Emergent phenomena and partonic structure in hadrons

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What & where is mass?
Whence Mass?

- Classical chromodynamics ... non-Abelian local gauge theory
- Remove the current mass ... no energy scale left
- *No dynamics in a scale-invariant theory*; only kinematics ... the theory looks the same at all length-scales ... there can be no clumps of anything ... *hence bound-states are impossible*.

- *Our Universe can’t exist*
- *Higgs boson doesn’t solve this problem* ... normal matter is constituted from light-quarks & the mass of protons and neutrons, the kernels of all visible matter, are 100-times larger than anything the Higgs can produce

- *Where did it all begin?* ... becomes ... *Where did it all come from?*
Whence Mass?

- Poincaré invariance entails that the Energy-Momentum Tensor is divergence-free, i.e. it defines a conserved current:

\[ \partial_{\mu} T_{\mu\nu} = 0 \]

- Noether current associated with a global scale transformation:

\[ x \rightarrow e^{-\sigma} x \]

is the dilation current: \( D_{\mu\nu} = T_{\mu\nu} x_{\nu} \)

- In a scale invariant theory, the dilation current is conserved

\[ \partial_{\mu} D_{\mu} = 0 = [\partial_{\mu} T_{\mu\nu}] x_{\nu} + T_{\mu\nu} \delta_{\mu\nu} = T_{\mu\mu} \]

- Consequently, in a scale invariant theory the energy-momentum tensor must be traceless.
Trace Anomaly

- Classical chromodynamics is meaningless ... must be quantised
- Regularisation and renormalisation of (ultraviolet) divergences introduces a mass-scale
  ... *dimensional transmutation*: mass-dimensionless quantities become dependent on a mass-scale, $\zeta$
- $\alpha \rightarrow \alpha(\zeta)$ in QCD’s (massless) Lagrangian density, $\mathcal{L}(m=0)$
  Under a scale transformation $\zeta \rightarrow e^\sigma \zeta$, then $\alpha \rightarrow \sigma \alpha \beta(\alpha)$
  $\mathcal{L} \rightarrow \sigma \alpha \beta(\alpha) \frac{d\mathcal{L}}{d\alpha}$
  $\Rightarrow \partial_\mu \mathcal{D}_\mu = \frac{\delta \mathcal{L}}{\delta \sigma} = \alpha \beta(\alpha) \frac{d\mathcal{L}}{d\alpha} = \beta(\alpha) \frac{\alpha}{4} g_{\mu\nu} G_{\mu\nu} = T_{\rho\rho} =: \Theta_0$
- Straightforward, nonperturbative derivation, without need for diagrammatic analysis ...

quantisation of renormalisable four-dimensional theory forces nonzero value for trace of energy-momentum tensor
Where is the mass?
Knowing that a trace anomaly exists does not deliver a great deal ... indicates only that a mass-scale exists

Can one compute and/or understand the magnitude of that scale?

One can certainly *measure* the magnitude ... consider proton:

\[
\langle p(P) | T_{\mu\nu} | p(P) \rangle = -P_\mu P_\nu
\]

\[
\langle p(P) | T_{\mu\mu} | p(P) \rangle = -P^2 = m_p^2
\]

\[
= \langle p(P) | \Theta_0 | p(P) \rangle
\]

In the chiral limit the entirety of the proton’s mass is produced by the trace anomaly, \( \Theta_0 \)

... In QCD, \( \Theta_0 \) measures the strength of gluon self-interactions

... so, from one perspective, \( m_p \) is completely generated by glue.
On the other hand...
In the chiral limit

\[ \langle \pi(q) | T_{\mu\nu} | \pi(q) \rangle = -q_\mu q_\nu \Rightarrow \langle \pi(q) | \Theta_0 | \pi(q) \rangle = 0 \]

Does this mean that the scale anomaly vanishes trivially in the pion state, \textit{i.e.} gluons contribute nothing to the pion mass?

That is a difficult way to obtain “zero”

Easier, perhaps, to imagine that “zero” owes to cancellations between different operator-component contributions to \( \Theta_0 \).

