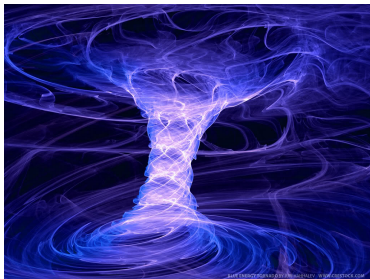


Confinement from Center Vortices

a review of old and new results

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Old (and very old) work, reviewed in

- *The confinement problem in lattice gauge theory*, Prog. Part. Nucl. Phys. 51 (2003) 1, hep-lat/0301023.
- *An introduction to the confinement problem*, Springer Lect.Notes Phys. 821 (2011).
- Michael Engelhardt, *Lattice 2004 plenary*, Nucl.Phys.Proc.Suppl. 140 (2005) 92-105, hep-lat/0409023.

Newer work:

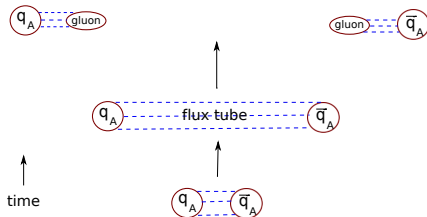
- Center vortices and chiral symmetry breaking (*Leinweber, Kamleh, and Trewartha*)
- Double-winding loops and implications for monopole/dyon confinement mechanisms (*Höllwieser & J.G*)

Motivation I

In any theory with a nonvanishing asymptotic string tension, the tension σ_r of static charges in color representation r depends only on the *N-ality* of the representation, not on r itself.

Reason: gluons can bind to color charges, reducing the dimension but not the N-ality of the effective charge.

This is a “*particle*” explanation.



Is there a purely “*field*” explanation for the N-ality dependence, in terms of configurations that dominate the vacuum state?

Focus on large vacuum fluctuations at all (large) scales: **Confinement as the phase of magnetic disorder.**

Which means:

- Wilson loop area law for all sufficiently large Wilson loops.
- Non-zero asymptotic string tension.

Defined in this way, confined and non-confined phases are distinguished by a symmetry, where

Confinement is the phase of unbroken center symmetry

Examples:

- gauge theories with Higgs fields in the adjoint representation
- the finite temperature deconfinement transition for pure gauge theories
- "string breaking:" matter fields which break center symmetry explicitly

Order parameters:

- Polyakov lines: $\langle P \rangle = 0$ in the confined phase.
- 't Hooft loops $B(C)$ are center vortex creation operators, which are “dual” to Wilson loops operators. They have a perimeter-law falloff in the confined phase.
- Center vortex free energy. Impose twisted boundary conditions to define the partition function Z_V , the vortex is oriented in the $L_z - L_t$ plane. Then

$$F_V = -\log \frac{Z_V}{Z_0} \quad , \quad \text{confinement if: } F_V = L_z L_t e^{-\rho L_x L_y}$$

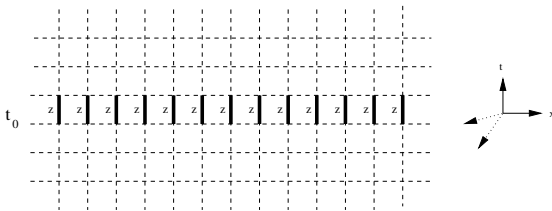
(Tomboulis and Jaffe (1985))

Center Symmetry

The center $Z_N \in SU(N)$ is the set of all elements $z_n \mathbb{1}$, $z_n = e^{2\pi i n/N}$ which commute with every member of $SU(N)$.

Let $R[g]$ be a representation of $SU(N)$. In general $R[zg] = z^M R[g]$. M is called the “N-ality” of the representation; it is also equal to the number of boxes in the Young tableau mod N .

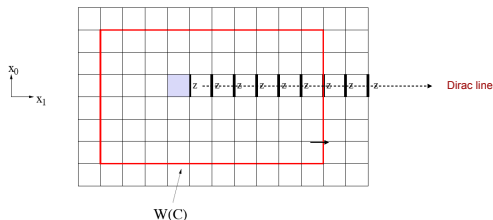
A lattice gauge action is center symmetric when it is invariant under the global transformation shown below.



