XIIth Quark Confinement and the Hadron Spectrum (CONF 12)

Thessaloniki 02/09/2016

Confinement, nonAbelian monopoles, and 2D CP(N-1) model on finite space interval

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Plan of the talk

- Confinement as dual superconductor of nonAbelian variety

review

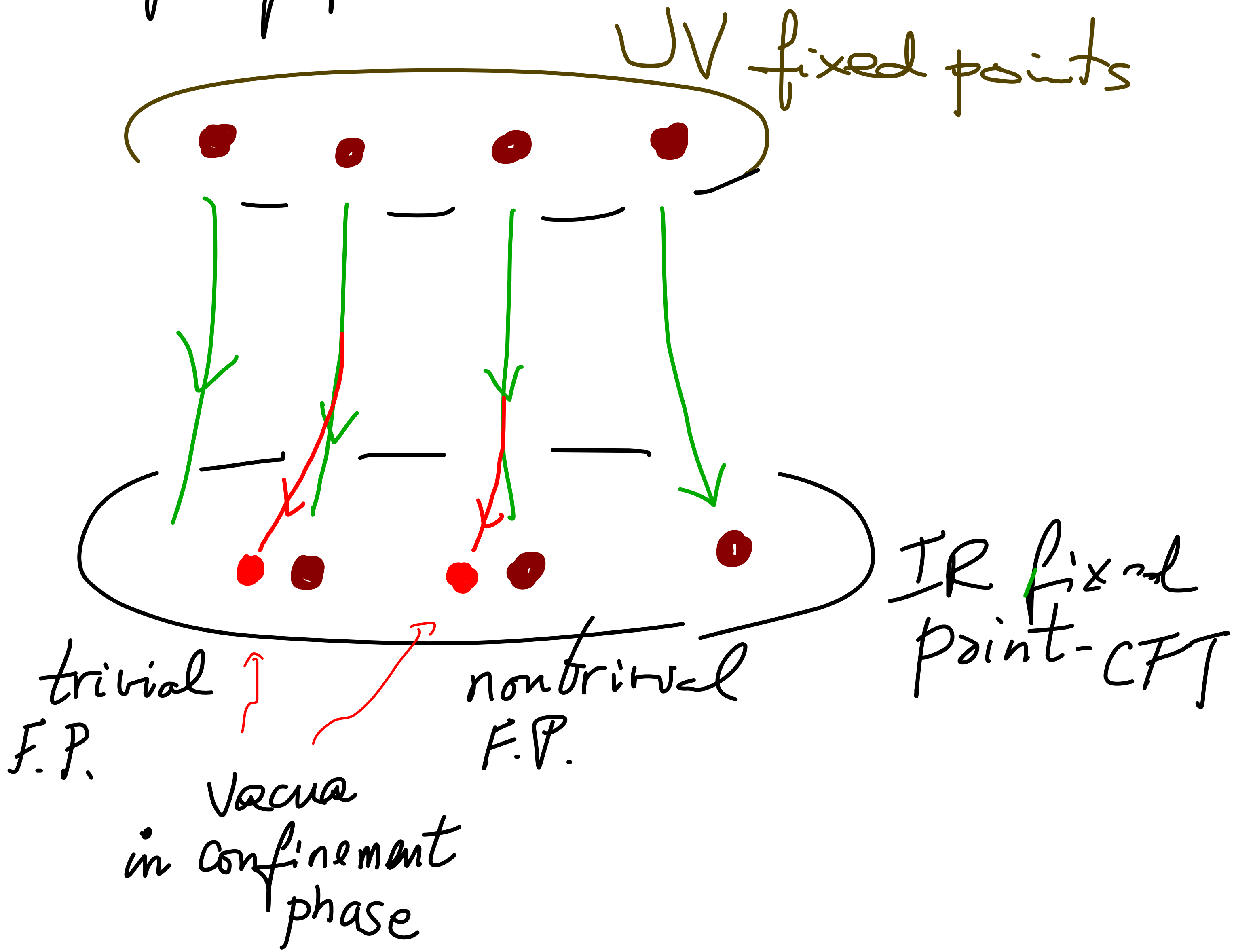
- Status of nonAbelian monopoles. Hints from N=2 Susy gauge theories and how to understand them semiclassically

review and recent results

- Quantum CP^{N-1} model on finite-width worldstrip

new

Renormalization-group flow (RG)



Confinement is a "failed" (or deformed) infrared-fixed point CFT

- Deformation by some relevant operator
- Confinement and symmetry breaking explained by the deg. of freedom describing CFT
- They are often topological solitons (kinks, monopoles - EM duality), rather than the particles in UV
- Type and details of the confinement phase ⟵
  the IR deg. of freedom and their interactions
- NonAbelian monopoles - strongly coupled CFT - could describe confinement (nonAbelian dual superconductor)

QCD?

- Abelian dual superconductor (dynamical Abelianization) ?

$$SU(3) \to U(1)^2 \to \mathbf{1}$$

‘t Hooft, Nambu, Mandelstam ’80

$$\langle M \rangle \neq 0$$

☞ Doubling of the meson spectrum (*)

$$\Pi_1(U(1)^2) = \mathbf{Z} \times \mathbf{Z}$$

☞ If confinement and XSB both induced by $\langle M_b^a \rangle = \delta_b^a \, \Lambda$

$$SU_L(N_F) \times SU_R(N_F) \to SU_V(N_F)$$

Accidental SU(N_F^2) : too many NG bosons (**)

- **Non-Abelian monopole condensation ?**

$$SU(3) \to SU(2) \times U(1) \to \mathbf{1}$$

☞ Problems (*) avoided $\Pi_1(SU(2) \times U(1)) = \mathbf{Z}$ but

Non-Abelian monopoles expected to be strongly coupled (no sign flip of b_0)

Hint that (**) also solved by non Abelian monopoles

Concept of nonAbelian monopoles turned out to be peculiarly evasive - Difficulties

- Difficulty for us, not for Nature …

but the fact is …

$\mathcal{N}$=2 SQCD (exact solutions): they are ubiquitous

- Abelian dual superconductivity ✔

  SU(2) with N_F = 0, 1,2,3

  monopole condensation ⇒ confinement & dyn symm. breaking

  SU(N) $\mathcal{N}$ =2 SYM : SU(N) ⇒ $U(1)^{N-1}$

Seiberg-Witten,

… … …

Beautiful, but don't look like QCD

- Non-Abelian monopole condensation ✔

  SU(N), N_F quarks

  SU(N) ⇒ SU(r) x U(1) x U(1) x .... $r \leq N_F/2$

  r vacua are local, IR free theories

  Solve apparently the problem of too-many-NG bosons!

Argyres,Plesser,Seiberg,'96
Hanany-Oz, '96

Carlino-Konishi-Murayama '00

Shifman-Yung '10-'13

Beautiful, but don't look like QCD

- **Non-Abelian monopoles interacting strongly** !

