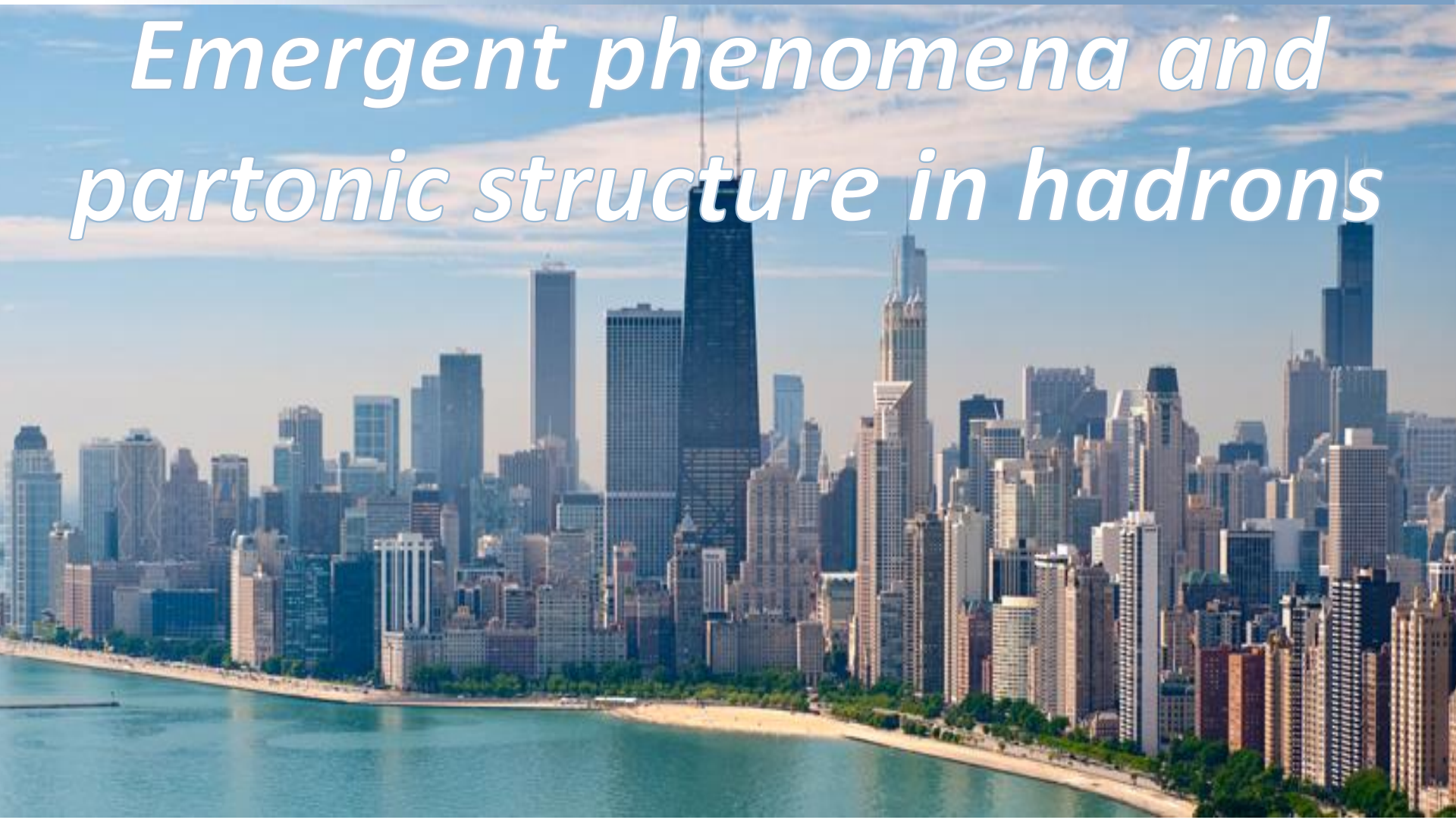
Argonne NATIONAL LABORATORY

Emergent phenomena and partonic structure in hadrons



Craig Roberts, Physics Division

U.S. DEPARTMENT OF ENERGY

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What & where is mass?

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Whence Mass?

- Classical chromdynamics … non-Abelian local gauge theory
- Remove the current mass … no energy scale left
- *No dynamics in a scale-invariant theory*; only kinematics … the theory looks the same at all length-scales … there can be no clumps of anything … *hence bound-states are impossible*.
- *Our Universe can't exist*
- *Higgs boson doesn't solve this problem* … normal matter is constituted from light-quarks & the mass of protons and neutrons, the kernels of all visible matter, are 100-times larger than anything the Higgs can produce
- *Where did it all begin?*
  … becomes … Where did it all come from?

Whence Mass?

- Poincaré invariance entails that the Energy-Momentum Tensor is divergence-free, *i.e*. it defines a conserved current:

$$\partial_\mu T_{\mu\nu} = 0$$

$T_{\mu\nu}$ can *always* be made symmetric

- Noether current associated with a global scale transformation:

  $x \to e^{-\sigma} x$

  is the dilation current: $D_{\mu\nu}=T_{\mu\nu}\, x_\nu$

- In a scale invariant theory, the dilation current is conserved

$$\partial_\mu \mathcal{D}_\mu = 0 = [\partial_\mu T_{\mu\nu}]x_\nu + T_{\mu\nu}\delta_{\mu\nu}$$
$$= T_{\mu\mu}\,,$$

- Consequently, in a scale invariant theory

  the *energy-momentum tensor must be traceless*.

Trace Anomaly

- Classical chromodynamics is meaningless … must be quantised
- Regularisation and renormalisation of (ultraviolet) divergences introduces a mass-scale

  … *dimensional transmutation*: mass-dimensionless quantities become dependent on a mass-scale, ζ
- $\alpha \to \alpha(\zeta)$ in QCD's (massless) Lagrangian density, $\mathcal{L}(m=0)$

  Under a scale transformation $\zeta \to e^{\sigma}\zeta$, then $\alpha \to \sigma\, \alpha\beta(\alpha)$

  $$\mathcal{L} \to \sigma\, \alpha\beta(\alpha)\, d\mathcal{L}/d\alpha$$

  $$\Rightarrow \partial_\mu \mathcal{D}_\mu = \delta\mathcal{L}/\delta\sigma = \alpha\beta(\alpha)\, d\mathcal{L}/d\alpha = \boxed{\beta(\alpha)\, \tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} = T_{\rho\rho} =: \Theta_0}$$

  Trace anomaly
- Straightforward, nonperturbative derivation, without need for diagrammatic analysis …

  quantisation of renormalisable four-dimensional theory forces nonzero value for trace of energy-momentum tensor

Where is the mass?

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- Knowing that a trace anomaly exists does not deliver a great deal … indicates only that a mass-scale exists
- Can one compute and/or understand the magnitude of that scale?
- One can certainly *measure* the magnitude … consider proton:

$$\langle p(P)|T_{\mu\nu}|p(P)\rangle = -P_\mu P_\nu$$

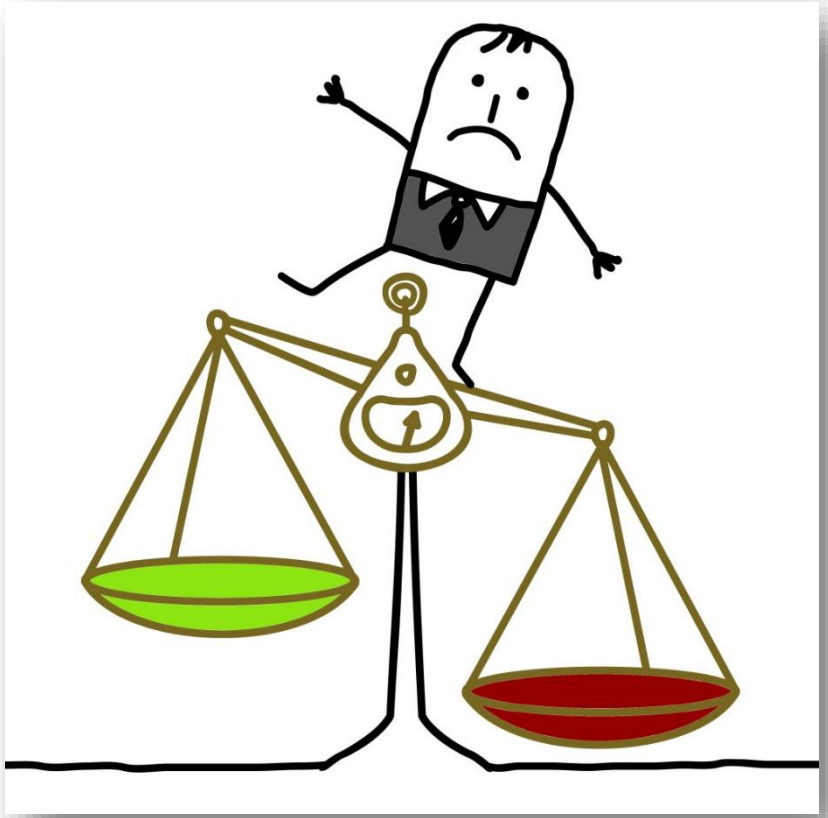
$$\langle p(P)|T_{\mu\mu}|p(P)\rangle = -P^2 = m_p^2$$

$$= \langle p(P)|\Theta_0|p(P)\rangle$$

- In the chiral limit the entirety of the proton's mass is produced by the trace anomaly, Θ_0

  … In QCD, Θ_0 measures the strength of gluon self-interactions

  … so, from one perspective, m_p is completely generated by glue.



On the other hand …

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- In the chiral limit

$$\langle\pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \;\Rightarrow\; \langle\pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- Does this mean that the scale anomaly vanishes trivially in the pion state, *i.e*. gluons contribute nothing to the pion mass?
- That is a difficult way to obtain "zero"
- Easier, perhaps, to imagine that "zero" owes to cancellations between different operator-component contributions to Θ_0.
- Of course, such precise cancellation should not be an accident.

It could only arise naturally because of some symmetry and/or symmetry-breaking pattern.

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- In the chiral limit

$$\langle\pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \;\Rightarrow\; \langle\pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- *No statement of the question*

"Whence the proton's mass?"

is complete without the additional clause

"Whence the absence *of a pion mass?"*

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Whence?

- Classical chromodynamics … non-Abelian local gauge theory
- Local gauge invariance; but there is no confinement without a mass-scale
  - Three quarks can still be colour-singlet
  - Colour rotations will keep them colour singlets
  - But they need have no proximity to one another
    … proximity is meaningless in a scale-invariant theory
- Whence mass … equivalent to whence a mass-scale … equivalent to whence a confinement scale
- *Understanding the origin and absence of mass in QCD is quite likely inseparable from the task of understanding confinement. Existence alone of a scale anomaly answers neither question*

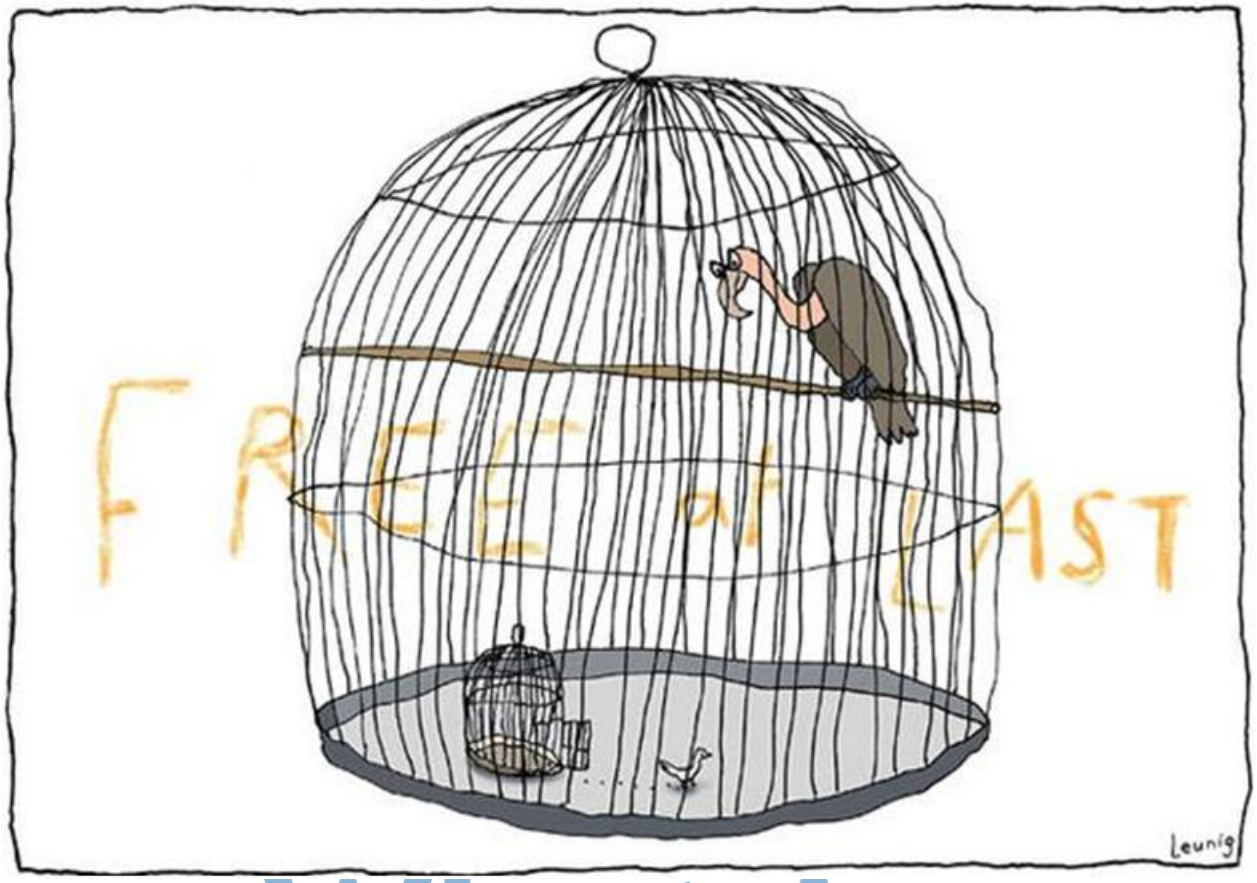
A New Era for hadro-particle physics

Overarching Science Challenges for the coming decade

- What is origin of mass in our Universe?
- What is the nature of confinement in real (dynamical-quarks) QCD?
- How are they connected?
- How can any
  - answers,
  - conjectures
  - and/or conclusions

  be empirically verified?