Polyakov lines $P \rightarrow zP$ are **not** invariant, and serve as order parameters, with $\langle P \rangle = 0$ in the center-symmetric phase.

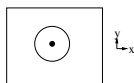
Center Vortex Creation I

Creation of a thin center vortex on an arbitrary background by a “bounded” center transformation. The vortex is the boundary.



A Wilson loop encircling the shaded plaquette picks up a factor of z .
“Encircling,” or in higher dimensions “linking,” is a topological concept.
A center vortex is point-like in $D=2$, line-like in $D=3$, surface-like in $D=4$.

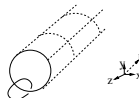
A loop can be linked to a point ($D=2$), another loop ($D=3$), or a surface ($D=4$)



(a)



(b)



(c)

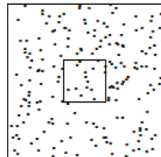
Creation of a vortex linked to a Wilson loop:

$$\text{Tr}U(C) \rightarrow z\text{Tr}U(C)$$

Confinement from Center Vortices

N vortices pierce a plane of area L^2 at random locations. The probability that n vortices will fall inside a loop of area A is

$$P_N(n) = \binom{N}{n} \left(\frac{A}{L^2}\right)^n \left(1 - \frac{A}{L^2}\right)^{N-n}$$



In $SU(2)$ each vortex contributes a factor of -1 , so

$$W(C) = \sum_{n=0}^N (-1)^n P_N(n) = \left(1 - \frac{2A}{L^2}\right)^N$$

Keep the vortex density $\rho = N/L^2$ fixed, taking $N, L \rightarrow \infty$, gives the Wilson loop area law falloff

$$W(C) = \lim_{N \rightarrow \infty} \left(1 - \frac{2\rho A}{N}\right)^N = e^{-2\rho A}$$

(The argument in this form is due to [Engelhardt, Reinhardt et al. \(1998\)](#).)

Finding Center Vortices

The confinement mechanism of Z_N lattice gauge theory is certainly the vortex mechanism (there are no other excitations), and $Z_N \in SU(N)$.

Strategy: Map $SU(N) \rightarrow Z_N$. The idea is that thin vortices of the Z_N theory, known as “*P-vortices*” may locate the thick vortices in the $SU(N)$ configuration.

- **Maximal Center Gauge:** Gauge fix so that $SU(N)$ link variables are as close as possible, on average to z_n center variables. The “direct” version is Landau gauge in the adjoint representation, and maximizes

$$R = \sum_{x, \mu} \text{Tr}[U_\mu(x)] \text{Tr}[U_\mu^\dagger(x)]$$

- **Center Projection:** Project each link variable to the closest Z_N variable. E.g. for $SU(2)$

$$z_\mu(x) = \text{sign Tr}[U_\mu(x)]$$

This is not guaranteed to work! Need to check that P-vortex locations are correlated with gauge-invariant observables.

A vortex-limited Wilson loop $W_n(C)$ is the VEV of Wilsons loops that are linked to n P-vortices.

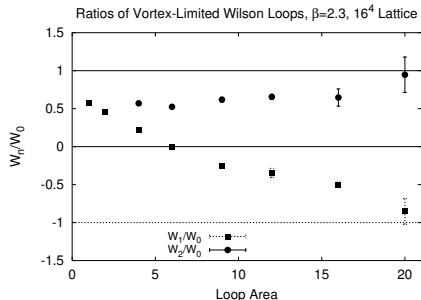
If P-vortices locate center vortices, then in SU(2) gauge theory, we expect for large loops,

$$\frac{W_n(C)}{W_0(C)} \rightarrow (-1)^n$$

Likewise, one finds for the asymptotic string tension σ

$$\sigma_n \rightarrow 0$$

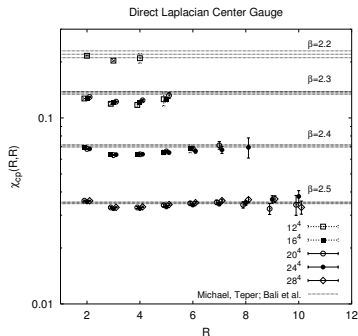
and plaquettes at P-vortex locations have a higher than average plaquette action.



