Detecting magnetic defects, ensembles, and gluon topological confinement

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Dual superconductivity ('t Hooft, Nambu, Mandelstam)

- Search for chromomagnetic quantum d.o.f. in pure YM that could capture the path integral measure:

  Abelian projected monopoles, center vortices, correlated chains formed by them: Di Giacomo, Engelhardt & Reinhardt, Faber, Greensite & Olejnik...

- Search for effective dual models in a Higgs phase where the chromoelectric confining string is represented by a classical vortex solution: Baker, Konishi, Shifman, Suzuki, Tong...

- Abelian ensembles of monopole loops lead to Abelian effective field models

$$\sum_n Z_{n-\mathrm{loops}} = \sum_n Z_{n-\mathrm{worldlines}} = 2^{\mathrm{nd}} - Q = \mathrm{Field-path-int}$$

(Halpern & Siegel '77, Bardakci & Samuel '78)

- In $SU(3)$ YM: Linearizing with $\Lambda_{\mu\nu}$, Abelian projection + Abelian dominance hypothesis (D. Antonov, 2000)

$$\Lambda_{\mu\nu} = (\partial_\mu \vec{\Lambda}_\nu - \partial_\nu \vec{\Lambda}_\mu) \cdot \vec{T} + \vec{B}_{\mu\nu} \cdot \vec{T} \quad , \quad \vec{\Lambda} \cdot \vec{T} = \Lambda^1_\mu T_1 + \Lambda^2_\mu T_2$$

$$(\partial_\nu \vec{B}_{\mu\nu} = 0)$$

$$\langle W \rangle = \int [D\Lambda]\, e^{-\int d^4x \, \frac{1}{4g^2} (\partial_\mu \vec{\Lambda}_\nu - \partial_\nu \vec{\Lambda}_\nu - \vec{J}_{\mu\nu})^2} \, Z_{\alpha_1} Z_{\alpha_2} \dots$$

Abelian model:

$$Z_\alpha = \int [D\psi_\alpha][D\bar{\psi}_\alpha]\, e^{-\int d^4x \, \bar{\psi}_\alpha [-D_\alpha^2 + m^2] \psi_\alpha} \quad , \quad D_\mu^\alpha = \partial_\mu - i\, \vec{\alpha} \cdot \vec{\Lambda}_\mu \; .$$

- $\vec{\alpha}$: the three positive roots (tuples) , $\vec{J}_{\mu\nu} = 2\pi \, 2N \, \vec{w}_{\rm e} \, s_{\mu\nu}$

  $\vec{w}_{\rm e}$ is a weight of the quark representation

  (A weight $\vec{w}$ is defined by the eigenvalues of the Cartan generators T_q corresponding to one common eigenvector.

  $$[T_q, T_p] = 0 \quad , \quad T_q \, eigenvector = \vec{w}|_q \, eigenvector)$$

- $g = 4\pi N / g_{\rm e}$
- SSB for $m^2 < 0$, after including density-density interactions
- Center vortices couple with $B_{\mu\nu}$
  Random surface models (Engelhardt & Reinhardt, 2000)

N-ality

Via ensembles in the $SU_{\rm e}(N)$ YM theory

- Center vortices (two-dimensional worldsheets) and chains formed by center vortices correlated with monopoles (loops).

$$W_{\rm f}[A^{\rm e}_\mu] = \left(e^{i\frac{2\pi}{N}}\right)^{\rm link} W_{\rm f}[P] \quad \text{vs.} \quad W_{\rm a}[A^{\rm e}_\mu] = W_{\rm a}[P]$$

Via effective dual YMH models

- $SU(N) \to Z(N)$ SSB
- $\mathcal{M} = SU(N)/Z(N) = Ad(SU(N))$
- $\Pi_1(Ad(SU(N))) = Z(N) \Rightarrow$ the confining string is a Center string: $3D$ static vortices that minimize an effective energy functional

  - they confine fundamental quarks to form normal hadrons

- M. Baker, J. S. Ball & F. Zachariasen '97, introduced a dual model with gauge group $SU(3)$ and three adjoint Higgs fields, and computed the interquark potential.

- A class of Yang-Mills-Higgs (YMH)

$$\frac{1}{2}\langle D_\mu \psi_I, D^\mu \psi_I \rangle + \frac{1}{4g^2}\langle F_{\mu\nu} - J_{\mu\nu}, F^{\mu\nu} - J^{\mu\nu} \rangle - V_{\rm Higgs}(\psi_I)$$

$$D_\mu = \partial_\mu - i\,[\Lambda_\mu, \;] \quad , \quad F_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu - i\,[\Lambda_\mu, \Lambda_\nu]$$

- $\psi_I \in \mathfrak{su}(N)$ is a set of adjoint Higgs fields. I is a flavour index

- For $SU(N) \to Z(N)$, the manifold of absolute minima $(\phi_1, \phi_2, \dots) \in \mathcal{M} \;/\; U\phi_I U^{-1} = \phi_I$ iff $U \in Z(N)$

