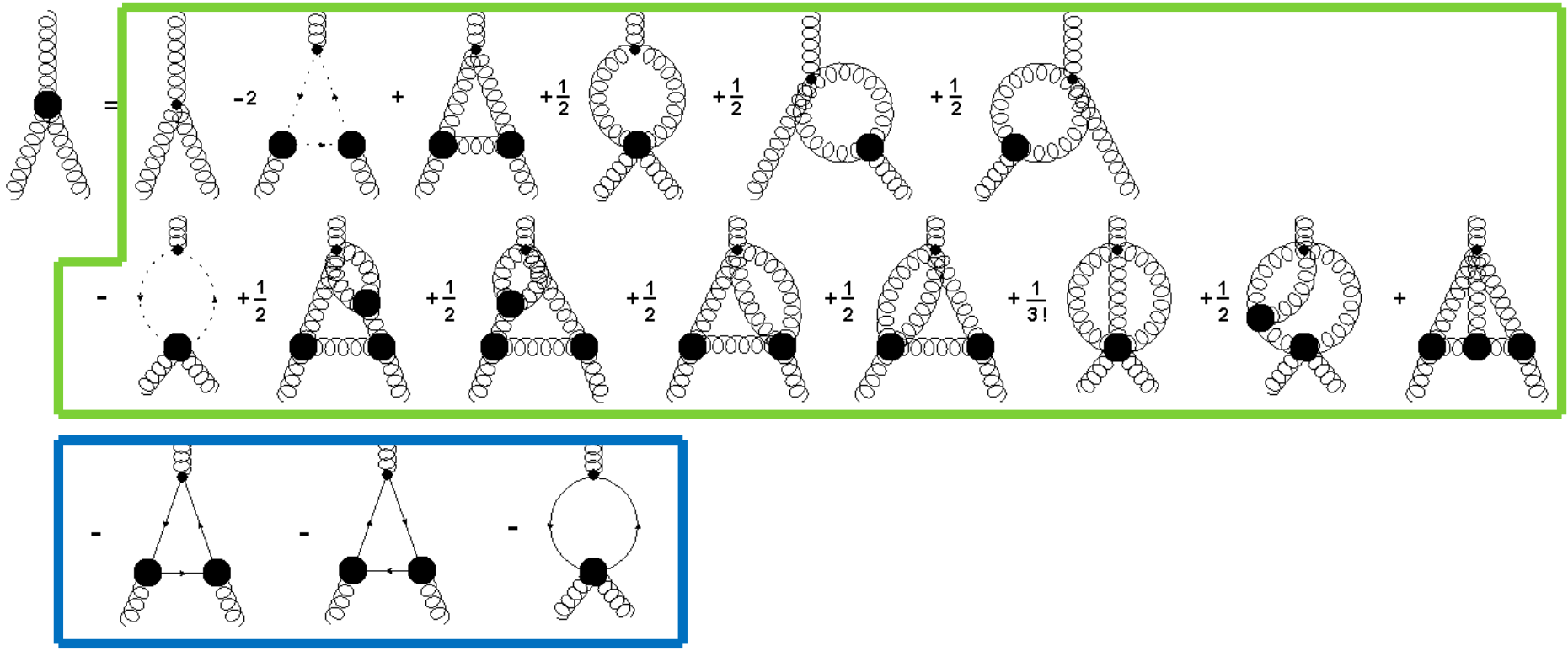


The three-gluon vertex DSE

Truncation is needed



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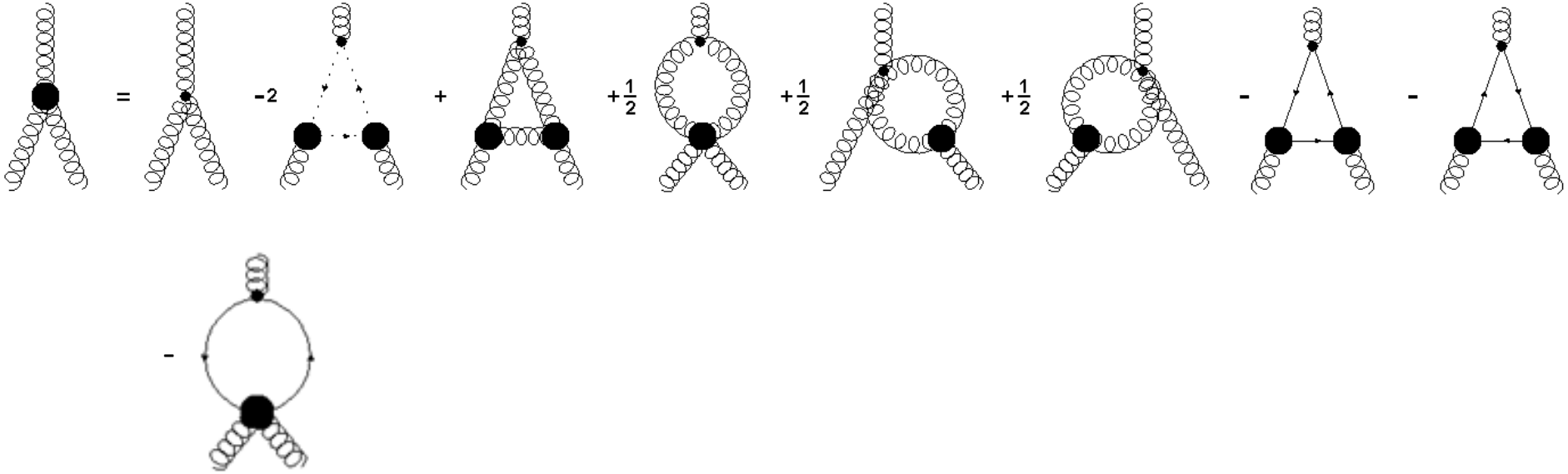
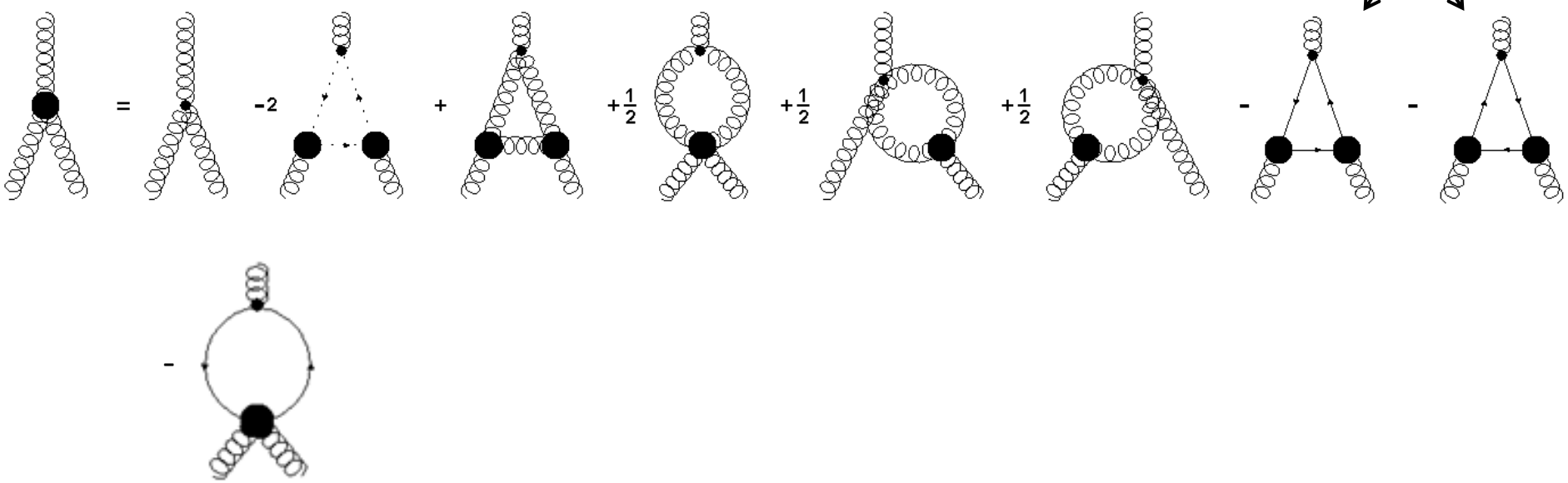


