

Confinement 2016, Thessaloniki, Greece

The Chiral Magnetic Effect:

from quark-gluon plasma to Dirac/Weyl semimetals

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RIKEN BNL
Research Center



Classical symmetries and Quantum anomalies

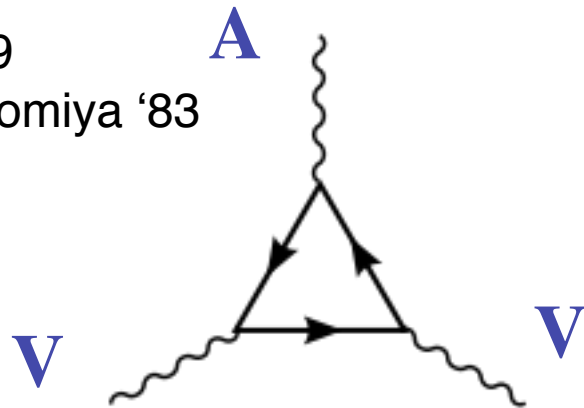
Anomalies: The classical symmetry of the Lagrangian is broken by quantum effects -

examples: chiral symmetry - chiral anomaly $\partial_\mu j_A^\mu = C_A \mathbf{E} \cdot \mathbf{B}$
scale symmetry - scale anomaly

Anomalies imply correlations between currents:

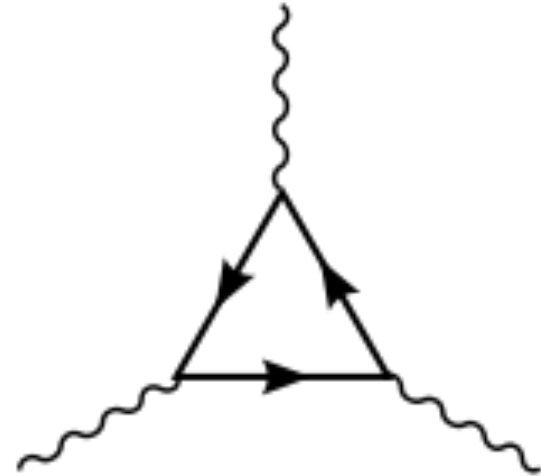
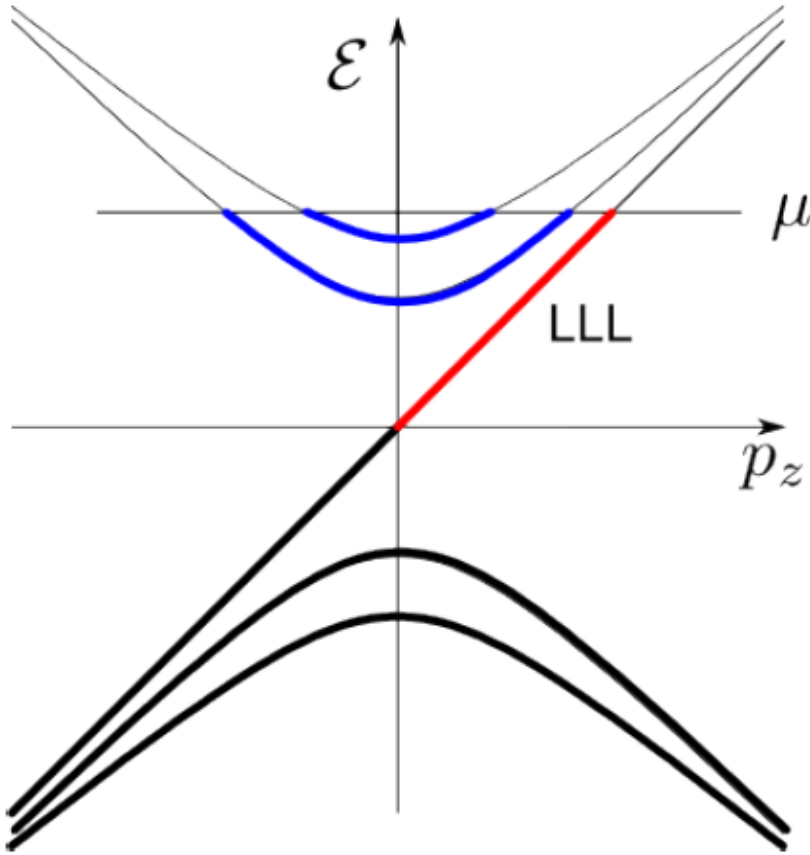
e.g. Adler; Bell, Jackiw '69
crystals: Nielsen, Ninomiya '83

$\pi^0 \rightarrow \gamma\gamma$
decay



**if A, V are
background fields,
V is generated!**

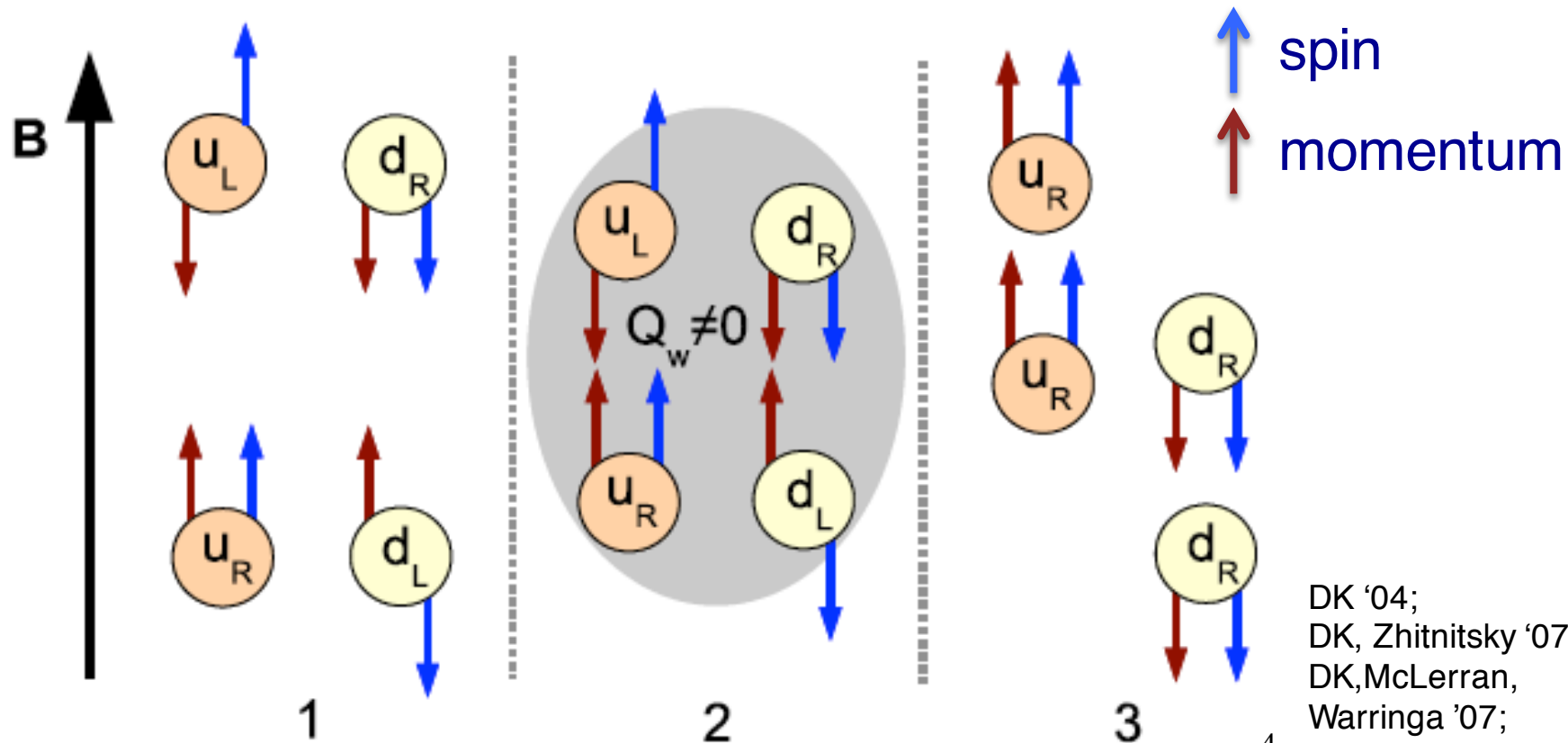
Chiral anomaly



In classical background fields (E and B), chiral anomaly induces a collective motion in the Dirac sea

Chirality in 3D: the Chiral Magnetic Effect

chirality + magnetic field = current



Review: DK, arxiv:1312.3348 (Prog.Part.Nucl.Phys'14)

DK '04;
DK, Zhitnitsky '07
DK, McLerran,
Warringa '07;
Fukushima,
DK, Warringa '08