- Minimum number of flavours is N

$SU(N)$: normal glue

Center strings can be labelled by the magnetic weights $\vec{\beta}$ of the different group representations, Konishi-Spanu (2001).
The asymptotic behavior is *locally* a pure gauge (but not *globally*),

$$S = e^{i\varphi\,\vec{\beta}\cdot\vec{T}} \quad , \quad \vec{\beta} = 2N\,\vec{w} \quad , \quad \vec{\beta}\cdot\vec{T} = \vec{\beta}|_q\,T_q$$

For the fundamental representation we have N weights $\vec{w}_i$ (fundamental colours), $\vec{\beta}_1 + \cdots + \vec{\beta}_N = \vec{0}$

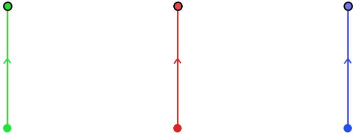
They are associated with the simplest center strings

$$e^{i2\pi\,\vec{\beta}_i\cdot\vec{T}} = e^{i\,2\pi/N}\,I$$

N-ality for **Infinite** adjoint center string: An asymptotic behavior $S \sim e^{i\varphi\,2N\,\vec{\alpha}\cdot\vec{T}}$ is a closed loop in $SU(N)$. As $\Pi_1(SU(N)) = 0$, it can be continuously deformed into $S \sim I$ at the origin.

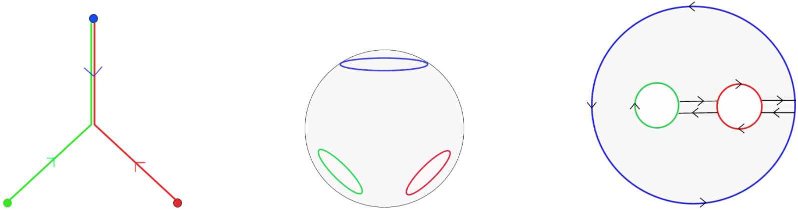
$SU(3)$: normal state

Finite center string induced by fundamental sources $\vec{w}, -\vec{w}$



or by three fundamental sources $\vec{w}_1, \vec{w}_2, \vec{w}_3$, $\quad S = e^{i\chi_1 \vec{\beta}_1 \cdot \vec{T}} \, e^{i\chi_2 \vec{\beta}_2 \cdot \vec{T}}$

$$\vec{\beta}_{c_1} + \vec{\beta}_{c_2} + \vec{\beta}_{c_3} = 0 \qquad (\vec{\beta}_1 + \vec{\beta}_2 = -\vec{\beta}_3)$$



$\vec{J}_{\mu\nu} = 2\pi\, 2N\, \vec{w}\, s_{\mu\nu}$. Static sources:

$$s_{0i} = 0 \;,\;\; s_{ij} = -\epsilon_{ijk} \int ds\, \frac{dx_k}{ds}\, \delta^{(3)}(x - x(s))$$

$$S = e^{i\varphi\,\vec{\beta}\cdot\vec{T}} \quad , \quad \vec{\beta} = 2N\,\vec{w} \quad , \quad \Lambda_i = a(\rho)\,i\,S\partial_i S^{-1} \quad \text{(locally)}$$

Adjoint sources $\vec{\alpha}, -\vec{\alpha}$ **at a finite distance.** $\vec{J}_{\mu\nu} = 2\pi\, 2N\, \vec{\alpha}\, s_{\mu\nu}$

$$S_{\mathrm{Abe}} = e^{i\varphi\, 2N\, \vec{\alpha} \cdot \vec{T}}$$

The roots are the weights of the adjoint representation, which acts via commutators

$$[T_q, E_\alpha] = \vec{\alpha}|_q \, E_\alpha \quad , \quad \vec{\alpha} = \vec{w} - \vec{w}'$$

$$S_{\rm non-Abe} = e^{i\varphi \, \vec{\beta}\cdot\vec{T}} \, W(\gamma) \, e^{-i\varphi \, \vec{\beta}'\cdot\vec{T}}$$

$$W(\gamma) = e^{i\gamma \, \sqrt{N} T_\alpha}$$

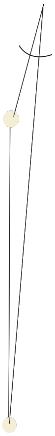
γ, φ are bipolar angles

N-ality in effective YMH models

$$e^{i\varphi\,\vec{\beta}\cdot\vec{T}}\,W(\gamma)\,e^{-i\varphi\,\vec{\beta}'\cdot\vec{T}}$$

$$W(\gamma) = e^{i\gamma\,\sqrt{N}\,T_\alpha}$$

- $\gamma \sim 0$: $S_{\mathrm{non-Abe}} \sim e^{i\varphi\,2N\,\vec{\alpha}\cdot\vec{T}}$