The three-gluon vertex DSE

Furry's Theorem



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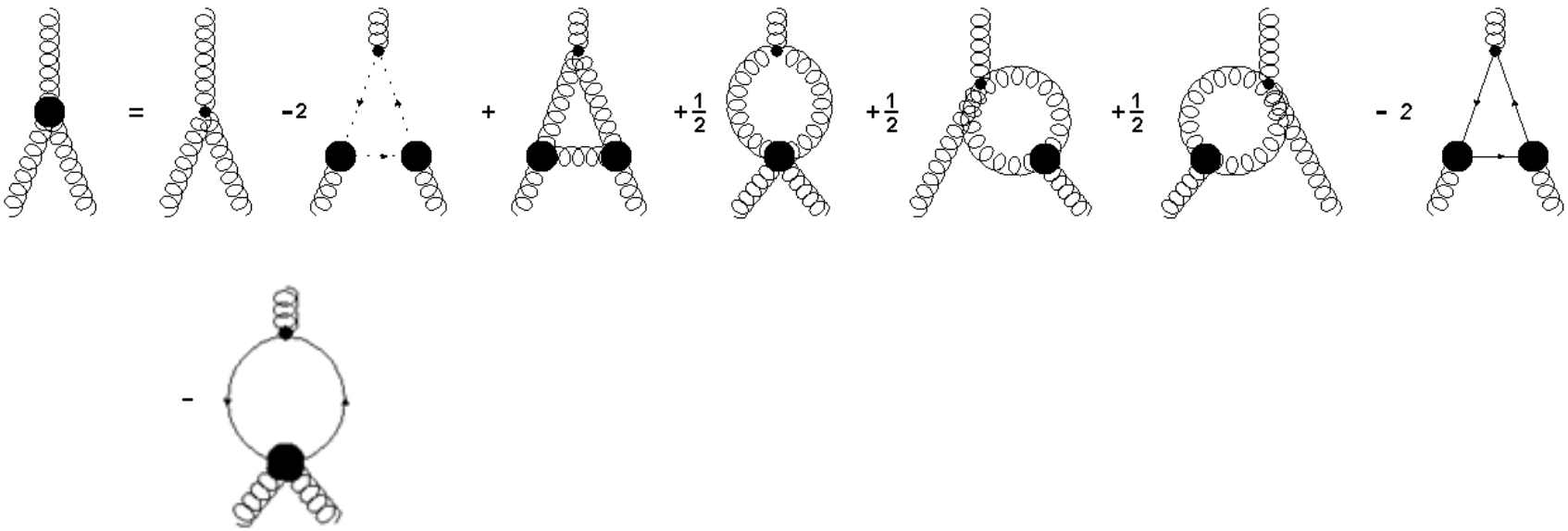
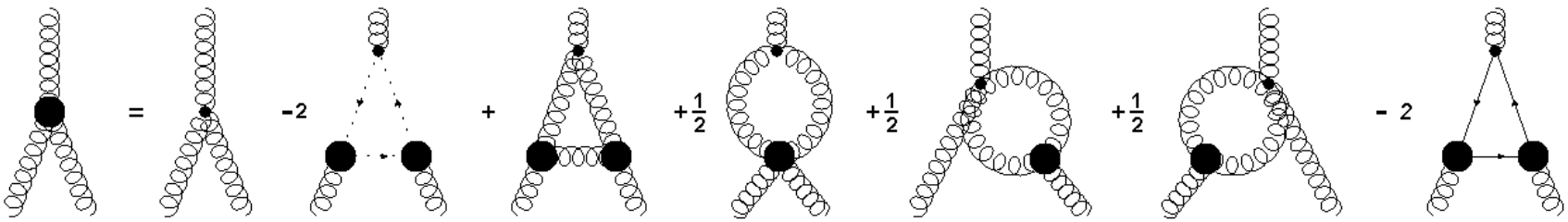
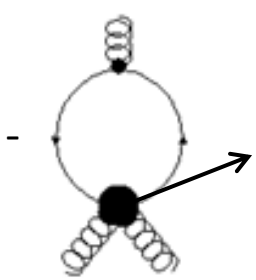


The three-gluon vertex DSE

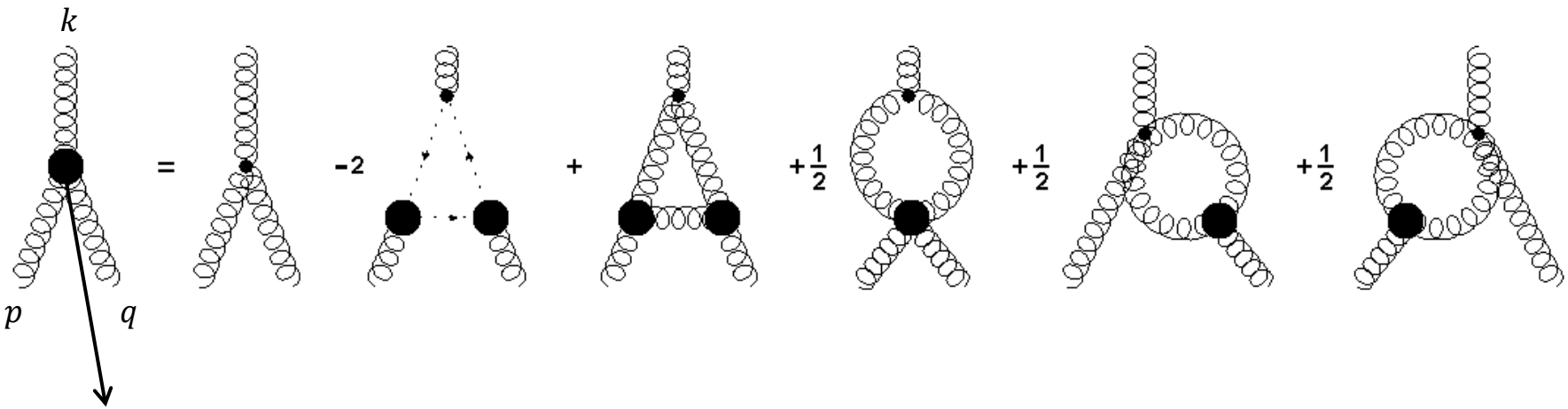




Employ model for 2q2gl-vertex[1]

1: M. Mitter et al., Phys. Rev. D **91**, 054035

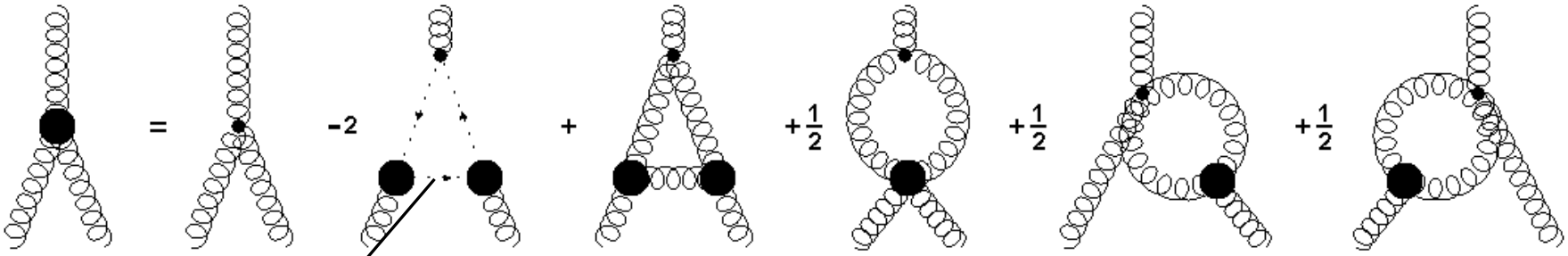
Parametrization of three-gluon vertex



- 4 transverse + 10 longitudinal tensors
- In Landau gauge full dynamics of theory described by transverse part
- dominant contribution from tree-level term[1,2]
- $\Gamma^{A^3}_{\mu\nu\varrho}(p,q,k) = D^{A^3}(p^2,q^2,\cos(\alpha))\,\Gamma^{A^3,0}_{\mu\nu\varrho}(p,q,k)$