Physics is an Empirical Science



What is Confinement?

CLAY MATHEMATICS INSTITUTE

Millennium prize of $1,000,000 for proving that $SU_c(3)$ gauge theory is mathematically well-defined, which will necessarily prove or disprove the confinement conjecture

YANG–MILLS EXISTENCE AND MASS GAP. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in* [**45, 35**].

5. Comments

An important consequence of the existence of a mass gap is clustering: Let $\vec{x} \in \mathbb{R}^3$ denote a point in space. We let H and $\vec{P}$ denote the energy and momentum, generators of time and space translation. For any positive constant $C < \Delta$ and for any local quantum field operator $\mathcal{O}(\vec{x}) = e^{-i\vec{P}\cdot\vec{x}}\mathcal{O}e^{i\vec{P}\cdot\vec{x}}$ such that $\langle\Omega, \mathcal{O}\Omega\rangle = 0$, one has

$$|\langle\Omega, \mathcal{O}(\vec{x})\mathcal{O}(\vec{y})\Omega\rangle| \leq \exp(-C|\vec{x} - \vec{y}|), \tag{2}$$

as long as $|\vec{x} - \vec{y}|$ is sufficiently large. Clustering is a locality property that, roughly speaking, may make it possible to apply mathematical results established on $\mathbb{R}^4$ to any 4-manifold, as argued at a heuristic level (for a supersymmetric extension of four-dimensional gauge theory) in [**49**]. Thus the mass gap not only has a physical significance (as explained in the introduction), but it may also be important in mathematical applications of four-dimensional quantum gauge theories to geometry. In addition the existence of a uniform gap for finite-volume approximations may play a fundamental role in the proof of existence of the infinite-volume limit.

There are many natural extensions of the Millennium problem. Among other things, one would like to prove the existence of an isolated one-particle state (an upper gap, in addition to the mass gap), to prove confinement, to

Confinement?

Craig Roberts. Emergence of Partonic Structure (60p)

CLAY MATHEMATICS INSTITUTE

142 MILLENNIUM PRIZE PROBLEMS

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Confinement?

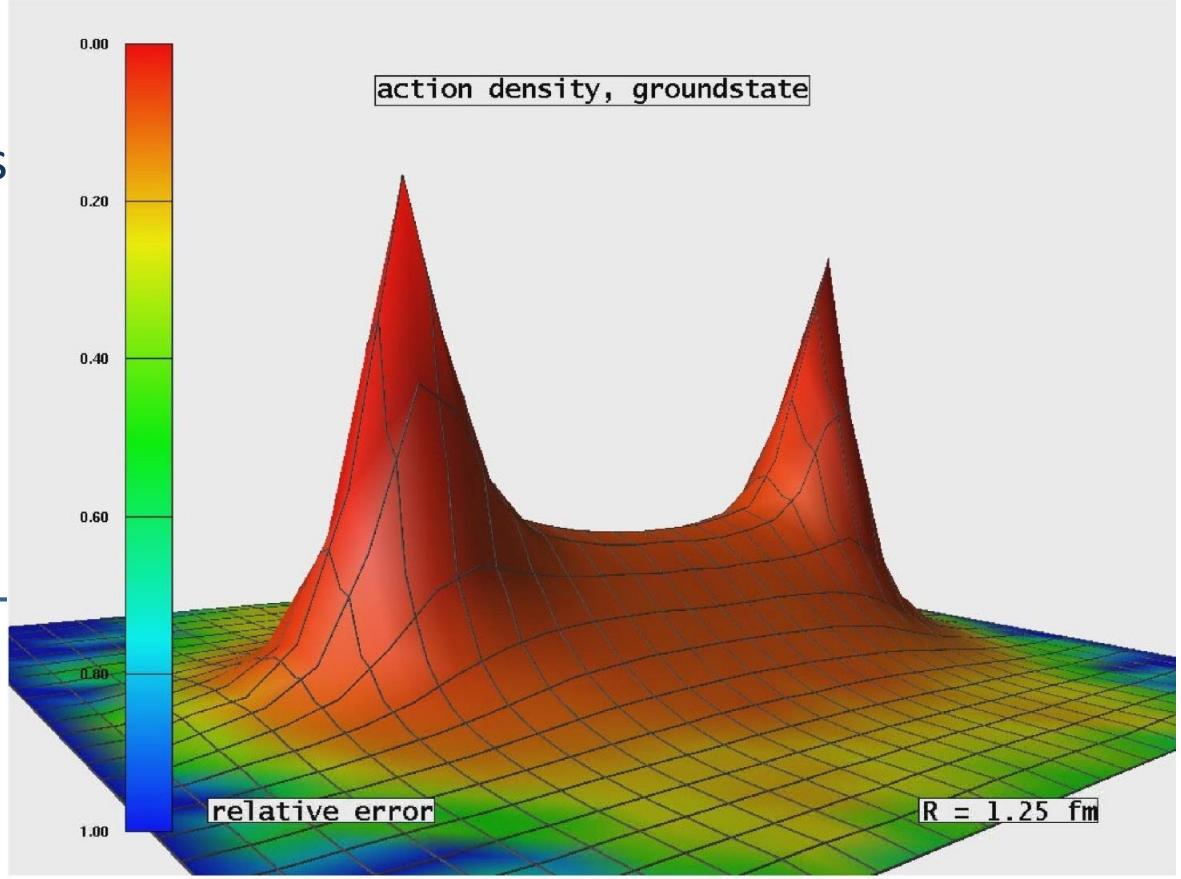
Light quarks & Confinement

- Folklore … ***Hall-D Conceptual Design Report(5)***

  "The color field lines between a quark and an anti-quark form flux tubes.

  A unit area placed midway between the quarks and perpendicular to the line connecting them intercepts a constant number of field lines, independent of the distance between the quarks.

  This leads to a constant force between the quarks – and a large force at that, equal to about 16 metric tons."



Light quarks & Confinement

- Problem:

  16 tonnes of force

  makes a lot of pions.

Light quarks & Confinement

- Problem:

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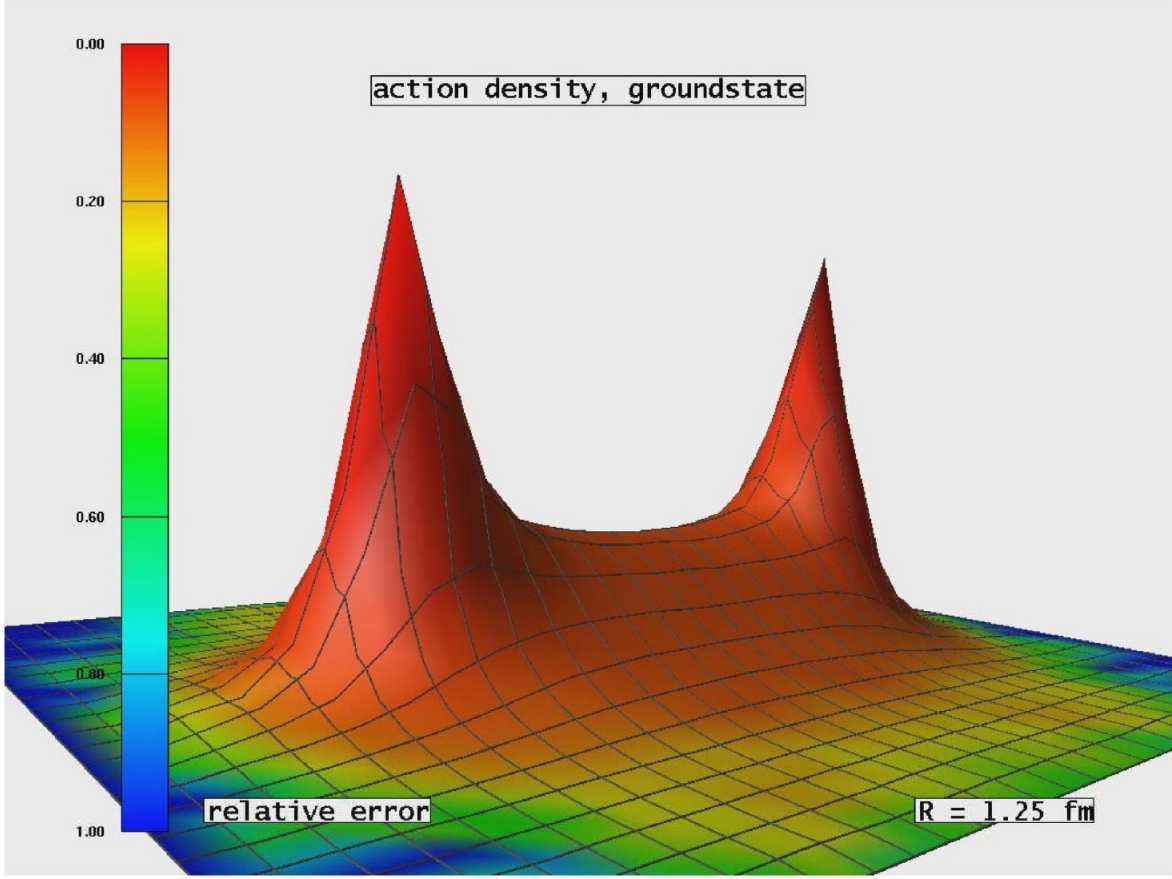
G. Bali et al., PoS LAT2005 (2006) 308

Light quarks & Confinement

- In the presence of light quarks, *pair creation seems to occur non-localized and instantaneously*
- No flux tube in a theory with light-quarks.
- ***Flux-tube is not the correct paradigm for confinement in hadron physics***



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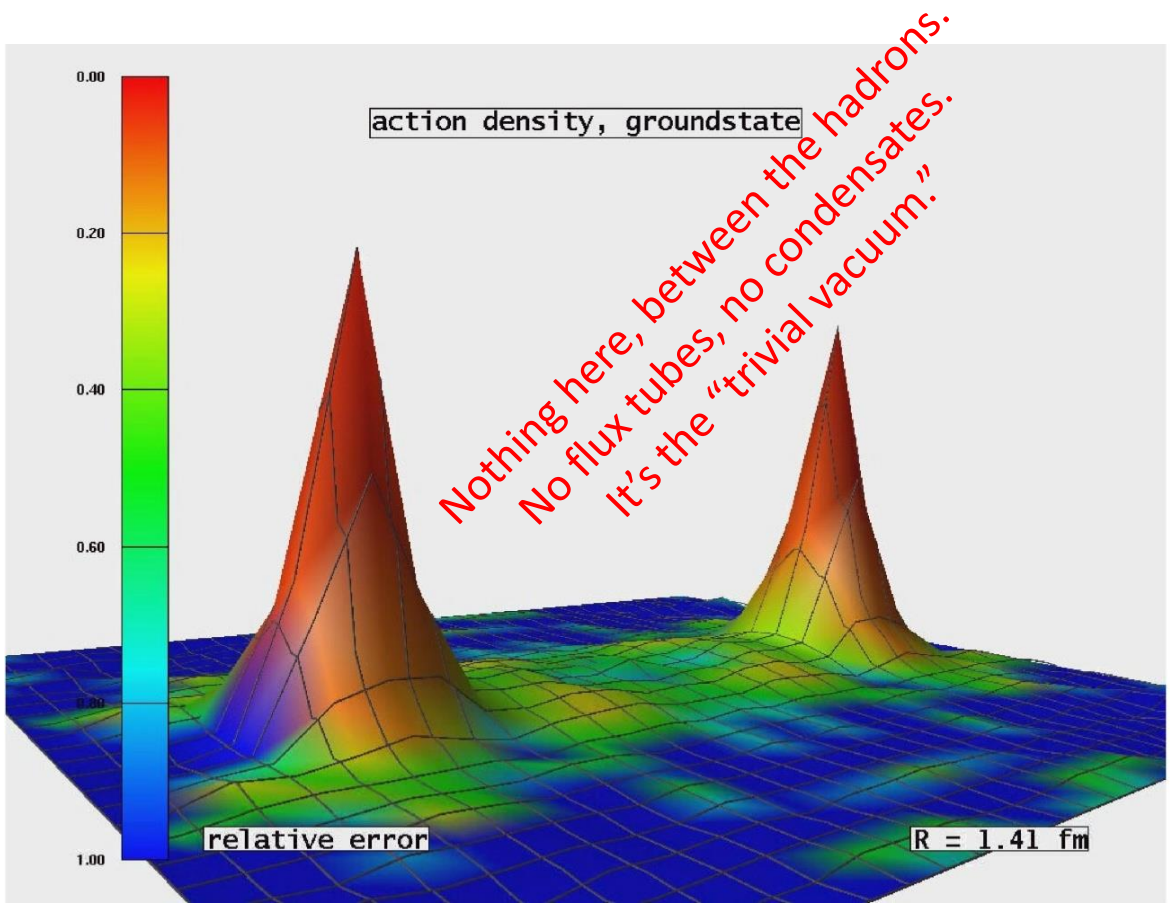
Confinement contains condensates
Brodsky, Roberts, Shrock,Tandy
arXiv:1202.2376 [nucl-th], Phys. Rev. C**85** (2012) 065202



- *Existence of mass-gap in pure-gauge theory*
- Strong evidence supporting this conjecture: lQCD predicts $\Delta \sim 1.5$ GeV
- But $\Delta^2/m_\pi^2 > 100$,

  So, can mass-gap in pure Yang-Mills play any role in understanding confinement when dynamical chiral symmetry breaking (DCSB) ensures existence of an almost-massless strongly-interacting excitation in our Universe?
- If *answer is* not *simply* ***no***, then it must be that one cannot claim to provide an understanding of confinement without simultaneously explaining its connection with DCSB.
- *Pion must play critical role in any explanation of real-world confinement; and any discussion that omits reference to the pion's role is practically irrelevant.*

YANG–MILLS EXISTENCE AND MASS GAP. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in* [**45, 35**].