  SCFT of highest criticality, EHIY (Eguchi-Hori-Ito-Yang) vacua

Beautiful and interesting

Argyres,Plesser,Seiberg,Witten,
Eguchi-Hori-Ito-Yang, '96

Argyres-Seiberg, Gaiotto-Seiberg-Tahcikawa '07, '11
Giacomelli-Konishi '12, '13,
Bolognesi, Giacomelli, Konishi, '15

Effective degrees of freedom in the quantum r vacuum of softly broken N=2 SQCD

($r \leq N_f$ / 2)

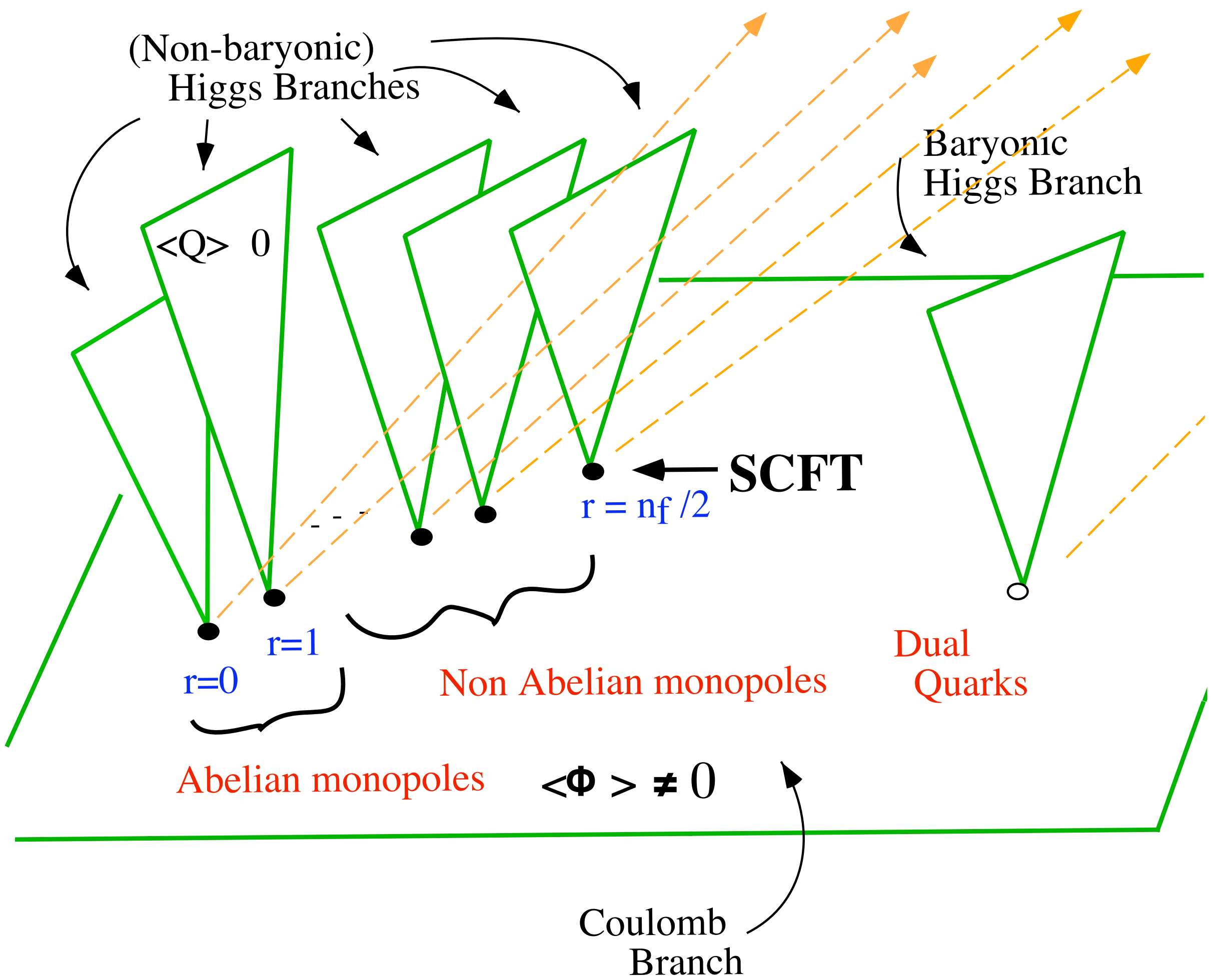
Seiberg-Witten '94
Argyres,Plesser,Seiberg,'96
Hanany-Oz, '96
Carlino-Konishi-Murayama '00

	$SU(r)$	$U(1)_0$	$U(1)_1$	$\ldots$	$U(1)_{N-r-1}$	$U(1)_B$
$N_F \times \mathcal{M}$	$\underline{\mathbf{r}}$	1	0	$\ldots$	0	0
M_1	$\underline{\mathbf{1}}$	0	1	$\ldots$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
M_{N-r-1}	$\underline{\mathbf{1}}$	0	0	$\ldots$	1	0

The massless non-Abelian and Abelian monopoles and their charges at the r vacua

- "Colored dyons" do exist !!!
- they carry flavor q.n.
- $\langle q^i{}_\alpha \rangle = v\ \delta^i{}_\alpha \Rightarrow \quad U(N_f) \Rightarrow U(r) \times U(N_f - r)$

QMS of N=2 SQCD (SU(n) with n_f quarks)



● N=1 Confining vacua (with Φ^2 perturbation)

○ N=1 vacua (with Φ^2 perturbation) in free magnetic pha

$m = m^{cr}$

next
slide

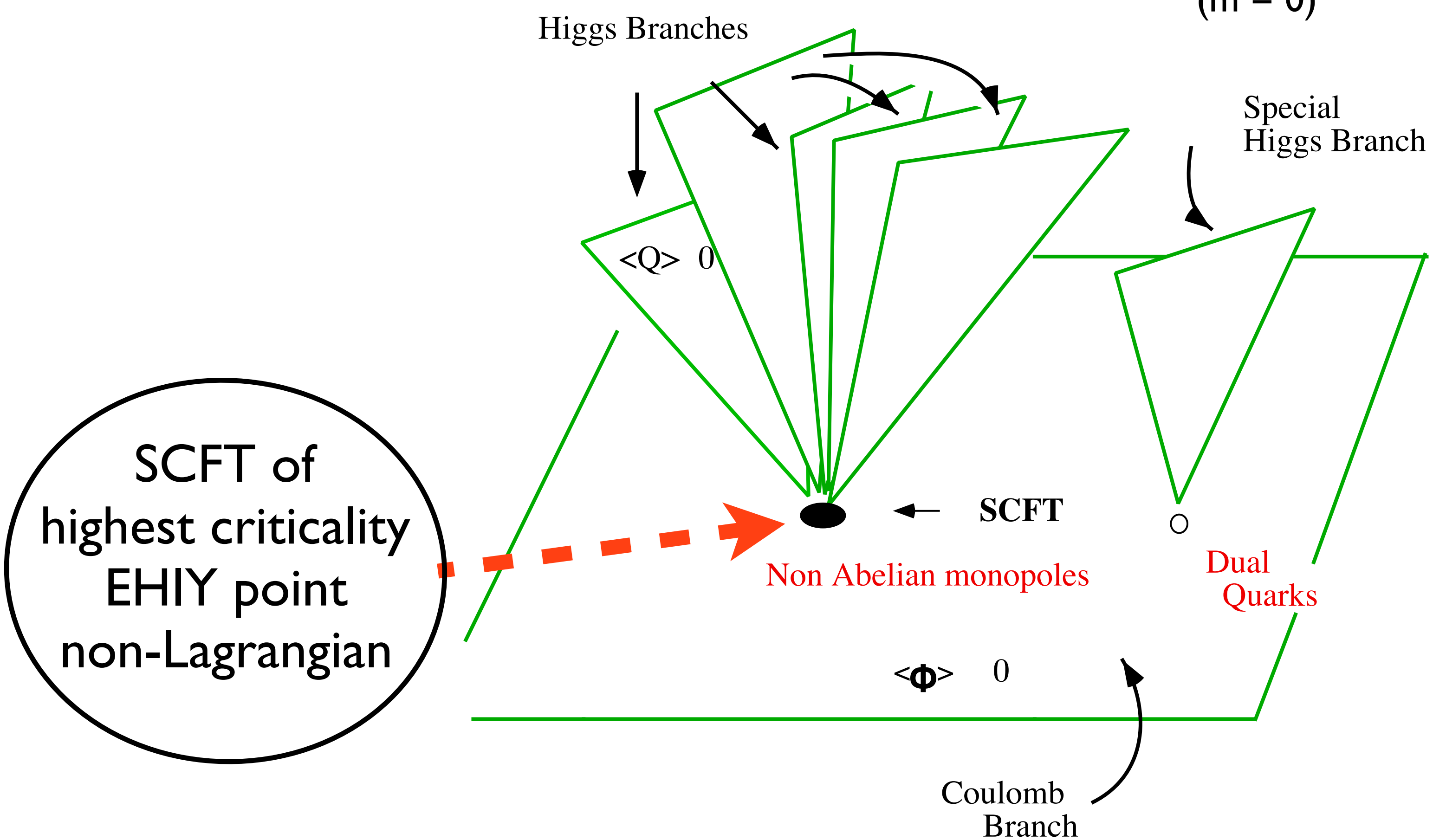
Di Pietro, Giacomelli '11

previous slide (Universality)