With a method in hand to find vortices, we can carry out various tests.

- 1 Center Dominance: What string tension is obtained from P-vortices?
- 2 Vortex Removal: What is the effect of removing vortices, identified by center projection, from the lattice configuration?
- 3 Scaling: Does the density of vortices scale according to the asymptotic freedom prediction?
- 4 Finite Temperature
- 5 Topological Charge
- 6 Chiral symmetry breaking

A comparison of the asymptotic string tension of center-projected Creutz ratios with the asymptotic string tension of the unprojected theory at various β , for SU(2) gauge theory.

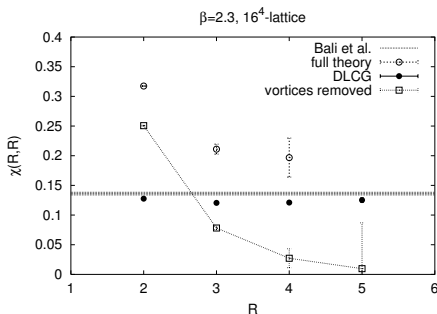


For SU(3), the Adelaide group finds similarly good agreement after some smoothing steps.
(Kamleh, Leinweber, Trewartha (2015))

Center vortices can be removed from a lattice configuration very simply:

$$U_\mu(x) \rightarrow U'_\mu(x) = Z_\mu^*(x) U_\mu(x)$$

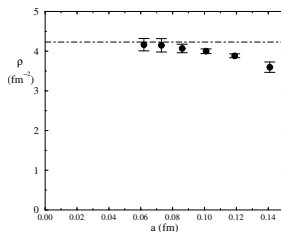
This procedure leaves all local gauge-invariant observables untouched, except exactly at P-vortex locations, where the action is increased. When the vortices are removed, confinement is also removed.



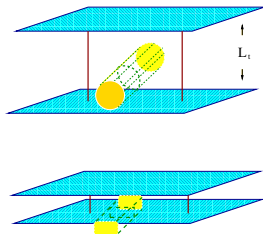
Vortex density in physical units

$$\rho = \frac{\text{vortex area}}{\text{lattice volume}} \frac{1}{a(\beta)^2}$$

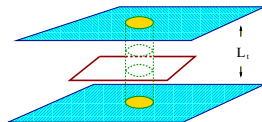
seems to have a finite continuum limit (Gubarev et al)



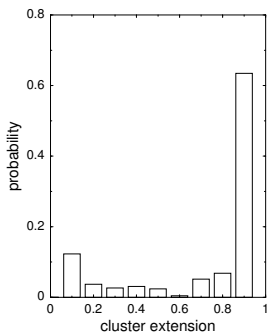
Deconfinement: static quark potential goes flat, but spacelike Wilson loops still have an area law.



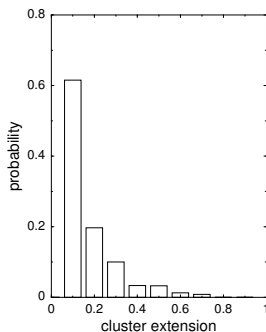
Vortices running in the spacelike directions disorder Polyakov loops. When the time extension L_t is smaller than the diameter of the vortex (high temperature case), then spacelike vortices are "squeezed" and cease to percolate.



Vortices running in the timelike direction disorder spacelike Wilson loops. The vortex cross section is not constrained by a small extension in the time direction.