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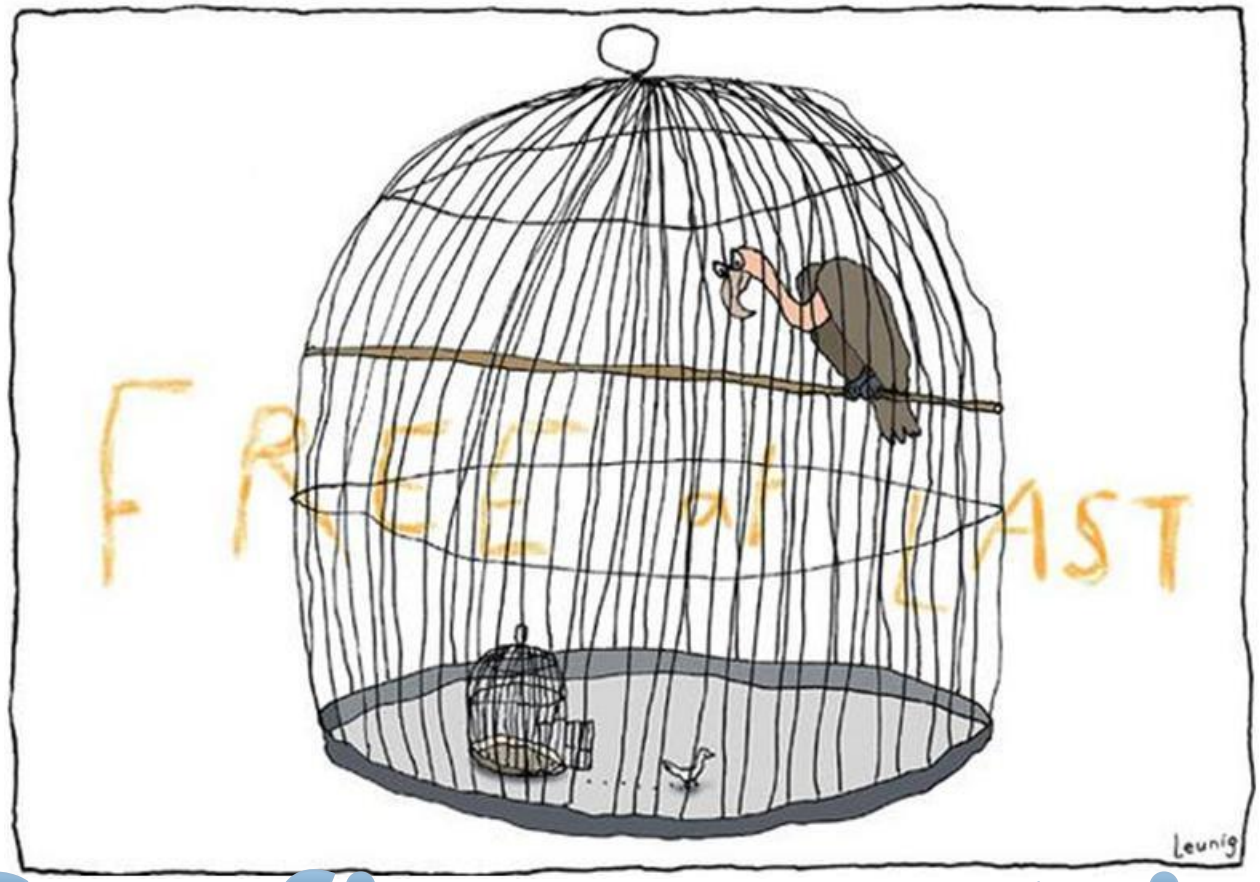


Confinement is dynamical

Confinement

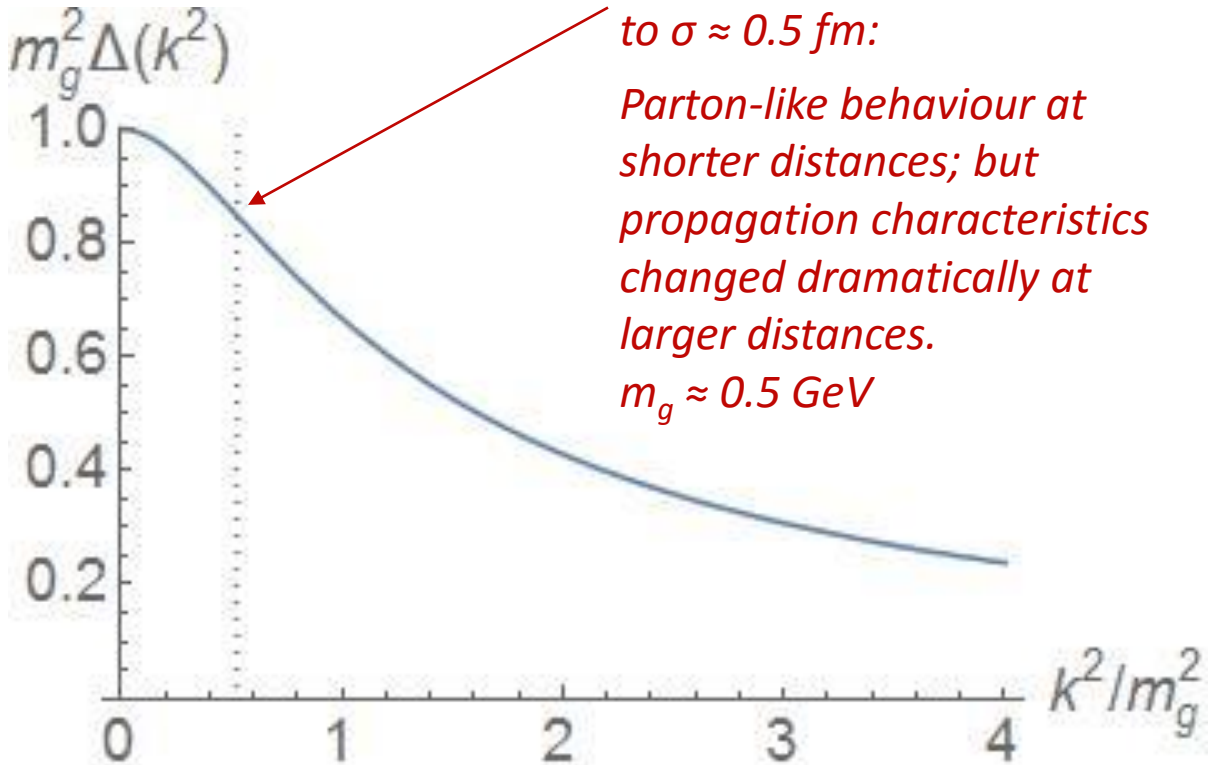
- All continuum and lattice predictions for Landau-gauge gluon & quark propagators exhibit an inflection point in k^2

⇒ Violate reflection positivity = sufficient for confinement

⇒ Such states have negative norm

⇒ Negative norm states are not observable

⇒ All observable states of a physical Hamiltonian have positive norm



Inflexion point corresponds to σ ≈ 0.5 fm:

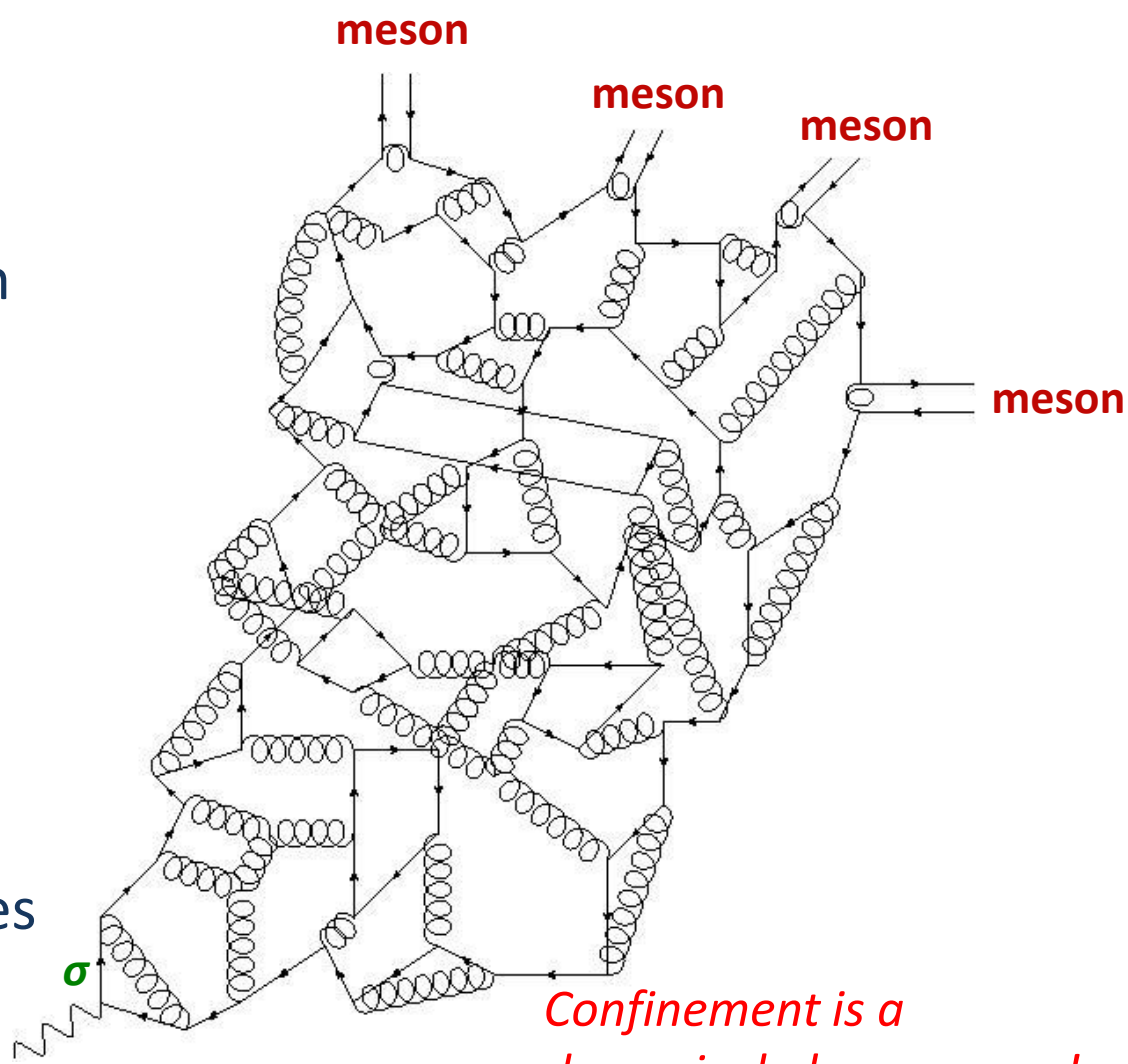
Parton-like behaviour at shorter distances; but propagation characteristics changed dramatically at larger distances.

m_g ≈ 0.5 GeV

Test: compute fragmentation functions & TMDs ⇒ compare with data

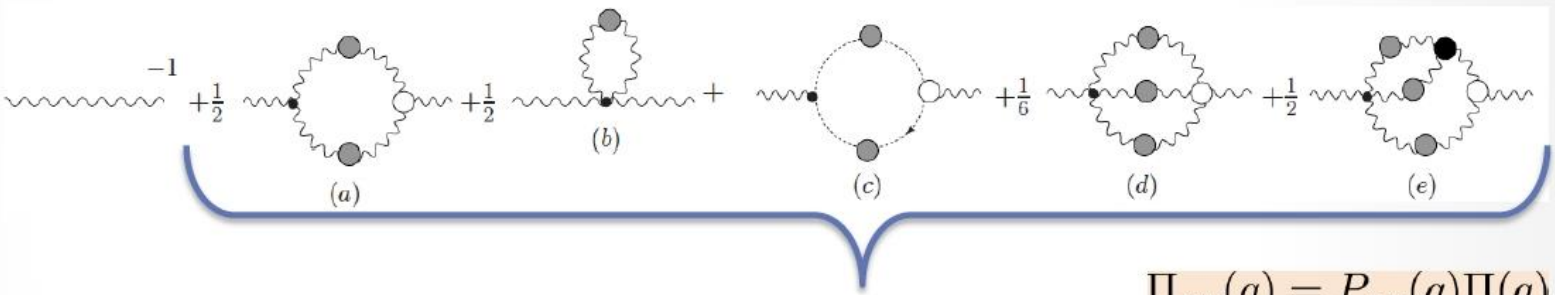
Quark Fragmentation

- A quark begins to propagate
- But after each "step" of length ***σ***, on average, an interaction occurs, so that the quark *loses* its identity, sharing it with other partons
- Finally, a cloud of partons is produced, which coalesces into colour-singlet final states



Confinement is a dynamical phenomenon!

$$\Delta^{-1}_{\mu\nu}(q) =$$



$$\Pi_{\mu\nu}(q)$$

$$\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$$

$$P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$$

Gluon Gap Equation

Bridging a gap between continuum-QCD and ab initio predictions of hadron observables, D. Binosi *et al.*, arXiv:1412.4782 [nucl-th], Phys. Lett. B742 (2015) 183-188

In QCD: Gluons also become massive!

