

# Landau gauge Yang-Mills correlation functions

Anton Konrad Cyrol

Ruprecht-Karls-Universität Heidelberg

based on

- AKC, Fister, Mitter, Pawłowski, Strodthoff, PRD, arXiv:1605.01856 [hep-ph]
- AKC, Mitter, Pawłowski, Strodthoff,  $T > 0$  Yang-Mills, in preparation

August 1, 2016

# QCD phase diagram with functional methods

## fQCD-collaboration:

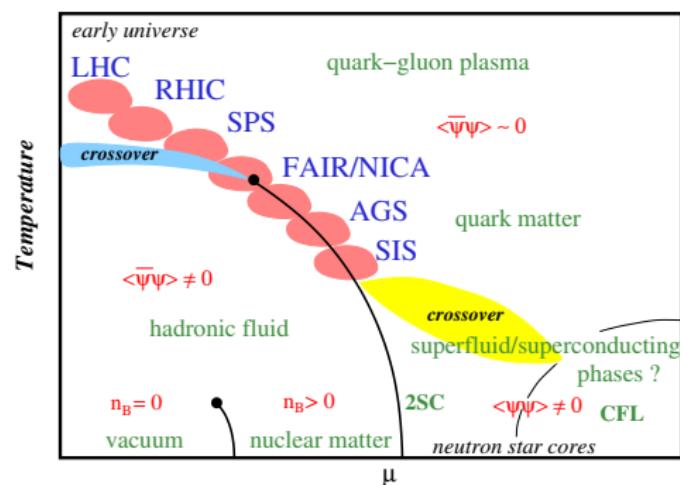
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## This talk:

- Vacuum Yang-Mills theory
- Preliminary  $T > 0$  results

## Aim:

- Qualitative understanding
- Quantitative precision



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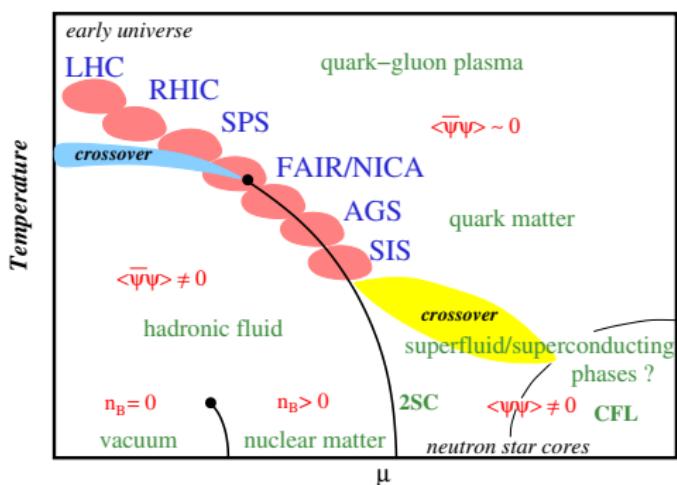
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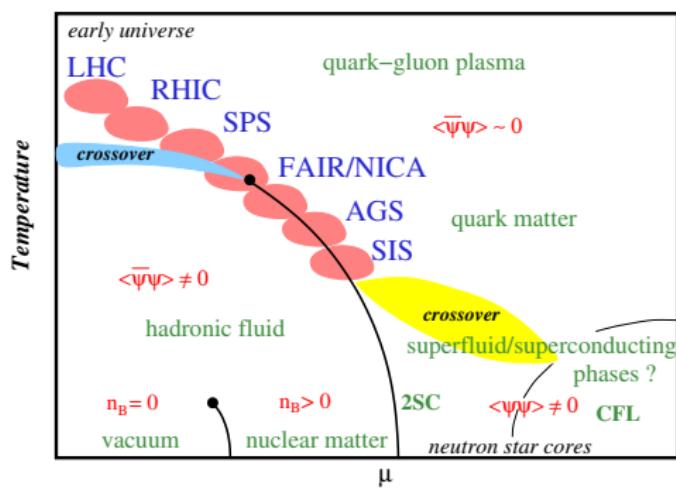
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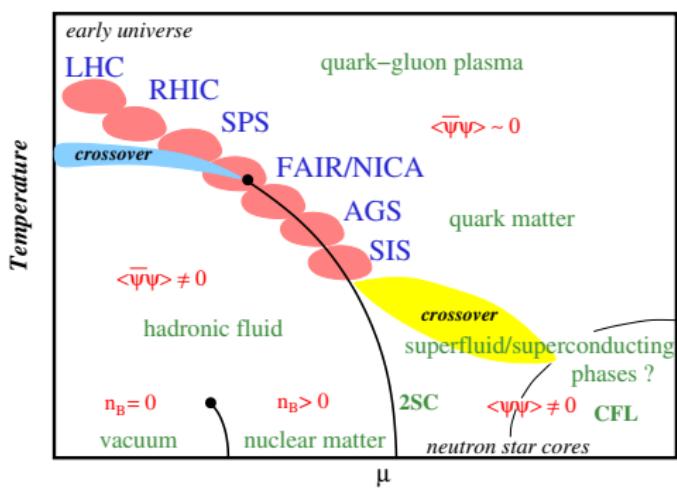
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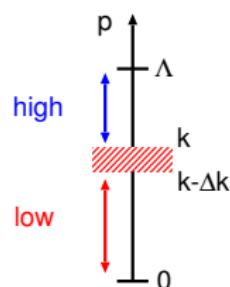
- Only perturbative QCD input
  - $\alpha_s(\mu = \mathcal{O}(10) \text{ GeV})$
  - $m_q(\mu = \mathcal{O}(10) \text{ GeV})$
- Wetterich equation with initial condition  $S[\Phi] = \Gamma_\Lambda[\Phi]$
- Effective action  $\Gamma[\Phi] = \lim_{k \rightarrow 0} \Gamma_k[\Phi]$
- Exact equation
- $\partial_t$ : integration of momentum shells controlled by regulator
- Full field-dependent equation with  $(\Gamma^{(2)}[\Phi])^{-1}$  on rhs

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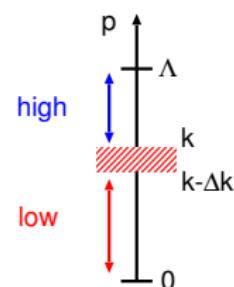


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# Vertex expansion

- Approximation necessary – vertex expansion:

$$\Gamma[\Phi] = \sum_n \int_{p_1, \dots, p_{n-1}} \Gamma_{\Phi_1 \dots \Phi_n}^{(n)}(p_1, \dots, p_{n-1}) \Phi^1(p_1) \dots \Phi^n(-p_1 - \dots - p_{n-1})$$

- Wanted: “apparent convergence” of  $\Gamma[\Phi] = \lim_{k \rightarrow 0} \Gamma_k[\Phi]$

- Current state-of-the-start truncation:

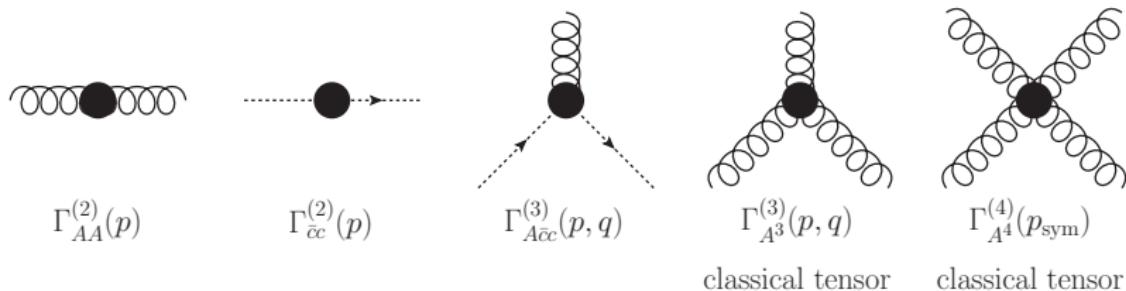
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# Truncation – closed set of equations

$$\partial_t \text{---} \text{---}^{-1} = \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---}$$

$$\partial_t \text{---} \text{---}^{-1} = \text{---} \text{---} \text{---} \text{---} - 2 \text{---} \text{---} \text{---} \text{---} - \frac{1}{2} \text{---} \text{---} \text{---} \text{---}$$

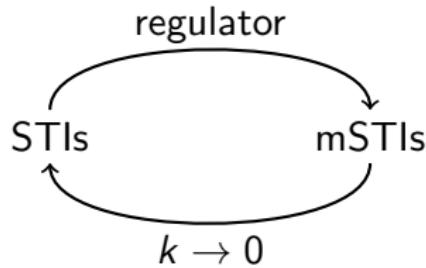
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# Regulator breaks BRST symmetry

- Breaking BRST symmetry  $\rightarrow$  modified STIs
- mSTIs reduce to STIs at  $k = 0$
- $\Rightarrow$  solve mSTIs to get initial action at  $k = \Lambda$
- More practical solution: choose  $\Gamma_\Lambda \approx S$  such that STIs are fulfilled  $k = 0$



$$\alpha_{A\bar{c}c}(p) = \frac{\alpha(\mu)}{4\pi} \frac{Z_{A\bar{c}c}^2(p)}{Z_A(p) Z_c^2(p)}$$

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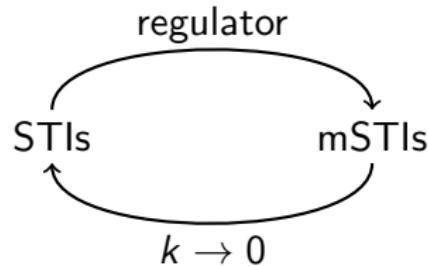
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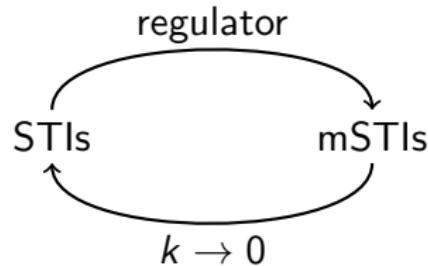
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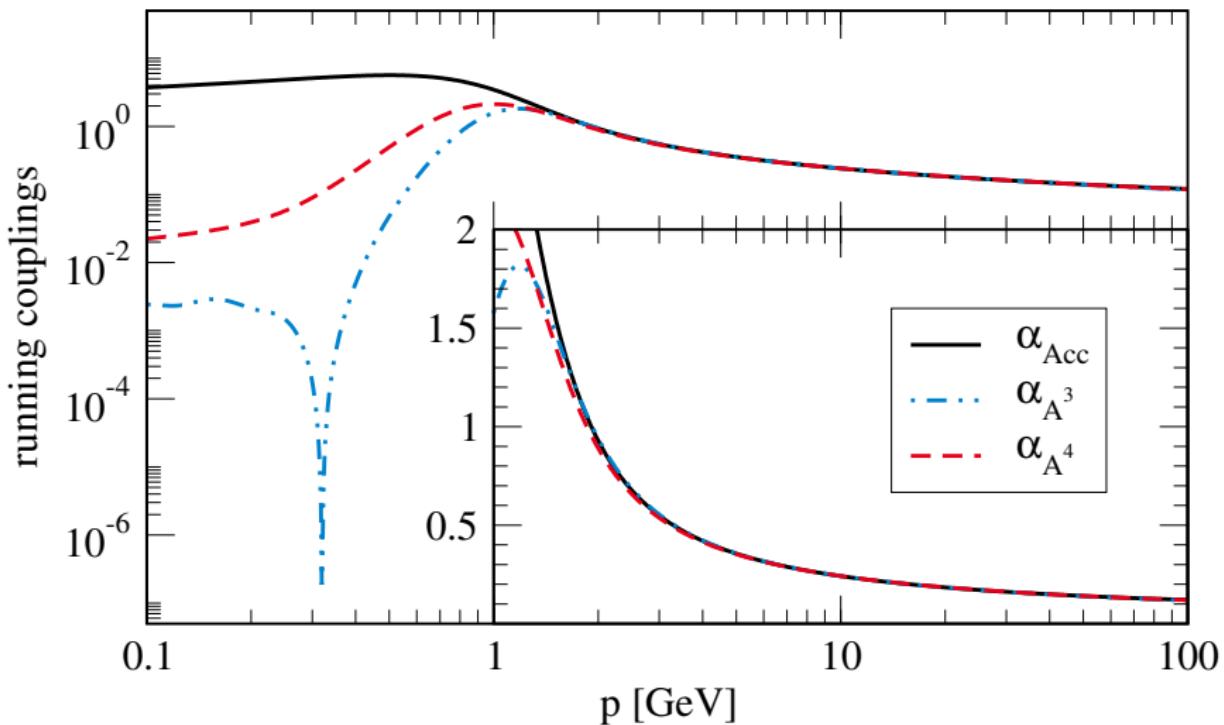
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# Running couplings (scaling solution)