1. G. Eichmann et al., Phys. Rev. D 89, 105014 (2014)
2. A. Sternbeck et al., to be published

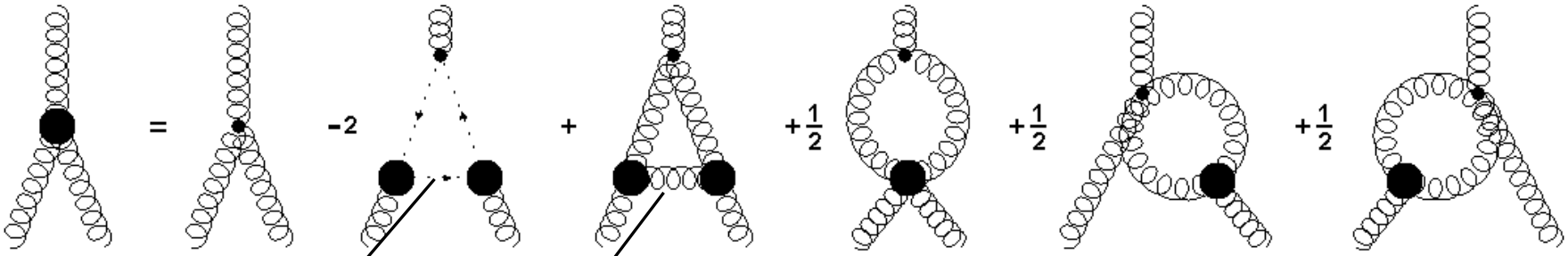
The ghost and gluon propagator



ghost propagator:

$$D^G(p^2) = -\frac{\mathrm{G}(p^2)}{p^2}$$

The ghost and gluon propagator

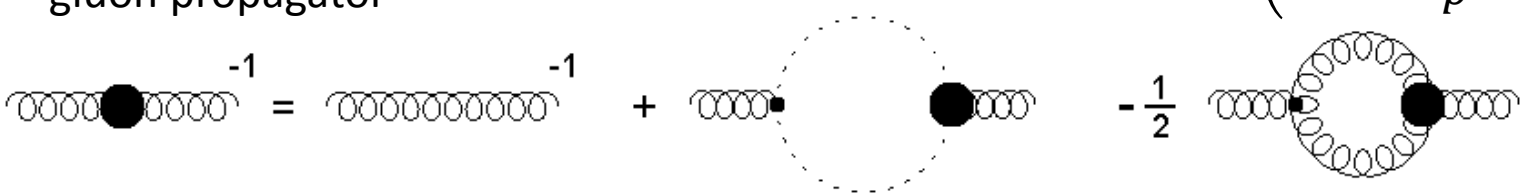


ghost propagator:

$$D^G(p^2) = -\frac{G(p^2)}{p^2}$$

gluon propagator

$$D_{\mu\nu}(p^2) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$$



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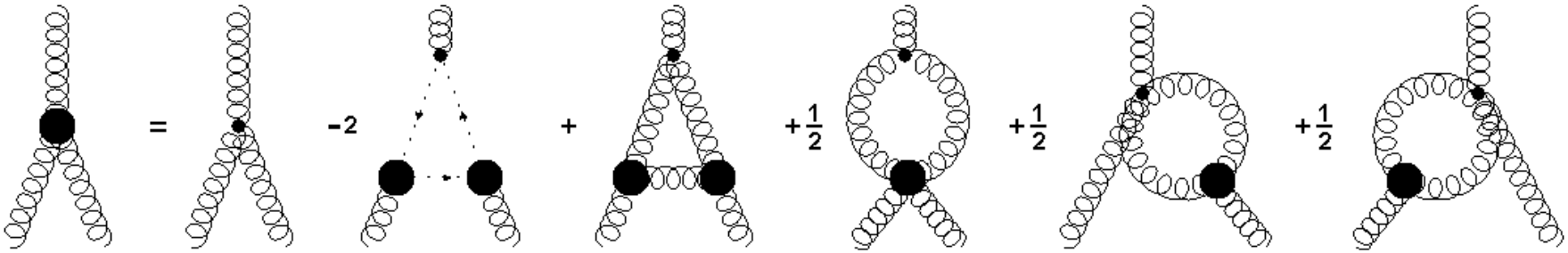
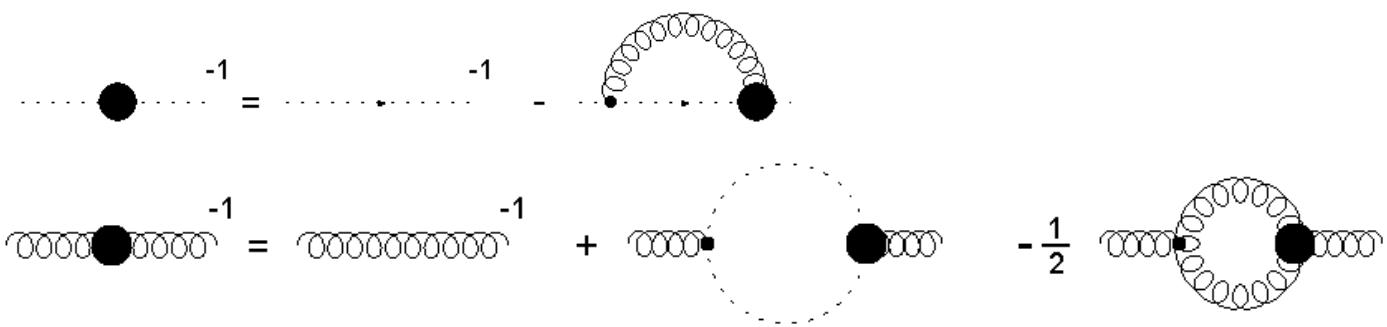
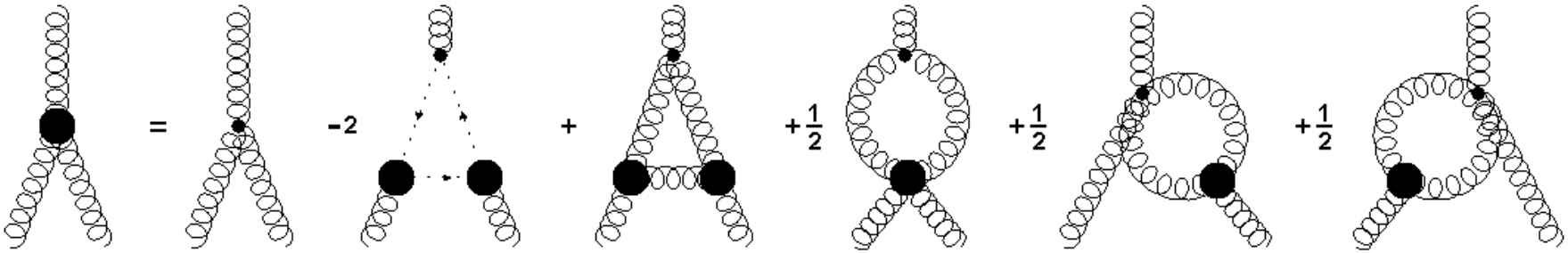


- Solve coupled system of YM-propagators
- Use model for 3gl-vert tuned in such a way to effectively include contribution from two-loop diagrams[1]