$m \neq 0$

QMS of N=2 USp(2n) Theory with n_f Quarks

(m = 0)



Carlino-Konishi-Murayama '00

● N=1 Confining vacua (with Φ^2 perturbation)

○ N=1 vacua (with Φ^2 perturbation) in free magnetic pha

Recent developments

N=2 SCFT's

Witten,Gaiotto, Seiberg, Argyres, Tachikawa, Moore, Maruyoshi, ... '09 - '12

- S-duality in SCFT at $g = \infty$ e.g. SU(N) w/ $N_F = 2N$

  Argyres-Seiberg '07

- Argyres-Seiberg S-duality applied to SCFT (IR f.p.) of highest criticality (EHIY)

  Gaiotto-Seiberg-Tachikawa '11

- GST duality generalized to USp(2N), SO(N)

  Giacomelli '12

- Colliding r-vacua and EHIY in SU(N)

  Giacomelli, Di Pietro '11

- GST duality and confinement

  Giacomelli-Konishi '12, '13

- N=1 deformation of AD vacua and confinement

  Bolognesi, Giacomelli, Konishi, '15

Two interesting hints:

- N=2 USp(2N) with NF =4 with GST duality broken to N=1

$$\langle M_0 \rangle \neq 0 \rightarrow \langle M_i \rangle \neq 0, \qquad (i = 1, 2, \ldots 4)$$

confinement XSB

"New Confinement Phases from Singular SCFT"
Giacomelli-Konishi '12, '13

- N=2 SU(N) with N_F at singular AD vacua, deformed to N=1

Bolognesi, Giacomelli, Konishi, '15

Strongly-coupled nontrivial SCFT

$\xrightarrow{\text{N=1}}$ Free SCFT described by massless mesons

$\xrightarrow{\text{Off AD}}$ Massless mesons ~ NG bosons

The key task:

Understand NA monopoles from semiclassical picture

(How the old “difficulties” solved)

Hierarchical (gauge) symmetry breaking

$$G \xrightarrow{\langle\phi\rangle = v_1} H \xrightarrow{\langle q\rangle = v_2} \mathbf{1}, \qquad v_1 \gg v_2$$

- Color-flavor locked H_{C+F} symmetric vacuum

- Low-energy system ($v_1 \approx \infty$) has vortices if $\pi_1(H) \neq 1$

absent in the full theory if $\pi_1(G)=1$

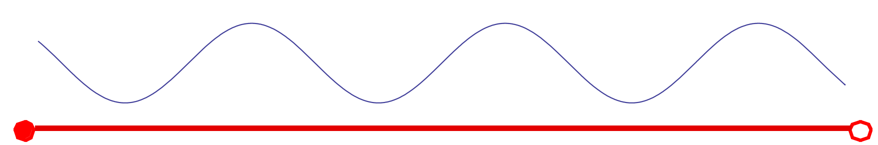
- High-energy system ($v_2 \approx 0$) has regular monopoles if $\pi_2(G/H) \neq 1$

absent in the full theory as $\pi_2(G)=1$

⇨ Vortex ends at monopoles / monopoles confined

- NonAbelian vortex e.g. CP^{N-1}, H=SU(N)

⇨ NonAbelian monopoles

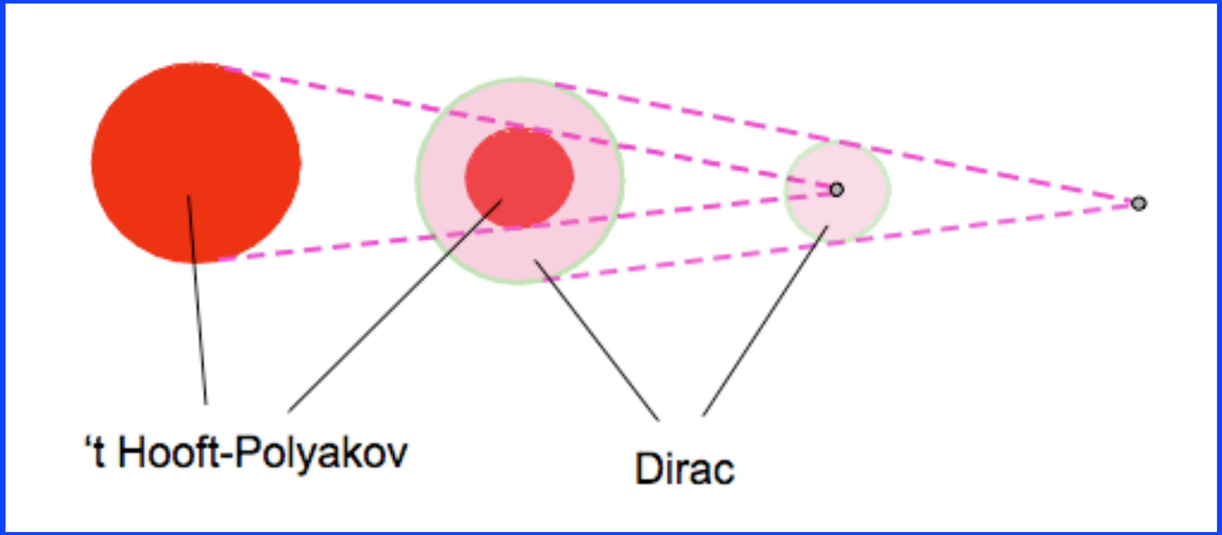


Make these ideas concrete

'03-'13
Hanany, Tong,
Auzzi, Bolognesi, Evslin, Konishi, Yung
Shifman, Yung, Sakai. NItta et. al.

Homotopy group exact sequence

$$\cdots \longrightarrow \pi_2(G) \longrightarrow \pi_2(G/H) \longrightarrow \pi_1(H) \longrightarrow \pi_1(G) \longrightarrow \cdots$$



e.g. G=SU(N), USp(2N): $\pi_1 = 1 \Rightarrow$ No Dirac monopoles (Wu-Yang)

G=SO(N) $\pi_1 = Z_2$, Z_2 monopoles; G=SU(N)/Z_N : Z_N monopoles;

Apply to physics: $G \xrightarrow{v_1} H \xrightarrow{v_2} 1$

't Hooft
SO(3)/U(1)

The model: SU(N+1) (N=2 susy inspired), N_F =N

Chandrasekhar Chatterjee & KK '14

adjoint fundamental

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |\mathcal{D}_\mu \phi|^2 + \sum_{I=1}^{N_F} |\mathcal{D}_\mu q_I|^2 - V(\phi, q)\,,$$

Scalar VEVS

$$\langle \phi \rangle = \langle \phi^A T^A \rangle = \frac{v_1}{N+1} \begin{pmatrix} N & \\ & -\mathbb{1}_N \end{pmatrix}$$

$$\langle q_I^a \rangle = v_2\, \delta_I^a = v_2\, \mathbb{1}_N \qquad\qquad v_1 \gg v_2$$