(a)



(b)

Figure : Histograms of vortex extension in a space-slice at finite temperature, both below (left figure, $T = 0.7T_c$) and above (right figure, $T = 1.85T_c$) the deconfinement phase transition. From Engelhardt et al. (2000)

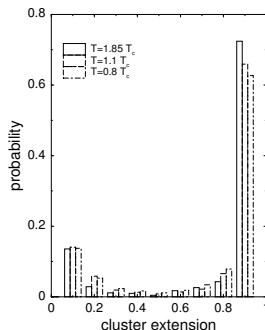
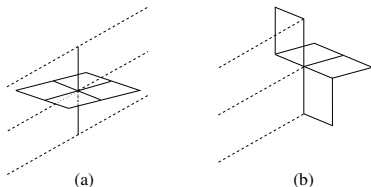


Figure : Histograms of vortex extension in a time slice at finite temperature. Vortices are identified by center projection at $\beta = 2.4$ in direct maximal center gauge. Data at three temperatures are shown on the same figure. From Engelhardt et al. (2000)

Topological Charge, Coulomb Confinement

- Vortex surfaces carry fractional topological charge at surface intersection points (a), and “writhing” points (b) (Engelhardt and Reinhardt)
- Numerical simulations of an effective vortex action (Engelhardt) gets the topological susceptibility about right (Bertle, Engelhardt, and Faber (2001)).
- .
- Strong connections to confinement scenarios in Coulomb gauge (Olejník, Zwanziger and JG). See also the talk of Hugo Reinhardt.



Some (relatively) new results from the Adelaide group (Kamleh, Leinweber, and Trewartha (2015)) in SU(3). They calculate the Landau gauge quark propagator using the overlap Dirac operator, for

- full (“untouched”)
- vortex-removed
- center projected (“vortex only”) after some cooling steps

and fit to the form

$$S(p) = \frac{Z(p)}{i\not{p} + M(p)}$$

It is found that after some smoothing, *the non-perturbative properties of full and vortex-only configurations are about the same!*

Here is what they find for full and vortex-removed:

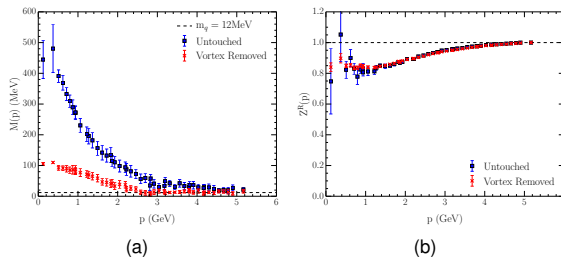


Figure : The mass (a) and renormalisation (b) functions on the original (untouched) (squares) and vortex-removed (crosses) configurations. Removal of the vortex structure from the gauge fields spoils dynamical mass generation and thus dynamical chiral symmetry breaking.

In contrast, cooled vortex-only looks about the same as full:

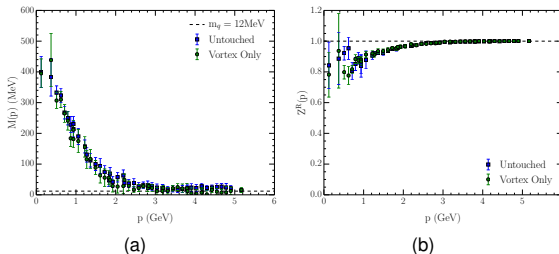


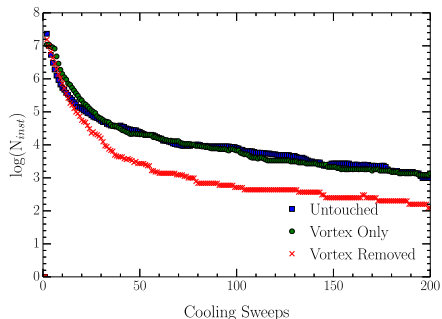
Figure : The mass(a) and renormalisation (b) functions on the original (untouched) (squares) and vortex-only (circles) configurations after 10 sweeps of three-loop $\mathcal{O}(a^4)$ -improved cooling, at an input bare quark mass of 12 MeV.

String tensions also agree, and the hadron spectra are very similar.

Vortices and Instantons?