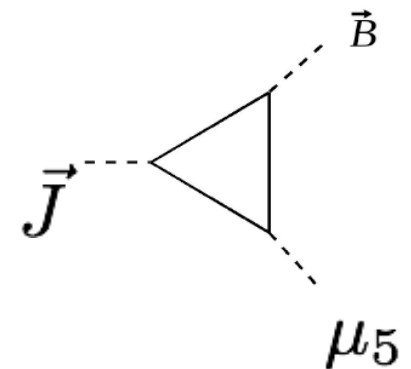
Chiral Magnetic Effect

DK'04; K.Fukushima, DK, H.Warringa, PRD'08;
Review and list of refs: DK, arXiv:1312.3348

Chiral chemical potential is formally
equivalent to a background chiral gauge field: $\mu_5 = A_5^0$

In this background, and in the presence of \vec{B} ,
vector e.m. current is generated:

$$\partial_\mu J^\mu = \frac{e^2}{16\pi^2} \left(F_L^{\mu\nu} \tilde{F}_{L,\mu\nu} - F_R^{\mu\nu} \tilde{F}_{R,\mu\nu} \right)$$



Compute the current through

$$J^\mu = \frac{\partial \log Z[A_\mu, A_\mu^5]}{\partial A_\mu(x)}$$

The result:

$$\vec{J} = \frac{e^2}{2\pi^2} \mu_5 \vec{B}$$

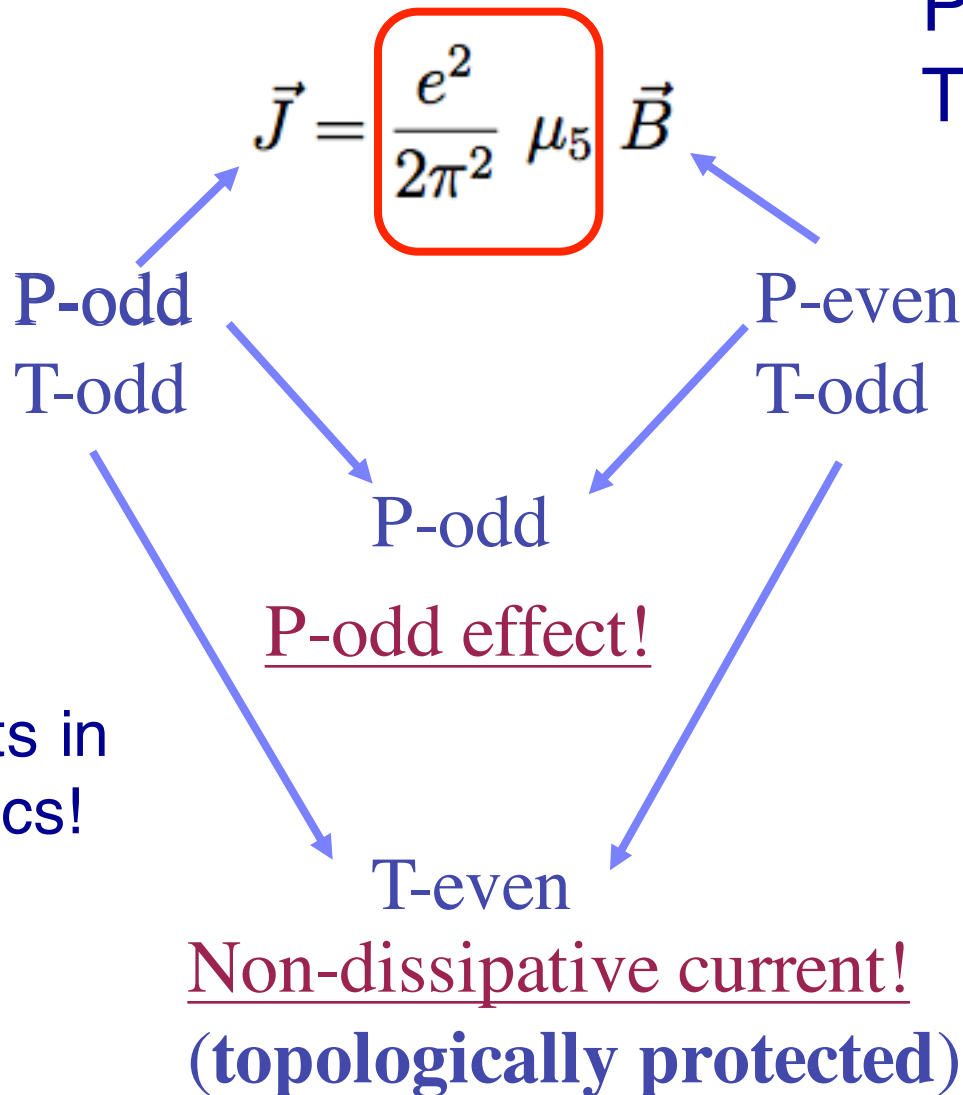
Coefficient is fixed by
the axial anomaly, no
corrections

Phases at finite μ_5 :
Talks by A.Andrianov,
V. Braguta

Chiral magnetic conductivity: discrete symmetries

P – parity

T – time reversal



Effect persists in
hydrodynamics!

cf Ohmic
conductivity:

$$\vec{J} = \sigma \vec{E}$$

T-odd,
dissipative

Systematics of anomalous conductivities

Magnetic field

Vorticity

Vector
current

$$\frac{\mu_A}{2\pi^2}$$

$$\frac{\mu\mu_A}{2\pi^2}$$

Axial
current

$$\frac{\mu}{2\pi^2}$$

$$\frac{\mu^2 + \mu_A^2}{4\pi^2} + \frac{T^2}{12}$$

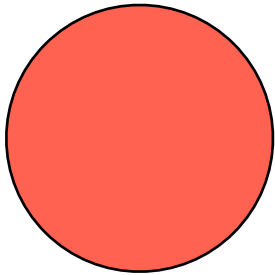
Hydrodynamics and symmetries

- Hydrodynamics: an effective low-energy TOE. States that the response of the fluid to slowly varying perturbations is completely determined by conservation laws (energy, momentum, charge, ...)
- Conservation laws are a consequence of symmetries of the underlying theory
- What happens to hydrodynamics when these symmetries are broken by quantum effects (anomalies of QCD and QED)?

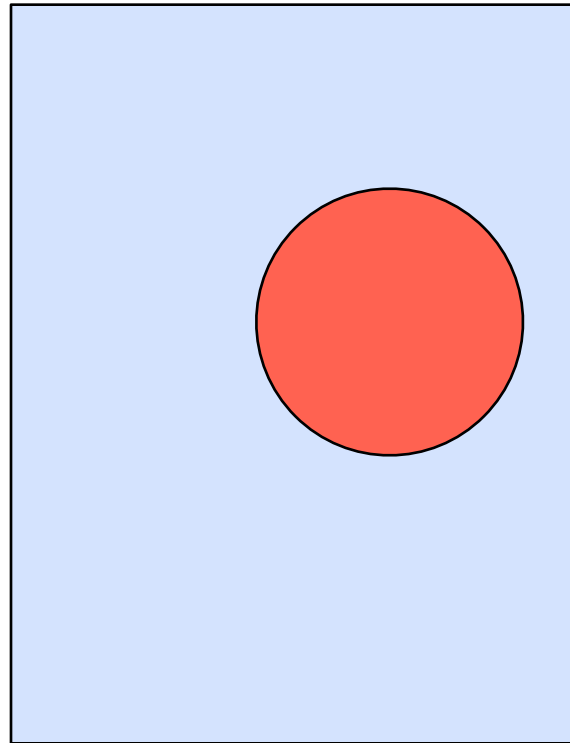
No entropy production from P-odd anomalous terms

DK and H.-U. Yee, 1105.6360

Entropy grows



$$\partial_{\mu} s^{\mu} \geq 0$$



Mirror reflection:
entropy decreases ?

$$\partial_{\mu} s^{\mu} \leq 0$$

Decrease is ruled
out by 2nd law of
thermodynamics



$$\partial_{\mu} s^{\mu} = 0$$

Allows to compute analytically 13 out of 18
anomalous transport coefficients in 2nd order
relativistic hydrodynamics

The CME in relativistic hydrodynamics:

The Chiral Magnetic Wave

DK, H.-U. Yee,
arXiv:1012.6026 [hep-th];
PRD

$$\vec{j}_V = \frac{N_c e}{2\pi^2} \mu_A \vec{B}; \quad \vec{j}_A = \frac{N_c e}{2\pi^2} \mu_V \vec{B},$$

CME

Chiral separation

$$\begin{pmatrix} \vec{j}_V \\ \vec{j}_A \end{pmatrix} = \frac{N_c e \vec{B}}{2\pi^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \mu_V \\ \mu_A \end{pmatrix}$$

Propagating chiral wave: (if chiral symmetry
is restored)

$$\left(\partial_0 \mp \frac{N_c e B \alpha}{2\pi^2} \partial_1 - D_L \partial_1^2 \right) j_{L,R}^0 = 0$$

Gapless collective mode is the carrier of CME current in MHD:

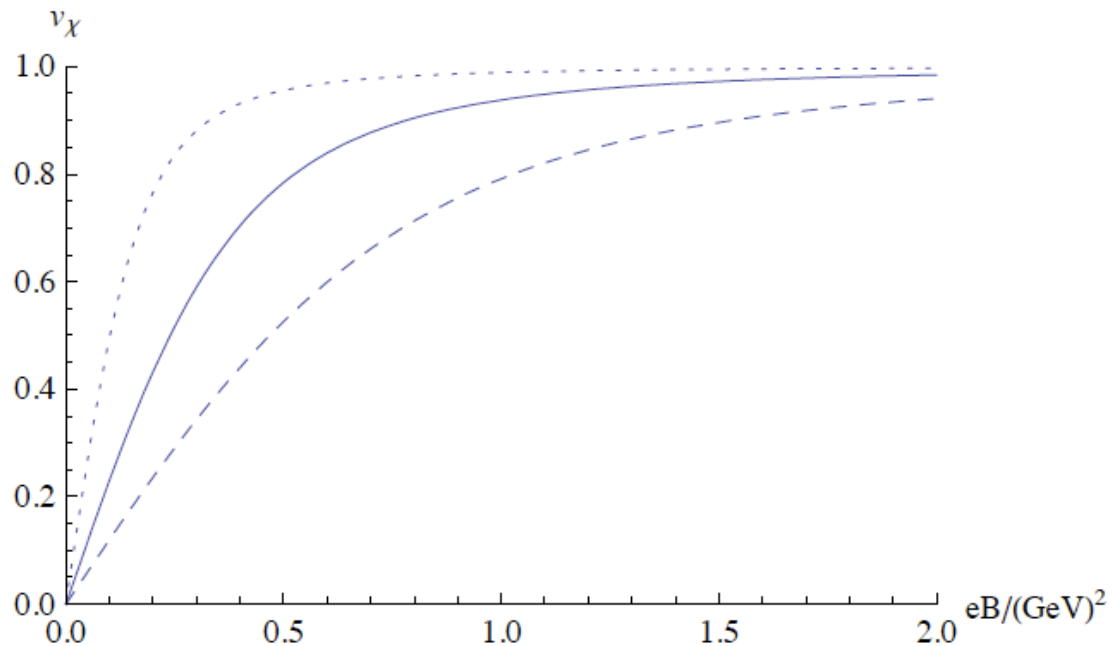
$$\omega = \mp v_\chi k - i D_L k^2 + \dots$$



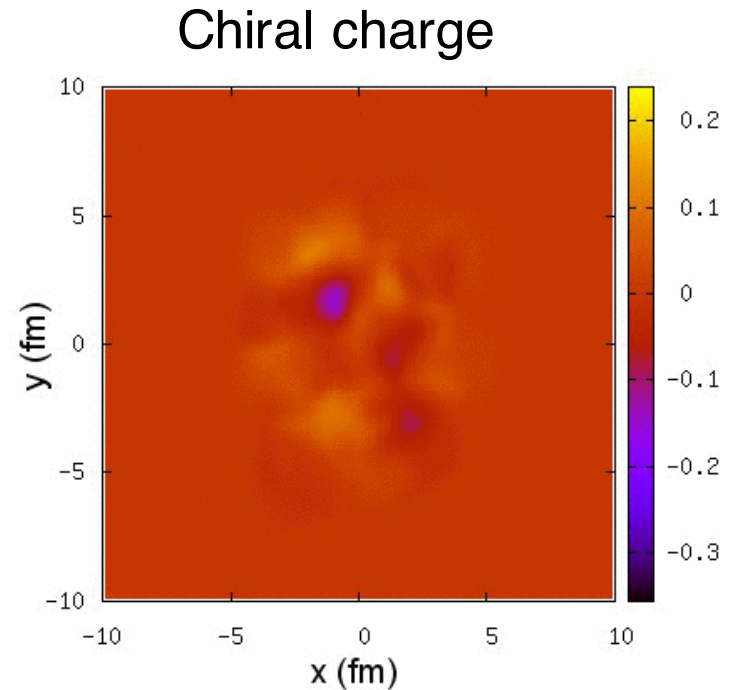
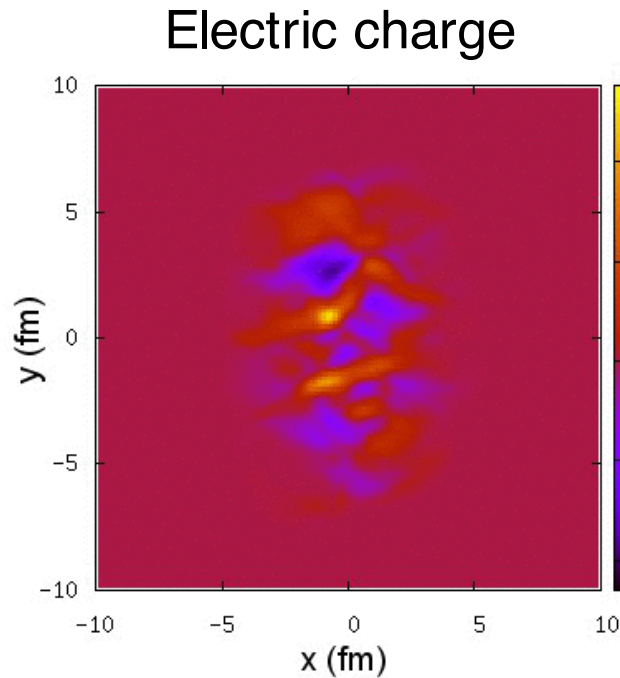
The Chiral Magnetic Wave:

oscillations of electric and chiral charges
coupled by the chiral anomaly

In strong magnetic field, CMW
propagates with the speed of light!



CMHD



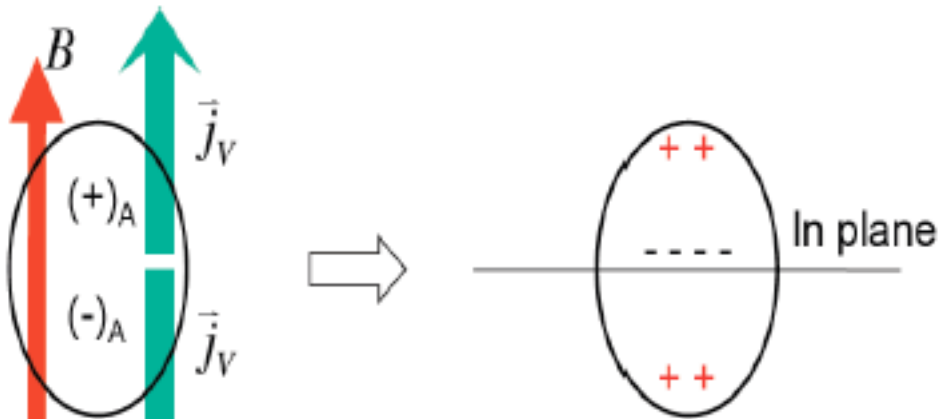
Y.Hirono, T.Hirano, DK, (Stony Brook – Tokyo), arxiv:1412.0311
(3+1) ideal CMHD (Chiral MagnetoHydroDynamics)

Pre-equilibrium CME: see poster by S.Schlichting, S.Sharma

Testing the Chiral Magnetic Wave

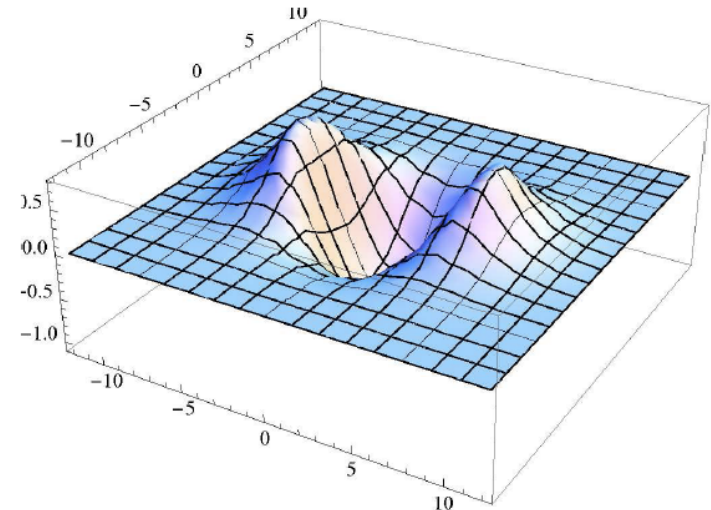
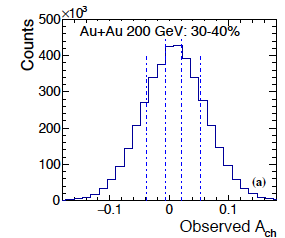
Finite baryon density + CMW = electric quadrupole moment of QGP