# Gluon mass gap

Scaling solution

$$\lim_{p \rightarrow 0} Z_c(p^2) \propto (p^2)^\kappa$$

$$\lim_{p \rightarrow 0} Z_A(p^2) \propto (p^2)^{-2\kappa}$$

Decoupling solution

$$\lim_{p \rightarrow 0} Z_c(p^2) \propto 1$$

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- Landau Gauge gluon STI requires longitudinally mass term to vanish:

$$p_\mu \left( [\Gamma_{AA,L}^{(2)}]_{\mu\nu}^{ab}(p) - [S_{AA,L}^{(2)}]_{\mu\nu}^{ab}(p) \right) = 0$$

- Splitting between longitudinal and transverse mass term necessary
- Splitting occurs "naturally" for scaling solution
- Decoupling solution requires irregular vertices,  
e.g. a pole in the longitudinal sector
- Unphysical gluon mass parameter present at  $k = \Lambda$ ,  
 $\implies$  can be uniquely determined

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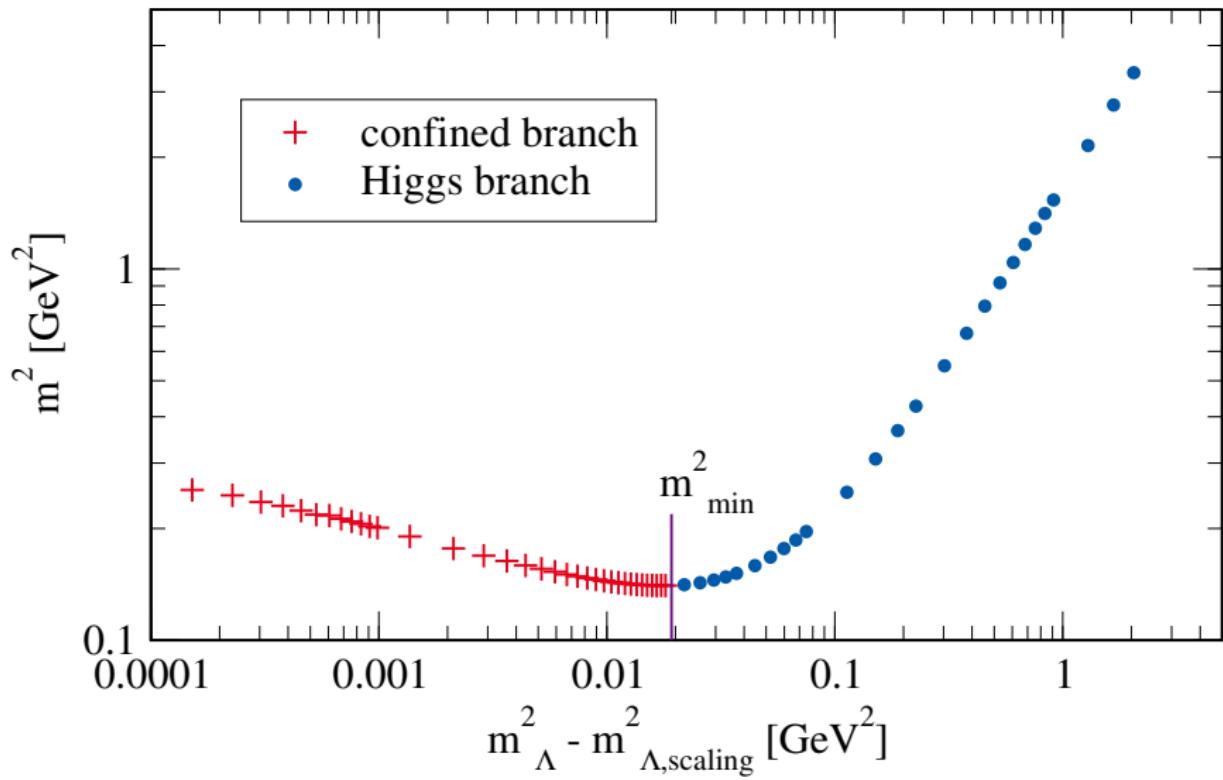
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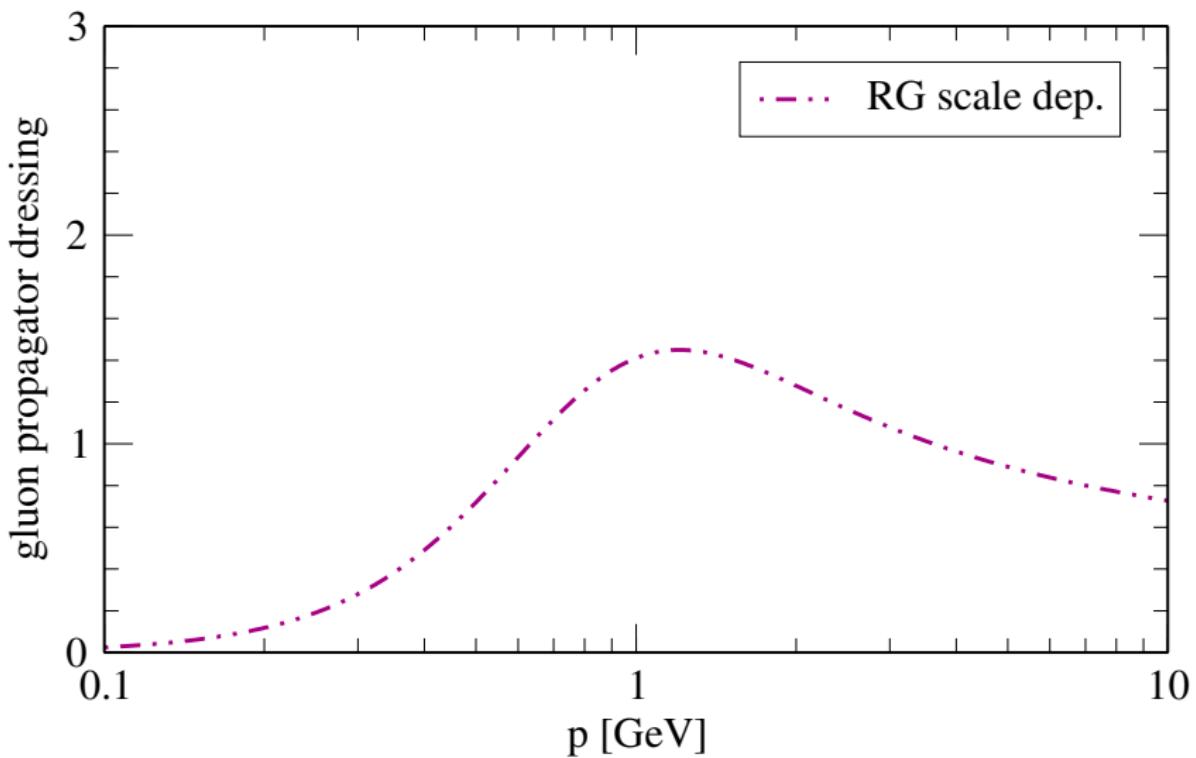
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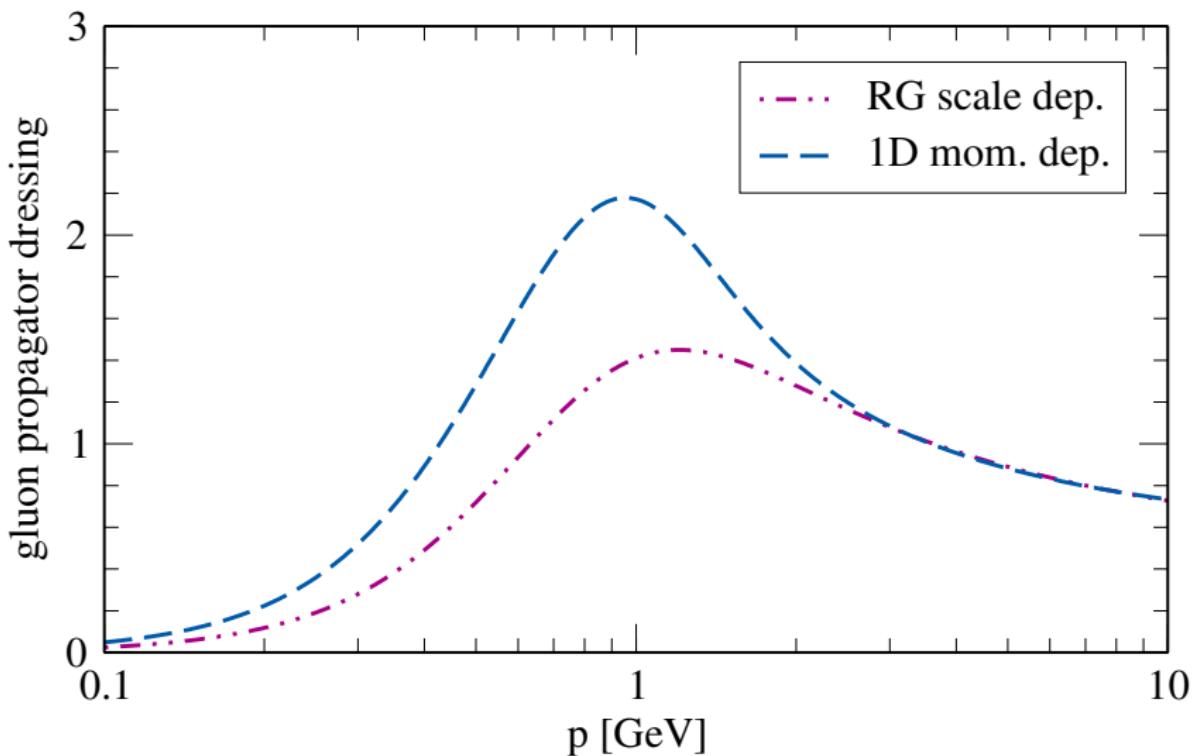
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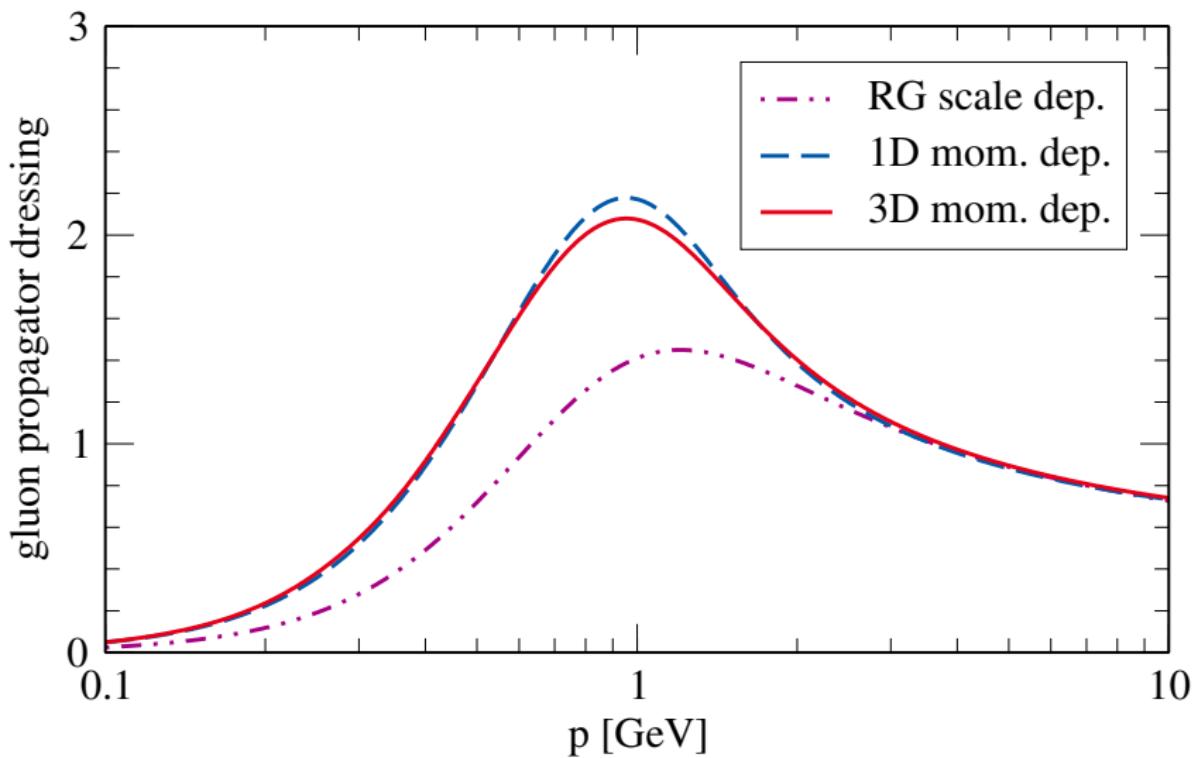
# Truncation dependence of the gluon propagator