From Leinweber et al.:

“By examining the local maxima of the action density on vortex-only configurations during cooling, we find that after just 10 sweeps of cooling these local maxima stabilize, and begin to resemble classical instantons in shape and corresponding topological charge density at the center.”



The authors speculate that center vortices contain the “seeds” of instantons, which are reproduced upon cooling.

The Adelaide group also computed low-lying hadron masses in vortex-only and vortex-removed ensembles. Results:

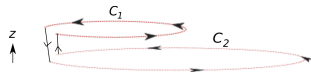
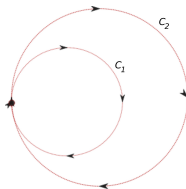
- The vortex-only spectrum is very similar to full QCD.
- The vortex-removed spectrum
 - shows chiral symmetry restoration for light quarks; pion is no longer a Goldstone boson.
 - is a weakly-interacting theory of constituent quarks at heavy quark masses.

Double-winding loops and abelian confinement mechanisms

We consider, in $SU(2)$ gauge theory, double-winding Wilson loops, coplanar and shifted:

computed according to

- 1 monopole/dyon plasma
- 2 dual superconductor
- 3 center vortex



theories of confinement.

Loops C_1 , C_2 have areas A_1 , A_2 respectively.
Ignore, initially, effects of “W”-bosons. We will return to them.

In the dual superconductor, by the dual Meissner effect, there are two flux sheets bounded by C_1 and C_2 . For loops in the RT-plane, there are two flux tubes in a time-slice.



This leads to a *sum-of-areas* falloff for the double-winding loop

$$W(C) \sim \exp[-\sigma(A_1 + A_2)]$$

In the case of a monopole or dyon plasma, following Polyakov and Diakonov-Petrov, there is a soliton spanning A_1 and another soliton spanning A_2 . *This leads again to a sum-of-areas law.*

Intuitively: two current loops in a monopole plasma are screened by two monopole-antimonopole sheets along A_1 and A_2 .

In particular, following Polyakov's classic calculation for U(1) gauge theory in D=3:

$$\langle W(C) \rangle = \frac{1}{Z_{mon}} \int D\chi(r) \exp \left[-\frac{g^2}{4\pi} \int d^3r \left(\frac{1}{2} (\partial_\mu (\chi - \eta_{S(C)})^2 - M^2 \cos \chi(r) \right) \right]$$

where

$$-\partial^2 \eta_{S(C)} = 2\pi \delta'(z) \theta_{S_2}(x, y) + 2\pi \delta'(z - \delta z) \theta_{S_1}(x, y)$$

and $\theta_{S_{1(2)}}(x, y) = 1$ if x, y lie in the minimal area of C_1 (C_2), and is zero otherwise. Assuming $\delta z \gg 1/M$, an approximate saddlepoint solution is the superposition

$$\begin{aligned} \chi = & \text{sign } z \cdot 4 \arctan(e^{-M|z|}) \theta_{S_2}(x, y) \\ & + \text{sign}(z - \delta z) \cdot 4 \arctan(e^{-M|z - \delta z|}) \theta_{S_1}(x, y) \end{aligned}$$

leading to the sum-of-areas law.

In contrast, the prediction of the vortex mechanism is a *difference-of-areas* falloff

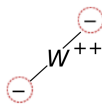
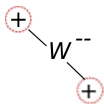
$$W(C) \sim \exp[-\sigma |A_2 - A_1|]$$

This is because a vortex only multiplies by a center element if it passes through the larger loop, but not the smaller loop

The numerical evidence is very clearly in favor of *difference-of-areas* falloff. (Roman Höllwieser and J.G., [arXiv:1411.5091](https://arxiv.org/abs/1411.5091))

What about the W's?

To get a difference-in-areas law, the abelian models must appeal to screening by W-particles (gluons charged in the abelian subgroup)



But this raises the following question: in such models, *what happens if we integrate out the W's?* What are then the confining configurations?

They can't be simply the monopole/dyon plasma or dual superconductor, since those by themselves give the wrong answer for double-winding loops.

Suggestion: After integrating out the W's, the result is center vortices.

There is one example where this can be shown to occur.