Signature - difference of elliptic flows of positive and negative pions determined by total charge asymmetry of the event A :
 at $A > 0$, $v_2(-) > v_2(+)$; at $A < 0$, $v_2(+) > v_2(-)$



$$v_2^- - v_2^+ = C + 2\left(\frac{q_e}{\bar{\rho}_e}\right)A_{\pm}$$

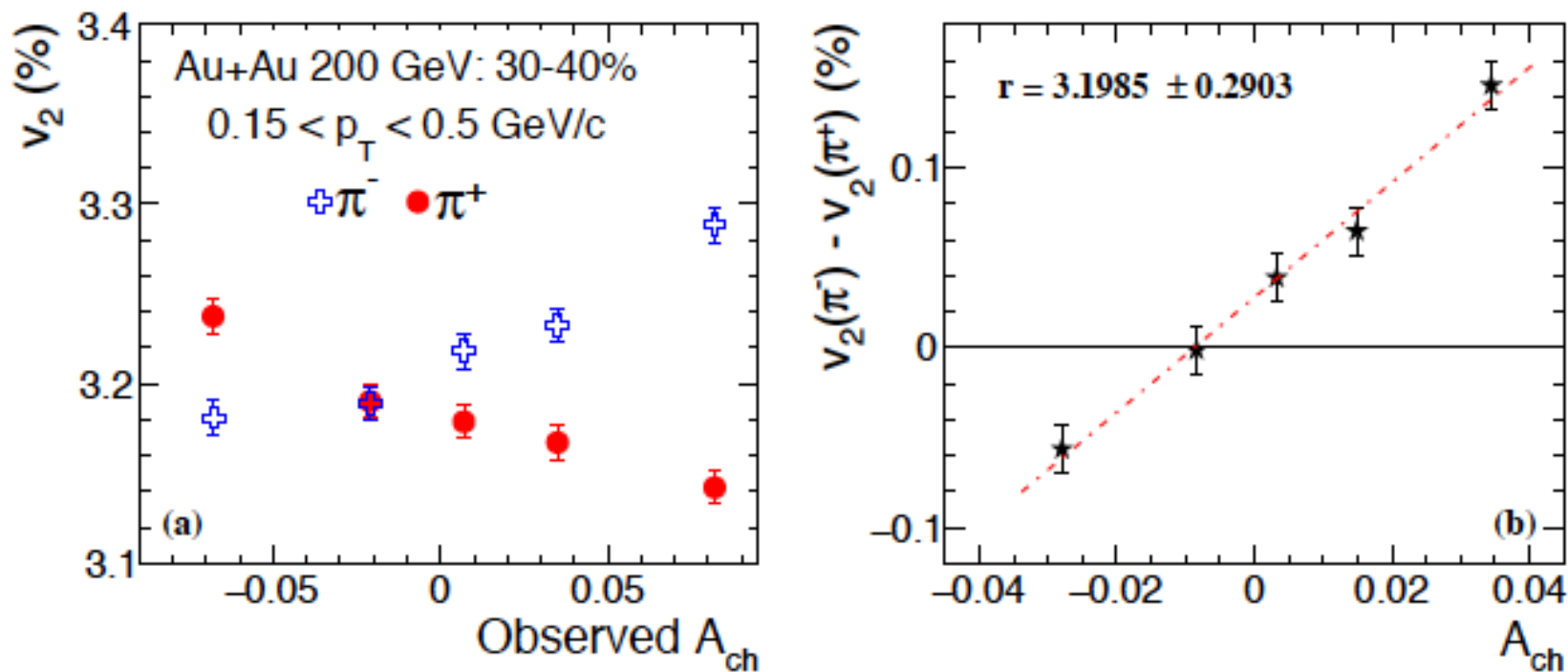
$$A_{\pm} = (\bar{N}_+ - \bar{N}_-)/(\bar{N}_+ + \bar{N}_-)$$



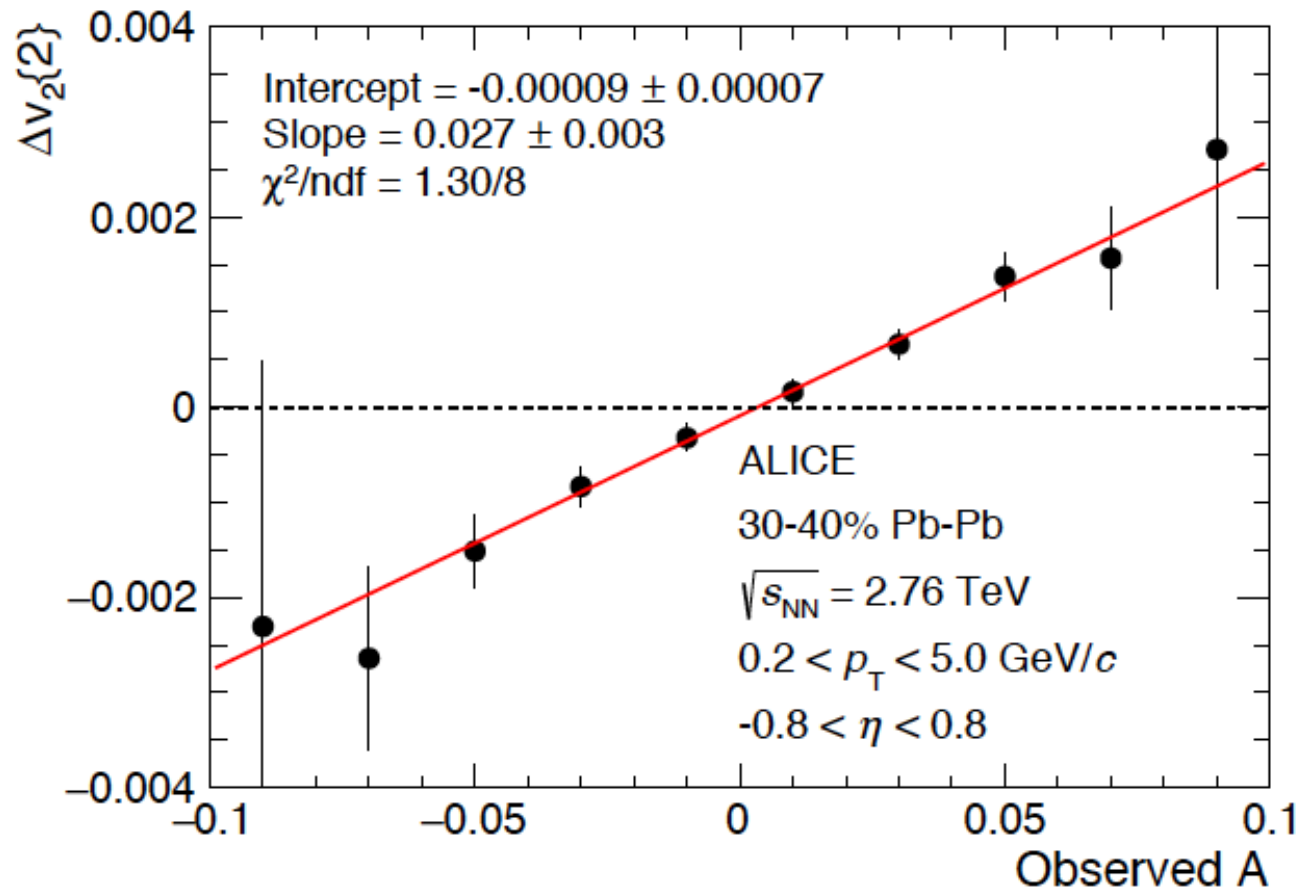
Y. Burnier, DK, J. Liao, H. Yee,
PRL 2011

Observation of charge asymmetry dependence of pion elliptic flow and the possible
chiral magnetic wave in heavy-ion collisions