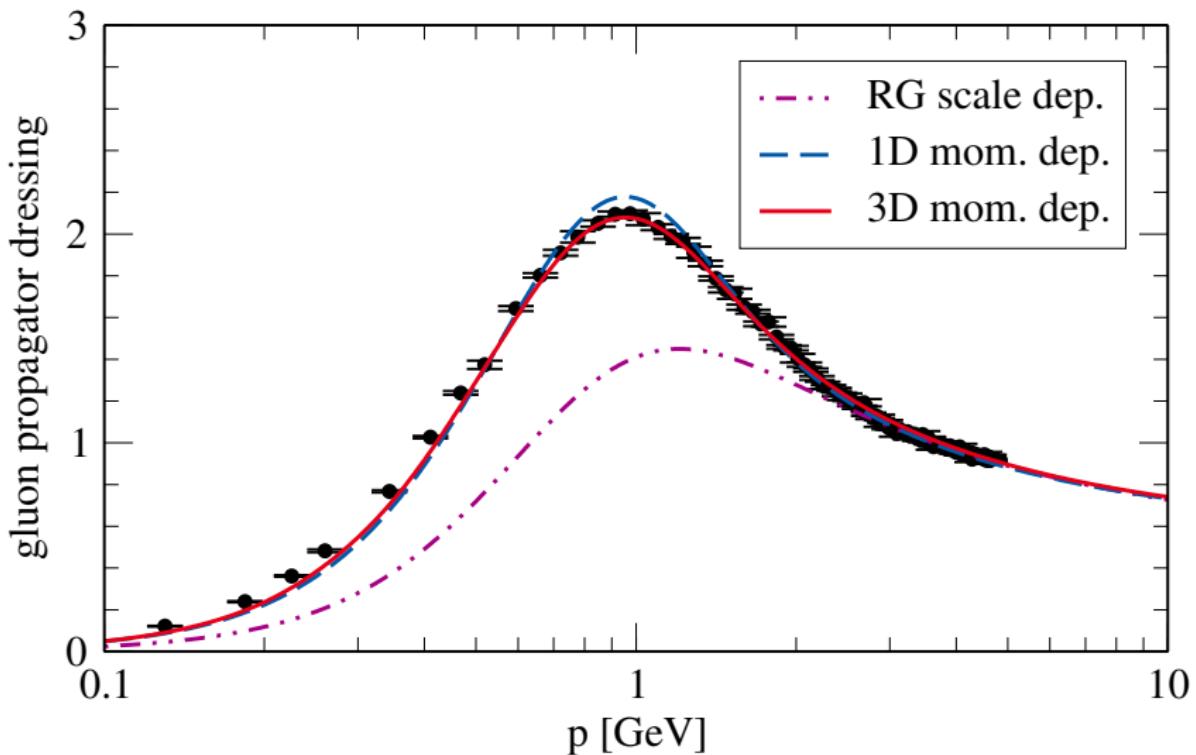
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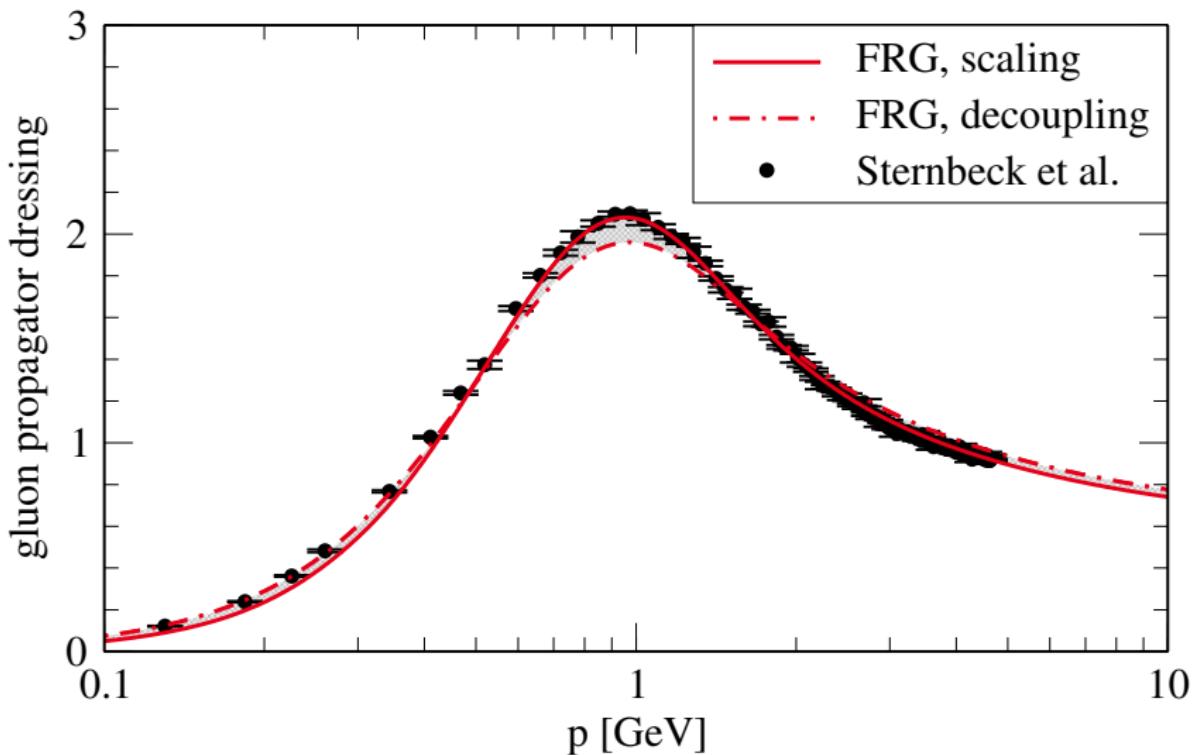


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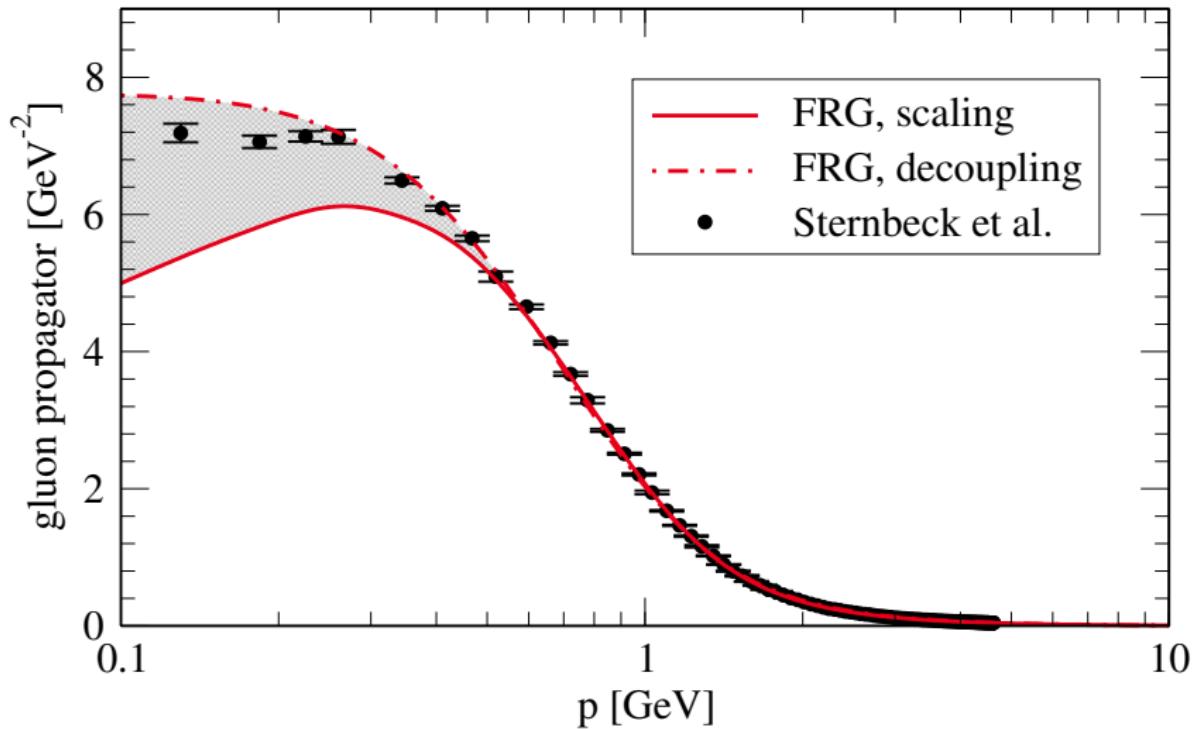
Lattice results: Sternbeck et al. 2006