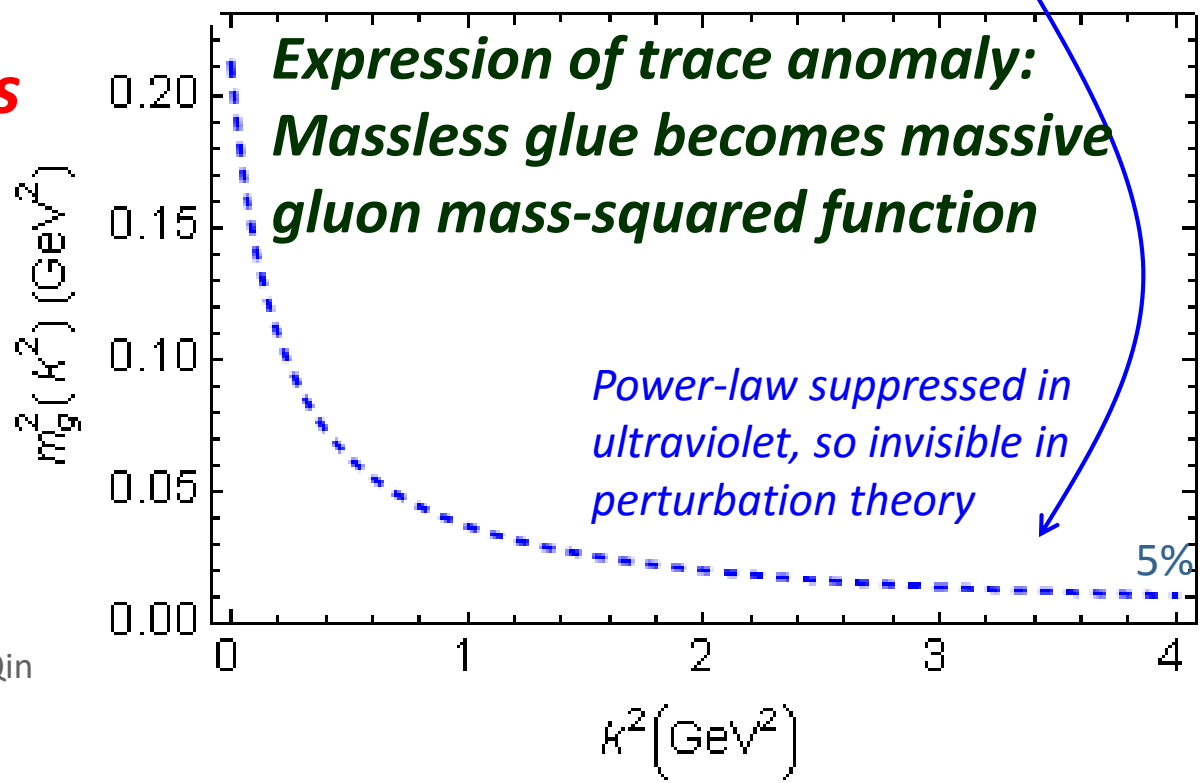
- Running gluon mass

$$d(k^2) = \frac{\alpha(\zeta)}{k^2 + m_g^2(k^2;\zeta)}$$

$$\alpha_s(0) = 2.77 \approx 0.9\pi\,,\ m_g^2(0) = (0.46\,\mathrm{GeV})^2$$

$$m_g^2(k^2) \approx \frac{\mu_g^4}{\mu_g^2 + k^2}$$

- Gluons are ***cannibals*** – a particle species whose members become massive by eating each other!



Interaction model for the gap equation, S.-x.Qin *et al.*, arXiv:1108.0603 [nucl-th], Phys. Rev. C **84** (2011) 042202(R) [5 pages]

Massive Gauge Bosons!

- Gauge boson cannibalism

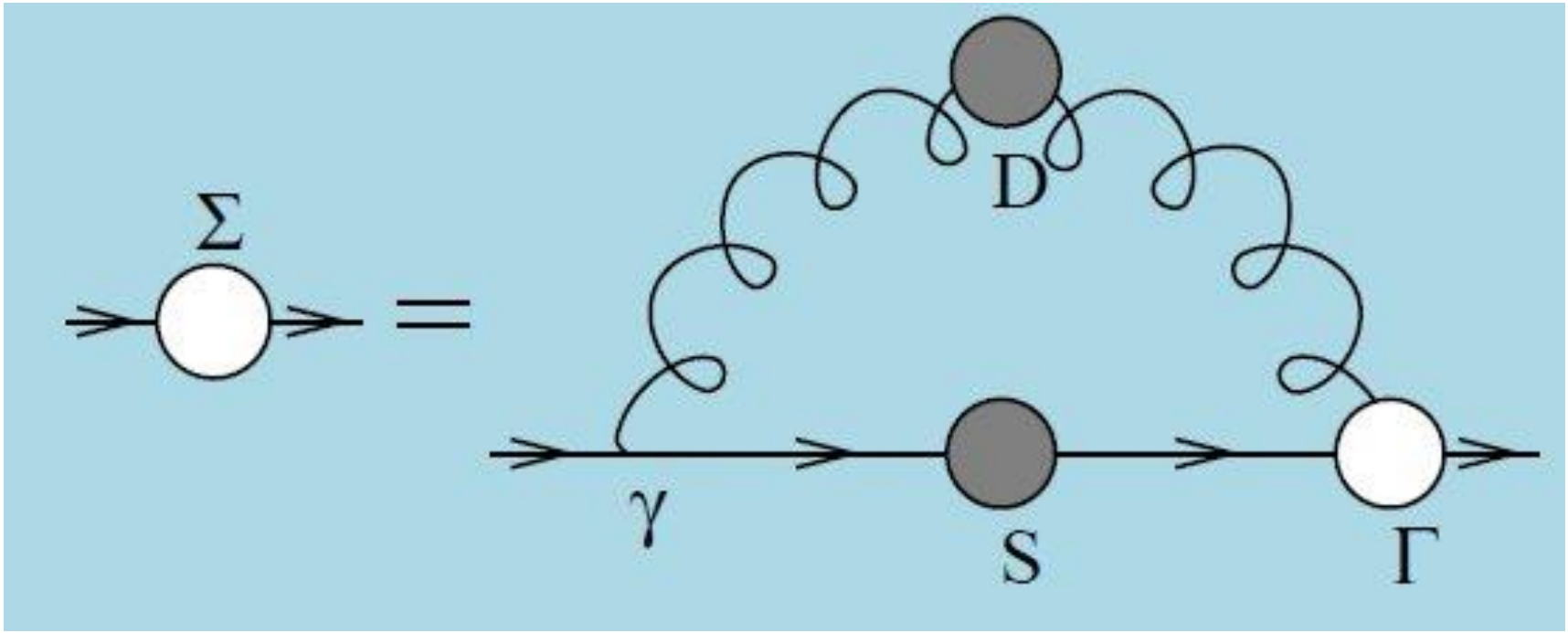
  … a new physics frontier … within the Standard Model
- Asymptotic freedom means
  … ultraviolet behaviour of QCD is controllable
- Dynamically generated masses for gluons and quarks means that QCD dynamically generates its own infrared cutoffs
  - Gluons and quarks with

    wavelength $\lambda > 2/\text{mass} \approx 1$ fm

    decouple from the dynamics … ***Confinement?!***
- How does that affect observables?
  - It will have an impact in any continuum study
  - Probably plays a role in gluon saturation … In fact, a harbinger of gluon saturation?

Electron Ion Collider: The Next QCD Frontier

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

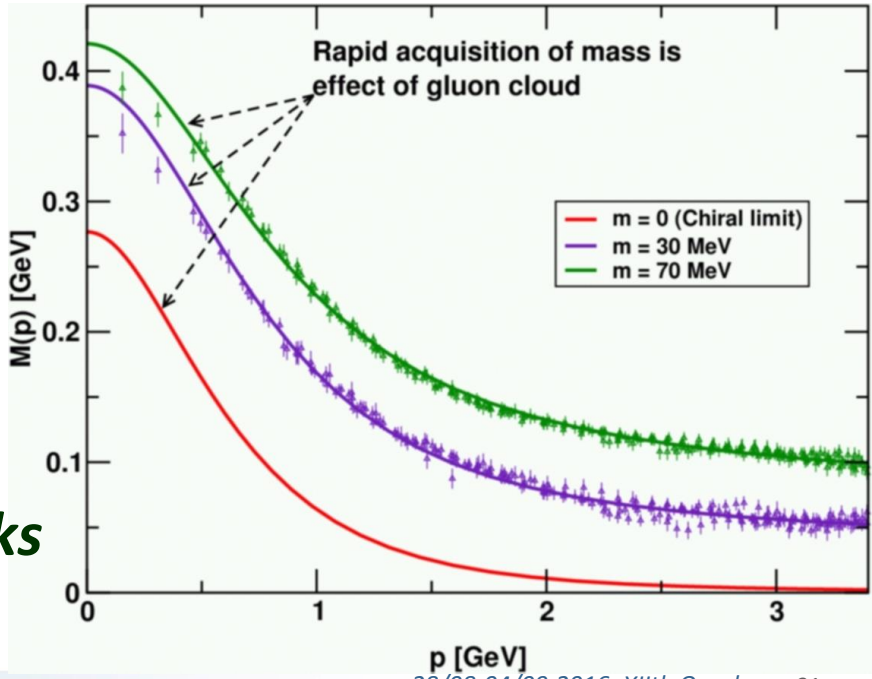


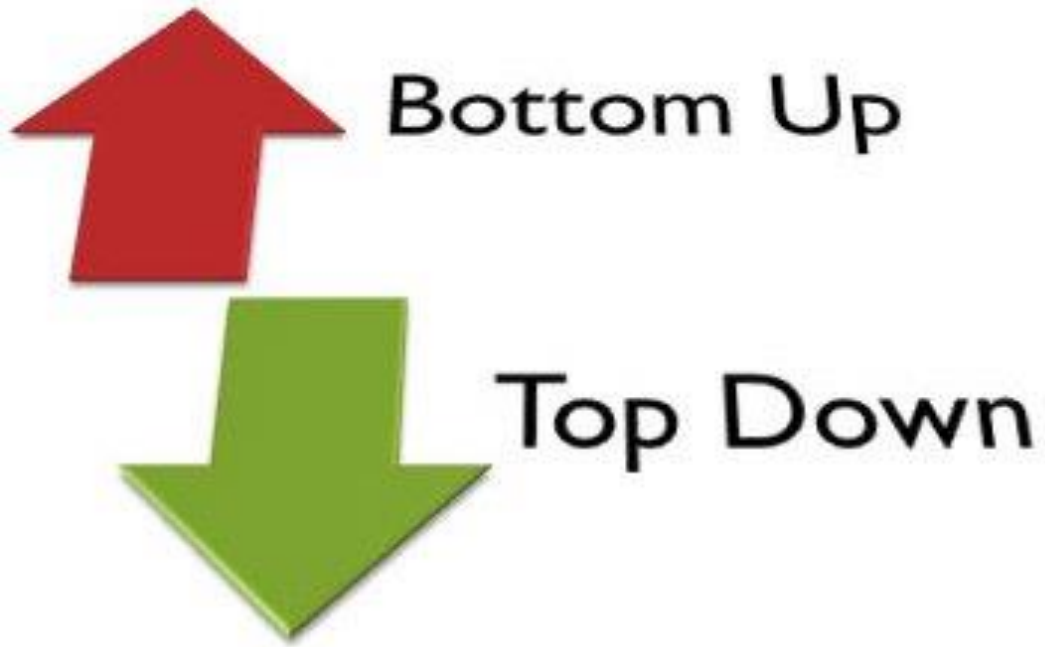
Quark Gap Equation

DCSB

- Dynamical chiral symmetry breaking (DCSB) is a critical emergent phenomenon in QCD
- Expressed in hadron wave functions not in vacuum condensates
- Contemporary theory indicates that DCSB is responsible for more than 98% of the visible mass in the Universe; namely, given that classical massless-QCD is a conformally invariant theory, then DCSB is the origin of *mass from nothing*.
- ***Dynamical***, not spontaneous
  - Add nothing to *QCD* ,

    No Higgs field, nothing!
    Effect achieved purely through quark+gluon dynamics.
  - ✓ ***Trace anomaly: massless quarks become massive***





Continuum-QCD & *ab initio* predictions

Bridging a gap between continuum-QCD & ab initio *predictions of hadron observables*

Top down & Bottom up

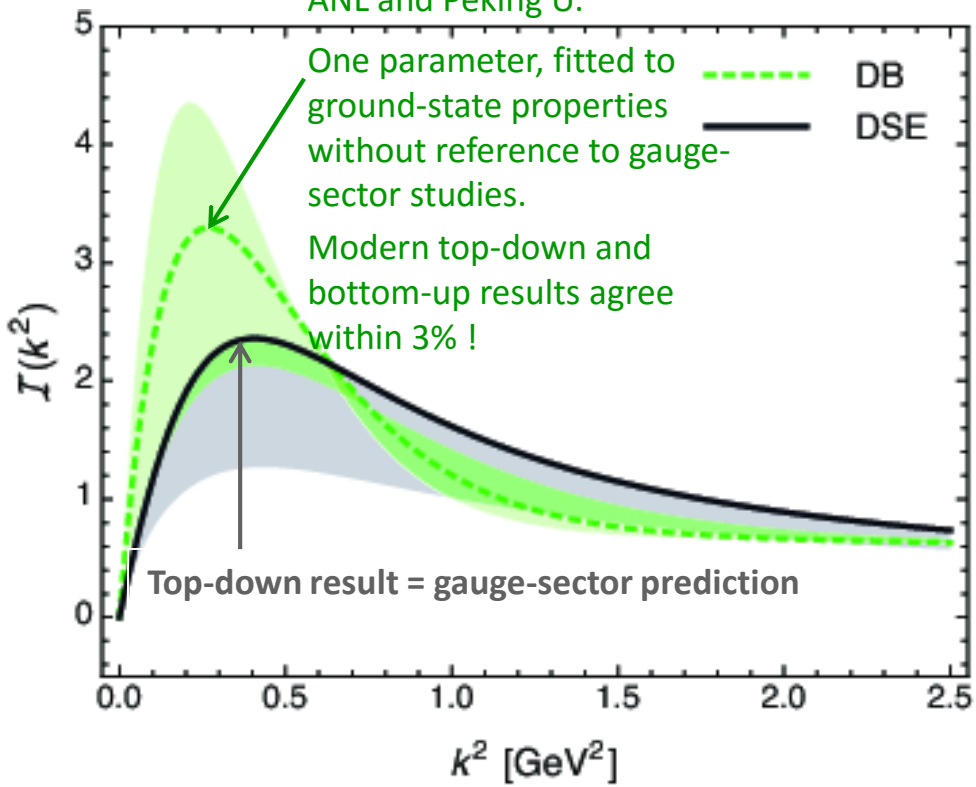
D. Binosi (Italy), L. Chang (Australia), J. Papavassiliou (Spain),
C. D. Roberts (US), arXiv:1412.4782 [nucl-th], *Phys. Lett. B* ***742*** *(2015) 183*

- *Top-down approach* – ab initio *computation of the interaction via direct analysis of the gauge-sector gap equations*
- *Bottom-up scheme – infer interaction by fitting data within a well-defined truncation of the matter sector DSEs that are relevant to bound-state properties.*
- *Serendipitous collaboration, conceived at one-week ECT* Workshop on* DSEs in Mathematics and Physics, *has united these two approaches*

Modern kernels and interaction, developed at ANL and Peking U.