(STAR Collaboration) arXiv:1504.02175



ALICE Coll. at the LHC



ALICE Coll, Phys. Rev. C93 (2016) 044903

Broader implications: Dirac semimetals



SOVIET PHYSICS JETP

VOLUME 32, NUMBER 4

APRIL, 1971

POSSIBLE EXISTENCE OF SUBSTANCES INTERMEDIATE BETWEEN METALS AND DIELECTRICS

A. A. ABRIKOSOV and S. D. BENESLAVSKIĬ

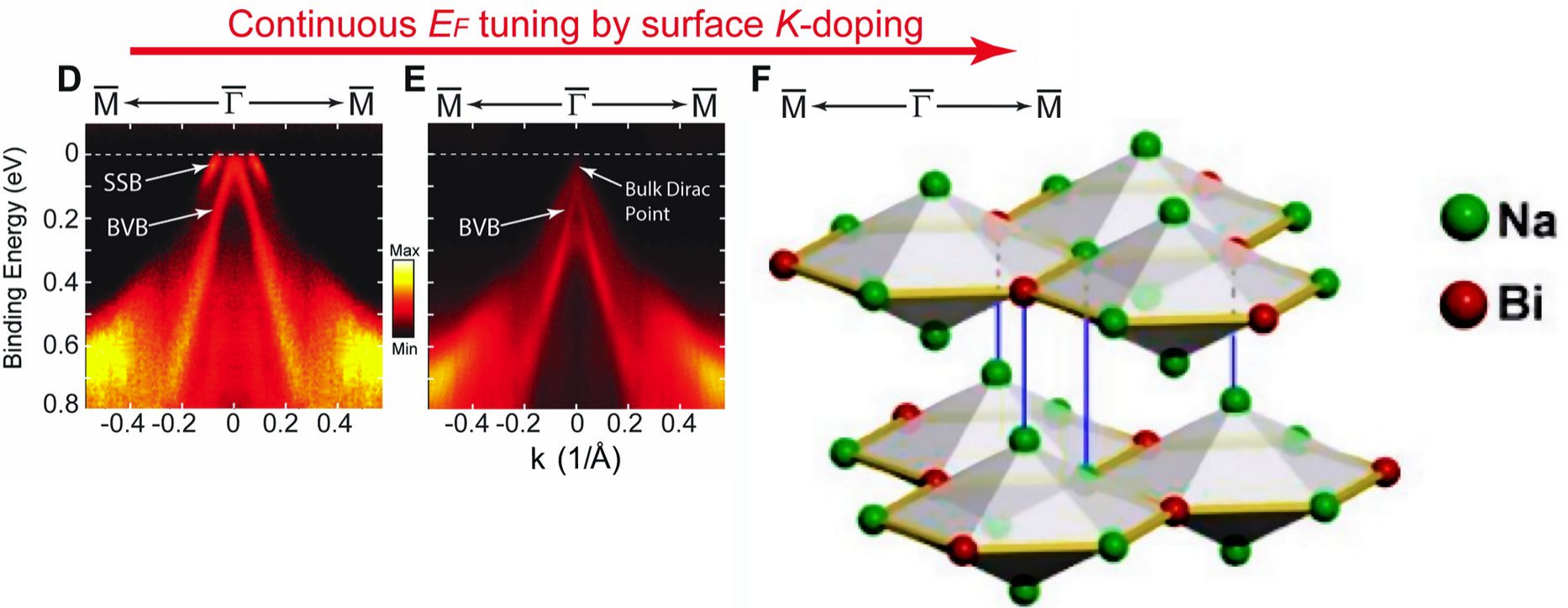
L. D. Landau Institute of Theoretical Physics

Submitted April 13, 1970

Zh. Eksp. Teor. Fiz. 59, 1280—1298 (October, 1970)

The question of the possible existence of substances having an electron spectrum without any energy gap and, at the same time, not possessing a Fermi surface is investigated. First of all the question of the possibility of contact of the conduction band and the valence band at a single point is investigated within the framework of the one-electron problem. It is shown that the symmetry conditions for the crystal admit of such a possibility. A complete investigation is carried out for points in reciprocal lattice space with a little group which is equivalent to a point group, and an example of a more complicated little group is considered. It is shown that in the neighborhood of the point of contact the spectrum may be linear as well as quadratic.

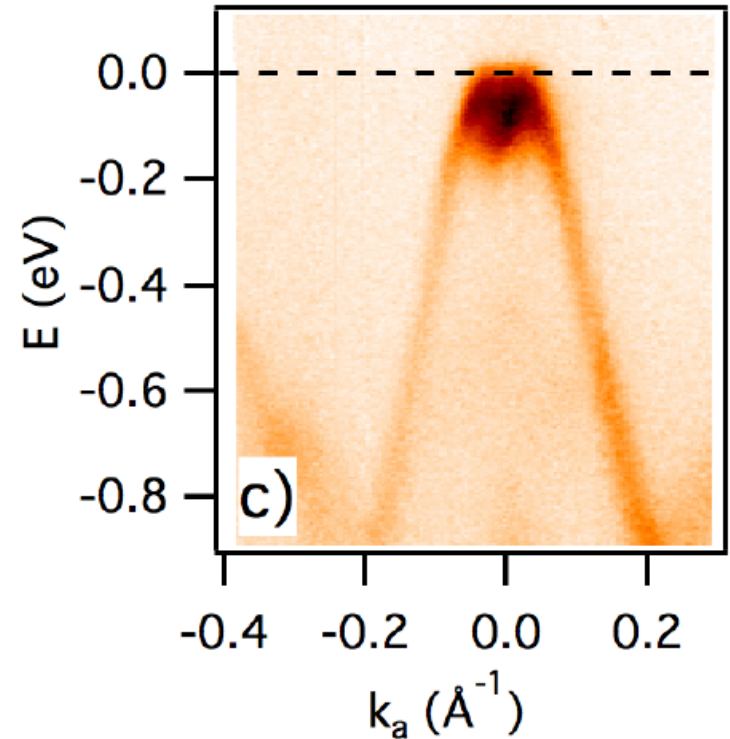
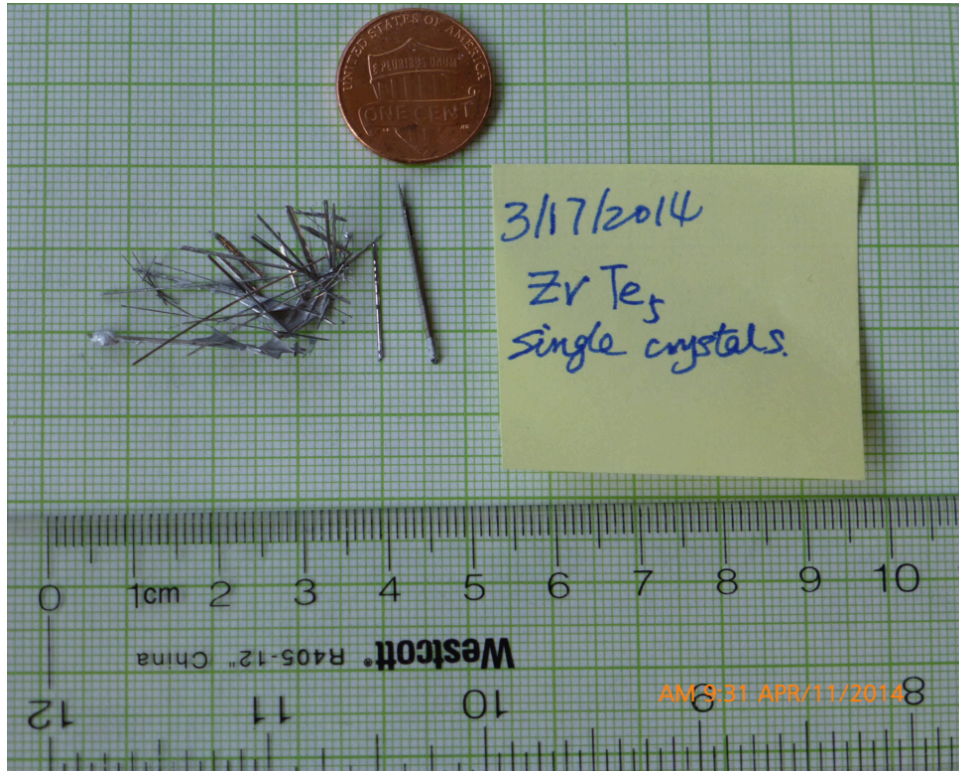
The discovery of Dirac semimetals – 3D chiral materials



Z.K.Liu et al., Science 343 p.864 (Feb 21, 2014)

Observation of the chiral magnetic effect in ZrTe_5

Qiang Li,¹ Dmitri E. Kharzeev,^{2,3} Cheng Zhang,¹ Yuan Huang,⁴ I. Pletikosić,^{1,5}
A. V. Fedorov,⁶ R. D. Zhong,¹ J. A. Schneeloch,¹ G. D. Gu,¹ and T. Valla¹



arXiv:1412.6543 (December 2014); Nature Physics **12**, 550 (2016)

Talk by Qiang Li

Observation of the chiral magnetic effect in ZrTe_5

Qiang Li,¹ Dmitri E. Kharzeev,^{2,3} Cheng Zhang,¹ Yuan Huang,⁴ I. Pletikosić,^{1,5}
A. V. Fedorov,⁶ R. D. Zhong,¹ J. A. Schneeloch,¹ G. D. Gu,¹ and T. Valla¹

Put the crystal in parallel \mathbf{E} , \mathbf{B} fields – the anomaly generates chiral charge:

$$\frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2\hbar^2c} \vec{E} \cdot \vec{B} - \frac{\rho_5}{\tau_V}.$$

and thus the chiral chemical potential:

$$\mu_5 = \frac{3}{4} \frac{v^3}{\pi^2} \frac{e^2}{\hbar^2c} \frac{\vec{E} \cdot \vec{B}}{T^2 + \frac{\mu^2}{\pi^2}} \tau_V.$$