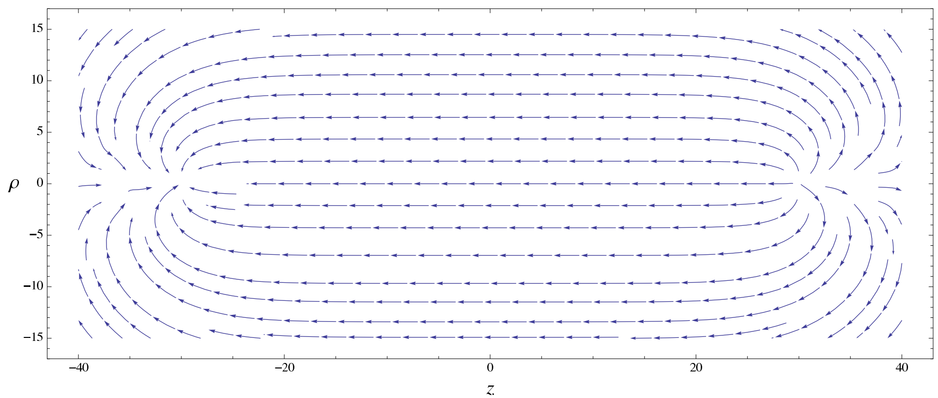
$$SU(N+1)_{\rm color} \otimes SU(N)_{\rm flavor} \xrightarrow{v_1} (SU(N) \times U(1))_{\rm color} \otimes SU(N)_{\rm flavor} \xrightarrow{v_2} SU(N)_{C+F}$$

monopole vortex

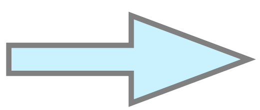
The monopole-vortex soliton complex at large distances

The monopole is pointlike

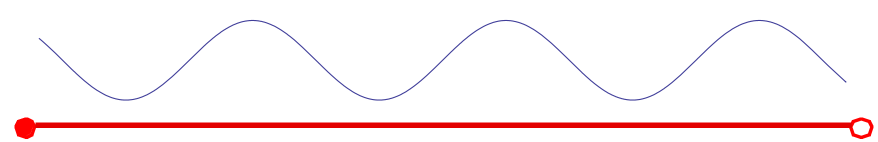
The vortex is a line



Cipriani, Dorigoni, Gudnason, Konishi, Michelini, ‘11



B(z,t)



- The monopole degrees of freedom ~ the winding

$$\phi(x) = v_1\, M(x) \qquad M(x) = \sqrt{\frac{2N}{N+1}}\, U(x)\, T^{(0)}\, U^\dagger(x), \qquad \mathrm{Tr}\,(T^{(0)})^2 = \frac{1}{2}\,.$$

$$SU(N+1)/U(N) \sim CP^N$$

$$T^{(0)} = \frac{1}{\sqrt{2N(N+1)}} \begin{pmatrix} N & \\ & -\mathbb{1}_N \end{pmatrix}$$

- Write down the vortex solutions

$$U^\dagger\, q_I = U^\dagger(z\chi_I + e\eta_I) = \begin{pmatrix} \chi_I \\ \eta_I^1 \\ \vdots \\ \eta_I^N \end{pmatrix}$$

η's = massless components of q

$$(\eta)_I^i = v_2 \begin{pmatrix} e^{i\psi} & 0 \\ 0 & \mathbb{1}_{N-1} \end{pmatrix}$$

- Study the vortex-monopole solutions of generic orientation

$$\eta \to \mathcal{U}\,\eta\,\mathcal{U}^\dagger, \qquad P_\mu \to \mathcal{U}\,P_\mu\,\mathcal{U}^\dagger, \qquad b_\mu \to \mathcal{U}\,b_\mu\,\mathcal{U}^\dagger, \qquad \mathcal{U} \in SU(N)$$

$$\mathcal{U} = \mathcal{U}(z,t) \subset \frac{SU(N)}{SU(N-1)\times U(1)} \sim CP^{N-1}$$

- Study low-energy effective action for $\mathcal{U}(z,t)$

For pure vortex '03-'13
Hanany,Tong,
Auzzi, Bolognesi,Evslin,Konishi,Yung
Shifman,Yung,
Gudnason, Jiang, Konishi '10

The result:

Chandrasekhar Chatterjee & KK '14

The monopole-vortex soliton complex described by a low-energy action

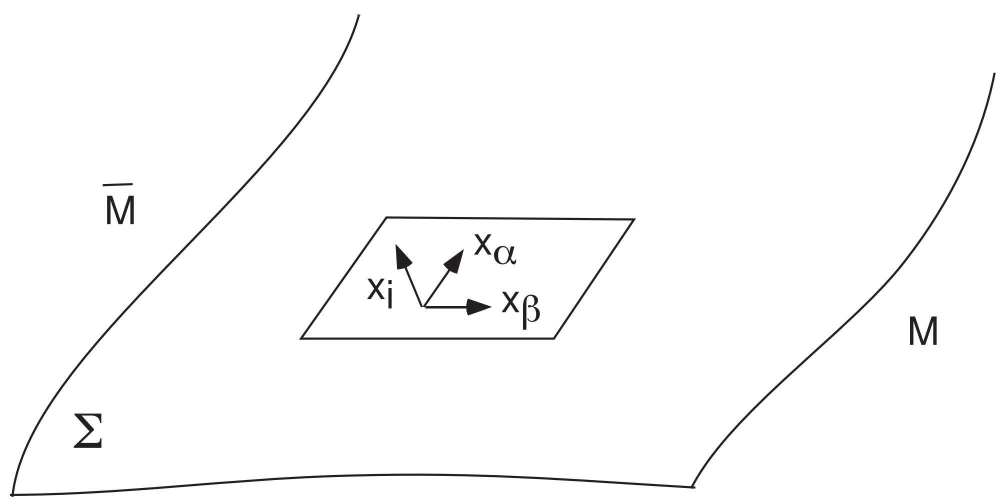
$$S = S^{2D} + S^{1D}_{M} + S^{1D}_{\bar{M}}$$

$$S^{2D} = 2f \int_{\Sigma} d^2x \, \mathcal{D}_\alpha n^{c\,\dagger} \mathcal{D}_\alpha n^c, \qquad \mathcal{D}_\alpha n^c \equiv \{\partial_\alpha - (n^\dagger \partial_\alpha n)\} n^c, \qquad n^\dagger n = 1$$

$$S^{1D} = \gamma \int_{K=M,\bar{M}} dt \, \mathcal{D}_0 n_K^{c\,\dagger} \mathcal{D}_0 n_K^c,$$

$$f = \frac{2\pi}{m^2} \int_0^\infty d\rho \, \rho \, (\partial_\rho \mathcal{B}^{(vor)})^2$$

$$\gamma \sim \frac{2\pi v_1}{g^3 v_2^2} \sim \frac{M}{m^2} \, ,$$



CP^N vs CP^{N-1}

$SU(N)_F \times SU(N+1)_C \quad \supset SU(2)_C$

$\langle \phi \rangle \sim v_1 \quad \downarrow \quad CP^N \quad \downarrow$ monopole

$SU(N)_F \times (SU(N) \times U(1))_C \quad \supset U(1)$

naive "nonAbelian monopole"

$\langle q \rangle \sim v_2 \, \mathbf{1}_N \quad \downarrow$

$SU(N)_{C+F}$

$\downarrow \quad CP^{N-1}$

monopole
-vortex
complex

$SU(N-1) \times U(1)$

SU(N)