One parameter, fitted to ground-state properties without reference to gauge-sector studies.

Modern top-down and bottom-up results agree within 3% !



– Interaction predicted by modern analyses of QCD's gauge sector coincides with that required to describe ground-state observables using a sophisticated matter-sector DSE truncation

Bridging a gap between continuum-QCD & ab initio predictions of hadron observables

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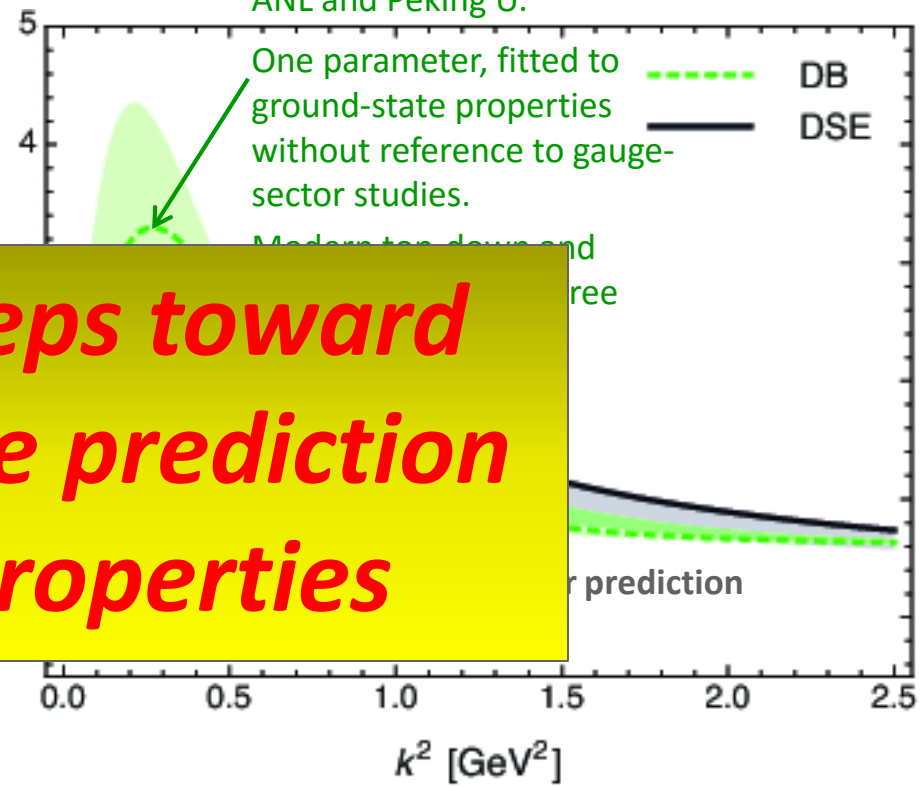


Significant steps toward parameter-free prediction of hadron properties

– Interaction predicted by modern analyses of QCD's gauge sector coincides with that required to describe ground-state observables using a sophisticated matter-sector DSE truncation

Natural constraints on the gluon-quark vertex,
Binosi, Chang, Papavassiliou, Qin & Roberts,
nearing completion

Reconciliation demands dressed-gluon-quark vertex

- Significant progress since 2009:
  dressed Γ_μ in gap- and Bethe-Salpeter equations …
- In principle, ∃ unique form of Γ_μ , but it's still obscure.
- To improve this situation, used the top-down/bottom-up RGI running-interaction
  - Computed gap equation solutions with
    1,660,000 distinct *Ansätze* for Γ_μ
- Each one of the solutions tested for compatibility with three physical criteria
- Remarkably, merely 0.55% of solutions survive the test

⇒ Even a small selection of observables places extremely tight bounds on the domain of acceptable, realistic vertex *Ansätze*

$$\tau_1^{qk} = a_1 \frac{\Delta_B^{qk}}{q^2+k^2} \qquad \tau_3^{qk} = -a_3 2\Delta_A^{qk}, \qquad T_\nu^1 = \frac{i}{2} t_\nu^{\mathrm{T}}, \qquad T_\nu^3 = \gamma_\nu^{\mathrm{T}},$$

$$\tau_4^{qk} = a_4 \frac{4\Delta_B^{qk}}{t^{\mathrm{T}} \cdot t^{\mathrm{T}}}, \quad \tau_5^{qk} = a_5 \Delta_B^{qk}, \qquad T_\nu^4 = -iT_\nu^1 \sigma_{\alpha\beta} q_\alpha k_\beta, \quad T_\nu^5 = \sigma_{\nu\rho} p_\rho,$$

$$\tau_8^{qk} = a_8 \Delta_A^{qk}, \qquad T_\nu^8 = q_\nu \gamma \cdot k - k_\nu \gamma \cdot q + i\gamma_\nu \sigma_{\alpha\beta} q_\alpha k_\beta, \qquad \text{(A1)}$$

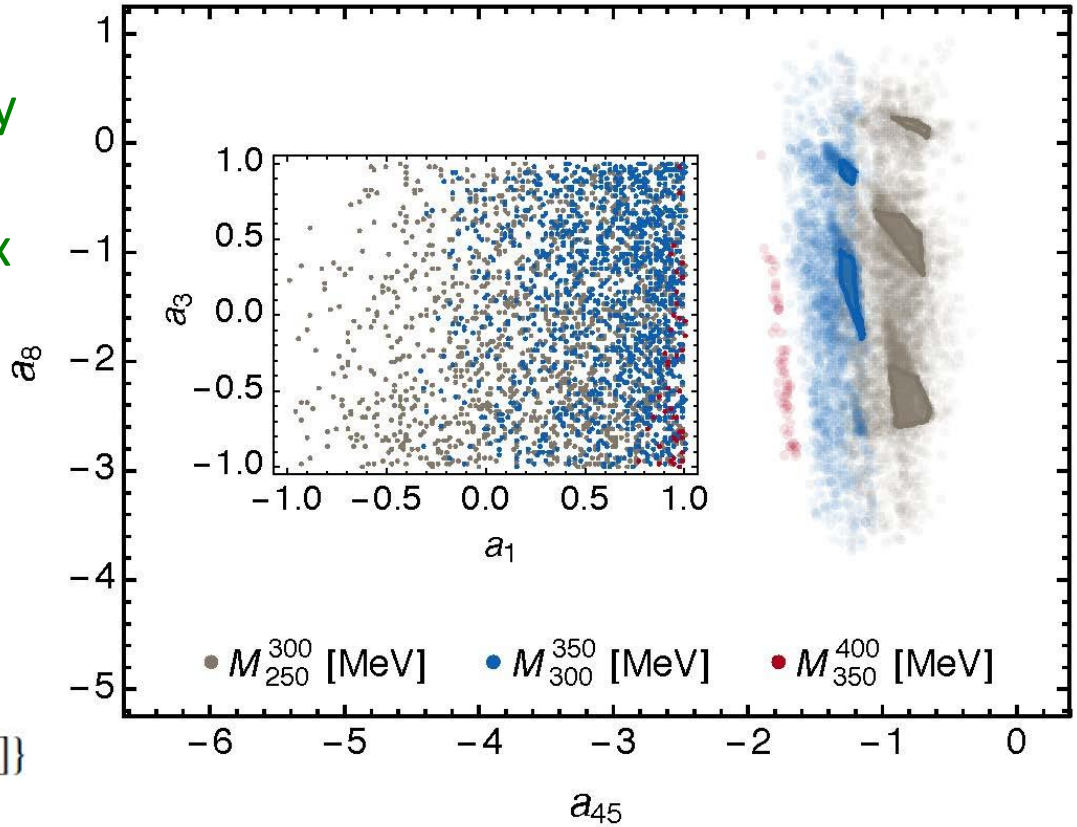
⇒ Even a small selection of observables places extremely tight bounds on the domain of acceptable, realistic vertex *Ansätze*

- Meson spectrum ⇒ $a_{2,6,7} = 0$

  (Sixue Qin *et al.*)

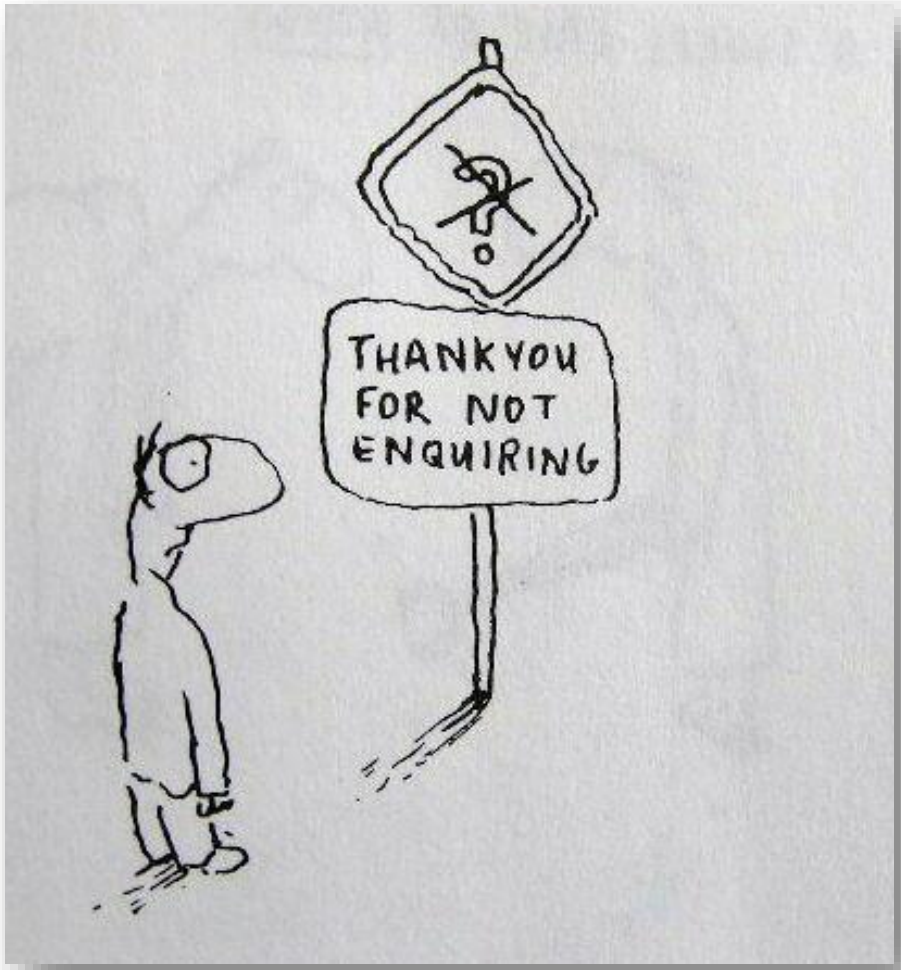
- In $\mathbf{R}^4$ … subset of (almost) zero measure

$$\mathbb{G}_4 \subset \{(a_1, a_3, a_{\hat{45}}, a_8) \,|\, a_1 \in [-0.5, 1], \; a_3 \in [-1, 1], a_{\hat{45}} \in [-2, -0.4], a_8 \in [-4, 1]\}$$



Dressed-gluon-quark vertex

Gap equation only "feels" $a_{45}=a_4-3a_5$



Enigma of Mass

Maris, Roberts and Tandy
nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273

Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

$$f_\pi E_\pi(k;P=0) = B(k^2)$$

Owing to DCSB & Exact in Chiral QCD

***Miracle*: two body problem solved, almost completely, once solution of one body problem is known**

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) = B(p^2)$$

The most fundamental expression of Goldstone's Theorem and DCSB

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) \Leftrightarrow B(p^2)$$

Pion exists if, and only if, mass is dynamically generated

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) \Leftrightarrow B(p^2)$$

This is why $m_\pi = 0$ in the absence of a Higgs mechanism

$f_\pi E_\pi(p^2) = B(p^2)$

Enigma of mass

This algebraic identity is why QCD's pion is massless in the chiral limit

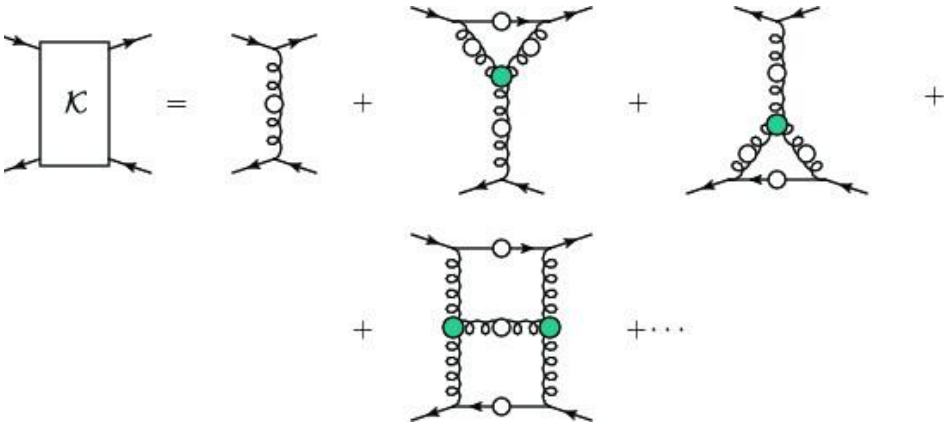
- The quark level Goldberger-Treiman relation shows that DCSB has a very deep and far reaching impact on physics within the strong interaction sector of the Standard Model; viz.,

  Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel.