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A. V. Fedorov,⁶ R. D. Zhong,¹ J. A. Schneeloch,¹ G. D. Gu,¹ and T. Valla¹

so that there is a chiral magnetic current:

$$\vec{J}_{\text{CME}} = \frac{e^2}{2\pi^2} \mu_5 \vec{B}.$$

resulting in the quadratic dependence of CME conductivity on B:

$$J_{\text{CME}}^i = \frac{e^2}{\pi\hbar} \frac{3}{8} \frac{e^2}{\hbar c} \frac{v^3}{\pi^3} \frac{\tau_V}{T^2 + \frac{\mu^2}{\pi^2}} B^i B^k E^k \equiv \sigma_{\text{CME}}^{ik} E^k.$$

adding the Ohmic one – negative magnetoresistance

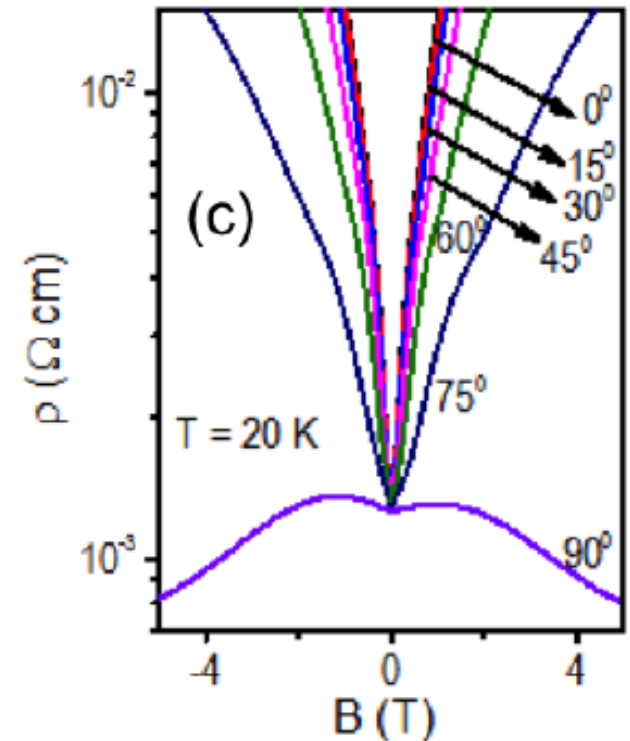
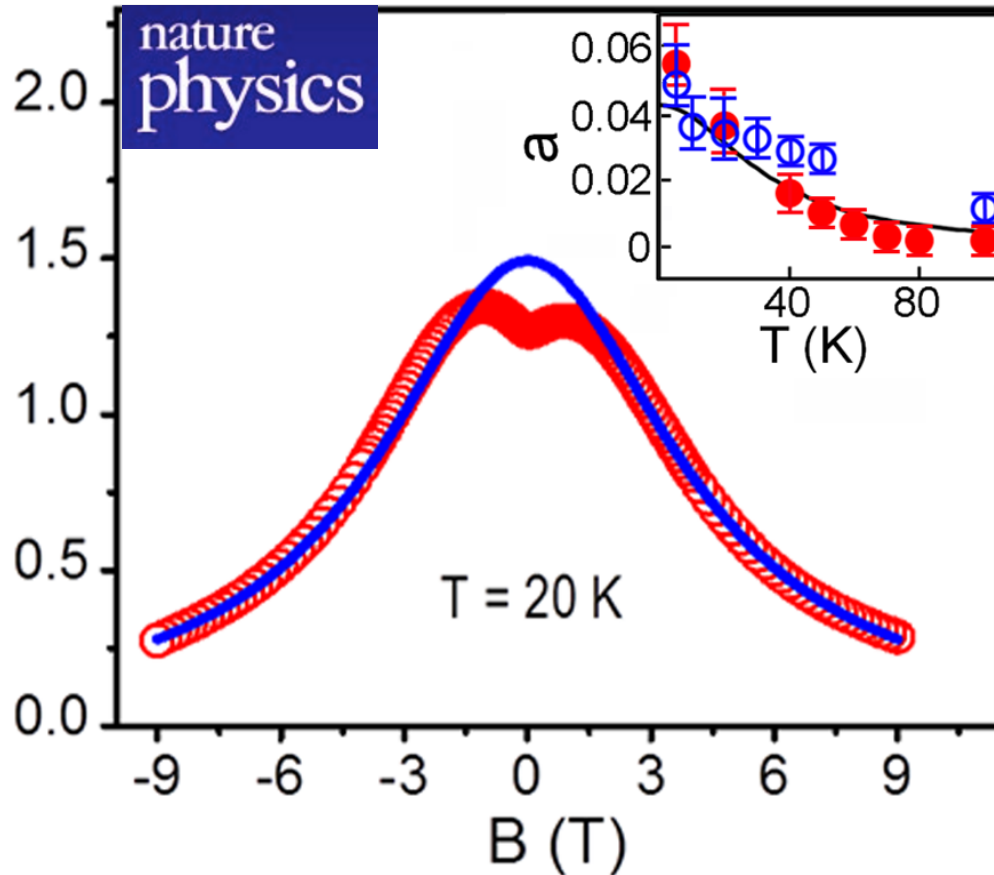
Son, Spivak, 2013

Chiral Magnetic Effect Generates Quantum Current

Separating left- and right-handed particles in a semi-metallic material produces anomalously high conductivity

February 8, 2016

Nature Physics **12**, 550 (2016)



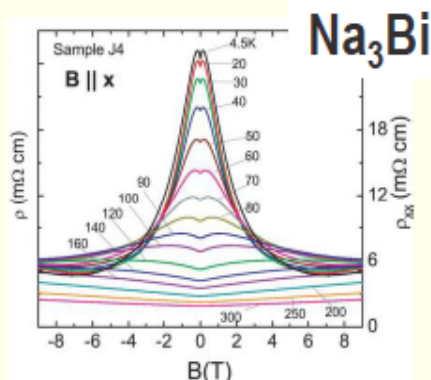
Qiang Li's Distinguished CQM lecture at Simons Center, Feb 19, 2016

on video:

http://scgp.stonybrook.edu/video_portal/video.php?id=2458

Chiral magnetic effect in Dirac/Weyl semimetals

Dirac semimetals:

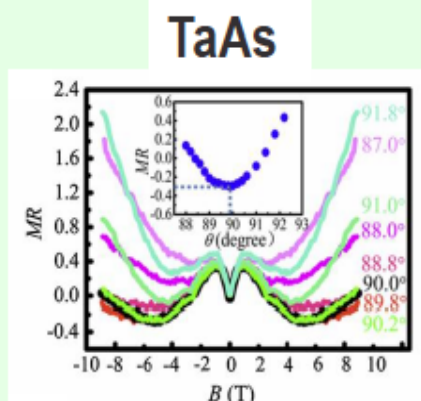


ZrTe₅ - Q. Li, D. Kharzeev, et al (BNL and Stony Brook Univ.)
arXiv:[1412.6543](#); doi:10.1038/NPHYS3648

Na₃Bi - J. Xiong, N. P. Ong et al (Princeton Univ.)
arxiv:[1503.08179](#); Science 350:413,2015

Cd₃As₂ - C. Li et al (Peking Univ. China)
arxiv:[1504.07398](#); Nature Commun. 6, 10137 (2015).

Weyl semimetals



TaAs - X. Huang et al (IOP, China)
arxiv:[1503.01304](#); Phys. Rev. X 5, 031023

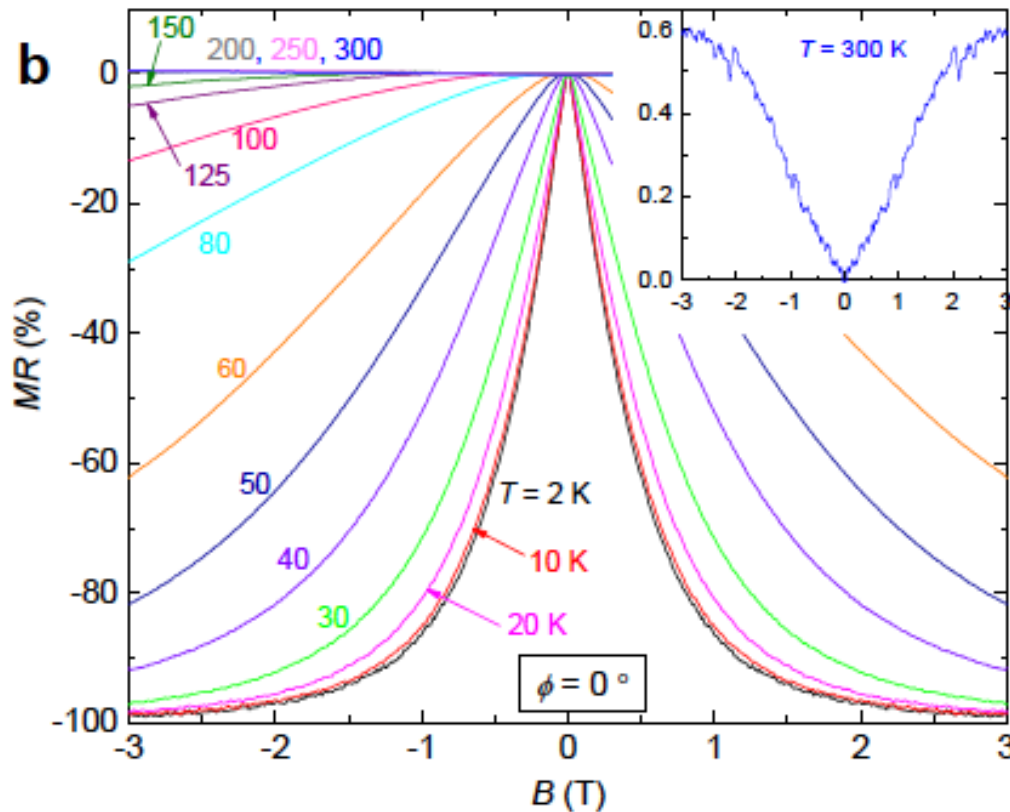
NbAs - X. Yang et al (Zhejiang Univ. China)
arxiv:[1506.02283](#)