N-ality in effective YMH models

$$e^{i\varphi\,\vec{\beta}\cdot\vec{T}}\,W(\gamma)\,e^{-i\varphi\,\vec{\beta}'\cdot\vec{T}}$$

$$W(\gamma) = e^{i\gamma\,\sqrt{N}\,T_\alpha}$$

- $\gamma \sim \pi$:

  $$W^{-1}(\pi)\,\vec{w}\cdot\vec{T}\,W(\pi) = \vec{w}'\cdot\vec{T}$$

  Weyl transformation

  $$S_{\rm non-Abe} \sim W(\pi)$$

(complete screening
by dynamical adjoint monopoles)

Hybrid QCD spectrum

- Lattice calculations predict a rich spectrum of exotic mesons
- Some of them correspond to $qg\bar{q}'$ hybrid mesons: a nonsinglet colour pair and a valence gluon that form a colourless state.
- Currently searched by a collaboration based at the Jefferson Lab (GlueX)

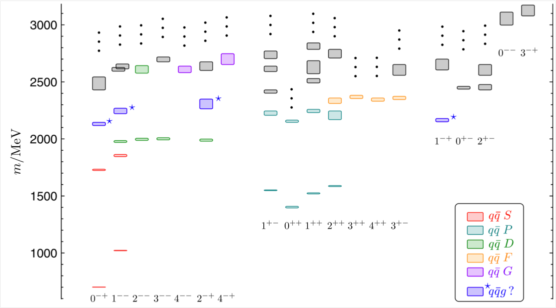
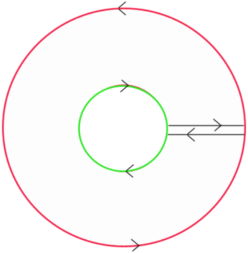
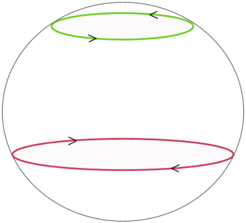


Figure: From B. Ketzer 2012

Induced by fundamental sources $\vec{w}, -\vec{w}'$, with $\vec{w} \neq \vec{w}'$





LEO, 2012

$SU(N)$: hybrid center string/dual monopole/center string

Infinite object: (θ, ϕ are spherical angles)

$$S = e^{i\varphi\,\vec{\beta}\cdot\vec{T}}\, W(\theta) \quad , \quad W(\theta) = e^{i\theta\,\sqrt{N}T_\alpha} \quad , \quad \vec{\alpha} = \vec{w} - \vec{w}'$$

Around the north pole,

$$S \sim e^{i\varphi\,\vec{\beta}\cdot\vec{T}}$$

Around the south pole, $W(\pi)$ is a Weyl reflection,

$$S \sim W(\pi)\, e^{i\varphi\,\vec{\beta}'\cdot\vec{T}}$$

(gauge transformations act on the left)

- A well-defined mapping for the local Cartan directions on S^2

$$n_q = S T_q S^{-1} \quad , \quad q = 1, \ldots, N-1$$

- The gauge invariant monopole charge is obtained from the (dual) field strength projection along the *local* Cartan directions n_q

$$\vec{Q}_m = 2\pi\, 2N\, (\vec{w} - \vec{w}') = 2\pi\, 2N\, \vec{\alpha}$$

- This monopole can be identified with a valence gluon with adjoint colour $\vec{\alpha} = \vec{w} - \vec{w}'$

- **Valence gluons are confined**: $\mathcal{M}$ is a compact group.

  $\Pi_2(\mathcal{M}) = \Pi_2(Ad(G)) = 0$, $G = SU(N) \Rightarrow$ there are no isolated *dynamical adjoint* monopoles

- Stable: One-to-one mapping between $\Pi_2\left(\frac{Ad\,(G)}{Ad\,(H)}\right)$ and loops in $\Pi_1(Ad\,(H))$, $H = U(1)^{N-1}$, that are trivial when seen as loops in $\Pi_1(Ad\,(G)) = Z(N)$

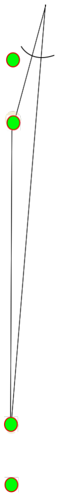
Two negative (up) and two positive (down) fundamental sources:

$$S_{\mathrm{non-Abe}} = e^{i\varphi\,\beta T_1}\, W(\gamma)\, e^{i\varphi\,\beta T_1}$$

- $\gamma \sim 0$: $S_{\mathrm{non-Abe}} \sim e^{i\varphi\, 2\beta T_1}$
- $\gamma \sim \pi$: $S_{\mathrm{non-Abe}} \sim W(\pi)$

$$W^{-1}(\pi)\, T_1\, W(\pi) = -T_1$$

The inner fundamental quarks are partially screened by dynamical adjoint monopoles



Gauge fixing in pure YM

Laplacian-center gauge

Faber, Greensite & Olejnik; de Forcrand & Pepe (2001)

- $$\mathcal{A}_\mu \to f_I \to Ad(S)$$

- $$U_{\rm e}\,\mathcal{A}_\mu\,U_{\rm e}^{-1} + i\,U_{\rm e}\partial_\mu U_{\rm e}^{-1} \to U_{\rm e}\,f_I\,U_{\rm e}^{-1} \to Ad(U_{\rm e}S)$$