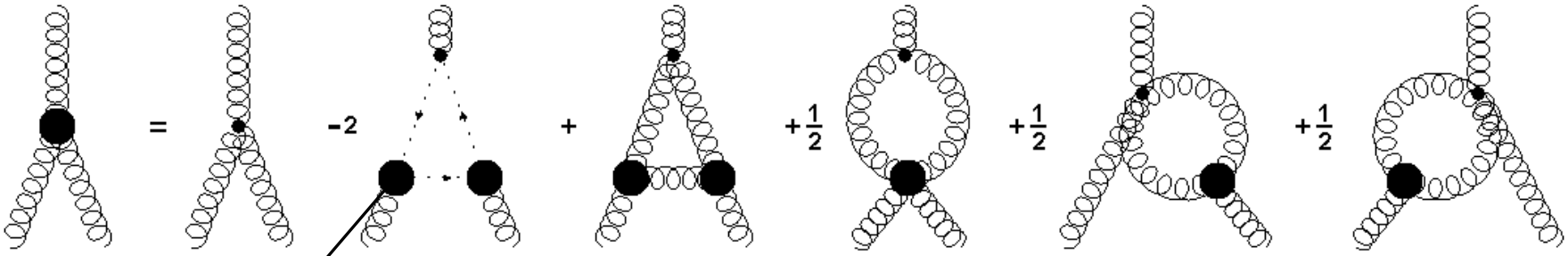


1 M.Q. Huber and L. von Smekal, J. High Energ. Phys. (2013) 2013: 149

The three-gluon vertex DSE

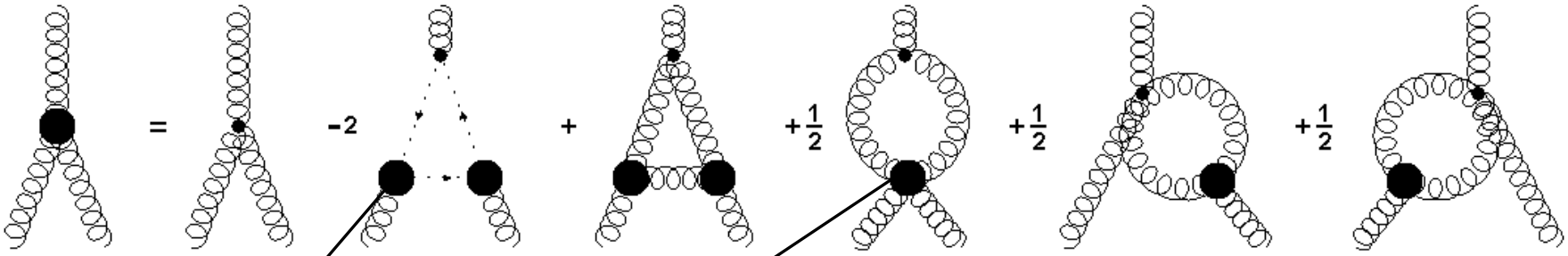


The ghost-gluon and four-gluon vertex



gh-gl vertex: good approximation -> bare vertex

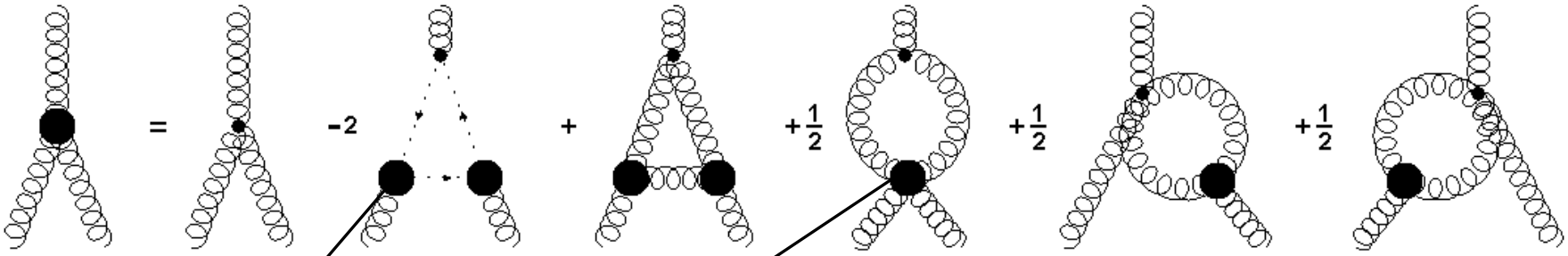
The ghost-gluon and four-gluon vertex



gh-gl vertex: good approximation -> bare vertex

4gl vertex: employ a model

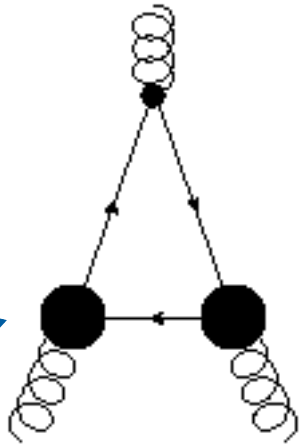
The ghost-gluon and four-gluon vertex



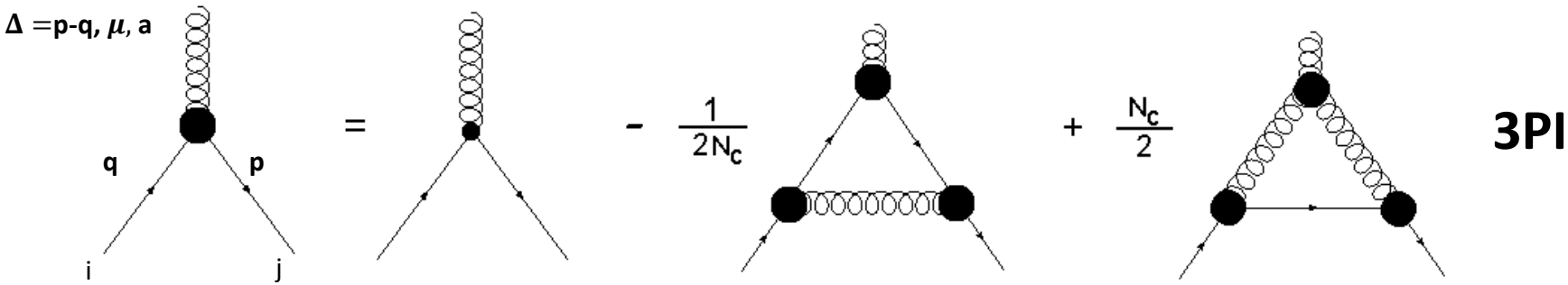
gh-gl vertex: good approximation -> bare vertex

4gl vertex: employ a model

unquenching: quark-gluon vertex is needed



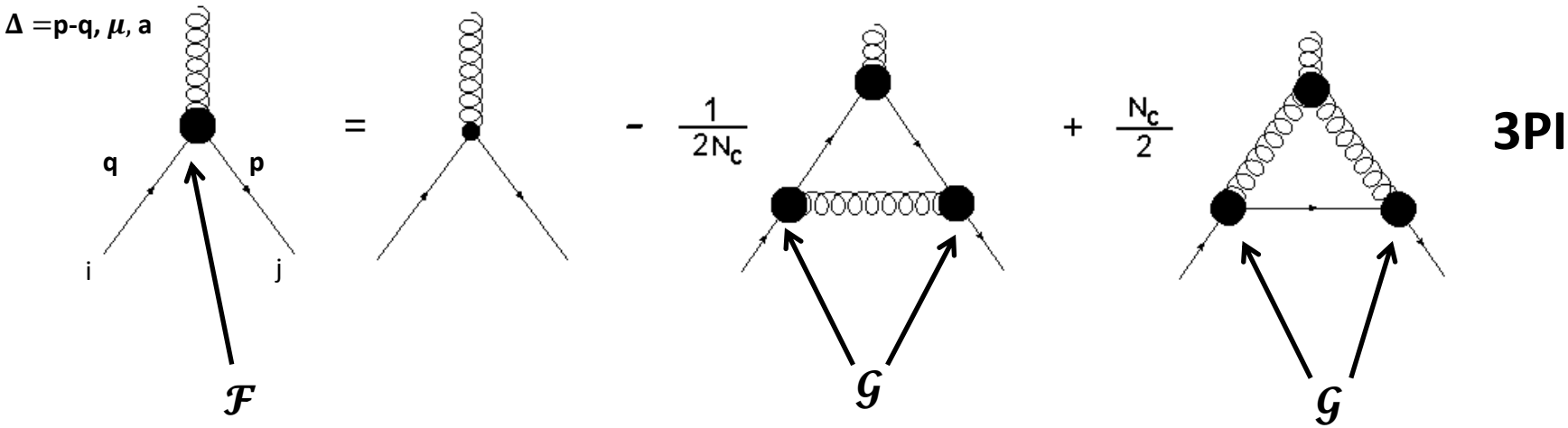
The quark-gluon vertex



- the quark-gluon vertex can be decomposed into 8 trans. + 4 long. basis tensors

$$\Gamma_\mu^{qgv}(q,p;\Delta) = \sum_{i=1}^{8} g_i\left(p^2, q^2; p\cdot q/(\sqrt{p^2q^2}\,)\right) b_\mu^{(i)}$$