Of course, such precise cancellation should not be an accident. It could only arise naturally because of some symmetry and/or symmetry-breaking pattern.
In the chiral limit

\[
\langle \pi(q) | T_{\mu\nu} | \pi(q) \rangle = -q_\mu q_\nu \Rightarrow \langle \pi(q) | \Theta_0 | \pi(q) \rangle = 0
\]

No statement of the question

“Whence the proton's mass?”

is complete without the additional clause

“Whence the absence of a pion mass?”
Classical chromodynamics ... non-Abelian local gauge theory

Local gauge invariance; but there is no confinement without a mass-scale

- Three quarks can still be colour-singlet
- Colour rotations will keep them colour singlets
- But they need have no proximity to one another
  ... proximity is meaningless in a scale-invariant theory

Whence mass ... equivalent to whence a mass-scale ...
equivalent to whence a confinement scale

Understanding the origin and absence of mass in QCD is quite likely inseparable from the task of understanding confinement. Existence alone of a scale anomaly answers neither question
A New Era for hadro-particle physics
What is origin of mass in our Universe?

What is the nature of confinement in real (dynamical-quarks) QCD?

How are they connected?

How can any
   – answers,
   – conjectures
   – and/or conclusions
be empirically verified?

Physics is an Empirical Science
What is Confinement?
Yang–Mills Existence and Mass Gap. Prove that for any compact simple gauge group \( G \), a non-trivial quantum Yang–Mills theory exists on \( \mathbb{R}^4 \) and has a mass gap \( \Delta > 0 \). Existence includes establishing axiomatic properties at least as strong as those cited in [45, 35].

5. Comments

An important consequence of the existence of a mass gap is clustering: Let \( \vec{x} \in \mathbb{R}^3 \) denote a point in space. We let \( H \) and \( \vec{P} \) denote the energy and momentum, generators of time and space translation. For any positive constant \( C < \Delta \) and for any local quantum field operator \( \mathcal{O}(\vec{x}) = e^{-i\vec{P} \cdot \vec{x}} e^{i\vec{P} \cdot \vec{x}} \) such that \( \langle \Omega, \mathcal{O} \Omega \rangle = 0 \), one has

\[
|\langle \Omega, \mathcal{O}(\vec{x}) \mathcal{O}(\vec{y}) \Omega \rangle| \leq \exp(-C|\vec{x} - \vec{y}|),
\]

as long as \( |\vec{x} - \vec{y}| \) is sufficiently large. Clustering is a locality property that, roughly speaking, may make it possible to apply mathematical results established on \( \mathbb{R}^4 \) to any 4-manifold, as argued at a heuristic level (for a supersymmetric extension of four-dimensional gauge theory) in [49]. Thus the mass gap not only has a physical significance (as explained in the introduction), but it may also be important in mathematical applications of four-dimensional quantum gauge theories to geometry. In addition the existence of a uniform gap for finite-volume approximations may play a fundamental role in the proof of existence of the infinite-volume limit.

There are many natural extensions of the Millennium problem. Among other things, one would like to prove the existence of an isolated one-particle state (an upper gap, in addition to the mass gap), to prove confinement to...
Yang–Mills Existence and Mass Gap. Prove that for any compact simple gauge group $G$, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in $[45, 35]$. 

Confinement?
Light quarks & Confinement

Folklore ... *Hall-D Conceptual Design Report*(5)

“The color field lines between a quark and an anti-quark form flux tubes.

A unit area placed midway between the quarks and perpendicular to the line connecting them intercepts a constant number of field lines, independent of the distance between the quarks.

This leads to a constant force between the quarks – and a large force at that, equal to about 16 metric tons.”
Light quarks & Confinement

Problem:

16 tonnes of force makes a lot of pions.
Light quarks & Confinement

Problem:

16 tonnes of force makes a lot of pions.
In the presence of light quarks, pair creation seems to occur non-localized and instantaneously.

No flux tube in a theory with light-quarks.

Flux-tube is not the correct paradigm for confinement in hadron physics.
In the presence of light quarks, pair creation seems to occur non-localized and instantaneously.

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Confinement contains condensates
Brodsky, Roberts, Shrock, Tandy
Existence of mass-gap in pure-gauge theory

Strong evidence supporting this conjecture: lQCD predicts $\Delta \sim 1.5$ GeV

But $\Delta^2/m_{\pi}^2 > 100$, So, can mass-gap in pure Yang-Mills play any role in understanding confinement when dynamical chiral symmetry breaking (DCSB) ensures existence of an almost-massless strongly-interacting excitation in our Universe?

If answer is not simply no, then it must be that one cannot claim to provide an understanding of confinement without simultaneously explaining its connection with DCSB.

Pion must play critical role in any explanation of real-world confinement; and any discussion that omits reference to the pion's role is practically irrelevant.