# Gluon propagator dressing



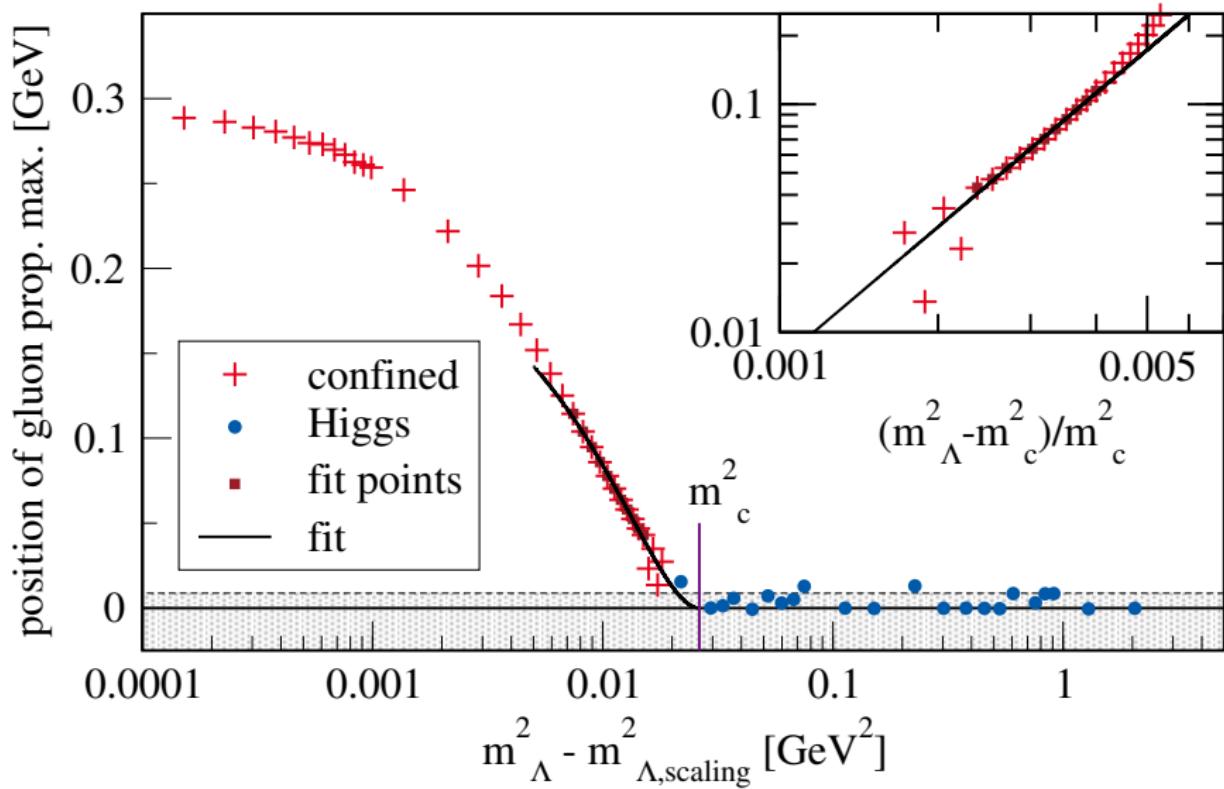
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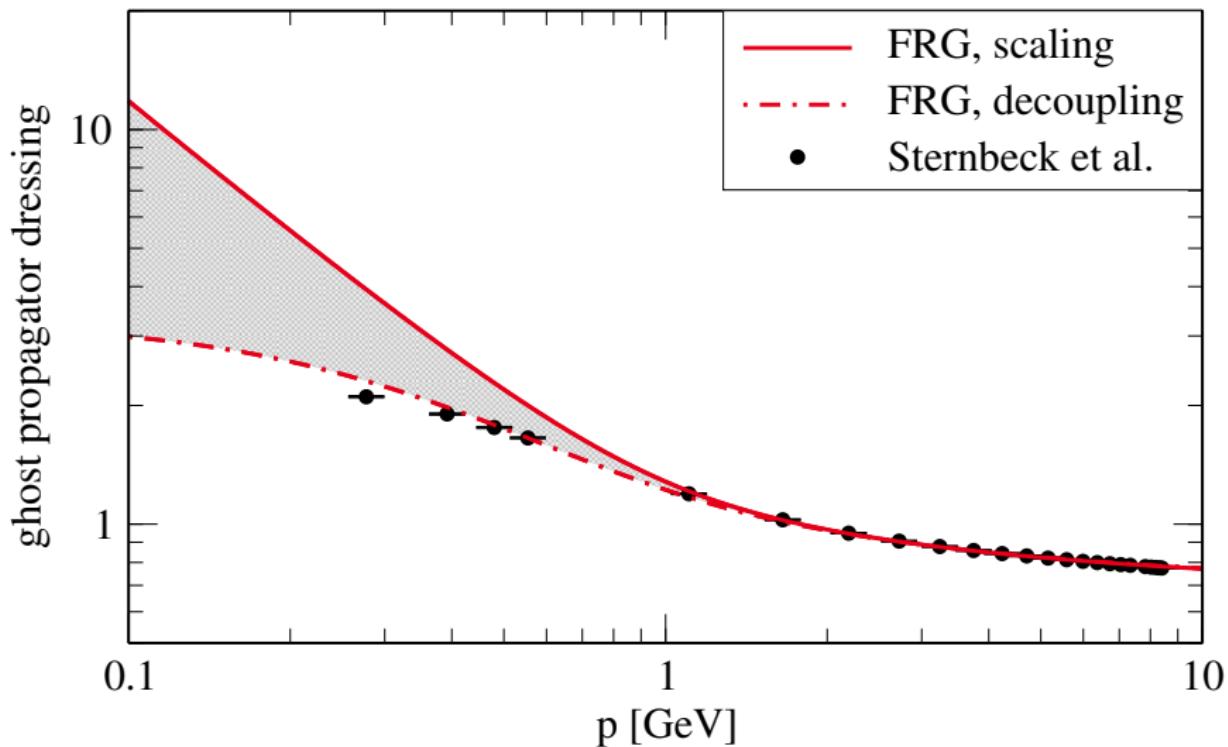


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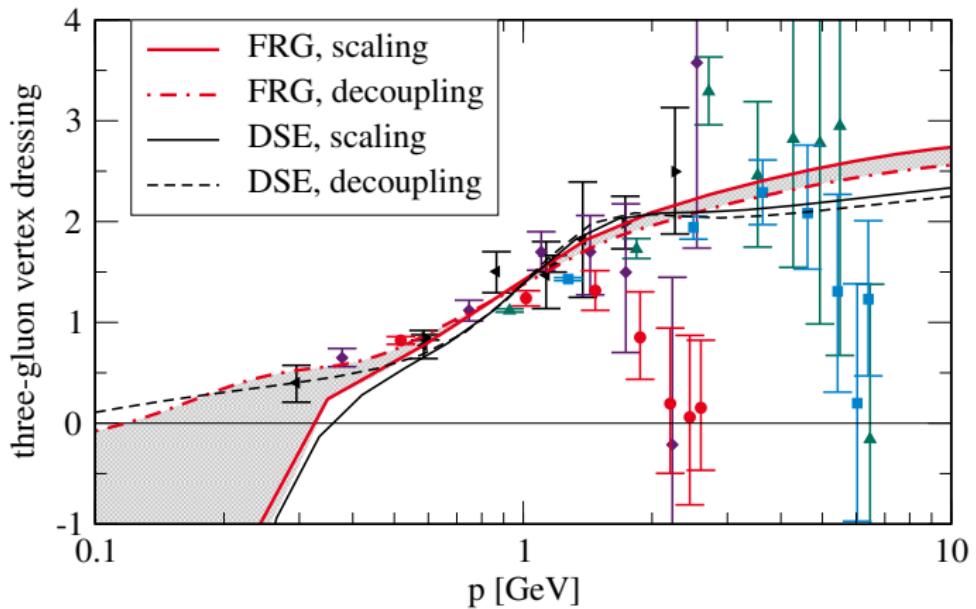
# Gluon propagator maximum over UV mass parameter



# Ghost propagator dressing



# Three-gluon vertex dressing (symmetric point)



Lattice: Cucchieri, Maas, and T. Mendes, 2006-8

DSE: Blum, Huber, Mitter, Smekal, 2014

- FRG: zero crossing between 0.1 GeV to 0.33 GeV

# Finite Temperature

## Going to finite temperature:

- Introduce Matsubara frequencies:

$$\int \frac{d^4 p}{(2\pi)^4} \rightarrow T \sum_{\omega_n} \int \frac{d^3 p}{(2\pi)^3}$$

- Thermal Debye mass
- Same parameter-free truncation as in vacuum YM
- Upcoming: full splitting of magnetic and electric components

Splitting of propagators only: Fister, Pawłowski, 2011

$$P_{\mu\nu}^T(p) = (1 - \delta_{0\mu})(1 - \delta_{0\nu}) \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \quad P_{\mu\nu}^L(p) = \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) - P_{\mu\nu}^T(p)$$