Finding a basis for the quark-gluon vertex



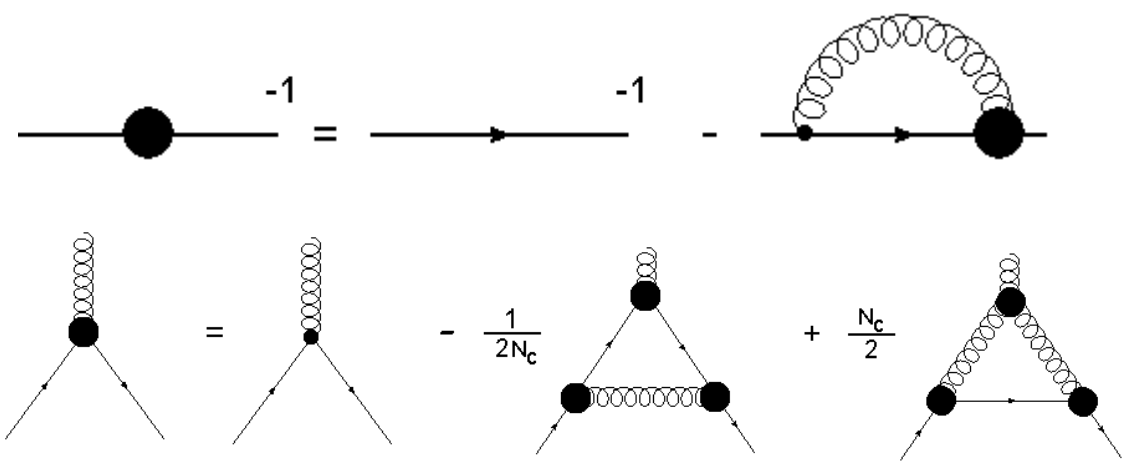
- Externally: use **orthonormal basis $\mathcal{F}$**
- Internally: use **transversal basis $\mathcal{G}$**
- convert from one basis set to the other in each iteration step

The quark propagator DSE

- dressed quark propagator: $S(p) = \frac{1}{-i\not{p}\,A(p^2)+B(p^2)} = Z_f(p^2)\,\frac{i\not{p}+M(p^2)}{p^2+M^2(p^2)}$
- quark wave function renormalization: $Z_f(p^2) = 1/A(p^2)$
- quark mass function: $M(p^2) = B(p^2)/A(p^2)$

Solve coupled system of quark propagator + quark-gluon vertex DSE

(use model for 3glvert)



The two-quark-two-gluon vertex

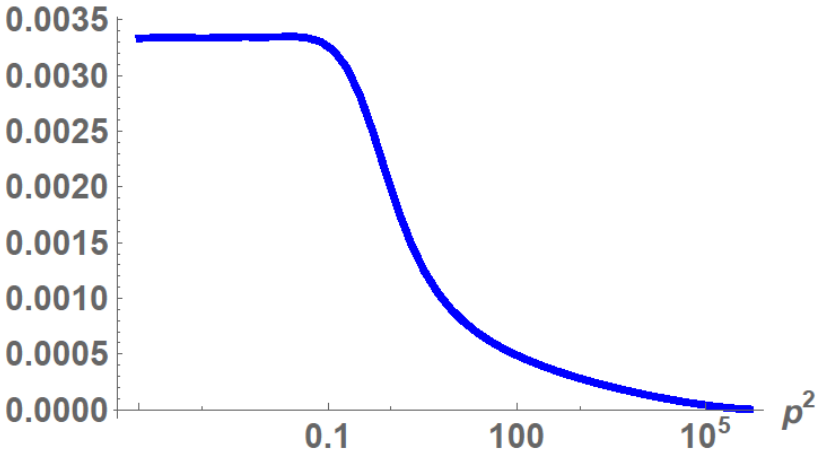
Use model for 2q2gl-vertex:[1]

- Each of the 8 tensor structures alone breaks gauge invariance
- Together with further terms T_{gauge} gauge invariance is guaranteed
- These terms contain contributions : $T_{gauge} \propto \mathcal{O}(\bar{q}Aq) + \mathcal{O}(\bar{q}AAq) + \mathcal{O}(\bar{q}AAAq)$
- Assume one dressing function λ_{gauge} for T_{gauge} and determine it from comparison with dressing function of quark-gluon vertex
- Focus on dominant qgv contribution (beyond tree-level dressing function).

1: M. Mitter et al., Phys. Rev. D **91**, 054035

The quark-swordfish diagram

Calculation of the quark-swordfish diagram with this model



Symmetrc point: $p^2 = q^2$, cos(α)=-0.5

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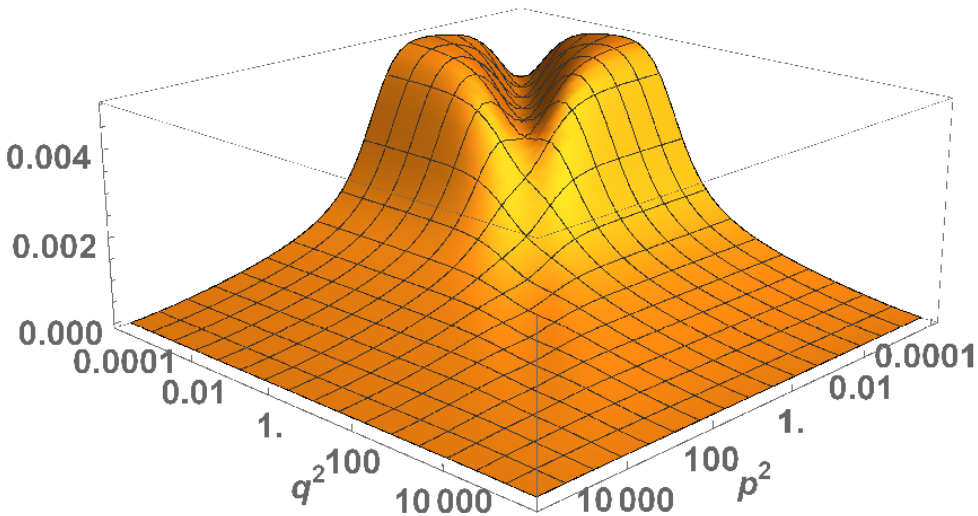
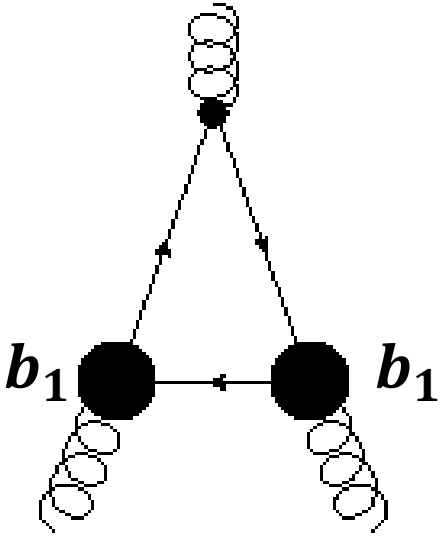


Unquenching the three-gluon vertex: A first step

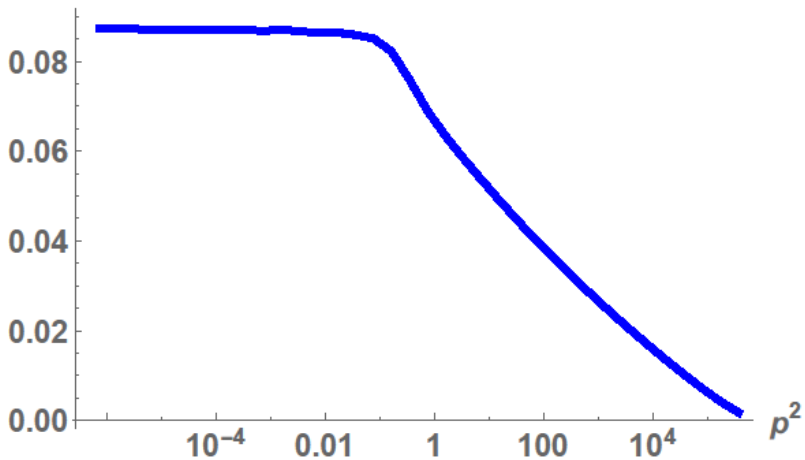
- Quark triangle + swordfish: static diagrams
  Calculate those first and include them in iteration of 3glvert