- This emphasises that Goldstone's theorem has a pointwise expression in QCD
- Hence, pion properties are an almost direct measure of the dressed-quark mass function.
- Thus, enigmatically, the properties of the *massless* pion are the cleanest expression of the mechanism that is responsible for almost all the visible mass in the universe.

Pion masslessness

- Renormalisation scale: ζ = 2GeV =: ζ_2
- Pion's Poincaré-invariant mass and Poincaré-covariant wave function are obtained by solving a Bethe-Salpeter equation.
- This is a scattering problem
- In chiral limit
  - two massless fermions interact via exchange of massless gluons … initial system is massless; and it remains massless at every order in perturbation theory
- But, complete the calculation using an enumerable infinity of dressings and scatterings



Munczek, H. J., Phys. Rev. D 52 (1995) pp. 4736-4740
Bender, A., Roberts, C.D. and von Smekal, L., Phys. Lett. B 380 (1996) pp. 7-12
Maris, P. , Roberts, C.D. and Tandy, P.C., Phys. Lett. B 420 (1998) pp. 267-273

Pion masslessness

- Produces a coupled set of gap- and Bethe-Salpeter equations
  - Bethe-Salpeter Kernel:
    - valence-quarks with a momentum-dependent running mass produced by self-interacting gluons, which have given themselves a running mass
    - Interactions of arbitrary but enumerable complexity involving these "basis vectors"
  - Chiral limit:
    - Algebraic proof that, at any finite order in a symmetry-preserving construction of the kernels for the gap (quark dressing) and Bethe-Salpeter (bound-state) equations, there is a precise cancellation between the mass-generating effect of dressing the valence-quarks and the attraction introduced by the scattering events
    - Cancellation guarantees that the simple system, which began massless, becomes a complex system, with a nontrivial bound-state wave function attached to a pole in the scattering matrix, which remains at $P^2=0$ … remains massless
- Quantum field theory statement: in the pseudsocalar channel, the dynamically generated mass of the two fermions is precisely cancelled by the attractive interactions between them – iff –

$$f_\pi E_\pi(p^2) = B(p^2)$$

Pion masslessness

DCSB Paradigm

$$\langle\pi(q)|\theta_0|\pi(q)\rangle \overset{\zeta\gg\zeta_2}{=} \langle\pi(q)|\tfrac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}|\pi(q)\rangle$$

$$\overset{\zeta\simeq\zeta_2}{\to} \langle\pi(q)|\sum_{f=u,d} M_f(\zeta)\,\bar{\mathcal{Q}}_f(\zeta)\mathcal{Q}_f(\zeta) + \tfrac{1}{4}[\beta(\alpha(\zeta))\mathcal{G}^a_{\mu\nu}\mathcal{G}^a_{\mu\nu}]_{2\mathrm{PI}}|\pi(q)\rangle$$

- Parton-basis chiral-limit expression of the expectation-value of the trace-anomaly in the pion at $\zeta \gg \zeta_2$
- Metamorphoses into a new expression, written in terms of a nonperturbatively-dressed quasi-particle basis
  - 1st term = *positive* = one-body dressing content of the trace anomaly … Plainly, a massless valence-quark acquiring a large mass through interactions with its own gluon field is an expression of the trace-anomaly in the one-body subsector of the complete pion wave function
  - 2nd term = *negative* (attraction) = 2-particle-irreducible scattering event content of the scale-anomaly … Plainly, acquires a scale because the couplings, and the gluon- and quark-propagators in the 2PI processes have all acquired a mass-scale
- Away from the chiral limit, and in other channels, the cancellation is incomplete.

Observing Mass

Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

- 2012 … methods were developed that enable direct computation of the pion's light-front wave function
- $\varphi_\pi(x)$ = twist-two parton distribution amplitude = projection of the pion's Poincaré-covariant wave-function onto the light-front

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\, \gamma_5 \gamma\cdot n\, S(k)\Gamma_\pi(k;P) S(k-P)$$

- Results have been obtained with the DCSB-improved DSE kernel, which unifies matter & gauge sectors

$$\varphi_\pi(x) \propto x^\alpha (1-x)^\alpha, \text{ with } \alpha \approx 0.5$$

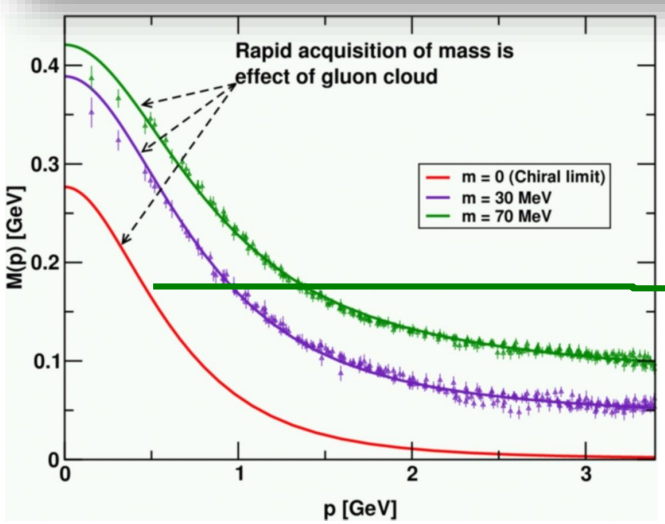
Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, *et al.*, arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

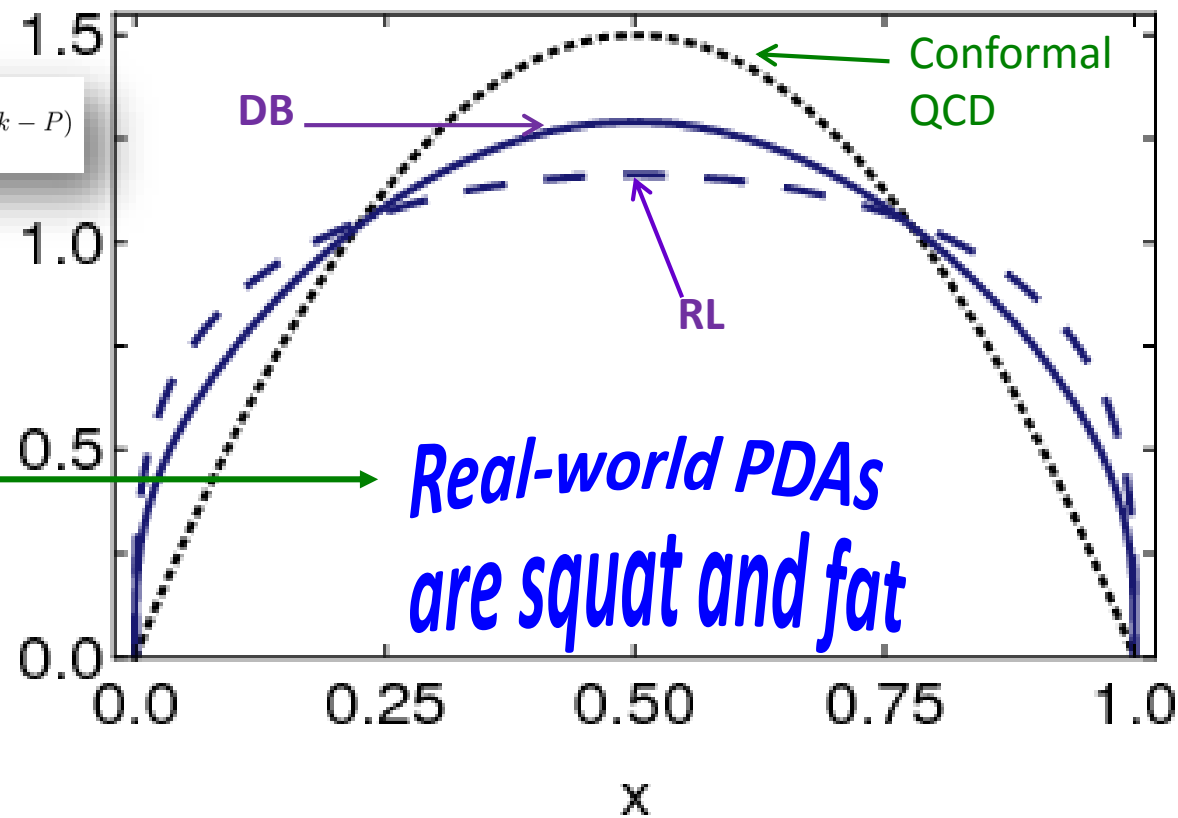
Pion's valence-quark Distribution Amplitude

- Continuum-QCD prediction:

  marked broadening of $\varphi_\pi(x)$, which owes to DCSB

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\, \gamma_5 \gamma\cdot n\, S(k) \Gamma_\pi(k;P) S(k-P)$$



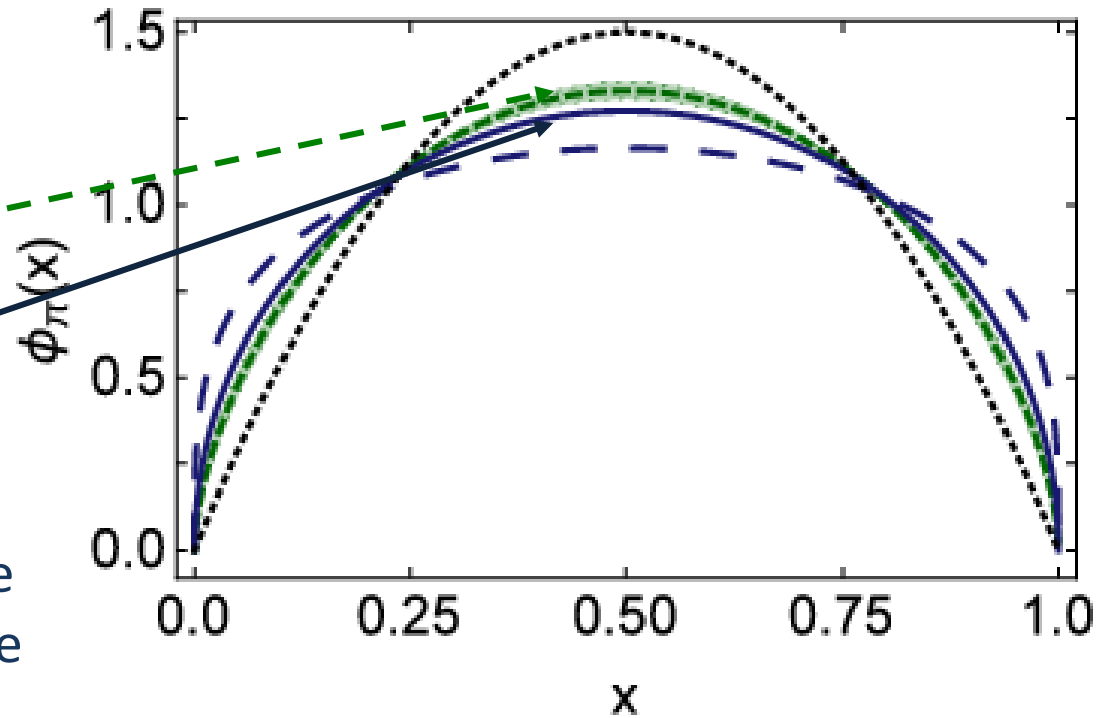


Pion distribution amplitude from lattice-QCD
I. C. Cloët *et al., arXiv:1306.2645 [nucl-th], Phys. Rev. Lett. 111 (2013) 092001 [5 pages]*

Flavour symmetry breaking in the kaon parton distribution amplitude, Chao Shi *et al., arXiv:1406:3353 [nucl-th], Phys. Lett. B 738 (2014) pp. 512–518*

Lattice-QCD & Pion's valence-quark PDA

- Isolated dotted curve = conformal QCD
- Green curve & band = result inferred from the single pion moment computed in lattice-QCD
- Blue solid curve = DSE prediction obtained with DB kernel
- DSE & lQCD predictions are practically indistinguishable