- Impose a condition on S
- S is extracted from a polar decomposition $f_I = Sq_IS^{-1}$ in term of "modulus" q_I and "phase" S variables

- For three adjoint fields f_1, f_2, f_3 in $SU(2)$ this corresponds to the usual polar decomposition of a 3×3 matrix

Lattice:

- Lowest eigenfunctions of the adjoint covariant Laplacian

$$\mathcal{D}_\mu \mathcal{D}_\mu f_I = \lambda_I f_I \qquad , \qquad \mathcal{D}_\mu = \partial_\mu - i\,[\mathcal{A}_\mu,\ ]$$

Continuum: LEO & Santos-Rosa, 2015

$$\mathcal{D}_\mu \mathcal{D}_\mu f_I = \dots \qquad \text{set of coupled eqs.} \qquad \frac{\delta S_{\rm aux}}{\delta f_I} = 0$$

$$1 = \int [Df_I] \, \det \left(\frac{\delta^2 S_{\rm aux}}{\delta f_I \, \delta f_J} \right) \delta \left(\frac{\delta S_{\rm aux}}{\delta f_I} \right)$$

$$[Df_I] = [Dq_I]\,[DS] \qquad , \qquad \sum_I [q_I, u_I] = 0$$

- S could have defects
- $S \sim S'$ if there is a regular $U_{\rm e}$ / $S' = U_{\rm e}\, S$
- Gauge fixed config. become partitioned into sectors $\mathcal{V}(S_0)$
- They are labelled by representatives S_0 that could be singular

$$A^{\rm e}_\mu \to {\rm Ad}(S_0) = R_0$$

If $P_\mu \in \mathcal{V}(I)$, then, something that is *locally*,

$$A^{\rm e}_\mu|_{\rm loc} = S_0\, P_\mu S_0^{-1} + i\, S_0 \partial_\mu S_0^{-1}$$

(P_μ / $A^{\rm e}_\mu$ is regular) is in $\mathcal{V}(S_0)$. Defining $\langle , \rangle$ as ${\rm Tr}$ in the adjoint:

$$S_{\rm YM} = \int d^4x\, \frac{1}{4g_{\rm e}}\, {\rm Tr}\,(F^{\rm e}_{\mu\nu} F^{\rm e}_{\mu\nu})$$

$$F_{\mu\nu}(A^{\rm e}) = R_0 \Big(F_{\mu\nu}(P) - H_{\mu\nu}(R_0) \Big)\, R_0^{-1} \quad , \quad H_{\mu\nu}(R_0) = i\, R_0^{-1} [\partial_\mu, \partial_\nu] R_0$$

Gauge transf. (**left** action with regular $U_{\rm e}$): $\qquad S_0 \to U_{\rm e}\, S_0$

$F_{\mu\nu}(A^{\rm e})$ rotates and $H_{\mu\nu}(R_0)$ is left invariant

Inequiv. config. (**right** action with regular $\tilde{U}^{-1}$): $\quad S_0' = S_0\, \tilde{U}^{-1}$

- if $\nexists$ regular $U_{\rm e}$ / $S_0\, \tilde{U}^{-1} = U_{\rm e}\, S_0$, $\qquad \mathcal{V}(S_0) \to \mathcal{V}(S_0')$
- $H_{\mu\nu}(R_0') = \tilde{R}\, H_{\mu\nu}(R_0)\, \tilde{R}^{-1}$

$SU(N)$: magnetic defects

- Center vortices are also labelled by the weights of the fundamental representation. Example $S_0 = e^{i\chi\,\vec{\beta}\cdot\vec{T}}$,

$$-\mathcal{H}_{\mu\nu}(R_0) = 2\pi\,\vec{\beta}\cdot\vec{M}\oint d^2\sigma_{\mu\nu}\,\delta^{(4)}(x-\bar{y}(\sigma_1,\sigma_2)) \quad , \quad M_q = \mathrm{Ad}(T_q)$$

$(\mathcal{H}_{\mu\nu} = \frac{1}{2}\,\epsilon_{\mu\nu\rho\sigma}H_{\mu\nu})$

- Open center vortex worldsheets with different weights $\vec{\beta}$, $\vec{\beta}'$ can be matched by monopoles. Only using a Weyl transf., at each junction, there is a contribution

$$-\,\partial_\nu\mathcal{H}_{\mu\nu}(R_0) = 2\pi\,2N\,\vec{\alpha}\cdot\vec{M}\oint_C dy_\mu\,\delta^{(4)}(x-y)$$

- Inequivalent configuration (right action): at each junction,

$$-D_\nu(\tilde{Z})\,\mathcal{H}_{\mu\nu}(R_0') = 2\pi\,2N\,\tilde{R}\,\vec{\alpha}\cdot\vec{M}\,\tilde{R}^{-1}\oint_C dy_\mu\,\delta^{(4)}(x-y)$$

$$D_\mu(\tilde{Z}) = \partial_\mu - i\,[\tilde{Z}_\mu,\cdot\,] \quad , \quad \tilde{Z}_\mu = i\,\tilde{R}\,\partial_\mu\tilde{R}^{-1}$$

Ensemble of monopoles with non-Abelian d.o.f.