NbP - Z. Wang et al (Zhejiang Univ. China)
arxiv:[1504.07398](#)

TaP - Shekhar, C. Felser, B. Yang et al (MPI-Dresden)
arxiv:[1506.06577](#)

Bi_{1-x}Sb_x at x ≈ 0.03 - Kim, et al. "Dirac versus Weyl Fermions in Topological Insulators: Adler-Bell-Jackiw Anomaly in Transport Phenomena. Phys. Rev. Lett., 111, 246603 (2013).

Negative MR in TaAs₂



Y.Luo et al, 1601.05524;
updated on June 8, 2016

Ta:
Z=73,
discovered
in 1802



Anders Gustaf Ekeberg
(1767-1813)

"This metal I call *tantalum* ... partly in allusion to its incapacity, when immersed in acid, to absorb any and be saturated."



CME as a new type of superconductivity

London theory of superconductors, '35:

$$\vec{J} = -\lambda^{-2} \vec{A} \quad \nabla \cdot \vec{A} = 0$$



Fritz and Heinz London

$$\vec{E} = -\dot{\vec{A}}$$

$$\vec{E} = \lambda^2 \dot{\vec{J}}$$

assume that chirality
is conserved:

$$\mu_5 \sim \vec{E} \vec{B} \, t$$

CME:

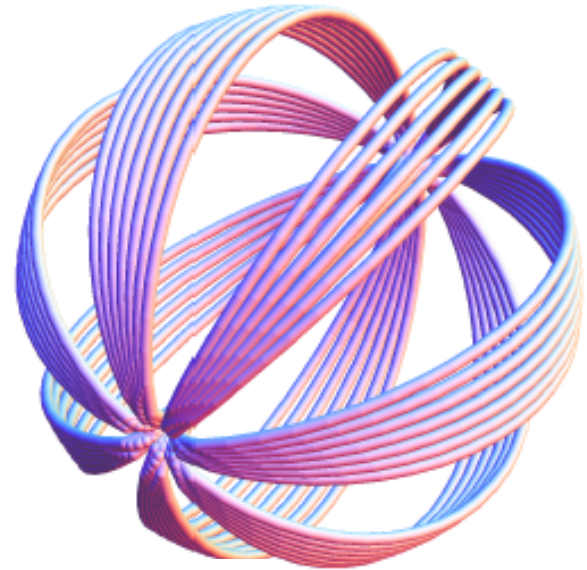
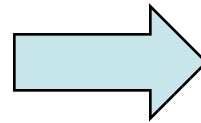
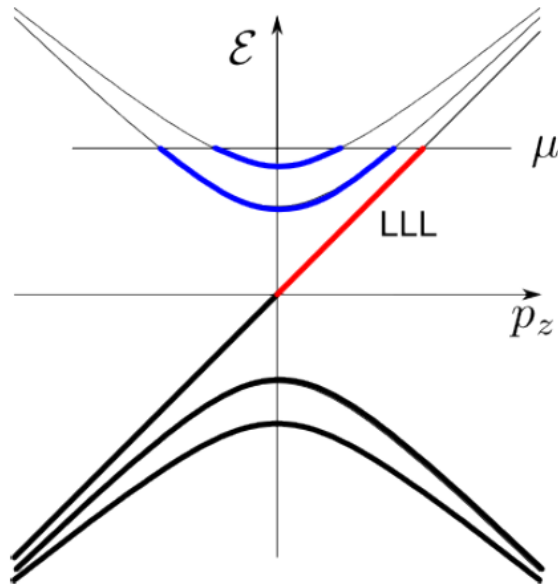
$$\vec{J} \sim \mu_5 \vec{B}$$

for $\vec{E} \parallel \vec{B}$

$$\vec{E} \sim B^{-2} \dot{\vec{J}}$$

superconducting
current, tunable
by magnetic field!

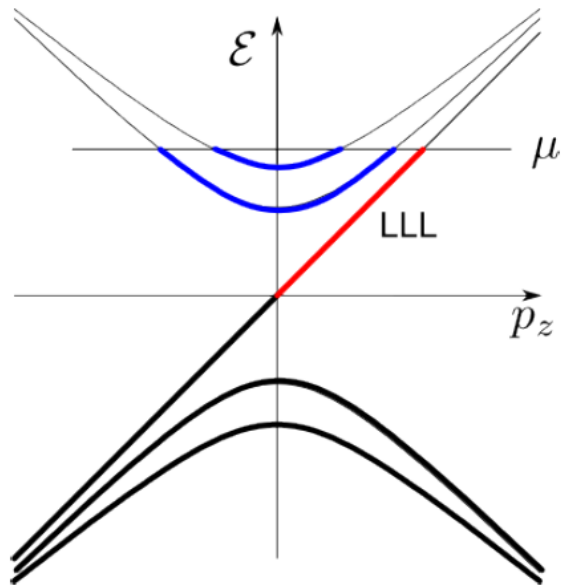
Chirality transfer from fermions to magnetic helicity



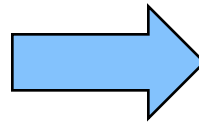
$$h_m \equiv \int d^3x \mathbf{A} \cdot \mathbf{B} \quad \partial_\mu j_A^\mu = C_A \mathbf{E} \cdot \mathbf{B}$$

$$h_0 \equiv h_m + h_F = \text{const} \quad \int d^3x \mathbf{E} \cdot \mathbf{B} = -\frac{1}{2} \frac{\partial h_m}{\partial t}$$

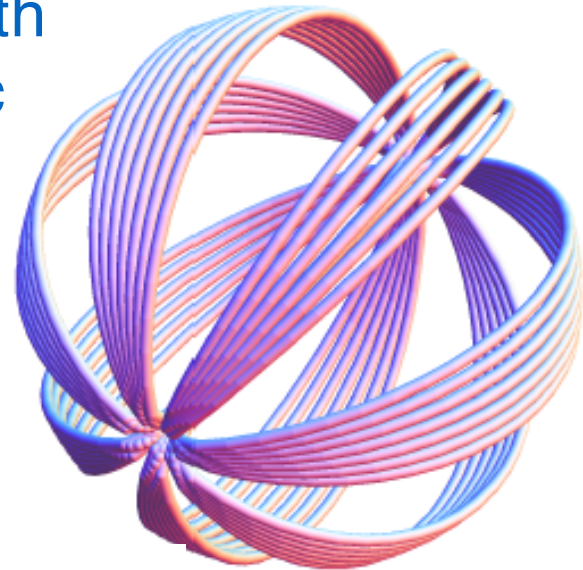
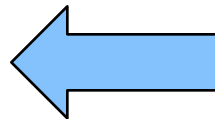
Self-similar inverse cascade