An important consequence of the existence of a mass gap is clustering: Let $\vec{x} \in \mathbb{R}^3$ denote a point in space. We let $H$ and $P$ denote the energy and momentum, generators of time and space translation. For any positive constant $C < \Delta$ and for any local quantum field operator $O(x) = e^{-iP \cdot \vec{x}} O e^{iP \cdot \vec{x}}$ such that $\langle \Omega, O \Omega \rangle = 0$, one has

$$|\langle \Omega, O(x) O(y) \Omega \rangle| \leq \exp(-C|x-y|),$$

as long as $|\vec{x} - \vec{y}|$ is sufficiently large. Clustering is a locality property that, roughly speaking, may make it possible to apply mathematical results established on $\mathbb{R}^4$ to any 4-manifold, as argued at a heuristic level (for a supersymmetric extension of four-dimensional gauge theory) in [49]. Thus the mass gap not only has a physical significance (as explained in the introduction), but it may also be important in mathematical applications of four-dimensional quantum gauge theories to geometry. In addition the existence of a uniform gap for finite-volume approximations may play a fundamental role in the proof of existence of the infinite-volume limit.

There are many natural extensions of the Millennium problem. Among other things, one would like to prove the existence of an isolated one-particle state (an upper gap, in addition to the mass gap), to prove confinement, to
Confinement is dynamical
All continuum and lattice predictions for Landau-gauge gluon & quark propagators exhibit an inflection point in $k^2$

$\Rightarrow$ Violate reflection positivity = sufficient for confinement

$\Rightarrow$ Such states have negative norm

$\Rightarrow$ Negative norm states are not observable

$\Rightarrow$ All observable states of a physical Hamiltonian have positive norm

Inflexion point corresponds to $\sigma \approx 0.5 \text{ fm}$:
Parton-like behaviour at shorter distances; but propagation characteristics changed dramatically at larger distances.
$m_g \approx 0.5 \text{ GeV}$
A quark begins to propagate

But after each “step” of length $\sigma$, on average, an interaction occurs, so that the quark loses its identity, sharing it with other partons

Finally, a cloud of partons is produced, which coalesces into colour-singlet final states

Confinement is a dynamical phenomenon!

Test: compute fragmentation functions & TMDs $\Rightarrow$ compare with data
\[ \Delta^{-1}_{\mu\nu}(q) = \Pi_{\mu\nu}(q) \]

\[ \Pi_{\mu\nu}(q) = P_{\mu\nu}(q) \Pi(q) \]

\[ P_{\mu\nu}(q) = g_{\mu\nu} - q_{\mu} q_{\nu} / q^2 \]
In QCD: Gluons also become massive!

Running gluon mass

\[ d(k^2) = \frac{\alpha(\zeta)}{k^2 + m_g^2(k^2; \zeta)} \]

\[ \alpha_s(0) = 2.77 \approx 0.9\pi, \quad m_g^2(0) = (0.46 \text{ GeV})^2 \]

Gluons are cannibals – a particle species whose members become massive by eating each other!

Expression of trace anomaly: Massless glue becomes massive gluon mass-squared function

Power-law suppressed in ultraviolet, so invisible in perturbation theory


Craig Roberts. Emergence of Partonic Structure (60p)
Gauge boson cannibalism
... a new physics frontier ... within the Standard Model

Asymptotic freedom means
... ultraviolet behaviour of QCD is controllable

Dynamically generated masses for gluons and quarks means that QCD dynamically generates its own infrared cutoffs

- Gluons and quarks with wavelength \( \lambda > 2/\text{mass} \approx 1 \text{ fm} \)
decouple from the dynamics ... Confinement?!

How does that affect observables?
- It will have an impact in any continuum study
- Probably plays a role in gluon saturation ...
In fact, a harbinger of gluon saturation?
\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]
Dynamical chiral symmetry breaking (DCSB) is a critical emergent phenomenon in QCD

Expressed in hadron wave functions not in vacuum condensates

Contemporary theory indicates that DCSB is responsible for more than 98% of the visible mass in the Universe; namely, given that classical massless-QCD is a conformally invariant theory, then DCSB is the origin of mass from nothing.

**Dynamical**, not spontaneous

- Add nothing to \( QCD \),

  *No Higgs field, nothing!*  
  Effect achieved purely through quark+gluon dynamics.

✓ **Trace anomaly**: massless quarks become massive
Continuum-QCD & ab initio predictions
Top down & Bottom up

- **Top-down approach** – ab initio computation of the interaction via direct analysis of the gauge-sector gap equations

- **Bottom-up scheme** – infer interaction by fitting data within a well-defined truncation of the matter sector DSEs that are relevant to bound-state properties.