Example: U(1) gauge theory with charge 2 matter fields

Fixed modulus $|\rho| = 1$ double-charged matter field, $\beta \ll 1$ (confinement), $\lambda \gg 1$.

$$Z = \int D\rho D\theta_\mu \exp \left[\beta \sum_p \cos(\theta(p)) + \frac{1}{2} \lambda \sum_{x,\mu} \left\{ \rho^*(x) e^{2i\theta_\mu(x)} \rho(x + \hat{\mu}) + \text{c.c.} \right\} \right]$$

In the absence of matter fields we have confinement via a Coulomb gas of magnetic monopoles, and string tension is proportional to charge. But with those matter fields, even charged Wilson loops $\langle U(C)^{2n} \rangle$ have zero string tension, and odd-charged loops have the same string tension.

This changes the monopole plasma to an ensemble of Z_2 center vortices.

Reason: go to unitary gauge $\rho = 1$, and make the field decomposition

$$\exp[i\theta_\mu(x)] = z_\mu(x) \exp[i\tilde{\theta}_\mu(x)]$$

where

$$z_\mu(x) \equiv \text{sign}[\cos(\theta_\mu(x))]$$

and

$$Z = \prod_{x,\mu} \sum_{z_\mu(x)=\pm 1} \int_{-\pi/2}^{\pi/2} \frac{d\tilde{\theta}_\mu(x)}{2\pi} \exp \left[\beta \sum_p Z(p) \cos(\tilde{\theta}(p)) + \lambda \sum_{x,\mu} \cos(2\tilde{\theta}_\mu(x)) \right]$$

One can easily show, for $\beta \ll 1$, $\lambda \gg 1$, that

$$\langle \exp[in\theta(C)] \rangle \approx \langle Z^n(C) \rangle \langle \exp[in\tilde{\theta}(C)] \rangle$$

with

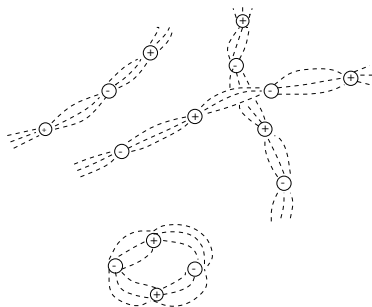
$$\begin{aligned} \langle Z^n(C) \rangle &= \begin{cases} \exp[-\sigma A(C)] & n \text{ odd} \\ 1 & n \text{ even} \end{cases} \\ \langle \exp[in\tilde{\theta}(C)] \rangle &= \exp[-\mu n^2 P(C)] \end{aligned}$$

where $Z(C)$ is the product of $z_\mu(x)$ link variables around the loop C .

This establishes that the *confining fluctuations are thin Z_2 center vortices* identified by the z_μ variables in unitary gauge.

One way to picture what has happened: the field of monopoles and antimonopoles is collimated into Z_2 vortices.

Something like this:



Whatever your favorite “confiners” are, perhaps this is what you get after integrating out the W 's.

In fact, this is what is seen in $SU(2)$ gauge theory, on abelian-projected lattices.

Conclusions: On the one hand...

Center vortices provide a plausible & well-motivated mechanism for

- 1 confinement
- 2 deconfinement
- 3 chiral symmetry breaking
- 4 generation of topological charge

It is not just a model. Center vortices are found in lattices generated by computer simulations, and

- 1 Vortex density scales according to asymptotic freedom.
- 2 Vortex-only configurations account (more-or-less accurately) for the observed asymptotic string tension, chiral symmetry breaking, instanton density, and hadron spectrum.
- 3 i.e. smoothed vortex-only configurations are *almost identical*, in their non-perturbative properties, to full configurations.
- 4 Vortex-removed configurations are completely different (no confinement, no chiral symmetry breaking, etc.)

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- 1 This mechanism does not lend itself to analytical treatment.
- 2 A simple, effective theory of center vortices, having many of the features of infrared QCD, can be simulated numerically (Michael Engelhardt). But then it can be argued...
- 3 you might as well just simulate QCD.

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