- So far only tree-level tensor structure b_1 for quark-gluon vertex considered

- Should give the major contribution

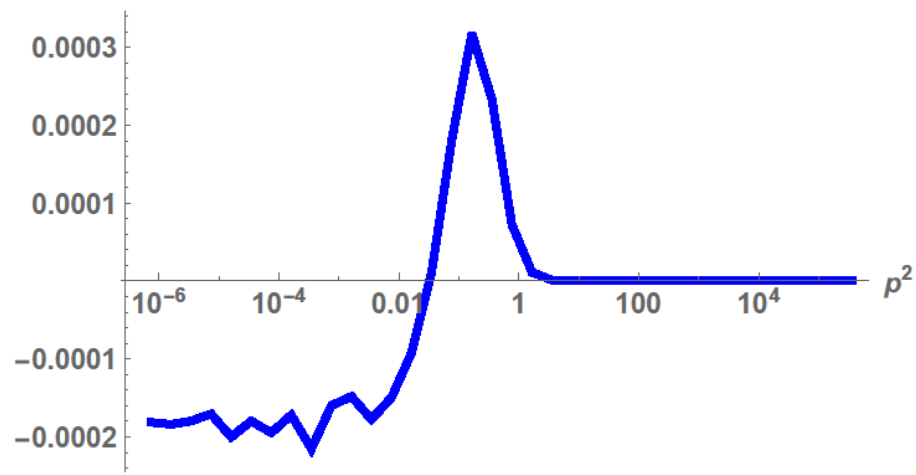


Results: Quark Triangle

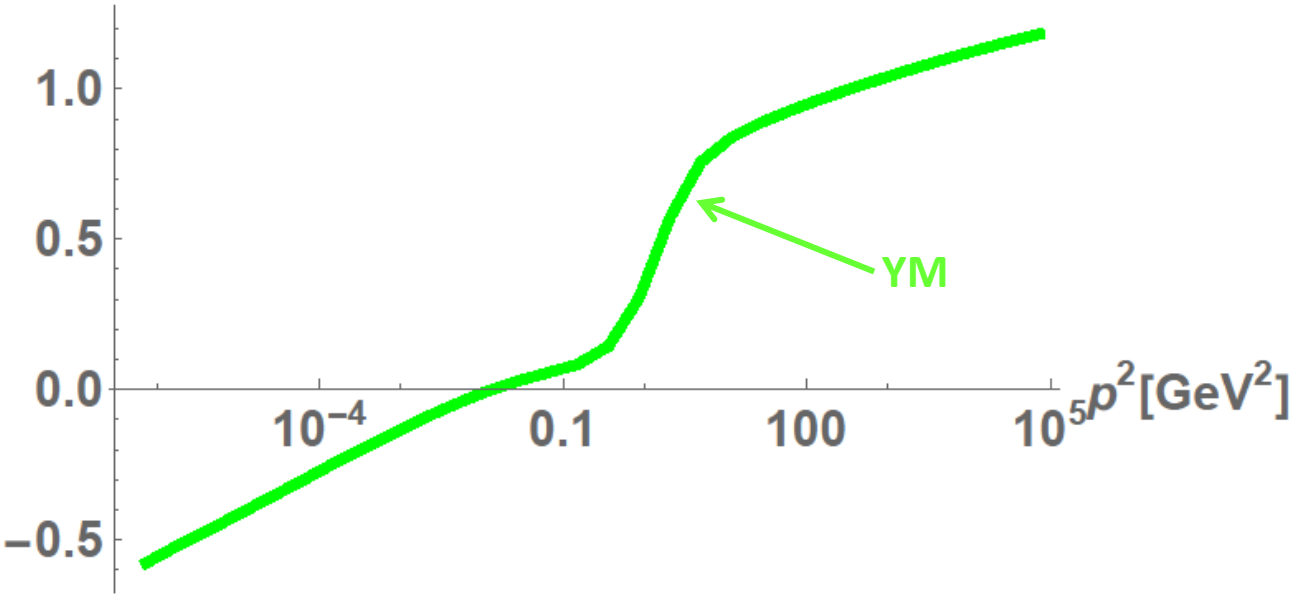


combination: b1b1

combination: b1b2



Results: three-gluon vertex



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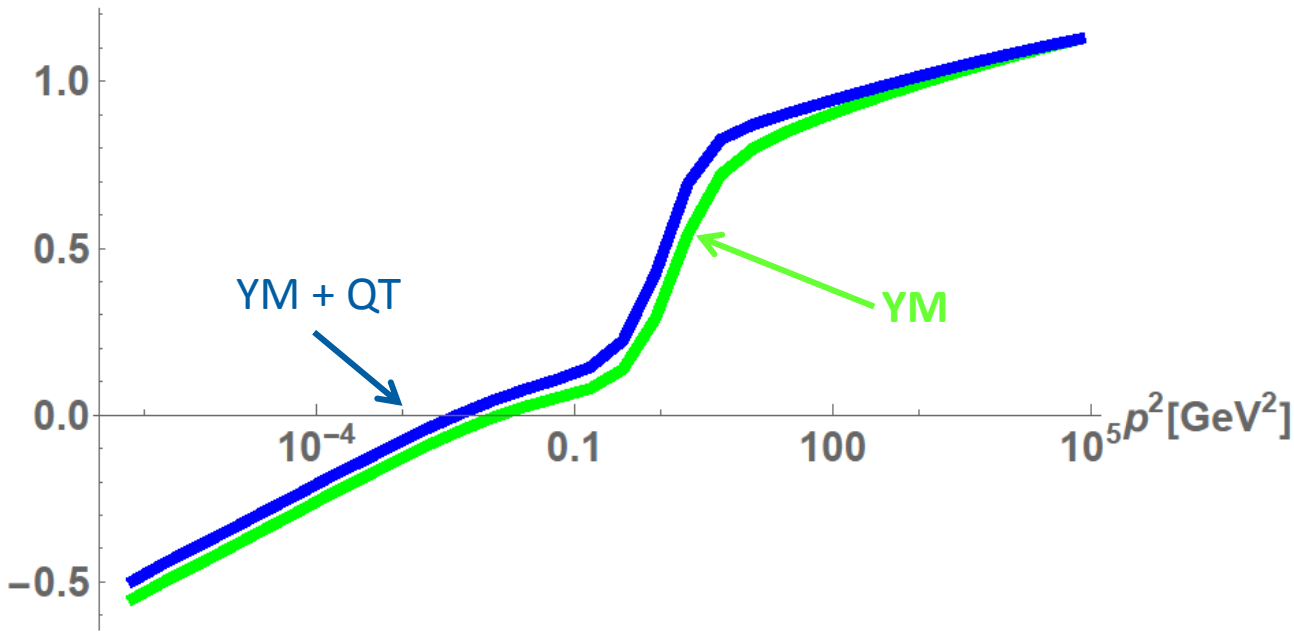

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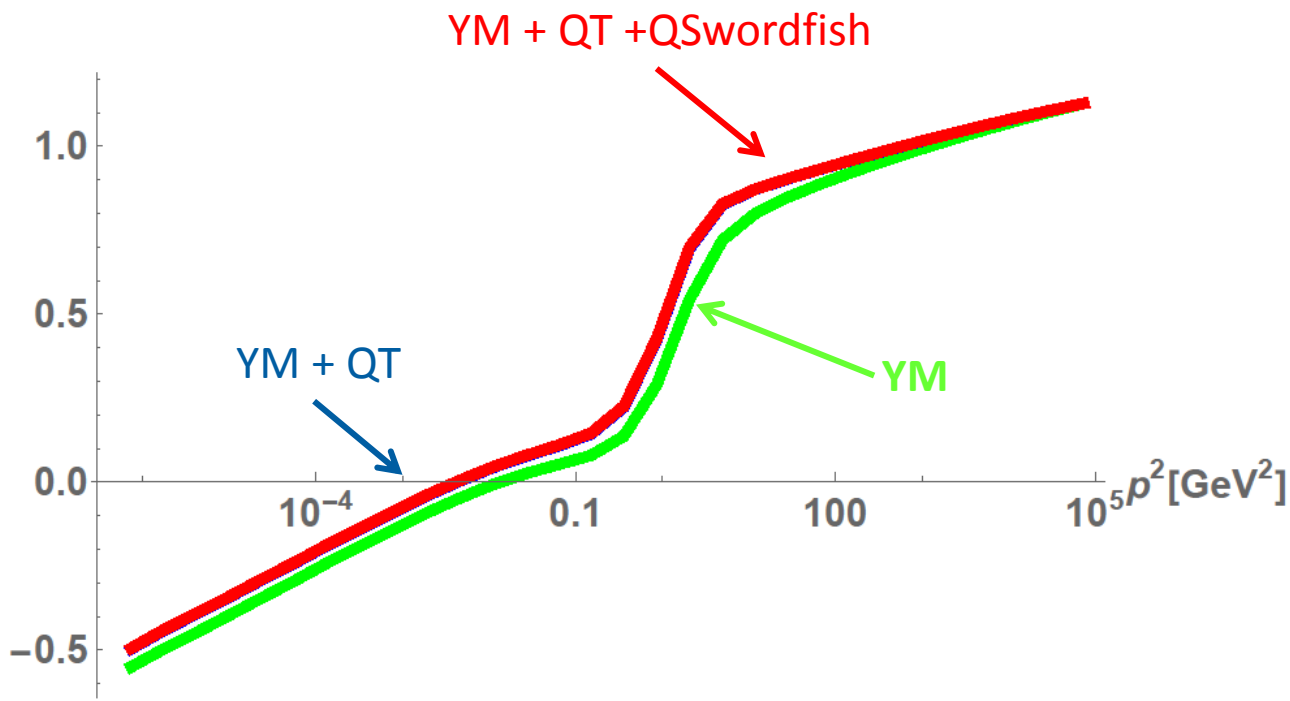