- Also upcoming: nonzero Matsubara modes

Following results are preliminary and based on

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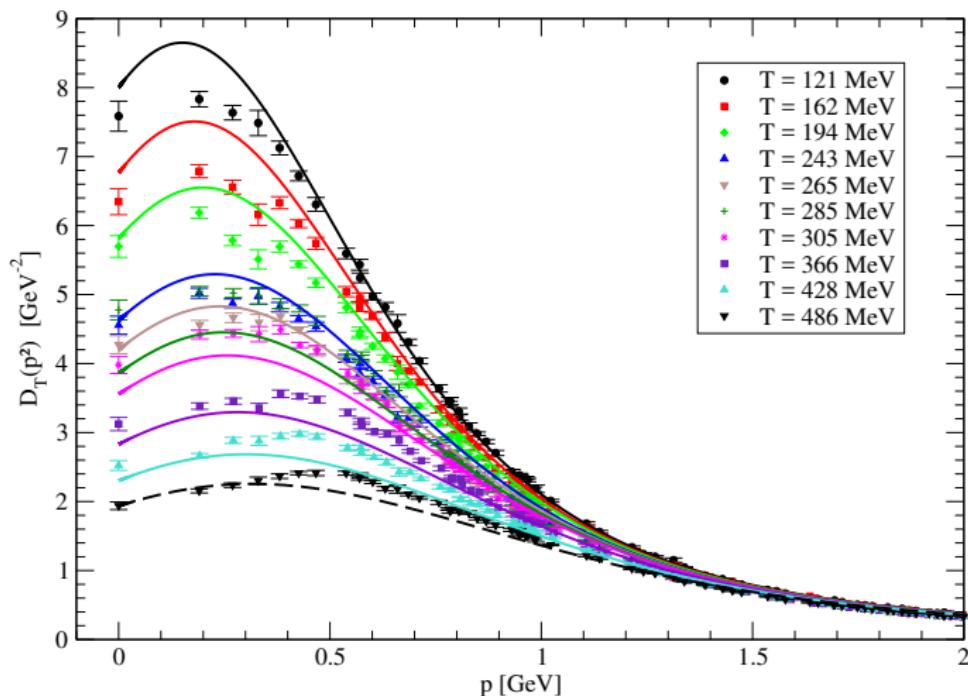
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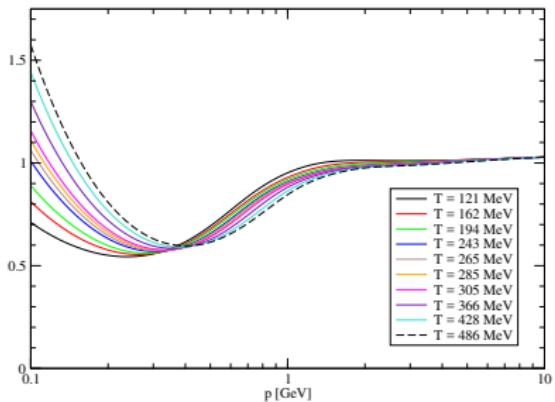
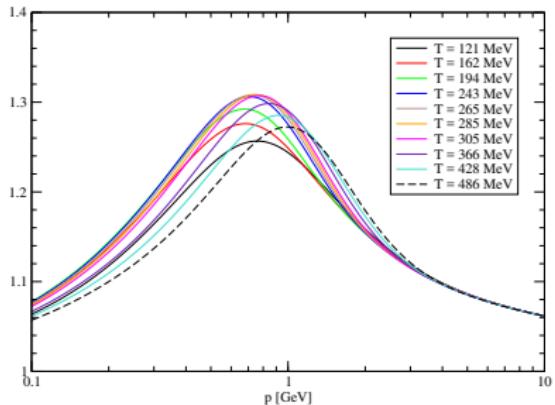
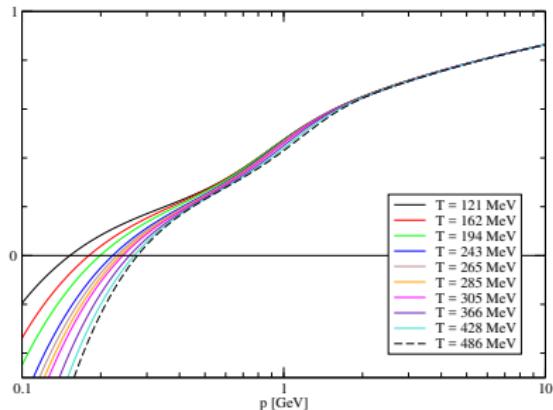
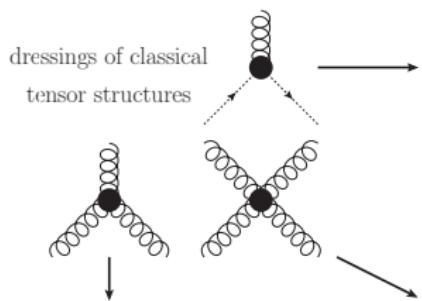
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# Temperature Dependence of the Gluon Propagator

Magnetic component from Silva et. al. Phys.Rev. D89 (2014)  
compared to averaged components from FRG (zeroth mode):



# Temperature Dependence of Vertices



## Conclusion

- FRG first principal approach to QCD, complementary to lattice QCD
- BRST symmetry is broken by regulator, proper care needs to be taken
- STI consistent solution computed
- Evidence for dynamical mass generation
- Very good agreement with lattice results

## Outlook

- Unquenched  $N_f = 2$  QCD, in preparation
- $T > 0$  YM with splitting of el. and mag. components, in preparation
- Bound states (Bethe-Salpeter eq.), decay widths, ...
- Nonzero Matsubara modes, gluon spectral function, ...

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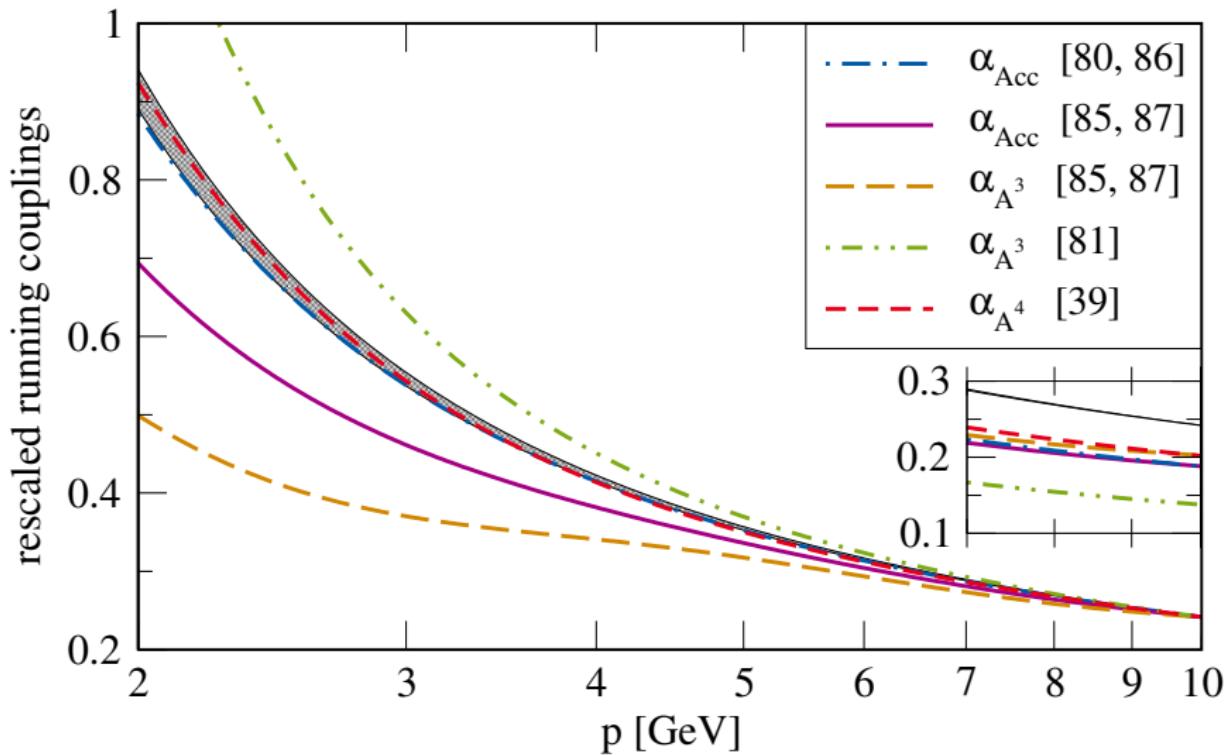
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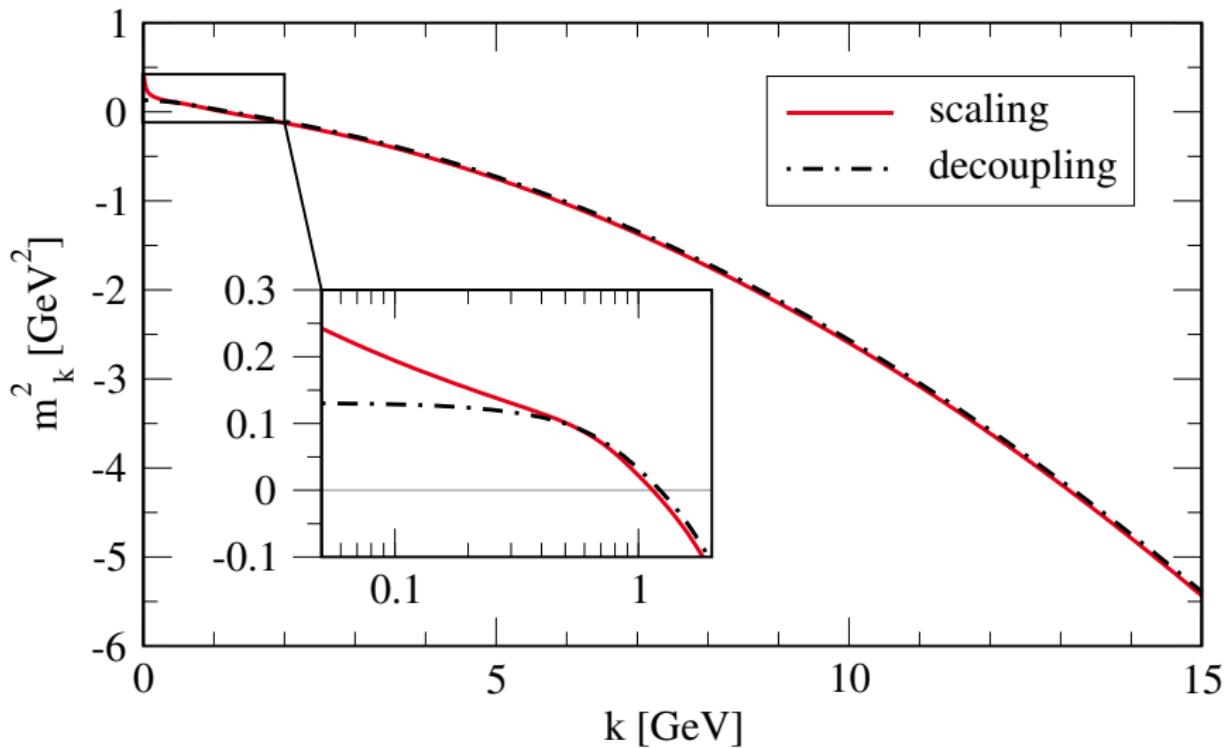
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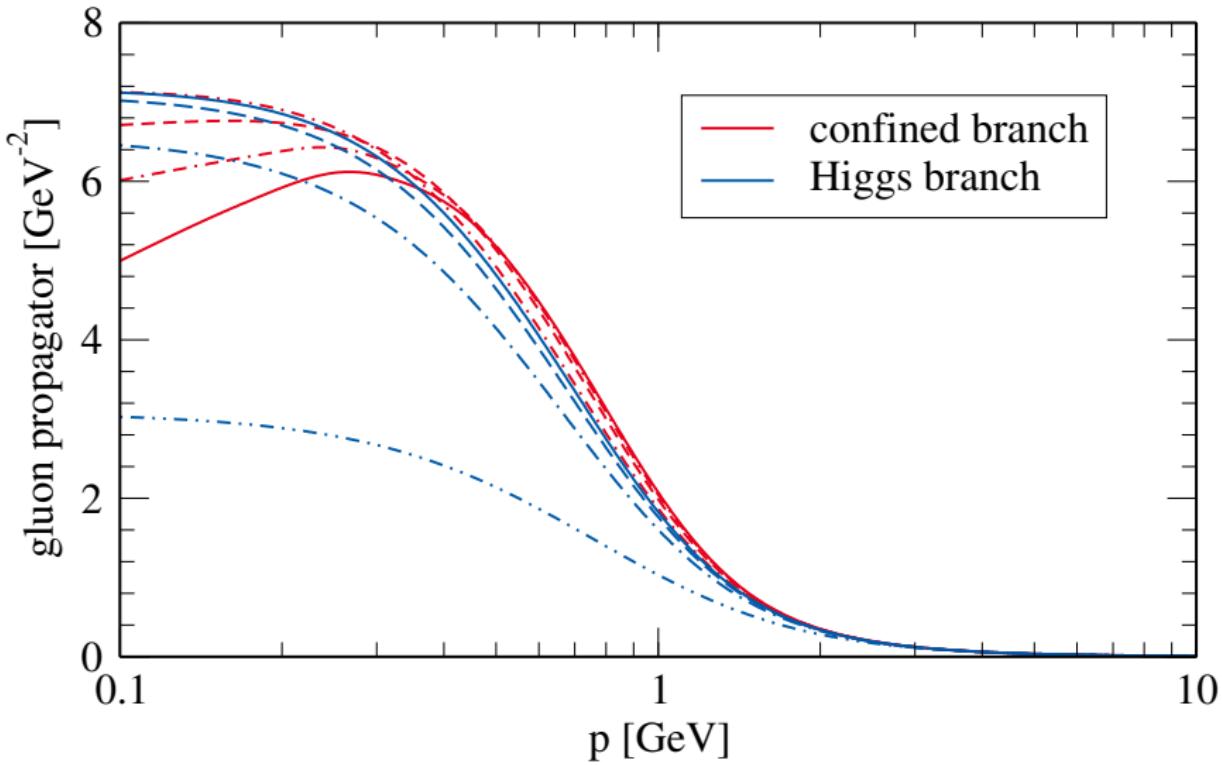
# Running couplings in comparison with DSE results