- **Serendipitous collaboration**, conceived at one-week ECT* Workshop on DSEs in Mathematics and Physics, has united these two approaches

  – Interaction predicted by modern analyses of QCD’s gauge sector coincides with that required to describe ground-state observables using a sophisticated matter-sector DSE truncation

Modern kernels and interaction, developed at ANL and Peking U.

One parameter, fitted to ground-state properties without reference to gauge-sector studies.

Modern top-down and bottom-up results agree within 3%!
Top down & Bottom up

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- **Serendipitous collaboration**, conceived at one-week ECT* Workshop on DSEs in Mathematics and Physics, has united these two approaches.

- Interaction predicted by modern analyses of QCD's gauge sector coincides with that required to describe ground-state observables using a sophisticated matter-sector DSE truncation.
Reconciliation demands dressed-gluon-quark vertex

- Significant progress since 2009: dressed $\Gamma_\mu$ in gap- and Bethe-Salpeter equations ...
- In principle, $\exists$ unique form of $\Gamma_\mu$, but it’s still obscure.
- To improve this situation, used the top-down/bottom-up RGI running-interaction
  - Computed gap equation solutions with 
    1,660,000 distinct Ansätze for $\Gamma_\mu$
- Each one of the solutions tested for compatibility with three physical criteria
- Remarkably, merely 0.55% of solutions survive the test

$\Rightarrow$ Even a small selection of observables places extremely tight bounds on the domain of acceptable, realistic vertex Ansätze
Even a small selection of observables places extremely tight bounds on the domain of acceptable, realistic vertex Ansätze

- Meson spectrum \( \Rightarrow a_{2,6,7} = 0 \) (Sixue Qin et al.)
- In \( \mathbb{R}^4 \) ... subset of (almost) zero measure

\[ G_4 \subset \{ (a_1, a_3, a_4, a_8) \mid a_1 \in [-0.5, 1], \]
\[ a_3 \in [-1, 1], a_4 \in [-2, -0.4], a_8 \in [-4, 1] \]
Enigma of Mass
Pion’s Goldberger-Treiman relation

- Pion’s Bethe-Salpeter amplitude
  Solution of the Bethe-Salpeter equation
  \[
  \Gamma_{\pi j}(k; P) = \tau_{\pi j} \gamma_5 \left[ iE_\pi(k; P) + \gamma \cdot P F_\pi(k; P) 
  + \gamma \cdot k k \cdot P G_\pi(k; P) + \sigma_{\mu\nu} k_\mu P_\nu H_\pi(k; P) \right]
  \]

- Dressed-quark propagator
  \[
  S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}
  \]

- Axial-vector Ward-Takahashi identity entails
  \[
  f_\pi E_\pi(k; P = 0) = B(k^2)
  \]

Owing to DCSB & Exact in Chiral QCD

Miracle: two body problem solved, almost completely, once solution of one body problem is known.
Rudimentary version of this relation is apparent in Nambu’s Nobel Prize work.

\[ f_{\pi} E_{\pi}(p^2) = B(p^2) \]

The most fundamental expression of Goldstone’s Theorem and DCSB.
Rudimentary version of this relation is apparent in Nambu’s Nobel Prize work.

\[ f_\pi E_\pi(p^2) \iff B(p^2) \]

Pion exists if, and only if, mass is dynamically generated.
Rudimentary version of this relation is apparent in Nambu’s Nobel Prize work.

\[ f_\pi E_\pi(p^2) \iff B(p^2) \]

Model independent
Gauge independent
Scheme independent

This is why \( m_\pi = 0 \) in the absence of a Higgs mechanism.
The quark level Goldberger-Treiman relation shows that DCSB has a very deep and far reaching impact on physics within the strong interaction sector of the Standard Model; viz.,

Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel.

This emphasises that Goldstone's theorem has a pointwise expression in QCD.

Hence, pion properties are an almost direct measure of the dressed-quark mass function.