Pion's electromagnetic form factor

- Broadening has enormous impact on understanding $F_\pi(Q^2)$

A: Internally-consistent DSE prediction

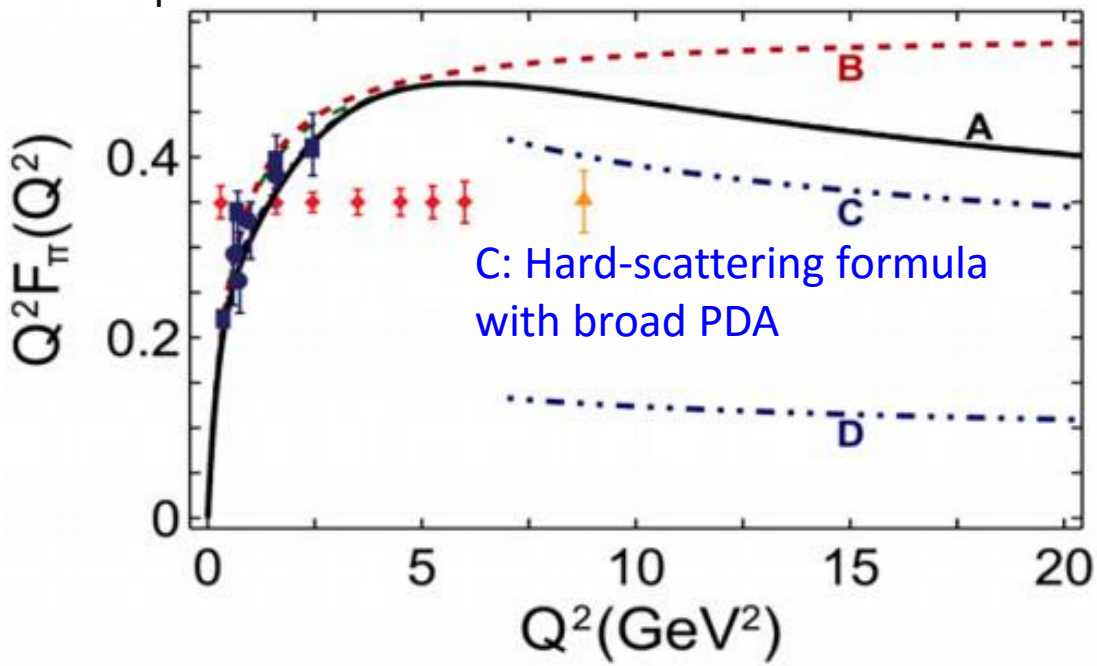
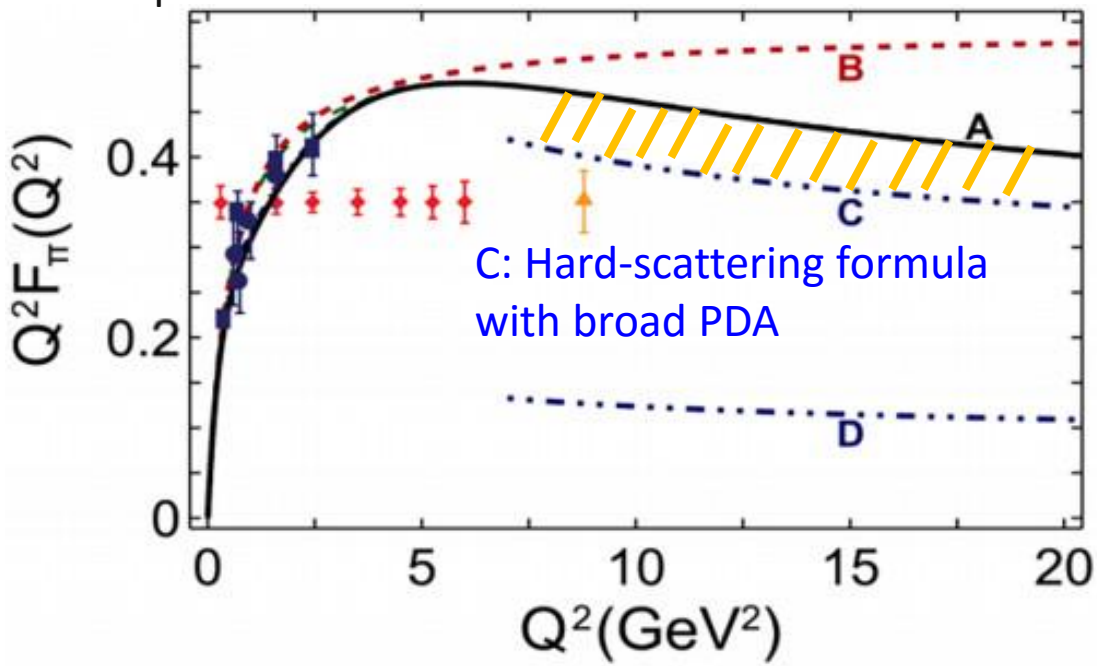


Figure 2.2: *Existing (dark blue) data and projected (red, orange) uncertainties for future data on the pion form factor. The solid curve (A) is the QCD-theory prediction bridging large and short distance scales. Curve B is set by the known long-distance scale—the pion radius. Curves C and D illustrate calculations based on a short-distance quark-gluon view.*

Pion's electromagnetic form factor

- Broadening has enormous impact on understanding $F_\pi(Q^2)$
- Appears that JLab12 is within reach of first verification of a QCD hard-scattering formula

A: Internally-consistent DSE prediction



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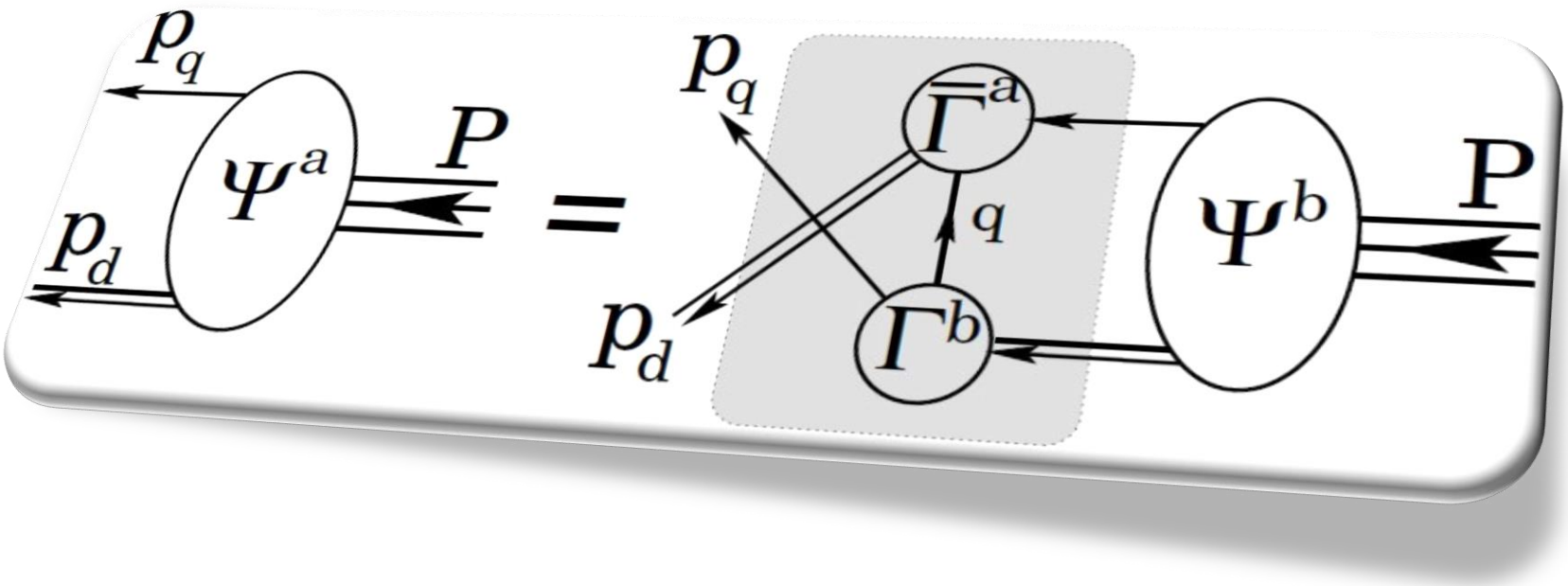
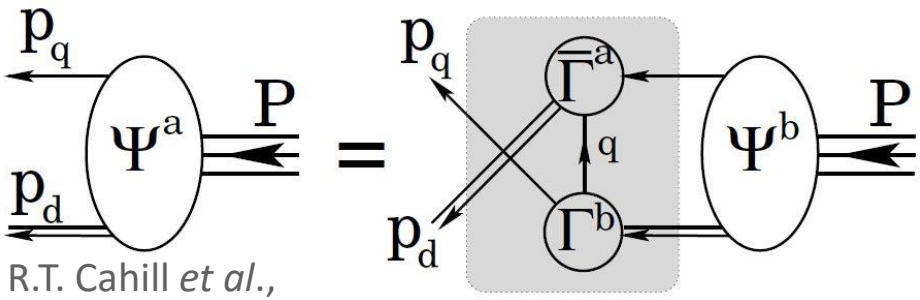


Structure of Baryons



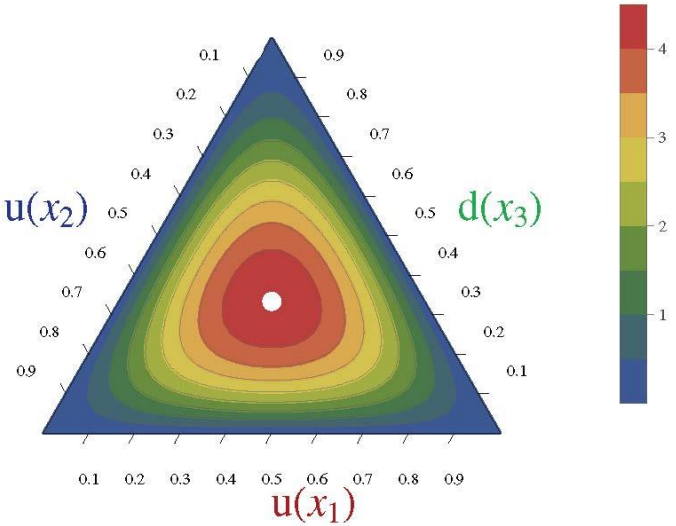
R.T. Cahill *et al.*, Austral. J. Phys. 42 (1989) 129-145

Baryon Structure

- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
- Confinement and DCSB are readily expressed
- ***Prediction***: owing to *DCSB in QCD, strong diquark correlations exist within baryons*
- Diquark correlations are not pointlike
  - Typically, $r_{0+} \sim r_\pi$ & $r_{1+} \sim r_\rho$ (actually 10% larger)
  - They have soft form factors

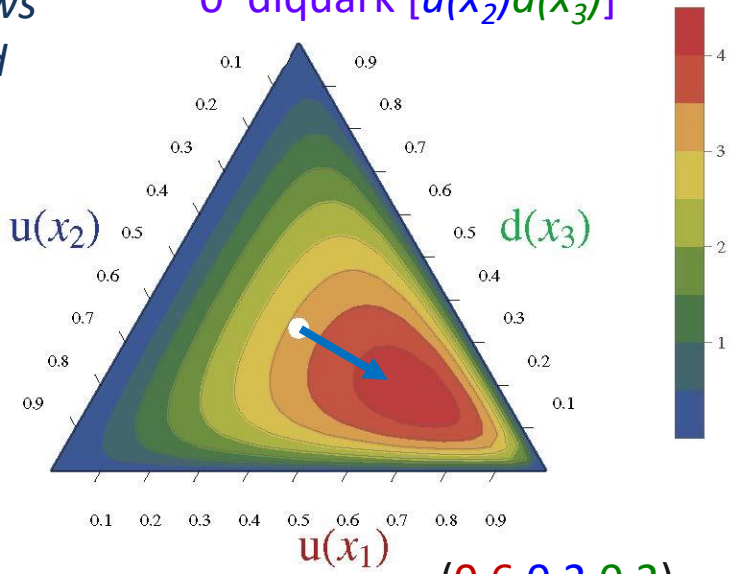
Nucleon Parton Distribution Amplitudes

- Computations underway. First results available.



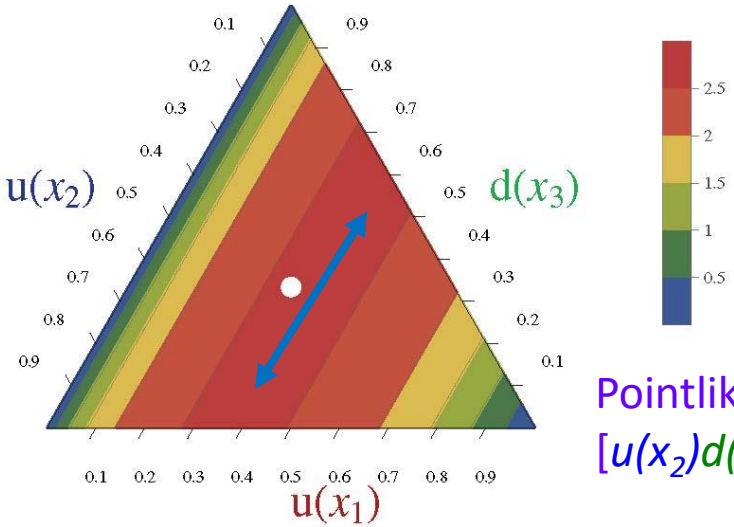
Diquark clustering skews the distribution toward the dressed-quark bystander, which therefore carries more of the proton's light-front momentum

Realistic, finite size (0.7fm)
0^+ diquark [$u(x_2)d(x_3)$]



(0.6,0.2,0.2)

conformal limit:
120 $x_1x_2x_3$
$\langle x_i \rangle = ⅓$ … peak of the distribution



Pointlike 0^+ diquark
[$u(x_2)d(x_3)$]

Light-cone distribution amplitudes of the nucleon and negative parity nucleon resonances from lattice QCD
V. M. Braun *et al.*, Phys. Rev. D 89 (2014) 094511

Light-cone distribution amplitudes of the baryon octet
G. S. Bali *et al.* JHEP 1602 (2016) 070