Motivated by:

$$e^{-\int d^4x\, \frac{1}{4g_{\rm e}^2}\, {\rm Tr}\,(F^{\rm e}_{\mu\nu}F^{\rm e}_{\mu\nu})} = \int [D\Lambda_{\mu\nu}]\, e^{-\int d^4x\, \frac{1}{4g^2}\, {\rm Tr}\,\Lambda^2_{\mu\nu}}\, e^{\frac{i}{2}\, \frac{1}{4\pi N} \int d^4x\, {\rm Tr}\,\Lambda_{\mu\nu}\, (\mathcal{F}_{\mu\nu}(P) - \mathcal{H}_{\mu\nu}(R'_0))}$$

- A non-Abelian Hodge decomposition

$$\Lambda_{\mu\nu} = D_\mu(\tilde{Z})(\Lambda_\nu - \tilde{Z}_\nu) - D_\nu(\tilde{Z})(\Lambda_\mu - \tilde{Z}_\mu) + B_{\mu\nu} \quad , \quad D_\nu(\tilde{Z})\, B_{\mu\nu} = 0$$

- Only monopoles (attached to unobservable Dirac worldsheets)
- Dress the monopoles with phenomenological information
- Simplest properties of looplike objects: tension, stiffness and interactions ($S_{\rm m}$)

Let us consider

$$Z = \sum_{\text{ensemble}} e^{-S_{\rm m}}\, e^{-S_G - S_H}$$

$$S_G = \int d^4x\, \frac{1}{4g^2}\, \mathrm{Tr}\, \left(D_\mu(\tilde{Z})(\Lambda_\nu - \tilde{Z}_\nu) - D_\nu(\tilde{Z})(\Lambda_\mu - \tilde{Z}_\mu) \right)^2$$

$$S_H = -i \int ds\, \frac{dx_\mu}{ds}\, \mathrm{Tr}\, (\Lambda_\mu - \tilde{Z}_\mu)\, \tilde{R}\, (\vec{\alpha} \cdot \vec{M})\, \tilde{R}^{-1}$$

$\vec{\alpha}$: $N(N-1)/2$ positive values.

The variable $\tilde{U}$ is similar to that appearing in the Skyrme model through the combination $\tilde{Z}_\mu$. Under,

$$\tilde{U} \to U\, \tilde{U} \qquad\qquad (\tilde{R} \to R\, \tilde{R})$$

$$\Lambda_\mu \to R\, \Lambda_\mu R^{-1} + i\, R\, \partial_\mu R^{-1} \;,$$

S_G and S_H are left invariant.

Non-Abelian coupling

- Analyze one $\vec{\alpha}$-sector. For the highest weight $\vec{\alpha}$,

$$\frac{dx_\mu}{ds}\,\mathrm{Tr}\,(\Lambda_\mu - \tilde{Z}_\mu)\,\tilde{R}\,(\vec{\alpha}\cdot\vec{M})\,\tilde{R}^{-1}\,\Lambda^A_\mu = \frac{dx_\mu}{ds}\,I_A\,\Lambda^A_\mu - i\,z_c\dot{\bar{z}}_c$$

$$I_A = M_A|_{cd}\,\bar{z}_c z_d \quad , \quad z = \tilde{R}\,u_\alpha \quad , \quad u_\alpha \text{ is the weight vector}$$

- A worldline with non-Abelian coupling: Balachandran et al. '77 : $I_A = R(T_A)|_{cd}\,\bar{z}_c z_d$
- An ensemble with non-Abelian coupling:

  Santos Rosa, Teixeira & LEO (2014)

$$Z = \int [D\phi]\, e^{-W[\phi]} \sum_n Z_n$$

$$Z_n = \int [Dm]_n \exp\left[-\sum_{k=1}^{n}\int_0^{L_k} ds_k\,(\,\cdot\,)_k\right] \quad , \quad u_\mu = \frac{dx_\mu}{ds} \in S^3$$

$$(\,\cdot\,) = \mu + \frac{1}{2}(\bar{z}_c\dot{z}_c - \dot{\bar{z}}_c z_c) + \frac{1}{2\kappa}\,\dot{u}_\mu\dot{u}_\mu - i\,u_\mu\,I_A\,\Lambda^A_\mu(x) + \phi(x) + I_A\,\phi^A(x)$$

$$[Dm]_n \equiv \frac{1}{n!}\int_0^\infty \frac{dL_1}{L_1}\frac{dL_2}{L_2}\ldots\frac{dL_n}{L_n}\int dv_1 dv_2\ldots dv_n \int [Dv(s_1)]_{v_1,v_1}\ldots[Dv(s_n)]_{v_n,v_n}$$

$$v : x, u, z \quad , \quad dv = d^4x\, d^3u\, dz\, d\bar{z}$$

For smooth closed monopole worldlines

$$\sum_n Z_n = e^{\int_0^\infty \frac{dL}{L} \int dv\, q(v,v,L)} \quad , \quad q(v, v_0, L) = \int [Dv(s)]_{v,v_0}\, e^{-\int_0^L ds\,(\cdot)}$$