Back to p.24

The quantum fluctuations of n

S. Bolognesi, K. Ohashi & KK '16
arXiv:1604.05630

☞ CP^{N-1} sigma model on finite worldstrips

Mini-review: The standard CP^{N-1} sigma model in 2D :

d'Dadda, DiVevcchia, Luscher,'78
Witten '79

$$S = \int dx dt \left((D_\mu n_i)^* D^\mu n_i - \lambda (n_i^* n_i - r) \right) \ , \qquad (i = 1, 2, \ldots, N)$$

$$D_\mu = \partial_\mu - i A_\mu .$$

$$\int \mathcal{D}\lambda(x) \quad \Rightarrow \quad n_i^* n_i = r \ , \qquad r = \frac{4\pi}{g^2} \ .$$

$$Z = \int [dA_\mu][d\lambda][dn_i][dn_i^*] \, e^{iS} = \int [dA_\mu][d\lambda] \, e^{iS_{\rm eff}} \ , \qquad S_{\rm eff} = \int d^2x \left(N \, {\rm tr} \log(-D_\mu^* D^\mu + \lambda) - \lambda r \right)$$

gap equation

- Asymptotically free
- n field mass generated $\quad \lambda = m^2 \qquad r - N \, {\rm tr} \left(\dfrac{1}{-\partial_\tau^2 - \partial_x^2 + m^2} \right) = 0$
- n fields confined by the Coulomb force
- Model of QCD?

$$r = N \int_0^{\Lambda_{\rm UV}} \frac{kdk}{2\pi} \frac{1}{k^2 + m^2} = \frac{N}{4\pi} \log \left(\frac{\Lambda_{\rm UV}^2 + m^2}{m^2} \right)$$

$$r(\mu) = \frac{4\pi}{g(\mu)^2} \simeq \frac{N}{2\pi} \log \left(\frac{\mu}{\Lambda} \right)$$

$$m = \Lambda$$

Finite strings and generalized gap equations

Boundary conditions:

$$\text{D-D}: \qquad n_1(0) = n_1(L) = \sqrt{r}\ , \qquad n_i(0) = n_i(L) = 0\ , \quad i > 1\ .$$

$$\text{N-N}: \qquad \partial_x n_i(0) = \partial_x n_i(L) = 0\ .$$

Separating $n_1 = \sigma$ and integrating over n_i , (i=2,3,.. N) (Large N approximation)

$$S_{\text{eff}} = \int d^2x\, \big((N-1)\,\mathrm{tr}\log(-D_\mu D^\mu + \lambda) + (D_\mu\sigma)^* D^\mu\sigma - \lambda(|\sigma|^2 - r)\big)\ .$$

$$E = N\sum_n \omega_n + \int_0^L \big((\partial_x\sigma)^2 + \lambda(\sigma^2 - r)\big)\,dx\ , \qquad \boxed{\big(-\partial_x^2 + \lambda(x)\big)\, f_n(x) = \omega_n^2\ f_n(x)\ ,}$$

Generalized gap equations

$$\boxed{\begin{aligned} &\partial_x^2\sigma(x) - \lambda(x)\sigma(x) = 0 \qquad (*) \\ &\frac{N}{2}\sum_n \frac{f_n(x)^2}{\omega_n} + \sigma(x)^2 - r = 0 \end{aligned}}$$

- Coupled equations for $\lambda(x),\ \sigma(x)$

$\lambda(x) \to \{\omega_n, f_n(x)\},\ \ n = 1, 2, \ldots, \infty$

$\{\omega_n, f_n(x)\} \to \sigma(x)$, regularizing UV div and renormalizing r (g)

$\sigma(x) \to \lambda(x)$

Strategy: solve the eqs recursively

cfr. Hartree's equation in atomic physics

k= iteration steps

(1) Give some trial potential $\lambda_k(x), \quad (k = 1, 2, ...)$

(2) Solve for $\{\omega_n, f_n(x)\}_k \qquad (n = 1, 2, \ldots n_{max})$

UV cutoff $\mu(n_{\max}) = \frac{n_{\max}\pi}{\mathrm{L}}$

(3) Cancel the UV divergences against r

$$r = \frac{4\pi}{g^2} = \frac{N}{2\pi} \log \frac{2\mu(n_{\max})}{\Lambda}$$

(4) Finite equation for $\sigma(x)$

$$\sigma_k^2 = -\frac{N}{2} \sum_n \frac{1}{\omega_n} \left(f_{n,k}(x)^2 - \frac{1}{L} \right) - \frac{N}{2L} \sum_{n=1}^{n_{\max}} \frac{1}{\omega_{n,k}} + \frac{N}{2\pi} \log \left(\frac{2\mu(n_{\max})}{\Lambda} \right) .$$

(5) Use Eq. (*) to find $\lambda_{k+1}(x)$

(6) Go back to the step (2) … (5) to find $\lambda_{k+2}(x)$, and repeat the procedure, until

(7) A self consistent solution $\{\sigma_k(x), \lambda_k(x)\} \simeq \{\sigma_{k+1}(x), \lambda_{k+1}(x)\}$

is reached

Mathematica Version 10.3

A subtlety: the behavior of $\lambda(x), \sigma(x)$ near x=0, L

D-D or N-N boundary cds ➪ : (by e.g. WKB)

$$\lambda(x) \sim \frac{1}{2x^2 \log \frac{1}{x}} \; ; \quad \sigma^2(x) \sim \frac{N}{2\pi} \log \frac{1}{x} \, , \qquad x \sim 0 \qquad (\%)$$

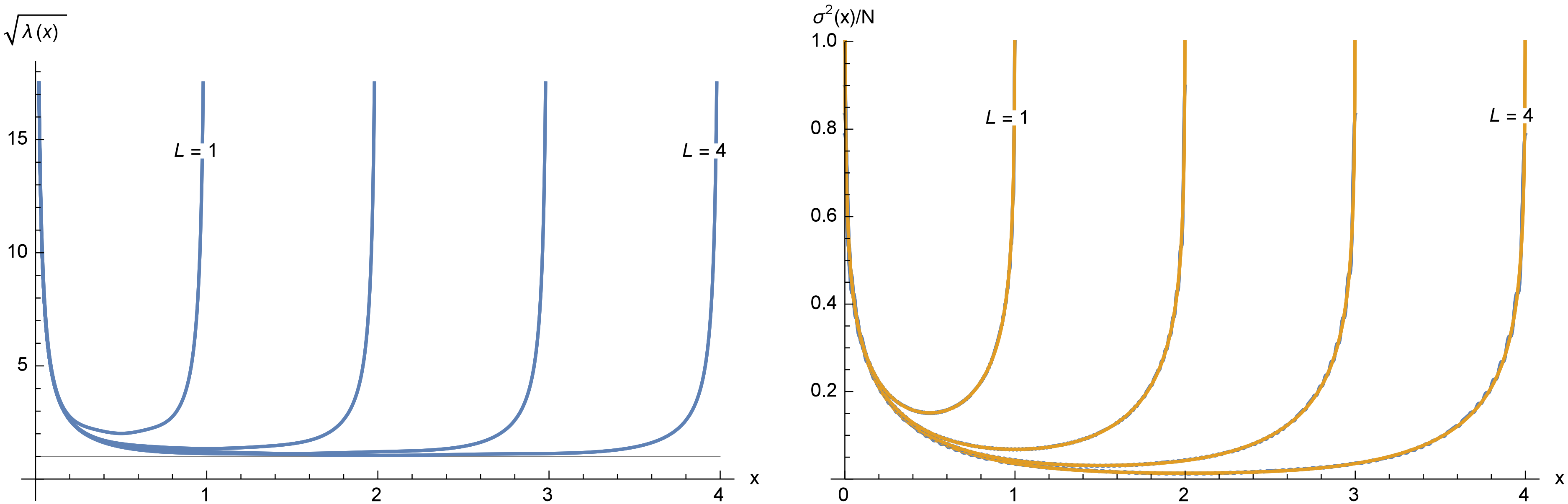
- At finite x (far from the boundaries) the UV divergences in

$$\frac{N}{2} \sum_n \frac{f_n(x)^2}{\omega_n} \quad \text{is cancelled by } r \text{ in} \quad \frac{N}{2} \sum_n \frac{f_n(x)^2}{\omega_n} + \sigma(x)^2 - r = 0$$

(as in the standard 2D CP(N-1) model: UV div is a local phenomenon !)