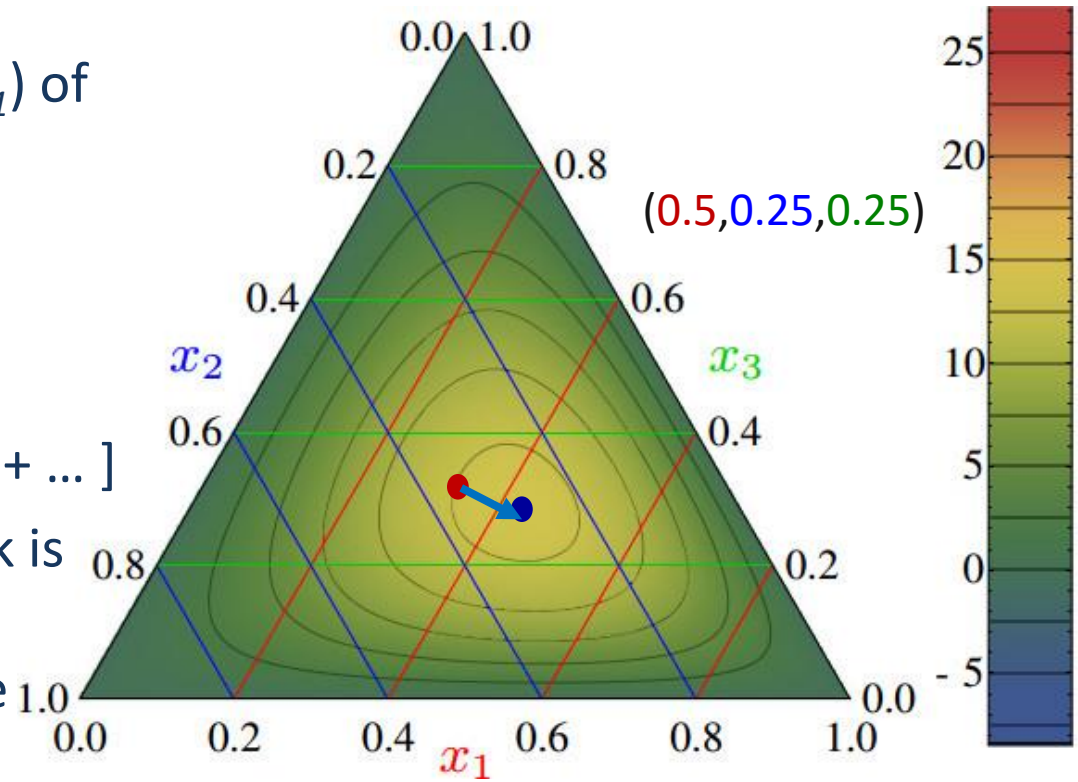
Nucleon PDAs & lQCD

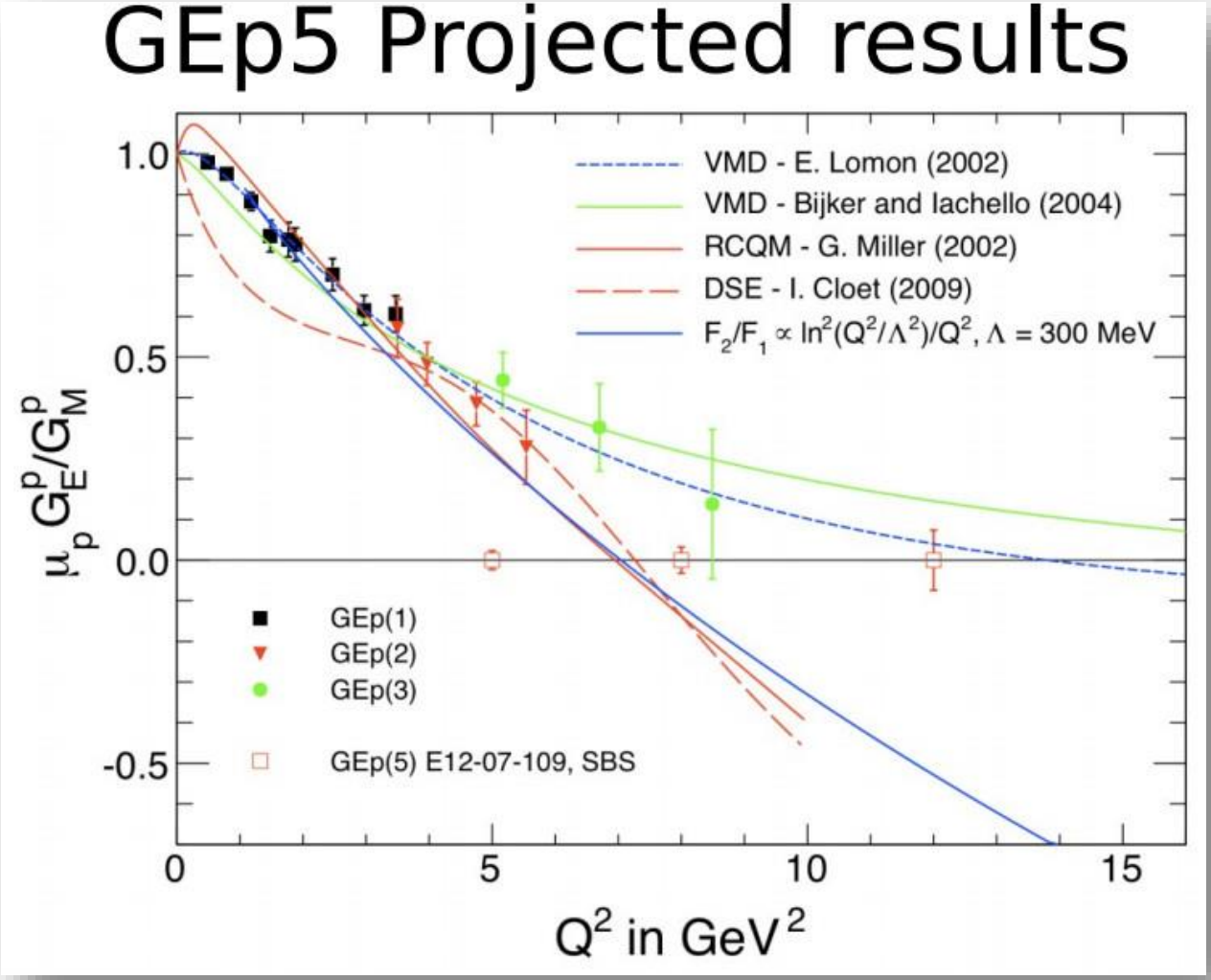
- First lQCD results for n=0, 1 moments of the leading twist PDA of the nucleon are available
- Used to constrain strength (a_{11}) of the leading-order term in a conformal expansion of the nucleon's PDA:

$\Phi(x_1,x_2,x_3)$

$= 120\, x_1\, x_2\, x_3\, [\, 1 + a_{11}\, P_{11}(x_1,x_2,x_3) + \ldots \,]$

- Shift in location of central peak is consistent with existence of diquark correlations within the nucleon

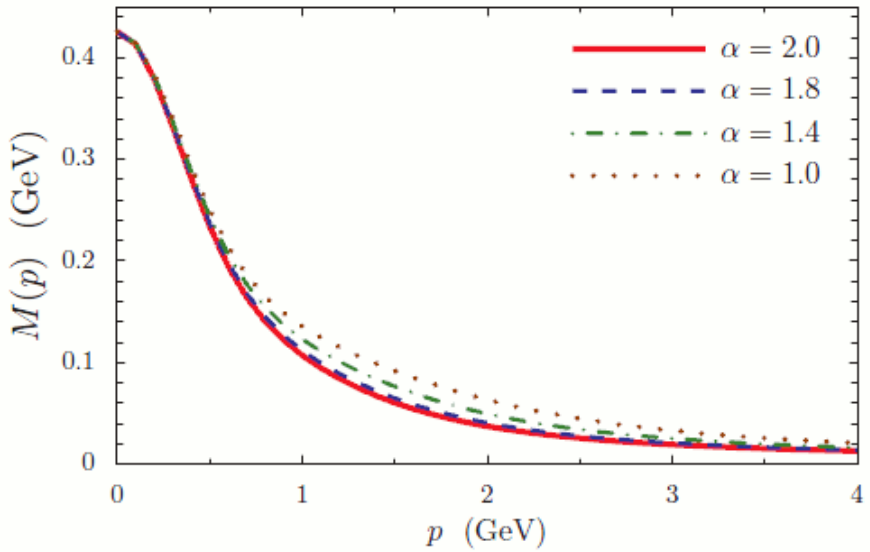




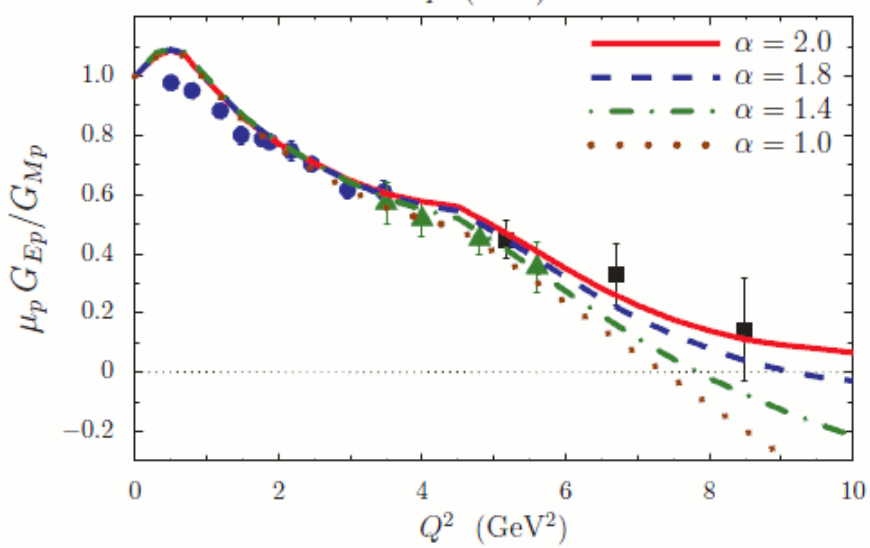
Nucleon Elastic FFs

I.C. Cloët, C.D. Roberts, A.W. Thomas: Revealing dressed-quarks via the proton's charge distribution, arXiv:1304.0855 [nucl-th], Phys. Rev. Lett. **111** (2013) 101803

Visible Impacts of DCSB



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



- Possible existence and location of a zero in the ratio of proton elastic form factors

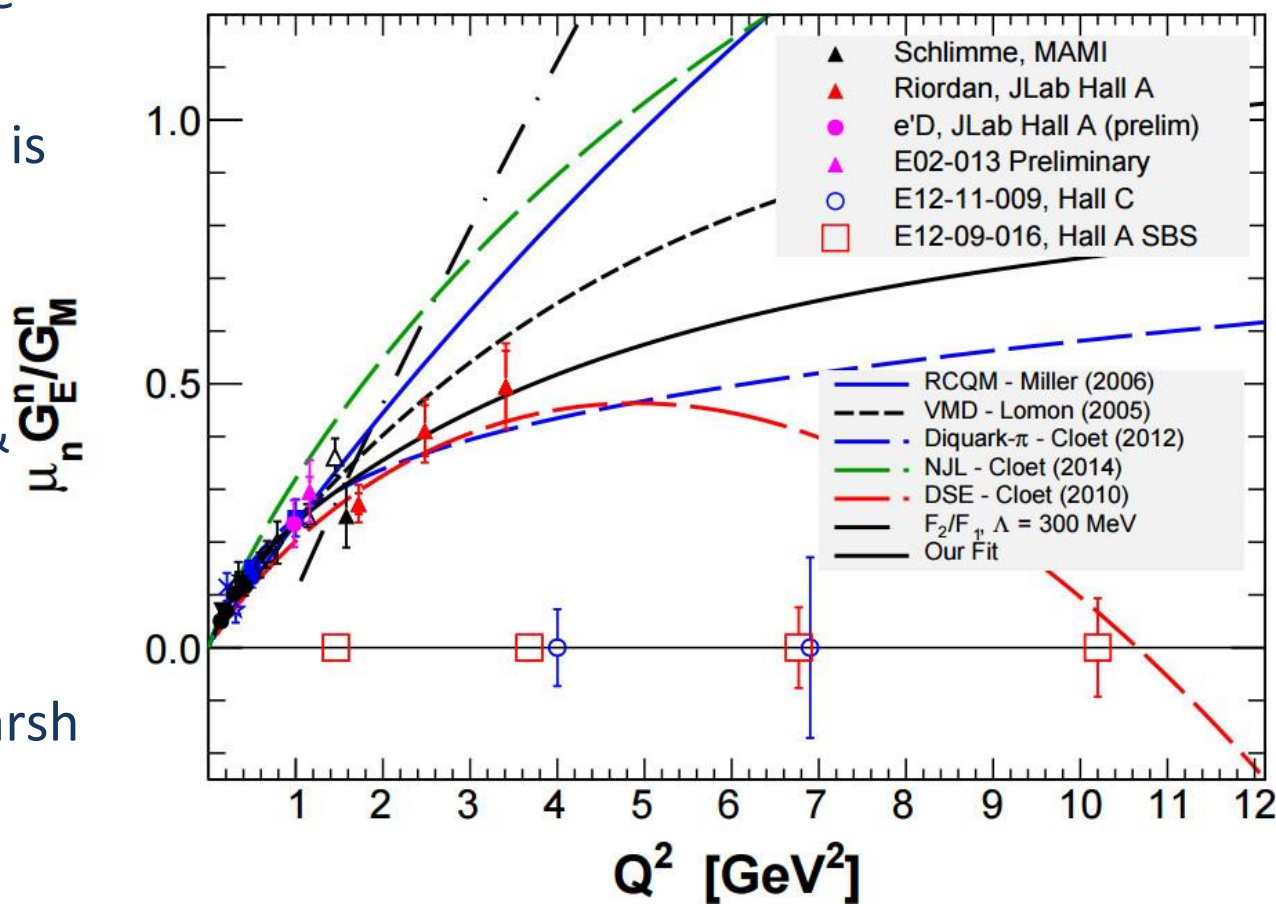
  $[\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)]$

  are a direct measure of the rate at which dressed-quarks become partons again,

  i.e. character of strong interactions in the Standard Model.

G_{En}

- Numerous calculations on this figure; but only one viable prediction
- DSE result (2008/2010) is not fitted to any data
  - Predicts zero in G_{En}
  - Owes to presence of running quark-mass & strong diquark correlations
  - Verifiable at JLab12
- G_{En} promises to be a harsh discriminator between descriptions of nucleon structure



Seamus Riordan, ECT, April 2016*

Epilogue

Epilogue

- Conformal anomaly … *gluons and quarks acquire momentum-dependent masses* … values are large in the infrared $m_g \propto 500$ MeV & $M_q \propto 350$ MeV … underlies DCSB; and has numerous observable consequences
- Universe with light quarks ⇒ confinement is a dynamical phenomenon *Confinement and DCSB are intimately linked* in real-QCD
- Origin and distribution of mass depend on the observer's preferred frame of reference and scale … Contemporary and planned experiments, DCSB paradigm is the best way to explicate and understand the associated, emerging phenomena. Numerous verifiable predictions accessible
  - form factors, PDAs and PDFs, GPDs and TMDs, etc.
- What can experiments at an EIC add to this?
  - Valence-quark region will be accessible
  - However, focus is on low-*x*, where gluons dominate … mass generation in the gluon sector … must affect potential for gluon saturation; how?
- Ability to compute valence-quark PDAs and PDFs has provided many new insights … must now begin to do the same for sea-quarks *and glue*