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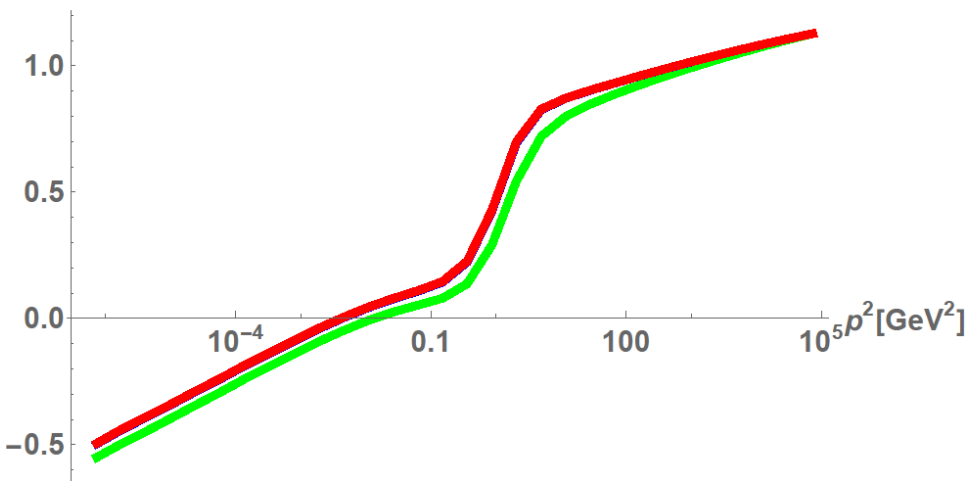
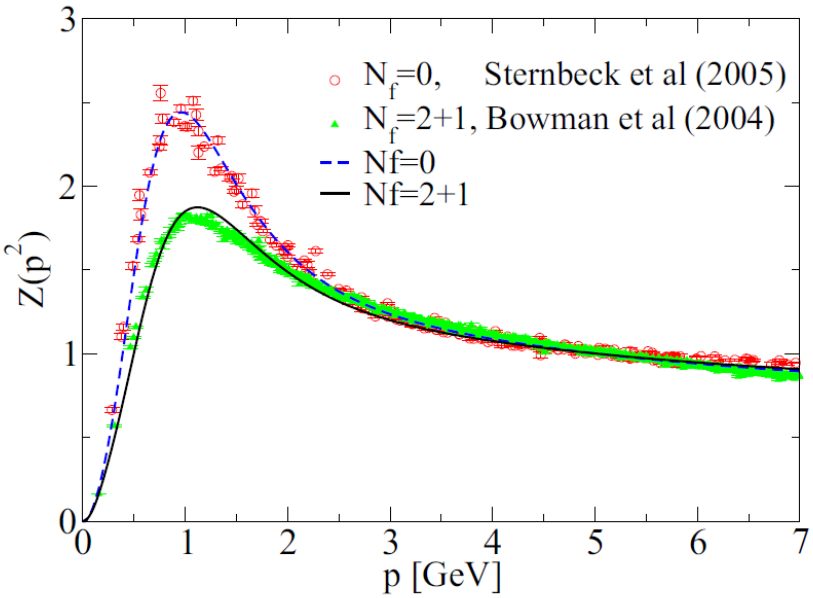
same behaviour observed in: R. Williams et al., Phys. Rev. D **93**, 034026

Results: three-gluon vertex





(M. Hopfer et al., J. High Energ. Phys. (2014) 2014: 35)

Summary and Conclusion

- unquenching of three-gluon vertex by including quark triangle and quark-swordfish diagram

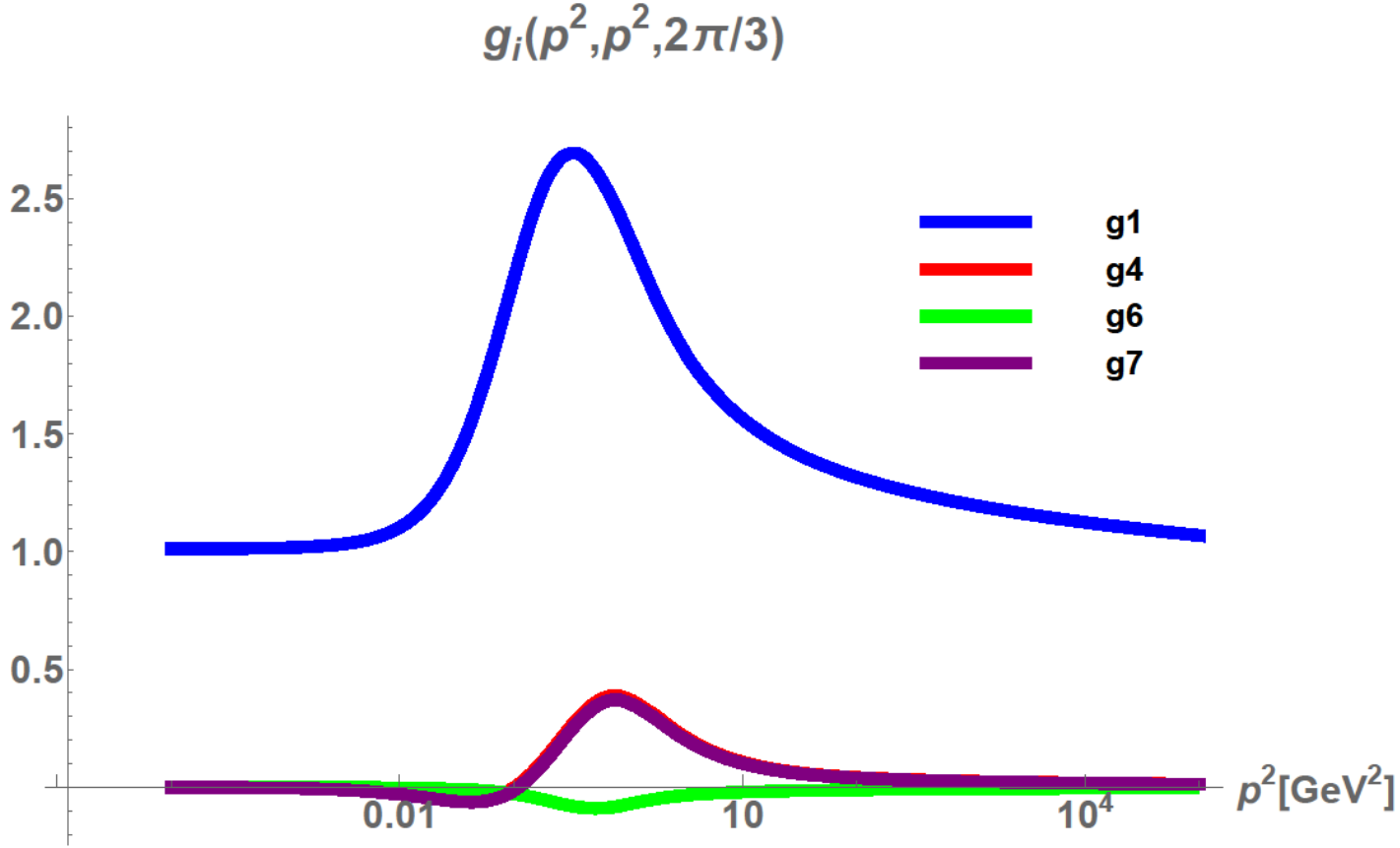
- quark-swordfish: model for 2q2gl vertex