- Near x ~ 0, L, $\frac{N}{2} \sum_n \frac{f_n(x)^2}{\omega_n}$ cannot diverge (no 2D "space")
- There the classical filed $n_1(x) = \sigma(x)$ comes to rescue! ➪ (%)

The result



or

Figure 4: On the left $\sqrt{\lambda(x)}$, on the right $\sigma(x)^2/N$. These plots are rescaled in order to keep $\Lambda = 1$ fixed and for $L = 1, 2, 3, 4$. The constant line on the left figure is Λ.

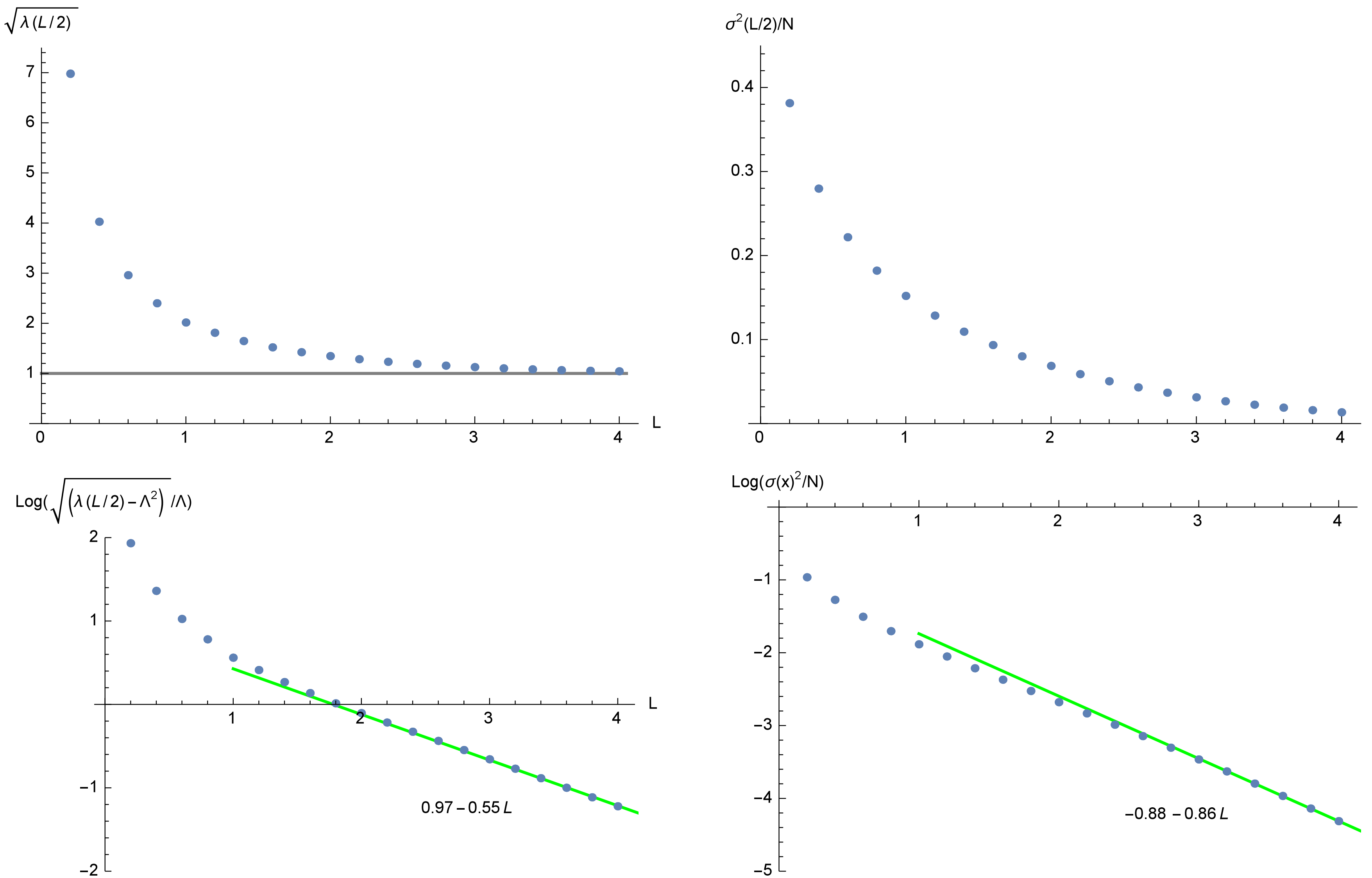


Figure 5: These plots are: on the top-left is $\sqrt{\lambda(L/2)}$, on the top-right $\sigma(L/2)^2/N$, on the bottom-left $\log\left(\sqrt{\lambda(L/2)-\Lambda^2}/\Lambda\right)$ and on the bottom-right $\log\left(\sigma^2(L/2)/N\right)$. These plots are obtained for various values of L keeping $\Lambda = 1$.

CP^{N-1} sigma model on finite worldstrip: summary

- Unique solution of the generalized gap equation (funct. saddle point eqn)
- No "Higgs" phase ($\langle \lambda \rangle = 0; \quad \langle \sigma(x) \rangle = \mathrm{const} \sim \Lambda$) exists
- No phase transition at L ~ $\frac{1}{\Lambda}$

cfr. Milekhin '14

- The standard 2D CP^{N-1} system emerging at large L ($\lambda \sim \Lambda^2 , \quad \sigma = 0$)
- Exactly the same result holds for D-D and N-N boundary conditions
- **Interpolates a 1D system (QM) at L < $\frac{1}{\Lambda}$ to a 2D QFT at L= ∞**
- The dynamical breaking of SU(N) symmetry does not occur Cfr. p. 6
- With the periodic boundary condition, a phase transition at L~ $\frac{1}{\Lambda}$

Shifman - Yung '15

Conclusion:

- NonAbelian monopole concept consistent with quantum mechanics ✔
- Monopoles ~ N of unbroken SU(N)= isometry group of CP(N-1) (Fig p. 17) ✔
- Nonlocal transformations - typical of EM duality ✔

The idea

- Quark confinement = dual Higgs phase of nonAbelian variety:
- Monopoles (nonAbelian and strongly coupled) condense, induce confinement and trigger XSB

to be pursued

- Difficulty for us, not for Nature !!