Thus, enigmatically, the properties of the massless pion are the cleanest expression of the mechanism that is responsible for almost all the visible mass in the universe.
Pion masslessness

- Renormalisation scale: $\zeta = 2\text{GeV} =: \zeta_2$
- Pion’s Poincaré-invariant mass and Poincaré-covariant wave function are obtained by solving a Bethe-Salpeter equation.
- This is a scattering problem
- In chiral limit
  - two massless fermions interact via exchange of massless gluons
    ... initial system is massless; and it remains massless at every order in perturbation theory
- But, complete the calculation using an enumerable infinity of dressings and scatterings

\[ \mathcal{K} = \quad \quad \quad \quad + \quad \quad + \quad + \quad \quad \ldots \]
Pion masslessness

- Produces a coupled set of gap- and Bethe-Salpeter equations
  - Bethe-Salpeter Kernel:
    - valence-quarks with a momentum-dependent running mass produced by self-interacting gluons, which have given themselves a running mass
    - Interactions of arbitrary but enumerable complexity involving these “basis vectors”
  - Chiral limit:
    - Algebraic proof that, at any finite order in a symmetry-preserving construction of the kernels for the gap (quark dressing) and Bethe-Salpeter (bound-state) equations, there is a precise cancellation between the mass-generating effect of dressing the valence-quarks and the attraction introduced by the scattering events
    - Cancellation guarantees that the simple system, which began massless, becomes a complex system, with a nontrivial bound-state wave function attached to a pole in the scattering matrix, which remains at $P^2=0$ … remains massless

- Quantum field theory statement: in the pseudoscalar channel, the dynamically generated mass of the two fermions is precisely cancelled by the attractive interactions between them – iff –

$$f_\pi E_\pi(p^2) = B(p^2)$$
Pion masslessness

\[
\langle \pi(q) | \theta_0 | \pi(q) \rangle \overset{\zeta \gg \zeta_2}{=} \langle \pi(q) | \frac{1}{4} \beta(\alpha(\zeta)) G^{a}_{\mu\nu} G^{a}_{\mu\nu} | \pi(q) \rangle
\]

\[
\overset{\zeta \approx \zeta_2}{=}
\sum_{f=u,d} M_f(\zeta) \bar{Q}_f(\zeta) Q_f(\zeta) + \frac{1}{4} [\beta(\alpha(\zeta)) G^{a}_{\mu\nu} G^{a}_{\mu\nu}]_{2PI} | \pi(q) \rangle
\]

- Parton-basis chiral-limit expression of the expectation-value of the trace-anomaly in the pion at \( \zeta \gg \zeta_2 \)
- Metamorphoses into a new expression, written in terms of a nonperturbatively-dressed quasi-particle basis
  - 1\textsuperscript{st} term = positive = one-body dressing content of the trace anomaly ... Plainly, a massless valence-quark acquiring a large mass through interactions with its own gluon field is an expression of the trace-anomaly in the one-body subsector of the complete pion wave function
  - 2\textsuperscript{nd} term = negative (attraction) = 2-particle-irreducible scattering event content of the scale-anomaly ... Plainly, acquires a scale because the couplings, and the gluon- and quark-propagators in the 2PI processes have all acquired a mass-scale
- Away from the chiral limit, and in other channels, the cancellation is incomplete.
Observing Mass
2012 ... methods were developed that enable direct computation of the pion’s light-front wave function

\( \varphi_\pi(x) = \text{twist-two parton distribution amplitude} = \text{projection of the pion’s Poincaré-covariant wave-function onto the light-front} \)

\[
\varphi_\pi(x) = Z_2 \text{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n \cdot k - xn \cdot P) \gamma_5 \gamma \cdot n S(k) \Gamma_\pi(k; P) S(k - P)
\]

Results have been obtained with the DCSB-improved DSE kernel, which unifies matter & gauge sectors

\[
\varphi_\pi(x) \propto x^\alpha (1-x)^\alpha, \text{ with } \alpha \approx 0.5
\]

Pion’s valence-quark Distribution Amplitude

- Continuum-QCD prediction: marked broadening of $\varphi_\pi(x)$, which owes to DCSB

$$\varphi_\pi(x) = Z_\pi tr_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n \cdot k - x n \cdot P) \gamma_5 \gamma \cdot n S(k) \Gamma_\pi(k; P) S(k - P)$$

Real-world PDAs are squat and fat
Lattice-QCD & Pion’s valence-quark PDA

- Isolated dotted curve = conformal QCD
- Green curve & band = result inferred from the single pion moment computed in lattice-QCD
- Blue solid curve = DSE prediction obtained with DB kernel
- DSE & lQCD predictions are practically indistinguishable

Pion distribution amplitude from lattice-QCD
Pion electromagnetic form factor at spacelike momenta
L. Chang et al., arXiv:1307.0026 [nucl-th],