- quark triangle: focus on tree-level tensor structure b_1

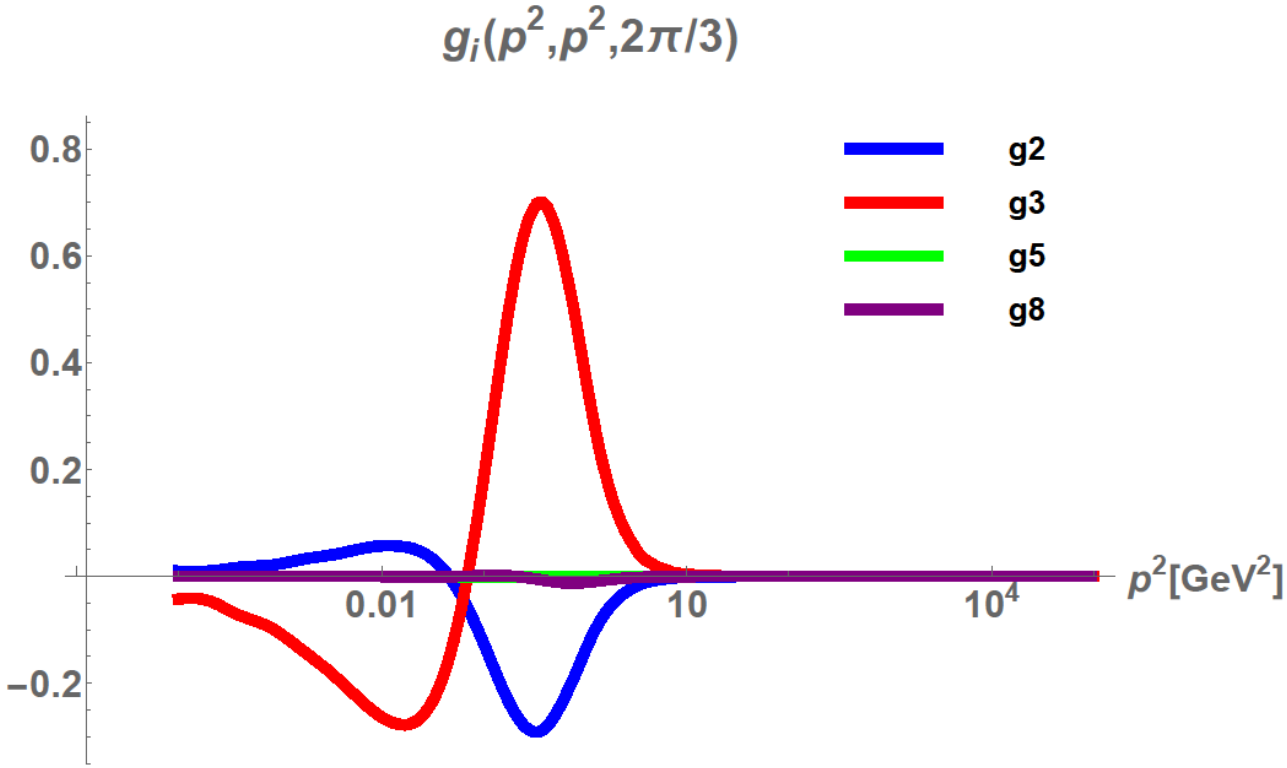
- unquenched three-gluon vertex above quenched three-gluon vertex:
  opposite behaviour compared to gluon propagator

- zero-crossing shifted towards infrared regime

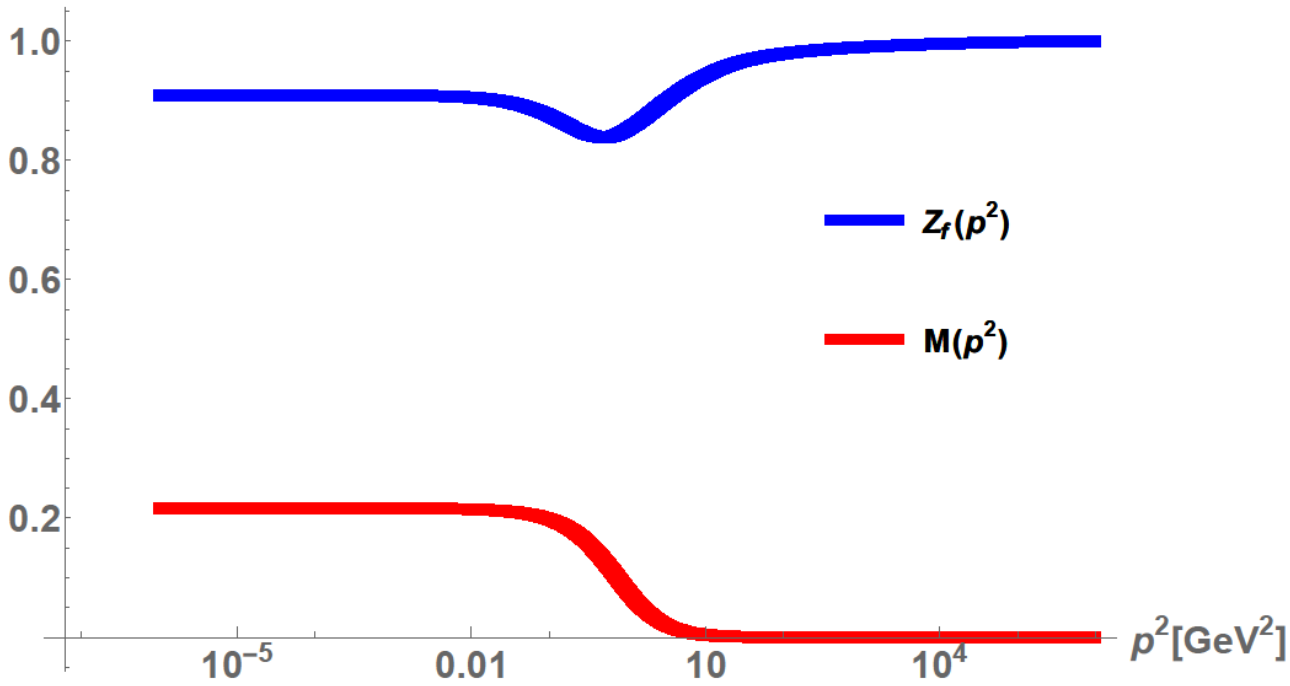
Backup



Chirally symmetric



Chirally antisymmetric


1.0
0.8
0.6
0.4
0.2
$Z_f(p^2)$
$M(p^2)$
10^{-5}
0.01
10
10^4
$p^2[\mathrm{GeV}^2]$

Dressed propagators and vertices in Landau gauge

the four-gluon vertex model:

- cancellations between gluon-triangle and swordfish diagrams
- model must take into account the balance between these diagrams

➡ especially strength in **midmomentum** regime important

- we make the following ansatz:

$$\Gamma^{A^4,abcd}_{\mu\nu\varrho\sigma}(p,q,k,r) = (a\tanh(b/\bar{p}^2_{A^4}) + 1)D^{A^4,UV}(p,q,k,r)\Gamma^{(0)A^4,abcd}_{\mu\nu\varrho\sigma}(p,q,k,r)$$

$$\bar{p}^2_{A^4} = (p^2 + q^2 + k^2 + r^2)/2$$

- parameters a,b can be varied ➡ produces a band of solutions

3glvert-Model

Decoupling:

$$\alpha = 3 + \frac{1}{\delta}$$

$$\beta = 0$$

$$D^{A^3,UV}(p,q,-p-q) = G\left(\frac{p^2+q^2+(p+q)^2}{2}\right)^{\alpha} Z\left(\frac{p^2+q^2+(p+q)^2}{2}\right)^{\beta}$$

$$D^{A^3,IR}(p,q,-p-q) = h_{IR}\, G(p^2+q^2+(p+q)^2)^3\, (f^{3g}(p^2) f^{3g}(q^2)\, f^{3g}((p+q)^2)^4$$

$$f^{3g}(x) = \frac{\Lambda^2}{\Lambda^2 + x}$$

$$D^{A^3}(p,q,-p-q) = \left(D^{A^3,IR}(p,q,-p-q) + D^{A^3,UV}(p,q,-p-q)\right)\frac{1}{Z_1} D^{A^3,UV}(p,q,-p-q)$$