THE END

Non-Abelian monopoles

$$G \xrightarrow{\langle\phi\rangle\neq 0} H$$

$$SU(3) \xrightarrow{\langle\phi\rangle} \frac{SU(2)\times U(1)}{\mathbb{Z}_2}, \qquad \langle\phi\rangle = \begin{pmatrix} v & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & -2v \end{pmatrix}$$

H: non-Abelian

root vectors

Goddard-Nuyts-Olive, E.Weinberg, Lee,Yi, Bais, Schroer, ... '77-80

$$F_{ij} = \epsilon_{ijk}\frac{r_k}{r^3}(\beta \cdot \mathrm{T}), \qquad 2\,\beta\cdot\alpha \in \mathbf{Z}$$

cfr. (Dirac)

$2\ \mathrm{m}\cdot\mathrm{e} \in \mathrm{Z}$

"Monopoles are multiplets of $\widetilde{\mathrm{H}}$ (GNO)"

$\widetilde{\mathrm{H}}$ generated by $\alpha^* \equiv \dfrac{\alpha}{\alpha\cdot\alpha}.$

H	$\tilde{\mathrm{H}}$
U(N)	U(N)
SU(N)	SU(N)/Z^N
SO(2N)	$\widetilde{\mathrm{SO}(2\mathrm{N})}$
SO(2N+1)	USp(2N)

$<\Phi> = \mathrm{v} = \mathrm{h}\ \cdot\mathrm{T}$

$$A_i(\mathrm{r}) = A_i^a(\mathrm{r}, \mathrm{h}\cdot\alpha)\, S_a; \quad \phi(\mathrm{r}) = \chi^a(\mathrm{r}, \mathrm{h}\cdot\alpha)\, S_a + [\,\mathrm{h} - (\mathrm{h}\cdot\alpha)\,\alpha^*] \cdot \mathrm{T},$$

$$S_1 = \frac{1}{\sqrt{2\alpha^2}}(E_\alpha + E_{-\alpha}); \quad S_2 = -\frac{i}{\sqrt{2\alpha^2}}(E_\alpha - E_{-\alpha}); \quad S_3 = \alpha^* \cdot \mathrm{T},$$

Back to p.4

Difficulties

① Topological obstructions

Abouelsaad et.al.'83

$\Phi = \mathrm{diag}(v,v,-2v)$

e.g., $SU(3) \to SU(2)\times U(1) \quad \Rightarrow$

$\nexists$ monopoles $\sim (2, 1^*)$

"No colored dyons exist"

Coleman, ... '81

② Non-normalizable gauge zero modes:

Monopoles not multiplets of H

Weinberg, '82,'96 Coleman, Nelson, '84 Dorey... '96

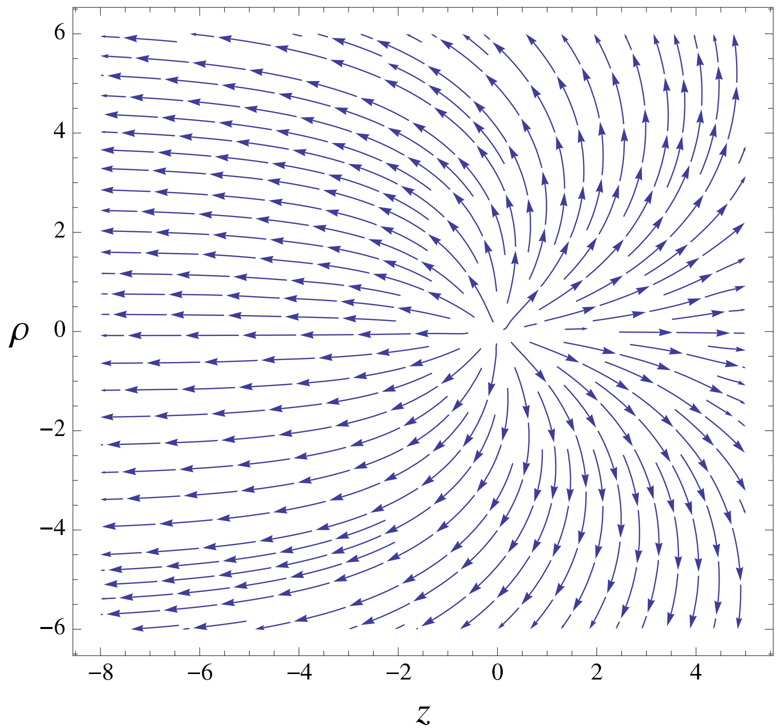
cfr.
Jackiw-Rebbi
Flavor Q.N. of monopoles
via
fermion zeromodes

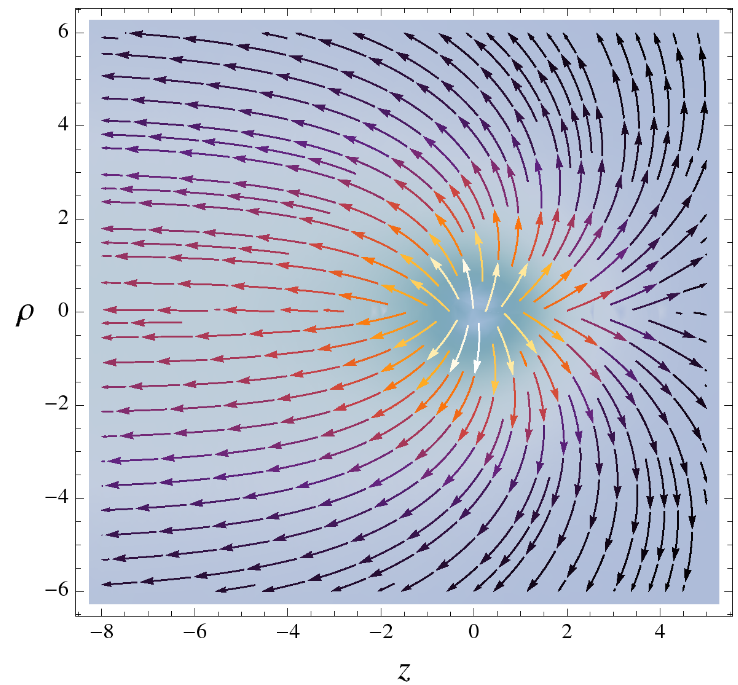
The real issue:
how do they transform under $\tilde{H}$?

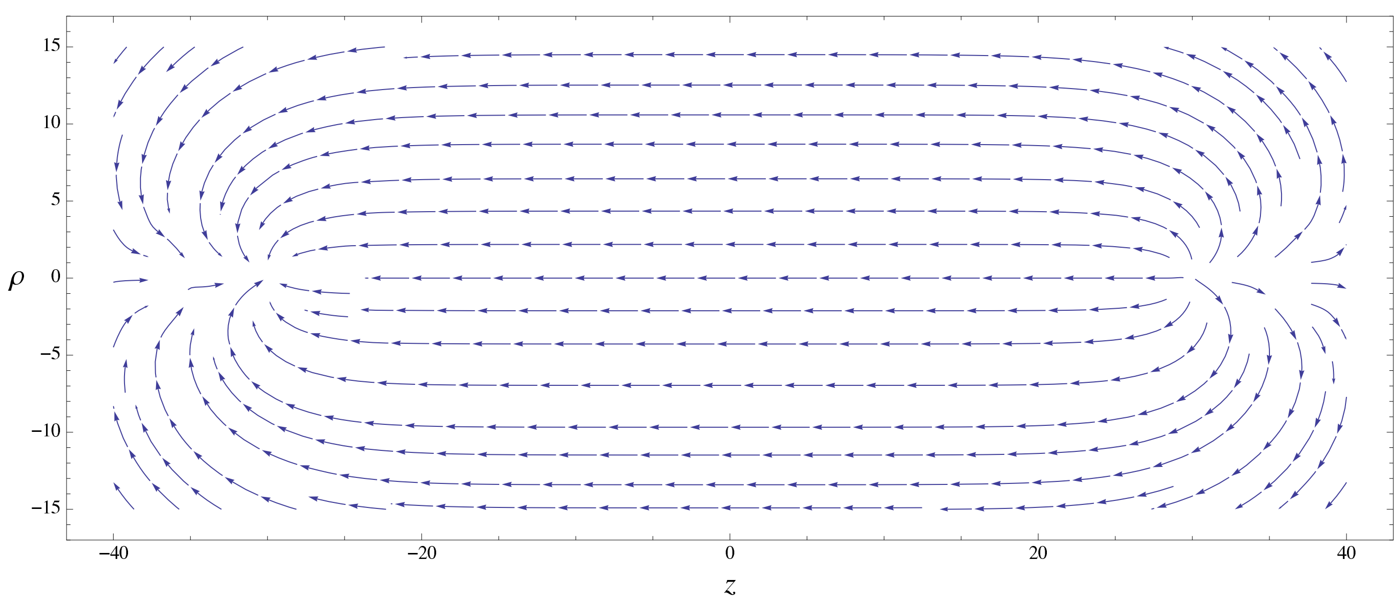
N.B. : H and $\tilde{H}$ relatively nonlocal