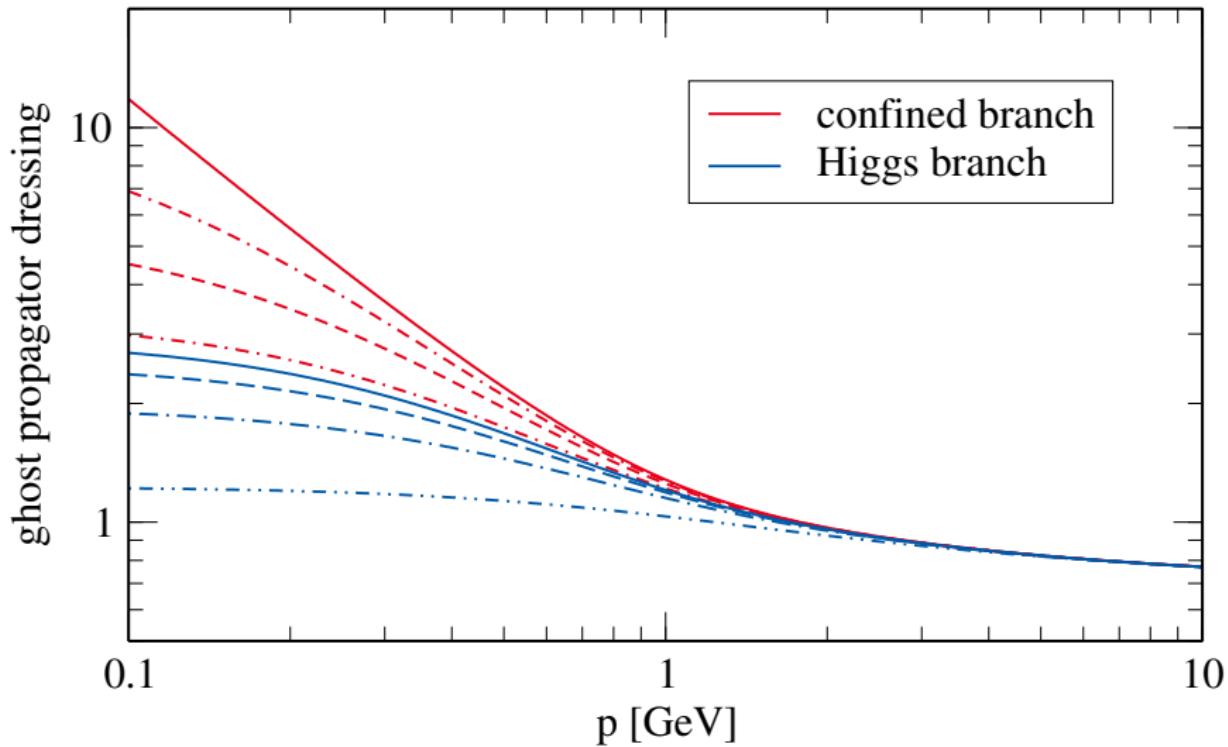
# Running of the gluon mass parameter



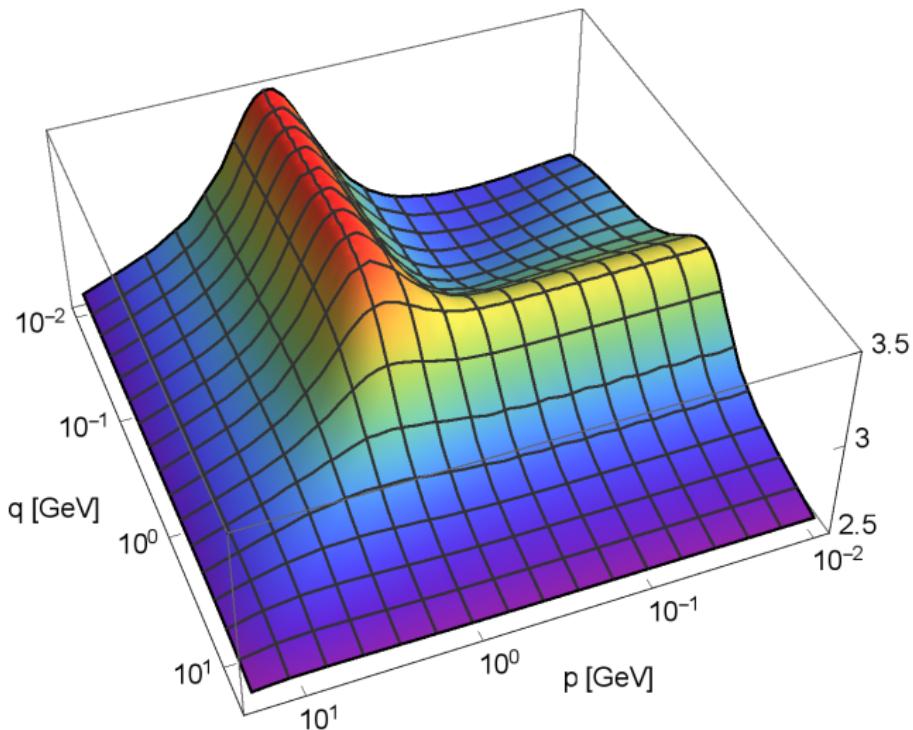
# Gluon propagator



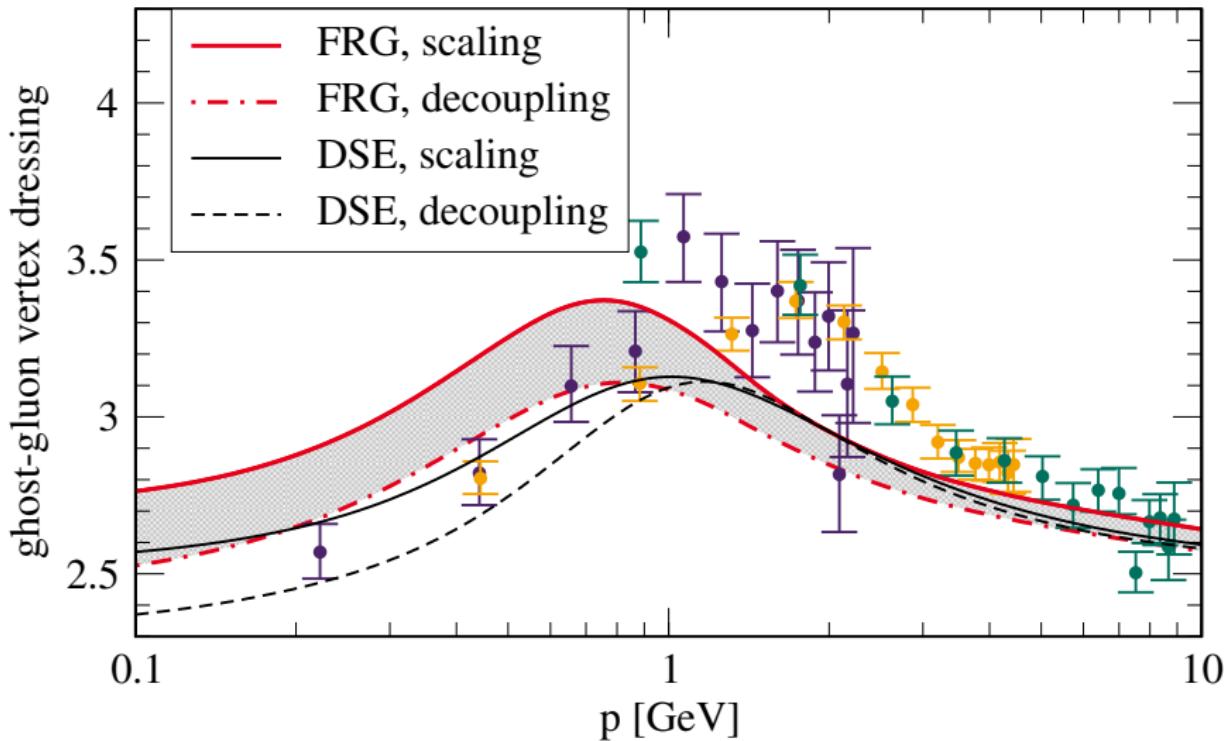
# Ghost propagator dressing



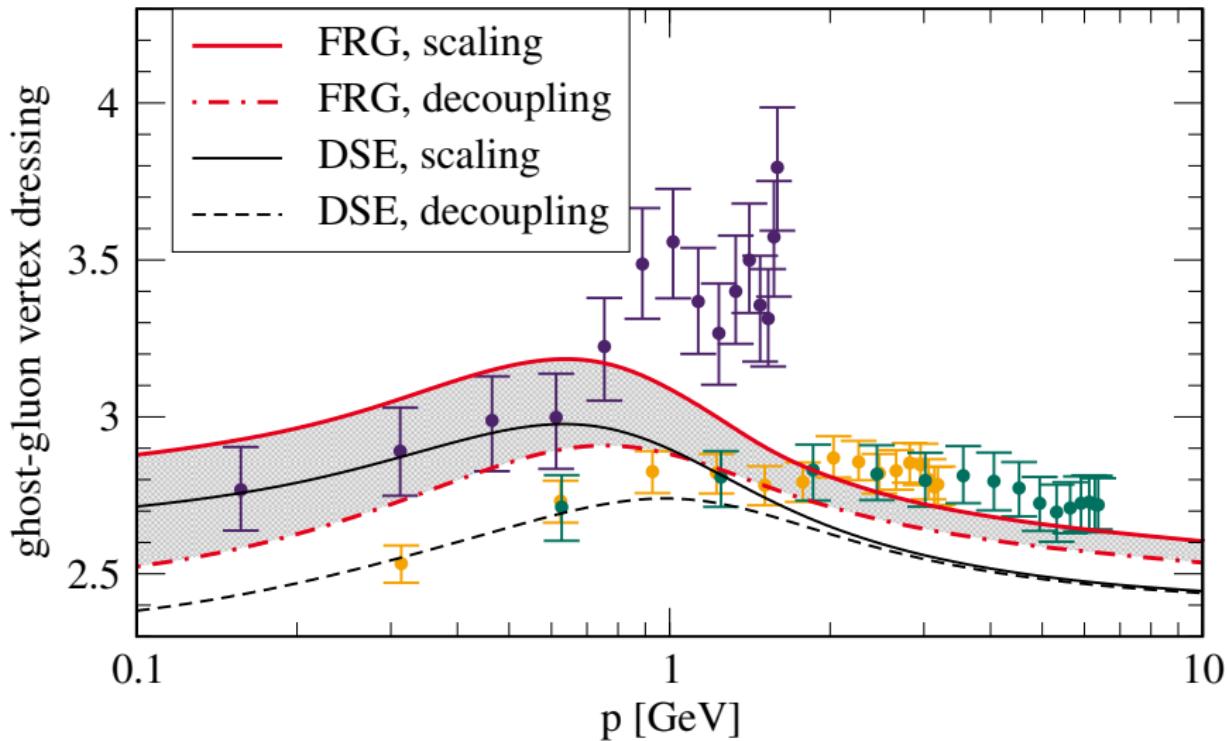