Instability at $k < C_A \mu_A$ leads to the growth of magnetic helicity

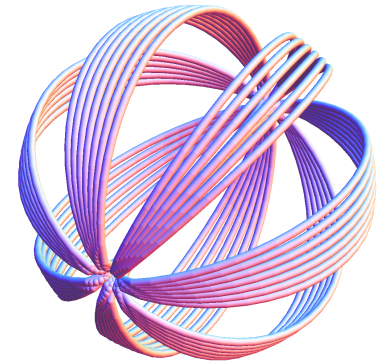
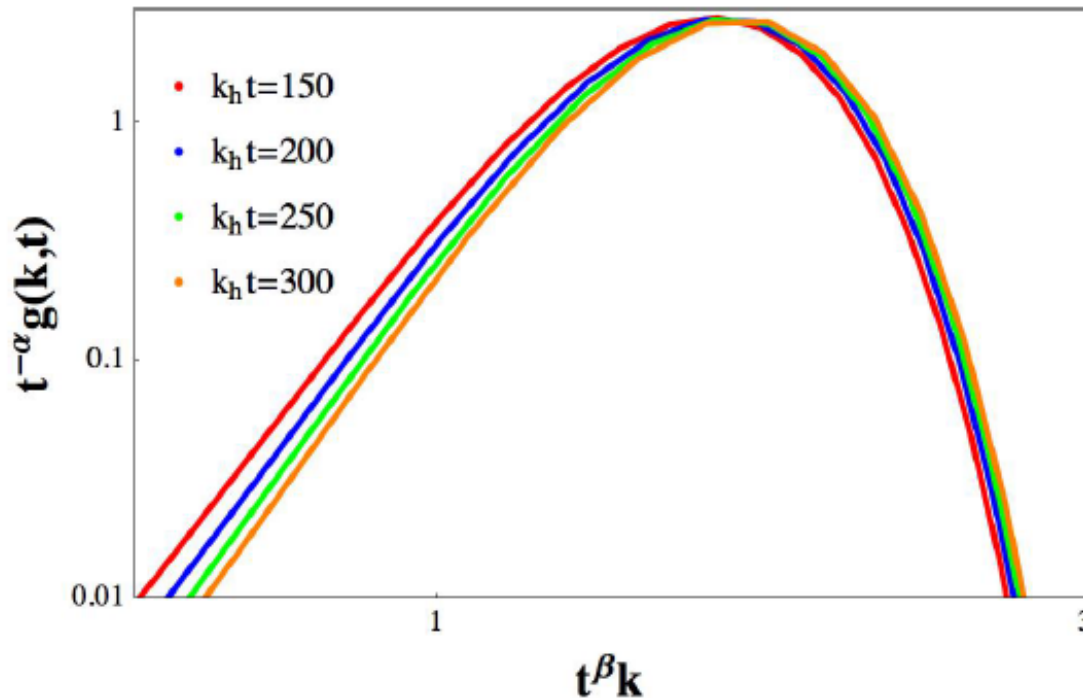


Increase of magnetic helicity reduces μ_A



Inverse cascade itself was noted earlier, see refs in
Y. Hirono, DK, Y. Yin, Phys.Rev.D92 (2015) 125031₂₆

Self-similar cascade of magnetic helicity driven by CME

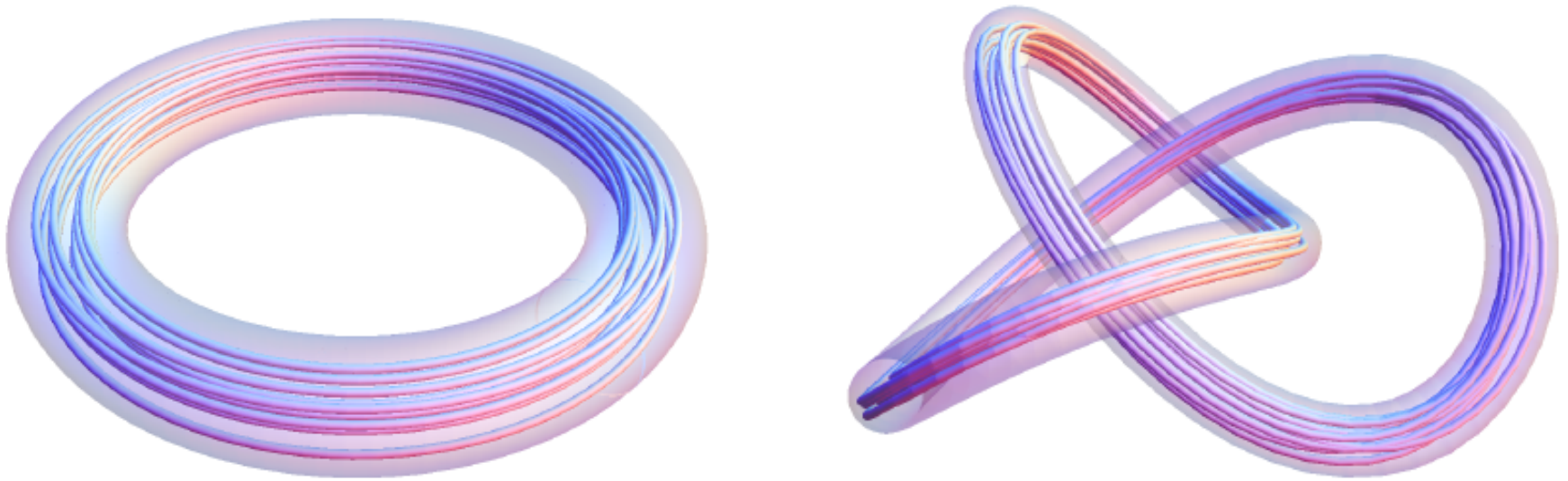


$$g(k, t) \sim t^{\alpha} \tilde{g}(t^{\beta} k) \quad \alpha = 1, \quad \beta = 1/2$$

Y. Hirono, DK, Y. Yin, Phys.Rev.D92 (2015) 125031;
N. Yamamoto, Phys.Rev.D93 (2016) 125016

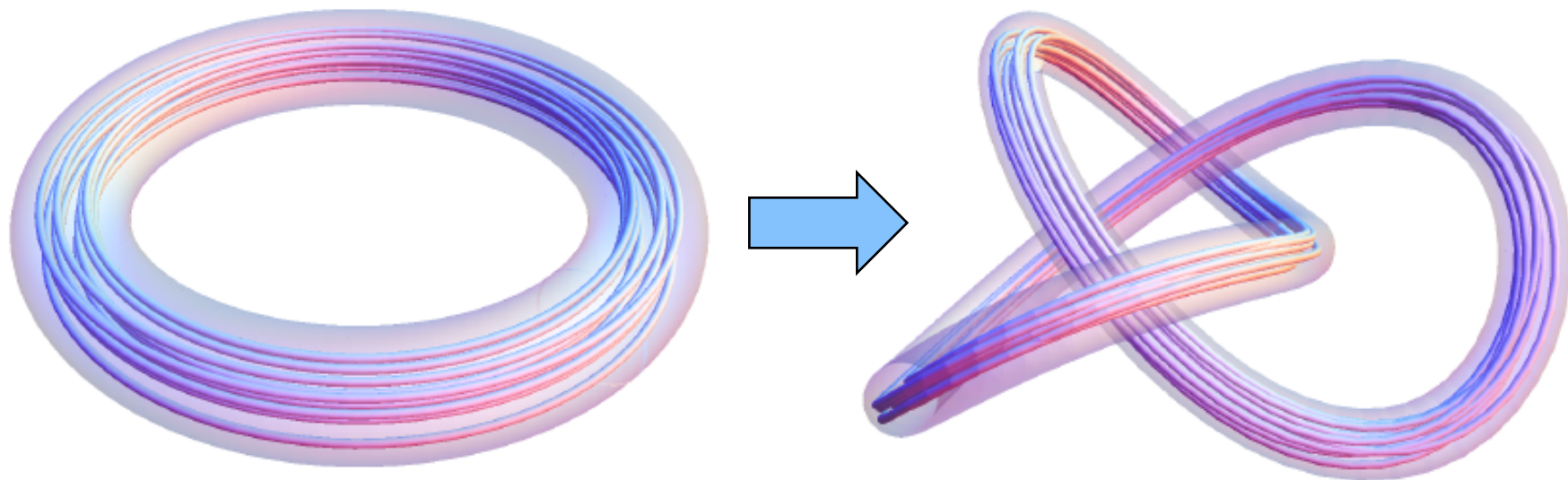
Quantized CME from knot reconnections

Y. Hirono, DK, Y. Yin, arXiv:1606.09611



Consider a tube (unknot) of magnetic flux, with chiral fermions localized on it.

To turn it into a (chiral) knot, we need a magnetic reconnection.
What happens to the fermions during the reconnection?



Changing magnetic flux through the area spanned by the tube will generate the electric field (Faraday's induction):

$$\frac{d}{dt}\Phi_B = - \oint_C \mathbf{E} \cdot d\mathbf{x}$$

The electric field will generate electric current of fermions (chiral anomaly in 1+1 D):

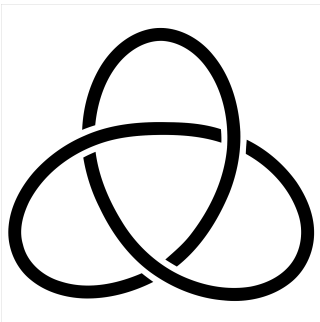
$$\Delta J = \Delta J_R + \Delta J_L = \frac{q^3 \Phi^2}{2\pi^2 L}$$

Y. Hirono, DK, Y. Yin,
arXiv:1606.09611



Helicity change per magnetic reconnection is $\Delta\mathcal{H} = 2\Phi^2$.

Multiple magnetic reconnections leading to non-chiral knots do not induce net current (need to break left-right symmetry).



For N_+ positive and N_- negative crossings on a planar knot diagram, the total magnetic helicity is:

$$\mathcal{H} = 2(N_+ - N_-)\Phi^2$$

The total current induced by reconnections to a chiral knot:

$$J = \frac{q^3 \mathcal{H}}{4\pi^2 L}$$

Summary