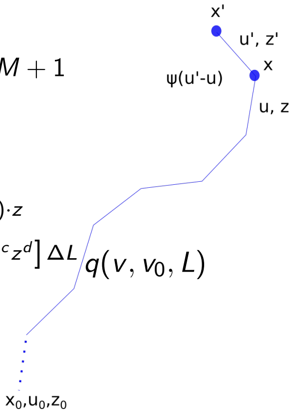
$q(v, v_0, L)$: end-to-end probability, for a line of length L, to start at x_0, with tangent u_0 and z_0, and end at x with u, z

- $[Dz(s)]_{z,z_0}$ gives $\langle z|e^{-\hat{H}L}|z_0\rangle$

$$\hat{H} = -i\, u_\mu \Lambda_\mu^A\, M_{cd}^A\, \hat{a}_c^\dagger \hat{a}_d + \phi^A\, M_{cd}^A\, \hat{a}_c^\dagger \hat{a}_d$$

- $\Longrightarrow$ in the exponent of $\sum_n Z_n$, there is a $\mathrm{Tr}\, e^{-\hat{H}L}$
- $\langle \phi|e^{-\hat{H}L}|\psi\rangle$:
  $\int dz d\bar{z}\, dz_0 d\bar{z}_0\, e^{-\frac{\bar{z}\cdot z}{2}} e^{-\frac{\bar{z}_0\cdot z_0}{2}}\, \bar{\phi}(z)\, \psi(\bar{z}_0)\, q(v, v_0, L)$
- $\langle b|e^{-\hat{H}L}|a\rangle$:
  $Q^{ba} = \int dz d\bar{z}\, dz_0 d\bar{z}_0\, e^{-\frac{\bar{z}\cdot z}{2}} e^{-\frac{\bar{z}_0\cdot z_0}{2}}\, z^b\, \bar{z}_0^a\, q(v, v_0, L)$
-
$$\sum_n Z_n = e^{\int_0^\infty \frac{dL}{L}\, \int d^4x\, d^3u\, \mathrm{tr}\, [Q(x,x,u,u,L)] + \dots}$$
- $q(v, v_0, L)$ as continuum limit of a Chapman-Kolmogorov recurrence relation for diffusion in v-space
- The equilibrium theory of inhomogeneous polymers, G. H. Fredrickson (2006)

From the probability
to get x, u, z with M monomers to the
probability to get $x' = x + u'\,\Delta L$, u', z' with $M+1$



$$q(v', v_0, , L + \Delta L) = \int d^3u\, dz\, d\bar{z}$$
$$\times\, e^{-\mu\Delta L}\, e^{-\frac{1}{2\kappa}\Delta L\left(\frac{u'-u}{\Delta L}\right)^2}\, e^{(\bar{z}'-\bar{z})\cdot z}$$
$$\times\, e^{-\left[\phi - i\, u'_\mu \Lambda^A_\mu\, M^A_{cd}\, \bar{z}'^c z^d + \phi^A\, M^A_{cd}\, \bar{z}'^c z^d\right]\Delta L}\, q(v, v_0, L)$$

First order in ΔL with finite $\kappa \to$ Fokker-Plank equation:

$$\partial_L q = \left[-\mu - \phi(x) + \frac{\kappa}{\pi}\,\hat{L}^2_u - u_\mu \partial_\mu + (i\, u_\mu \Lambda^A_\mu - \phi^A)\, M^A_{cd}\, \bar{z}^c \frac{\partial}{\partial \bar{z}^d}\right] q$$

Reduced Fokker-Plank equation

$$\left[\left(\partial_L - (\kappa/\pi)\,\hat{L}_u^2 + (\mu+\phi)\,1 + u\cdot D\right]\,Q(x,x_0,u,u_0,L) = 0$$

$$Q(x,x_0,u,u_0,0) = \delta(x-x_0)\,\delta(u-u_0)\,1$$

where $Q|^{cd} = Q^{cd}$, 1 is a $\mathcal{D}\times\mathcal{D}$ identity matrix ($\mathcal{D} = N^2-1$)

$$D_\mu = 1\,\partial_\mu - i\,\Lambda_\mu^A\,M^A$$

Semiflexible limit:

Small stiffness: disregard the angular momenta $l \geq 2$ in an expansion of spherical harmonics on S^3 (memory loss)

Effective field representation

$$\int d^4x\, d^3u\, Q(x,x,u,u,L) \approx \int d^4x\, \langle x|e^{-LO}|x\rangle$$

$$O = -\frac{\pi}{12\kappa} D_\mu D_\mu + (\phi + \mu)\, 1 + \phi^A M_A$$

$$\begin{aligned} Z &= \int [D\phi]\, e^{-W}\, e^{-\mathrm{Tr}\ \ln O} = \int [D\phi]\, e^{-W}\, (\mathrm{Det}\ O)^{-1} \\ &= \int [D\phi]\, e^{-W} \int [D\zeta][D\bar{\zeta}]\, e^{-\int d^4x\, \zeta^\dagger O \zeta} \end{aligned}$$