# Momentum dependence of the ghost-gluon vertex



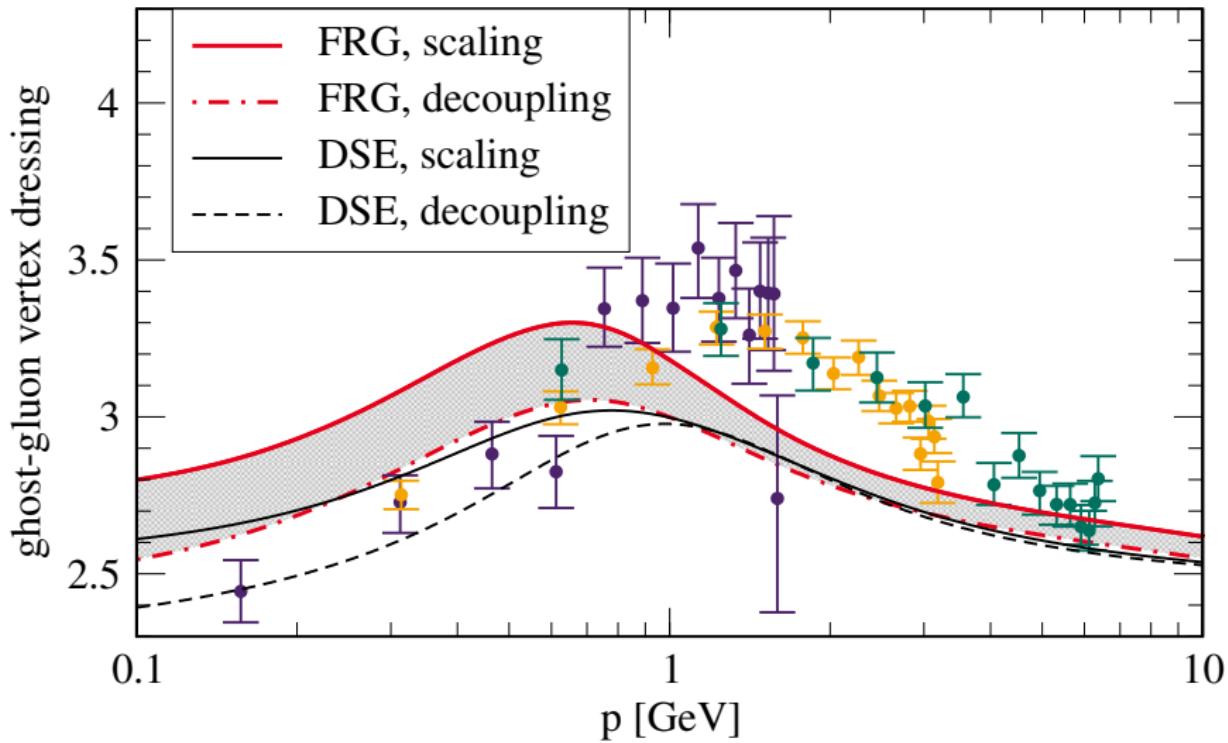
# Ghost-gluon vertex at the symmetric point



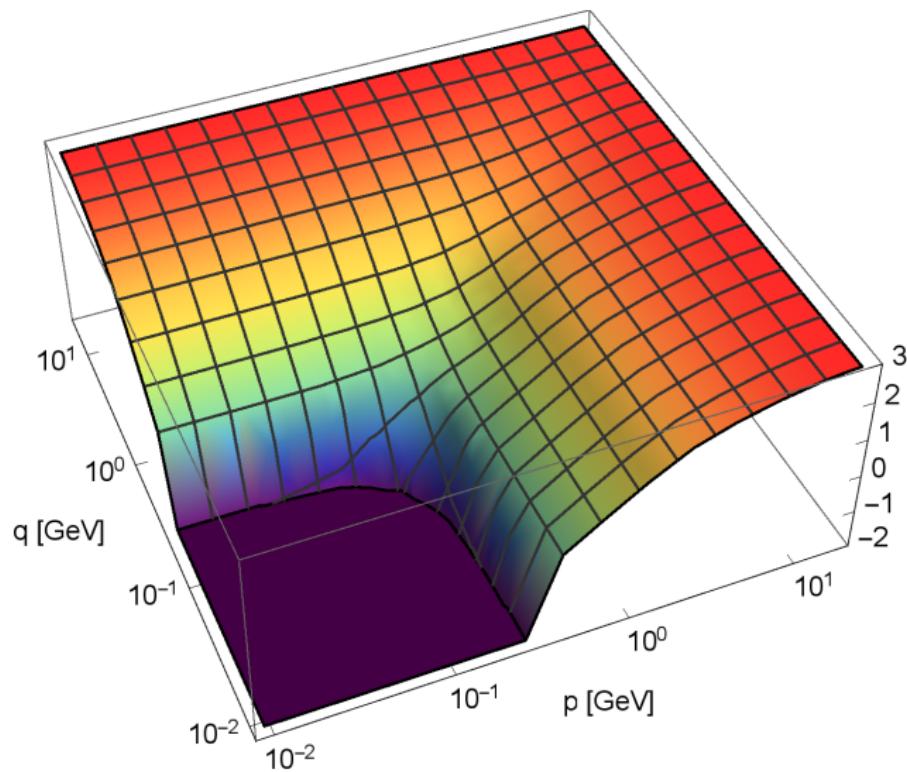
# Ghost-gluon vertex with vanishing gluon momentum



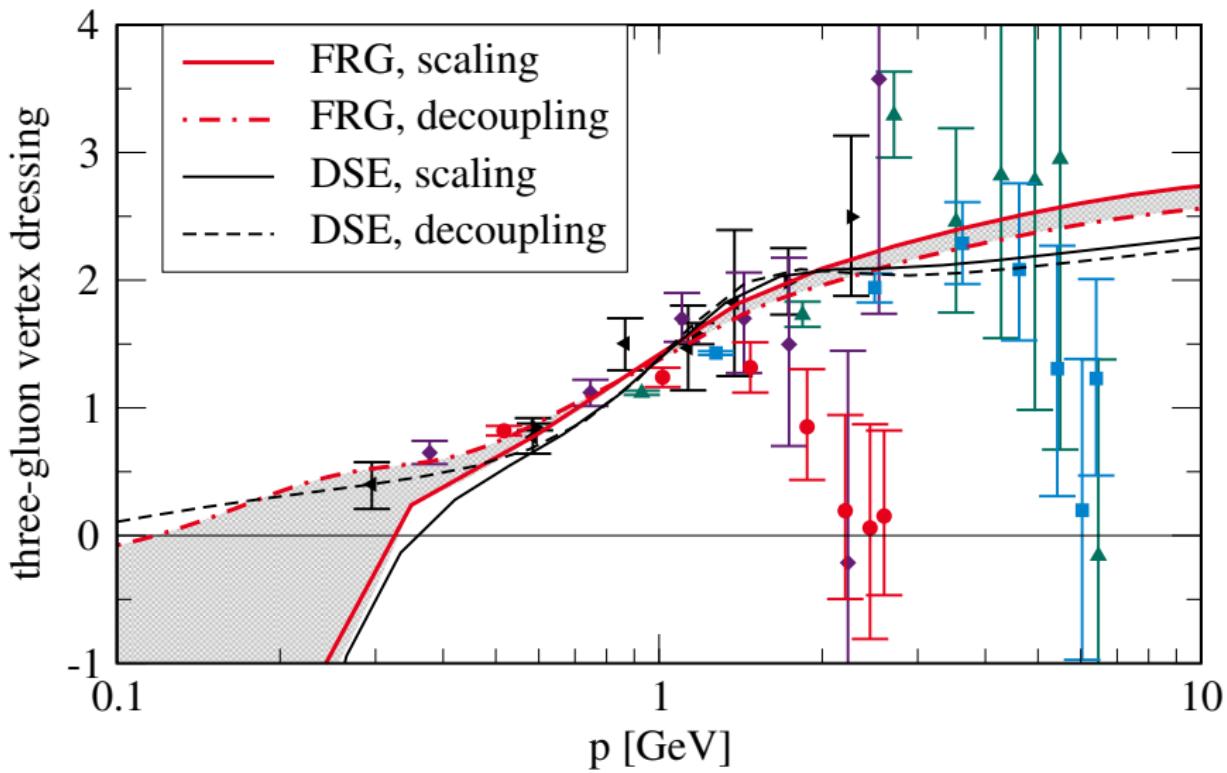
# Ghost-gluon vertex with orthogonal momenta