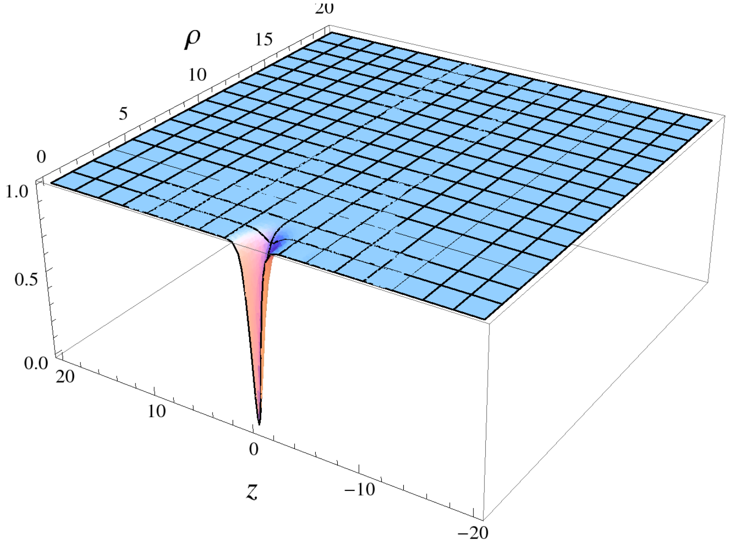
Back to p.5

Cipriani, Gudnason, ... '11

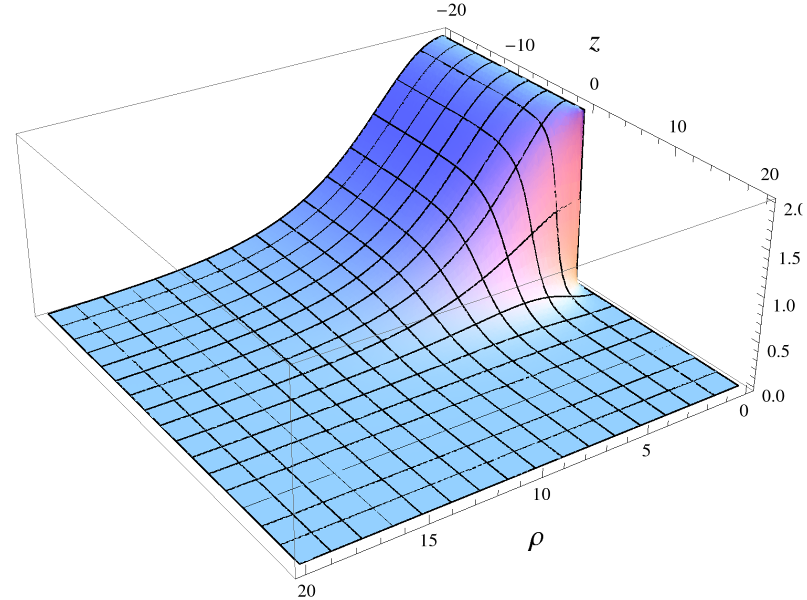




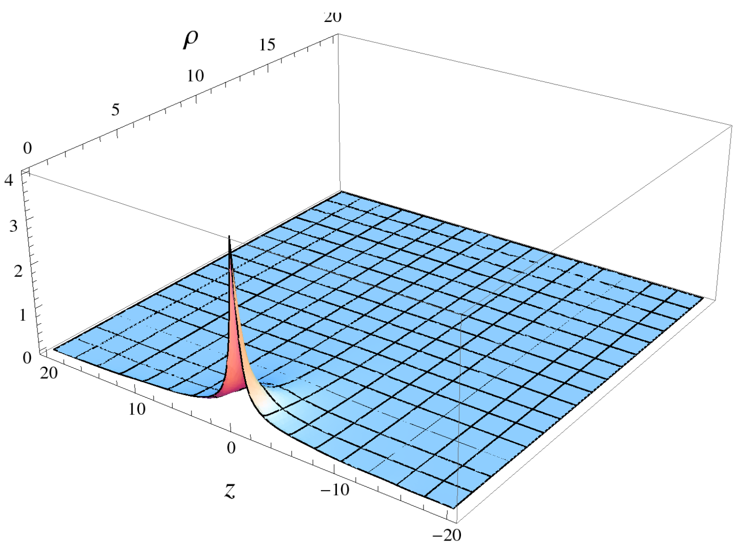




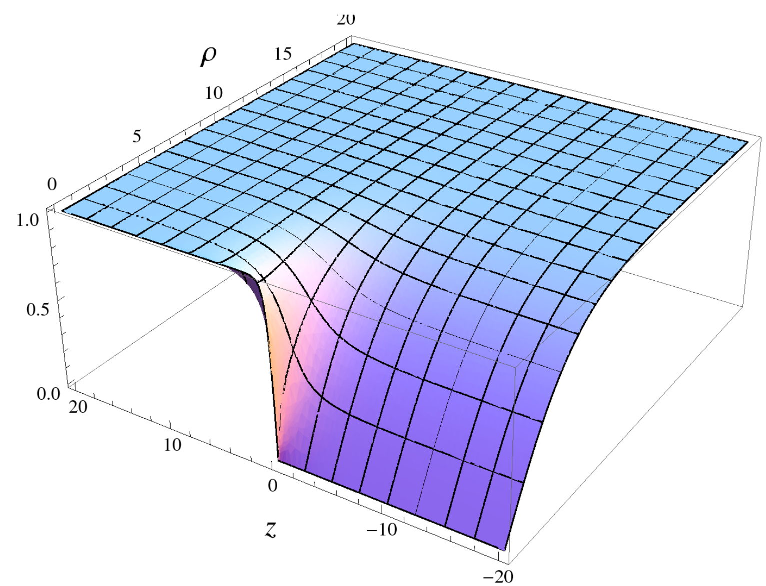
(a) gauge profile function $f(\rho, z)$



(b) gauge profile function $\ell(\rho, z)$



(c) scalar profile function $\frac{1}{\sqrt{2}}s(\rho, z)$



(d) squark profile function $q(\rho, z)$

But this allows a direct description of IR physics !!

For $\quad USp(2N),\ N_f = 4$

the GST dual is (both the A and B sectors are free doublets) :

Giacomelli, Konishi '12,'13

$$\boxed{1} - SU(2) - \boxed{4}\,.$$

U(1)

the effects of m_i and $\mu\,\Phi^2$ perturbation can be studied with the superpotential:

$$\sqrt{2}\,Q_0 A_D \tilde{Q}^0 + \sqrt{2}\,Q_0 \phi \tilde{Q}^0 + \sum_{i=1}^{4} \sqrt{2}\,Q_i \phi \tilde{Q}^i + \mu A_D \Lambda + \mu\,\mathrm{Tr}\phi^2 + \sum_{i=1}^{4} m_i\, Q_i \tilde{Q}^i\ ,$$

cfr. UV Lagrangian:

$$W = \mu \mathrm{Tr}\Phi^2 + \frac{1}{\sqrt{2}} Q_a^i \Phi_b^a Q_c^i\ J^{bc} + \frac{m_{ij}}{2} Q_a^i Q_b^j\ J^{ab}$$

$$m = -i\sigma_2 \otimes \mathrm{diag}\,(m_1, m_2, \ldots, m_{n_f})\,.$$

Correct flavor symmetry for all {m}

- $m_i = m$: SU(4) x U(1) ;
- $m_i = 0$: SO(8) ; etc.,

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