- Integrating ϕ, ϕ^A with Gaussian weight

$$Z = \int [D\psi]\, e^{-\int d^4x\, \mathcal{L}_{\rm eff}}$$

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\langle D_\mu \psi_I, D^\mu \psi_I\rangle + \frac{m^2}{2}\langle \psi_I, \psi_I\rangle + \frac{\lambda}{4}\langle \psi_I \wedge \psi_J, \psi_I \wedge \psi_J\rangle + \frac{\eta}{4}\langle \psi_I, \psi_I\rangle\langle \psi_J, \psi_J\rangle$$

- ψ_I : a pair of Hermitian adjoint Higgs fields
- $m^2 \propto \mu\, \kappa$

- Natural invariant terms to form $V_{\rm Higgs}(\psi_I)$

$$\langle \psi_I , \psi_J \rangle \quad , \quad \langle \psi_I , \psi_J \wedge \psi_K \rangle$$

$$\langle \psi_I \wedge \psi_J , \psi_K \wedge \psi_L \rangle \quad , \quad \langle \psi_I , \psi_J \rangle \langle \psi_K , \psi_L \rangle$$

where $\psi_I \wedge \psi_J = -i[\psi_I , \psi_J]$.

- Flavour symmetric Higgs potential, $I \to A = 1, \dots , N^2 - 1$

LEO, 2012

$$c + \frac{m^2}{2} \langle \psi_A , \psi_A \rangle + \frac{\gamma}{3} f_{ABC} \langle \psi_A \wedge \psi_B , \psi_C \rangle + \frac{\lambda}{4} \langle \psi_A \wedge \psi_B , \psi_A \wedge \psi_B \rangle$$

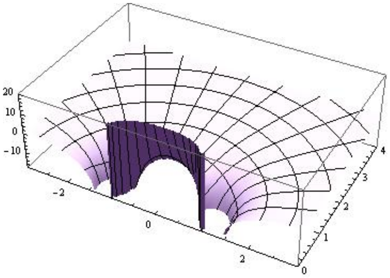
- Degenerate potential

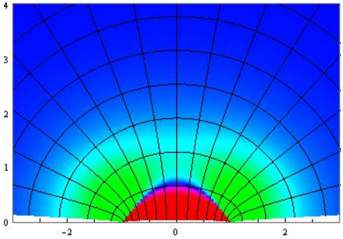
$$\frac{\lambda}{4} \langle \psi_A \wedge \psi_B - f_{ABC} \; v \, \psi_C \rangle^2 \qquad , \qquad \psi_A \wedge \psi_B - v \, f_{ABC} \, \psi_C = 0$$

For $m^2 < \frac{2}{9} \frac{\gamma^2}{\lambda}$, the absolute minima are Lie bases:

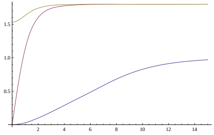
$SU(N) \to Z(N)$

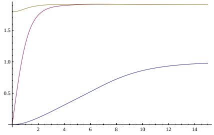
Numerical solutions: LEO & D. Vercauteren (2016)

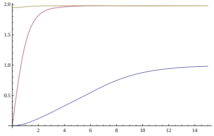




- $m^2 = 0$: Eqs. of motion get Abelianized (all N), a, h, h_1







- BPS point $\lambda = 1$ ($N = 2$): Starting point to explore parameter space

Normal potential: R. Höllwieser, LEO & D. Vercauteren

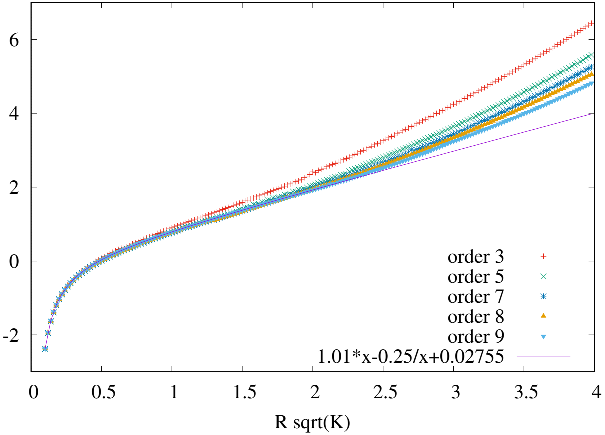


Figure: Normal potential at BPS point in $SU(2)$

Heavy quark hybrid potentials in lattice YM

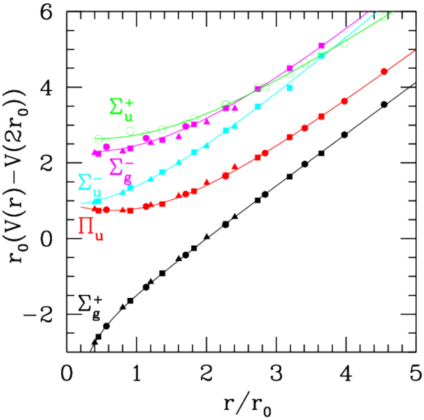


Figure:
Hybrid potentials. From K. J. Juge, J. Kuti & C. J. Morningstar 1997, Orlando Oliveira 2007