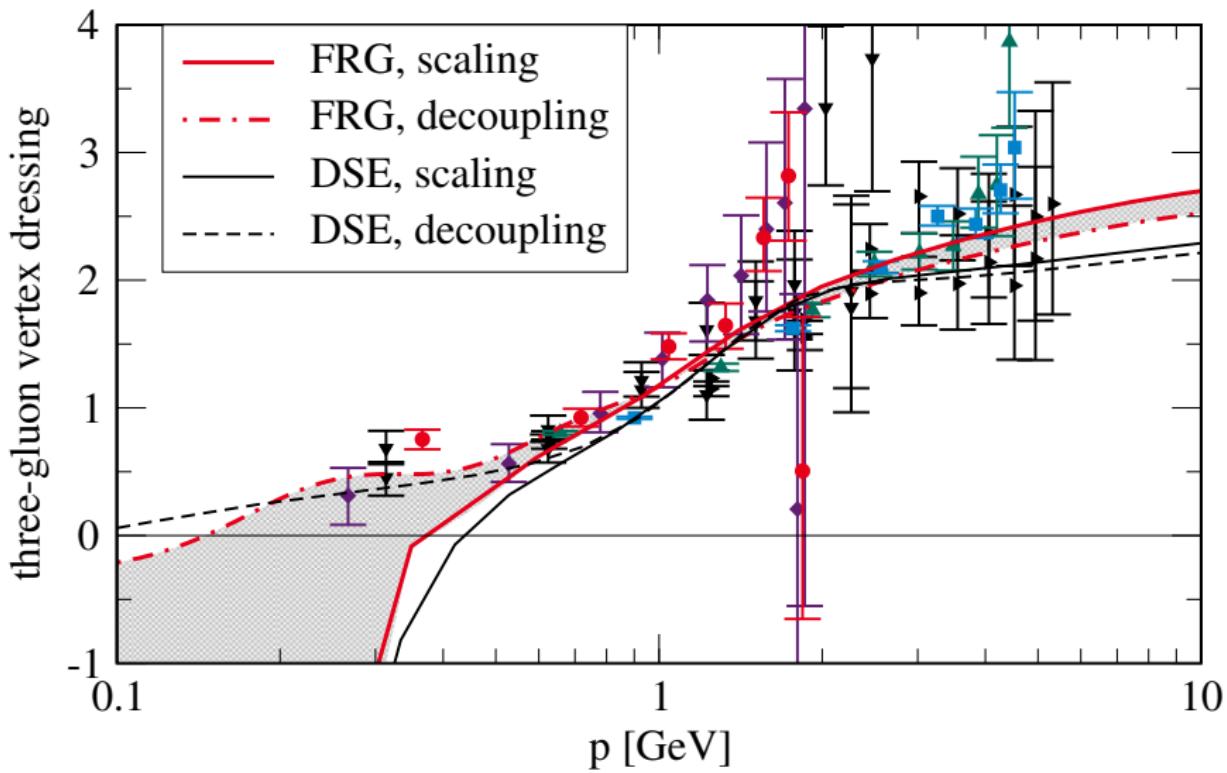
# Momentum dependence of the three-gluon vertex



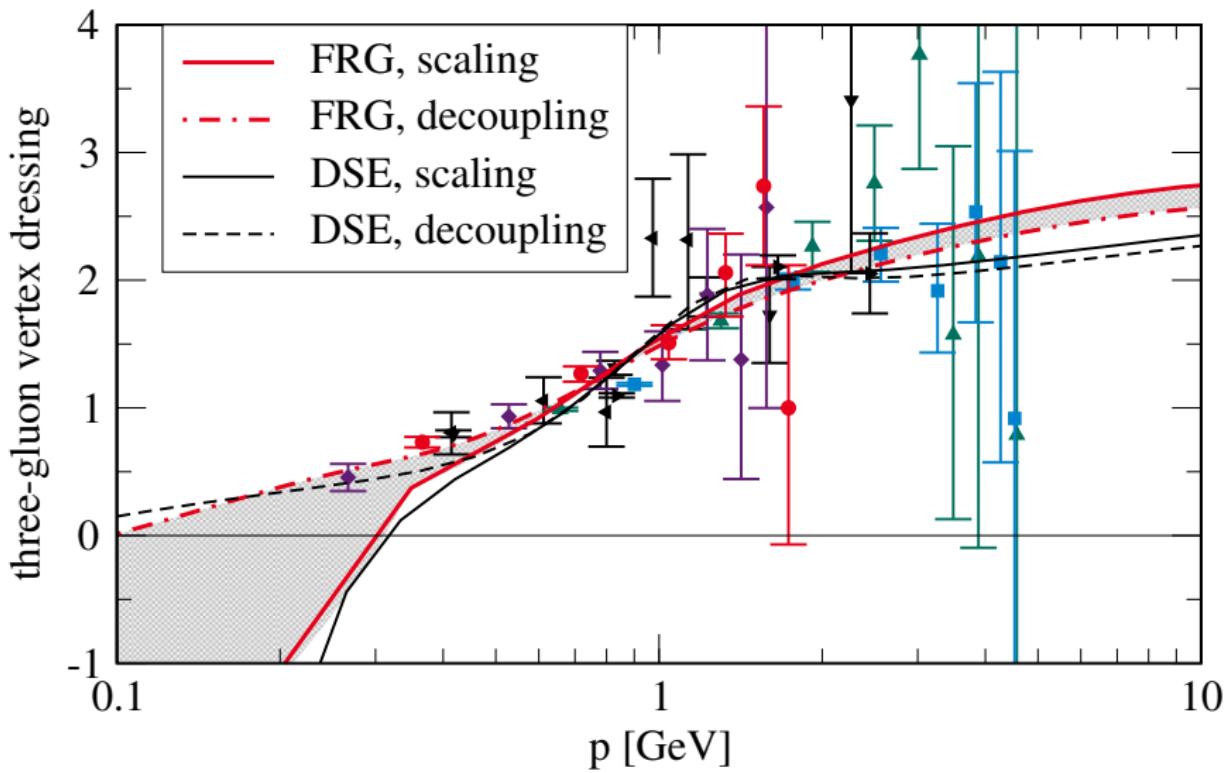
# Three-gluon vertex at the symmetric point



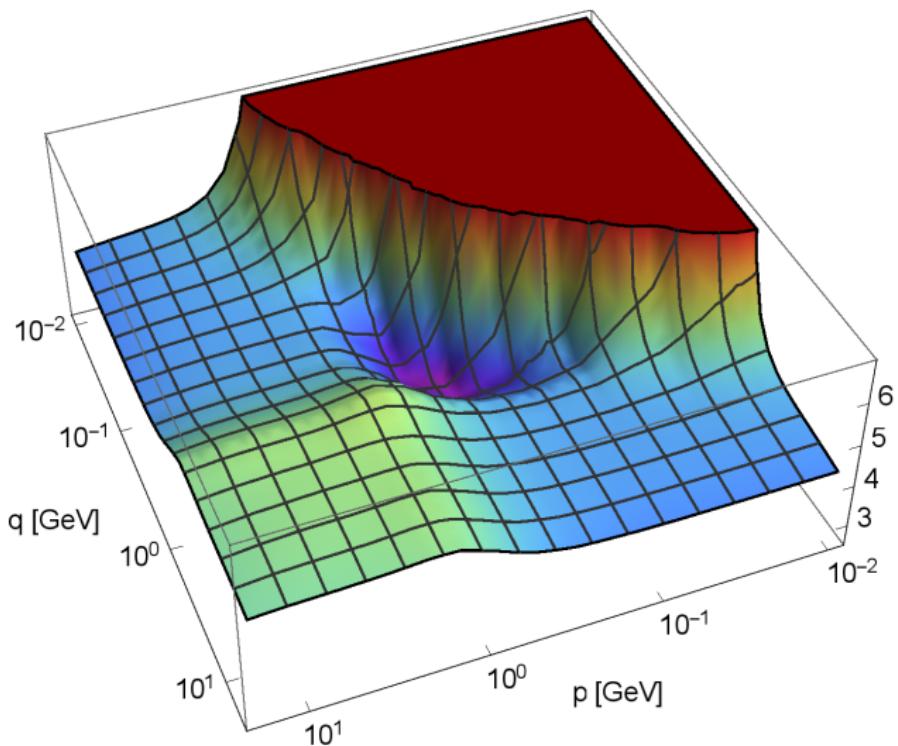
# Three-gluon vertex with vanishing gluon momentum



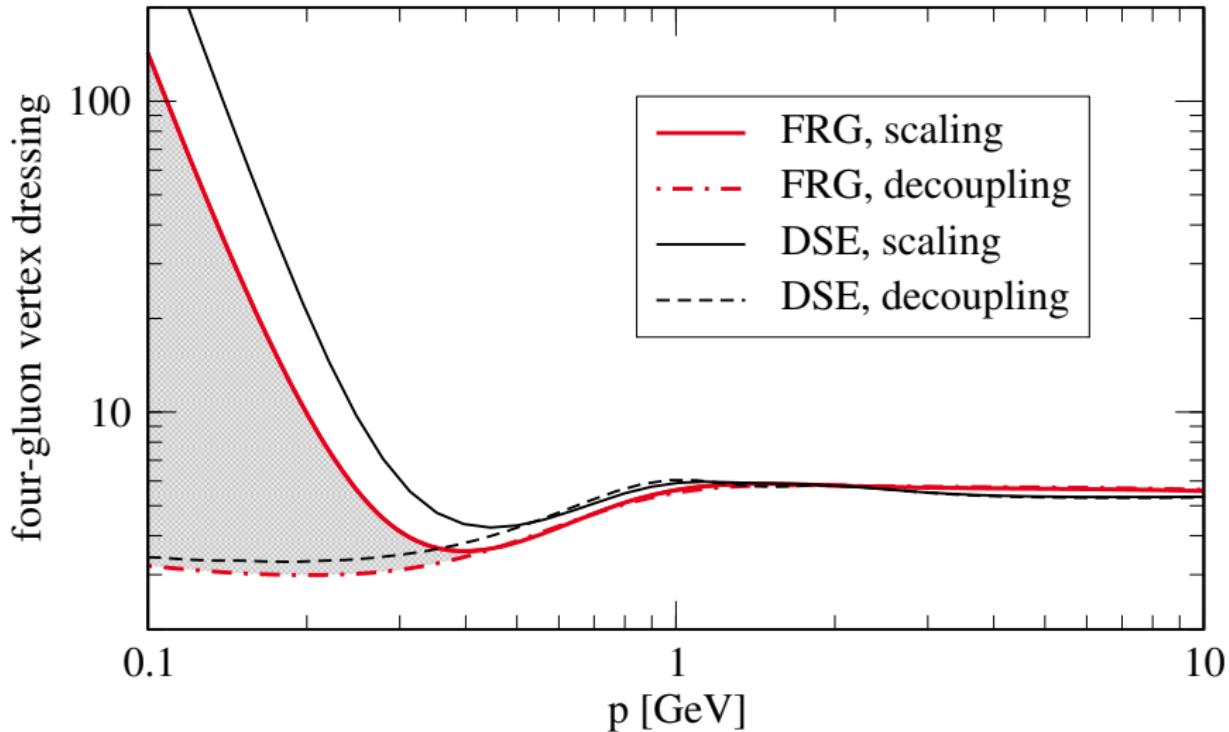
# Three-gluon vertex with orthogonal momenta



# Momentum dependence of the four-gluon vertex



# Four-gluon vertex at the symmetric point



# Regulator dressing