Broadening has enormous impact on understanding $F_\pi(Q^2)$

A: Internally-consistent DSE prediction

C: Hard-scattering formula with broad PDA

**Figure 2.2:** Existing (dark blue) data and projected (red, orange) uncertainties for future data on the pion form factor. The solid curve (A) is the QCD-theory prediction bridging large and short distance scales. Curve B is set by the known long-distance scale—the pion radius. Curves C and D illustrate calculations based on a short-distance quark-gluon view.
Pion's electromagnetic form factor

- Broadening has enormous impact on understanding $F_\pi(Q^2)$
- Appears that JLab12 is within reach of first verification of a QCD hard-scattering formula

**Figure 2.2:** Existing (dark blue) data and projected (red, orange) uncertainties for future data on the pion form factor. The solid curve (A) is the QCD-theory prediction bridging large and short distance scales. Curve B is set by the known long-distance scale—the pion radius. Curves C and D illustrate calculations based on a short-distance quark-gluon view.
Structure of Baryons

\[ \Psi^a \quad \Gamma^a \quad \Gamma^b \quad \Psi^b \]

\[ P \quad p_d \quad p_q \]

Craig Roberts. Emergence of Partonic Structure (60p)
- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
- Confinement and DCSB are readily expressed

**Prediction**: owing to DCSB in QCD, strong diquark correlations exist within baryons

- Diquark correlations are not pointlike
  - Typically, \( r_{0^+} \sim r_{\pi} \) & \( r_{1^+} \sim r_{\rho} \) (actually 10% larger)
  - They have soft form factors
Nucleon Parton Distribution Amplitudes

Computations underway. First results available.

Diquark clustering skews the distribution toward the dressed-quark bystander, which therefore carries more of the proton’s light-front momentum.

Conformal limit:

$$120 x_1 x_2 x_3$$

$$\langle x_i \rangle = \frac{1}{3} \ldots \text{peak of the distribution}$$

Realistic, finite size (0.7fm)

$$0^+ \text{ diquark } [u(x_2)d(x_3)]$$

Pointlike $$0^+ \text{ diquark } [u(x_2)d(x_3)]$$
First IQCD results for $n=0,1$ moments of the leading twist PDA of the nucleon are available

Used to constrain strength ($a_{11}$) of the leading-order term in a conformal expansion of the nucleon’s PDA:

$$\Phi(x_1,x_2,x_3) = 120 x_1 x_2 x_3 [ 1 + a_{11} P_{11}(x_1,x_2,x_3) + ... ]$$

Shift in location of central peak is consistent with existence of diquark correlations within the nucleon
GEP5 Projected results

\[ \mu_p \frac{G_E^p}{G_M} \]

- VMD - Bijker and Iachello (2004)
- DSE - I. Cloet (2009)
- \( F_2/F_1 \propto \ln^2(Q^2/\Lambda^2)/Q^2, \Lambda = 300 \text{ MeV} \)

Nucleon Elastic FFs

Craig Roberts. Emergence of Partonic Structure (60p)
Visible Impacts of DCSB

Possible existence and location of a zero in the ratio of proton elastic form factors

\[ \left[ \mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2) \right] \]

\textit{are} a direct measure of the rate at which dressed-quarks become partons again,

\textit{i.e.} character of strong interactions in the Standard Model.
Numerous calculations on this figure; but only one viable prediction

DSE result (2008/2010) is not fitted to any data
- Predicts zero in $G_{En}$
- Owes to presence of running quark-mass & strong diquark correlations
  - Verifiable at JLab12

$G_{En}$ promises to be a harsh discriminator between descriptions of nucleon structure
Conformal anomaly ... *gluons and quarks acquire momentum-dependent masses* ... values are large in the infrared $m_g \propto 500$ MeV & $M_q \propto 350$ MeV ... underlies DCSB; and has numerous observable consequences

Universe with light quarks $\Rightarrow$ confinement is a dynamical phenomenon *Confinement and DCSB are intimately linked* in real-QCD

Origin and distribution of mass depend on the observer’s preferred frame of reference and scale ... Contemporary and planned experiments, DCSB paradigm is the best way to explicate and understand the associated, emerging phenomena. Numerous verifiable predictions accessible

- form factors, PDAs and PDFs, GPDs and TMDs, etc.

What can experiments at an EIC add to this?

- Valence-quark region will be accessible
- However, focus is on low-$x$, where gluons dominate ... mass generation in the gluon sector ... must affect potential for gluon saturation; how?

Ability to compute valence-quark PDAs and PDFs has provided many new insights ... must now begin to do the same for sea-quarks *and glue*