Conclusions

- Motivated an ensemble of monopoles with non-Abelian d. o. f.
- Obtained an effective model based on a set of interacting adjoint fields (after assuming phenomenological properties like tension, stiffness, etc.)
- Depending on the parameters: $SU(N) \to Z(N)$
- N-ality

  (via Weyl transformations)
- Hybrid glue
- Topological confinement of (valence) gluons
- Correct minimum energy for doubled fundamental pairs

Disorder Parameters in $3D$

't Hooft (1978)

- Vortex creation operators. At a given time t:

$$\hat{W}(\mathcal{C})\hat{V}(\mathbf{x}) = e^{i2\pi\,\mathrm{link}/N}\,\hat{V}(\mathbf{x})\hat{W}(\mathcal{C})$$

$$\hat{W}|\mathcal{A}\rangle = W|\mathcal{A}\rangle \quad , \quad \hat{W}\left(\hat{V}|\mathcal{A}\rangle\right) = e^{i2\pi\,\mathrm{link}/N}\,W\left(\hat{V}|\mathcal{A}\rangle\right)$$

$$\hat{V}|\mathcal{A}\rangle = |\mathcal{A} + \mathit{defect}\rangle$$

- defect + defect + ... defect = nothing $\rightarrow$ Z(N) symmetry
- Criteria for confinement

$$\langle\hat{V}(\mathbf{x})\rangle = \mathrm{const} \neq 0$$

- Understanding the criterion in $3D$:

$$\hat{V}(\mathbf{x})|V\rangle = V|V\rangle$$

$$\hat{V}(\mathbf{x})\left(\hat{W}(\mathcal{C})|V\rangle\right) = e^{-i2\pi\,\mathrm{link}/N}\,V\left(\hat{W}(\mathcal{C})|V\rangle\right)$$

$$\hat{W}(\mathcal{C})|V\rangle = |V\ \textit{rotated inside}\ \mathcal{C}\rangle$$

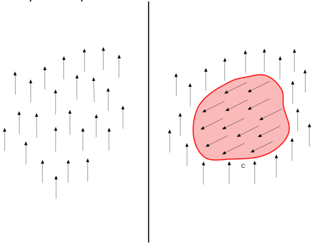


Figure: If there is SSB in the dual theory, $\langle \mathrm{fund}|\hat{W}(\mathcal{C})|\mathrm{fund}\rangle \sim e^{-\sigma Area}$

$3D$ dual superconductor model proposed by t'Hooft

$$\partial_\mu \bar{V} \partial_\mu V + m^2\, \bar{V} V + \alpha\, (\bar{V} V)^2 + \beta\, (V^N + \bar{V}^N)$$

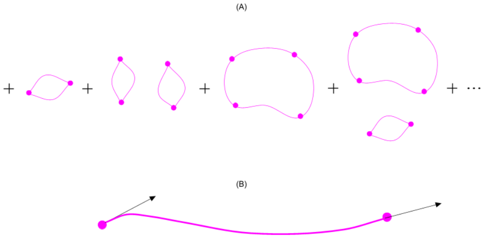
where V represents the vortex sector. Displays a global magnetic $Z(N)$ symmetry.

- when $m^2 < 0 \Rightarrow$ SSB $\Rightarrow$ Domain walls (confining strings)

Chains of monopoles and center vortices in $3D$

Lemos, Teixeira & LEO (2011)

$N = 2$



$$S_d = \sum_{k=1}^{2n} \int_0^{L_k} ds \left[\mu + \frac{1}{2\kappa}\, \dot{u}_\alpha^{(k)} \dot{u}_\alpha^{(k)} \right] + \frac{1}{2} \sum_{k,k'} \int_0^{L_k} \int_0^{L_k'} ds\, ds'\, V\left(x^{(k)}(s), x^{(k')}(s') \right)$$

Building block

$$Q(x,x_0) \;=\; \int_0^\infty dL\, e^{-\mu L} \int [Dx(s)]\, e^{-\int_0^L ds\, \left[\frac{1}{2\kappa}\, \dot{u}_\alpha \dot{u}_\alpha + \phi(x(s))\right]}$$

Small stiffness:

$$\left[-\frac{1}{3\kappa}\nabla^2 + \phi(x) + \mu\right] \mathcal{Q} = \delta(x - x_0)$$

- Summing over the ensemble of closed loops and chains $\rightarrow$ 't Hooft vortex model with $m^2 \propto \mu\,\kappa$
- Chains are essential to describe typical Z(2) processes that lead to the Z(2) terms!